CMSC 303 Introduction to Theory of Computation
Final Exam Review

Here is what you should know:

1. Short answer.
   (a) What is a language
   (b) What is a *configuration* of a Turing machine
   (c) What is the input alphabet Σ and tape alphabet Γ for a Turing machine
   (d) What counts as one "computational step" on a Turing machine?
   (e) Properties and running of a DFA and length of strings.
   (f) Languages generated by a CFG.
2. Properties of the following:
   (a) Regular languages
   (b) Context-free languages
   (c) Regular expressions
   (d) Non-deterministic finite automaton
   (e) Turing-recognizable and Turing-decidable languages
   (f) $A \leq_m B$ and what we can then determine
   (g) $A \leq_P B$ and what we can then determine
   (h) NP-hard and NP-complete.

3. Philosophical implications encountered in this course.
   (a) What is a universal Turing machine and how it changed this view on computing?
   (b) What does the Church-Turing thesis state about the Turing machine model of computing, and why it is important?
   (c) What does it mean for a language to be *undecidable*? Examples of undecidable languages.
4. Regular and context-free languages.
   (a) Draw the state diagram for a DFA recognizing the language

   $$L = \{x \mid x \text{ contains the substring 1010 and ends with a 0}\}.$$

   (b) Give a regular expression for the language

   $$L = \{x \mid x \text{ begins with a 1, contains the substring 001, and ends with a 0}\}.$$

(c) Give a CFG generating the language $\{01x10 \mid 0{,}1 \in \Sigma, x \in \Sigma^*\}$.

(d) Define language, $L = \{ww^R | w \in \{a, b\}^*\}$. Recall that w^R means w written backwards. Use the Pumping Lemma to prove that a language is not regular.

5. Turing machines.

(a) What does it mean that a Turing machine model of computing robust?

(b) Let

$$L = \{ww^R \mid w \in \{a, b\}^*\}$$

Recall that w^R means w written backwards. Give an implementation-level description of a TM that decides L.

(c) What happens if we alter our model of a Turing machine in some way?

(d) What does it mean for a non-deterministic Turing machine to accept an input?

6. Undecidability.

(a) What does it mean for a set to be countable?

(b) Show that the set of Fibonacci numbers are countable.

(c) Show that the set Σ^*, where Σ = {English alphabet} is uncountable using a proof by diagonalization.

(d) The proof of a mapping reduction as we showed for $HALT_{TM}$ (e) What the mapping reduction provides that reductions do not?

(e) Use Rice's theorem to prove L = {<M>| M is a TM and $1011 \in L(M)$} is undecidable.

7. P and NP.

(a) What it means for a Turing machine to run in polynomial time?

(b) Let

$$L = \{ww^R \mid w \in \{a, b\}^*\}$$

Recall that w^R means w written backwards. State the runtime of L.

(c) Use the definition of NP that states each NP language has a polynomial time verifier to prove that HAM_{PATH} = {<G,u,v> | there is a Hamilton path from u to v in graph G}, where a Hamilton path is a path within a graph from one vertex to another that visits every vertex exactly once. Find the non-deterministic TM and the verifier.

(d) Prove if $55x^3 + 2x^2 - 66 = O(n^3)$ using a value for c and n_0.

8. NP-completeness.

(a) The difference between NP-hard and NP-complete.

(b) What is the relationship if any between undecidable problems and problems in P and NP?