CMSC303 Introduction to Theory of Computation, VCU
Assignment 2

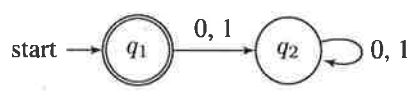
Turned in electronically in PDF, PNG or Word format before the start of class

Total marks: 44 marks + 3 bonus marks for all the answers typed out.

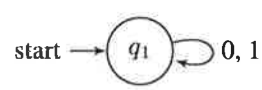
Unless otherwise noted, the alphabet for all questions below is assumed to be $\Sigma = \{0, 1\}$.

1. (6 marks) This question tests your comfort with "boundary cases" of DFA's. Draw the state diagrams of DFA's recognizing each of the following languages.

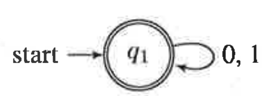
   (a) (2 marks) $L = \{\epsilon\}$ for ϵ the empty string.
   Solution:



   (b) (2 marks) $L = \emptyset$.
   Solution:

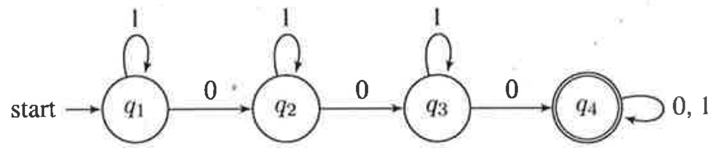


   (c) (2 marks) $L = \{0, 1\}^*$.
   Solution:

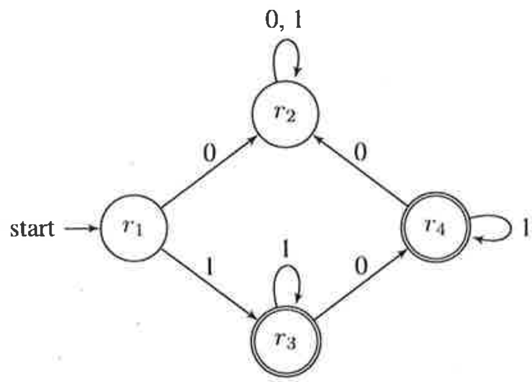


2. (12 marks)

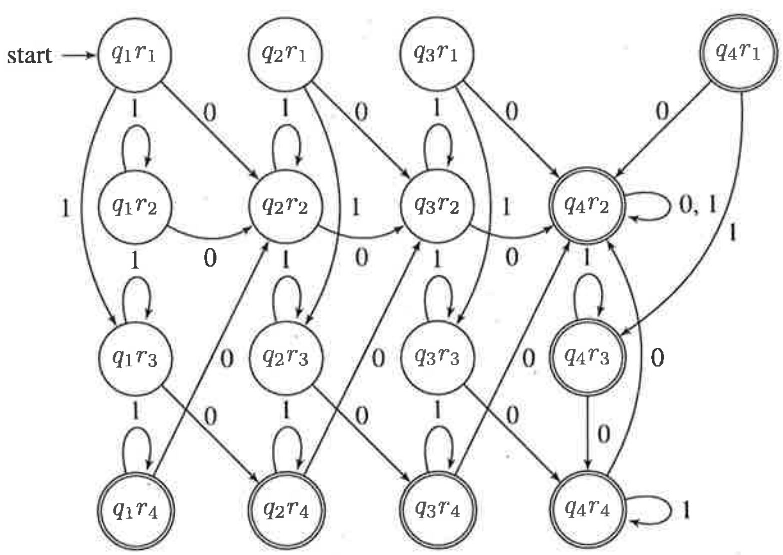
(a) (2 marks) Draw the state diagram for a DFA recognizing language $L_1 = \{x \mid x \text{ contains at least three 0s}\}$.
Solution:



(b) (2 marks) Draw the state diagram for a DFA recognizing language $L_2 = \{x \mid x$ starts with a 1 and contains at most one 0
Solution:

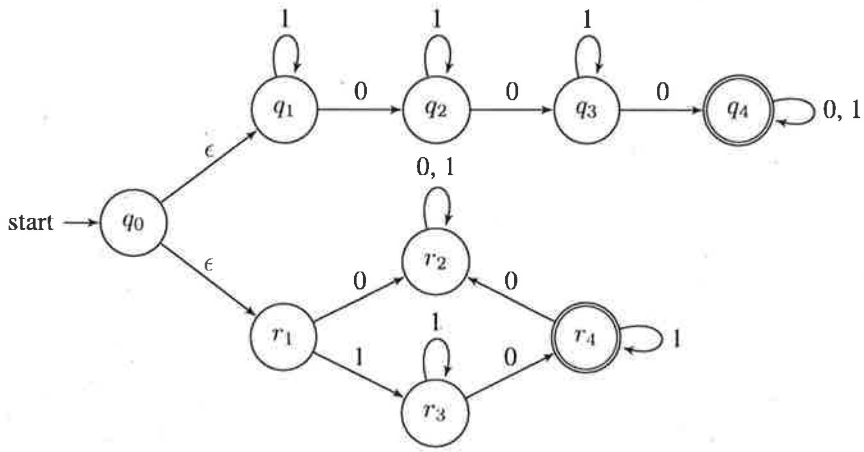


(c) (4 marks) Draw the state diagram for a DFA recognizing language $L_3 = L_1 \cup L_2$. Hint: One option is to use the construction of Theorem 1.25 in the text.
Solution:

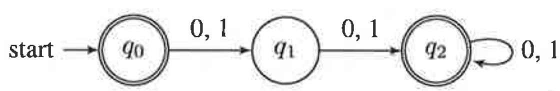


(d) (4 marks) Draw the state diagram for a NFA recognizing language $L_3 = L_1 \cup L_2$.

Solution:



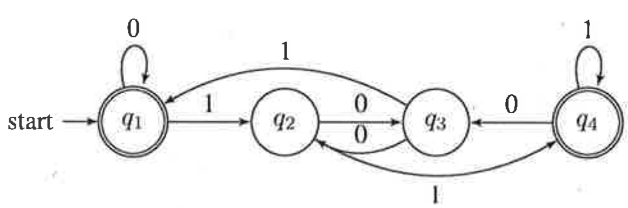
3. (4 marks) Draw the state diagram for a DFA recognizing language $L_4 = \{x \mid x \text{ is any string except 0 and 1}\}$.
Solution:



4. (2 marks) Consider DFA $M = (\{q_1, q_2, q_3, q_4\}, \{0, 1\}, \delta, q_1, \{q_1, q_4\})$ for δ defined as the following table. Give the state diagram of this machine.

	0	1
q_1	q_1	q_2
q_2	q_3	q_4
q_3	q_2	q_1
q_4	q_3	q_4

Solution:



5. (8 marks) This question illustrates differences and commonalities between NFAs and DFAs.

(a) (4 marks) Show that if M is a DFA that recognizes language B, then swapping the accept and nonaccept states in M yields a new DFA M' recognizing the complement of B, $\overline{B}$. Which operation does this imply the regular languages are closed under?

Solution: Let $x \in \{0,1\}^*$ be an arbitrary string, and suppose that running M on x takes us to some state final state q_x in M. The key observation is that running M' on x takes us to the *same* final state, q_x — this follows because M and M' are DFA's, and hence they have only one computation branch which acts deterministically, regardless of where the accept states are placed. The claim now follows, since q_x is accepting in M' iff q_x is non-accepting in M. This implies the set of regular languages is closed under complement.

(b) (4 marks: 2 marks for counterexample, 2 marks for question on closure under complement) Prove by counterexample that if M is a NFA that recognizes languages B, then swapping the accept and nonaccept states in M does not necessarily yield an NFA recognizing $\overline{B}$. With this in mind, is the class of languages recognized by NFA's closed under complement (explain your answer!)?

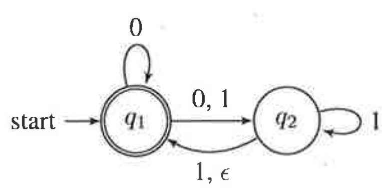
Solution: A counterexample is given by swapping the accept and non-accept states of the NFA sketched in the solutions for question 2d. Specifically, in this case, the original NFA and the swapped version both accept ϵ. Note that the proof from 5(a) above does not work here, as in the NFA, there are multiple states we could stop in if $x = \epsilon$. The class of languages recognized by NFA's is nevertheless closed under complement, since NFA's recognize the class of regular languages, and by question 5(a), the regular languages are closed under complement. (One simply needs a different construction to recognize the complement for NFA's than the one from 5(a).)

6. (12 marks) This question tests your understanding of the equivalence between DFAs and NFAs. Consider NFA $M = (\{q_1, q_2\}, \{a, b\}, \delta, q_1, \{q_1\})$ for δ defined as:

δ	0	1	ϵ
q_1	$\{q_1, q_2\}$	$\{q_2\}$	$\emptyset$
q_2	$\emptyset$	$\{q_1, q_2\}$	$\{q_1\}$

(a) (4 marks) Draw the state diagram for M.

Solution:



(b) (2 marks) Based on the construction of Theorem 1.39 in the text, start to build the DFA M' that is equivalent to M by identifying the number of DFA states and listing them.

Solution: Number of states for the DFA M' = 2^2 = 4. The states are: $\{\emptyset, q_1, q_2, q_{12}\}$.

(c) (2 marks) Identify the DFA M' starting and acceptance states.

Solution: The start state is q_1. The accept states are: $\{q_1, q_{12}\}$.

(d) (4 marks) Draw the state diagram for the DFA M' equivalent to M based on the construction of Theorem 1.39 in the text (recall the latter proves that DFAs and NFAs are equivalent).

Solution: