Chapter 7.1 Practice Key

Find the upper bound in terms of O(n^x) for the problems below. Identify a value for c and n_0.

1. $3n^2 + n^3 \log n - 32n^{3/2} + 12n^4 + 17$?

   O(n^4), where c = 33 or 13 and n_0= 1 or any value greater than 1.

2. $30{,}000n^6 + 10n^{-25} - 32n^5 + 1{,}999n^2 - 5$?

   O(n^6), where c = 32009 or 30001 and n_0= 1 or any value greater than 1.

3. Sort the following functions from asymptotically smallest to asymptotically largest, indicating ties if there are any.

$\boldsymbol{n}$	$\mathbf{lg}\ (\boldsymbol{n})$	$\boldsymbol{n^2}$	$\mathbf{1}$
$(\mathbf{lg}\ \boldsymbol{n})^{\boldsymbol{n}}$	$(\mathbf{lg}\ \boldsymbol{n})^{\boldsymbol{lg\ n}}$	$\mathbf{n}^{\boldsymbol{lg\ n}}$	$\mathbf{n\ lg}\ (\boldsymbol{n})$
$(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{1000}})^{\boldsymbol{n}}$	$(\mathbf{1} + \frac{\mathbf{1}}{\boldsymbol{n}})^{\boldsymbol{n}}$	$\mathbf{lg}\ (\boldsymbol{n}^{\mathbf{1000}})$	$\boldsymbol{n}^{\mathbf{1}/\boldsymbol{lg\ n}}$

$$1 \equiv \left(\mathbf{1} + \frac{\mathbf{1}}{\boldsymbol{n}}\right)^{\boldsymbol{n}} \equiv \boldsymbol{n}^{\mathbf{1}/\boldsymbol{lg\ n}} \ll \mathbf{lg}\ (\boldsymbol{n}) \equiv \mathbf{lg}\ (\boldsymbol{n}^{\mathbf{1000}}) \ll \boldsymbol{n} \ll \mathbf{n\ lg}\ (\boldsymbol{n}) \ll \boldsymbol{n^2}$$
$$\ll (\mathbf{lg}\ \boldsymbol{n})^{\boldsymbol{lg\ n}} \ll \mathbf{n}^{\boldsymbol{lg\ n}} \ll (\mathbf{1} + \frac{\mathbf{1}}{\mathbf{1000}})^{\boldsymbol{n}} \ll (\mathbf{lg}\ \boldsymbol{n})^{\boldsymbol{n}}$$

Determine the runtime for the implementation level descriptions of the following turning machines.

4. M = “On input <G>, the encoding of a graph G:
   - a Select the first node of G and mark it. O(1)
   - b Repeat the following stage until no new nodes are marked: O(n)
     - i For each node in G, mark it if it is attached by an edge to node that is already marked. O(n)

c Scan all the nodes of G to determine if they all are marked. If they are, accept; otherwise, reject. O(n)

Total: O(1) + O(n) * O(n) + O(n) = O(n^2)

5. Old MacDonald(animals[1..n], noise[1..n]):

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for i = 1 to n:  O(n)
    Sing "Old MacDonald had a farm, E I E I O"
    Sing "And on this farm he had some animals[i], E I E I O"
    Sing "With a noise[i] noise[i] here, and a noise[i] noise[i] there"
    Sing "Here a noise[i], there a noise[i], everywhere noise[i] noise[i]"
    for j = i − 1 down to 1:  O(n)
        Sing "noise[j] noise[j] here, noise[j] noise[j] there"
        Sing "Here noise[j], there noise[j], everywhere noise[j] noise[j]"
        Sing "Old MacDonald had a farm, E I E I O."?
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Total: O(n) * O(n) = O(n^2)

6. TM M which decides the language $C = \{a^i b^j c^k \mid i \times j = k \text{ and } i, j, k \geq 1\}$.

M = "On input string w:

a Scan the input from left to right to determine whether it is a member of $a^+b^+c^+$; if not, reject. O(n)

b Return the head to the left end of the tape. O(n)

c Cross off an a and scan to the right until a b occurs. Shuttle between the b's and c's, crossing off one of each until all b's are crossed off. If all c's have been crossed off and some b's remain, reject. O(n)

d Restore all the crossed off b's and repeat Step c if there is another a to cross off. If all a's have been crossed off, determine whether all c's also have been crossed off. If yes, accept; otherwise reject." O(n)

Total: O(n) + O(n) + O(n) * O(n) = O(n^2)