CMSC303 Introduction to Theory of Computation, VCU
Assignment 2

Turned in electronically in PDF, PNG or Word format

Total marks: 44 marks $+$ 3 bonus marks for all the answers typed out.

Unless otherwise noted, the alphabet for all questions below is assumed to be $\Sigma = \{0, 1\}$. Also note that all DFA's in your solutions should have one transition for each state in the DFA for each character in the alphabet.

1. (6 marks) This question tests your comfort with "boundary cases" of DFA's. Draw the state diagrams of DFA's recognizing each of the following languages.
   (a) (2 marks) $L = \{\epsilon\}$ for ϵ the empty string.
   (b) (2 marks) $L = \emptyset$.
   (c) (2 marks) $L = \{0, 1\}^*$.
2. (12 marks)
   (a) (2 marks) Draw the state diagram for a DFA recognizing language $L_1 = \{x \mid x$ contains at least three 0's$\}$.
   (b) (2 marks) Draw the state diagram for a DFA recognizing language $L_2 = \{x \mid x$ starts with a 1 and contains at most one 0$\}$.
   (c) (4 marks) Draw the state diagram for a DFA recognizing language $L_3 = L_1 \cup L_2$. Hint: One option is to use the construction of Theorem 1.25 in the text.
   (d) (4 marks) Draw the state diagram for a NFA recognizing language $L_3 = L_1 \cup L_2$.
3. (4 marks) Draw the state diagram for a DFA recognizing language $L_4 = \{x \mid x$ is any string except 0 and 1$\}$.
4. (2 marks) Consider DFA $M = (\{q_1, q_2, q_3, q_4\}, \{0, 1\}, \delta, q_1, \{q_1, q_4\})$ for δ defined as the following table. Give the state diagram of this machine.

	0	1
q_1	q_1	q_2
q_2	q_3	q_4
q_3	q_2	q_1
q_4	q_3	q_4

5. (8 marks) This question illustrates differences and commonalities between NFAs and DFAs.
   (a) (4 marks) Show that if M is a DFA that recognizes language B, then swapping the accept and nonaccept states in M yields a new DFA M' recognizing the complement of B, $\overline{B}$. Which operation does this imply the regular languages are closed under?
   (b) (4 marks: 2 marks for counterexample, 2 marks for question on closure under complement) Prove by counterexample that if M is a NFA that recognizes languages B, then swapping the accept and nonaccept states in M does not necessarily yield an NFA recognizing $\overline{B}$. With this in mind, is the class of languages recognized by NFA's closed under complement (explain your answer!)?

6. (12 marks) This question tests your understanding of the equivalence between DFAs and NFAs. Consider NFA $M = (\{q_1, q_2\}, \{0, 1\}, \delta, q_1, \{q_1\})$ for δ defined as:

δ	0	1	ϵ
q_1	$\{q_1, q_2\}$	$\{q_2\}$	$\emptyset$
q_2	$\emptyset$	$\{q_1, q_2\}$	$\{q_1\}$

(a) (4 marks) Draw the state diagrams for M.

(b) (2 marks) Based on the construction of Theorem 1.39 in the text, start to build the DFA M' that is equivalent to M by identifying the number of DFA states and listing them.

(c) (2 marks) Identify the DFA M' starting and acceptance states.

(d) (4 marks) Draw the state diagram for the DFA M' equivalent to M based on the construction of Theorem 1.39 in the text (recall the latter proves that DFAs and NFAs are equivalent).