QUEEN'S UNIVERSITY FINAL EXAMINATION
FACULTY OF Arts and Science
DEPARTMENT OF Economics

Micro Theory I, Econ 212 001-002 – Barber & Gong
April 14th 2022

INSTRUCTIONS TO STUDENTS:
This examination is 3 HOURS in length.
There is 1 section to this examination.
Please answer all questions in the answer booklets

The following aids are allowed:
Casio FX-991 calculator

Put your student number on all pages of all answer booklets, including the front.

GOOD LUCK!

PLEASE NOTE:
Proctors are unable to respond to queries about the interpretation of exam questions.
Do your best to answer exam questions as written.

1. Logan owns an independent cafe in Kingston, Onterrible. She uses two inputs in the production of her coffee: labour (x_1) and capital (x_2). Her production function is $f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{3}}$. The price of a coffee is \$6, the cost of labour is \$2, and the cost of capital is \$1.

   (a) Are returns to scale increasing, decreasing, or constant for this firm? (**2 points**)

   (b) Suppose in the short run the amount of capital she has is fixed at 8. How much labour should she use? How much coffee will she produce? How much profit does her cafe make? (**4 points**)

   (c) Using an isoprofit line, as well as the production function, draw a diagram of your solution from (b). Be sure to label all intercepts and slopes using your answer from (b). (**3 points**)

   (d) In the long run, how much labour should she use? how much capital? How much coffee will she make? What will her profits be? (**4 points**)

   (e) The local government implements a program to encourage employment in Kingston by providing a \$1 subsidy per unit of labour. What is the profit maximizing amount of each input to use? How much does the firm produce when it is profit maximizing? How much profit does the firm make? (**2 points**)

2. Lynn operates a firm that produces butter from 2 types of milk, 1% milk and 2% milk. Her technology can be described by the following production function: $f(x_1, x_2) = \sqrt{x_1 + 4x_2}$, where x_1 denotes the amount of 1% milk and x_2 denotes the amount of 2% milk. This production function means that the 1% milk and the 2% milk can be used interchangeably to produce butter according to a certain ratio.

   (a) Lynn wants to produce 1 unit of butter. If she uses only the 1% milk, how many units of 1% milk does she need? If she uses only the 2% milk, how many units of 2% milk does she need? (**2 points**)

   (b) Does the production function have increasing, decreasing, or constant marginal product of the 1% milk? Derive the marginal product MP_1 of the 1% milk and the marginal product MP_2 of the 2% milk. Calculate the technical rate of substitution TRS. (**4 points**)

   (c) If the price of 1% milk is \$3 and the price of 2% milk is \$5, to product 2 units of butter, how much of the two inputs should Lynn use to minimize her cost? Draw a graph of the isoquant and isocost line to show this optimal bundle of inputs. Carefully label the intercepts and slopes. (**4 points**)

   (d) Suppose the price of 1% milk is w_1 and the price of 2% milk is w_2. Let y denote the amount of butter produced. Derive Lynn's cost function $c(w_1, w_2, y)$. (**3 points**)

   (e) Suppose Lynn can purchase a new type of milk, the 3% milk, at the price w_3. The butter transformed from 1 unit of 3% milk is three times as much as that from 1 unit of 1% milk. Derive Lynn's new cost function $c(w_1, w_3, y)$. (**3 points**) (*Hint: the new production function should have the form* $f(x_1, x_3) = \sqrt{x_1 + ax_3}$ *where* x_3 *denotes the 3% milk.*)

3. Ricky and Julian have decided to start their own small business growing plants. They need to use two inputs, labour and water to grow the plants. Their firm has the production function: $f(x_1, x_2) = (min\{x_1, 6x_2\})^{\frac{1}{2}}$ where x_1 is the amount of labour and x_2 is the amount of water.

   (a) If the price of labour is \$5, and the price of water is \$1, what is the cost of growing "y" plants? **(4 points)**

   (b) What is the firms marginal cost of growing another plant? **(2 points)**

   (c) Suppose the plant growing business is a perfectly (pure) competitive market. The firm has quasi-fixed costs of \$5. If the price that they can sell each plant for is \$60, how many plants should they grow? how much profit will they make? **(4 points)**

   (d) At what price should Ricky and Julian not grow any plants at all? **(1 point)**