1 Part-A

SSE=$\frac{1}{2}(y - X\beta)'W(y - X\beta)$

$\rightarrow \frac{1}{2}(y' - \beta'X')W(y - X\beta)$

$\rightarrow \frac{1}{2}(y'W - \beta'X'W)(y - X\beta)$

$\rightarrow \frac{1}{2}(y'Wy - y'WX\beta - \beta'X'Wy + \beta'X'WX\beta)$

Now, $\nabla(\frac{1}{2}(y'Wy - y'WX\beta - \beta'X'Wy + \beta'X'WX\beta))$

$\rightarrow \frac{1}{2}(-y'WX - X'Wy + 2X'WX\beta) = 0$

$\rightarrow \frac{1}{2}(-2X'Wy + 2X'WX\beta) = 0$

$\rightarrow X'WX\beta = X'Wy$

2 Part-B

$X'WX\beta = X'Wy$.
Let $X'W^{1/2}$=P and $W^{1/2}y$=u.
Therefore the new system of linear equation is: $P'P\beta = P'u$.
It seems that $P'P$ is a square matrix. The above equation can be viewed as Ax=b form. So this A ($P'P$) matrix can be decomposed via LU decomposition method. The LU decomposition factorizes a matrix into a lower triangular matrix L and an upper triangular matrix U. This decomposition summarizes the process of Gaussian elimination in matrix form.

Singular Value Decomposition: Suppose the system of linear equations: Ax=b.
The case where A is an n x n square matrix is of particular interest. In this case, the Singular Value Decomposition of A is given as A=USV^T, where V and U are orthogonal matrices.

QR Decomposition: Any real square matrix A may be decomposed as A=QR. Where Q is an orthogonal matrix (its columns are orthogonal unit vectors meaning $Q^TQ =$ I) and R is an upper triangular matrix (also called right triangular matrix). If A is invertible, then the factorization is unique if we require the diagonal elements of R to be positive. If instead A is a complex square matrix, then there is a decomposition A = QR where Q is a unitary matrix.