CMSC 303 Introduction to Theory of Computation, VCU
Assignment 7

Turned in electronically in PDF, PNG or Word format

Total marks: 65 marks + 3 marks bonus for all the answers typed out.

This assignment focuses on the complexity classes P and NP, as well as polynomial-time reductions.

1. We begin looking at Complexity Theory

   (a) [3 marks] Let $f(n) = 6n^3 + 15n^2 + 20n - 5$. Prove that $f(n) \in O(n^3)$. In your proof, give explicit values for c and n_0.
   Solution: Observe that

   $$6n^3 + 15n^2 + 20n - 5 \leq 6n^3 + 15n^2 + 20n \leq 6n^3 + 15n^3 + 20n^3 \leq 41n^3,$$

   where the second inequality holds for $n \geq 1$. Setting $c = 41$ and $n_0 = 1$, we hence have that $f(n) \leq cg(n)$ for all $n \geq n_0$, i.e. $f \in O(g(n))$, as required.

   (b) [4 marks] Is $f(n) = n^{100} \in o(g(2^n))$? Explain why or why not.

   (c) [10 marks] Let M be a Turing Machine to decide the language Z = $\{a^i b^j c^k \mid$ the number of c's is greater than the number of a's and b's$\}$. What is the runtime of M?

2. We now will look at some terms we discussed

   (a) [3 marks] What does it mean for a Turing Machine to run in polynomial time?

   (b) [3 marks] What does A $\leq_p$ B mean?

   (c) [2 marks] What does it mean if A is NP-hard?

   (d) [2 marks] What does it mean if A is NP-complete?

   (e) [3 marks] Is the halting problem in NP? Explain your reasoning?

3. [10 marks] Consider a puzzle, PUZZLE, where you start with a n x n matrix with a random allocation of three characters, $X's, Y's$, and $Z's$, in the grid locations. The goal of the puzzle is to remove the characters, one at a time, so that each row contains only characters of one letter, such as only $X's$, and each column contains at least one character. Show that PUZZLE $\in$ NP.

   Solution: Need to show that PUZZLE $\in$ NP. Start with a non-deterministic TM. We can choose three characters from the puzzle grid locations. Then we can check that each character is in its own row and column. If so, "accept", else "reject". We can also,build the verifier by passing in three characters. We can check that each character is in the grid and that each character is in its own row and column. If so, "accept", else "reject".

4. [10 marks] Let X $= \{< G, k >|$ G has a a subset of k nodes where every other node in G is adjacent to one of the k nodes$\}$. Show that X is NP-complete.

5. [10 marks] Read Sections 8.1 and 8.2 in the course text and, using the material we discussed in Chapter 7, prove that NP is contained in PSPACE. Fully elaborate your answer.

6. [5 marks] Research the Traveling Salesman Problem and discuss this problem in terms of complexity theory.