Monte Carlo Exercises

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We have used Monte Carlo simulation throughout this course to demonstrate statistical concepts; namely, sampling from the population. We mostly applied this to demonstrate the statistical properties related to inference on differences in averages. Here, we will consider examples of how Monte Carlo simulations are used in practice.

Monte Carlo Exercises #1

1 punto posible (calificable)

Imagine you are William Sealy Gosset and have just mathematically derived the distribution of the t-statistic when the sample comes from a normal distribution. Unlike Gosset, you have access to computers and can use them to check the results.

Let's start by creating an outcome.

Set the seed at 1, then use `rnorm()` to generate a random sample of size 5, $X_1, \ldots, X_5$ from a standard normal distribution, then compute the t-statistic $t = \sqrt{5}\,\bar{X}/s$ with s the sample standard deviation. What value do you observe?

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Monte Carlo Exercises #2

1 punto posible (calificable)

You have just performed a Monte Carlo simulation using `rnorm()` , a random number generator for normally distributed data. Gosset's mathematical calculation tells us that the t-statistic defined in the previous exercises, a random variable, follows a t-distribution with $N-1$ degrees of freedom. Monte Carlo simulations can be used to check the theory: we generate many outcomes and compare them to the theoretical result. Set the seed to 1, then generate $B = 1000$ t-statistics as done in exercise 1. What proportion is larger than 2?

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Monte Carlo Exercises #3

0/1 punto (calificado)

The answer to exercise 2 is very similar to the theoretical prediction: `1-pt(2,df=4)`. We can check several such quantiles using the `qqplot` function.

To obtain quantiles for the t-distribution we can generate percentiles from just above 0 to just below 1: `B=100; ps = seq(1/(B+1), 1-1/(B+1),len=B)`, and compute the quantiles with `qt(ps,df=4)`. Now we can use `qqplot()` to compare these theoretical quantiles to those obtained in the Monte Carlo simulation. Use Monte Carlo simulation developed for exercise 2 to corroborate that the t-statistic $t = \sqrt{N}\,\bar{X}/s$ follows a t-distribution for several values of N (try `Ns <- seq(5,30,5)`).

For which sample sizes does the approximation best work?

Ocultar Notas

- [x] Larger sample sizes.
- [ ] Smaller sample sizes.
- [ ] The approximations are spot on for all sample sizes.
- [ ] None. We should use CLT instead.

✖

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Monte Carlo Exercises #4

0/1 punto (calificado)

Use Monte Carlo simulation to corroborate that the t-statistic comparing two means and obtained with normally distributed (mean 0 and sd) data follows a t-distribution. In this case we will use the `t.test()` function with `var.equal=TRUE`. With this argument the degrees of freedom will be `df=2*N-2` with `N` the sample size. For which sample sizes does the approximation best work?

- [x] Larger sample sizes.
- [ ] Smaller sample sizes.
- [ ] The approximations are spot on for all sample sizes.
- [ ] None. We should use CLT instead.

✖

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Monte Carlo Exercises #5

1 punto posible (calificable)

Is the following statement true or false? If instead of generating the sample with `X=rnorm(15)` we generate it with binary data (either positive or negative 1 with probability 0.5) `X =sample(c(-1,1), 15, replace=TRUE)` then the t-statistic

```
tstat <- sqrt(15)*mean(X) / sd(X)
```

is approximated by a t-distribution with 14 degrees of freedom.

- [ ] true
- [ ] false

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Monte Carlo Exercises #6

Ocultar Notas

1 punto posible (calificable)
Is the following statement true or false ? If instead of generating the sample with `X=rnorm(N)` with `N=1000`, we generate the data with binary data `X= sample(c(-1,1), N, replace=TRUE)`, then the t-statistic `sqrt(N)*mean(X)/sd(X)` is approximated by a t-distribution with 999 degrees of freedom.

- ○ true
- ○ false

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Monte Carlo Exercises #7

0/1 punto (calificado)
We can derive approximation of the distribution of the sample average or the t-statistic theoretically. However, suppose we are interested in the distribution of a statistic for which a theoretical approximation is not immediately obvious.

Consider the sample median as an example. Use a Monte Carlo to determine which of the following best approximates the median of a sample taken from normally distributed population with mean 0 and standard deviation 1.

- ○ Just like for the average, the sample median is approximately normal with mean 0 and SD $1/\sqrt{N}$.
- ○ The sample median is not approximately normal.
- ● The sample median is t-distributed for small samples and normally distributed for large ones.
- ○ The sample median is approximately normal with mean 0 and SD larger than $1/\sqrt{N}$.

✖

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