Linear Algebra Practice Test Review – Chapters 1–3

Based on Gilbert Strang, *Introduction to Linear Algebra (5th Edition)*

Chapter 1: Introduction to Vectors

1. Write the vector form of the line through the points $P(1, 2)$ and $Q(4, 5)$.

2. Let $v = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ and $w = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$. Compute:

(a) $v + w$

(b) $3v - 2w$

(c) $v \cdot w$

(d) the angle between v and w

3. A vector x is perpendicular to both $a = (1, 1, 0)$ and $b = (0, 1, 1)$. Find a unit vector in the direction of x.

4. Determine whether the vectors $(1, 2, 3), (2, 4, 6), (1, 0, 1)$ are linearly independent.

5. Describe geometrically what it means for two vectors in $\mathbb{R}^2$ to be linearly dependent.

6. Compute the projection of $b = (3, 4)$ onto $a = (1, 2)$. Verify that the error vector $e = b - \text{proj}_a b$ is perpendicular to a.

7. If $u = (2, -1, 1)$, find a vector v such that $\|v\| = 5$ and v is parallel to u.

8. The plane $x + 2y + 3z = 6$ passes through $(0, 0, 2)$. Find the normal vector and two direction vectors lying in the plane.

Chapter 2: Solving Linear Equations

9. Solve the system using elimination:

$$\begin{cases} 2x + y = 5 \\ 4x + 3y = 11 \end{cases}$$

10. Write the following system in matrix form $Ax = b$ and determine if it is consistent:

$$\begin{cases} x + 2y + 3z = 6 \\ 2x + 5y + 8z = 15 \\ 3x + 4y + 7z = 14 \end{cases}$$

11. Perform Gaussian elimination on $A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & 0 & 3 \\ -1 & -2 & 3 & 0 \end{bmatrix}$ to reach echelon form. Identify pivot columns.

12. For the same matrix, determine the rank.

13. Suppose a system $Ax = b$ has a unique solution. What must be true about the pivots of A?

14. Describe geometrically when a system has: no solution, exactly one solution, or infinitely many solutions.

15. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Does $Ax = b$ have a solution? Explain.

16. Solve $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & 5 \end{bmatrix} x = \begin{bmatrix} 3 \\ 2 \\ 8 \end{bmatrix}$ using elimination.

17. Find $\det \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ and interpret its meaning.

18. Explain why elimination fails when a pivot is zero and how row swaps fix the problem.

Chapter 3: Vector Spaces and Subspaces

19. State the three requirements for a subspace of $\mathbb{R}^n$.

20. Which of the following are subspaces of $\mathbb{R}^3$?

(a) $\{(x, y, z) : x + y + z = 0\}$

(b) $\{(x, y, z) : x = 1\}$

(c) $\text{Span}\{(1, 0, 0), (0, 1, 1)\}$

21. Find a basis for the column space of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 0 & 1 \end{bmatrix}$.

22. Determine the nullspace of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$.

23. Verify that every vector x in the nullspace satisfies $Ax = 0$.

24. If the columns of A are linearly independent, what can be said about the nullspace of A?

25. State and interpret the Rank–Nullity Theorem.

26. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$. Find the dimensions of its column space and nullspace.

27. Describe the relationship among the four fundamental subspaces (for A and A^T).

28. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$. Identify its rank, basis for column space, and basis for nullspace.

29. True or False (justify):

(a) The zero vector is in every subspace.

(b) The nullspace of A contains all solutions to $Ax = b$.

(c) The column space of A is a subspace of $\mathbb{R}^m$.

30. Suppose the columns of A are a_1, a_2, a_3. Explain what it means for b to be in the column space of A.