Chapter 2.2 Practice Key

For each language below, construct a CFG generating it.

1. The strings with either all 1s or all 0s.

$S \rightarrow 1T \mid 0U$
$T \rightarrow 1T \mid \varepsilon$
$U \rightarrow 0U \mid \varepsilon$

2. The strings with a substring of 0s followed by a substring of 1s.

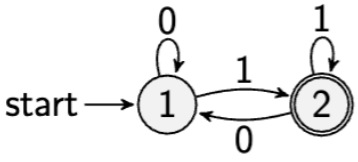
$S \rightarrow TU$
$T \rightarrow 0T \mid \varepsilon$
$U \rightarrow 1U \mid \varepsilon$

3. The strings with 0 or more 1's and 0's.

$S \rightarrow \varepsilon \mid ST$
$T \rightarrow 1T \mid 0T \mid \varepsilon$

Convert the following NFAs to CFGs.

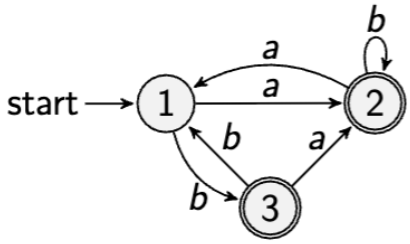
4.



$R_1 \rightarrow 0R_1|1R_2$

$R_2 \rightarrow 0R_1|1R_2|\varepsilon$

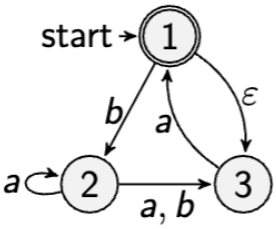
5.



$R_1 \rightarrow aR_2|bR_3$

$R_2 \rightarrow aR_1|bR_2|\varepsilon$

$R_3 \rightarrow aR_2|bR_1|\varepsilon$

6. 

$R_1 \rightarrow bR_2|R_3|\varepsilon$

$R_2 \rightarrow aR_2|aR_3|bR_3$

$R_3 \rightarrow aR_1$

Convert the following CFGs to Chomsky Normal Form.

7. S → aSa | bY | Ya
   Y → bY | aY | ε

New Start State:
S_0 → S
S → aSa | bY | Ya
Y → bY |aY | ε

Remove Y→ ε:
S_0 → S
S → aSa | bY | Ya | b | a
Y → bY | aY | b | a

Remove S_0 → S:
S_0 → aSa | bY | Ya | b | a
S → aSa | bY | Ya | b | a
Y → bY |aY | b | a

Convert each rule to form A → BC or A → a:
S_0 → AC | AY | YA | b | a
S → AC | BY | YA | b | a
Y → BY |AY | b | a
A → a
B → b
C → SA

8. R → S | T
   S → aSb | ab
   T → aTbb | abb

No need for a new start state since have: R → S | T
No ε transitions to remove
Remove R → S | T: R → aSb | ab | aTbb | abb
S → aSb | ab
T → aTbb | abb

Convert each rule to form A → BC or A → a:
R → AC | AB | ED | AD
S → AC | AB
T → ED | AD
A → a
B → b
C → SB
D → BB
E → AT

9. P → 0P | 1Q | ε
   Q → 0Q | 1P

New Start State: S_0 → P
P → 0P | 1Q | ε
Q → 0Q | 1P

Remove P→ ε: S_0 → P | ε (OK since at start)
P → 0P | 1Q | 0
Q → 0Q | 1P | 1

Remove S_0 → P: S_0 → 0P | 1Q | 0 | ε
P → 0P | 1Q | 0
Q → 0Q | 1P | 1

Convert each rule to form A → BC or A → a:
S_0 → AP | BQ | 0 | ε
P → AP | BQ | 0
Q → AQ | BP | 1
A → 0
B → 1

10. A → BAB | B | ε
    B → 00 | ε

New Start State:
S_0 → A
A → BAB | B | ε
B → 00 | ε

Remove B → ε:
S_0 → A
A → BAB | B | AB | BA | A | ε
B → 00

Remove A → ε:
S_0 → A | ε
A → BAB | B | AB | BA | A | BB
B → 00

Remove A → A:
S_0 → A | ε
A → BAB | B | AB | BA | BB
B → 00

Remove S_0 → A:
S_0 → BAB | B | AB | BA | BB | ε
A → BAB | B | AB | BA | BB
B → 00

Remove A→ B:
S_0 → BAB | B | AB | BA | BB | ε
A → BAB | 00 | AB | BA | BB
B → 00

Remove S_0 → B:
S_0 → BAB | 00 | AB | BA | BB | ε
A → BAB | 00 | AB | BA | BB
B → 00

Convert each rule to form A → BC or A → a:
S_0 → BY | XX | AB | BA | BB | ε
A → BY | XX | AB | BA | BB
B → XX
X → 0
Y → AB