

Maximization problem by Simplex method

Steps to perform the Simplex method:

1. **Set up the problem.** That is, write the objective function and the inequality constraints.
2. **Convert the inequalities into equations.** This is done by adding one slack variable for each inequality.
3. **Construct the initial simplex tableau.** Write the objective function as the bottom row.
4. **The most negative entry in the bottom row identifies the pivot column.**
5. **Calculate the quotients. The smallest quotient identifies a row. The element in the intersection of the column identified in step 4 and the row identified in this step is identified as the pivot element.** The quotients are computed by dividing the far right column by the identified column in step 4. A quotient that is a zero, or a negative number, or that has a zero in the denominator, is ignored.
6. **Perform pivoting to make all other entries in this column zero.** This is done the same way as we did with the Gauss-Jordan method.
7. **When there are no more negative entries in the bottom row, we are finished; otherwise, we start again from step 4.**
8. **Read off your answers.** Get the variables using the columns with 1 and 0s. All other variables are zero. The maximum value you are looking for appears in the bottom right hand corner.

Question: Find a solution using the Simplex Method:

MAX Z = $3x_1 + 5x_2 + 4x_3$

subject to

$$2x_1 + 3x_2 \leq 8$$

$$2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

and $x_1, x_2, x_3 \geq 0$

Solution:

Problem is

$$\text{Max } Z = 3x_1 + 5x_2 + 4x_3$$

subject to

$$2x_1 + 3x_2 \leq 8$$

$$2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

and $x_1, x_2, x_3 \geq 0$;

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropriate

1. As the constraint-1 is of type ' \leq ' we should add slack variable S_1
2. As the constraint-2 is of type ' \leq ' we should add slack variable S_2
3. As the constraint-3 is of type ' \leq ' we should add slack variable S_3

After introducing slack variables

$$\text{Max } Z = 3x_1 + 5x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3$$

subject to

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3. As the constraint-3 is of type ' \leq ' we should add slack variable S_3

After introducing slack variables

$$\text{Max } Z = 3x_1 + 5x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3$$

subject to

$$2x_1 + 3x_2 + S_1 = 8$$

$$2x_2 + 5x_3 + S_2 = 10$$

$$3x_1 + 2x_2 + 4x_3 + S_3 = 15$$

and $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Iteration-1		C_j	3	5	4	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_2}$
S_1	0	8	2	(3)	0	1	0	0	$\frac{8}{3} = 2.6667 \rightarrow$
S_2	0	10	0	2	5	0	1	0	$\frac{10}{2} = 5$
S_3	0	15	3	2	4	0	0	1	$\frac{15}{2} = 7.5$
$Z = 0$		Z_j	0	0	0	0	0	0	
		$Z_j - C_j$	-3	-5 ↑	-4	0	0	0	

Negative minimum $Z_j - C_j$ is -5 and its column index is 2. So, the entering variable is x_2 .

Minimum ratio is 2.6667 and its row index is 1. So, the leaving basis variable is S_1 .

\therefore The pivot element is 3.

Entering = x_2 , Departing = S_1 , Key Element = 3

— $R_1(\text{new}) = R_1(\text{old}) \div 3$

$R_1(\text{old}) =$	8	2	3	0	1	0	0
$R_1(\text{new}) = R_1(\text{old}) \div 3$	2.6667	0.6667	1	0	0.3333	0	0

+ $R_2(\text{new}) = R_2(\text{old}) - 2R_1(\text{new})$

+ $R_3(\text{new}) = R_3(\text{old}) - 2R_1(\text{new})$

Iteration-2		C_j	3	5	4	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_3}$
x_2	5	2.6667	0.6667	1	0	0.3333	0	0	---
S_2	0	4.6667	-1.3333	0	(5)	-0.6667	1	0	$\frac{4.6667}{5} = 0.9333 \rightarrow$
S_3	0	9.6667	1.6667	0	4	-0.6667	0	1	$\frac{9.6667}{4} = 2.4167$
$Z = 13.3333$		Z_j	3.3333	5	0	1.6667	0	0	
		$Z_j - C_j$	0.3333	0	-4 ↑	1.6667	0	0	

Negative minimum $Z_j - C_j$ is -4 and its column index is 3. So, the entering variable is x_3 .

Minimum ratio is 0.9333 and its row index is 2. So, the leaving basis variable is S_2 .

\therefore The pivot element is 5.

Entering = x_3 , Departing = S_2 , Key Element = 5

+ $R_2(\text{new}) = R_2(\text{old}) \div 5$

+ $R_1(\text{new}) = R_1(\text{old})$

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⊕ $R_3(\text{new}) = R_3(\text{old}) - 4R_2(\text{new})$

Iteration-3		C_j	3	5	4	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio $\frac{X_B}{x_1}$
x_2	5	2.6667	0.6667	1	0	0.3333	0	0	$\frac{2.6667}{0.6667} = 4$
x_3	4	0.9333	-0.2667	0	1	-0.1333	0.2	0	---
S_3	0	5.9333	(2.7333)	0	0	-0.1333	-0.8	1	$\frac{5.9333}{2.7333} = 2.1707 \rightarrow$
$Z = 17.0667$		Z_j	2.2667	5	4	1.1333	0.8	0	
		$Z_j - C_j$	-0.7333 ↑	0	0	1.1333	0.8	0	

Negative minimum $Z_j - C_j$ is -0.7333 and its column index is 1. So, **the entering variable is x_1** .

Minimum ratio is 2.1707 and its row index is 3. So, **the leaving basis variable is S_3** .

∴ **The pivot element is 2.7333.**

Entering = x_1 , Departing = S_3 , Key Element = 2.7333

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⊕ $R_3(\text{new}) = R_3(\text{old}) \div 2.7333$

⊕ $R_1(\text{new}) = R_1(\text{old}) - 0.6667R_3(\text{new})$

⊕ $R_2(\text{new}) = R_2(\text{old}) + 0.2667R_3(\text{new})$

Iteration-4		C_j	3	5	4	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	MinRatio
x_2	5	1.2195	0	1	0	0.3659	0.1951	-0.2439	
x_3	4	1.5122	0	0	1	-0.1463	0.122	0.0976	
x_1	3	2.1707	1	0	0	-0.0488	-0.2927	0.3659	
$Z = 18.6585$		Z_j	3	5	4	1.0976	0.5854	0.2683	
		$Z_j - C_j$	0	0	0	1.0976	0.5854	0.2683	

Since all $Z_j - C_j \geq 0$

Hence, optimal solution is arrived with value of variables as :

$x_1 = 2.1707, x_2 = 1.2195, x_3 = 1.5122$

$Z = 18.6585$