Chapter 1.6 Practice

Use the pumping lemma to prove the languages below are not regular.

1. $L = \{a^i b^{3i} | \; i \geq 1\}$

   Let the pumping length be p > 0

   Pick the string $s = a^p b^{3p}$

   $s \in L$ and $|s| = 4p \geq p$

   3. $|xy| \leq p$, so x and y contain only a's

   2. $|y| > 0$, so y contains at least one a

   1. For all $i \geq 0$, $xy^i z \in A$, so let $xy^2 z = a^{i'} b^{3i}$, which would not keep the relationship of $L = a^i b^{3i}$

   You can think of $xyz = ab^3$, when $p = 1$, and $x = \varepsilon, y = a, z = b^3$. If we pump y, we get $xy^2 z = a^2 b^3$, which is not in L.

2. $L = \{a^j \, b^k \, a^{jk} \; | \; j \geq 1, k \geq 1\}$

   Let the pumping length be p > 0

   Pick the string $s = a^p b^{p+1} a^{2p+1}$

   $s \in L$ and $|s| = 4p + 2 \geq p$

   3. $|xy| \leq p$, so x and y contain only a's

   2. $|y| > 0$, so y contains at least one a

   1. For all $i \geq 0$, $xy^i z \in A$, so let $xy^2 z = a^{j'} \, b^k \, a^{jk}$, where $j' \neq j$, which would not keep the relationship of $L = a^j \, b^k \, a^{jk}$

   You can think of $xyz = a \, b^2 \, a^2$, when $p = 1$, and $x = \varepsilon, y = a, z = b^2 a^2$. If we pump y, we get $xy^2 z = a^2 b^2 a^2$, which is not in L.

3. $L = \{a^{k^3}\}$

Let the pumping length be p > 0

Pick the string $s = a^{p^3}$

$s \in L$ and $|s| = p^3 \geq p$

3. $|xy| \leq p$, so x and y contain only a's

2. $|y| > 0$, so y contains at least one a

1. For all $i \geq 0$, $xy^iz \in A$, so let $xy^2z = a^{p^3+1}$, where $p^3 + 1 \neq p^3$, which would not keep the relationship of $L = a^{k^3}$

You can think of $xyz = a^{1^3} = a$, when $p = 1$, and $x = \varepsilon, y = a, z = \varepsilon$. If we pump y, we get $xy^2z = a^2$, which is not in L.

Is the following language regular? Can you use the pumping lemma to prove that it is?

4. $L = \{a^2b^ma^nb^3 | m \geq 0, n \geq 0\}$

This language is regular. The pumping lemma cannot be used to show a language is regular. We can instead use a DFA, NFA, or regular expression. RE = $a^2b^*a^*b^3$