Chapter 4 Practice

Show each pair of sets have equal cardinalities.

1. $\mathbb{Z}$ and S = $\{\dots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16, \dots\}$

2. $\{0, 1\} \times \mathbb{N}$ and $\mathbb{N}$

3. $\{0, 1\} \times \mathbb{N}$ and $\mathbb{Z}$

4. Odd integers

5. A = $\{3k : k \in \mathbb{Z}\}$ and B = $\{7k : k \in \mathbb{Z}\}$.

Prove each of the following.

6. The union of any two countably infinite sets is countably infinite.

7. The union of any three countably infinite sets is countably infinite.

8. The union of any four countably infinite sets is countably infinite.

9. The union of countably many infinite sets is countable. (Hint: Think about the proof that the set of all rationals is countable.)

10. The set of all irrationals is uncountable. (Hint: Try a proof by contradiction.)