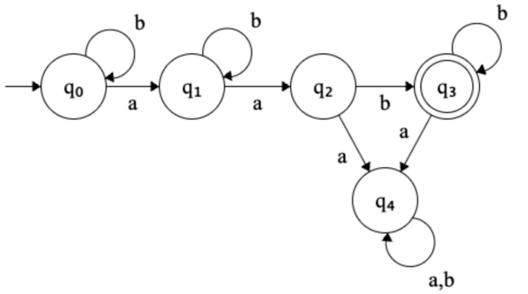
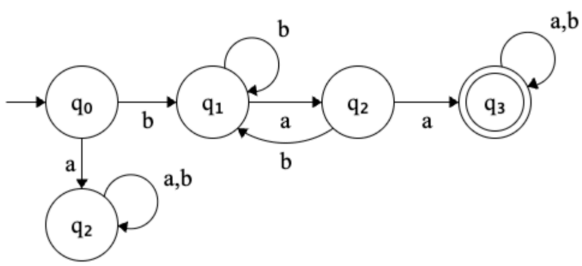
Chapter 1 Review Key

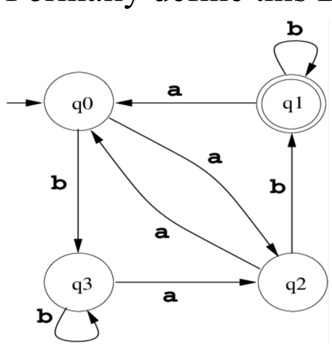
1. Give the state diagram for a DFA with $\Sigma = \{a, b\}$ that accepts precisely the strings that contain exactly two a's and ends with a b.



2. Give the state diagram for a DFA with $\Sigma = \{a, b\}$ that accepts precisely the strings that starts with b and contains the string aa.



3. Formally define this DFA:



$Q = \{q_0, q_1, q_2, q_3\}$

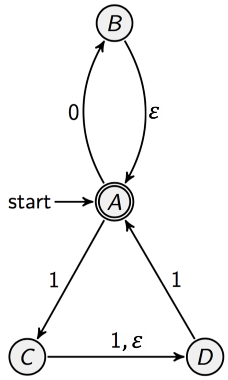
$\Sigma = \{a, b\}$

$q_0 = q_0$

$F = \{q_1\}$

δ	a	b
q_0	q_2	q_3
q_1	q_0	q_1
q_2	q_0	q_1
q_3	q_2	q_3

4. Formally define this NFA:



$Q = \{A, B, C, D\}$

$\Sigma = \{0,1\}$

$\Sigma_\varepsilon = \{0,1,\varepsilon\}$

$q_0 = A$

$F = \{A\}$

δ	0	1	ε
A	{B}	{C}	∅
B	∅	∅	{A}
C	∅	{D}	{D}
D	∅	{A}	∅

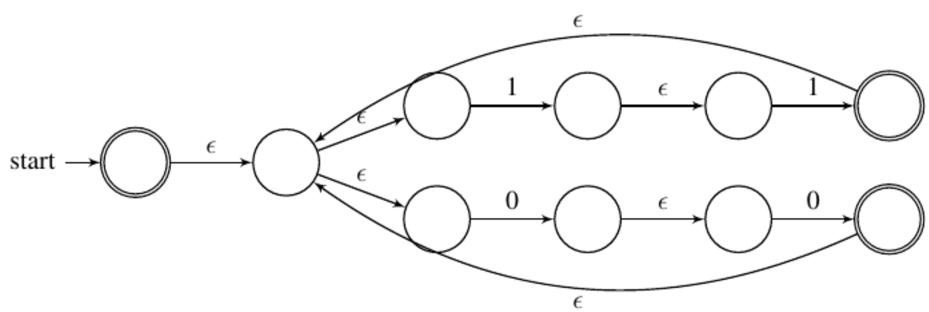
5. {w | w is all strings containing two 0s followed by a 1 (It does not have to be consecutive, just in that order.)}

   $\Sigma^*0\ \Sigma^*0\ \Sigma^*1\Sigma^*$

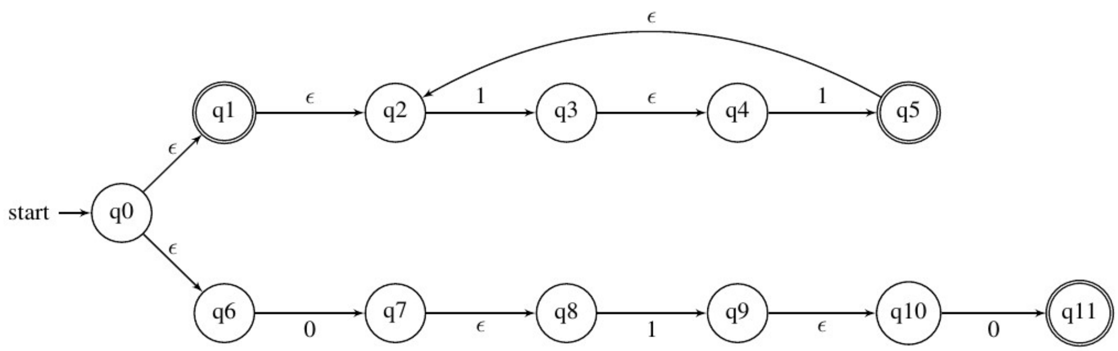
6. {w | w has only two 0s and has at least one 1}

   $1^*01^*01^+ \cup 1^*01^+01^* \cup 1^+01^*01^*$

7. Construct an NFA from the following regular expressions:
   a. (11 ∪ 00)*



b. (11)* ∪ 010



8. Use the pumping lemma to prove $L = \{a^j\, b^k\, a^{jk} \mid j \geq 1, k \geq 1\}$ is not a regular language.

   Let the pumping length be p > 0

   Pick the string $s = a^p b^{p+1} a^{2p+1}$

   $s \in L$ and $|s| = 4p + 2 \geq p$

   3. $|xy| \leq p$, so x and y contain only a's

   2. $|y| > 0$, so y contains at least one a

   1. For all $i \geq 0$, $xy^iz \in A$, so let $xy^2z = a^{j'}\, b^k\, a^{jk}$, where $j' \neq j$, which would not keep the relationship of $L = a^j\, b^k\, a^{jk}$

   You can think of $xyz = a\, b^2\, a^2$, when $p = 1$, and $x = \varepsilon, y = a, z = b^2a^2$. If we pump y, we get $xy^2z = a^2b^2a^2$, which is not in L.