DESIGN AND PERFORMANCE ANALYSIS OF TWO STAGE EJECTOR SYSTEM

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NOMENCLATURE

pc = cavity pressure
ga = adiabatic constant
mc = cavity Mach number
At = Throat area
Pp = Plenum pressure
L1 = converging duct length
L2 = constant area duct length
re = Recovery ratio
Cd = Discharge coefficient
mf = Motive gas flow rate
L = Length
P = Pressure
ms = Suction gas flow rate
w = entrainment ratio

Pc = cavity pressure
Pob = Stagnation temperature
P = Pressure
thd, theta = convergence angle

To = Stagnation temperature of suction gas
Ds = Suction gas diameter
en = Entrainment ratio
Pr = Pressure ratio
Tr = Temperature ratio
Pxo = Constant pressure to stagnation pressure ratio
A = Area
T = Temperature
Ti = Stagnation temperature of suction gas for second stage
G = adiabatic constant
To_s = Suction gas stagnation temperature
mm = Motive gas flow rate
M = Mach number
D = diameter

ABSTRACT

An ejector system is a highly efficient system for providing the required Pressure instauration in Sub-atmospheric systems. A supersonic ejector systems are widely utilized in bulwark, industrial and commercial applications, ergo the field utilization of the ejectors provides congruous pressure recuperation in various vacuum demanding systems. There are two possible modes of motive gas injection into the ejector system. The present work deals with the design of 2 stage ejectors for both modes of injection for a sub atmospheric system. A MATLAB code has been developed to find the sundry parameters of flow for both stages of ejector. The design of the ejector for both stages has been estimated by calculating the dimensions of sundry components of ejector. The results show an appreciable elevate in the static and stagnation pressure at the terminus of both stages. The developed design imparts a static pressure of ~922 Torr and stagnation pressure of ~1245 Torr at the cessation of second stage in case of Peripheral mode and a static pressure of ~900 Torr and stagnation pressure of ~1010 Torr at the end of second stage for Central mode . Conclusively, utilizing the same code, the transmutations in Recuperated pressure and geometry of the ejector are calculated for transmuting test chamber pressure and Mach number.

1. INTRODUCTION

1.1 MOTIVATION FOR WORK

The main quandary with the sub-atmospheric systems is atmospheric exhaust, which if not felicitously done, reduces their efficiency . Ergo, there is desideratum of finding some designates of direct atmospheric exhaust at sea level. Though there are sundry methods to achieve identically tantamount, but they have certain drawbacks. The most opportune method is to utilize the congruous efficient pressure recuperation system. These systems involve the utilization of pressure instauration components such as Supersonic ejectors and diffusers.

The advantages of ejector systems lie in the simplicity of their construction and operation, and that it requires no separate puissance, when a source of high pressure primary fluid is yarely available. Ejectors have been widely utilized for variety of applications from engendering vacuum in steel power plants, aircraft thrust augmentation, green refrigeration technologies and recently in fuel cell applications. Ejectors can be efficacious as recompression contrivances, especially in high speed wind tunnels, where the pressure within the test chambers can be low and has to be diffused to the ambient pressures. The utilization of ejectors can result in reduced power utilization.

A recent breakthrough in the development of sub-atmospheric systems is the utilization of advanced ejector nozzles [5], which not only provide excellent gain, but withal sanction the generation of high total pressure active medium. The employment of ejector nozzles for achieving high pressure laser operation and implementation of transonic and supersonic iodine injection mechanism has provided efficiencies of approximately 30%. The condition achieved through such type of nozzles are astronomically propitious for design of ejector predicated pressure recuperation system.

Ejectors have a lot industrial applications. Especially they are utilized in Purloin power plants, refineries and chemical engenderment companies. They are capable of supplying the required vacuum in power plant condensers or transfer gas and vapor specially the corrosive ones in refineries. These contrivances are withal utilized in aliment industries for thickening the solutions. They work steadily in their operating range and their engenderment cost is low as compared to vacuum pumps.

Hence, with the wide range of applications of ejector systems and their advantages over other vacuum generation contrivances, it’s inevitably ineluctable to utilize the ejectors as the most congruous and efficient contrivances for engendering vacuum or achieve the atmospheric exhaust in sub-atmospheric system for better efficiencies.

1.2 OBJECTIVE

To design an efficient two stage ejector system for establishing the felicitous atmospheric discharge in sub atmospheric systems.

1) Design of Ejector system for Peripheral mode of motive gas injection and analyze the Pressure recovery achieved
2) Design of Ejector system for Central mode of motive gas injection and analyze the Pressure recovery achieved
3) Analysis of Recovery ratio for increasing entrainment and pressure ratio.
4) Analysis of change in pressure recovery and ejector geometry with varying test chamber conditions.

2. LITERATURE REVIEW

One major approach to achieve the required atmospheric discharge or pressure recovery is the utilization of Ejectors, which ascertain the opportune atmospheric discharge and required Pressure recuperation. Several studies and research have been conducted to design efficient ejector systems and analyze their performance. In the present work, the ejectors with two different modes have been designed and performance is recorded. It has been hypothesized that employment of ejector systems avail get the required pressure instauration. The following five Literature review endeavor to demonstrate and support the hypothesis.

J. H. Keenan, 1948 Next, the topic of affirming in cognation to the investigation of Ejector design by performance and analysis [4] is presented to analyze the performance of ejectors. The results for both constant area commixing and constant pressure commixing have been calculated. Conclusively, the comparison of analytical and experimental results and a comparison of results for both commixing has been made. The process in the ejector is postulated to be isentropic. It is hypothesized that constant pressure commixing gives better results.
The suction flow is injected through the periphery. The design of ejector consists of constant pressure commixing chamber followed by constant area chamber. At the exit, there is a subsonic diffuser. The primary and suction flow are injected from their stagnation states and the velocity at the exit of diffuser becomes zero. The commixing is always consummate at the terminus of constant area section. For constant pressure commixing design, the results have been calculated for commixing at the exit of constant pressure commixing chamber. In case of design of ejector employing constant area commixing, the results at the terminus of constant area commixing chamber have been calculated. Conclusively, the pressure curves for both commixing have been analyzed and results are compared. The diffuser length and position of nozzle has been estimated.
For constant pressure commixing, the calculated values match with experimental values up to a precision of 93%. It is observed that the distinction between calculated and experimental values increases as the entrainment ratio increases. The results are in accordance with the hypothesis made at the commencement.

B.J. Huang, 1999 Next, the topic of annexation in cognation to the 1-D analysis of ejector performance [3] is addressed. The ejectors can be categorized into "Constant pressure commixing" and "Constant area commixing". The focus of the research is to design a "constant pressure commixing" ejector system and analyze its performance. It has been hypothesized that the system with constant pressure commixing provide better efficiency.
The commixing takes place inside the constant area section at a constant pressure. The primary flow expedites the suction flow by inducing a converging duct. The suction flow reaches the sonic conditions at the point, which is called as Hypothetical throat. The entrained flow is verbally expressed to be choked here. The work for calculation of Mach number, velocity, temperature and pressure of both flows has been done by making utilization of conservation equations. The commixed flow is observed to undergo a transverse shock at the exit of constant area section and determinately peregrinates to a subsonic diffuser. The experiment is performed on 2 different nozzles and 11 different ejector designs to estimate the precise results.
The results and observations show that for better ejector performance, the position of nozzle is major concern. It is observed that the ejector exit to nozzle throat area ratio increase as the pressure of primary flow increases. The system has a drawback of superheating of primary and suction flow, which reduces the ejector performance. Though, in present system, the heating is minuscule enough to be neglected.

Singhal Gaurav, 2006 In the Research article on pressure recovery system for 2 stage ejector based system for peripheral mode of motive gas injection [1], the need of employing ejector systems in high power supersonic chemical laser, has been discussed, as these lasers require sufficient atmospheric discharge for better efficiencies.

There are two main modes of motive gas injection into the ejector i. e. Peripheral and Central mode. The article deals with the study of Peripheral mode of motive gas injection. The focus of the paper is to analyze the efficiency of ejector and record its performance and conclusively match with experimental results. A supersonic diffuser has additionally been employed at the terminus of ejector to reach the desired subsonic conditions. The whole process inside the ejector, which includes injection of motive and suction gas into the ejector, commixing of the two streams in constant pressure and constant area commixing chamber, the flow coming across transverse shock which results is the transmutation of the properties of flow, has been described. The system has been studied for two stages and pressure at the terminus of 2nd stage is recorded. The results of the study support the hypothesis and it has been observed that numerical result match with the experimental ones up to a precision of 80%. The total static pressure recuperated at the cessation of second stage is 900 Torr and at the corresponding stagnation pressure is 1100 Torr. Though the calculations have certain constraints, as these do not take into account the sundry losses, verbalize frictional, shear and shock losses, along the length of ejector as the stream flows. So some postulations have been made to take those losses into account. At the cessation of the second stage, the static pressure is calculated to be 19 times the pressure at the commencement of second stage.

S. Daneshmand, 2009 Next, the research article describing Analytical and experimental methods of design of ejectors [5] expounds the study and comparison of analytical and experimental results of Supersonic Ejectors. As observed in the antecedent research works, because of the high viscosity, turbulence and 3D nature of fluid, the numerical methods are not capable of giving 100% precise results, hence analytical methods are required to calculate the ejector performance.

The article describes two types of ejectors. The distinction between two types is only the position of Power unit of Wind tunnel, for which the ejector systems are employed. First ejector has power unit placed at the ingression and the other types has it placed at the cessation of ejector. The high energy flow enters the ejector from center and is driven to diffuser with high energy air permeate nozzle. The ejectors discussed here has four main components -- Active nozzle, Passive nozzle, commixing chamber and Diffuser. If the ejectors are utilized in series, the output of one ejector is the input of the following ejector. In subsonic ejectors, the flow velocity reaches the haste of sound, whereas in Supersonic ejectors, the flow velocity may be more than the haste of sound. A FORTRAN code has been developed to calculate the results and conclusively the analytical results are compared with experimental values.

It has been observed that analytical solutions match with the experimental ones to a good approximation. It is observed that If the pressure ratio, supplied by the potency unit is not according to the area ratio, then the performance of ejector is greatly reduced because of mundane shock wave. The results show that, more the energy of fluid in commixing chamber, more will be the efficiency and hence more will be vacuum engendered.

Singhal Gaurav, 2010 After the discussion and performance analysis of peripheral mode, another research article on pressure instauration studies with central injection mode [2] describes the central mode of motive gas injection. The whole process inside the ejector is kindred to the peripheral mode, the difference lies in the injection of motive gas into the ejector.

The motive gas is made to enter a convergent-divergent nozzle placed at the center of ingression of ejector. Here, suction flow from the cavity enters the ejector through periphery. Both the gases undergo same process as in the case of Peripheral mode i.e. injection, commixing, Transverse shock and determinately discharge through a subsonic diffuser. The system with the central mode is studied for 1 stage, but it's suggested that for better efficiencies, two stages are required. The geometry of the ejector has been estimated by calculating areas and diameters of sundry components of the ejector.

The calculated static pressure at the exit is around 70 Torr and corresponding stagnation pressure is 80 Torr. The experimentally observed value of static pressure is 63 Torr, which again might be because of the sundry losses inside the ejector. One constraint of the system is that in the commixing chamber, the motive gas forms an Aerodynamic choke, because of which suction flow gets expedited and reach the sonic conditions, which engenders variations in the haste and pressure of commixing flow. But overall, the results match with experimental values.

The book “**Modern Compressible Flow” by J. D. Anderson** [6] provides a brief overview of Fundamental thermodynamic concepts along with the exhaustive study of concepts of compressible flow.
Initially, the rudimentary understanding of compressible and incompressible flow has been provided. It’s explicated how fluid properties change with the postulation of compressibility and incompressibility. Paramount thermodynamic concepts, terms and equations have been expounded and worked exhaustively. Further, integral form of conservation equations for inviscid flow has been brought into light. Making a posit of steady state flow, final results utilizing finite control volume, infinitesimal finite element and molecular approach have been calculated. The integral form of conservation equations is derived for 1-D flow. It has been again surmised that flow is in steady state and frictional and shear forces are negligible. Further Mundane shock cognations for 1-D flow are derived. The Hugoniot equation and concept of heat additament in 1-D flow, which expounds how the properties of fluid change when heat is integrated to it, has been described. Peregrinating from the concept of 1-D flow to Quasi 1-D flow, it has been expounded how area of flow changes along the flow direction and how does it affect the results of conservation equations. The area-velocity cognations, explicating transmutation in velocity of flow with area, are described. The concept of “Isentropic flow of Calorically perfect gas”, which is a consequential concept in the study of ejector and diffuser systems has been explicated.
Determinately, it provides a brief review of diffusers along with congruous cognations for diffuser analysis.

3. THEORETICAL BACKGROUND

3.1 EJECTOR

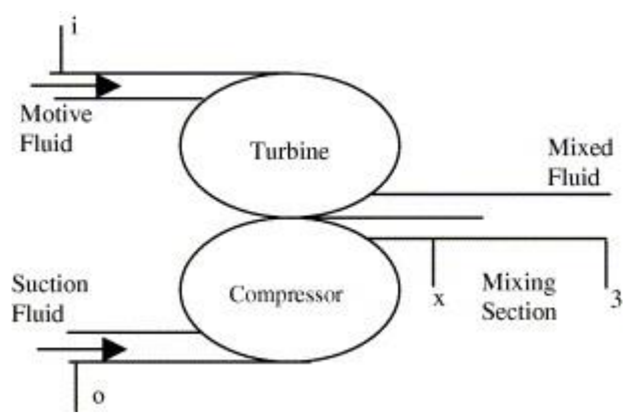
Ejector is a pumping contrivance with no moving components. It utilizes a gas and a fluid as a motive force. When the motive flow is a steam, then the system is called “Steam Jet ejector”. Fundamental components of an ejector are Nozzle, suction, throat and a diffuser. [5] Two major functions of Ejectors are :

- Thermocompressors
- Vacuum engenderers

Though, in the present work, ejectors have been utilized as Vacuum engenderers for sub-atmospheric systems. The major distinction between an ejector and vacuum pump is that it doesn't have moving components.
Ejectors are relegated as "Mass augmenters" and "Jet pumps". Mass augmenters have a sizably voluminous ratio of inlet area of suction gas to inlet area of motive gas. But they are not capable of pumping suction permeate an immensely colossal pressure elevate. The ejectors of "Jet pump" type are capable of pumping the suction flow from a low stagnation pressure to sizably voluminous back pressure. Jet pumps type ejectors accommodate the purport of pressure recuperation in vacuum demanding systems.

- **Principle**

In an ejector system, the motive gas imparts its energy to the low energy suction fluid thus raising its pressure to provide the indispensable pressure instauration. Thus an ejector system can be thought of as a turbine-compressor system, where the motive gas acts a turbine. It expands from a low pressure to high pressure and performs work on suction gas, and hence compresses it from low inlet pressure to high exit pressure. The whole process is postulated to be adiabatic isentropic expansion. A low pressure zone is engendered afore the commixing chamber due to pressure drop of motive fluid. And because of the low pressure zone, the suction fluid will be expedited towards it and commences commixing with the motive fluid in commixing chamber. From the commixing chamber, which is a constant area section, withal called as throat of the ejector, the flow is driven to the diffuser, where its pressure increases and velocity decreases at the exit.



The capacity of an ejector is decided by its dimensions. According to the requisite of vacuum generation or Pressure instauration, ejectors are made to work in series and parallel. Ejectors range from single to five or six stage units. The number of stages required are conventionally decided by the level of vacuum required. Sundry stages are described as :

- **Single stage ejectors**

These cover vacuum ranges upto atmospheric pressure. All single stage ejectors are designed to discharge at or either marginally above the atmospheric pressure.

- **Two stage ejectors**

In operation, a two stage system consists of a primary High Vacuum ejector and a secondary low vacuum ejector. Initially the LV ejector is operated to pull vacuum down from the commencement pressure to an intermediate pressure. Once this pressure is reached the HV ejector is then operated in conjunction with the LV ejector to determinately pull vacuum to the required pressure.

- **Three stage ejectors**

A three stage ejector system consists of a Primary booster, a secondary High Vacuum ejector and a tertiary low vacuum ejector. initially the LV ejector is operated to pull vacuum down from the commencement pressure to an intermediate pressure. Once this pressure is reached the HV ejector is then operated in conjunction with the LV ejector to pull vacuum to the lower intermediate pressure. Conclusively the Booster is operated to pull vacuum to the required pressure.

- **Four, five and Six stage ejectors**

In contrast to three stage ejector system, these include adscititious boosters. In order to increment the efficiency and reduce the motive gas consumption, these are conventionally condensing type.

The present work deals with the design of Two Stage Ejector system for Pressure recuperation in SCOIL system.

➢ Basic terminology for Ejector design

- **Mach number :** Ratio of speed of flow to the speed of sound. Mach number is a dimensionless quantity.
- **Static pressure :** The pressure of fluid measured at a point while moving with the fluid i.e. keeping the point of consideration at rest w.r.t the fluid
- **Stagnation pressure :** The pressure of the fluid at stagnation state i.e. when it's velocity is zero and all kinetic energy of fluid has been converted into pressure energy
- **Recovery Ratio :** Ratio of pressure achieved at the exit to the pressure at the inlet of ejector
- **Entrainment Ratio :** Ratio of Suction gas flow rate to Motive gas flow rate
- **Pressure ratio :** Ratio of Motive to Suction gas stagnation pressure
- **Temperature Ratio :** Ratio of Suction to motive gas stagnation temperature
- **Stagnation temperature :** Temperature of fluid at stagnation state

3.2 Sub-atmospheric systems with ejectors

In the present design, high flow rates with a low test chamber pressure of 3 Torr are taken. The system has been optimized for a flow rate of 3 gm/s. The operating temperature and pressure of engenderer is 300 K and 25 Torr respectively. High capacity vacuum systems are required for exhaust of outgoing gases.

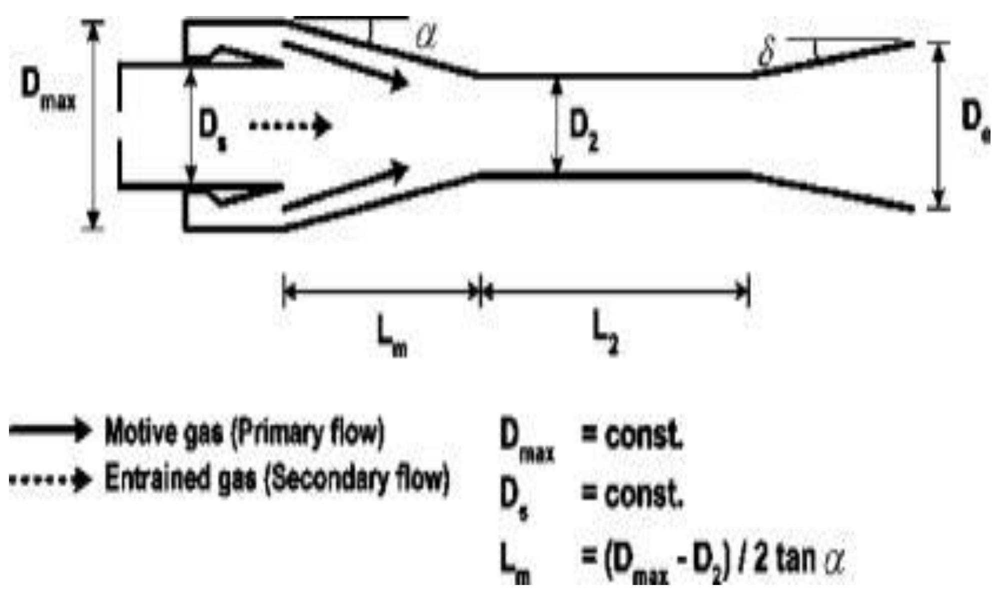
Atmospheric discharge is achieved by the utilization of Supersonic ejector and diffuser system . The design of a 2 stage ejector predicated pressure recuperation system has been described. High energy motive gas and suction gas are injected into the ejector, which are then commixed inside a gradually converging commixing chamber under a constant pressure, which is the more efficacious than constant area commixing [3]. The high energy motive gas imparts its energy to Suction Gas and makes the flow supersonic, which

then comes across a transverse shock leading to a static pressure elevate. The commixed flow then goes into a subsonic diffuser to achieve the required subsonic conditions of flow. The output of the first stage is inputted into the second stage ejector and the flow goes through the whole process again, which leads to the higher pressure gain as the output of second stage ejector.
In an ejector system, primarily two modes of motive gas injection are possible :

- Peripheral mode of Injection
- Central mode of injection

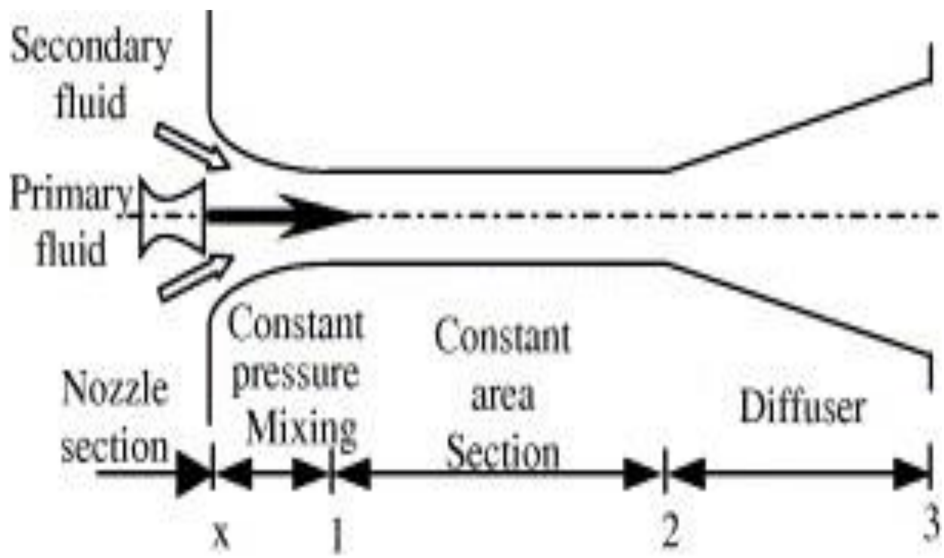
3.2.1 Peripheral mode :

In the peripheral mode [1] of injection, the motive gas is injected through periphery of ejector walls and suction gas is supplied through the center. The motive gas permeates a throat and commixes with a marginally expedited suction flow under a constant pressure commixing in a commixing chamber. The constant pressure commixing chamber is followed by a constant area section which is conventionally the throat of ejector. A transverse shock occurs at the exit of constant area section followed by a subsonic diffuser which avails in ameliorating the recuperated pressure and lower the velocity at exhaust. Usually the peripheral mode is preferred over central mode, because in this case, the suction flow doesn't perturb the flow properties. Since the flow near the walls have high energy, it can handle the adverse pressure gradients in a better manner. Also, the peripheral mode provides a higher pressure recovery as compared to the central mode.



3.2.2 Central mode :

In the central mode of injection [2], the motive gas is injected into the nozzle placed at the center of ejector and suction gas is supplied through the area between nozzle and shroud boundary. The whole process of commixing and exhaust is same as that of peripheral mode except for the injection of motive gas. It is experimentally observed that the central mode shows lesser performance as compared to Peripheral mode. Since the energy and momentum of fluid near the walls is less, it produces a thicker boundary layers and hence, it's not capable of handling adverse pressure gradients. Though the pressure recovery is diminished in both cases for increasing values of entrainment ratios, but the central mode indicates a higher pressure drop for higher values of entrainment ratio. One advantage of using central mode is that the drop in recovery pressure is lesser compared to peripheral mode, if the nozzle position is changed. Thus, it can be inferred that there will be lesser change in recovered pressure as compared to peripheral mode, if geometry is disturbed.



The present work deals with the study and design of both modes of motive gas injection along with their performance and efficiency calculations.

4. THEORETICAL CALCULATIONS

4.1 ALGORITHM

- Stage 1

⇒ Define Input parameters

1) Call the function to calculate Mach number of cavity flow from Area Ratio
2) Calculate Stagnation Pressure of Cavity flow using formula

$$Po = pc\left[1 + \frac{ga-1}{2}mc^2\right]^{\frac{ga}{ga-1}}$$

3) Calculate pseudo throat area using formula

$$A_{pt} = (P_p A_t)/P_o$$

4) Calculate Mach number downstream of shock using formula

$$M_b = sqrt\left[\frac{\frac{2}{ga-1} + M_a^2}{\frac{2(ga)}{ga-1}M_a^2 - 1}\right]$$

5) Calculate stagnation pressure downstream of shock using formula

$$P_{ob} = P_o\left[1 + \frac{2(ga)}{ga+1}(M_a^2 - 1)\right]\left[\frac{1+\frac{ga-1}{2}M_b^2}{1+\frac{ga-1}{2}M_a^2}\right]^{\frac{ga}{ga-1}}$$

6) Calculate static pressure downstream of shock using formula

$$P_b = \frac{P_{ob}}{\left[1+\frac{ga-1}{2}M_b^2\right]^{\frac{ga}{ga-1}}}$$

7) Calculate Mach number of suction and motive gas using formulae

$$M_{x1} = sqrt\left(\left((P_{ix}{}^{\frac{ga-1}{ga}} - 1\right) * \left({}^{2}\!/_{ga-1}\right)\right)$$

$$M_{x2} = sqrt\left(\left((\frac{1}{P_{xo}})^{\frac{ga-1}{ga}} - 1\right) * \left({}^{2}\!/_{ga-1}\right)\right)$$

8) Calculate characteristic Mach number for both gases using formulae

$$M_{x1_c} = sqrt\left[\frac{\frac{ga+1}{2}{M_{x1}}^2}{1+\frac{ga-1}{2}{M_{x1}}^2}\right]$$

$$M_{x2\_c} = sqrt\left[\frac{\frac{ga+1}{2}{M_{x2}}^2}{1+\frac{ga-1}{2}{M_{x2}}^2}\right]$$

9) Calculate characteristic Mach number of mixed gas using formula

$$M_{1c} = {}^{(M_{x1_c} + en * M_{x2_c} * sqrt(T_r))}\Big/_{sqrt((1+en)*(1+en*T_r))}$$

10) Calculate Mixed stream Mach number using formula

$$M_1 = sqrt\left[\frac{2{M_{1c}}^2}{(ga+1)-(ga-1){M_{1c}}^2}\right]$$

11) Calculate Mach number downstream of Transverse shock using formula

$$M_{1b} = sqrt\left[\frac{\frac{2}{ga-1} + M_1^2}{\frac{2(ga)}{ga-1}M_1^2 - 1}\right]$$

12) Calculate Static pressure downstream of shock using formula

$$P_{1b} = P_x\left[\frac{2(ga)}{ga+1}M_1^2 - \frac{ga-1}{ga+1}\right]$$

$P_{1b\_1} = \frac{P_{1b}*80}{100}$; % After considering Frictional loss

13) Calculate stagnation pressure downstream of shock using formula

$$P_{o1b} = P_{1b\_1}\left[1 + \frac{ga-1}{2}{M_{1b}}^2\right]^{\frac{ga}{ga-1}}$$

14) Calculate suction to motive gas area ratio using formula

$$A_{sm} = (P_r)\frac{(P_{xi})^{\frac{1}{ga}}\sqrt{1-(P_{xi})^{\frac{ga-1}{ga}}}}{(P_{xo})^{\frac{1}{ga}}\sqrt{1-(P_{xo})^{\frac{ga-1}{ga}}}}\, en * \sqrt{T_r}$$

15) Calculate throat to motive gas area ratio using formula

$$A_{tm} = \frac{\left(\frac{1}{P_{ix}}\right)^{\frac{1}{ga}}\sqrt{1-\left(\frac{1}{P_{ix}}\right)^{\frac{ga-1}{ga}}}}{\left(\frac{2}{ga+1}\right)\sqrt{1-\frac{2}{ga+1}}}$$

16) Calculate throat to constant area duct ratio using formula

$$A_{t2} = \frac{(P_{3o})}{(P_r)}\sqrt{\frac{1}{(1+en)(1+en*T_r)}}\frac{(P_{23})^{\frac{1}{ga}}\sqrt{1-\left(\frac{p_2}{p_3}\right)^{\frac{ga-1}{ga}}}}{\left(\frac{2}{ga+1}\right)^{\frac{1}{ga-1}}\sqrt{1-\frac{2}{ga+1}}}$$

17) Calculate throat area at specified conditions using formula

$$A_t = \frac{mf\sqrt{T_o}}{0.0404 * P_i * C_d}$$

$$A_t = A_t * 10^6;$$

18) Calculate motive gas area using formula

$$A_s = \frac{3.14 * {D_s}^2}{4}$$

$$A_m = \frac{A_s}{A_{sm}};$$

19) Calculate constant area duct area using formula

$$A_{t1} = A_{tm} * A_m$$

$$A_2 = A_{t1}/A_{t2}$$

20) Calculate motive gas diameter using formula

$$D_m = \sqrt{\frac{4*A_m}{3.14}}$$

21) Calculate constant area duct diameter using formula

$$D_c = \sqrt{\frac{4*A_2}{3.14}}$$

22) Calculate throat diameter using formula

$$D_t = \sqrt{\frac{4*A_{t1}}{3.14}}$$

23) Calculate diameter of Outer tube using formula

$$A_w = A_s + A_m$$

$$D_w = \sqrt{\frac{4*A_w}{3.14}}$$

24) Calculate Convergence angle using formula

$$thd = 2 * atand\left(\frac{\left(\sqrt{\frac{A_w}{A_2}}-1\right)}{\left(\frac{L_1+L_2}{D}\right)}\right)$$

- Stage 2

⇒ Define Input parameters

  - Repeat the steps from 3 to 25 used in the calculation for stage 1

- Define function to calculate Mach number from Area-velocity Relation

- ⇒ Body of the function
- ⇒ Start
- ⇒ Define Area ratio and isentropic constant as input variables
- ⇒ Apply the formula to calculate Mach number

$$fval = \frac{1}{M} * \left(\frac{2}{ga+1} + \frac{ga-1}{ga+1} * M^2\right)^{\frac{ga+1}{2*(ga-1)}} - A_{ratio}$$

- ⇒ End function

4.2 PERIPHERAL MODE OF INJECTION

4.2.1 Stage 1 :

The first and foremost decision that is to be made for an ejector design is to find the number of stages required to achieve the optimum pressure recovery. The number of stages can be determined by using the following equation :

$$\frac{P_{amb}}{P_{suc}} = R^n \qquad (1)$$

$'P_{amb}'$ is the atmospheric pressure, also called as the ambient Pressure and $'P_{suc}'$ is the suction gas static pressure. R is the pressure recovery ratio, which for the present case is 18 to be achieved. n is the number of stages required to achieve the pressure recovery ratio. Hence with P_{amb} equal to 760 Torr and P_{suc} equal to 3 Torr, the required number of stages are 2.

In the present work, the exit area of Suction gas supplier is taken to be 75 mm X 12 mm. The test chamber Mach number is taken to be 1.5. The subsonic flow from a chamber prior to venturi nozzle has a pressure of 16 Torr. The throat height of nozzle is taken to be 10 mm. The flow from nozzle with an exit dimension of (75 X 24)mm^2 expands into a duct of diameter 75mm. The Mach number after an Isentropic expansion is calculated as

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (2)$$

$\frac{A}{A^*}$ is the ratio of duct area to nozzle exit area which is calculated to be 2.45 from the above given dimensions. The area ratio gives the value of Mach number equal to 2.6198. The pseudo throat area A^*_{pt} can be calculated using the equation

$$p_p A^* = p_o A^*_{pt} \qquad (3)$$

Where p_p is the plenum pressure, A^* is the throat area and p_o is the Stagnation pressure of suction gas, which is calculated from the equation

$$\frac{p_o}{p} = \left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}} \qquad (4)$$

Here, M in the above equation is cavity Mach number equal to 1.5. The Stagnation pressure calculated from above equation comes out to be 11.0131 Torr. Because the flow is unstable, oblique shocks occur in the duct and make the flow subsonic. Therefore, the Mach number M_b downstream the shock is given by

$$M_b^2 = \frac{\frac{2}{\gamma-1} + M_a^2}{\frac{2\gamma}{\gamma-1}M_a^2 - 1} \qquad (5)$$

M_a is the Mach number upstream the shock. Subscripts a and b represents upstream and downstream conditions respectively. Similarly, the stagnation pressure on the downstream can be calculated as

$$\frac{P_{ob}}{P_{oa}} = \left[\frac{1+\frac{\gamma-1}{2}M_b^2}{1+\frac{\gamma-1}{2}M_a^2}\right]^{\frac{\gamma}{\gamma-1}} \left[1+\frac{2\gamma}{\gamma+1}(M_a^2-1)\right] \tag{6}$$

Here, P_{oa} equals to 11 Torr is stagnation pressure upstream, which gives downstream stagnation pressure P_{ob} equals to 4.9858 Torr. The static pressure P_b downstream the shock can be calculated from this stagnation pressure from the equation

$$\frac{P_{ob}}{P_b} = \left[1+\frac{\gamma-1}{2}M_b^2\right]^{\frac{\gamma}{\gamma-1}} \tag{7}$$

This value of P_b comes out to be 4.197 Torr and this gives the value of Pressure for constant pressure mixing of motive and suction flow, which is taken to be ~ 4 Torr. The mixed gas undergoes a transverse shock leading to rise in static pressure and decrease in the total pressure of mixed gas, which generally occurs at the inlet of subsonic diffuser.

For the given values of Entrainment ratio(ω), Pressure Ratio (P_r) and Temperature ratio (T_r), the values of $\frac{p_x}{p_o}$ is estimated to be 0.875. The Mach number values M_x' and M_x'' for Motive and Suction flow respectively, are determined using the following equations

$$p_i = p_x\left[1+\frac{\gamma-1}{2}M_x'^2\right]^{\frac{\gamma}{\gamma-1}} \tag{8}$$

$$p_o = p_x\left[1+\frac{\gamma-1}{2}M_x''^2\right]^{\frac{\gamma}{\gamma-1}} \tag{9}$$

The above equation give the values of M_x' and M_x'' to be 4.8326 and 0.44096 respectively . In the regions of high speed flows, where the Mach number approaches infinity, it's convenient to work with characteristic Mach number, which is calculated using the equation

$$M^{*2} = \frac{\frac{\gamma+1}{2}M^2}{1+\frac{\gamma-1}{2}M^2} \tag{10}$$

The values of characteristic Mach numbers from above equation for both primary and suction flow are calculated to be 2.2231 and 0.47392 respectively. The mixed flow characteristic Mach number M_1^* is given by

$$M_1^* = \frac{M_x'^* + \omega M_x''^*\sqrt{T_r}}{\sqrt{(1+\omega)(1+\omega T_r)}} \tag{11}$$

The value of mixed stream characteristic Mach number given by this equation is 2.1804. Corresponding to this, the mixed stream Mach number M_1 calculated using

$$M^2 = \frac{M^{*2}}{\frac{\gamma+1}{2}-\frac{\gamma-1}{2}M^{*2}} \tag{12}$$

comes out to be 4.368. The mixed stream Mach number downstream of shock is calculated using Eq. 5 and it's value comes out to be 0.429. The static pressure at the exit of mixing chamber and downstream of Transverse shock is calculated from equation

$$P_b = P_a\left[\frac{2\gamma}{\gamma+1}M_a^2 - \frac{\gamma-1}{\gamma+1}\right] \tag{13}$$

After considering frictional losses, the static pressure downstream the shock is calculated to be ~70 Torr. The stagnation pressure downstream the shock is calculated using Eq. 4 and it's value comes out to be 80 Torr.

To estimate the geometry of the ejector system, the relevant area ratios can be estimated using following equations

$$\frac{a_x''}{a_x'} = \left(\frac{p_i}{p_o}\right)\frac{\left(\frac{p_x}{p_i}\right)^{\frac{1}{\gamma}}\sqrt{1-\left(\frac{p_x}{p_i}\right)^{\frac{\gamma-1}{\gamma}}}}{\left(\frac{p_x}{p_o}\right)^{\frac{1}{\gamma}}\sqrt{1-\left(\frac{p_x}{p_o}\right)^{\frac{\gamma-1}{\gamma}}}}\,\omega\sqrt{T_r} \tag{14}$$

$$\frac{a_t}{a_x'} = \frac{\left(\frac{p_x}{p_i}\right)^{\frac{1}{\gamma}}\sqrt{1-\left(\frac{p_x}{p_i}\right)^{\frac{\gamma-1}{\gamma}}}}{\left(\frac{2}{\gamma+1}\right)\sqrt{1-\frac{2}{\gamma+1}}} \tag{15}$$

$$\frac{a_t}{a_2} = \frac{\left(\frac{p_3}{p_o}\right)}{\left(\frac{p_i}{p_o}\right)}\sqrt{\frac{1}{(1+\omega)(1+\omega T_r)}}\,\frac{\left(\frac{p_2}{p_3}\right)^{\frac{1}{\gamma}}\sqrt{1-\left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}}}}{\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\sqrt{1-\frac{2}{\gamma+1}}} \tag{16}$$

Here $a_x'',\ a_x',\ a_t,\ a_2$ represents suction flow area, primary flow area, throat area and Constant area duct area respectively. The values of these ratios $\frac{a_x''}{a_x'}, \frac{a_t}{a_x'}, \frac{a_t}{a_2}$ are estimated to be 0.64013, 0.034834 and 0.026105 respectively. For the given suction flow area of 4416 mm^2, corresponding to a duct diameter of 75 mm, the values of area a_x', a_t, a_2 are calculated as 6898.0384 mm^2, 240.29 mm^2 and 9204.697 mm^2. The throat area calculated using the following isentropic relation for mass flow rate

$$A_d^* = \frac{m\sqrt{T_o}}{0.0404\,P_0\,C_d} \tag{17}$$

is 118.32 mm^2. Here m, T_o, P_o, C_d represent Motive mass flow rate, Motive gas Stagnation temperature, Motive gas stagnation pressure and Discharge coefficient. In order to achieve the near constant pressure mixing, one another parameter taken into account is the Convergence angle θ for the mixing chamber, which is calculated using the equation

$$tan\frac{\theta}{2} = \frac{\left(\sqrt{\frac{a_x}{a_2}}-1\right)}{\left(\frac{L_1+L_2}{D}\right)} \tag{18}$$

And it's value comes out to be 1.0376°. a_x here, represents the area of outer tube having diameter equals to 120.0513 mm^2. L_1 and L_2 represent the length of mixing chamber and constant area duct respectively. D is minimum diameter of mixing chamber, which is equal to the diameter of constant area chamber. From these area values, the value of motive gas diameter and constant duct diameter is calculated to be 93.7407 mm and 108.2854 mm respectively.

Hence, the first stage has been designed for recovered static pressure of 70 Torr and stagnation pressure of 80 Torr and an exit Mach number of ∼0.427.

4.2.2 Stage 2 :

The value of static pressure recovered from the first stage is 70 Torr. The practically achievable stagnation pressure from stage 1 is around 80% of the calculated value, which comes out to be around ~60 Torr. These serve as the input parameters for the second stage. The Mach number as achieved in the first stage is taken to be 0.429. The static pressure corresponding to a stagnation pressure of 60 Torr is calculated using

$$\frac{P_{ob}}{P_b} = \left[1 + \frac{\gamma - 1}{2} M_b^2\right]^{\frac{\gamma}{\gamma-1}} \tag{19}$$

M_b here is 0.429. The value of P_b comes out to be 52.9549 Torr and this gives the value of Pressure for constant pressure mixing of motive and suction flow, which is taken to be ~ 50 Torr. The mixed gas undergoes a transverse shock leading to rise in static pressure and decrease in the total pressure of mixed gas, which generally occurs at the inlet of subsonic diffuser.

For the given values of Entrainment ratio(ω), Pressure Ratio (P_r) and Temperature ratio (T_r), the values of $\frac{p_x}{p_o}$ is estimated to be 0.833. The Mach number values M_x' and $M_x^{"}$ for Motive and Suction flow respectively, are determined using the following equations

$$p_i = p_x \left[1 + \frac{\gamma - 1}{2} M_x'^2\right]^{\frac{\gamma}{\gamma-1}} \tag{20}$$

$$p_o = p_x \left[1 + \frac{\gamma - 1}{2} M_x^{"2}\right]^{\frac{\gamma}{\gamma-1}} \tag{21}$$

The above equation give the values of M_x' and $M_x^{"}$ to be 4.8999 and 0.51707 respectively . In the regions of high speed flows, where the Mach number approaches infinity, it's convenient to work with characteristic Mach number, which is calculated using the equation

$$M^{*2} = \frac{\frac{\gamma + 1}{2} M^2}{1 + \frac{\gamma - 1}{2} M^2} \tag{22}$$

The values of characteristic Mach numbers from above equation for both primary and suction flow are calculated to be 2.2284 and 0.55186 respectively. The mixed flow characteristic Mach number M_1^* is given by

$$M_1^* = \frac{M_x'^* + \omega M_x''^* \sqrt{T_r}}{\surd(1 + \omega)(1 + \omega T_r)} \tag{23}$$

The value of mixed stream characteristic Mach number given by this equation is 2.1796. Corresponding to this, the mixed stream Mach number M_1 calculated using

$$M^2 = \frac{M^{*2}}{\frac{\gamma+1}{2} - \frac{\gamma-1}{2} M^{*2}} \tag{24}$$

comes out to be 4.3602. The mixed stream Mach number downstream of shock is calculated using Eq. 5 and it's value comes out to be 0.42637. The static pressure at the exit of mixing chamber and downstream of Transverse shock is calculated from equation

$$P_b = P_a \left[\frac{2\gamma}{\gamma+1} M_a^2 - \frac{\gamma-1}{\gamma+1}\right] \tag{25}$$

The static pressure downstream the shock is calculated to be ~1100.6577 Torr, which is nearly 22 times the constant mixing pressure. The stagnation pressure downstream the shock is calculated using Eq. 4 and it's value comes out to be 1247.2056 Torr.

To estimate the geometry of the ejector system, the relevant area ratios can be estimated using following equations

$$\frac{a_x''}{a_x'} = \left(\frac{p_i}{p_o}\right) \frac{\left(\frac{p_x}{p_i}\right)^{\frac{1}{\gamma}} \sqrt{1 - \left(\frac{p_x}{p_i}\right)^{\frac{\gamma-1}{\gamma}}}}{\left(\frac{p_x}{p_o}\right)^{\frac{1}{\gamma}} \sqrt{1 - \left(\frac{p_x}{p_o}\right)^{\frac{\gamma-1}{\gamma}}}} \omega \sqrt{T_r} \tag{26}$$

$$\frac{a_t}{a_x'} = \frac{\left(\frac{p_x}{p_i}\right)^{\frac{1}{\gamma}} \sqrt{1 - \left(\frac{p_x}{p_i}\right)^{\frac{\gamma-1}{\gamma}}}}{\left(\frac{2}{\gamma+1}\right) \sqrt{1 - \frac{2}{\gamma+1}}} \tag{27}$$

$$\frac{a_t}{a_2} = \frac{\left(\frac{p_3}{p_o}\right)}{\left(\frac{p_i}{p_o}\right)} \sqrt{\frac{1}{(1+\omega)(1+\omega T_r)}} \frac{\left(\frac{p_2}{p_3}\right)^{\frac{1}{\gamma}} \sqrt{1 - \left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}}}}{\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \sqrt{1 - \frac{2}{\gamma+1}}} \tag{28}$$

Here $a_x'',\ a_x',\ a_t,\ a_2$ represents suction flow area, primary flow area, throat area and Constant area duct area respectively. The values of these ratios $\frac{a_x''}{a_x'}, \frac{a_t}{a_x'}, \frac{a_t}{a_2}$ are estimated to be 0.66716, 0.032982 and 0.023875 respectively. For the given suction flow area of 15386 mm^2,corresponding to a duct diameter of 140 mm, the values of area a_x' , a_t , a_2 are calculated as 23062.0356 mm^2, 760 mm^2 and 25456.2331 mm^2. The throat area calculated using the following isentropic relation for mass flow rate

$$A_d^* = \frac{m\sqrt{T_o}}{0.0404\, P_0\, C_d} \tag{29}$$

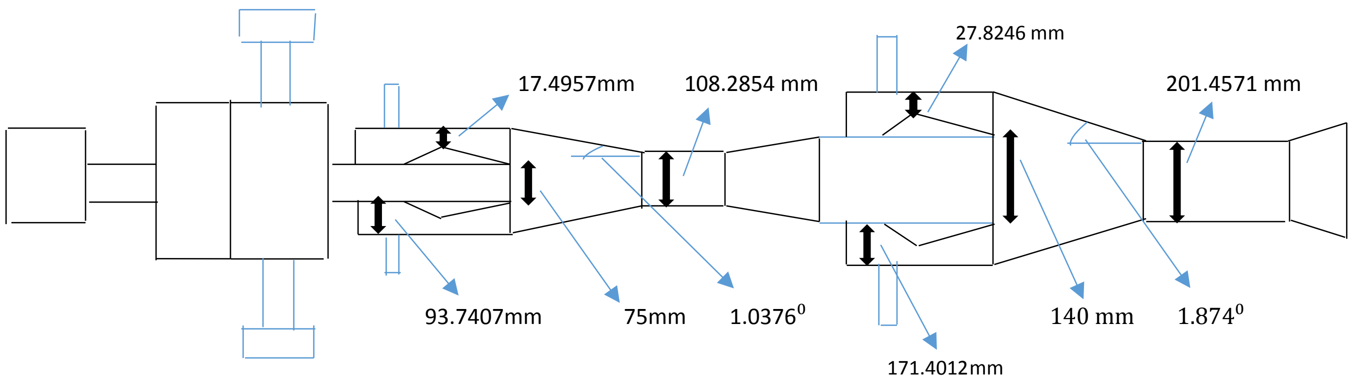
is 607 mm^2. Here m, T_o, P_o, C_d represent Motive mass flow rate, Motive gas Stagnation temperature, Motive gas stagnation pressure and Discharge coefficient. In order to achieve the near constant pressure mixing, one another parameter taken into account is the Convergence angle θ for the mixing chamber, which is calculated using the equation

$$tan\frac{\theta}{2} = \frac{\left(\sqrt{\frac{a_x}{a_2}} - 1\right)}{\left(\frac{L_1 + L_2}{D}\right)} \tag{30}$$

And it's value comes out to be 1.8739°. a_x here, represents the area of outer tube having diameter equals to 221.3106 mm^2. L_1 and L_2 represent the length of mixing chamber and constant area duct respectively. D is minimum diameter of mixing chamber, which is equal to the diameter of constant area chamber. From these area values, the value of motive gas diameter and constant duct diameter is calculated to be 171.4013 mm and 180.0787 mm respectively.

Hence, the first stage has been designed for recovered static pressure of 1100.6577 Torr and stagnation pressure of 1247.2056 Torr. And an exit Mach number of ~0.4263

- Calculated Results



4.3 CENTRAL MODE OF INJECTION

4.3.1 Stage 1 :

The central mode of ejection is different from the peripheral mode in the injection of motive and suction gas only. Here, Suction flow is supplied through the periphery and motive gas through a central nozzle placed inside an outer tube. Central mode is designed for a suction gas flow rate (M_s) of 3 gs^{-1} and mixing chamber pressure of ~ 4 Torr.

The dimensions of Suction flow supplier are 75 mm X 12 mm. The supersonic nozzle, to which the suction flow is transported, has a throat height of 10 mm. The test chamber Mach number is 1.5. The subsonic flow, from a chamber prior to nozzle, has a pressure of 16 Torr. The flow from nozzle with an exit dimension of (75 X 24)mm^2 expands into a duct of diameter 75mm. The Mach number after an Isentropic expansion is calculated as

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$\frac{A}{A^*}$ is the ratio of duct area to nozzle exit area which is calculated to be 2.45 from the above given dimensions. The area ratio gives the value of Mach number equal to 2.6198. The pseudo throat area A_{pt}^* can be calculated using the equation

$$p_p A^* = p_o A^*_{pt}$$

Where p_p is the plenum pressure, A^* is the throat area and p_o is the Stagnation pressure of suction gas, which is calculated from the equation

$$\frac{p_o}{p} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}$$

Here, M in the above equation is cavity Mach number equal to 1.5. The Stagnation pressure calculated from above equation comes out to be 11.0131 Torr. Because the flow is unstable, oblique shocks occur in the duct and make the flow subsonic. Therefore, the Mach number M_b downstream the shock is given by

$$M_b^2 = \frac{\frac{2}{\gamma - 1} + M_a^2}{\frac{2\gamma}{\gamma - 1} M_a^2 - 1}$$

M_a is the Mach number upstream the shock. Subscripts a and b represents upstream and downstream conditions respectively. Similarly, the stagnation pressure on the downstream can be calculated as

$$\frac{P_{ob}}{P_{oa}} = \left[\frac{1 + \frac{\gamma - 1}{2} M_b^2}{1 + \frac{\gamma - 1}{2} M_a^2}\right]^{\frac{\gamma}{\gamma-1}} \left[1 + \frac{2\gamma}{\gamma + 1}(M_a^2 - 1)\right]$$

Here, P_{oa} equals to 11 Torr is stagnation pressure upstream, which gives downstream stagnation pressure P_{ob} equals to 4.9858 Torr. The static pressure P_b downstream the shock can be calculated from this stagnation pressure from the equation

$$\frac{P_{ob}}{P_b} = \left[1 + \frac{\gamma - 1}{2} M_b^2\right]^{\frac{\gamma}{\gamma-1}}$$

This value of P_b comes out to be 4.197 Torr and this gives the value of Pressure for constant pressure mixing of motive and suction flow, which is taken to be ~ 4 Torr. The mixed gas undergoes a transverse shock leading to rise in static pressure and decrease in the total pressure of mixed gas, which generally occurs at the inlet of subsonic diffuser.

The motive gas flow rate can be calculated using the formula

$$M_m = \frac{M_s}{\omega}$$

where ω is the entrainment ratio given for central mode. The value of Mm from the above equation comes out to be 120 g/s. Motive gas stagnation pressure (Pm) can be calculated using

$$P_m = P_s \, X \, P_r$$

where Pr is the pressure ratio. The value of Pm is calculated to be 1496 Torr. The Mach number values M_x' and M_x'' for Motive and Suction flow respectively, are determined using the following equations

$$P_m = P_x \left[1 + \frac{\gamma - 1}{2} M_x'^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$P_s = P_x\left[1 + \frac{\gamma - 1}{2}M_x^{"2}\right]^{\frac{\gamma}{\gamma-1}}$$

Here, P_x is the constant mixing pressure. The above equation give the values of M_x' and $M_x^{"}$ to be 4.71 and 0.5737 respectively . In the regions of high speed flows, where the Mach number approaches infinity, it's convenient to work with characteristic Mach number, which is calculated using the equation

$$M^{*2} = \frac{\frac{\gamma + 1}{2}M^2}{1 + \frac{\gamma - 1}{2}M^2}$$

The values of characteristic Mach numbers from above equation for both primary and suction flow are calculated to be 2.21 and 0.605 respectively. The mixed flow characteristic Mach number M_1^* is given by

$$M_1^* = \frac{M_x'^* + \omega M_x''^*\sqrt{T_r}}{\sqrt{(1+\omega)(1+\omega T_r)}}$$

The value of mixed stream characteristic Mach number given by this equation is 2.16. Corresponding to this, the mixed stream Mach number M_1 calculated using

$$M^2 = \frac{M^{*2}}{\frac{\gamma + 1}{2} - \frac{\gamma - 1}{2}M^{*2}}$$

comes out to be 4.24. The mixed stream Mach number downstream of shock is calculated using Eq. 5 and it's value comes out to be 0.42888. The static pressure at the exit of mixing chamber and downstream of Transverse shock is calculated from equation

$$P_b = P_a\left[\frac{2\gamma}{\gamma + 1}M_a^2 - \frac{\gamma - 1}{\gamma + 1}\right]$$

After considering frictional losses, the static pressure downstream the shock is calculated to be ~70 Torr. The stagnation pressure downstream the shock is calculated using Eq. 4 and it's value comes out to be 80 Torr.

To estimate the geometry of the ejector system, the relevant area ratios can be estimated using following equations

$$\frac{a_x''}{a_x'} = \left(\frac{p_i}{p_o}\right)\frac{\left(\frac{p_x}{p_i}\right)^{\frac{1}{\gamma}}\sqrt{1 - \left(\frac{p_x}{p_i}\right)^{\frac{\gamma-1}{\gamma}}}}{\left(\frac{p_x}{p_o}\right)^{\frac{1}{\gamma}}\sqrt{1 - \left(\frac{p_x}{p_o}\right)^{\frac{\gamma-1}{\gamma}}}}\,\omega\sqrt{T_r}$$

$$\frac{a_t}{a_x'} = \frac{\left(\frac{p_x}{p_i}\right)^{\frac{1}{\gamma}}\sqrt{1 - \left(\frac{p_x}{p_i}\right)^{\frac{\gamma-1}{\gamma}}}}{\left(\frac{2}{\gamma + 1}\right)\sqrt{1 - \frac{2}{\gamma + 1}}}$$

$$\frac{a_t}{a_2} = \frac{\left(\frac{p_3}{p_o}\right)}{\left(\frac{p_i}{p_o}\right)} \sqrt{\frac{1}{(1+\omega)(1+\omega T_r)}} \frac{\left(\frac{p_2}{p_3}\right)^{\frac{1}{\gamma}} \sqrt{1-\left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}}}}{\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \sqrt{1-\frac{2}{\gamma+1}}}$$

Here a_x'', a_x', a_t, a_2 represents suction flow area, primary flow area, throat area and Constant area duct area respectively. The values of these ratios $\frac{a_x''}{a_x'}$, $\frac{a_t}{a_x'}$, $\frac{a_t}{a_2}$ are estimated to be 0.538, 0.0385 and 0.0317 respectively. For the given suction flow area of 4416 mm^2,corresponding to a duct diameter of 75 mm, the values of area a_x' , a_t , a_2 are calculated as 8207 mm^2, 316.66 mm^2 and 10476 mm^2. The throat area calculated using the following isentropic relation for mass flow rate

$$A_d^* = \frac{m\sqrt{T_o}}{0.0404\, P_0\, C_d}$$

is 287.24 mm^2. Here m, T_o, P_o, C_d represent Motive mass flow rate, Motive gas Stagnation temperature, Motive gas stagnation pressure and Discharge coefficient. In order to achieve the near constant pressure mixing, one another parameter taken into account is the Convergence angle θ for the mixing chamber, which is calculated using the equation

$$tan\frac{\theta}{2} = \frac{\left(\sqrt{\frac{a_x}{a_2}}-1\right)}{\left(\frac{L_1+L_2}{D}\right)}$$

And it’s value comes out to be 0.0079605°. a_x here, represents the area of outer tube having diameter equal to 127.0621 mm^2. L_1 and L_2 represent the length of mixing chamber and constant area duct respectively and their values are 695.6023 each. D is minimum diameter of mixing chamber, which is equal to the diameter of constant area chamber. From these area values, the value of motive gas diameter and constant duct diameter is calculated to be 102.566 mm and 115.9337 mm respectively. The diameter of throat is calculated from throat area to be 20.1267 mm.
Hence, the first stage of central mode has been designed for recovered static pressure of 70 Torr and stagnation pressure of 80 Torr. And an exit Mach number of ∼0.42888.

4.3.2 Stage 2

The value of static pressure recovered from the first stage is 70 Torr. The practically achievable stagnation pressure from stage 1 is around 80% of the calculated value, which comes out to be around ~60 Torr. These serve as the input parameters for the second stage. The Mach number as achieved in the first stage is taken to be 0.429. The static pressure corresponding to a stagnation pressure of 60 Torr is calculated using

$$\frac{P_{ob}}{P_b} = \left[1+\frac{\gamma-1}{2}M_b^2\right]^{\frac{\gamma}{\gamma-1}}$$

M_b here is 0.429. The value of P_b comes out to be 50.5842 Torr and this gives the value of Pressure for constant pressure mixing of motive and suction flow, which is taken to be ~ 50 Torr. The mixed gas undergoes a transverse shock leading to rise in static pressure and decrease in the total pressure of mixed gas, which generally occurs at the inlet of subsonic diffuser.

For the given values of Entrainment ratio(ω), Pressure Ratio (P_r) and Temperature ratio (T_r), the values of $\frac{p_x}{p_o}$ is estimated to be 0.83338. The Mach number values M_x' and M_x'' for Motive and Suction flow respectively, are determined using the following equations

$$p_i = p_x \left[1 + \frac{\gamma - 1}{2} M_x'^2\right]^{\frac{\gamma}{\gamma - 1}}$$

$$p_o = p_x \left[1 + \frac{\gamma - 1}{2} M_x''^2\right]^{\frac{\gamma}{\gamma - 1}}$$

The above equation give the values of M_x' and M_x'' to be 4.8998 and 0.517 respectively . In the regions of high speed flows, where the Mach number approaches infinity, it's convenient to work with characteristic Mach number, which is calculated using the equation

$$M^{*2} = \frac{\frac{\gamma + 1}{2} M^2}{1 + \frac{\gamma - 1}{2} M^2}$$

The values of characteristic Mach numbers from above equation for both primary and suction flow are calculated to be 2.2284 and 0.55178 respectively. The mixed flow characteristic Mach number M_1^* is given by

$$M_1^* = \frac{M_x'^* + \omega M_x''^* \sqrt{T_r}}{\sqrt{(1 + \omega)(1 + \omega T_r)}}$$

The value of mixed stream characteristic Mach number given by this equation is 2.1716. Corresponding to this, the mixed stream Mach number M_1 calculated using

$$M^2 = \frac{M^{*2}}{\frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} M^{*2}}$$

comes out to be 4.2853. The mixed stream Mach number downstream of shock is calculated using Eq. 5 and it's value comes out to be 0.42799. The static pressure at the exit of mixing chamber and downstream of Transverse shock is calculated from equation

$$P_b = P_a \left[\frac{2\gamma}{\gamma + 1} M_a^2 - \frac{\gamma - 1}{\gamma + 1}\right]$$

The static pressure downstream the shock is calculated to be ~900 Torr, which is nearly 22 times the constant mixing pressure. The stagnation pressure downstream the shock is calculated using Eq. 4 and it's value comes out to be 1014.2487 Torr.

To estimate the geometry of the ejector system, the relevant area ratios can be estimated using following equations

$$\frac{a_x''}{a_x'} = \left(\frac{p_i}{p_o}\right) \frac{\left(\frac{p_x}{p_i}\right)^{\frac{1}{\gamma}} \sqrt{1 - \left(\frac{p_x}{p_i}\right)^{\frac{\gamma-1}{\gamma}}}}{\left(\frac{p_x}{p_o}\right)^{\frac{1}{\gamma}} \sqrt{1 - \left(\frac{p_x}{p_o}\right)^{\frac{\gamma-1}{\gamma}}}} \omega\sqrt{T_r}$$

$$\frac{a_t}{a_x'} = \frac{\left(\frac{p_x}{p_i}\right)^{\frac{1}{\gamma}} \sqrt{1 - \left(\frac{p_x}{p_i}\right)^{\frac{\gamma-1}{\gamma}}}}{\left(\frac{2}{\gamma+1}\right) \sqrt{1 - \frac{2}{\gamma+1}}}$$

$$\frac{a_t}{a_2} = \frac{\left(\frac{p_3}{p_o}\right)}{\left(\frac{p_i}{p_o}\right)} \sqrt{\frac{1}{(1+\omega)(1+\omega T_r)}} \frac{\left(\frac{p_2}{p_3}\right)^{\frac{1}{\gamma}} \sqrt{1 - \left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}}}}{\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \sqrt{1 - \frac{2}{\gamma+1}}}$$

Here $a_x'',\ a_x',\ a_t,\ a_2$ represents suction flow area, primary flow area, throat area and Constant area duct area respectively. The values of these ratios $\frac{a_x''}{a_x'}, \frac{a_t}{a_x'}, \frac{a_t}{a_2}$ are estimated to be 0.76951, 0.032983 and 0.023719 respectively. For the given suction flow area of 15386 mm^2,corresponding to a duct diameter of 140 mm, the values of area a_x', a_t, a_2 are calculated as 19994.6486 mm^2, 659.4796 mm^2 and 27803.5702 mm^2. The throat area calculated using the following isentropic relation for mass flow rate

$$A_d^* = \frac{m\sqrt{T_o}}{0.0404\, P_0\, C_d}$$

is 607.5967 mm^2. Here m, T_o, P_o, C_d represent Motive mass flow rate, Motive gas Stagnation temperature, Motive gas stagnation pressure and Discharge coefficient. In order to achieve the near constant pressure mixing, one another parameter taken into account is the Convergence angle θ for the mixing chamber, which is calculated using the equation

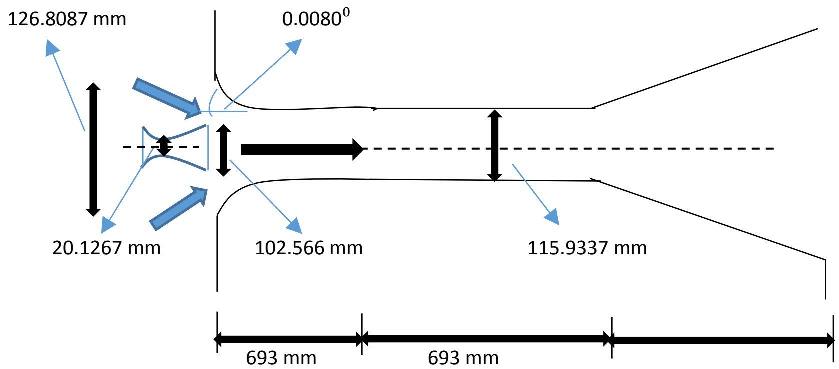
$$tan\frac{\theta}{2} = \frac{\left(\sqrt{\frac{a_x}{a_2}} - 1\right)}{\left(\frac{L_1 + L_2}{D}\right)}$$

And it's value comes out to be 0.0055696°. a_x here, represents the area of outer tube having diameter equals to 212.2991 mm^2. L_1 and L_2 represent the length of mixing chamber and constant area duct respectively. D is minimum diameter of mixing chamber, which is equal to the diameter of constant area chamber. From these area values, the value of motive gas diameter and constant duct diameter is calculated to be 159.596 mm and 188.1982 mm respectively.

Hence, the first stage has been designed for recovered static pressure of 900 Torr and stagnation pressure of 1014.2487 Torr. And an exit Mach number of $\sim$0.42799

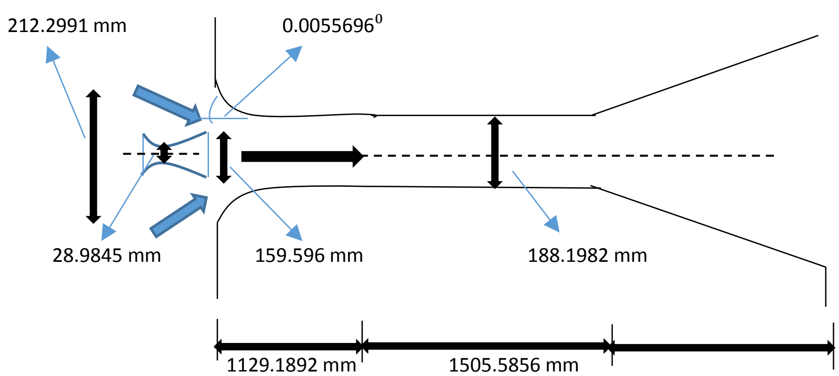
Stage 1 :

- Calculated Results



Stage 2 :

- Calculated Values



5. RESULTS AND DISCUSSIONS

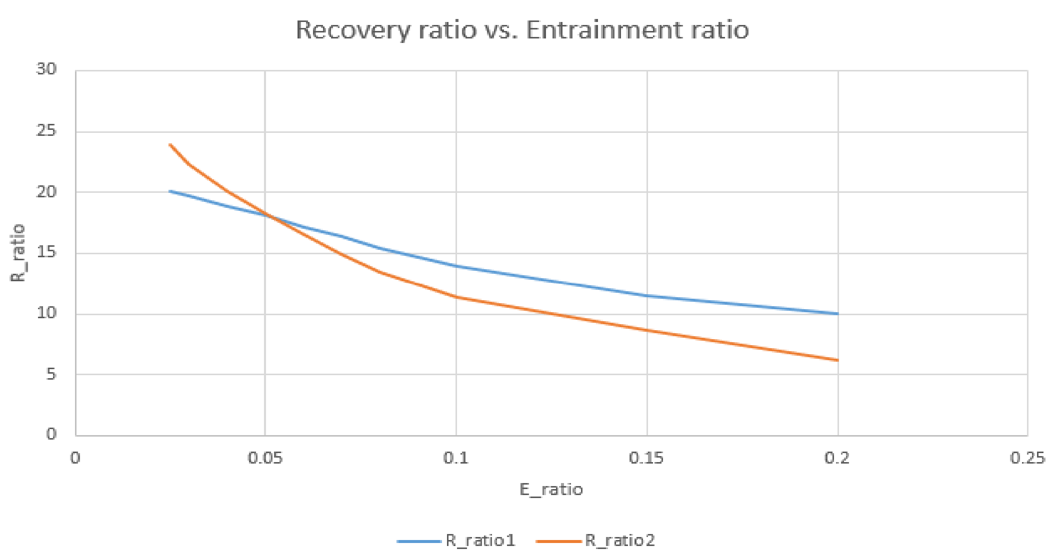
For the peripheral mode of injection, the flow at the exit of first stage acquires a static pressure of 70 Torr and a stagnation pressure of about 80 Torr. Though the virtually achieved stagnation pressure is observed to be around 80 percent of the calculated value. Hence the Stagnation pressure at the exit of first stage is considered to be around 60 Torr. The Mach number at the exit of first stage is found to be 0.4. Then the output of first stage goes to the second stage of ejector as input. The whole process of injection and commixing is reiterated. The Static pressure achieved at the terminus of the second stage is around 1000 Torr. The corresponding stagnation pressure is around 1100 Torr. Though there are discrepancies in the calculated and observed results which have been shown in the diagrams presented in calculations.

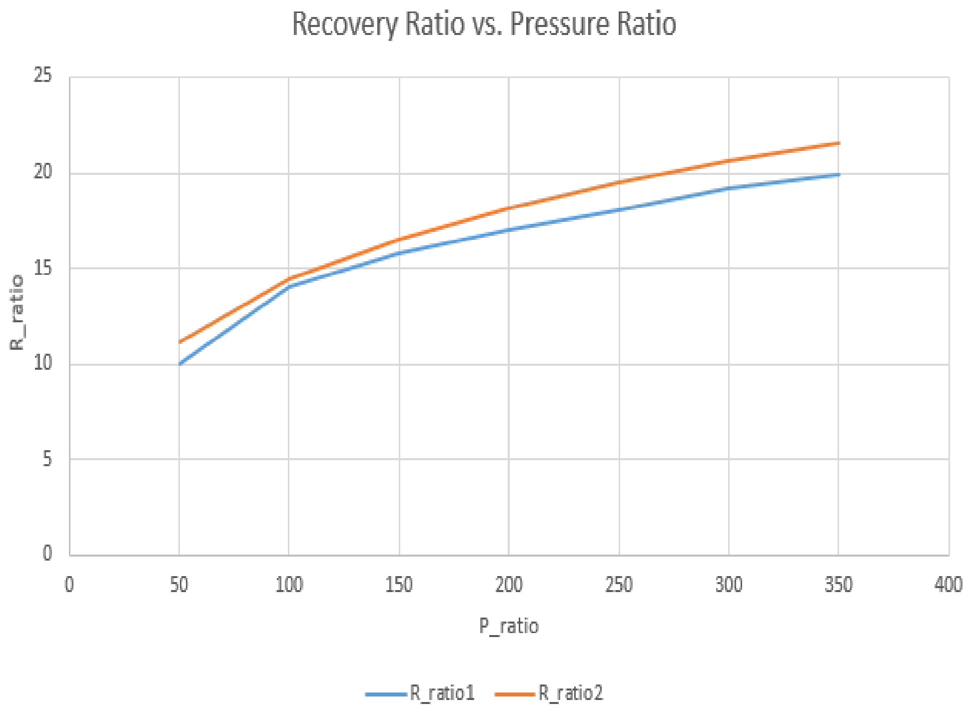
For the central mode of injection, the flow at the exit of first stage acquires a static pressure of around 70 Torr, where the observed value is 63 Torr. The corresponding stagnation pressure is 80 Torr and the observed value is around 80 percent of the calculated value. At the cessation of second stage, the static pressure is around 900 Torr and the stagnation pressure is around 100 Torr.

It has been observed that the calculated values for central and peripheral mode match to some extent, though the values of some input parameters are different for both modes. This might be the reason of discrepancy in the calculated values for both modes. One constraint is that sundry losses occurring inside ejectors i.e. Frictional, Shear and commixing losses can't be taken into account and that's why certain postulation for some percent decrement in the calculated results have to be made in order to consider those losses. Due to mismatch in the calculated and experimental results, the observed pressure instauration is different than what is achieved virtually.

5.1 Recovery Ratio

The variation in the recovery ratio with changing Pressure ratio and entrainment ratio has been studied. A MATLAB code is developed to study the variations and results are plotted. It is observed that recuperation ratio decreases with the incrementation in Entrainment ratio and increment with the incrementation in Pressure ratio.





The above graphs present the Recovery ratio variations with transmuting Pressure and Entrainment ratios. Here, R_ratio represents the Recovery ratio, P_ratio represents Pressure ratio and E_ratio represents Entrainment ratio.

The two plots in each graph represent a comparison between values of Recovery ratio from a standard literature and the values of Recovery ratio calculated in the present work.

R_ratio1 = Recuperation ratio plot of values from standard literature

R_ratio2 = Plot of values calculated in present work

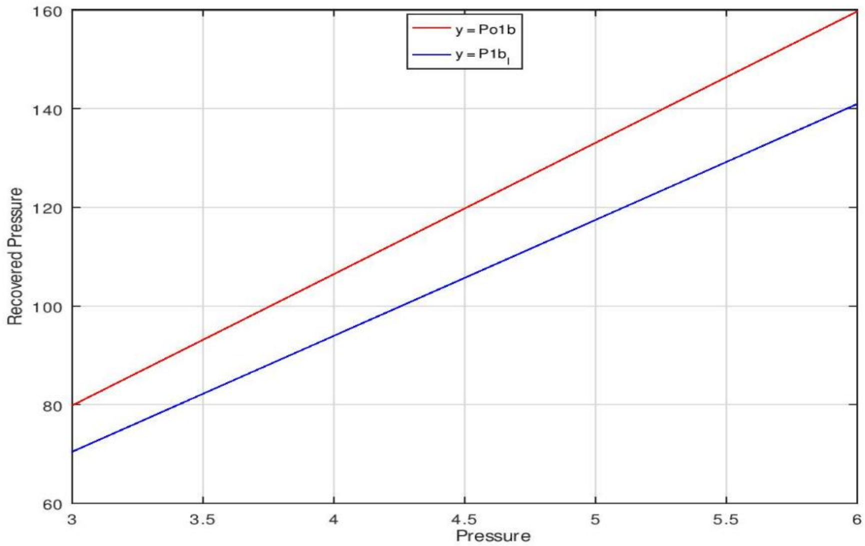
It can be deciphered from the above graphs A 5-10% difference in the values of two plots in each has been observed, which might be due to the difference in the values of Mach numbers calculated . Ergo, for a higher pressure recuperation, the ratio of motive to suction gas stagnation pressure should be kept higher. And the value of entrainment ratio should be as low as possible.

Further, it is observed that if the test chamber conditions are transmuted, then values of static and stagnation pressure achieved at the final stage supplementally change.

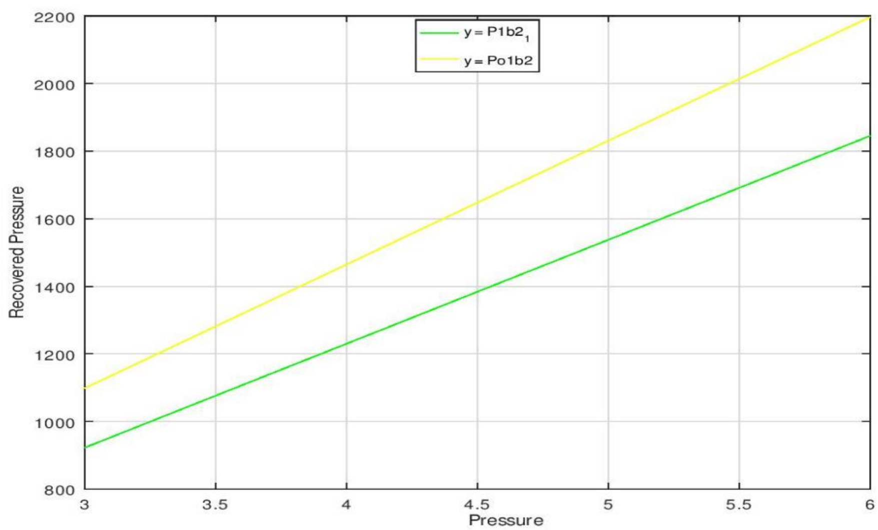
5.2 Recovered pressure vs. Test chamber pressure

Here, the vicissitude in pressure instauration for different values of cavity pressure has been studied and the results have been plotted for both the stages of the ejector. It is observed that as the values of cavity parameters are incremented, the higher pressure recuperation is achieved at the final stage.

- **Stage 1**



- **Stage 2**



The above two graphs show the elevation in Recuperated pressure with incrementing test chamber pressure keeping the Mach number constant. The first shows the variation in recuperated pressure for stage 1 of ejector design and second graph shows the variation in recuperated pressure for stage 2.

Polb = Stagnation pressure at the terminus of stage 1

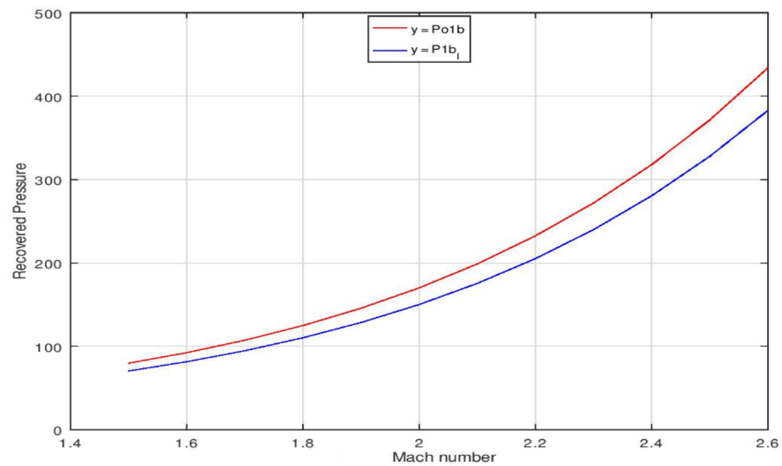
P1b1 = Static pressure at the terminus of stage 1

P1b2 = Static pressure at the terminus of stage 2

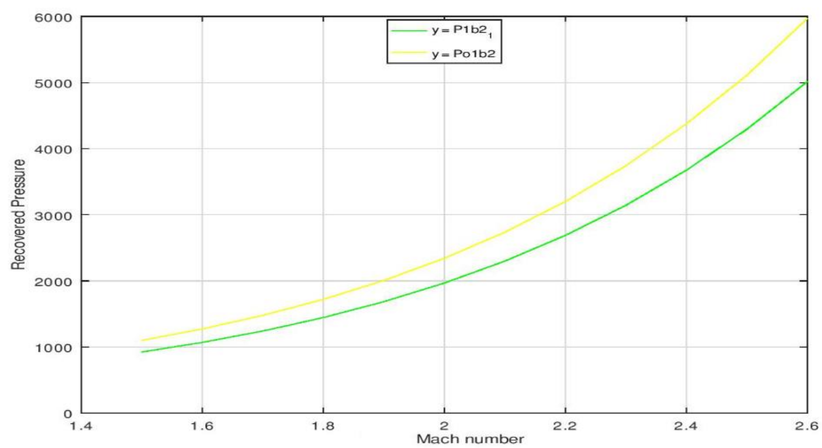
Polb2 = Stagnation pressure at the terminus of stage 2

- **Recovered Pressure vs. Test chamber Mach number**

  - **Stage 1**



  - **Stage 2**



Hence from the above graphs, it's discernible that if test chamber Mach number is incremented and pressure is kept constant this time, then recuperated pressure at the exit increases. Though the graph in case of Mach number variation is not linear as in the case of test chamber pressure variation. This shows that Recuperated pressure at the exit increases more rapidly if cavity pressure is incremented.

Withal, the geometry or design of the ejector system additionally, changes with transmuting test chamber conditions. Hence, we can look forward for more compact designs of ejectors by transmuting test chamber conditions. Additionally, from the calculated values, it can be observed that recuperated pressure and convergence angle increases much rapidly with an increment of half only in test chamber Mach number as compared to incrementing cavity pressure. Consequently, if we optate a higher increment in recuperated pressure, then it can be achieved by a slightest transmutation in Mach number. It withal shows how high convergence of commixing chamber is required to cope with the Mach number of flow emanating from cavity.

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APPENDIX

➢ PROGRAM

A MATLAB code has been developed for the design of two stage ejector system for both Peripheral and central mode. This includes calculation of areas and diameters of various parts of ejector. The code for the same is given below

A.1 Peripheral Mode

%% study of pressure recovery
%% study of pressure recovery

disp('For first stage : '); fprintf('\n');

%% Writing Cavity conditions and given values
pc = 3; ga = 1.4; mc = 1.5; At = 10; Pp = 16; L1 = 6;
re = 18; Cd = 0.9; mf = 0.12; To = 400; Ds = 75; L2 = 6;
en = 0.025; Pr = 380; Tr = 1; Pxo = 0.875;

%% Stage 1

%% Calculating Mach number
Ma = fzero(@(M) flowisentropic(M), [1; 4]);
disp(['mach number is Ma :', num2str(Ma)]);

%% Calculating stagnation pressure

$$Po = pc\left[1 + \frac{ga-1}{2}mc^2\right]^{\frac{ga}{ga-1}}$$

disp(['Stagnation pressure is : ',num2str(Po)]);

%% Calculating Pseudo throat area
$A_{pt} = (P_p A_t)/P_o$;
disp(['Pseudo throat height is : ',num2str(A_{pt})]);

%% Calculating Mach number downstream of shock

$$M_b = sqrt\left[\frac{\frac{2}{ga-1} + M_a^2}{\frac{2(ga)}{ga-1}M_a^2 - 1}\right];$$

disp(['Mach number downstream is : ',num2str(M_b)]);

%% Stagnation pressure downstream of shock

$$P_{ob} = P_o\left[1 + \frac{2(ga)}{ga+1}(M_a^2 - 1)\right]\left[\frac{1+\frac{ga-1}{2}M_b^2}{1+\frac{ga-1}{2}M_a^2}\right]^{\frac{ga}{ga-1}};$$

disp(['Stagnation pressure downstream is : ',num2str(P_{ob})]);

%% Calculating static pressure downstream

$$P_b = \frac{P_{ob}}{\left[1+\frac{ga-1}{2}M_b^2\right]^{\frac{ga}{ga-1}}};$$

disp(['Static pressure downstream is : ',num2str(P_b)]);

$P_x = (P_b * 95)/100;$
$P_{ix} = \frac{P_r}{P_{xo}};$
$P_{xi} = \frac{1}{P_{ix}};$
$P_{3o} = re * (P_{xo});$

%% Calculating Mach number of Suction and Motive gas at x
$M_{x1} = sqrt\left(\left((P_{ix}{}^{\frac{ga-1}{ga}} - 1\right) * \left(2/_{ga-1}\right)\right);$
disp(['Motive Gas Mach number at x is : ',num2str(M_{x1}1)]);

$M_{x2} = sqrt\left(\left((\frac{1}{P_{xo}})^{\frac{ga-1}{ga}} - 1\right) * \left(2/_{ga-1}\right)\right);$
disp(['Suction Gas Mach number at x is : ',num2str(M_{x2})]);

%% Calculating Characteristic Mach number for both gases
$M_{x1\_c} = sqrt\left[\frac{\frac{ga+1}{2}M_{x1}{}^2}{1+\frac{ga-1}{2}M_{x1}{}^2}\right];$
disp(['Motive Gas characteristic Mach number at x is : ',num2str(M_{x1_c})]);

$M_{x2\_c} = sqrt\left[\frac{\frac{ga+1}{2}M_{x2}{}^2}{1+\frac{ga-1}{2}M_{x2}{}^2}\right];$
disp(['Suction Gas characteristic Mach number at x is : ',num2str(M_{x2_c})]);

%% Calculating characteristic Mach number of mixed stream from individual mach numbers
$M_{1c} = {(M_{x1_c} + en * M_{x2_c} * sqrt(T_r))}\Big/{sqrt((1+en)*(1+en*T_r))};$
disp(['Mixed Stream characteristic Mach number : ',num2str(M_{1c})]);

%% Calculating Mixed Stream Mach number
$M_1 = sqrt\left[\frac{2M_{1c}{}^2}{(ga+1)-(ga-1)M_{1c}{}^2}\right];$
disp(['Mixed Stream Mach number : ',num2str(M_1)]);

%% Calculating Mach number downstream of Transverse shock
$M_{1b} = sqrt\left[\frac{\frac{2}{ga-1}+M_1^2}{\frac{2(ga)}{ga-1}M_1^2-1}\right];$
M1b = sqrt((2/(ga-1) + M1^2)/(((2*ga)/(ga-1))*(M1^2) - 1));
disp(['Mach number downstream of Transverse shock : ',num2str(M_{1b})]);

%% Calculating Static pressure at the exit
$P_{1b} = P_x\left[\frac{2(ga)}{ga+1}M_1^2 - \frac{ga-1}{ga+1}\right];$
disp(['Static pressure downstream of Transverse shock : ',num2str(P_{1b})]);

$P_{1b\_1} = \frac{P_{1b}*80}{100};$ % After considering Frictional loss
disp(['After considering frictional losses, Static pressure at exit is :', num2str(P_{1b_1})]);

%% Calculating Stagnation pressure downstream of Transverse shock
$P_{o1b} = P_{1b\_1}\left[1 + \frac{ga-1}{2}M_{1b}{}^2\right]^{\frac{ga}{ga-1}};$
disp(['Stagnation pressure downstream of Transverse shock : ',num2str(P_{o1b})]);

%%Calculating suction to motive gas area ratio

$A_{sm} = (P_r)\frac{(P_{xi})^{\frac{1}{ga}}\sqrt{1-(P_{xi})^{\frac{ga-1}{ga}}}}{(P_{xo})^{\frac{1}{ga}}\sqrt{1-(P_{xo})^{\frac{ga-1}{ga}}}}\ en * \sqrt{T_r}$;

disp(['Suction to motive gas area ratio :',num2str(A_{sm})]);

%% Calculating Throat to motive gas area ratio

$A_{tm} = \frac{\left(\frac{1}{P_{ix}}\right)^{\frac{1}{ga}}\sqrt{1-\left(\frac{1}{P_{ix}}\right)^{\frac{ga-1}{ga}}}}{\left(\frac{2}{ga+1}\right)\sqrt{1-\frac{2}{ga+1}}}$;

disp(['Throat to motive gas area ratio :',num2str(A_{tm})]);

%% Calculating throat to constant area duct ratio

$P_{23} = \frac{P_{1b\_1}}{P_{o1b}}$;

$A_{t2} = \frac{(P_{3o})}{(P_r)}\sqrt{\frac{1}{(1+en)(1+en*T_r)}}\frac{(P_{23})^{\frac{1}{ga}}\sqrt{1-\left(\frac{p_2}{p_3}\right)^{\frac{ga-1}{ga}}}}{\left(\frac{2}{ga+1}\right)^{\frac{1}{ga-1}}\sqrt{1-\frac{2}{ga+1}}}$;

disp(['Throat to constant duct area ratio :',num2str(A_{t2})]);

%% Calulating throat area at specified conditions

$P_i = (P_r * P_o) * 133$;

$A_t = \frac{mf\sqrt{T_o}}{0.0404 * P_i * C_d}$;

$A_t = A_t * 10^6$;

disp(['Throat area at specified conditions :',num2str(A_t)]);

%% Calculating motive gas area

$A_s = \frac{3.14 * {D_s}^2}{4}$;

$A_m = \frac{A_s}{A_{sm}}$;

disp(['motive gas area :',num2str(A_m)]);

%% Calculating area of constant duct

$A_{t1} = A_{tm} * A_m$;

disp(['Throat area :',num2str(A_{t1})]);

$A_2 = A_{t1}/A_{t2}$;

disp(['constant duct area :',num2str(A_2)]);

%% Calculating motive gas diameter

$D_m = \sqrt{\frac{4*A_m}{3.14}}$;

disp(['motive gas diameter :',num2str(D_m)]);

%% Calculating constant duct diameter

$D_c = \sqrt{\frac{4*A_2}{3.14}}$;

disp(['constant duct diameter :',num2str(D_c)]);

%% Calculating throat diameter

$D_t = \sqrt{\frac{4*A_{t1}}{3.14}}$;
disp(['throat diameter :',num2str(D_t)]);

%% Calculating diameter of outer tube
$A_w = A_s + A_m$;
$D_w = \sqrt{\frac{4*A_w}{3.14}}$;
disp(['outer tube diameter :',num2str(D_w)]);

%% Calculating convergence angle
$thd = 2 * atand\left(\frac{\left(\sqrt{\frac{A_w}{A_2}}-1\right)}{\left(\frac{L_1+L_2}{D}\right)}\right)$;
disp(['convergence angle :',num2str(thd)]); fprintf('\n');

disp('Stage 2: '); fprintf('\n');

%% Writing given conditions

re = 18; ga = 1.4; mf = 4; Ti = 300; L1 = 6;
Cd = 0.9; Ds = 140; P_o = (P_{o1b}*75)/100; Mb = M1b; L2 = 8;
en = 0.030; Pr = 392; Tr = 1;

disp(['Mach number at the inlet Mb is : ',num2str(M_b)]);
disp(['Practically achievable Stagnation pressure at inlet P0 is : ',num2str(P_o)]);

%% Calculating static pressure downstream
$P_b = \frac{P_o}{\left[1+\frac{ga-1}{2}M_b^2\right]^{\frac{ga}{ga-1}}}$;
disp(['Static pressure downstream is : ',num2str(P_b)]);

$P_x = (P_b * 95)/100$;
$P_{ix} = \frac{P_r}{P_{xo}}$;
$P_{xo} = \frac{P_x}{P_o}$;
$P_{3o} = re * (P_{xo})$;

%% Calculating Mach number of Suction and Motive gas at x
$M_{x1} = sqrt\left(\left((P_{ix}{}^{\frac{ga-1}{ga}} - 1\right) * \left({}^{2}/_{ga-1}\right)\right)$;
disp(['Motive Gas Mach number at x : ',num2str(M_{x1})]);

$M_{x2} = sqrt\left(\left((\frac{1}{P_{xo}})^{\frac{ga-1}{ga}} - 1\right) * \left({}^{2}/_{ga-1}\right)\right)$;
disp(['Suction Gas Mach number at x is : ',num2str(M_{x2})]);

%% Calculating Characteristic Mach number for both gases
$M_{x1\_c} = sqrt\left[\frac{\frac{ga+1}{2}{M_{x1}}^2}{1+\frac{ga-1}{2}{M_{x1}}^2}\right]$;
disp(['Motive Gas characteristic Mach number at x : ',num2str(M_{x1_c})]);

$M_{x2\_c} = sqrt\left[\frac{\frac{ga+1}{2}{M_{x2}}^2}{1+\frac{ga-1}{2}{M_{x2}}^2}\right];$
disp(['Suction Gas characteristic Mach number at x : ',num2str(M_{x2_c})]);

%% Calculating characteristic Mach number of mixed stream from individual mach numbers
$M_{1c} = {(M_{x1\_c} + en * M_{x2\_c} * sqrt(T_r))}\Big/{sqrt((1+en)*(1+en*T_r))}\ ;$
disp(['Mixed Stream characteristic Mach number : ',num2str(M_{1c})]);

%% Calculating Mixed Stream Mach number
$M_1 = sqrt\left[\frac{2{M_{1c}}^2}{(ga+1)-(ga-1){M_{1c}}^2}\right];$
disp(['Mixed Stream Mach number : ',num2str(M_1)]);

%% Calculating Mach number downstream of Transverse shock
$M_{1b} = sqrt\left[\frac{\frac{2}{ga-1}+M_1^2}{\frac{2(ga)}{ga-1}M_1^2-1}\right];$
disp(['Mach number downstream of Transverse shock : ',num2str(M_{1b})]);

%% Calculating Static pressure at the exit
$P_{1b} = P_x\left[\frac{2(ga)}{ga+1}M_1^2 - \frac{ga-1}{ga+1}\right];$
disp(['Static pressure downstream of Transverse shock : ',num2str(P_{1b})]);

$P_{1b\_1} = \frac{P_{1b}*84}{100};$ % After considering Frictional loss
disp(['Static pressure after considering frictional losses : ',num2str(P_{1b_1})]);

%% Calculating Stagnation pressure downstream of Transverse shock
$P_{o1b} = P_{1b\_1}\left[1+\frac{ga-1}{2}{M_{1b}}^2\right]^{\frac{ga}{ga-1}};$
disp(['Stagnation pressure downstream of Transverse shock : ',num2str(P_{o1b})]);

%%Calculating suction to motive gas area ratio
$A_{sm} = (P_r)\frac{(P_{xi})^{\frac{1}{ga}}\sqrt{1-(P_{xi})^{\frac{ga-1}{ga}}}}{(P_{xo})^{\frac{1}{ga}}\sqrt{1-(P_{xo})^{\frac{ga-1}{ga}}}}\ en*\sqrt{T_r}\ ;$
disp(['Suction to motive gas area ratio :',num2str(A_{sm})]);

%% Calculating Throat to motive gas area ratio
$A_{tm} = \frac{\left(\frac{1}{P_{ix}}\right)^{\frac{1}{ga}}\sqrt{1-\left(\frac{1}{P_{ix}}\right)^{\frac{ga-1}{ga}}}}{\left(\frac{2}{ga+1}\right)\sqrt{1-\frac{2}{ga+1}}}\ ;$
disp(['Throat to motive gas area ratio :',num2str(A_{tm})]);

%% Calculating throat to constant area duct ratio
$P_{23} = \frac{P_{1b\_1}}{P_{o1b}}\ ;$

$$A_{t2} = \frac{(P_{3o})}{(P_r)} \sqrt{\frac{1}{(1+en)(1+en*T_r)}} \frac{(P_{23})^{\frac{1}{ga}}\sqrt{1-\left(\frac{p_2}{p_3}\right)^{\frac{ga-1}{ga}}}}{\left(\frac{2}{ga+1}\right)^{\frac{1}{ga-1}}\sqrt{1-\frac{2}{ga+1}}};$$

disp(['Throat to constant duct area ratio :',num2str(A_{t2})]);

%% Calulating throat area at specified conditions

$P_i = (P_r * P_o) * 133;$

$A_{t1} = \frac{mf\sqrt{T_i}}{0.0404*P_i*C_d};$

$A_{t1} = A_{t1} * 10^6;$

disp(['Throat area at specified conditions :',num2str(A_{t1})]);

%% Calculating motive gas area

$A_s = \frac{3.14*D_s{}^2}{4};$

$A_m = \frac{A_s}{A_{sm}};$

disp(['motive gas area :',num2str(A_m)]);

%% Calculating area of constant duct

$A_t = A_{tm} * A_m;$

disp(['Throat area :',num2str(A_t)]);

$A_2 = A_{t1}/A_{t2};$

disp(['constant duct area :',num2str(A_2)]);

%% Calculating motive gas diameter

$D_m = \sqrt{\frac{4*A_m}{3.14}};$

disp(['motive gas diameter :',num2str(D_m)]);

%% Calculating constant duct diameter

$D_c = \sqrt{\frac{4*A_2}{3.14}};$

disp(['constant duct diameter :',num2str(D_c)]);

%% Calculating throat diameter

$D_t = \sqrt{\frac{4*A_{t1}}{3.14}};$

disp(['throat diameter :',num2str(D_t)]);

%%Calculating diameter of outer tube

$A_w = A_s + A_m;$

$D_w = \sqrt{\frac{4*A_w}{3.14}};$

disp(['outer tube diameter :',num2str(D_w)]);

%%Calculating convergence angle

$$thd = 2 * atand\left(\frac{\left(\sqrt{\frac{A_w}{A_2}}-1\right)}{\left(\frac{L_1+L_2}{D}\right)}\right);$$

disp(['convergence angle :',num2str(thd)]); fprintf('\n');

A.2 Central Mode

% to study the central injection mode of pressure recovery

%% Stage 1
%% Input parameters
ms = 3; Px = 4; Tr = 1.33; G = 1.4; Cd = 0.9; mc = 1.5;
w = 0.025; Pr = 300; To_s = 400; re = 18; Ds = 75; Pc = 3;

% Calculating parameters
%% Motive gas flow rate
$m_m = \frac{m_s}{w}$;
disp(['Motive gas flow rate mm : ',num2str(m_m)]);

%% Calculating Mach number
M_c = fzero(@(M) flowisentropic(M), [1; 4]); %% function to calculate Mach number
disp(['Mach number of flow from cavity M_c : ',num2str(M_c)]);

%% Calculating stagnation pressure
$Po = Pc\ \left[1 + \frac{ga-1}{2} mc^2\right]^{\frac{ga}{ga-1}}$;
disp(['Stagnation pressure of flow Po : ',num2str(Po)]);

%% Calculating Mach number downstream of shock
$M_b = sqrt\left[\frac{\frac{2}{ga-1} + M_{\_c}^2}{\frac{2(ga)}{ga-1} M_{\_c}^2 - 1}\right]$;
disp(['Mach number downstream of shock Mb : ',num2str(M_b)]);

%% Stagnation pressure downstream of shock
$P_{ob} = P_o\left[1 + \frac{2(ga)}{ga+1}\left(M_{\_c}^2 - 1\right)\right]\left[\frac{1+\frac{ga-1}{2}M_b^2}{1+\frac{ga-1}{2}M_{\_c}^2}\right]^{\frac{ga}{ga-1}}$;
disp(['stagnation pressure downstream of shock Pob : ',num2str(P_{ob})]);

%% Calculating static pressure downstream
$P_{sb} = \frac{P_{ob}}{\left[1+\frac{ga-1}{2}M_b^2\right]^{\frac{ga}{ga-1}}}$;
disp(['Static pressure downstream of shock Psb : ',num2str(P_{sb})]);

%% Motive gas stagnation pressure
$P_{o\_m} = P_r * P_{ob}$;
disp(['Motive gas stagnation pressure Po_m : ',num2str(P_{o_m})]);

%% Motive gas Mach number
$P_{ix} = \frac{P_{o\_m}}{P_x}$; disp(['Pix : ',num2str(P_{ix})]);
$P_{xi} = \frac{1}{P_{ix}}$; disp(['Pix : ',num2str(P_{xi})]);

$M_{xm} = sqrt\left(\left((P_{ix}{}^{\frac{ga-1}{ga}} - 1\right) * \left({}^{2}/_{ga-1}\right)\right)$;
disp(['Motive gas Mach number Mx_m : ',num2str(M_{xm})]);

$M_{xm\_c} = sqrt\left[\frac{\frac{ga+1}{2}{M_{xm}}^2}{1+\frac{ga-1}{2}{M_{xm}}^2}\right]$;
disp(['Motive gas characteristic Mach number Mxm_c : ',num2str(M_{xm_c})]);

%% Suction gas Mach number
$P_{ox} = \frac{P_{o_s}}{P_x}$; disp(['Pox : ',num2str(P_{ox})]);
$P_{xo} = \frac{1}{P_{ox}}$; disp(['Pxo : ',num2str(P_{xo})]);

$M_{x\_s} = sqrt\left(\left((P_{ox})^{\frac{ga-1}{ga}} - 1\right) * \left({}^{2}/_{ga-1}\right)\right)$;
disp(['Suction gas Mach number Mx_s : ',num2str(M_{x_s})]);

$M_{xs\_c} = sqrt\left[\frac{\frac{ga+1}{2}{M_{x\_s}}^2}{1+\frac{ga-1}{2}{M_{x\_s}}^2}\right]$;
disp(['Suction gas characteristic Mach number Mxs_c : ',num2str(M_{xs_c})]);

%% Mixed stream Mach number
$M_{1\_c} = {}^{(M_{xm\_c} + en * M_{xs\_c} * sqrt(T_r))}\Big/_{sqrt((1+en)*(1+en*T_r))}$;
disp(['Mixed stream characteristic Mach number : ',num2str(M_{1_c})]);

$M_1 = sqrt\left[\frac{2{M_{1\_c}}^2}{(ga+1)-(ga-1){M_{1\_c}}^2}\right]$;
disp(['Mixed stream Mach number M1 : ',num2str(M_1)]);

%% Mach number downstream shock
$M_{1b} = sqrt\left[\frac{\frac{2}{ga-1}+M_1^2}{\frac{2(ga)}{ga-1}M_1^2-1}\right]$;
disp(['Mach number downstream of Transverse shock : ',num2str(M_{1b})]);

%% Static pressure after the shock
$P_b = P_x\left[\frac{2(ga)}{ga+1}M_1^2 - \frac{ga-1}{ga+1}\right]$;
disp(['Static pressure after the shock Pb : ',num2str(P_b)]);
P_b = (P_b*84)/100;
disp(['after considering frictional losses static pressure Pb : ',num2str(P_b)]);

%% Stagnation pressure downstream
$P_{bo} = P_b\left[1 + \frac{ga-1}{2}{M_{1b}}^2\right]^{\frac{ga}{ga-1}}$;
disp(['Stagnation pressure after the shock Pbo : ',num2str(P_{bo})]);

%% Suction to motive gas area ratio

$A_{sm} = (P_r) * en * \sqrt{T_r} \frac{(P_{xi})^{\frac{1}{ga}} \sqrt{1-(P_{xi})^{\frac{ga-1}{ga}}}}{(P_{xo})^{\frac{1}{ga}} \sqrt{1-(P_{xo})^{\frac{ga-1}{ga}}}}$;
disp(['Suction to Motive gas area ratio Asm : ',num2str(A_{sm})]);

%% Motive throat to ejector entry area ratio
$A_{tm} = \frac{(P_{xi})^{\frac{1}{ga}} \sqrt{1-\left(\frac{1}{P_{ix}}\right)^{\frac{ga-1}{ga}}}}{\left(\frac{2}{ga+1}\right) \sqrt{1-\frac{2}{ga+1}}}$;
disp(['Motive throat to Ejector entry area ratio Atm : ',num2str(A_{tm})]);

%% Motive throat to constant area duct ratio
$P_{23} = \frac{P_b}{P_{bo}}$;
$P_{3o} = re * (P_{xo})$;
$A_{t2} = \frac{(P_{3o})}{(P_r)} \sqrt{\frac{1}{(1+en)(1+en*T_r)}} \frac{(P_{23})^{\frac{1}{ga}} \sqrt{1-\left(\frac{p_2}{p_3}\right)^{\frac{ga-1}{ga}}}}{\left(\frac{2}{ga+1}\right)^{\frac{1}{ga-1}} \sqrt{1-\frac{2}{ga+1}}}$;
disp(['Motive throat to constant area duct ratio At2 : ',num2str(A_{t2})]);

%% Suction gas inlet area
$A_s = \frac{3.14 * {D_s}^2}{4}$;
disp(['Suction gas inlet area As : ',num2str(A_s)]);

%% Motive gas inlet area
$A_m = \frac{A_s}{A_{sm}}$;
disp(['Motive gas inlet area Am : ',num2str(A_m)]);

$D_m = \sqrt{\frac{4*A_m}{3.14}}$;
disp(['Motive gas diameter Dm : ',num2str(D_m)]);

%% Calulating throat area
$P_i = P_{o\_m} * 133.3$;
$T_{o\_m} = \frac{T_{o_s}}{T_r}$;

$m_m = m_m * 10^{-3}$;
$A_{t\_c} = \frac{m_m \sqrt{T_{o\_m}}}{0.0404 * P_i * C_d}$;
$A_{t\_c} = A_{t\_c} * 10^6$;
disp(['Throat area at specified conditions At_c : ',num2str(A_{t_c})]);

$A_t = A_{tm} * A_m$;
disp(['Throat area At : ',num2str(A_t)]);

$D_t = \sqrt{\frac{4*A_t}{3.14}}$;

disp(['Throat diameter Dt : ',num2str(D_t)]);

%% Calculating constant duct area
$A_2 = \frac{A_t}{A_{t2}}$;
disp(['Constant duct area A2 : ',num2str(A_2)]);

$D_2 = \sqrt{\frac{4*A_2}{3.14}}$;
disp(['constant duct diameter : ',num2str(D_2)]);

%% Convergence angle
$A_x = A_s + A_m$;
$D_x = \sqrt{\frac{4*A_x}{3.14}}$; disp(['Outer tube diameter : ',num2str(D_x)]);
$L_1 = 6 * D_2$; disp(['Length of constant pressure mixing chamber L1 : ',num2str(L_1)]);
$L_2 = 6 * D_2$; disp(['Length of constant area duct L2 : ',num2str(L_2)]);
$theta = 2 * atand\left(\frac{\left(\sqrt{\frac{A_x}{A_2}}-1\right)}{\left(\frac{L_1+L_2}{D}\right)}\right)$; disp(['Convergence angle theta : ',num2str($theta$)]);

%% Stage 2
%% Input parametres
ms = 3; Pob = (Pbo*75.6)/100; Tr = 1.33; G = 1.4; Cd = 0.9; mm = 4;
w = 0.030; Pr = 392; To_s = 400; re = 18; Ds = 140;

%% Calculating static pressure downstream
$P_{sb} = \frac{P_{ob}}{\left[1+\frac{ga-1}{2}M_b^2\right]^{\frac{ga}{ga-1}}}$;
disp(['Static pressure downstream of shock Psb : ',num2str(P_{sb})]);

$P_x = \frac{P_{sb}*99}{100}$
P_{ob}
%% Motive gas stagnation pressure
$P_{o\_m} = P_r * P_{ob}$;
disp(['Motive gas stagnation pressure Po_m : ',num2str(P_{o_m})]);

%% Motive gas Mach number
$P_{ix} = \frac{P_{o\_m}}{P_x}$; disp(['Pix : ',num2str(P_{ix})]);
$P_{xi} = \frac{1}{P_{ix}}$; disp(['Pix : ',num2str(P_{xi})]);

$M_{x\_m} = sqrt\left(\left((P_{ix}{}^{\frac{ga-1}{ga}} - 1\right) * \left({}^{2}/_{ga-1}\right)\right)$;
disp(['Motive gas Mach number Mx_m : ',num2str(M_{x_m})]);

$M_{xm\_c} = sqrt\left[\frac{\frac{ga+1}{2}M_{x\_m}{}^2}{1+\frac{ga-1}{2}M_{x\_m}{}^2}\right]$;
disp(['Motive gas characteristic Mach number Mxm_c : ',num2str(M_{xm_c})]);

%% Suction gas Mach number

$P_{ox} = \frac{P_{o_s}}{P_x}$; disp(['Pox : ',num2str(P_{ox})]);

$P_{xo} = \frac{1}{P_{ox}}$; disp(['Pxo : ',num2str(P_{xo})]);

$M_{x\_s} = sqrt\left(\left((P_{ox})^{\frac{ga-1}{ga}} - 1\right) * \left(2/_{ga-1}\right)\right)$;

disp(['Suction gas Mach number Mx_s : ',num2str(M_{x_s})]);

$M_{xs\_c} = sqrt\left[\frac{\frac{ga+1}{2}M_{x\_s}{}^2}{1+\frac{ga-1}{2}M_{x\_s}{}^2}\right]$;

disp(['Suction gas characteristic Mach number Mxs_c : ',num2str(M_{xs_c})]);

%% Mixed stream Mach number

$M_{1\_c} = {(M_{xm\_c} + w * M_{xs\_c} * sqrt(T_r))}\Big/{sqrt((1+w)*(1+w*T_r))}$;

disp(['Mixed stream characteristic Mach number : ',num2str(M_{1_c})]);

$M_1 = sqrt\left[\frac{2M_{1\_c}{}^2}{(ga+1)-(ga-1)M_{1\_c}{}^2}\right]$;

disp(['Mixed stream Mach number M1 : ',num2str(M_1)]);

%% Mach number downstream shock

$M_{1b} = sqrt\left[\frac{\frac{2}{ga-1}+M_1^2}{\frac{2(ga)}{ga-1}M_1^2-1}\right]$;

disp(['Mach number downstream of Transverse shock : ',num2str(M_{1b})]);

%% Static pressure after the shock

$P_b = P_x\left[\frac{2(ga)}{ga+1}M_1^2 - \frac{ga-1}{ga+1}\right]$;

disp(['Static pressure after the shock Pb : ',num2str(P_b)]);

P_b = (P_b*84)/100;

disp(['after considering frictional losses static pressure Pb : ',num2str(P_b)]);

%% Stagnation pressure downstream

$P_{bo} = P_b\left[1 + \frac{ga-1}{2}M_{1b}{}^2\right]^{\frac{ga}{ga-1}}$;

disp(['Stagnation pressure after the shock Pbo : ',num2str(P_{bo})]);

%% Suction to motive gas area ratio

$A_{sm} = (P_r) * w * \sqrt{T_r}\,\frac{(P_{xi})^{\frac{1}{ga}}\sqrt{1-(P_{xi})^{\frac{ga-1}{ga}}}}{(P_{xo})^{\frac{1}{ga}}\sqrt{1-(P_{xo})^{\frac{ga-1}{ga}}}}$;

disp(['Suction to Motive gas area ratio Asm : ',num2str(A_{sm})]);

%% Motive throat to ejector entry area ratio

$A_{tm} = \frac{(P_{xi})^{\frac{1}{ga}}\sqrt{1-\left(\frac{1}{P_{ix}}\right)^{\frac{ga-1}{ga}}}}{\left(\frac{2}{ga+1}\right)\sqrt{1-\frac{2}{ga+1}}}$;

disp(['Motive throat to Ejector entry area ratio Atm : ',num2str(A_{tm})]);

%% Motive throat to constant area duct ratio
$P_{23} = \frac{P_b}{P_{bo}}$;
$P_{3o} = re * (P_{xo})$;

$$A_{t2} = \frac{(P_{3o})}{(P_r)} \sqrt{\frac{1}{(1+w)(1+w*T_r)}} \frac{(P_{23})^{\frac{1}{ga}} \sqrt{1- \left(\frac{p_2}{p_3}\right)^{\frac{ga-1}{ga}}}}{\left(\frac{2}{ga+1}\right)^{\frac{1}{ga-1}} \sqrt{1- \frac{2}{ga+1}}};$$

%% Suction gas inlet area
$A_s = \frac{3.14 * D_s{}^2}{4}$;
disp(['Suction gas inlet area As : ',num2str(A_s)]);

%% Motive gas inlet area
$A_m = \frac{A_s}{A_{sm}}$;
disp(['Motive gas inlet area Am : ',num2str(A_m)]);

$D_m = \sqrt{\frac{4*A_m}{3.14}}$;
disp(['Motive gas diameter Dm : ',num2str(D_m)]);

%% Calulating throat area
$P_i = P_{o\_m} * 133.3$;
$T_{o\_m} = \frac{T_{o_s}}{T_r}$;

$m_m = m_m * 10^{-3}$;
$A_{t\_c} = \frac{m_m \sqrt{T_{o\_m}}}{0.0404 * P_i * C_d}$;
$A_{t\_c} = A_{t\_c} * 10^6$;
disp(['Throat area at specified conditions At_c : ',num2str(A_{t_c})]);

$A_t = A_{tm} * A_m$;
disp(['Throat area At : ',num2str(A_t)]);

$D_t = \sqrt{\frac{4*A_t}{3.14}}$;
disp(['Throat diameter Dt : ',num2str(D_t)]);

%% Calculating constant duct area
$A_2 = \frac{A_t}{A_{t2}}$;
disp(['Constant duct area A2 : ',num2str(A_2)]);

$D_2 = \sqrt{\frac{4*A_2}{3.14}}$;
disp(['constant duct diameter : ',num2str(D_2)]);

%% Convergence angle
$A_x = A_s + A_m$;

$D_x = \sqrt{\frac{4*A_x}{3.14}}$; disp(['Outer tube diameter : ',num2str(D_x)]);
$L_1 = 6 * D_2$; disp(['Length of constant pressure mixing chamber L1 : ',num2str(L_1)]);
$L_2 = 8 * D_2$; disp(['Length of constant area duct L2 : ',num2str(L_2)]);
$theta = 2 * atand\left(\frac{\left(\sqrt{\frac{A_x}{A_2}}-1\right)}{\left(\frac{L_1+L_2}{D}\right)}\right)$; disp(['Convergence angle theta : ',num2str($theta$)]);

A.3 Program for Calculating Recovery ratio Variation

- **Recovery Ratio vs. Entrainment Ratio**

w = [0.2,0.15,0.1,0.08,0.07,0.06,0.05,0.04,0.03,0.025];
disp(w);

%% Input values
Mx1 = 2.22; Mx2 = 0.52; Tr = 1; ga = 1.4; px = 4;

$m_{1\_c} = {(M_{x1\_c} + w * M_{x2\_c} * sqrt(T_r))} \Big/ {sqrt((1+w)*(1+w*T_r))}$;
%disp(['m1c : ',num2str(m_{1_c})]);

$m_1 = sqrt\left[\frac{2{m_{1\_c}}^2}{(ga+1)-(ga-1){m_{1\_c}}^2}\right]$;
%disp(['m1 : ',num2str(m_1)]);

$m_{1b} = sqrt\left[\frac{\frac{2}{ga-1}+m_1^2}{\frac{2(ga)}{ga-1}m_1^2-1}\right]$;
%disp(['m1b : ',num2str(m_{1b})]);

$p_{1b} = p_x\left[\frac{2(ga)}{ga+1}m_1^2 - \frac{ga-1}{ga+1}\right]$;
%disp(['p1b : ',num2str(p_{1b})]);

R = p_{1b}./ p_x;
disp(['R : ',num2str(R)]);

plot (w, R)
xlabel (' ER ');
ylabel (' RR ');

fprintf('\n');

- **Recovery ratio vs. Pressure ratio**

p_r = [50,100,150,200,250,300,350,400,450,500];

disp(['p_r : ',num2str(p_r)]);

%%Input values

Pxo = 0.875; ga = 1.4; en = 0.025; Tr = 1; px1 = 4;

$P_{ix} = p_r * \left(\frac{1}{P_{xo}}\right)$;   %disp(['Pix : ',num2str(P_{ix})]);

$M_{x1} = sqrt\left(\left((P_{ix}{}^{\frac{ga-1}{ga}} - 1\right) * \left(2/_{ga-1}\right)\right)$;   %disp(['Mx1 : ',num2str(Mx1)]);

$M_{x2} = sqrt\left(\left((\frac{1}{P_{xo}})^{\frac{ga-1}{ga}} - 1\right) * \left(2/_{ga-1}\right)\right)$;   %disp(['Mx2 : ',num2str(Mx2)]);

$M_{x1\_c} = sqrt\left[\frac{\frac{ga+1}{2}{M_{x1}}^2}{1+\frac{ga-1}{2}{M_{x1}}^2}\right]$;   %disp(['Mx1_c : ',num2str(Mx1_c)];

$M_{x2\_c} = sqrt\left[\frac{\frac{ga+1}{2}{M_{x2}}^2}{1+\frac{ga-1}{2}{M_{x2}}^2}\right]$;   %disp(['Mx2_c : ',num2str(Mx2_c)]);

$M_{1c} = {}^{(M_{x1\_c} + en * M_{x2\_c} * sqrt(T_r))}\Big/_{sqrt((1+en)*(1+en*T_r))}$;

$M_1 = sqrt\left[\frac{2{M_{1c}}^2}{(ga+1)-(ga-1){M_{1c}}^2}\right]$;   %disp(['M1_c : ',num2str(M1_c)]);

$p_b = p_{x1}\left[\frac{2(ga)}{ga+1}M_1^2 - \frac{ga-1}{ga+1}\right]$;   %disp(['pb : ',num2str(pb)]);

R = p_b./ p_{x1};

disp(['R : ',num2str(R)]);

plot (p_r, R);

xlabel (' PR ');

ylabel (' RR ');