

# Chapter 7.2 Practice

Show that the following problems are in NP by describing a non-deterministic TM and a verifier for each.

1. VERTEX COVER: A graph  $G = (V, E)$  and a positive integer  $k \leq |V|$  where there is a subset  $V' \subseteq V$  such that  $|V'| \leq k$  and, for each edge  $uv \in E$ , at least one of  $u$  and  $v$  belongs to  $V'$ .
2. HAMILTON CIRCUIT: A graph  $G = (V, E)$ .  $G$  contain a Hamiltonian circuit, that is, an ordering  $\langle v_1, v_2, \dots, v_n \rangle$  of the vertices of  $G$ , where  $n = |V|$ , such that  $v_n v_1 \in E$  and  $v_i v_{i+1} \in E$  whenever  $1 \leq i \leq n - 1$ .
3. PARTITION: A multiset  $A$  of positive integer, and there is a subset  $A' \subseteq A$  such that the sum of the elements of  $A'$  = the sum of the elements of  $A$  that are not in  $A'$ .
4. 3-DIMENSIONAL MATCHING: A set  $M \subseteq W \times X \times Y$ , where  $W$ ,  $X$ , and  $Y$  are disjoint sets with the same number  $q$  of elements, where  $M$  contain a subset  $M' \subseteq M$  such that  $|M'| = q$  and no two elements of  $M'$  agree in any coordinate.
5. FEEDBACK VERTEX SET: A directed graph  $G = (V, A)$ , integer  $k \leq |V|$  where there is a subset  $V' \subseteq V$  with  $|V'| \leq k$  such that every directed cycle in  $G$  includes at least one vertex from  $V'$ .