

# Chapter 4 Practice Key

Show each pair of sets have equal cardinalities.

1.  $\mathbb{Z}$  and  $S = \{\dots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16, \dots\}$

$\dots, -2, -1, 0, 1, 2, 3, 4, 5, \dots$

$\dots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16, \dots$

2.  $\{0, 1\} \times \mathbb{N}$  and  $\mathbb{N}$

$(0, 1), (1, 1), (0, 2), (1, 2), (0, 3), (1, 3), \dots$

$1, 2, 3, 4, 5, 6, \dots$

3.  $\{0, 1\} \times \mathbb{N}$  and  $\mathbb{Z}$

$\dots, (0, 4), (0, 3), (0, 2), (0, 1), (1, 1), (1, 2), (1, 3), (1, 4), \dots$

$\dots, -3, -2, -1, 0, 1, 2, 3, 4, \dots$

4. Odd integers and even integers

$\dots, -3, -1, 1, 3, 5, 7, 9, \dots$

$\dots, -4, -2, 0, 2, 4, 6, 8, \dots$

5.  $A = \{3k : k \in \mathbb{Z}\}$  and  $B = \{7k : k \in \mathbb{Z}\}$ .

$\dots, -6, -3, 0, 3, 6, 9, \dots$

$\dots, -14, -7, 0, 7, 14, 21, \dots$

Prove each of the following.

6. The union of any two countably infinite sets is countably infinite.

$$\mathbb{N} \cup \mathbb{N} = 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, \dots$$

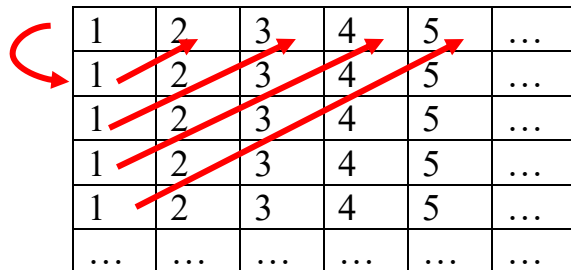
7. The union of any three countably infinite sets is countably infinite.

$$\mathbb{N} \cup \mathbb{N} \cup \mathbb{N} = 1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4, 5, 5, 5, \dots$$

8. The union of any four countably infinite sets is countably infinite.

$$\mathbb{N} \cup \mathbb{N} \cup \mathbb{N} \cup \mathbb{N} = 1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, \dots$$

9. The union of countably many infinite sets is countable. (Hint: Think about the proof that the set of all rationals is countable.)



|     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 3   | 4   | 5   | ... |
| 1   | 2   | 3   | 4   | 5   | ... |
| 1   | 2   | 3   | 4   | 5   | ... |
| 1   | 2   | 3   | 4   | 5   | ... |
| 1   | 2   | 3   | 4   | 5   | ... |
| ... | ... | ... | ... | ... | ... |

10. The set of all irrationals is uncountable.

Cantor's Diagonalization Argument

0.00110854...

0.11987654...

0.22889615...

0.33985724...

...

If I pick  $0.0303030\dots$ , I have found a number that is not in the set and not the same as the diagonalization of  $0.0188\dots$ . I can always find a number like this that is not the same as what is in the set above and not on the diagonal.