CMSC 303 Introduction to Theory of Computation, VCU
Assignment 7

Turned in electronically in PDF, PNG or Word format

Total marks: 65 marks + 3 marks bonus for all the answers typed out.

This assignment focuses on the complexity classes P and NP, as well as polynomial-time reductions.

1. We begin looking at Complexity Theory
   (a) [3 marks] Let $f(n) = 6n^3 + 15n^2 + 20n - 5$. Prove that $f(n) \in O(n^3)$. In your proof, give explicit values for c and n_0.
   (b) [4 marks] Is $f(n) = n^{100} \in o(g(2^n))$? Explain why or why not.
   (c) [10 marks] Let M be a Turing Machine to decide the language Z = $\{a^i b^j c^k \mid$ the number of c's is greater than the number of a's and b's$\}$. What is the runtime of M?
2. We now will look at some terms we discussed
   (a) [3 marks] What does it mean for a Turing Machine to run in polynomial time?
   (b) [3 marks] What does A $\leq_p$ B mean?
   (c) [2 marks] What does it mean if A is NP-hard?
   (d) [2 marks] What does it mean if A is NP-complete?
   (e) [3 marks] Is the halting problem in NP? Explain your reasoning?
3. [10 marks] Consider a puzzle, PUZZLE, where you start with a n x n matrix with a random allocation of three characters, $X's, Y's$, and $Z's$, in the grid locations. The goal of the puzzle is to remove the characters, one at a time, so that each row contains only characters of one letter, such as only $X's$, and each column contains at least one character. Show that PUZZLE $\in$ NP.
4. [10 marks] Let X $= \{< G, k >| \; G$ has a a subset of k nodes where every other node in G is adjacent to one of the k nodes$\}$. Show that X is NP-complete.
5. [10 marks] Read Sections 8.1 and 8.2 in the course text and, using the material we discussed in Chapter 7, prove that NP is contained in PSPACE. Fully elaborate your answer.
6. [5 marks] Research the Traveling Salesman Problem and discuss this problem in terms of complexity theory.