Solutions: Linear Algebra Review – Chapters 1–3

Chapter 1 Solutions

1. Line: $\mathbf{r} = (1,2) + t(3,3) = (1+3t, 2+3t)$.

2. (a) $(3,-1,7)$ (b) $(4,-3,-2)$ (c) $v \cdot w = 14$ (d) $\theta = \cos^{-1}\left(\frac{14}{\sqrt{14}\sqrt{17}}\right)$.

3. $x = a \times b = (1,1,0) \times (0,1,1) = (1,-1,1)$, unit vector $\frac{1}{\sqrt{3}}(1,-1,1)$.

4. Not independent: second vector is $2(1,2,3)$.

5. Linearly dependent $\Leftrightarrow$ one vector is a scalar multiple of the other (same or opposite direction).

6. $\text{proj}_a b = \frac{a \cdot b}{a \cdot a} a = \frac{11}{5}(1,2)$; $e = b - \text{proj}_a b = (-\frac{8}{5}, -\frac{2}{5})$, $a \cdot e = 0$.

7. $v = 5\frac{u}{\|u\|} = 5\frac{(2,-1,1)}{\sqrt{6}}$.

8. Normal $(1,2,3)$; two directions in plane: e.g. $(1,0,-\frac{1}{3})$ and $(0,1,-\frac{2}{3})$.

Chapter 2 Solutions

9. $x = 2$, $y = 1$.

10. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 4 & 7 \end{bmatrix}$, $b = \begin{bmatrix} 6 \\ 15 \\ 14 \end{bmatrix}$; system is consistent (rows not contradictory).

11. Row reduce to echelon form: $\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}$, pivots in cols 1 and 3.

12. Rank $= 2$.

13. Each column has a pivot (no free variables).

14. 1 solution $\rightarrow$ unique intersection, 0 solutions $\rightarrow$ parallel/no intersection, ∞ solutions $\rightarrow$ same line/plane.

15. Dependent rows, rank 1, system has solutions when b lies in column space; here it does.

16. Solve: $x = 1$, $y = 1$, $z = 1$.

17. det $= 0$, so matrix is singular $\rightarrow$ no unique solution.

18. A zero pivot prevents elimination; swap rows to create a nonzero pivot.

Chapter 3 Solutions

19. Contains 0, closed under addition and scalar multiplication.

20. (a) Subspace; (b) Not (fails zero); (c) Subspace.

21. Cols 1 and 3 independent $\to$ basis $\{(1,2,1),(3,6,1)\}$.

22. Nullspace spanned by $(1,2,-1)$.

23. $A(1,2,-1)^T = 0$, verified.

24. Nullspace contains only zero vector.

25. $\text{rank}(A) + \text{nullity}(A) = n$.

26. Rank $= 2$, nullity $= 1$.

27. Column space and row space have same rank; nullspace and left nullspace orthogonal complements.

28. Rank 2, Col space basis $\{(1,0,0),(2,1,0)\}$, Nullspace basis $\{(-2,-4,1)\}$.

29. (a) True (b) False (c) True.

30. b is in the column space if $\exists x$ such that $Ax = b$, i.e., b is a linear combination of the columns.