QUEEN'S UNIVERSITY FINAL EXAMINATION
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF ECONOMICS

ECON 212 001-002 – Professors: Mohsen Bakhshi-Moghaddam and Ming Xu
December 21, 2024

INSTRUCTIONS TO STUDENTS:
This examination is 3 HOURS in length.
The exam consists of 4 long questions. Each question is worth 25 marks for a total of 100 marks. You should answer all the long questions. Marks will be awarded on the basis of the logical arguments given to support your answers.

Please answer all questions in the answer booklets.

The following aids are allowed:
Casio FX-991 calculator

Put your student number on all pages of all answer booklets, including the front.
GOOD LUCK!

PLEASE NOTE:
Proctors are unable to respond to queries about the interpretation of exam questions.
Do your best to answer exam questions as written.

1. A firm uses labour and machines to produce output according to the production function

$$f(M, L) = M^{\frac{1}{4}} L^{\frac{1}{2}}$$

Where L is the number of units of labour used and M is the number of machines. The cost of labour is \$20 per unit, and the cost of using a machine is \$10. The output sells for \$100 per unit.

a) Does the firm have constant return to scale, decreasing return to scale, or increasing return to scale **(4 points)**

b) Does the firm have constant, increasing, or diminishing marginal product in M and L? **(6 points)**

c) Suppose in the short run, the number of machines is fixed at 16. How much labour should the firm use to maximize the profit? How much profit does the firm make? **(7 points)**

d) Using an isoprofit line as well as the production function, draw a diagram of your solution to (c). Be sure to write out the isoprofit function clearly, and label all intercepts and slopes in your diagram. **(8 points)**

2. Joe makes little deer ornaments for lawns. He fully automates the production process, so the only input he uses is wood and plastic (no labour). Joe's production function is given by $f(x_1, x_2) = (2x_1 + x_2)^{\frac{1}{2}}$ where x_1 is the amount of plastic used, and x_2 is the amount of wood used.

a) What is the technical rate of substitution for this production function? **(4 points)**

b) If the price of plastic is $\$w_1$ and the price of wood is $\$w_2$, what are the conditional factor demand functions of each input to produce y deer ornaments? What is the cost function $c(w_1, w_2, y)$? **(8 points)**

c) If $w_1 = 8, w_2 = 16$, how much of each input should the firm use to produce y deer ornaments? Draw a diagram of your solution, make sure you carefully show the isoquant and isocost curves, and label all intercepts and slopes of your lines. **(6 points)**

d) Given the input prices from (c) and a price P, what is Joe's supply function? **(7 points)**

3. Julia knows that for her cakes to turn out perfectly, she must use eggs (x_1) and sugar (x_2) in fixed proportions. She has determined that she can bake 4 perfect cakes when she combines 3 units of eggs with 2 units of sugar.

a) What is Julia's production function, $f(x_1, x_2)$? **(5 points)**

b) Suppose Julia wants to produce y units of output, and the cost of using per unit of egg and sugar is w_1 and w_2 , respectively. What are the conditional factor demand functions for eggs and sugar? **(6 points)**

c) Using the results above, derive Julia's cost function, $c(w_1, w_2, y)$. **(6 points)**

d) Imagine $w_1 = 8$ and $w_2 = 4$. Draw a diagram of an isoquant for producing 4 units of output, as well as an isocost line representing the lowest cost of producing this output level. Show the optimum point on the diagram. Be sure to label the axes, all the intercepts, the kink point and the slope of the isocost line. **(8 points)**

4. A software company develops mobile apps using the following production function:

$$f(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$$

where x_1(Programming Hours) represents the number of hours spent by programmers writing code, and x_2 (Design Assets) represents the quantity of design assets used in app development. Let w_1 denote the hourly wage rate and w_2 denote the price of using design assets. Let p denote the market price of the mobile app.

a) Given w_1 and w_2, derive the cost function $c(w_1, w_2, y)$ for this company. **(8 points)**

b) Let $w_1 = 16$ and $w_2 = 25$. Suppose the software company operates in a competitive market. What's the company's supply function $S(p)$? **(8 points)**

c) The market price of the mobile app is \$240. How much output should the company produce to maximize its profit? How much profit does the company make at this optimum? **(3 points)**

d) Now, suppose the software company has to pay a quasi-fixed cost of \$250 for electricity only when it produces a positive amount of output. Also, the market price of the mobile app drops from \$240 to \$160. How much output should the company produce to maximize its profit? How much profit does the company make at this optimum? **(6 points)**