

# Chapter 7.1 Practice

Find the upper bound in terms of  $O(n^x)$  for the problems below. Identify a value for  $c$  and  $n_0$ .

1.  $3n^2 + n^3 \log n - 32n^{3/2} + 12n^4 + 17$ ?
2.  $30,000n^6 + 10n^{-25} - 32n^5 + 1,999n^2 - 5$ ?
3. Sort the following functions from asymptotically smallest to asymptotically largest, indicating ties if there are any.

$n$	$\lg(n)$	$n^2$	1
$(\lg n)^n$	$(\lg n)^{\lg n}$	$(\lg n)^n$	$n \lg(n)$
$(1 + \frac{1}{1000})^n$	$(1 + \frac{1}{n})^n$	$\lg(n^{1000})$	$n^{1/\lg n}$

Determine the runtime for the implementation level descriptions of the following turning machines.

4.  $M =$  “On input  $\langle G \rangle$ , the encoding of a graph  $G$ :
  - a Select the first node of  $G$  and mark it.
  - b Repeat the following stage until no new nodes are marked:
    - i For each node in  $G$ , mark it if it is attached by an edge to node that is already marked.
  - c Scan all the nodes of  $G$  to determine if they all are marked. If they are, accept; otherwise, reject.
5. Old MacDonald(animals[1..n], noise[1..n]):
 

for  $i = 1$  to  $n$ :

Sing “Old MacDonald had a farm, E I E I O”

Sing “And on this farm he had some animals[i], E I E I O”

Sing “With a noise[i] noise[i] here, and a noise[i] noise[i] there”

Sing “Here a noise[i], there a noise[i], everywhere noise[i] noise[i]”  
for  $j = i - 1$  down to 1:

Sing “noise[j] noise[j] here, noise[j] noise[j] there”

Sing “Here noise[j], there noise[j], everywhere noise[j] noise[j]”

Sing “Old MacDonald had a farm, E I E I O.”?

6. TM  $M$  which decides the language  $C = \{a^i b^j c^k \mid i \times j = k \text{ and } i, j, k \geq 1\}$ .

$M =$  “On input string  $w$ :

- a Scan the input from left to right to determine whether it is a member of  $a^+ b^+ c^+$ ; if not, reject.
- b Return the head to the left end of the tape.
- c Cross off an  $a$  and scan to the right until a  $b$  occurs. Shuttle between the  $b$ 's and  $c$ 's, crossing off one of each until all  $b$ 's are crossed off. If all  $c$ 's have been crossed off and some  $b$ 's remain, reject.
- d Restore all the crossed off  $b$ 's and repeat Step c if there is another  $a$  to cross off. If all  $a$ 's have been crossed off, determine whether all  $c$ 's also have been crossed off. If yes, accept; otherwise reject.”