Chapter 3 Classwork Key

1. Show how the following Turing Machine M accepts or rejects this string: aabbbccccccc. M decides the language $C = \{a^i b^j c^k | i \times j = k \text{ and } i, j, k \geq 1\}$.

   TM M = "on inputs w ∈ {a, b, c}*

   i. Scan the input from left to right to determine whether it is a member of $a^+b^+c^+$, reject if not.

   ii. Scan left.

   iii. Mark an unmarked a. If there are no a's left to mark, go to Step vi.

   iv. Scan to the right until a b occurs. Shuttle between the b's and c's, marking one of each until all b's are marked. If all c's have been marked and some b's remain, reject.

   v. Unmark all the marked b's and repeat Step ii.

   vi. Scan right to see if all c's are marked. If so, accept; otherwise, reject.

Start	~~aa~~~~bb~~b~~c~~cccc	~~aab~~bb~~ccccc~~c
a̲abbbccccccc	~~aa~~~~bbb~~b~~cc~~ccc	~~aab~~b̲~~cccc~~cc
~~aa~~a̲bbbccccccc	~~aa~~~~bbb~~b̲~~cc~~cccc	~~aabb~~b̲~~cccc~~cc
~~aa~~a̲bbbccccccc	~~aa~~~~bbbbcc~~c̲~~ccc	~~aabbb~~b̲~~cccc~~c
~~aa~~~~b~~b̲bbccccccc	~~aa~~~~bbbbccc~~c̲~~cc	~~aabbb~~b̲~~cccccc~~
~~aa~~~~bbb~~c̲ccccc	~~aa~~a̲~~bbb~~c~~ccc~~ccc	~~aabbb~~b̲~~cccccc~~
~~aa~~~~bbb~~c̲~~c~~cccc	~~aa~~b̲~~bbc~~c~~ccc~~ccc	~~aabbbcccccc~~
~~aa~~~~b~~b̲~~bc~~ccccc	~~aab~~b̲~~bccc~~ccc	~~aabbbcccccc~~␣̲
~~aa~~~~bbb~~b̲~~cccccc —↑	~~aab~~~~bbc~~c~~c~~c~~c̲~~c —↑	accept

Note: Underlined letter is where we are, and crossed through letter is marked.

2. Give an implementation-level description of a Turing Machine that describes L, where L accepts the alphabet Σ = {0, 1} and triples the string on the tape, for example, given 0100 it would write 010001000100. L = {0, 1}*.

   TM M = "on inputs w ∈ {0,1}*

   i. If the first symbol is ⊔, accept.

   ii. Mark the first unmarked symbol. Scan to the right to the first empty cell and write the unmarked symbol, and mark it with a different mark. Repeat for all unmarked symbols.

   iii. Unmark all the marked original symbols. Scan left.

   iv. Mark the first unmarked symbol. Scan to the right to the first empty cell and write the unmarked symbol, and mark it with a different mark. Repeat for all unmarked symbols.

   v. Accept.

3. Give an implementation-level description of a Turing Machine that describes L, where L accepts strings that contain ***aba*** where the alphabet is Σ = {a, b}.

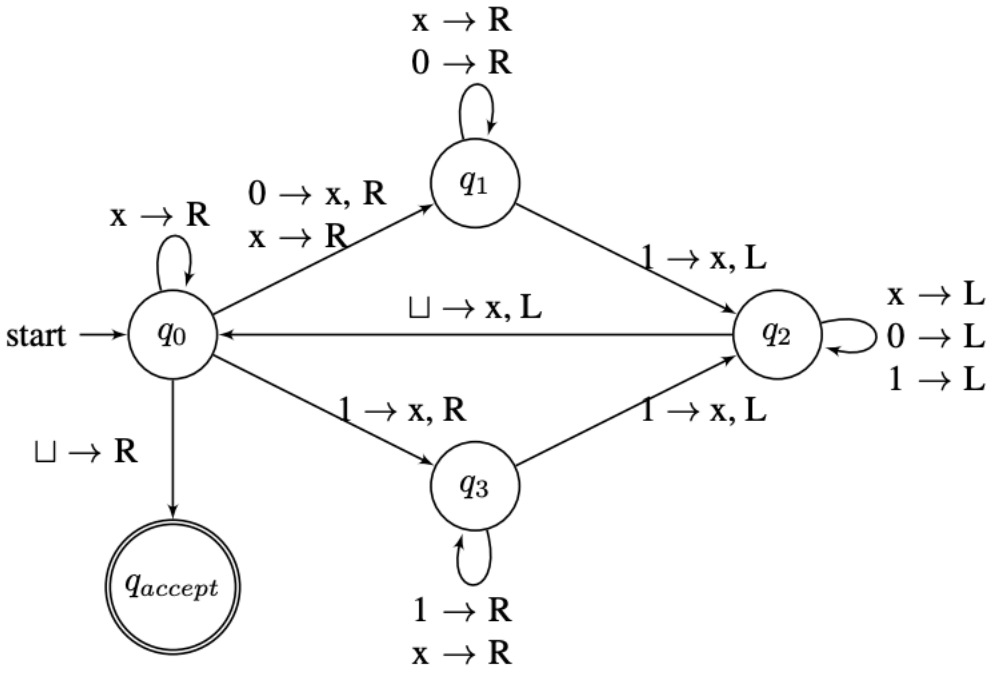
   TM M = "on inputs w ∈ {a, b}*

   i. If the first symbol is ⊔, reject.

   ii. Scan to the right. If an *a* occurs, move right to the next cell. If that symbol is a *b*, move right to the next cell. If that symbol is an *a*, accept. If not, move one cell to the right.

   iii. If that cell is empty, reject, else go to Step ii.

Given the Turing Machine below, decide if the given string is in the language. Show the configuration at each step.



4. String 01101

Tape:

0	1	1	0	1	⊔	⊔	…

Tape:	Configuration:
$\underline{0}1101$	$q_0 01101$
x$\underline{1}$101	x$q_1$1101
$\underline{x}$x101	$q_2$xx101 What happens now? This string will **not** be accepted.

5. String ε

Tape:

⊔	⊔	⊔	⊔	⊔	⊔	⊔	…

Tape:	Configuration:
⊔	q_0 ⊔
⊔⊔	⊔ q_{accept} ⊔
	This string will be accepted.

6. String 1111

Tape:

1	1	1	1	⊔	⊔	⊔	…

Tape:	Configuration:
$\underline{1}111$	$q_0 1111$
x$\underline{1}11$	x$q_3 111$
x1$\underline{1}1$	x1$q_3 11$
x11$\underline{1}$	x11$q_3 1$
x1$\underline{1}$x	x1$q_2$1x
x$\underline{1}$1x	x$q_2$11x
$\underline{x}$11x	q_2x11x
	What happens now? This string will **not** be accepted.