Chapter 2.1 Practice Key

For each language below, construct a CFG generating it.

1. $\{a^i b^j \mid i = j\}$

   S → aSb | ε

2. $\{a^i b^j \mid i \neq j\}$

   S → aSb | B
   B → aC | Db
   C → aC | ε
   D → Db | ε

3. All strings of the form $0^a 1^b 0^c$ where $a + c = b$. Note: this language is the concatenation of 2 simpler languages.

   S → AB
   A → 0A1 | ε
   B → 1B0 | ε

4. $\{w \mid w$ contains at least two $1s\}$

   S → T1T1T
   T → 0T | 1T | ε

5. All binary strings with both an even number of zeros and an even number of ones.

   E → 00E | 11E | 10D | 01D | ε
   D → 01E | 10E | 00D 11D

6. Show the derivation or parse tree of the following string, aababbabb, using the grammar G:

   S → aSb | bSa | ε

   S → aSb → aaSbb → aabSabb → aababSbabb → aababεbabb → <u>aababbabb</u>

7. Show the derivation or parse tree of the following string, aaaaaabbb using the grammar G:

   S → AB
   A → aaA | ε
   B → Bb| ε

   S → AB → aaAB → aaaaAB → aaaaaaAB → aaaaaaεB → aaaaaaB → aaaaaaBb → aaaaaaBbb → aaaaaaBbbb → aaaaaaεbbb → <u>aaaaaabbb</u>

8. Show the derivation or parse tree of the following string, 10101110, using the grammar G:

   S → A | B | C
   A → D10D
   D → 0D | 1D | ε
   B → 0B | 0
   C → 1C | 1

   S → A → D10D → D10D → 1D10D → 10D10D → 10ε10D → 1010D → 10101D → 101011D → 1010111D → 10101110D → 10101110ε → <u>10101110</u>

9. Use the following CFG to parse the string, aababb:

   R → XRX | S
   S → aTb | bTa
   T → XTX | X | ε
   X → a | b

   R → XRX → aRX → aXRXX → aaRXX → aaSXX → aabTaXX→ aabεaXX → aabaXX → aababX → aababb

   Or

   R → S → aTb → aXTXb → aaTXb → aaXTXXb → aabTXXb → aabεXXb → aabXXb → aabaXb → aababb

10. Choose another string from the CFG in question 9 to parse and show the parse tree. Is this grammar ambiguous?

abab →		or	
	R		R
	S		XRX
	aTb		aRX
	aXTXb		aSX
	abTXb		abTaX
	abεXb		abεaX
	abXb		abaX
	abab		abab

This language is ambiguous since strings can be parsed using two different left-most derivations.