QUEEN'S UNIVERSITY FINAL EXAMINATION
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF ECONOMICS

Econ 212 001-002 Barber & Bakhshi-Moghaddam
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INSTRUCTIONS TO STUDENTS:
This examination is 3 HOURS in length.
There is 1 section to this examination.
Please answer all questions in the answer booklets.

The following aids are allowed:
Casio FX-991 calculator

GOOD LUCK!

PLEASE NOTE:
Proctors are unable to respond to queries about the interpretation of exam questions.
Do your best to answer exam questions as written.

1. A firm uses two inputs, labour (x_1) and capital (x_2) to produce output. The production function for the firm is $f(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{2}}$ and the price of output is \$8.

   (a) Suppose the amount of capital is fixed at 36 in the short run. If the price of labour is w and the price of capital is r how much labour should the firm use to maximize profits in the short run? **(8 points)**

   (b) Suppose the price of labour is \$1.50 and the price of capital is \$1. How much labour should the firm use? How much would they produce? What are their profits? **(5 points)**

   (c) Given the prices in (b), suppose the government announces a 10% tax on business profit, as well as a \$0.50 tax on each unit of capital a firm uses. What is the firm's profit maximization problem now? How does this influence how much labour a firm uses? What about production and profits? Carefully explain. **(6 points)**

   (d) Draw a diagram of your solution to (b) and (c). Be sure to carefully label all intercepts and slopes in your diagram. **(6 points)**

2. Raheem owns a firm that produces bags of dog treats. He can produce 1 bag of dog treats with either 2 units of input 1 (chicken) **or** can produce 1 bag with a $\frac{1}{4}$ unit of input 2 (beef).

   (a) If x_1 is the amount of chicken and x_2 is the amount of beef, what is the firm's production function? Does it have increasing, decreasing, or constant returns to scale? **(5 points)**

   (b) Suppose the price of one unit of chicken is $\$w_1$ and the price of one unit of beef is $\$w_2$. What are the conditional factor demand functions of each input to produce "y" units of output? What is the cost function? **(10 points)**

   (c) If $w_1 = \frac{1}{2}$ and $w_2 = 2$, how much of each input should the firm use to produce "y" units of output? Draw a diagram of your solution. **(6 points)**

   (d) Is the average cost for this firm increasing, decreasing or constant? What about marginal costs? Draw a diagram of the cost functions with output along the x-axis and AC/MC on the y-axis. **(4 points)**

3. Julia has decided to start her own small business growing plants in Kingston. She needs to use two inputs, water, x_1, and labour, x_2 in fixed proportions to grow plants. In particular, to grow 3 plants, Julia needs to combine $\frac{1}{2}$ a unit of labour with 3 units of water.

   (a) What is her production function, $f(x_1, x_2)$ for plants? **(5 points)**

   (b) Suppose Julia wants to produce y units of output, and the cost of using per unit of labour and water is w_1 and w_2, respectively. What are Julia's conditional factor demand functions for labour, $x_1(w_1, w_2, y)$, and water $x_2(w_1, w_2, y)$? **(6 points)**

   (c) What is the cost function, $c(w_1, w_2, y)$? **(6 points)**

   (d) Imagine $w_1 = 6$ and $w_2 = 2$. Draw a diagram of an isoquant for producing 3 plants, as well as an isocost line representing the lowest cost of producing this output level. Show the optimum point on the diagram. Place labour, x_1, on the X-axis and water, x_2, on the Y-axis. Be sure to label the axes, all the intercepts, the kink point and the slope of the isocost line. **(8 points)**

4. A firm produces its output using the production function: $f(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}$, where x_1 and x_2 are the amounts of input 1 and input 2, respectively. Let w_1 denote the cost of using per unit of input 1 and w_2 denote the cost of using per unit of input 2. Let p denote the market price of output.

   (a) Given w_1 and w_2, derive the firm's cost function $c(w_1, w_2, y)$. **(7 points)**

   (b) Let $w_1 = 4$ and $w_2 = 9$. Suppose the firm is operating in a competitive market. What is the firm's supply function S(p)? **(8 points)**

   (c) The market price of output is \$6. How much output should the firm produce to maximize its profit? How much profit does the firm make at this optimum? **(5 points)**

   (d) Now, suppose the firm has to pay a quasi-fixed cost of \$5 for electricity. Also, the market price of output increases from \$6 to \$12. How much output should the firm produce to maximize its profit? How much profit does it make at this optimum? **(5 points)**