CMSC303 Introduction to Theory of Computation, VCU
Assignment 1

Turned in electronically in PDF, PNG or Word format before the start of class

Total marks: 26 marks + 3 bonus marks for all the answers typed out.

NOTE: As this is a warmup assignment intended to refresh your memory on background material, it will be marked only for completeness, *not* correctness. It is your responsibility to compare your answers with the solutions (to be posted after the due date) to gauge how well you understand the concepts on this assignment.

1 Exercises

1. (6 marks) Let A be the set $\{x, y\}$ and B be the set $\{x, y, z\}$.

   (a) (1 mark) Is A a subset of B?
   (b) (1 mark) Is B a subset of A?
   (c) (1 mark) What is $A \cup B$?
   (d) (1 mark) What is $A \cap B$?
   (e) (1 mark) What is $A \times B$?
   (f) (1 mark) What is the power set of B?

2. (2 marks) Sipser, Ex. 0.4: If A has a elements and B has b elements, how many elements are in $A \times B$? Explain your answer.

3. (2 marks) Sipser, Ex. 0.5: If C is a set with c elements, how many elements are in the power set of C? Explain your answer.

4. (7 marks) Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 10\}$. The unary function $f : X \mapsto Y$ and the binary function $g : X \times Y \mapsto Y$ are described in the following tables.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

   (a) (1 mark) What is the value of $f(3)$?
   (b) (2 marks) What are the domain and co-domain of f?
   (c) (1 mark) What is the value of $g(3, 9)$?
   (d) (2 marks) What are the domain and co-domain of g?
   (e) (1 mark) What is the value of $g(5, f(5))$?

5. (3 marks) Sipser, Ex. 0.8. Consider the undirected graph $G = \{V, E\}$, where V, the set of nodes, is $\{1, 2, 3, 4\}$ and E, the set of edges, is $\{\{1, 2\}, \{2, 3\}, \{1, 3\}, \{2, 4\}, \{1, 4\}\}$. Draw the graph G. What are the degrees of each node? Indicate a path from node 3 to node 4 on your drawing of G.

2 Problems

1. (2 marks) Sipser, Prob. 0.12 (0.11 in 2nd edition): Find the error in the following proof that all horses are the same color.
   CLAIM: In any set of h horses, all horses are the same color.
   PROOF: By induction on h.

   Base Case: For $h = 1$. In any set containing just one horse, all horses clearly are the same color.

   Induction Step: For $k \geq 1$ assume that the claim is true for $h = k$ and prove that it is true for $h = k + 1$. Take any set H of $k + 1$ horses. We show that all the horses in this set are the same color. Remove one horse from this set to obtain the set H_1 with just k horses. By the induction hypothesis, all the horses in H_1 are the same color. Now replace the removed horse and remove a different one to obtain the set H_2. By the same argument, all the horses in H_2 are the same color. Therefore all horses in H must be the same color, and the proof is complete.

2. (4 marks) Prove using induction that
$$\sum_{m=0}^{n} m = \frac{n(n+1)}{2}.$$