THE JOURNAL OF FINANCE • VOL. XLVIII, NO. 4 • SEPTEMBER 1993

Imperfect Information and Cross-Autocorrelation among Stock Prices

KALOK CHAN*

ABSTRACT

I develop a model to explain why stock returns are positively cross-autocorrelated. When market makers observe noisy signals about the value of their stocks but cannot instantaneously condition prices on the signals of other stocks, which contain marketwide information, the pricing error of one stock is correlated with the other signals. As market makers adjust prices after observing true values or previous price changes of other stocks, stock returns become positively cross-autocorrelated. If the signal quality differs among stocks, the cross-autocorrelation pattern is asymmetric. I show that both own- and cross-autocorrelations are higher when market movements are larger.

SHORT-HORIZON EQUITY PORTFOLIO or index returns are shown in the literature to be positively autocorrelated (Fisher (1966), Scholes and Williams (1977), Dimson (1979), Cohen *et al.* (1980), Perry (1985)). Since autocorrelations in individual stock returns are only weakly positive or negative (Fama (1965), French and Roll (1986), Lo and MacKinlay (1990a)), positive cross-autocorrelations among stock returns are largely responsible for the positive index return autocorrelation.

While cross-autocorrelation among stock returns is well documented, its sources are puzzling. The most common explanation is that the time series of stock prices are not sampled simultaneously, but rather nonsynchronously, which induces spurious cross-effects among stocks. Several researchers (e.g., Atchison, Butler, and Simonds (1987) and Lo and MacKinlay (1990b)) point out, however, that while some of the cross-autocorrelations may be due to nonsynchronous trading problems, claiming all of them would require unrealistically thin markets.

In this paper I develop a model to explain cross-autocorrelations among stock returns. I assume that in each period each market maker observes a noisy signal about the value of his stock but cannot instantaneously observe

* Arizona State University. I thank Hank Bessembinder, James Booth, K. C. Chan, Michael Hertzel, Tim Mech, Richard Smith, Jiang Wang, an anonymous referee, *Journal* editors David Mayers and René Stulz, and seminar participants at Arizona State University and the 1991 Western Finance Association Meeting for valuable comments, and Beth Baugh for editorial assistance. Any errors are, of course, my responsibility.

signals about the value of other stocks.[1] Each signal contains marketwide information and uncorrelated noise, so that an extra signal diversifies the noise and provides more precise marketwide information. When market makers condition prices only on their signals, the pricing error of one stock is correlated with signals of other stocks. Further, although each stock price is an unbiased estimate of the true stock value conditional on its own signal, the aggregate of stock prices (the index price) is not an unbiased estimate of the true aggregate value conditional on all signals. Consequently, if market makers correct pricing errors based upon additional signals inferred from previous price changes of other stocks, stock returns will be positively cross-autocorrelated.

The intuition behind cross-autocorrelation can be illustrated with a simple example. Suppose there are two stocks, A and B. In the first period, the market maker in stock A receives a favorable signal about his stock but cannot observe the signal about stock B. Since the signal is noisy, the market maker adjusts his stock's price only partially upward in response to the favorable information. In the second period, he examines previous price movement of stock B because it also contains marketwide information. If it increases, he is more confident about the favorable information and adjusts his stock's price further upward. If it decreases, he is less confident about the favorable information and revises the stock's price downward. Therefore, the price change of stock A in the second period is positively correlated with the price change of stock B in the first period.

The model has several implications. First, stock returns are serially uncorrelated individually but positively cross-autocorrelated. Second, assuming that the signal quality of large firms is better than that of small firms, the covariances of current returns of small firms with past returns of large firms are larger than the covariances of current returns of large firms with past returns of small firms. Third, both own- and cross-autocovariances are larger when market movements are larger. Existing empirical evidence supports the first two implications, which can supplement nonsynchronous trading as an explanation for observed cross-autocorrelations. I provide empirical evidence consistent with the third implication. Using data from the Center for Research in Security Prices (CRSP) daily stock price files from 1980 to 1989, I find that both the own- and cross-autocorrelation coefficients of daily stock returns are significantly higher under large market movements than under small market movements.

Section I develops the model and discusses the implications. I demonstrate how this leads to positive cross-autocorrelation among securities in a multiperiod framework when market makers readjust prices after observing true values or previous price changes of other stocks. Section II presents evi-

[1] This model is related to papers by Bossaerts (1991), Caballe and Krishnan (1992), and Bhasin (1992), who study strategic informed trading in a multiple-correlated asset environment in which market makers condition prices on order flows. In this paper, volume is not explicitly modeled, although the signals can be thought of as representing order flows.

dence on own- and cross-autocorrelation patterns. A conclusion follows in Section III.

I. The Model

A. One-Period Model

Assume N stocks are traded in the market. The value of each stock is given by:

$$V_i = \bar{v} + W + S_i, \qquad i = 1, 2 \ldots N,$$

where

$\bar{v}$ = a constant, normalized to be the same for each stock,
W = a marketwide information component that affects all stocks, which is normally distributed with mean zero and variance σ_w^2, and
S_i = a stock-specific information component that affects stock i, which is normally distributed with mean zero, variance σ_s^2, and $E(S_i, S_j) = 0$, $i \neq j$.

W is uncorrelated with S_i.

There are N market makers, one for each stock. Each market maker observes a noisy signal about the value of his stock, $\theta_i = W + S_i + \epsilon_i$, where ϵ_i is normally distributed with mean zero, variance σ_ϵ^2 and $E(\epsilon_i, \epsilon_j) = 0$. Note that market makers do not observe separately the marketwide and stock-specific information components. This assumption is plausible because Seyhun (1988) finds that even insiders cannot always distinguish between the two. The assumption is made for convenience and is not crucial for deriving the implications of the model. Essential to the model, however, is the assumption that market makers do not instantaneously observe signals about other stocks. This is defensible because market makers naturally receive information about their stocks more quickly, and further, the costs of processing price information prevents them from instantaneously retrieving other signals from current prices.

If market makers earn zero expected profits, each stock price will be set equal to the expected value conditional on their signals:

$$\begin{aligned} P_i &= E(V_i | \theta_i) \\ &= E(V_i) + \frac{\text{Cov}(V_i, \theta_i)}{\text{Var}(\theta_i)}(\theta_i - E(\theta_i)) \\ &= \bar{v} + k\theta_i \\ &= \bar{v} + k(W + S_i + \epsilon_i) \qquad (1) \end{aligned}$$

where

$$k = \frac{\sigma_w^2 + \sigma_s^2}{\sigma_w^2 + \sigma_s^2 + \sigma_\epsilon^2}$$

The price adjustment coefficient k is between zero and one, reflecting the partial price adjustment made by market makers in response to the noisy signals. If the signal contains no noise ($\sigma_\epsilon^2 = 0$), k will be equal to one, and prices will adjust completely to the signals.[2] Unlike the real world, where trades by informed traders determine prices, the model here assumes that market makers set prices based on their own information. Thus, the model is related to Kyle's (1985), where market makers do not observe information directly, but instead observe order flows composed of an informed traders component and a liquidity traders random component, and determine prices conditional on those order flows.

Two results occur when market makers condition prices on their own signals. First, the pricing error of one stock is correlated with signals of other stocks. The pricing error in stock i is $e_i = V_i - P_i = (1-k)(W + S_i) - k\epsilon_i$. The linear projection rule ensures that the pricing error is orthogonal to its own signal, i.e., $\mathrm{Cov}(e_i, \theta_i) = 0$. However, the pricing error is correlated with signals of other stocks, $\mathrm{Cov}(e_i, \theta_j) = (1-k)\sigma_w^2$, $i \neq j$, because of the partial price adjustment to the marketwide component. Second, although each stock price is an unbiased estimate of the true stock value conditional on its own signal, the aggregate of stock prices is not an unbiased estimate of the true aggregate value conditional on all signals. Without loss of generality, suppose there are only two stocks. If market makers condition prices on two signals separately, as in equation (1), the aggregate of stock prices is

$$\begin{aligned} P_1 + P_2 &= E(V_1|\theta_1) + E(V_2|\theta_2) \\ &= 2\bar{v} + k(\theta_1 + \theta_2) \end{aligned} \tag{2}$$

However, if they condition prices on two signals simultaneously, the aggregate of stock prices is

$$\begin{aligned} P_1 + P_2 &= E(V_1 + V_2|\theta_1, \theta_2) \\ &= 2\bar{v} + k'(\theta_1 + \theta_2) \end{aligned} \tag{3}$$

where

$$k' = \frac{2\sigma_w^2 + \sigma_s^2}{2\sigma_w^2 + \sigma_s^2 + \sigma_\epsilon^2}.$$

The proof is in Appendix A. Since $k' > k$, the aggregate of stock prices conditional on two signals separately in equation (2) is not equal to that conditional on two signals simultaneously in equation (3). When the signals contain a common marketwide component with uncorrelated noise, two signals are more precise than one. Consequently, if market makers observe an extra signal, they will adjust prices more aggressively.[3]

[2] Since the signals are identically distributed for all stocks, the partial adjustment coefficient k is the same for all stocks. This restriction is relaxed later in the section.

[3] The difference between conditioning prices on one signal versus two signals is analogous to the difference between simple regression and multiple regression. Unless the independent variables (signals) are orthogonal to each other, the slope coefficients from a simple regression will not equal those from a multiple regression.

B. Multiperiod Model

The model is extended to multiple periods to examine the implications of the pricing rules for intertemporal return relations. The stocks are traded over T periods, and the stock value in period t is given by

$$V_{i,t} = \bar{v} + \sum_{\tau=1}^{t} \Delta V_{i,\tau} \tag{4}$$

where

$$\Delta V_{i,\tau} = W_\tau + S_{i,\tau}.$$

Note that $\Delta V_{i,\tau}$ represents the change in the value at period τ. W_τ and $S_{i,\tau}$ are serially uncorrelated, mutually independent, and multivariate normally distributed with zero mean and variances σ_w^2 and σ_s^2. At period $t-1$, the market maker in stock i does not observe $\Delta V_{i,t-1}$ directly, but instead observes the signal $\theta_{i,t-1} = \Delta V_{i,t-1} + \epsilon_{i,t-1}$. At period t, he observes additional information about $\Delta V_{i,t-1}$ and readjusts prices. The additional information can be modeled in two scenarios. In the first, market makers observe the true value of $\Delta V_{i,t-1}$ at period t. This scenario is similar to the model in Admati and Pfleiderer (1988), where signals are assumed useful for only one period. In the second scenario, market makers at period t retrieve additional signals about $\Delta V_{i,t-1}$ from previous price changes of other stocks; signals are therefore useful for two periods. The first scenario requires that market makers observe $\Delta V_{i,t-1}$ perfectly at period t, whereas the second scenario requires only that market makers observe $\Delta V_{i,t-1}$ more precisely at period t.

B.1. First Scenario: Observing True Values Subsequently

Assume that at period t, the market maker observes the true values of $\Delta V_{i,\tau}$, $\tau = 1, 2 \ldots t-1$, and a current signal $\theta_{i,t}$. Conditional on his information set, the market maker sets the price as

$$\begin{aligned} P_{i,t} &= \bar{v} + \sum_{\tau=1}^{t-1} \Delta V_{i,\tau} + k\theta_{i,t} \\ &= \bar{v} + \sum_{\tau=1}^{t-1} (W_\tau + S_{i,\tau}) + k(W_t + S_{i,t} + \epsilon_{i,t}) \end{aligned} \tag{5}$$

where k is defined as in equation (1). The second term on the right-hand side of (5) reflects the publicly available information at period t, and the last term is similar to that in (1), reflecting the partial price adjustment made by market makers in response to current signals. The price change at period t, $\Delta P_{i,t} = P_{i,t} - P_{i,t-1}$, is equal to

$$\Delta P_{i,t} = (1-k)(W_{t-1} + S_{i,t-1}) - k\epsilon_{i,t-1} + k(W_t + S_{i,t} + \epsilon_{i,t}) \tag{6}$$

The own- and cross-autocovariance of the price changes can now be calculated and the expression evaluated by substituting for k,

$$\begin{aligned}\mathrm{Cov}(\Delta P_{i,t}, \Delta P_{j,t-1}) &= k(1-k)(\sigma_w^2 + \sigma_s^2) - k^2\sigma_\epsilon^2 = 0 \qquad & i = j \\ &= k(1-k)\sigma_w^2 & i \neq j \qquad (7)\end{aligned}$$

While the own-autocovariances are zero, the cross-autocovariances are positive.[4] This result follows from the earlier result that the pricing error of one stock is correlated with signals of other stocks. Consequently, when market makers revise pricing errors, the price readjustments are correlated with past price changes (signals) of other stocks.

Two conditions are critical for positive cross-autocorrelation. The first is that signals of different stocks contain positively correlated true values (marketwide information) and uncorrelated noises. The second is that market makers cannot instantaneously condition prices on signals of other stocks. If market makers condition prices on all signals instantaneously, the cross-autocorrelation is zero.[5]

B.2. Second Scenario: Observing Previous Price Changes

The assumption that market makers update prices based on observing the true values of $\Delta V_{i,t-1}$ at period t may not be reasonable if the period is short (for example, an hour or a day), where uncertainty about information may not be resolved in a single period. Even if true values are never revealed, positive cross-autocorrelation occurs if market makers can update prices based on signals inferred from previous price changes of other stocks.

At period t, the market maker observes his current signal $\theta_{i,t}$ and a past price series $\{P_{1,t-1} \dots P_{1,1}, P_{2,t-1} \dots P_{2,1} \dots P_{N,t-1} \dots P_{N,1}\}$, from which he can retrieve past price changes of other stocks to obtain more precise information about the marketwide component. Therefore, the market maker conditions the stock price not only on his current signal, but also on the past prices of other stocks.[6]

[4] Although the theoretical section of the paper is developed in terms of price changes, the model can be developed in terms of returns. Assuming initially that the logarithm of true stock values is normally distributed, the linear projection rule can be applied to determine the logarithm of stock prices. The changes in (logarithm of) stock prices are stock returns. Consequently, stock price changes and stock returns are synonymous in this paper.

[5] Suppose there are two stocks, and each market maker conditions prices on both signals. The aggregate of stock price (index price) changes is

$$\begin{aligned}\Delta P_{1,t} + \Delta P_{2,t} &= (1-k')(2W_{t-1} + S_{1,t-1} + S_{2,t-1}) \\ &\quad - k'(\epsilon_{1,t-1} + \epsilon_{2,t-1}) + k'(2W_t + S_{1,t} + S_{2,t} + \epsilon_{1,t} + \epsilon_{2,t})\end{aligned}$$

where k' is defined as in equation (3). The autocorrelation of index price changes is equal to zero (i.e., cross-autocovariance is zero).

[6] The formulation is similar to Kumar and Seppi's (1989) framework where they examine how market makers in the stock market and index futures market update prices based on profiles of lagged prices from other markets, and analyze the statistical properties of the arbitrage gap between futures and index prices.

This analysis is related to the information aggregation models of Grossman (1976), Diamond and Verrecchia (1981), and Admati (1985), in which each trader receives a piece of diverse information, and equilibrium prices are aggregates of different pieces of information. Each trader can therefore extract other traders' information from equilibrium prices. However, while these models use a one-period framework and analyze the efficiency of equilibrium prices, the model here uses a multiperiod framework and analyzes the intertemporal relation among stock returns.

To form the pricing strategies, assume without loss of generality there are only two stocks, stocks 1 and 2. The market maker in stock 1 can observe past prices of stock 2, and vice versa. To simplify the analysis, we can represent the conditioning information set faced by the market makers in terms of past price innovations. Define $\Delta P^*_{1,t}$ as the price innovation of stock 1, set by the market maker to be equal to the expected change in stock value conditional on the signal at time t, so that $\Delta P^*_{1,t} = E(\Delta V_{1,t}|\theta_{1,t}) = k\theta_{1,t}$, where k is defined as in equation (1). Similarly, define $\Delta P^*_{2,t}$ as the price innovation of stock 2, set to be equal to $k\theta_{2,t}$. As we will see later, the information set $\Phi = \{\Delta P^*_{1,t-1}, \Delta P^*_{1,t-2} \ldots \Delta P^*_{1,1}; \Delta P^*_{2,t-1}, \Delta P^*_{2,t-2} \ldots \Delta^*_{2,1}\}$ is identical to the information set $\phi = \{\Delta P_{1,t-1}, \Delta P_{1,t-2} \ldots \Delta P_{1,1}; \Delta P_{2,t-1}, \Delta P_{2,t-2} \ldots \Delta P_{2,1}\}$, that is, the information provided by past price innovations is equivalent to the information provided by past price changes. The pricing strategy of the market maker in stock 1 can therefore be represented as

$$\begin{aligned} P_{1,t} &= E(V_{1,t}|\theta_{1,t}, \Delta P^*_{1,t-1} \ldots \Delta P^*_{1,1}, \Delta P^*_{2,t-1} \ldots \Delta P^*_{2,1}) \\ &= \bar{v} + E(\Delta V_{1,t}|\theta_{1,t}) + \sum_{\tau=1}^{t-1} E(\Delta V_{1,\tau}|\Delta P^*_{1,\tau}, \Delta P^*_{2,\tau}) \end{aligned} \tag{8}$$

Equation (8) is stated as a sum of expectations of $\Delta V_{1,\tau}$ because $\Delta V_{1,\tau}$ is serially independent. The expectation of $\Delta V_{1,\tau}$, $\tau = 1, \ldots t-1$, is conditional on $\Delta P^*_{1,\tau}$ and $\Delta P^*_{2,\tau}$, which contain relevant information for inferring the true value of $\Delta V_{1,\tau}$. $P_{1,t-1}$ can be represented similarly, and the relation between $P_{1,t}$ and $P_{1,t-1}$ can be expressed as

$$\begin{aligned} P_{1,t} = P_{1,t-1} &+ E(\Delta V_{1,t}|\theta_{1,t}) + E(\Delta V_{1,t-1}|\Delta P^*_{1,t-1}, \Delta P^*_{2,t-1}) \\ &- E(\Delta V_{1,t-1}|\theta_{1,t-1}) \end{aligned} \tag{9}$$

$P_{1,t}$ consists of three components: (1) $P_{1,t-1}$, the information incorporated in the past price; (2) $E(\Delta V_{1,t}|\theta_{1,t})$, the price innovation made in response to the current signal $\theta_{1,t}$; and (3) $E(\Delta V_{1,t-1}|\Delta P^*_{1,t-1}, \Delta P^*_{2,t-1}) - E(\Delta V_{1,t-1}|\theta_{1,t-1})$, the price revision after observing the past price innovation of stock 2. The last term captures the difference between projections based on two signals and those based on one signal.

Equation (9) can be simplified as follows

$$\Delta P_{1,t} = \Delta P^*_{1,t} + \lambda[\Delta P^*_{2,t-1} - (1-m)\Delta P^*_{1,t-1}] \tag{10}$$

where

$$\lambda = \frac{(1-k)\sigma_w^2}{k(\sigma_s^2 + \sigma_\epsilon^2 + m\sigma_w^2)}$$

$$m = \frac{\sigma_s^2 + \sigma_\epsilon^2}{\sigma_w^2 + \sigma_s^2 + \sigma_\epsilon^2}$$

The proof is in Appendix B. Therefore, stock price changes can be decomposed into (1) a price innovation component, $\Delta P_{1,t}^*$, the price adjustment based on its own signal; and (2) a price revision component, $\lambda[\Delta P_{2,t-1}^* - (1-m)\Delta P_{1,t-1}^*]$, the price readjustment based on the past price innovation of stock 2. The price revision coefficient, λ, varies inversely with both k and m. The intuition is as follows. The noise-to-signal ratio is $(1-k)$. An increase in the proportion of noise (a smaller k) increases the effectiveness of an extra signal in reducing noise and improving information quality, so that prices adjust more. The marketwide information-to-signal ratio is $(1-m)$. An increase in the proportion of marketwide information (a smaller m) increases the effectiveness of an extra signal in improving marketwide information quality, so that prices adjust more.

Equation (10) demonstrates how the past price innovations of the stocks become observable and can be used in the information set in (8). Since the coefficients, λ and m, are publicly known, the previous price innovation of stock 1 can be retrieved by subtracting the previous price revision component from the previous price change of stock 1: $\Delta P_{1,t}^* = \Delta P_{1,t} - \lambda[\Delta P_{2,t-1}^* - (1-m)\Delta P_{1,t-1}^*]$. Through recursive substitutions, all past price innovations can be retrieved. Therefore, observing past price changes is identical to observing past price innovations.

Similarly, the market maker in stock 2 sets the price such that $\Delta P_{2,t} = \Delta P_{2,t}^* + \lambda[\Delta P_{1,t-1}^* - (1-m)\Delta P_{2,t-1}^*]$. The autocovariances of price changes are computed by evaluating the variance and covariance, and producing

$$\begin{aligned}
\mathrm{Cov}(\Delta P_{1,t}, \Delta P_{1,t-1}) &= \lambda\{\mathrm{Cov}(\Delta P_{1,t-1}^*, \Delta P_{2,t-1}^*) - (1-m)\mathrm{Var}(\Delta P_{1,t-1}^*)\} \\
&= 0 \\
\mathrm{Cov}(\Delta P_{1,t}, \Delta P_{2,t-1}) &= \lambda\{\mathrm{Var}(\Delta P_{2,t-1}^*) - (1-m)\mathrm{Cov}(\Delta P_{1,t-1}^*, \Delta P_{2,t-1}^*)\} \\
&= k(1-k)\sigma_w^2 \qquad (11)
\end{aligned}$$

Again, the own-autocovariance of stock price changes is zero, but the cross-autocovariance among stock price changes is positive. Therefore, even though the true values are not known, market makers can infer additional information from previous price innovations of other stocks, which results in positive cross-autocorrelation.

The model described for two securities can be generalized to a multisecurity model. When there are N securities, the price change behavior for stock 1 is

$$\Delta P_{1,t} = \Delta P^*_{1,t} + \delta \sum_{i=2}^{N} [\Delta P^*_{i,t-1} - (1-m)\Delta P^*_{1,t-1}] \tag{12}$$

where

$$\delta = \frac{(1-k)\sigma_w^2}{k(\sigma_s^2 + \sigma_\epsilon^2 + (N-1)m\sigma_w^2)}$$

The proof is in Appendix C. The coefficient δ is the price revision parameter in the multisecurity model. The own- and cross-autocovariances of stock price changes can be verified to be identical to the results in equation (11).

C. Heterogenous Signal Quality

In previous sections, the variances of the noise of each signal are assumed to be equal so that the quality of the observed signals is homogeneous across all stocks. However, Ho and Michaely (1988) suggest that large firms may have higher-quality information than small firms because the marginal information costs to large firms may be lower. In this section the assumption of homogeneous signal quality is relaxed, allowing a richer framework for examining the temporal relationship across securities.

Suppose the variance of ϵ_i is $\sigma_{i,\epsilon}^2$, $i = 1, 2 \ldots N$, and $\sigma_{i,\epsilon}^2$ can be different from $\sigma_{j,\epsilon}^2$, $i \neq j$. Thus, the noise-to-signal ratio can vary across stocks. The signal quality is better when the variance of ϵ_i is lower. Implications for the first scenario, i.e., when true values are publicly available next period, are examined. Implications for the second scenario, i.e., when signals are retrieved from previous price changes, are not discussed since the results are identical.

If the signal quality varies across stocks, an equation similar to (6) can be derived for price changes of stock i as follows:

$$\Delta P_{i,t} = (1-k_i)(W_{t-1} + S_{i,t-1}) - k_i \epsilon_{i,t-1} + k_i(W_t + S_{i,t} + \epsilon_{i,t}) \tag{13}$$

where

$$k_i = \frac{\sigma_w^2 + \sigma_s^2}{\sigma_w^2 + \sigma_s^2 + \sigma_{i,\epsilon}^2}$$

The partial price adjustment coefficient k_i now varies across stocks. If the signal quality is better (a smaller $\sigma_{i,\epsilon}^2$), the market maker will adjust the price more in response to his signal (a higher k_i). Based on the time series of stocks i and j, the autocovariances of price changes are

$$\begin{aligned} \text{Cov}(\Delta P_{i,t}, \Delta P_{j,t-1}) &= k_i(1-k_i)(\sigma_w^2 + \sigma_s^2) - k_i^2 \sigma_{i,\epsilon}^2 = 0 && i = j \\ &= k_j(1-k_i)\sigma_w^2 && i \neq j \end{aligned} \tag{14}$$

Price changes of individual stocks remain serially uncorrelated, and the cross-autocovariances among securities remain positive. However, since k_i

and k_j are not the same, $\mathrm{Cov}(\Delta P_{i,t}, \Delta P_{j,t-1}) \neq \mathrm{Cov}(\Delta P_{j,t}, \Delta P_{i,t-1})$; the covariance of current price changes of i with past price changes of j is not the same as the covariance of current price changes of j with past price changes of i. If $\sigma_{i,\epsilon}^2 > \sigma_{j,\epsilon}^2$ (the signal quality of stock i is lower than that of stock j), then $k_j > k_i$, so that

$$\mathrm{Cov}(\Delta P_{i,t}, \Delta P_{j,t-1}) = k_j(1 - k_i)\sigma_w^2 > \mathrm{Cov}(\Delta P_{j,t}, \Delta P_{i,t-1}) = k_i(1 - k_j)\sigma_w^2 \tag{15}$$

Therefore, if the quality signal of large firms is assumed to be better than that of small firms, the covariances of current returns of small firms with past returns of large firms will be larger than the covariances of current returns of large firms with past returns of small firms. This is consistent with Lo and MacKinlay (1990a), who find that the returns of large firms generally lead the returns of small firms.[7]

D. Conditional Autocovariances

In previous sections, implications about unconditional autocovariances are developed. This section shows that own- and cross-autocovariances vary with the size of the market movement. First, rewrite equation (12) as

$$\begin{aligned}\Delta P_{1,t} &= \Delta P_{1,t}^* + \delta \sum_{i=2}^{N} [\Delta P_{i,t-1}^* - (1 - m)\Delta P_{1,t-1}^*] \\ &= \Delta P_{1,t}^* + \delta' \left\{ mW_{t-1} + \frac{1}{N-1} \sum_{i=2}^{N} [S_{i,t-1} + \epsilon_{i,t-1} \right. \\ &\qquad \left. -(1 - m)(S_{1,t-1} + \epsilon_{1,t-1})] \right\}\end{aligned} \tag{16}$$

where

$$\delta' = (N - 1)\delta k$$

where N is large, $\Sigma(S_{i,t-1} + \epsilon_{i,t-1})/(N - 1)$ approaches zero, equation (16) can be reduced to

$$\Delta P_{1,t} = \Delta P_{1,t}^* + \delta'\{mW_{t-1} - (1 - m)(S_{1,t-1} + \epsilon_{1,t-1})\} \tag{17}$$

and the first-order own- and cross-autocovariances of stock price changes conditional on the information at period t are

$$\begin{aligned}E_t[\Delta P_{1,t}, \Delta P_{1,t-1}] &= \delta' k\left\{mE_t\left[W_{t-1}^2\right] - (1 - m)\left[E_t\left[S_{1,t-1}^2\right] + E_t\left[\epsilon_{1,t-1}^2\right]\right]\right\} \\ E_t[\Delta P_{1,t}, \Delta P_{2,t-1}] &= \delta' k\left\{mE_t\left[W_{t-1}^2\right]\right\}\end{aligned} \tag{18}$$

[7] Lo and MacKinlay (1990b) show that if securities have different probabilities of nontrading, an asymmetry of cross-autocorrelation among securities returns is induced. Empirically, they demonstrate that when securities are grouped according to size (which is a proxy for relative market thinness), weekly returns of large firms lead those of small firms.

Equation (18) demonstrates that when the time series are partitioned based on the size of market movement at period $t - 1$, first-order autocovariance coefficients vary. This is obvious for the cross-autocovariance, which is simply a function of $E_t[W_{t-1}^2]$. When the market movement at period $t - 1$ is larger, $E_t[W_{t-1}^2]$ is higher so that the cross-autocovariance is higher. The own-autocovariance coefficients also vary positively with the size of market movement. Further, when the market movement is small (or large), $E_t[\Delta P_{1,t}, \Delta P_{1,t-1}]$ is likely to be negative (or positive). In other words, price reversal is more likely when the market movement is small, while price continuation is more likely when the market movement is large. To see this, note that at period $t - 1$ market makers partially adjust prices based on their own noisy signals. At period t, their signals can be verified by the past price changes of other stocks (or, simply, past market price movement). If the market movement is small at period $t - 1$, it is more likely that the market makers' own signals at that period are false. Therefore, there is a price reversal at period t to correct the pricing errors. On the other hand, if the market movement is large at period $t - 1$, it is more likely that the market makers' own signals at that period are correct. Therefore, there is a price continuation at period t to reinforce previous price movement. A similar implication holds for own- and cross-autocorrelations, which are autocovariances standardized by the variances of stock price changes, calculated from equation (17). Comparative statics show that own- and cross-autocorrelations increase when $E_t[W_{t-1}^2]$ is larger, holding $E_t[S_{t-1}^2]$ and $E_t[\epsilon_{t-1}^2]$ constant.

If market makers could continually retrieve signals from past price innovations of other stocks, the intraday cross-autocorrelation pattern would be uniform throughout the day. However, the New York Stock Exchange closes overnight. The behavior at its opening is different from the rest of the day. Amihud and Mendelson (1987, 1991) find higher volatility at the opening; Stoll and Whaley (1990) report a tendency for price reversal. An important feature about the opening is that it resembles a call auction market, where all market and limit orders are submitted to specialists who accumulate them and determine opening prices. The specialists see only the demand and supply for their stocks, not the orders for other stocks. The inability of a specialist in one stock to observe the simultaneous opening of other stocks may induce higher cross-autocorrelation at the opening. From equation (18), if there is more marketwide information accumulated overnight, it will appear as a higher $E_t[W_{t-1}^2]$, so that the cross-autocovariance is larger. The effect on cross-autocorrelation, however, is not clear. If $E_t[S_{t-1}^2]$ and $E_t[\epsilon_{t-1}^2]$ do not increase overnight, the cross-autocorrelation at the opening will increase unambiguously. If $E_t[S_{t-1}^2]$ and $E_t[\epsilon_{t-1}^2]$ increase, the effect on the cross-autocorrelation is ambiguous.

II. Empirical Evidence

To determine whether own- and cross-autocorrelations vary with the size of market movement, I obtain data from the CRSP daily stock price files from 1980 to 1989 and sort stocks into ten size deciles based on their equity values at the beginning of each year. For each stock, the first-order own-autocorrelation of daily returns $\rho(R_{i,t}, R_{i,t-1})$ and cross-autocorrelation with past returns $\rho(R_{i,t}, R_{m,t-1})$ are computed,[8] where $R_{i,t}$ and $R_{m,t}$ are daily returns of individual stocks and of the CRSP value-weighted market index, respectively. Averages of $\rho(R_{i,t}, R_{i,t-1})$ and $\rho(R_{i,t}, R_{m,t-1})$ are calculated for each size decile, and grand averages are calculated for the full ten years. Since Froot and Perold (1990) report that the time series properties of stock market indexes have changed in recent years, grand averages are also computed separately for two subperiods, 1980 to 1984 and 1985 to 1989, to check the consistency of the data.

The own- and cross-autocorrelations are presented in Table I. Results indicate that the own-autocorrelation coefficient is -9.87 percent for decile 1 (smallest firms) and 4.85 percent for decile 10 (largest firms) from 1980 to 1989. This may be due to larger bid-ask errors for small firms. Results also show that the own-autocorrelation in all deciles decreases between the 1980 to 1984 and 1985 to 1989 subperiods. For example, the autocorrelation coefficient decreases from -8.14 to -11.62 percent in decile 1, and from 6.87 to 2.80 percent in decile 10.

The cross-autocorrelation coefficients are significantly positive in all deciles. Results indicate that the cross-autocorrelation decreases between the 1980 to 1984 and 1985 to 1989 subperiods. The coefficient decreases from 7.41 to 5.75 percent in decile 1, and from 7.23 to 3.00 percent in decile 10. This is consistent with Froot and Perold (1990), who document a decrease in the autocorrelation of stock market indexes. They suggest that new trading practices have improved the processing of marketwide information.

To investigate the own-autocorrelation patterns conditional on the size of marketwide movement at period $t-1$, I sort trading days into five quintiles based on the magnitude of market movement at period $t-1$, which is measured by absolute CRSP (value-weighted) market returns. Quintile 1 contains trading days with the lowest 20 percent absolute market returns, representing small market movements at period $t-1$; quintile 5 contains trading days with the highest 20 percent absolute market returns, representing large market movements at period $t-1$. I then compare own-autocorrelations under small and large market movements. Since stock price fluctuations are affected by factors of the trading mechanism—such as bouncing between bid and ask prices (Roll (1984)), random arrival of orders to the market (Mendelson (1982)), and the transitory state of dealers' inventory position (Amihud and Mendelson (1980))—the own-autocorrelation may also be affected. However, so long as the effect of these factors is the same under small and large market movements, own-autocorrelations under large market movements will still be larger than those under small market movements.

[8] Although the theoretical discussion is in terms of autocovariances, the implications also hold for autocorrelations. Empirical results are presented only for autocorrelations since results are similar for autocovariances.

Table I

Grand Averages of Own- and Cross-Autocorrelations

Grand averages of first-order own-autocorrelations $\rho(R_{i,t}, R_{i,t-1})$ and cross-autocorrelations $\rho(R_{i,t}, R_{m,t-1})$ of daily returns in ten deciles of New York Stock Exchange and American Stock Exchange stocks from 1980 to 1989, where $R_{i,t}$ and $R_{m,t}$ are daily returns of individual stocks and the CRSP value-weighted market index at day t. Values in parentheses are standard errors.

Size	Own-Autocorrelations (in %) $\rho(R_{i,t}, R_{i,t-1})$			Cross-Autocorrelations (in %) $\rho(R_{i,t}, R_{m,t-1})$		
Decile	1980–84	1985–89	1980–89	1980–84	1985–89	1980–89
1	−8.14	−11.62	−9.87	7.41	5.75	6.59
(Small)	(0.45)	(0.50)	(0.34)	(0.23)	(0.28)	(0.18)
2	−3.87	−5.89	−4.87	8.50	8.10	8.30
	(0.42)	(0.47)	(0.32)	(0.23)	(0.32)	(0.20)
3	−1.91	−4.89	−3.39	9.68	8.85	9.27
	(0.40)	(0.46)	(0.31)	(0.24)	(0.33)	(0.21)
4	0.35	−3.06	−1.34	10.87	9.32	10.10
	(0.41)	(0.46)	(0.31)	(0.26)	(0.34)	(0.21)
5	1.29	−1.22	0.05	11.35	9.59	10.48
	(0.40)	(0.43)	(0.29)	(0.28)	(0.32)	(0.21)
6	3.16	0.40	1.79	12.78	10.85	11.82
	(0.35)	(0.39)	(0.26)	(0.27)	(0.33)	(0.22)
7	3.40	2.27	2.84	12.46	11.23	11.85
	(0.33)	(0.39)	(0.26)	(0.29)	(0.34)	(0.22)
8	4.90	2.41	3.67	13.12	9.56	11.35
	(0.31)	(0.36)	(0.24)	(0.29)	(0.34)	(0.23)
9	6.49	3.79	5.15	11.85	7.80	9.84
	(0.28)	(0.31)	(0.21)	(0.29)	(0.33)	(0.22)
10	6.87	2.80	4.85	7.23	3.00	5.13
(Large)	(0.29)	(0.28)	(0.21)	(0.29)	(0.31)	(0.22)
All	1.26	−1.50	−0.11	10.53	8.41	9.48
	(0.12)	(0.14)	(0.09)	(0.09)	(0.11)	(0.07)

Table II presents first-order own-autocorrelations $\rho(R_{i,t}, R_{i,t-1})$ conditional on small and large market movements at period $t - 1$. In all deciles, the own-autocorrelation is larger under large market movements. The differences between own-autocorrelations under large and small market movements are positive in all but one decile (decile 10 in the 1980 to 1984 subperiod). The difference is largest in the intermediate deciles and smallest in decile 10. The t-test shows that most of the differences are significant at the 0.01 percent level. A Wilcoxin signed-rank test examines for differences and confirms that the significance levels are robust.

Table III reports first-order cross-autocorrelations $\rho(R_{i,t}, R_{m,t-1})$ conditional on small and large market movements. The cross-autocorrelation is larger under large market movements than under small market movements, and the differences are significant at the 0.01 percent level. Similar to the results for own-autocorrelation, the difference in cross-autocorrelations is largest in the intermediate deciles and smallest in decile 10. This provides

Table II

Own-Autocorrelations Conditional on the Size of Market Movement

Grand averages of first-order own-autocorrelations $\rho(R_{i,t}, R_{i,t-1})$ of daily returns of individual stocks in ten size deciles, conditional on the size of market movement at day $t-1$. Small (or large) market movement is taken from the quintile of the lowest 20 percent (or highest 20 percent) absolute CRSP returns at day $t-1$.

		Own-Autocorrelations			Test for the Difference		
Size Decile	Sample Period	Small Market Movement (%)	Large Market Movement (%)	Difference (%)	t-Statistic	t-Test p-Value (%)	Wilcoxon p-Value (%)
1	1980–84	−9.81	−7.02	2.79	4.03	0.01	0.01
(Small)	1985–89	−13.38	−11.10	2.28	2.89	0.39	0.17
2	1980–84	−5.08	−2.46	2.62	3.76	0.01	0.01
	1985–89	−7.70	−4.02	3.68	4.63	0.01	0.01
3	1980–84	−3.67	1.63	5.29	7.73	0.01	0.01
	1985–89	−6.41	−3.26	3.15	3.88	0.01	0.02
4	1980–84	−0.92	3.85	4.78	7.34	0.01	0.01
	1985–89	−6.05	−0.53	5.52	6.83	0.01	0.01
5	1980–84	0.10	5.23	5.13	7.45	0.01	0.01
	1985–89	−4.15	1.78	5.93	7.46	0.01	0.01
6	1980–84	1.11	7.50	6.39	9.55	0.01	0.01
	1985–89	−4.00	4.40	8.40	10.64	0.01	0.01
7	1980–84	1.43	7.37	5.94	8.49	0.01	0.01
	1985–89	−1.44	5.82	7.26	9.29	0.01	0.01
8	1980–84	2.63	9.27	6.65	10.31	0.01	0.01
	1985–89	−0.20	5.38	5.57	7.29	0.01	0.01
9	1980–84	4.95	10.42	5.47	8.28	0.01	0.01
	1985–89	2.44	5.35	2.91	3.73	0.02	0.01
10	1980–84	5.61	7.16	1.56	2.50	1.24	1.26
(Large)	1985–89	3.78	1.67	−2.11	−2.69	0.72	1.88
All	1980–84	−0.36	4.30	4.66	21.92	0.01	0.01
	1985–89	−3.71	0.55	4.26	16.97	0.01	0.01

indirect evidence that variations of own- and cross-autocorrelations across deciles are closely related.

III. Conclusion

In this paper I develop a model to explain why stock returns are positively cross-autocorrelated. In the model market makers observe noisy signals about their stocks, but cannot instantaneously condition prices on the signals of other stocks, which contain marketwide information. As a result, the pricing error of one stock is correlated with signals of other stocks. In addition, although each stock price is an unbiased estimate of the true stock value conditional on one signal, the aggregate of stock prices (the index price)

Table III

Cross-Autocorrelations Conditional on the Size of Market Movement

Grand averages of first-order cross-autocorrelations $\rho(R_{i,t}, R_{m,t-1})$ of daily returns of individual stocks with market returns of the previous day in ten size deciles, conditional on the magnitude of market movement at day $t - 1$. Small (or large) market movement is taken from the quintile of the lowest 20 percent (or highest 20 percent) absolute CRSP returns at day $t - 1$.

		Cross-Autocorrelations			Test for the Difference		
Size Decile	Sample Period	Small Market Movement (%)	Large Market Movement (%)	Difference (%)	t-Statistic	t-Test p-Value (%)	Wilcoxon p-Value (%)
1	1980–84	3.34	14.01	10.67	17.71	0.01	0.01
(Small)	1985–89	−0.69	10.33	11.02	15.31	0.01	0.01
2	1980–84	4.36	15.62	11.25	19.19	0.01	0.01
	1985–89	0.07	14.15	14.07	19.91	0.01	0.01
3	1980–84	4.20	17.85	13.65	22.02	0.01	0.01
	1985–89	−0.31	14.98	15.29	20.69	0.01	0.01
4	1980–84	5.62	19.66	14.04	23.39	0.01	0.01
	1985–89	−1.17	15.68	16.85	22.83	0.01	0.01
5	1980–84	5.25	20.35	15.10	24.48	0.01	0.01
	1985–89	0.88	16.20	15.32	20.66	0.01	0.01
6	1980–84	6.75	21.91	15.15	23.99	0.01	0.01
	1985–89	−0.78	18.56	19.33	25.87	0.01	0.01
7	1980–84	7.14	21.61	14.47	22.32	0.01	0.01
	1985–89	0.40	18.48	18.08	23.63	0.01	0.01
8	1980–84	7.27	21.77	14.50	23.29	0.01	0.01
	1985–89	0.19	15.65	15.46	19.95	0.01	0.01
9	1980–84	6.67	19.30	12.63	20.70	0.01	0.01
	1985–89	−0.44	12.19	12.63	16.16	0.01	0.01
10	1980–84	5.78	11.41	5.63	9.57	0.01	0.01
(Large)	1985–89	−0.24	3.98	4.22	5.46	0.01	0.01
All	1980–84	5.64	18.35	12.71	65.01	0.01	0.01
	1985–89	−0.21	14.02	14.23	59.31	0.01	0.01

is not an unbiased estimate of the true aggregate value conditional on all signals.

I extend the model to multiple periods to analyze the implications of this pricing rule for intertemporal return relations under two scenarios. In the first, true stock values are known subsequently, and market makers readjust prices upon observing true values. In the second scenario, true stock values are never revealed, but market makers condition prices on past prices of other stocks. I show under both scenarios that stock returns are serially uncorrelated individually but positively cross-autocorrelated.

The model here supplements nonsynchronous trading as an explanation for cross-autocorrelations among securities. Even if no measurement error occurs, economic reasons can account for the zero autocorrelation of individual stock returns and positive cross-autocorrelation among securities. Further, if the signal quality of large firms is assumed to be better than that of small firms, the covariance of current returns of small firms with past returns of large firms is larger than the covariance of current returns of large firms with past returns of small firms. Thus, the model can explain why large firm returns tend to lead small firm returns. Another implication is that the autocorrelations vary with the size of market movement. This is supported by the evidence that both the own- and cross-autocorrelation coefficients of daily stock returns are significantly higher under large market movements than under small market movements. The evidence therefore suggests a link between index return serial correlation and market volatility.

Appendix A

Derivation of Equation (3)

Since $E(V_1 + V_2|\theta_1, \theta_2)$ is a linear projection of $(V_1 + V_2)$ on θ_1 and θ_2, multiple regression gives the slope coefficients as $E(V_1 + V_2|\theta_1, \theta_2) = B_0 + B_1\theta_1 + B_2\theta_2$, where

$$\begin{aligned} B_0 &= E(V_1 + V_2) \\ B_1 &= \frac{\mathrm{Cov}(V_1 + V_2, \theta_1)\mathrm{Var}(\theta_2) - \mathrm{Cov}(V_1 + V_2, \theta_2)\mathrm{Cov}(\theta_1, \theta_2)}{\mathrm{Var}(\theta_1)\mathrm{Var}(\theta_2) - [\mathrm{Cov}(\theta_1, \theta_2)]^2} \\ B_2 &= \frac{\mathrm{Cov}(V_1 + V_2, \theta_2)\mathrm{Var}(\theta_1) - \mathrm{Cov}(V_1 + V_2, \theta_1)\mathrm{Cov}(\theta_1, \theta_2)}{\mathrm{Var}(\theta_1)\mathrm{Var}(\theta_2) - [\mathrm{Cov}(\theta_1, \theta_2)]^2} \end{aligned} \tag{A1}$$

Evaluating the slope coefficients yields,

$$B_1 = B_2 = \frac{2\sigma_w^2 + \sigma_s^2}{2\sigma_w^2 + \sigma_s^2 + \sigma_\epsilon^2} \tag{A2}$$

Appendix B

Derivation of Equation (10)

Let $\mathbf{X}' = (\Delta V_{1,t-1}, \Delta P^*_{1,t-1}, \Delta P^*_{2,t-1})$, and Σ be the 3×3 variance-covariance matrix of $\mathbf{X}$ with the elements Σ_{ij}. Define $\Sigma_{13\cdot2} = \Sigma_{13} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{23}$, $\Sigma_{33\cdot2} = \Sigma_{33} - \Sigma_{32}\Sigma_{22}^{-1}\Sigma_{23}$, then

$$\begin{aligned} &E(\Delta V_{1,t-1}|\Delta P^*_{1,t-1}, \Delta P^*_{2,t-1}) \\ &\quad = E(\Delta V_{1,t-1}|\Delta P^*_{1,t-1}) + \Sigma_{13\cdot2}\Sigma_{33\cdot2}^{-1}[\Delta P^*_{2,t-1} - E(\Delta P^*_{2,t-1}|\Delta P^*_{1,t-1})] \end{aligned} \tag{B1}$$

Substitute the above expression into (9), and define $\Delta P_{1,t} = P_{1,t} - P_{1,t-1}$, $\Delta P^*_{1,t} = E(\Delta V_{1,t}|\theta_{1,t})$ to obtain

$$\Delta P_{1,t} = \Delta P^*_{1,t} + \Sigma_{13\cdot 2}\,\Sigma_{33\cdot 2}^{-1}[\Delta P^*_{2,t-1} - E(\Delta P^*_{2,t-1}|\Delta P^*_{1,t-1})] \quad \text{(B2)}$$

Substituting the following expressions into (B2) gives equation (11):

$$\begin{aligned}
\Sigma_{13\cdot 2} &= \mathrm{Cov}(\Delta V_{1,t-1}, \Delta P^*_{2,t-1}) \\
&\quad - \frac{\mathrm{Cov}(\Delta V_{1,t-1}, \Delta P^*_{1,t-1})\mathrm{Cov}(\Delta P^*_{1,t-1}, \Delta P^*_{2,t-1})}{\mathrm{Var}(\Delta P^*_{1,t-1})} \\
&= k\sigma_w^2 - \frac{k(\sigma_w^2 + \sigma_s^2)k^2\sigma_w^2}{k^2(\sigma_w^2 + \sigma_s^2 + \sigma_\epsilon^2)} \\
&= k(1-k)\sigma_w^2 \qquad \text{(B3)}
\end{aligned}$$

$$\begin{aligned}
\Sigma_{33\cdot 2} &= \mathrm{Var}(\Delta P^*_{2,t-1}) - \frac{[\mathrm{Cov}(\Delta P^*_{1,t-1}, \Delta P^*_{2,t-1})]^2}{\mathrm{Var}(\Delta P^*_{1,t-1})} \\
&= k^2(\sigma_w^2 + \sigma_s^2 + \sigma_\epsilon^2) - \frac{(k^2\sigma_w^2)^2}{k^2(\sigma_w^2 + \sigma_s^2 + \sigma_\epsilon^2)} \\
&= k^2(\sigma_s^2 + \sigma_\epsilon^2 + m\sigma_w^2) \qquad \text{(B4)}
\end{aligned}$$

$$\begin{aligned}
E(\Delta P^*_{2,t-1}|\Delta P^*_{1,t-1}) &= \frac{\mathrm{Cov}(\Delta P^*_{1,t-1}, \Delta P^*_{2,t-1})}{\mathrm{Var}(\Delta P^*_{1,t-1})}(\Delta P^*_{1,t-1} - E(\Delta P^*_{1,t-1})) \\
&= \frac{k^2\sigma_w^2}{k^2(\sigma_w^2 + \sigma_s^2 + \sigma_\epsilon^2)}\Delta P^*_{1,t-1} \\
&= (1-m)\Delta P^*_{1,t-1} \qquad \text{(B5)}
\end{aligned}$$

Appendix C

Derivation of Equation (12)

In the multisecurity case, market makers retrieve marketwide information from lagged price changes of the other $N-1$ securities. I examine the price behavior of a representative security, say stock 1. Let $\mathbf{X}' = (\Delta V_{1,t-1}, \Delta P^*_{1,t-1}, \mathbf{\Delta P}^{*\prime}_{t-1})$, where $\mathbf{\Delta P}^{*\prime}_{t-1} = (\Delta P^*_{2,t-1}, \Delta P^*_{3,t-1} \ldots \Delta P^*_{N,t-1})$. If Σ represents the $(N+1) \times (N+1)$ variance-covariance matrix of $\mathbf{X}$, the elements of Σ are

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ \Sigma_{21} & \Sigma_{22} & \Sigma_{23} \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \end{bmatrix}$$

where $\Sigma_{11}, \Sigma_{12}, \Sigma_{21}, \Sigma_{22}$ are scalars; Σ_{13}, Σ_{23} are $1 \times (N-1)$ vectors; Σ_{31}, Σ_{32} are $(N-1) \times 1$ vectors, and Σ_{33} is an $(N-1) \times (N-1)$ matrix. A version equivalent to equation (B2) for representing the price adjustment of stock 1 in a multisecurity model can be expressed as

$$\Delta P_{1,t} = \Delta P^*_{1,t} + \Sigma_{13\cdot 2}\,\Sigma^{-1}_{33\cdot 2}[\,\mathbf{\Delta P}^*_{t-1} - \mathbf{E}(\mathbf{\Delta P}^*_{t-1}|\Delta P^*_{1,t-1})] \tag{C1}$$

$\Sigma_{13\cdot 2}$, $\Sigma^{-1}_{33\cdot 2}$, and $E(\mathbf{\Delta P}^*_{t-1}|\Delta P^*_{1,t-1})$ are evaluated as follows:

$$\begin{aligned}
\Sigma_{13\cdot 2} &= \Sigma_{13} - \Sigma_{12}\,\Sigma^{-1}_{22}\,\Sigma_{23} \\
&= \begin{bmatrix} \mathrm{Cov}(\Delta V_{1,t-1}, \Delta P^*_{2,t-1}) \\ \mathrm{Cov}(\Delta V_{1,t-1}, \Delta P^*_{3,t-1}) \\ \vdots \\ \mathrm{Cov}(\Delta V_{1,t-1}, \Delta P^*_{N,t-1}) \end{bmatrix}' - \frac{\mathrm{Cov}(\Delta V_{1,t-1}, \Delta P^*_{1,t-1})}{\mathrm{Var}(\Delta P^*_{1,t-1})} \\
&\quad \times \begin{bmatrix} \mathrm{Cov}(\Delta P^*_{1,t-1}, \Delta P^*_{2,t-1}) \\ \mathrm{Cov}(\Delta P^*_{1,t-1}, \Delta P^*_{3,t-1}) \\ \vdots \\ \mathrm{Cov}(\Delta P^*_{1,t-1}, \Delta P^*_{N,t-1}) \end{bmatrix}' \\
&= k\sigma_w^2\mathbf{1}' - \frac{k(\sigma_w^2 + \sigma_s^2)}{k^2(\sigma_w^2 + \sigma_s^2 + \sigma_\epsilon^2)}k^2\sigma_w^2\mathbf{1}' \\
&= k(1-k)\sigma_w^2\mathbf{1}'
\end{aligned} \tag{C2}$$

$\Sigma_{33\cdot 2} = \Sigma_{33} - \Sigma_{32}\,\Sigma^{-1}_{22}\Sigma_{23}$. Σ_{33} is the $(N-1)\times(N-1)$ covariance matrix of $\mathbf{\Delta P}^{*\prime}_{t-1}$, and the ijth element of Σ_{33} is: $a_{ij} = k^2(\sigma_w^2 + \sigma_s^2 + \sigma_\epsilon^2)$, $i = j$; $a_{ij} = k^2\sigma_w^2$, $i \neq j$. $\Sigma_{22} = \mathrm{Var}(\Delta P^*_{1,t-1}) = k^2(\sigma_w^2 + \sigma_s^2 + \sigma_\epsilon^2)$. Since Σ_{22} is a scalar, $\Sigma_{32}\,\Sigma^{-1}_{22}\,\Sigma_{23} = \Sigma^{-1}_{22}\,\Sigma_{32}\,\Sigma_{23}$. The elements in the matrix $\Sigma_{32}\,\Sigma_{23}$ are $(k^2\sigma_w^2)^2$. Therefore, $\Sigma_{32}\,\Sigma^{-1}_{22}\,\Sigma_{23} = k^2(1-m)\sigma_w^2\mathbf{1}\mathbf{1}'$. Evaluating $\Sigma_{33\cdot 2} = \Sigma_{33} - \Sigma_{32}\,\Sigma^{-1}_{22}\,\Sigma_{23}$,

$$\Sigma_{33\cdot 2} = k^2 \times \begin{bmatrix} (\sigma_s^2 + \sigma_\epsilon^2 + m\sigma_w^2) & m\sigma_w^2 & \cdot & \cdot & m\sigma_w^2 \\ m\sigma_w^2 & (\sigma_s^2 + \sigma_\epsilon^2 + m\sigma_w^2) & m\sigma_w^2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ m\sigma_w^2 & m\sigma_w^2 & \cdot & m\sigma_w^2 & (\sigma_s^2 + \sigma_\epsilon^2 + m\sigma_w^2) \end{bmatrix}$$

To find $\Sigma^{-1}_{33\cdot 2}$, note that $\Sigma_{33\cdot 2} = k^2[(\sigma_s^2 + \sigma_\epsilon^2)\mathbf{I} + m\sigma_w^2\mathbf{1}\,\mathbf{1}']$. Define $\mathbf{A} = [k^2(\sigma_s^2 + \sigma_\epsilon^2)]^{-1}\Sigma_{33\cdot 2}$, then $\mathbf{A} = \mathbf{I} + h\mathbf{1}\,\mathbf{1}'$, where $h = m\sigma_w^2/(\sigma_s^2 + \sigma_\epsilon^2)$. The inverse of $\mathbf{A}$ is known to be $\mathbf{A}^{-1} = \mathbf{I} + g\mathbf{1}\,\mathbf{1}'$,
where

$$g = \frac{-h}{1 + (N-1)h} = \frac{-m\sigma_w^2}{\sigma_s^2 + \sigma_\epsilon^2 + (N-1)m\sigma_w^2}$$

The inverse of $\Sigma_{33 \cdot 2}$ can be found,

$$\Sigma_{33 \cdot 2}^{-1} = \left[k^2(\sigma_s^2 + \sigma_\epsilon^2)\right]^{-1}\mathbf{A}^{-1}$$
$$= \left[k^2(\sigma_s^2 + \sigma_\epsilon^2)\right]^{-1}\left[\mathbf{I} - \frac{m\sigma_w^2}{\sigma_s^2 + \sigma_\epsilon^2 + (N-1)m\sigma_w^2}\mathbf{1}\,\mathbf{1}'\right] \tag{C3}$$

$$E(\mathbf{\Delta P}^*_{t-1} | \Delta P^*_{1,t-1}) = \begin{bmatrix} E(\Delta P^*_{2,t-1}) \\ E(\Delta P^*_{3,t-1}) \\ \cdot \\ \cdot \\ E(\Delta P^*_{N,t-1}) \end{bmatrix} + \frac{\Delta P^*_{1,t-1} - E(\Delta P^*_{1,t-1})}{\mathrm{Var}(\Delta P^*_{1,t-1})}$$
$$\times \begin{bmatrix} \mathrm{Cov}(\Delta P^*_{1,t-1}, \Delta P^*_{2,t-1}) \\ \mathrm{Cov}(\Delta P^*_{1,t-1}, \Delta P^*_{3t-1}) \\ \cdot \\ \cdot \\ \mathrm{Cov}(\Delta P^*_{1,t-1}, \Delta P^*_{N,t-1}) \end{bmatrix}$$
$$= \frac{k^2\sigma_w^2}{k^2(\sigma_w^2 + \sigma_s^2 + \sigma_\epsilon^2)}\Delta P^*_{1,t-1}\mathbf{1}$$
$$= (1-m)\Delta P^*_{1,t-1}\mathbf{1} \tag{C4}$$

Substituting (C2), (C3), and (C4) into (C1) yields equation (12).

REFERENCES

Admati, A. R., 1985, A noisy rational expectations equilibrium for multi-asset securities markets, *Econometrica* 53, 629–657.

——— and P. Pfleiderer, 1988, A theory of intraday patterns: Volume and price variability, *Review of Financial Studies* 1, 1–40.

Amihud, Y., and H. Mendelson, 1980, Dealership market: Market-making with inventory, *Journal of Financial Economics* 8, 31–53.

———, 1987, Trading mechanisms and stock returns: An empirical investigation, *Journal of Finance* 8, 533–553.

———, 1991, Volatility, efficiency and trading: Evidence from the Japanese stock market, *Journal of Finance* 46, 1765–1789.

Atchison, M., K. Butler, and R. Simonds, 1987, Nonsynchronous security trading and market index autocorrelation, *Journal of Finance* 42, 111–118.

Bhasin, V., 1992, On interconnected financial asset markets, Working paper, Indiana University.

Bossaerts, P., 1991, Transaction prices when insiders trade portfolios, Working paper, California Institute of Technology.

Caballe, J., and M. Krishnan, 1992, Insider trading and asset pricing in an imperfectly competitive multi-security market, *Econometrica*, Forthcoming.

Cohen, K., G. Hawawini, S. Maier, R. Schwartz, and D. Whitcomb, 1980, Implications of microstructure theory for empirical research on stock price behavior, *Journal of Finance* 35, 249–257.

Diamond, D., and R. Verrecchia, 1981, Information aggregation in a noisy rational expectations economy, *Journal of Financial Economics* 9, 221–235.

Dimson, E. 1979, Risk measurement when shares are subject to infrequent trading, *Journal of Financial Economics* 7, 197–226.

Fama, E., 1965, The behavior of stock market prices, *Journal of Business* 38, 34–105.

Fisher, L., 1966, Some new stock market indexes, *Journal of Business* 39, 191–225.

French, K., and R. Roll, 1986, Stock return variances, The arrival of information and the reaction of traders, *Journal of Financial Economics* 17, 5–26.

Froot, K., and A. Perold, 1990, New trading practices and short-run market efficiency, NBER Working Paper No. 3948.

Grossman, S., 1976, On the efficiency of competitive stock markets when traders have diverse information, *Journal of Finance* 31, 573–585.

Ho, T., and R. Michaely, 1988, Information quality and market efficiency, *Journal of Financial and Quantitative Analysis* 23, 53–70.

Kumar, P., and D. Seppi, 1989, Information and index arbitrage, Working paper, Carnegie Mellon University.

Kyle, A., 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315–1335.

Lo, A., and A. C. MacKinlay, 1990a, When are contrarian profits due to stock market overreaction?, *Review of Financial Studies* 3, 175–206.

———, 1990b, An econometric analysis of nonsynchronous trading, *Journal of Econometrics* 45, 181–211.

Mendelson, H., 1982, Market behavior in a clearing house, *Econometrica* 50, 1505–1524.

Perry, P., 1985, Portfolio serial correlation and nonsynchronous trading, *Journal of Financial and Quantitative Analysis* 20, 517–523.

Roll, R. 1984, A simple implicit measure of the bid/ask spread in an efficient market, *Journal of Finance* 39, 1127–1139.

Scholes, M., and J. Williams, 1977, Estimating betas from nonsynchronous data, *Journal of Financial Economics* 5, 309–327.

Seyhun, H., 1988, The information content of aggregate insider trading, *Journal of Business* 61, 1–24.

Stoll, H., and R. Whaley, 1990, Stock market structure and volatility, *Review of Financial Studies* 3, 37–71.