CMSC 303 Introduction to Theory of Computing, VCU
Assignment 3

Turned in electronically in PDF, PNG or Word format

Total marks: 62 marks $+$ 3 bonus marks for all the answers typed out.

Unless otherwise noted, the alphabet for all questions below is assumed to be $\Sigma = \{0, 1\}$.

1. [12 marks] This question develops your ability to devise regular expressions, given an explicit definition of a language. For each of the following languages, prove they are regular by giving a regular expression which describes them. Justify your answers.

   (a) $L = \{x \mid x$ begins with one 1 and ends with two $0's\}$.

   (b) $L = \{x \mid x$ contains at least three $0's\}$.

   (c) $L = \{1, 111, \epsilon\}$.

   (d) $L = \{x \mid$ the length of x is at most $5\}$.

   (e) $L = \{x \mid x$ doesn't contain the substring $010\}$.

   (f) $L = \{x \mid |x| > 0$, i.e. x is non-empty$\}$.

2. This question tests your understanding of how to translate a regular expression into a finite automaton. Using the construction of Lemma 1.55, construct NFAs recognizing the languages described by the following regular expressions.

   (a) [5 marks] $R = \emptyset^*$.

   (b) [10 marks] $R = (0 \cup 1)^* 010 (0 \cup 1)^*$.

3. [15 marks] This question tests your understanding of how to translate a finite automaton into a regular expression. Consider DFA $M = (Q, \Sigma, \delta, q, F)$ such that $Q = \{q_1, q_2, q_3\}$, $q = q_1$, $F = \{q_1, q_3\}$, and δ is given by:

δ	0	1
q_1	q_2	q_2
q_2	q_3	q_2
q_3	q_2	q_1

   Draw the state diagram for M, and then apply the construction of Lemma 1.60 to obtain a regular expression describing $L(M)$.

4. [10 marks] This question allows you to practice proving a language is non-regular via the Pumping Lemma. Using the Pumping Lemma (Theorem 1.70), give formal proofs that the following languages are *not* regular:

   (a) $L = \left\{www \mid w \in \{0,1\}^*\right\}$.

   (b) $L = \{1^n 0^m 1^n \mid m, n \geq 0\}$.

5. This question further tests your understanding of the subtleties of the Pumping Lemma:

   (a) [5 marks] Let $B_1 = \left\{1^k y \mid y \in \{0,1\}^* \text{ and } y \text{ contains at least } k \text{ 1s, for } k \geq 1\right\}$. Show that B_1 is a regular language.

   (b) [5 marks] Let $B_2 = \left\{1^k y \mid y \in \{0,1\}^* \text{ and } y \text{ contains at most } k \text{ 1s, for } k \geq 1\right\}$. Show that B_2 is not a regular language.