

Chapter 7.1 Practice

Find the upper bound in terms of $O(n^x)$ for the problems below. Identify a value for c and n_0 .

1. $3n^2 + n^3 \log n - 32n^{3/2} + 12n^4 + 17?$
2. $30,000n^6 + 10n^{-25} - 32n^5 + 1,999n^2 - 5?$
3. Sort the following functions from asymptotically smallest to asymptotically largest, indicating ties if there are any.

n	$\lg(n)$	n^2	1
$(\lg n)^n$	$(\lg n)^{\lg n}$	$(\lg n)^n$	$n \lg(n)$
$(1 + \frac{1}{1000})^n$	$(1 + \frac{1}{n})^n$	$\lg(n^{1000})$	$n^{1/\lg n}$

Determine the runtime for the implementation level descriptions of the following turning machines.

4. $M = \text{"On input } <G>, \text{ the encoding of a graph } G:$
 - a Select the first node of G and mark it.
 - b Repeat the following stage until no new nodes are marked:
 - i For each node in G , mark it if it is attached by an edge to node that is already marked.
 - c Scan all the nodes of G to determine if they all are marked. If they are, accept; otherwise, reject.
5. Old MacDonald(animals[1..n], noise[1..n]):
for $i = 1$ to n :
 Sing “Old MacDonald had a farm, E I E I O”
 Sing “And on this farm he had some animals[i], E I E I O”
 Sing “With a noise[i] noise[i] here, and a noise[i] noise[i] there”

Sing “Here a noise[i], there a noise[i], everywhere noise[i] noise[i]”
for $j = i - 1$ down to 1:

Sing “noise[j] noise[j] here, noise[j] noise[j] there”

Sing “Here noise[j], there noise[j], everywhere noise[j] noise[j]”

Sing “Old MacDonald had a farm, E I E I O.”?

6. TM M which decides the language $C = \{a^i b^j c^k \mid i \times j = k \text{ and } i, j, k \geq 1\}$.

M = “On input string w:

- Scan the input from left to right to determine whether it is a member of $a^+ b^+ c^+$; if not, reject.
- Return the head to the left end of the tape.
- Cross off an a and scan to the right until a b occurs. Shuttle between the b's and c's, crossing off one of each until all b's are crossed off. If all c's have been crossed off and some b's remain, reject.
- Restore all the crossed off b's and repeat Step c if there is another a to cross off. If all a's have been crossed off, determine whether all c's also have been crossed off. If yes, accept; otherwise reject.”