The Binomial Expansion of $(x + y)^n$ Let x and y be any real numbers, then

$$(x + y)^n = \binom{n}{0} x^n y^0 + \binom{n}{1} x^{n-1} y^1 + \binom{n}{2} x^{n-2} y^2 + \cdots + \binom{n}{n} x^0 y^n$$

$$= \sum_{i=0}^{n} \binom{n}{i} x^{n-i} y^i.$$

The Sum of a Geometric Series Let r be a real number such that $|r| < 1$, and m be any integer $m \geq 1$

$$\sum_{i=0}^{\infty} r^i = \frac{1}{1-r}, \quad \sum_{i=1}^{\infty} r^i = \frac{r}{1-r}, \quad \sum_{i=0}^{m} r^i = \frac{1 - r^{m+1}}{1-r}.$$

The (Taylor) Series Expansion of e^x Let x be any real number, then

$$e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}.$$

Some useful formulas for particular summations follow. The proofs (omitted) are most easily established by using mathematical induction.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2} \right)^2.$$

Gamma Function Let $t > 0$, then $\Gamma(t)$ is defined by the following integral:

$$\Gamma(t) = \int_0^{\infty} y^{t-1} e^{-y} dy.$$

Using the technique of integration by parts, it follows that for any $t > 0$

$$\Gamma(t + 1) = t\Gamma(t)$$

and if $t = n$, where n is an integer,

$$\Gamma(n) = (n - 1)!.$$

Further,

$$\Gamma(1/2) = \sqrt{\pi}.$$

If $\alpha, \beta > 0$, the ***Beta function***, $B(\alpha, \beta)$, is defined by the following integral,

$$B(\alpha, \beta) = \int_0^1 y^{\alpha-1} (1 - y)^{\beta-1} dy$$

and is related to the gamma function as follows:

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)}.$$

Common Probability Distributions, Means, Variances, and Moment-Generating Functions

Table 1 Discrete Distributions

Distribution	Probability Function	Mean	Variance	Moment-Generating Function
Binomial	$p(y) = \binom{n}{y} p^y (1-p)^{n-y}$; $y = 0, 1, \ldots, n$	np	$np(1-p)$	$[pe^t + (1-p)]^n$
Geometric	$p(y) = p(1-p)^{y-1}$; $y = 1, 2, \ldots$	$\dfrac{1}{p}$	$\dfrac{1-p}{p^2}$	$\dfrac{pe^t}{1-(1-p)e^t}$
Hypergeometric	$p(y) = \dfrac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$; $y = 0, 1, \ldots, n$ if $n \le r$, $y = 0, 1, \ldots, r$ if $n > r$	$\dfrac{nr}{N}$	$n\left(\dfrac{r}{N}\right)\left(\dfrac{N-r}{N}\right)\left(\dfrac{N-n}{N-1}\right)$	does not exist in closed form
Poisson	$p(y) = \dfrac{\lambda^y e^{-\lambda}}{y!}$; $y = 0, 1, 2, \ldots$	λ	λ	$\exp[\lambda(e^t - 1)]$
Negative binomial	$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r}$; $y = r, r+1, \ldots$	$\dfrac{r}{p}$	$\dfrac{r(1-p)}{p^2}$	$\left[\dfrac{pe^t}{1-(1-p)e^t}\right]^r$

Table 2 Continuous Distributions

Distribution	Probability Function	Mean	Variance	Moment-Generating Function
Uniform	$f(y) = \dfrac{1}{\theta_2 - \theta_1}; \theta_1 \leq y \leq \theta_2$	$\dfrac{\theta_1 + \theta_2}{2}$	$\dfrac{(\theta_2 - \theta_1)^2}{12}$	$\dfrac{e^{t\theta_2} - e^{t\theta_1}}{t(\theta_2 - \theta_1)}$
Normal	$f(y) = \dfrac{1}{\sigma\sqrt{2\pi}}\exp\left[-\left(\dfrac{1}{2\sigma^2}\right)(y - \mu)^2\right]$ $-\infty < y < +\infty$	μ	σ^2	$\exp\left(\mu t + \dfrac{t^2\sigma^2}{2}\right)$
Exponential	$f(y) = \dfrac{1}{\beta}e^{-y/\beta}; \quad \beta > 0$ $0 < y < \infty$	β	β^2	$(1 - \beta t)^{-1}$
Gamma	$f(y) = \left[\dfrac{1}{\Gamma(\alpha)\beta^\alpha}\right]y^{\alpha-1}e^{-y/\beta};$ $0 < y < \infty$	$\alpha\beta$	$\alpha\beta^2$	$(1 - \beta t)^{-\alpha}$
Chi-square	$f(y) = \dfrac{(y)^{(v/2)-1}e^{-y/2}}{2^{v/2}\Gamma(v/2)};$ $y > 0$	v	$2v$	$(1 - 2t)^{-v/2}$
Beta	$f(y) = \left[\dfrac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)}\right]y^{\alpha-1}(1 - y)^{\beta-1};$ $0 < y < 1$	$\dfrac{\alpha}{\alpha + \beta}$	$\dfrac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$	does not exist in closed form