Math 310: Linear Algebra
Fall 2025
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Linear Algebra Test Review — Chapters 4 & 5
(Strang, *Introduction to Linear Algebra*, 5th ed.)
Prerequisite: Precalculus. No calculus required.

Part I — Concept Check (Short Answer)

1. Define orthogonal vectors.

2. Define orthonormal set.

3. Distinguish row space and column space.

4. What does Gram–Schmidt produce?

5. Give one geometric interpretation of the determinant.

6. True/False: "If $\det(A) = 0$ then the columns of A are linearly dependent."

7. Effect on det when: (a) rows swapped, (b) a row multiplied by 3, (c) a multiple of one row added to another.

8. If A is invertible, what must hold for $\det(A)$?

9. Define an orthogonal matrix.

10. Why are orthogonal matrices useful for numerical computations?

Part II — Practice Problems

Orthogonality & Projections

11. Compute the dot product

$$u = (3, -1, 2), \qquad v = (1, 4, -2).$$

12. Are these orthogonal?

$$a = (2, 1, -1), \qquad b = (1, -2, 3).$$

13. Find the projection of $y = (4, 1, 2)$ onto $a = (1, 2, 2)$.

14. Apply one Gram–Schmidt step to obtain an orthogonal pair from

$$v_1 = (1, 2, 0), \qquad v_2 = (2, 1, 1).$$

15. Let

$$Q = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

(a) Show Q is orthogonal. (b) What geometric transformation does Q represent?

Least Squares (no calculus)

16. Fit the points $(1,3),(2,4),(3,7)$ with a line $y = mx+b$ using the normal equations $A^T Ax = A^T b$. Show your work and solve for m, b.

Determinants

17. Compute $\det \begin{pmatrix} 2 & 1 \\ 5 & -3 \end{pmatrix}$.

18. Compute $\det \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 2 & 0 & 1 \end{pmatrix}$.

19. If $\det(C) = -6$, find:

(a) $\det(2C)$ (state answer in terms of $\dim C = n$),

(b) $\det(C^{-1})$,

(c) $\det(C^T)$.

20. Compute $\det \begin{pmatrix} 4 & 1 & 2 \\ 2 & 0 & 1 \\ 6 & 1 & 3 \end{pmatrix}$ using row operations.

21. True/False: If a square matrix has two equal rows then its determinant is zero.

Part III — Multi-Step Problems

22. Gram–Schmidt & Orthonormal Basis. For

$$v_1 = (1,1,0), \quad v_2 = (1,0,1), \quad v_3 = (0,1,1),$$

(a) use Gram–Schmidt to find an orthogonal basis $\{u_1, u_2, u_3\}$, and (b) normalize to obtain an orthonormal basis.

23. Determinant Properties. Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Compute $\det(A),\ \det(B),\ \det(AB)$ and verify $\det(AB) = \det(A)\det(B)$.

24. Invertibility Test (determinant). Without computing inverses decide whether each matrix is invertible:

$$(a)\ \begin{pmatrix} 4 & 2 \\ 6 & 3 \end{pmatrix} \qquad (b)\ \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 2 & 3 & 1 \end{pmatrix}.$$

25. Application (projection in 3D). For force $F = (5,-1,2)$ and direction $d = (2,1,1)$ compute:

(a) component of F in the direction of d (projection),

(b) component of F perpendicular to d.