Assignment No. 2

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Subject: Elasticity

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Submitted on: 1/5/2022

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QUESTION

STRAIN-DISPLACEMENT RELATION FROM CYLINDRICAL TO SPHERICAL COORDINATE SYSTEM

Answer:

The relation between Cylindrical and Spherical Coordinates is

$$r = \rho \sin\phi \qquad z = \rho \cos\phi \qquad \theta = \theta$$

Where

$$\rho = \sqrt{r^2 + z^2} \qquad \theta = \tan^{-1}\left[\frac{y}{x}\right]$$

$$\phi = \arccos\left(\frac{z}{\rho}\right)$$

The Partial derivatives for the above equation are

$$\frac{\partial}{\partial r} = \frac{\partial \rho}{\partial r} \cdot \frac{\partial}{\partial \rho} + \frac{\partial \phi}{\partial r} \cdot \frac{\partial}{\partial \phi}$$

$$= \sin\phi \frac{\partial}{\partial \rho} + \frac{r^2}{\sqrt{r^2 - z^2} \cdot \rho^{3/2}} \cdot \frac{\partial}{\partial \phi}$$

$$\frac{\partial}{\partial z} = \frac{\partial \rho}{\partial z} \cdot \frac{\partial}{\partial \rho} + \frac{\partial \phi}{\partial z} \cdot \frac{\partial}{\partial \phi}$$

$$= \cos\phi \frac{\partial}{\partial \rho} + \frac{rz}{\sqrt{r^2 - z^2} \cdot \rho^{3/2}} \frac{\partial}{\partial \phi}$$

Now,

$$U_r = U_\rho \sin\phi + U_\phi \frac{r^2}{\sqrt{r^2 - z^2} \cdot \rho^{3/2}} \quad ; \quad U_z = U_\rho \cos\phi + U_\phi \frac{rz}{\sqrt{r^2 - z^2} \cdot \rho^{3/2}}$$

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$\therefore u_\theta = u_\theta$

Calculating $e_\varsigma - \frac{\partial u_r}{\partial r}$

$$\hat{e}_\varsigma = \sin\phi \left[\frac{\partial}{\partial \varsigma} \left(u_\varsigma \sin\phi + u_\phi \frac{r^2}{\sqrt{r^2 - z^2} \cdot \varsigma^{3/2}} \right) \right.$$

$$\left. + \frac{r^2}{\sqrt{r^2 - z^2} \cdot \varsigma^{3/2}} \frac{\partial}{\partial \theta} \left[u_\varsigma \sin\phi + u_\phi \frac{r^2}{\sqrt{r^2 - z^2}\ \varsigma^{3/2}} \right] \right]$$

$$= \left[\frac{\partial u_\varsigma}{\partial \varsigma} \sin^2\phi + \frac{\partial u_\phi}{\partial \varsigma} \cdot \frac{r^2 \sin\phi}{\varsigma^{3/2} \sqrt{r^2 - z^2}} + \frac{u_\phi r^2}{\sqrt{r^2 - z^2}} \cdot \frac{\sin\phi}{\varsigma^{-5/2}} \right.$$

$$+ \frac{\partial u_\varsigma}{\partial \phi} \cdot \frac{\sin\phi\, r^2}{\sqrt{r^2 - z^2} \cdot \varsigma^{3/2}} + \frac{r^2 u_\varsigma \cos\phi}{\sqrt{r^2 - z^2}}$$

$$\left. + \frac{\partial u_\phi}{\partial \phi} \cdot \frac{r^4}{\varsigma^3 (r^2 - z^2)} \right]$$

$$\hat{e}_\varsigma = \frac{\partial u_\varsigma}{\partial \varsigma} \sin^2\phi + \left[\frac{\partial u_\phi}{\partial \varsigma} \cdot \frac{1}{\varsigma^{3/2}} + \frac{u_\phi}{\varsigma^{5/2}} \right.$$

$$\left. + \frac{\partial u_\varsigma}{\partial \phi} \cdot \frac{1}{\varsigma^{3/2}} \right) \frac{r^2 \sin\phi}{\sqrt{r^2 - z^2}} +$$

$$\left(\left(-u_\varsigma \cos\phi + \frac{\partial u_\phi}{\partial \phi} \cdot \frac{1}{\varsigma^3} \right) \frac{r^2}{\sqrt{r^2 - z^2}} \right) \frac{r^2}{\sqrt{r^2 - z^2}}$$

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$$\hat{e}_\phi = \frac{\partial u_z}{\partial z}$$

$$\hat{e}_\phi = \cos\phi \frac{\partial}{\partial \rho}\left\{ u_\rho \cos\phi + u_\phi \cdot \frac{rz}{\rho^{3/2} \cdot \sqrt{r^2 - z^2}} \right\} +$$

$$\frac{rz}{\sqrt{r^2-z^2}\,\rho^{3/2}} \frac{\partial}{\partial z}\left[u_\rho \cos\phi + u_\phi \frac{rz}{\sqrt{r^2-z^2} \cdot \rho^{3/2}} \right]$$

$$= \frac{\partial u_\rho}{\partial \rho} \cos^2\phi + \frac{\partial u_\phi}{\partial \rho} \cdot \frac{rz \cos\phi}{\rho^{3/2} \sqrt{r^2-z^2}} + \frac{u_\phi\, rz}{\sqrt{r^2-z^2}} \cdot \frac{\cos\phi}{\rho^{-5/2}}$$

$$+ \frac{\partial u_\rho}{\partial \phi} \frac{\cos\phi\, rz}{\rho^{3/2}\sqrt{r^2-z^2}} - \frac{\sin\phi \cdot rz\, u_\rho}{\sqrt{r^2-z^2} \cdot \rho^{3/2}} + \frac{\partial u_\phi}{\partial \phi}$$

$$\cdot \frac{r^2 z^2}{(r^2-z^2)\rho^3}$$

$$\hat{e}_\phi = \frac{\partial u_\rho}{\partial \rho} \cos^2\phi + \left(\frac{\partial u_\phi}{\partial \rho} \cdot \frac{1}{\rho^{3/2}} + \frac{u_\phi}{\rho^{-5/2}} + \frac{\partial u_\rho}{\partial \phi} \right.$$

$$\left. \frac{1}{\rho^{3/2}} \right) \frac{\cos\phi \cdot rz}{\sqrt{r^2-z^2}} + \left(\frac{\partial u_\phi}{\partial \phi} \cdot \frac{rz}{\sqrt{r^2-z^2} \cdot \rho^{3/2}} \right.$$

$$\left. - \frac{u_\rho \sin\phi}{\rho^{3/2}} \right) \frac{rz}{\sqrt{r^2-z^2}}$$

Therefore the strain displacement relation becomes

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$$e_{\rho} = \frac{\partial u_r}{\partial r} \qquad e_{\phi} = \frac{1}{\rho}\left(u_r + \frac{\partial u_z}{\partial \phi}\right)$$

$$e_{\theta} = \frac{1}{\rho\sin\phi}\left(\frac{\partial u_\theta}{\partial \theta} + \sin\phi\, u_r + \cos\phi\, u_z\right)$$

$$e_{\rho\phi} = \frac{1}{2}\left(\frac{1}{\rho}\frac{\partial u_r}{\partial \phi} + \frac{\partial u_z}{\partial \rho} - \frac{u_z}{\rho}\right)$$

$$e_{\phi\theta} = \frac{1}{2\rho}\left(\frac{1}{\sin\theta}\cdot\frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial \phi} - \cot\phi\, u_\theta\right)$$

$$e_{\theta\rho} = \frac{1}{2}\left(\frac{1}{\rho\sin\phi}\cdot\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial \rho} - \frac{u_\theta}{\rho}\right)$$