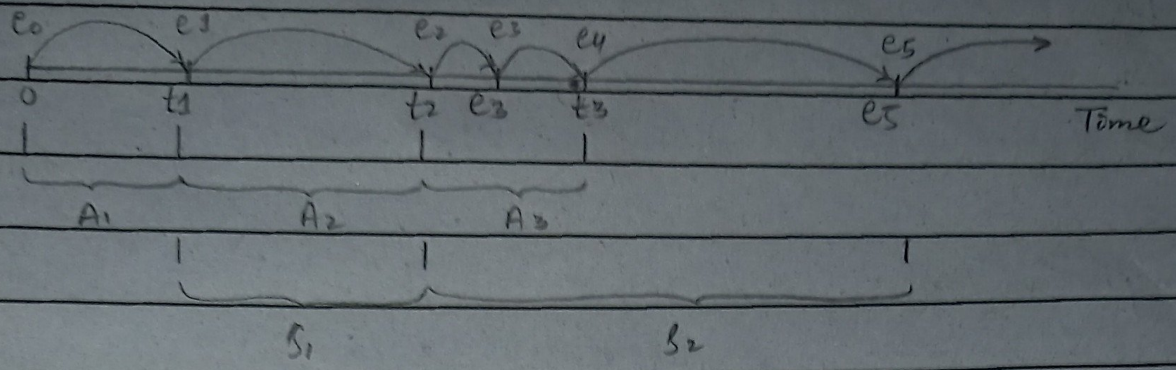
Date

Chapter : 1 : (fig 1.2)



Soln

t_i = time of arrival of the ith customer ($t_0 = 0$)

$A_i = t_i - t_{i-1}$ = interarrival time between (i-1)st and ith arrivals of customers.

S_i = time that server actually spends serving ith customer (exclusive of customer's delay in queue)

D_i = delay in queue of ith customer

$c_i = t_i + D_i + S_i$ = time that ith customer completes service and departs

e_i = time of occurrence of ith event of any type (ith value the simulation clock takes on, excluding the value $e_0 = 0$)

Let us assume :-

$A_1 = 1$, $A_2 = 7$, $A_3 = 4$

$S_1 = 3$, $S_2 = 9$, $S_3 = 2$

Initial time => $t_0 = 0$, $e_A = 0$

Arrival Time :

$t_1 = t_0 + A_1 \Rightarrow 0 + 1 = 1$

$t_2 = t_1 + A_2 \Rightarrow 1 + 7 = 8$

$t_3 = t_2 + A_3 \Rightarrow 8 + 4 = 12$

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Delay Time:

$D_1 = max(0, C_0 - t_1) \Rightarrow max(0, 0-1) \Rightarrow max(0,-1) \Rightarrow 0$

$D_2 = max(0, C_1 - t_2) \Rightarrow max(0, 4-8) \Rightarrow max(0,-4) \Rightarrow 0$

$D_3 = max(0, C_2, t_3) \Rightarrow max(0, 17-12) \Rightarrow max(0,5) \Rightarrow 5$

Completion Time:

$C_1 = t_1 + D_1 + S_1' \Rightarrow 1 + 0 + 3 \Rightarrow 4$

$C_2 = t_2 + D_2 + S_2 \Rightarrow 8 + 0 + 9 \Rightarrow 17$

$C_3 = t_3 + D_3 + S_3 \Rightarrow 12 + 5 + 2 \Rightarrow 19$

Customers	IA	Arrival Time	Service Time	Delay Time	Completion Time
1	1	1	3	0	4
2	7	8	9	0	17
3	4	12	2	5	19

Simulation Of Pre-emptive Priority Queuing Model (M/M/1)

Q. Solve the following with the help of simulation of pre-emptive priority queuing model & also with the help of gantt chart.

1. Avg Inter Arrival.
2. Avg Service Time $\bar{S}$ (General/Priority wise).
3. Avg Turn Around.
4. Avg Wait Time.
5. Utilization.
6. Avg Respone.

P#	A	ST	Priority	End	Start	E–A T.A	T.A.–ST W.T	Start – A Response
1	0	19	3	70	0	70	51	0
2	4	18	3	88	70	84	66	66
3	7	12	2	30	7	23	11	0
4	9	17	3	105	88	96	79	79
5	15	5	1	20	15	5	0	0
6	18	6	1	26	20	8	2	2
7	27	13	2	43	30	16	3	3
8	30	15	2	58	43	28	13	13
		$\Sigma = 105$			$\Sigma =$	330	225	163

Gantt Chart:-

P_1	P_3	P_5	P_6	P_3	P_7	P_8	P_1	P_2	P_4
0–7	7–15	15–20	20–26	26–30	30–43	43–58	58–70	70–88	88–105

Probability of waiting Patients $= \frac{7}{8} = 0.8 = 80\%$.

General :-

$$\text{Avg T.A} = \frac{330}{8} = 41.25$$

$$\text{Avg W.T} = \frac{225}{8} = 28.125$$

$$\text{Avg Respone Time} = \frac{103}{8} = 20.375$$

$$\text{Avg S.T} = \frac{105}{8} = 13.125$$

Priority Wise :-

Avg T.A :-

For Priority 1 $\Rightarrow$ 2 Patient $= \left(\frac{8+5}{2}\right) = 6.5$

For Priority 2 $\Rightarrow$ 3 Patient $= \left(\frac{23+16+28}{3}\right) = 22.33$

For Priority 3 $\Rightarrow$ 3 Patient $= \left(\frac{70+84+96}{3}\right) = 83.33$

Avg W.T :-

For Priority 1 $\Rightarrow$ 2 $= \left(\frac{0+2}{2}\right) = 1$

For Priority 2 $\Rightarrow$ 3 $= \left(\frac{11+13+3}{3}\right) = 9$

For Priority 3 $\Rightarrow$ 3 $= \left(\frac{51+66+79}{3}\right) = 65.33$

Avg Response Time :-

For Priority 1 $\Rightarrow$ 2 $= \left(\frac{0+2}{2}\right) = 1$

For Priority 2 $\Rightarrow$ 3 $= \left(\frac{3+13+0}{3}\right) = 5.33$

For Priority 3 $\Rightarrow$ 3 $= \left(\frac{0+66+79}{3}\right) = 48.33$

(M/M/2)

P#	A	ST	P	Server	End	Start	E-A T.A	T.A-ST W.T	Start-A R.T
1	0	19	3	A	47	0	47	28	0
2	4	18	3	B	28	4	24	6	0
3	7	12	2	A.	24	7	17	5	0
4	9	17	3	B	62	45	53	36	36
5	15	5	1	A	20	15	5	0	0
6	18	6	1	B	24	18	6	0	0
7	27	13	2	A	40	27	13	0	0
8	30	15	3	B	45	30	15	0	0
		Σ=105				Σ=	180	75	36

Server A:-

P_1	P_3	P_5	P_3	P_1	P_7	P_1

0 7 15 20 24 27 40 47

Server B:-

P_2	P_6	P_2	P_1	P_8	P_4

4 18 24 28 30 45 62

Probability of waiting Patient $= 4/8 = 0.5 = 50\%$

General:-

Avg T.A $= \frac{180}{8} = 22.5$

Avg W.T $= \frac{75}{8} = 9.375$

Avg R.T $= \frac{36}{8} = 4.5$

Avg S.T $= \frac{105}{8} = 13.125$

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Priority Wise :-

Avg T.A :-

1 $\Rightarrow \left(\frac{5+6}{2}\right) = 5.5$

2 $\Rightarrow \left(\frac{17+13+15}{3}\right) = 15$

3 $\Rightarrow \left(\frac{47+24+53}{3}\right) = 41.333$

Avg W.T :-

1 $\Rightarrow \left(\frac{0+0}{2}\right) = 0$

2 $\Rightarrow \left(\frac{5+0+0}{3}\right) = 1.666$

3 $\Rightarrow \left(\frac{28+6+36}{3}\right) = 23.33$

Avg R.T :-

1 $\Rightarrow \left(\frac{0+0}{2}\right) = 0$

2 $\Rightarrow \left(\frac{0+0+0}{3}\right) = 0$

3 $\Rightarrow \left(\frac{36+0+0}{3}\right) = 12$

Avg ST :-

1 $\Rightarrow \left(\frac{5+6}{2}\right) = 5.5$

2 $\Rightarrow \left(\frac{12+13+15}{3}\right) = 13.33$

3 $\Rightarrow \left(\frac{19+18+17}{3}\right) = 18$

LCG (Algo) (Linear Congruential Generator)

Let $m = 123$, $a = 5$, $b = 2$ choose $x_0 = 73$ $i = 5$

we have LCG algo as:

$$x_i = (a x_{i-1} + b)(\text{mod } m)$$

For x_1:

$$x_1 \equiv (a x_0 + b)(\text{mod } m) \equiv ((5)(73) + 2)(\text{mod } 123)$$

$$x_1 \equiv 367 (\text{mod } 123) \Rightarrow \boxed{x_1 \equiv 121}$$

For x_2:

$$x_2 \equiv (a x_1 + b)(\text{mod } m) \equiv ((5)(121) + 2)(\text{mod } 123)$$

$$x_2 \equiv 607 (\text{mod } 123) \Rightarrow \boxed{x_2 \equiv 115}$$

For x_3:

$$x_3 \equiv (a x_2 + b)(\text{mod } m) \equiv ((5)(115) + 2)(\text{mod } 123)$$

$$x_3 \equiv 577 (\text{mod } 123) \Rightarrow \boxed{x_3 \equiv 85}$$

For x_4:

$$x_4 \equiv (a x_3 + b)(\text{mod } m) \equiv ((5)(85) + 2)(\text{mod } 123)$$

$$x_4 \equiv 427 (\text{mod } 123) \Rightarrow \boxed{x_4 \equiv 58}.$$

For x_5:

$$x_5 \equiv (a x_4 + b)(\text{mod } m) \equiv ((5)(58) + 2)(\text{mod } 123)$$

$$x_5 \equiv 292 (\text{mod } 123) \Rightarrow \boxed{x_5 \equiv 46}$$

$\therefore$ 73, 121, 115, 85, 58, 46 are called Pseudo Random Numbers.

Patient Priority Modelling.

$1 \leq Y \leq 3$ $a = 1, b = 3$

Let $A = 55$ $m = 1994$ $z_0 = 10112166$ $C = 9$

$\Rightarrow$ z_i = value of LCG; $\Rightarrow$ $R(LCG)^{xi} = (A z_0 + C)(\text{mod } m)$

$\Rightarrow$ Random Number = $R(LCG)xi / m$

$\Rightarrow$ Generate priority = Random No. $(b-a)+a$

S.no	Zi (initial seed)	R(LCG) xi	Random Number	Generate Priority
1	10112166	665	0.333500502	2
2	665	692	0.347041123	2
3	692	183	0.091775326	1
4	183	104	0.052156469	1
5	104	1741	0.873119358	3
6	1741	52	0.026078235	1
7	52	875	0.438816449	2
8	875	278	0.139418255	1
9	278	1341	0.672517553	2
10	1341	1980	0.992978937	3

Q. Generate uniform variate U[0,1] using linear congruential generator (LCG) to emulate discrete priority variable (X) such that $1 \leq x \leq 3$; which classifies the emergency dept. (ED) patients based on the critical behavior of surgery.

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Empirical Distribution:

Q Suppose we observe a sample made of four observations

$$C_4 = [x_1 \quad x_2 \quad x_3 \quad x_4]$$

where, $x_1 = 3$, $x_2 = 2$, $x_3 = 5$, $x_4 = 2$

What is the value of the empirical distribution function of the definition above it is:

Since,

$$F_n(x) = \frac{1}{4} \sum_{i=1}^{n} 1\{x_i \le x\}$$

$$F_4(4) = \frac{1}{4} \sum_{i=1}^{4} 1\{x_i \le 4\}$$

$$= \frac{1}{4}\left(1_{\{x_1 \le 4\}} + 1_{\{x_2 \le 4\}} + 1_{\{x_3 \le 4\}} + 1_{\{x_4 \le 4\}}\right)$$

$$= \frac{1}{4}(1 + 1 + 0 + 1)$$

$$F_4(4) = 3/4$$

In other words, the proportion of observations that are less than or equal to 4 is 3/4.

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Probability Distribution From Empirical Data:

Q Let x be the no. of seminars a high school student will conduct in a given month.
A survey conducted at one particular high school in the month of December is given by the following table.

No. of seminars conducted	1	2	3	4	5
% of students	17	28	34	15	6

Soln:-
Using the weighted average formula:

$$E(x) = \sum x_i \cdot P(x_i)$$

$$= (1)\cdot(0.17) + (2)(0.28) + (3)(0.34) + (4)(0.15) + (5)(0.06)$$

$$E(x) = 2.95$$

So, we can expect the avg high school student to conduct 2.95 seminars per month.

Empirical Distribution Functions:

Q A random sample of $n=8$ people yields the following (ordered) counts of the number of times they swam in the past month.

0 1 2 2 4 6 6 7

Calculate the empirical distribution function $F_n(x)$.

Soln
As reported the data are ordered therefore the order statistics are $y_1=0$, $y_2=1$, $y_3=2$, $y_4=2$, $y_5=4$, $y_6=6$, $y_7=6$, $y_8=7$. Therefore using the definition of the empirical dist. function we have:

$F_n(x) = 0$ for $x < 0$

and

$F_n(x) = 1/8$ for $0 \leq x < 1$

$F_n(x) = 2/8$ for $1 \leq x < 2$

$F_n(x) = \frac{2}{8} + \frac{2}{8} = \frac{4}{8}$ for $2 \leq x < 4$

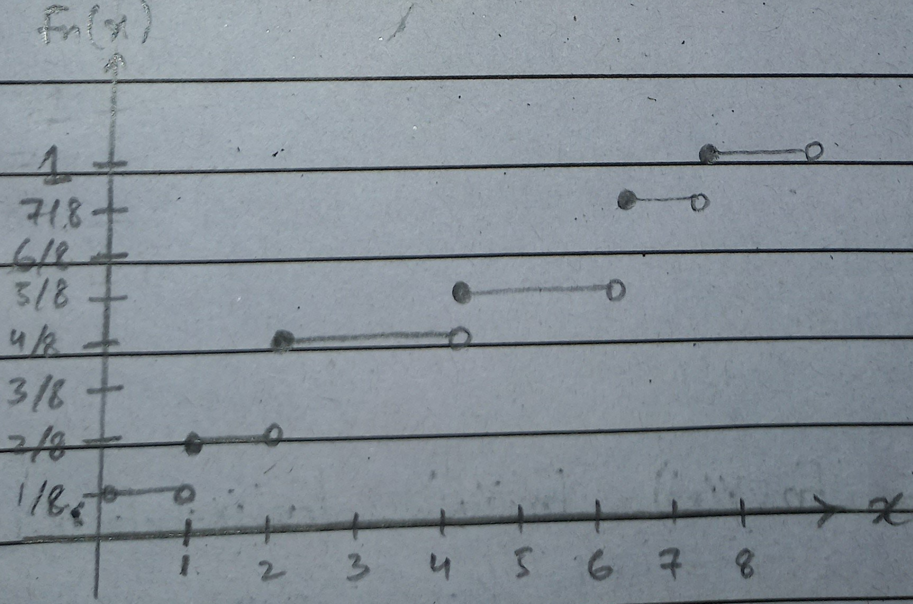
$F_n(x) = 5/8$ for $4 \leq x < 6$

$F_n(x) = 5/8 + 2/8 = 7/8$ for $6 \leq x < 7$

And finally:

$F_n(x) = 7/8 + 1/8 = 8/8 = 1$ for $x \geq 7$

Plotting the function:



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KS - Goodness Of Fit Test.

Q. Given the dataset is there any evidence to suggest that the data were not randomly sampled from a uniform [0, 2] distribution ? Apply KS - Goodness of fit test to identify the distribution of the data.

Sol.-

The pdf of the uniform [0,2] random variable (X) is $f(x) = 1/2$

For $0 \le x \le 2$

Therefore the probability that the X is less than or equal to x.

$$(P(x) \le x) = \int_0^x \frac{1}{2}\,dt = \frac{1}{2}x$$

For $0 < x < 2$

1- Setup Ho:

$H_0 : F(x) = F_0(x)$

$H_A : F(x) \ne F_0(x)$

K	Y(k)	$F_n(Y_{(k-1)})$	$F_0(Y(k))$	$F_n(Y_{(k)})$	$\lvert F_n(Y_{k-1}) - F_0(Y_k)\rvert$	$\lvert F_0(Y_k) - F_n(Y_k)\rvert$
1	0.26	1-1/8 = 0	½ (0.26) = 0.13	1/8 = 0.125	|0 - 0.13| = 0.13	|0.13 - 0.125| = 5×10^{-3}
2	0.33	1/8 = 0.125	0.165	2/8 = 0.25	0.04	0.085
3	0.55	2/8 = 0.25	0.275	3/8 = 0.375	0.025	0.1
4	0.77	3/8 = 0.375	0.385	4/8 = 0.5	0.01	0.115
5	1.18	4/8 = 0.5	0.59	5/8 = 0.625	0.09	0.035
6	1.41	5/8 = 0.625	0.705	6/8 = 0.75	0.08	0.045
7	1.46	6/8 = 0.75	0.73	7/8 = 0.875	0.02	0.145
8	1.97	7/8 = 0.875	0.985	8/8 = 1	0.11	0.015

$df = 8$

$(0.05, 8) = 0.45427$

$\Rightarrow 0.145 < 0.45427$

Conclusion:

Since calculated value < tabulated value, we accept H_0.
Hence, we conclude the data follows uniform distribution.