

CHI-SQUARE TEST OF INDEPENDENCE (SOLVED EXAMPLE)

QUESTION :

1600 Families were selected randomly in a city to test the belief that high-income families usually send their children to private schools and low-income families often send their children to Government schools. The following results were obtained:

| School | Private | Govt. | Total |
|---------------|---------|-------|-------|
| Income | | | |
| Low | 494 | 506 | 1000 |
| High | 162 | 438 | 600 |
| Total | 656 | 944 | 1600 |

Test whether income and type of school are independent at $\alpha = 0.05$

Solution :

1. H_0 : Income and type of schools are independent
2. H_1 : Income and type of schools are not independent
3. $\alpha = 0.05$
4. Test Statistic : $\chi^2 = \sum \left[\frac{(O_{ij} - e_{ij})^2}{e_{ij}} \right]$ where $e_{ij} = \frac{R_i C_j}{G}$

Since $O_{11} = 494$, $O_{12} = 506$, $O_{21} = 162$, $O_{22} = 438$

then $e_{11} = \frac{R_1 C_1}{G} = \frac{(1000)(656)}{1600} = 410$ $e_{12} = \frac{R_1 C_2}{G} = \frac{(1000)(944)}{1600} = 590$

$e_{21} = \frac{R_2 C_1}{G} = \frac{(600)(656)}{1600} = 246$ $e_{22} = \frac{R_2 C_2}{G} = \frac{(600)(944)}{1600} = 354$

| O | e | $\frac{(O - e)^2}{e}$ |
|----------------|------|-----------------------|
| $O_{11} = 494$ | 410 | 17.2 |
| $O_{12} = 506$ | 590 | 11.96 |
| $O_{21} = 162$ | 246 | 28.68 |
| $O_{22} = 438$ | 354 | 19.93 |
| Total | 1600 | 77.78 |

Then χ^2 - calculated = 77.78

5. Critical Region : The critical region at $\alpha = 0.05$ for right-tailed test with d.f. $v = (2 - 1)(2 - 1) = 1$ is $\chi^2 > \chi_{0.05(1)} = 3.84$

6. Conclusion: Since the calculated value is greater than the tabulated value(critical value), therefore we reject the null hypothesis and accept the alternative hypothesis and hence conclude that the income and type of schools are dependent.

ASSIGNMENT:

The following table shows the relation between the number of accidents in 1 year and the age of the driver in a random sample of 500 drivers between 18 and 50. Test at alpha = 0.05, the hypothesis that the number of accidents is independent of driver's age.

| | | Age of Driver | | | Total |
|------------------|---|---------------|---------|---------|-------|
| | | 18 - 25 | 26 - 40 | over 40 | |
| No. of Accidents | 0 | 75 | 115 | 110 | 300 |
| | 1 | 50 | 65 | 35 | 150 |
| | 2 | 25 | 20 | 5 | 50 |
| Total | | 150 | 200 | 150 | 500 |