CMSC 303 Introduction to Theory of Computing
Chapter 2 Practice

Key

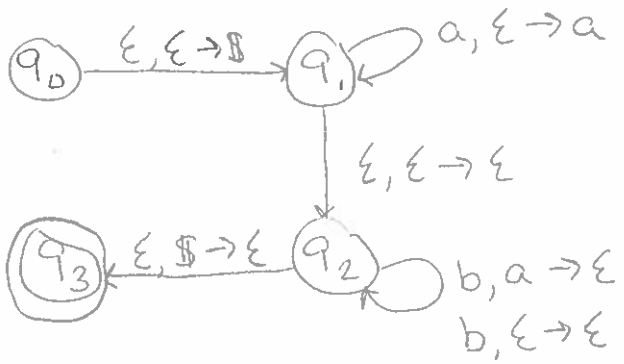
1. Define language $L = \{w \mid w$ starts and end with the same symbol and $w \in \{a, b\}^*\}$ Give a CFG generating L.

$S \to aTa \mid bTb \mid b \mid a \mid \varepsilon$

$T \to aT \mid bT \mid \varepsilon$

2. Create a state diagram for a PDA recognizing the language as defined below.
   $Q = \{q_0, q_1, q_2, q_3\}$,
   $\Sigma = \{a, b, \}$,
   $\Gamma = \{a, \$\}$,
   $q_0 = q_0$,
   $F = \{q_3\}$, and
   δ given by the following table, wherein blank entries signify $\emptyset$.

δ	a			b			ϵ		
pop	a	$	ϵ	a	$	ϵ	a	$	ϵ
q_0									$(q_1, \$)$
q_1			(q_1, a)						(q_2, ϵ)
q_2				(q_2, ϵ)		(q_2, ϵ)		(q_3, ϵ)	
q_3									



3. Describe the strings this PDA recognizes.

$L = \{a^n b^m \mid n \leq m\}$

4. Use the following CFG to parse the string, aababb:

$$\begin{aligned} R &\rightarrow XRX \mid S \\ S &\rightarrow aTb \mid bTa \\ T &\rightarrow XTX \mid X \mid \epsilon \\ X &\rightarrow a \mid b \end{aligned}$$

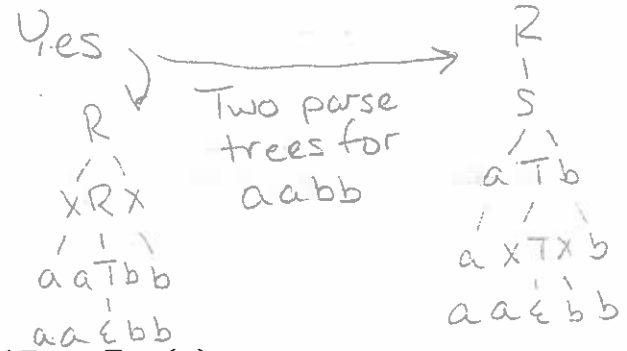
one way

$R \Rightarrow XRX \Rightarrow aRX \Rightarrow aXRXX \Rightarrow aaRXX \Rightarrow aaSXX$
$\Rightarrow aabTaXX \Rightarrow aab\epsilon aXX \Rightarrow aababXX \Rightarrow aababb$
$\Rightarrow aababb$

5. Choose another string from this CFG to parse and show the parse tree. Is this grammar ambiguous?

Your choice.

I pick aabb



6. Place the CFG below into Chomski Normal Form. $\Sigma = \{a\}$

$$\begin{aligned} A &\rightarrow BAB \mid B \mid \epsilon \\ B &\rightarrow aa \mid \epsilon \end{aligned}$$

① New Start State

$S \rightarrow A$
$A \rightarrow BAB \mid B \mid \epsilon$
$B \rightarrow aa \mid \epsilon$

② Get rid of $B \rightarrow \epsilon$

$S \rightarrow A$
$A \rightarrow BAB \mid B \mid A \mid BA \mid AB \mid \epsilon$
$B \rightarrow aa$

③ Get rid of $A \rightarrow \epsilon$

$S \rightarrow A \mid \epsilon$
$A \rightarrow BAB \mid B \mid A \mid BA \mid AB \mid BB$
$B \rightarrow aa$

④ Get rid of $A \rightarrow A$ & $S \rightarrow A$

$S \rightarrow BAB \mid B \mid BA \mid AB \mid BB \mid \epsilon$
$A \rightarrow BAB \mid B \mid BA \mid AB \mid BB$
$B \rightarrow aa$

⑤ Get rid of $S \rightarrow B$ & $A \rightarrow B$

$S \rightarrow BAB \mid aa \mid BA \mid AB \mid BB \mid \epsilon$
$A \rightarrow BAB \mid aa \mid BA \mid AB \mid BB$ $B \rightarrow aa$

4. Use the following CFG to parse the string, aababb:

$$
\begin{aligned}
R &\rightarrow XRX \mid S \\
S &\rightarrow aTb \mid bTa \\
T &\rightarrow XTX \mid X \mid \epsilon \\
X &\rightarrow a \mid b
\end{aligned}
$$

one way

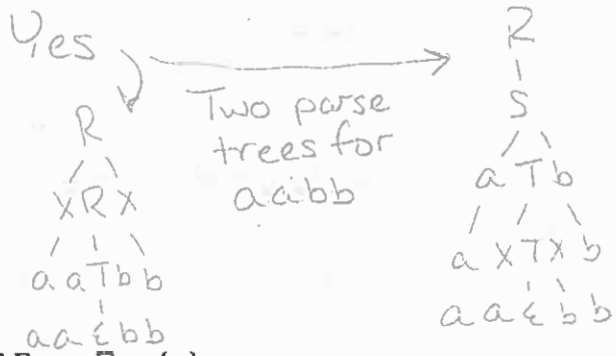
$R \Rightarrow XRX \Rightarrow aRX \Rightarrow aXRXX \Rightarrow aaRXX \Rightarrow aaSXX$
$\Rightarrow aabTaXX \Rightarrow aab\epsilon aXX \Rightarrow aababXX \Rightarrow aababab$
$\Rightarrow aababb$

5. Choose another string from this CFG to parse and show the parse tree. Is this grammar ambiguous?

Your choice.

I pick aabb

Yes



6. Place the CFG below into Chomski Normal Form. $\Sigma = \{a\}$

$$
\begin{aligned}
A &\rightarrow BAB \mid B \mid c \\
B &\rightarrow aa \mid c
\end{aligned}
$$

① New Start State

$S \rightarrow A$
$A \rightarrow BAB \mid B \mid \epsilon$
$B \rightarrow aa \mid \epsilon$

② Get rid of $B \rightarrow \epsilon$

$S \rightarrow A$
$A \rightarrow BAB \mid B \mid A \mid BA \mid AB \mid \epsilon$
$B \rightarrow aa$

③ Get rid of $A \rightarrow \epsilon$

$S \rightarrow A \mid \epsilon$
$A \rightarrow BAB \mid B \mid A \mid BA \mid AB \mid BB$
$B \rightarrow aa$

④ Get rid of $A \rightarrow A$ + $S \rightarrow A$

$S \rightarrow BAB \mid B \mid BA \mid AB \mid BB \mid \epsilon$
$A \rightarrow BAB \mid B \mid BA \mid AB \mid BB$
$B \rightarrow aa$

⑤ Get rid of $S \rightarrow B$ & $A \rightarrow B$

$S \rightarrow BAB \mid aa \mid BA \mid AB \mid BB \mid \epsilon$
$A \rightarrow BAB \mid aa \mid BA \mid AB \mid BB$ $B \rightarrow aa$

6 cont. $S \to BAB \mid aa \mid BA \mid AB \mid BB \mid \varepsilon$

$A \to BAB \mid aa \mid BA \mid AB \mid BB$

$B \to aa$

⑥ Place into correct form

$S \to XB \mid DD \mid BA \mid AB \mid BB \mid \varepsilon$

$A \to XB \mid DD \mid BA \mid AB \mid BB$

$B \to DD$

$D \to a$

$X \to BA$

All in form

$A \to BC$

$A \to a$

$A \to \varepsilon$