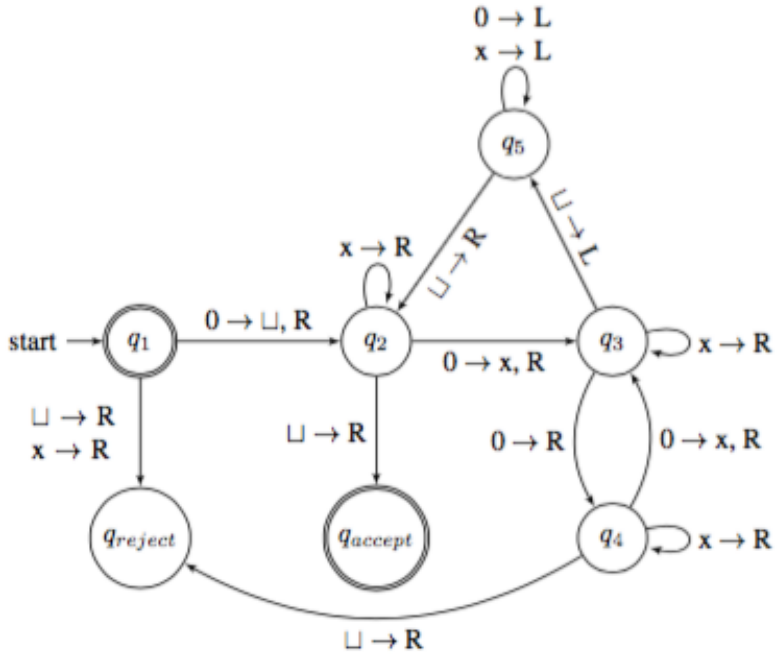
Chapter 3.1 Practice Key

Using the Turing Machine below, give the sequence of configurations leading to an accept or reject state.



1. 0
   Ans: $q_1 0 \sqcup \rightarrow \sqcup\, q_2 \sqcup \rightarrow \sqcup\sqcup\, q_{accept} \sqcup$

2. 000
   Ans: $q_1 000 \sqcup \rightarrow \sqcup\, q_2 00 \sqcup \rightarrow \sqcup\, x q_3 0 \sqcup \rightarrow \sqcup\, x 0 q_4 \sqcup \rightarrow \sqcup\, x0 \sqcup\, q_{reject} \sqcup$

3. 0000
   Ans: $q_1 0000 \sqcup \rightarrow \sqcup\, q_2 000 \sqcup \rightarrow \sqcup\, x q_3 00 \sqcup \rightarrow \sqcup\, x0 q_4 0 \sqcup \rightarrow \sqcup\, x0x q_3 \sqcup \rightarrow$
   $\sqcup\, x0 q_5 x \sqcup \rightarrow \sqcup\, x q_5 0x \sqcup \rightarrow \sqcup\, q_5 x0x \sqcup \rightarrow\, q_5 \sqcup\, x0x \sqcup \rightarrow$
   $\sqcup\, q_2 x0x \sqcup \rightarrow \sqcup\, x q_2 0x \sqcup \rightarrow \sqcup\, xx q_3 x \sqcup \rightarrow \sqcup\, xxx q_3 \sqcup \rightarrow$
   $\sqcup\, xx q_5 x \sqcup \rightarrow \sqcup\, x q_5 xx \sqcup \rightarrow \sqcup\, q_5 xxx \sqcup \rightarrow\, q_5 \sqcup\, xxx \sqcup \rightarrow$
   $\sqcup\, q_2 xxx \sqcup \rightarrow \sqcup\, x q_2 xx \sqcup \rightarrow \sqcup\, xx q_2 x \sqcup \rightarrow \sqcup\, xxx q_2 \sqcup \rightarrow$
   $\sqcup\, xxx \sqcup\, q_{accept} \sqcup$

4. 000000

Ans: $q_1000000 \sqcup\rightarrow\sqcup q_200000 \sqcup\rightarrow\sqcup xq_30000 \sqcup\rightarrow\sqcup x0q_4000 \sqcup\rightarrow$
$\sqcup x0xq_300 \sqcup\rightarrow \sqcup x0x0q_40 \sqcup\rightarrow\sqcup x0x0xq_3 \sqcup \rightarrow$
$\sqcup x0x0q_5x \sqcup\rightarrow\sqcup x0xq_50x \sqcup\rightarrow\sqcup x0q_5x0x \sqcup\rightarrow$
$\sqcup xq_50x0x \sqcup\rightarrow\sqcup q_5x0x0x \sqcup\rightarrow q_5 \sqcup x0x0x \sqcup\rightarrow$
$\sqcup q_2x0x0x \sqcup\rightarrow\sqcup xq_20x0x \sqcup\rightarrow\sqcup xxq_3x0x \sqcup\rightarrow$
$\sqcup xxxq_30x \sqcup\rightarrow\sqcup xxx0q_4x \sqcup\rightarrow\sqcup xxx0xq_4 \sqcup\rightarrow$
$\sqcup xxx0x \sqcup q_{reject} \sqcup$

Give implementation-level descriptions of Turing Machines that decide each of the languages below. (Hint: Start with a high-level recursive solution.)

5. All strings of a's and b's with an even number of a's and an even number of b's.

TM M = "on inputs w ∈ {a, b}*

i. If the first symbol is ⊔, accept.

ii. Place a mark on the top of the left-most unmarked tape symbol. If it is not an a or b, reject.

iii. Scan right to the next unmarked symbol that is the same and place a mark on it. If no same symbol is encountered before ⊔, reject.

iv. Scan left, if no unmarked characters remain, accept. Else, go to Step ii."

6. All strings of a's and b's with an odd number of a's and an odd number of b's.

TM M = "on inputs w ∈ {a, b}*

i. Find and mark an a symbol. Find and mark a b symbol. If either symbol is missing, reject. Else scan left.

ii. Place a mark on the top of the left-most unmarked tape symbol. If it is not an a or b, reject.

iii. Scan right to the next unmarked symbol that is the same and place a mark on it. If no same symbol is encountered before ⊔, reject.

iv. Scan left, if no unmarked characters remain, accept. Else, go to Step ii."

7. Every string w of a's and b's with at least as many a's as b's.

TM M = "on inputs w ∈ {a, b}*

i. If the first symbol is ⊔, accept.

ii. Place a mark on the top of the left-most unmarked a symbol. If no a symbols can be found, reject.

iii. Scan right to the next unmarked b symbol and place a mark on it. If no b symbols can be found, accept.

iv. Scan left, if no unmarked characters remain, accept. Else, go to Step ii."

8. $\{a^r b^s a^t | r \geq 0, s \geq 0, t \geq 0, s = 2r + t\}$

TM M = "on inputs w ∈ {a, b}*

i. If the first symbol is ⊔, accept.

ii. Place a mark on the top of the left-most unmarked a symbol. If no a symbols can be found, go to Step v.

iii. Scan right to the next 2 unmarked b symbols and place a mark on them. If 2 unmarked b symbols cannot be found, reject.

iv. Scan left and go to Step ii.

v. Place a mark on the top of the left-most unmarked b symbol. If no b symbols can be found, go to Step viii.

vi. Scan right to the next unmarked a symbol and place a mark on it. If an unmarked a symbol cannot be found, reject.

vii. Scan left and go to Step v.

viii. Scan the tape. If all symbols are marked, accept, else reject."

9. Palindromes on the alphabet {a, b}.

TM M = "on inputs w ∈ {a, b}*

i. If the first symbol is ⊔, accept.

ii. Place a mark on the top of the left-most unmarked symbol. If no unmarked symbols can be found, accept.

iii. Scan right to the last unmarked symbol. If that symbol is the same as the one in Step ii, mark this symbol. If this symbol is not the same, reject. If you cannot find an unmarked symbol, reject.

iv. Scan left and go to Step ii."

10. $\{c^r d^s c^r d^s \mid r \geq 0, s \geq 1\}$

TM M = "on inputs w ∈ {c, d}*

i. If the first symbol is ⊔, reject.

ii. Place a mark on the top of the left-most unmarked c symbol and go to Step iii. If no unmarked c symbols can be found, scan right to an unmarked d symbol. If no unmarked d symbols can be found, reject. Else, scan left and go to Step v.

iii. Scan right to a d symbol. If a d symbol cannot be found, reject. Scan right to the next unmarked c symbol, and mark it. If you cannot find an unmarked c symbol, reject.

iv. Scan left and go to Step ii.

v. Place a mark on the top of the left-most unmarked d symbol. If no unmarked d symbols can be found, go to Step viii.

vi. Scan right past marked c symbols to the first unmarked d symbol. If no marked c symbols can be found, place a mark on the top of the left-most unmarked d symbol. If an unmarked d symbol cannot be found, reject.

vii. Scan left and go to Step v.

viii. If all symbols are marked, accept."