Chapter 5.3 Practice Key

Use Rice's Theorem, if possible, to prove that the following languages are undecidable.

1. $L_1 = \{< M > \mid M$ is a TM and L(M) accepts some input$\}$
   The property P is "L(M) accepts some input"
   - P is non-trivial since there is a TM that accepts some input and a TM that accepts nothing
   - P is a property of the language of TM's since for any two machines M_1 and M_2 such that $L(M_1) = L(M_1)$

$$M_1 \in L_1 \Longleftrightarrow L(M_1) = \text{accepts some input}$$
$$\Longleftrightarrow L(M_2) = \text{accepts nothing}$$
$$\Longleftrightarrow M_2 \in L_1$$

2. $L_2 = \{< M > \mid M$ is a TM and accepts all even numbers$\}$
   The property P is "L(M) accepts all even numbers"
   - P is non-trivial since there is a TM that accepts all strings of even numbers and a TM that accepts all strings of odd numbers
   - P is a property of the language of TM's since for any two machines M_1 and M_2 such that $L(M_1) = L(M_1)$

$$M_1 \in L_2 \Longleftrightarrow L(M_1) = \text{accepts strings of even numbers}$$
$$\Longleftrightarrow L(M_2) = \text{accepts strings of odd numbers}$$
$$\Longleftrightarrow M_2 \in L_2$$

3. $L_3 = \{< M > \mid M$ is a TM and accepts at least two strings of different lengths$\}$
   The property P is "L(M) accepts at least two strings of different lengths"
   - P is non-trivial since there is a TM that accepts strings of even length and a TM that accepts strings of odd length
   - P is a property of the language of TM's since for any two machines M_1 and M_2 such that $L(M_1) = L(M_1)$

$$M_1 \in L_3 \Longleftrightarrow L(M_1) = \text{accepts strings of even length}$$
$$\Longleftrightarrow L(M_2) = \text{accepts strings of even length}$$
$$\Longleftrightarrow M_2 \in L_3$$

4. $L_4 = \{< M > \mid M$ is a TM and accepts when given <M> as an input$\}$

<M> given as an input is not a property P, so Rice's Theorem does not apply

5. $L_5 = \{< M > \mid$ whenever M is given two positive integers as input on its input tape it halts with their sum on its work tape $\}$

M is given two positive integers as input on its input tape and halting with their sum on its work tape is not a property of the language of TMs, since it refers to the contents of the work tape, so Rice's Theorem does not apply

6. $L_1 = \{< M > \mid M$ is a TM and accepts the string 001$\}$

The property P is "L(M) accepts the string 001"

- P is non-trivial since there is a TM that accepts the string 001 and a TM that accepts the empty string
- P is a property of the language of TM's since for any two machines M_1 and M_2 such that $L(M_1) = L(M_1)$

$$M_1 \in L_3 \Longleftrightarrow L(M_1) = \text{accepts the string } 001$$
$$\Longleftrightarrow L(M_2) = \text{accepts the empty string}$$
$$\Longleftrightarrow M_2 \in L_3$$