Operation Research
DCS – UOK
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Basic Probability Rules

1) Possible values for probabilities range from 0 to 1

    0 = impossible event
    1 = certain event

2) The sum of all the probabilities for all possible outcomes is equal to 1.

    Note the connection to the complement rule.

3) Addition Rule - the probability that one or both events occur

    mutually exclusive events: P(A or B) = P(A) + P(B)
    not mutually exclusive events: P(A or B) = P(A) + P(B) - P(A and B)

4) Multiplication Rule - the probability that **both** events occur together

    independent events: P(A and B) = P(A) * P(B)
    P(A and B) = P(A) * P(B|A)

5) Conditional Probability - the probability of an event happening **given** that another event has already happened

    P(A|B) = P(A and B) / P(B)
    *Note the line | means "given" while the slash / means divide

Key Terminology

Mutually Exclusive - this indicates that two events cannot happen at the same time.

For example, consider the following two events: A) rolling a 2 and B) rolling an odd number. Since 2 is an even number, it's not possible for me to roll a 2 and for that number to be odd. Therefore, these events are mutually exclusive.

Independent Events - the probability of one event does not change based on the outcome of the other event

Consider a basketball player shooting 2 free throws. If the player's probability of making the second shot changes based on whether or not they make the first shot, then these events are dependent. If the probability does not change, then they would be independent.

Single Event Probability

The sample space refers to possible event outcomes. Subsets of this sample space can be used to compute simple probabilities. See the following examples on how to use this approach.

Example 1

Let's start with a simple example to warm us up:

We're flipping a fair coin and we want to find the probability of getting "heads". We can begin our thought process by first determining all of the possible outcomes. In this case, we can 1) flip and get "**heads**" or 2) flip and get "**tails**". Therefore there are 2 ***possible outcomes = heads, tails***

Next, we want to identify which of these outcomes are our desired outcomes, meaning the outcome(s) we are trying to find the probability for. Since we want to find the probability of getting "heads" ***desired outcome = flipping heads***. Looking at our possible outcomes, we can see that only **one** outcome would be included in our desired outcome: ***heads***.

Now we have everything that we need to compute the probability:

Number of successful/desired outcomes = 1
Total number of possible outcomes = 2

Therefore, **the probability of flipping a fair coin and getting "heads" is 1/2**. We could also report this as a decimal (.5) or a percentage (50%).

Example 2

Let's look at another simple example that has a few more possible outcomes:

This time, let's roll a fair die. We want to find the probability of rolling a 3. We will again begin by first determining all of the possible outcomes. In this case, we can roll a one, a two, a three, a four, a five, or a six. Therefore there are 6 ***possible outcomes = 1, 2, 3, 4, 5, 6***

Next, we want to identify which of these outcomes are our desired outcomes, meaning the outcome(s) we are trying to find the probability for. Since we want to find the probability of rolling a 3, the ***desired outcome = 3***. Looking at our possible outcomes, we can see that only **one** outcome would be included in our desired outcome: ***3***.

Now we have everything that we need to compute the probability:

Number of successful/desired outcomes = 1
Total number of possible outcomes = 6

Therefore, **the probability of rolling a 3 is 1/6**. We could also report this as a decimal (.167) or a percentage (16.7%).

Example 3

Let's consider another die example:

This time, we want to know the probability of rolling an odd number. Applying the same thought process to this scenario, the ***possible outcomes*** remain the same: ***1, 2, 3, 4, 5, 6***. The ***desired outcomes*** now include ***1, 3,*** and ***5***. That means that there are now **three desired outcomes**. This means that **the probability of rolling an odd number is 3/6 = .5 = 50%**

Complement Rule

The **complement rule** works off of the idea that two parts make a whole. In probability, the "whole" refers to all possible outcomes. I find it's easiest to think of this as being 100%, which we know as a decimal value is simply 1. Thus, **the sum of the probabilities of all possible outcomes must equal 1.**

There are a few situations where this rule comes in handy. Let's take a look at those.

Example 1

The most common application of this rule is when we see probabilities that use the phrasing of "at least 1". For example, let's say a group of 25 students had to indicate if they were eating lunch at school or not (yes/no). You want to find the probability of getting at least 1 person that will be eating at school.

Applying the thought process we've been using, there are **26 possible outcomes**: 0-25.
The **desired outcomes** include outcomes 1 through 25. That means we would need to compute the probability for each of those value individually, which is tedious!

Looking at this another way, we can see that while 1-25 are desired outcomes, the outcome of 0 is not desired. Using the logic of two parts make a whole, we can apply the following logic to approach this problem:

[probability of desired outcomes] + [probability of not desired outcomes] = 1

We can rearrange this to state that the [probability of desired outcomes] = 1 - [probability of not desired outcomes]. Therefore, if we find the probability of getting 0 people, we can subtract that from 1 to find the probability for this scenario. Let's say the probability that no one each lunch at school is .06. That means **the probability of at least one person eating lunch at school is** 1 - .06 = **.94** or 94%.

This approach works with any situation where you can divide the outcomes into desired (successful) and not desired (failure) outcomes. We'll explore scenarios similar to this more on the binomial probability tab.

Example 2

Another handy time to use this rule is to find a missing probability from a probability distribution table. Let's take a look at the following example:

X	Probability
red	.21
orange	?
yellow	.17
green	.36
blue	.14

In this table, the "X" column denotes the possible outcomes. The "Probability" column denotes the probability of achieving each outcome. Notice that the table does not provide the probability for "orange".

Applying an understanding of basic probability rules, we can compute the missing probability. We know that the probability column must sum to 1 in order to be a proper probability distribution, therefore, we can add up the probabilities given and subtract from 1 to find the missing probability value:

$$.21 + .17 + .36 + .14 = .88$$
$$1 - .88 = .12$$

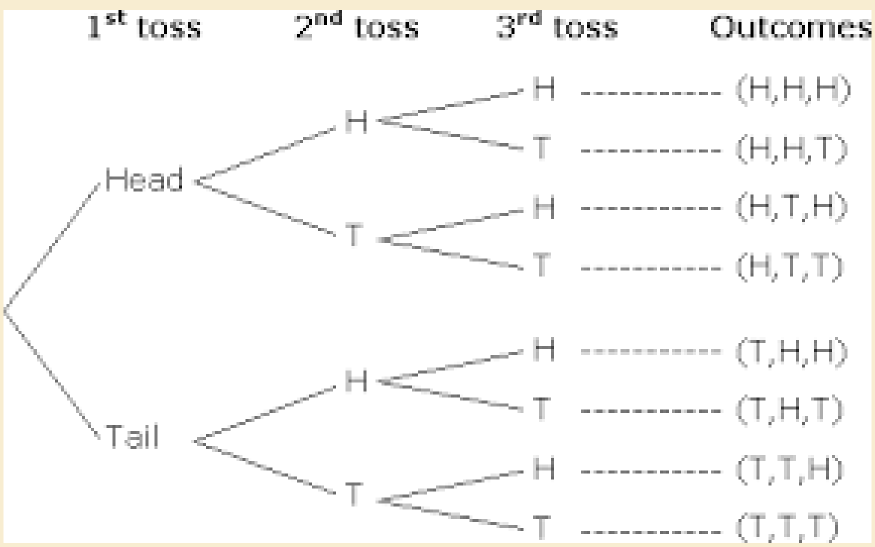
Therefore, **the probability of getting orange must be .12.**

Probability – Tree diagrams and Venn diagrams

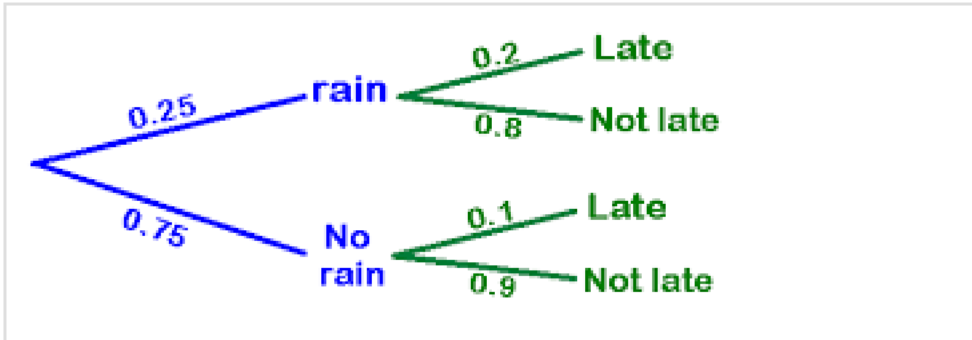
Probability can be calculated through the use of tree diagrams and Venn diagrams.

Tree diagrams

When probabilities of many events are to be sorted, a tree diagram could be used. The following is an example of a tree diagram:



For example, the probability that it will rain tomorrow is 0.25. If it rains, the probability that I will be late to school is 0.2. If it doesn't rain, the probability of me being late to school is 0.1. Draw a tree diagram for this information.



The tree diagram would look like the above illustration. To find out the probability of not raining, subtract the probability of it raining from 1:

Probability of raining: 0.25

Probability of not raining: 1 – 0.25 = 0.75

Then, if it rains, the probability of me getting to school is 0.2. To find the probability of me not being late to school, subtract 0.2 from 1 to get 0.8. So the probability of not being late is 0.8.

If it doesn't rain, the probability of me being late is 0.1. The probability of me not being late is: 1 – 0.1 = 0.9.

The tree diagram is now completed and can be drawn.

Venn diagrams

—A Venn diagram is an illustration that shows all possible and relevant relations between a finite collection of sets. —It is usually represented using two circles intersecting at a point, which have been put inside a rectangle. —Elements from a set are then put in one circle. Elements from another set are put in the second circle whereas elements in common are put in the area where the circles intersect.

For example;

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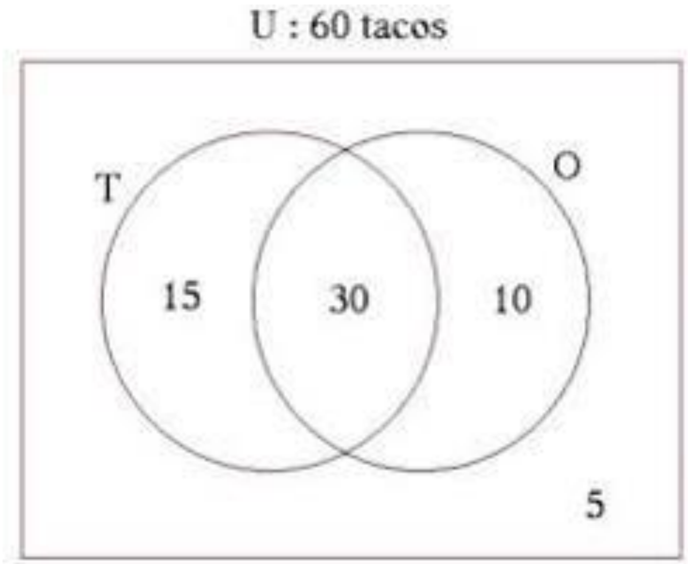

The Venn diagram above represents two sets; one with tomato tacos and another with orange tacos. The area in the middle has tacos which have both orange and tomato. The area outside the circle but within the rectangle has tacos without anything inside. The total amount of tacos is shown above the Venn diagram.

If we were to find probability, we would need to take the element in question and put it over the total number of elements. For example, the probability of Tomato tacos and not orange tacos is:

tomato tacos only / total amount of tacos

= 15 / 60

= 1 / 4.