

Repeated Measures ANOVA

We want to know if three different drugs lead to different reaction times. To test this, they measure the reaction time (in seconds) of five patients on each drug. The results are shown below:

Patients	Panadol	Paracetamol	Ponstan
Patient1	30	28	16
Patient2	14	18	10
Patient3	24	20	18
Patient4	38	34	20
Patient5	26	28	14

Since all of the three drugs (painkillers) are tested on each patient, therefore we can use one-way repeated measure ANOVA to conclude if the mean reaction time differs between drugs.

Calculation steps:

- Calculate SST:

$$SST = s^2_{total}(n_{total}-1)$$

- ✓ s^2_{total} : the variance for the entire dataset
- ✓ n_{total} : the total number of observations in the entire dataset
 therefore $SST = (64.2667)(15-1) = 899.7$

- Calculate SSB:

$$SSB = \sum n_j(x_j - x_{total})^2$$

Where

- ✓ Σ : a greek symbol that means "sum"
- ✓ n_j : the total number of observations in the j^{th} group
- ✓ x_j : the mean of the j^{th} group
- ✓ x_{total} : the mean of the entire dataset
 therefore $SSB = (5)(26.4-22.533)^2 + (5)(25.6-22.533)^2 + (5)(15.6-22.533)^2 = 362.1$

- Calculate SSS

$$SSS = (\sum r_k^2 / c) - (N^2 / rc)$$

- ✓ Σ : a greek symbol that means "sum"
- ✓ r_k^2 : squared sum of the k^{th} patient
- ✓ N : the grand total of the entire dataset
- ✓ r : total number of patients
- ✓ c : total number of groups

$$\text{therefore SSS} = ((74^2 + 42^2 + 62^2 + 92^2 + 68^2)/3) - (338^2/(5)(3)) = 441.1$$

- Calculate SSE

$$\text{We have SSE} = \text{SST} - \text{SSB} - \text{SSS}, \text{therefore SSE} = 899.7 - 362.1 - 441.1 = 96.5$$

Now fill in the repeated measure ANOVA table:

Source	Sum of Squares (SS)	df	Mean Squares (MS)	F
Between	362.1	2	181.1	15.006
Subject	441.1	4	110.3	
Error	96.5	8	12.1	

- **df between:** #groups – 1 = 3 – 1 = 2
- **df subject:** #participants – 1 = 5 – 1 = 4
- **df error:** df between * df subject = 2*4 = 8
- **MS between:** SSB / df between = 362.1 / 2 = 181.1
- **MS subject:** SSS / df subject = 441.1 / 4 = 110.3
- **MS error:** SSE / df error = 96.5 / 8 = 12.1
- **F:** MS between / MS error = 181.1 / 12.1 = 15.006

Interpret the results:

At 5% level of significance with df1 =2 and df2 = 8, we found the f-critical value from ANOVA table as 4.459, which is lesser than the f-calculated value, therefore, we reject the null hypothesis and concluded that we have sufficient evidence to say that there is a statistically significant difference between the mean response times of the drugs.