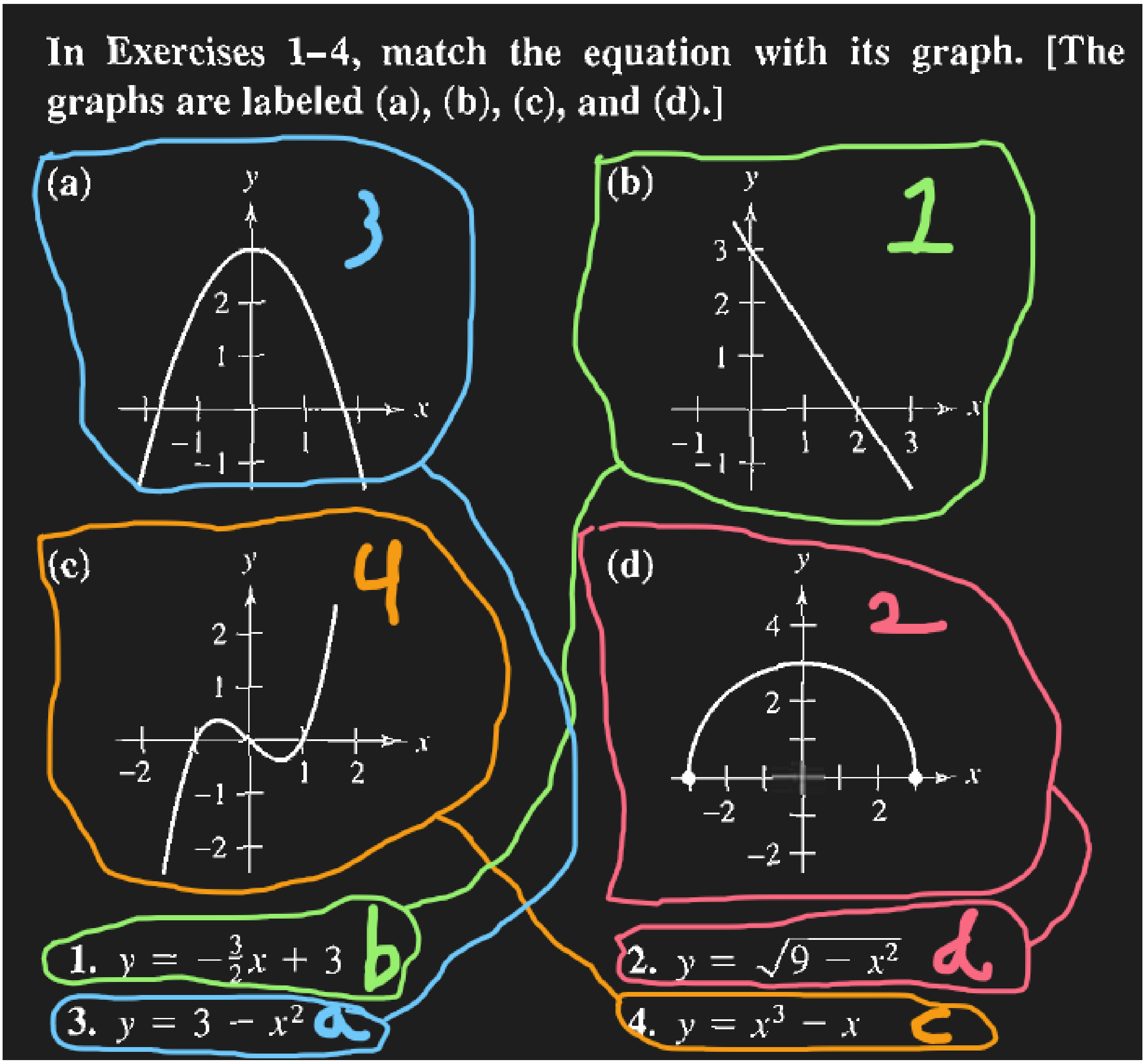
Chapter P.1 Homework

Homework for Chapter P.1:

This file may look wrong if you are not using Obsidian. Please take a look at the *PDF export* *instead.*

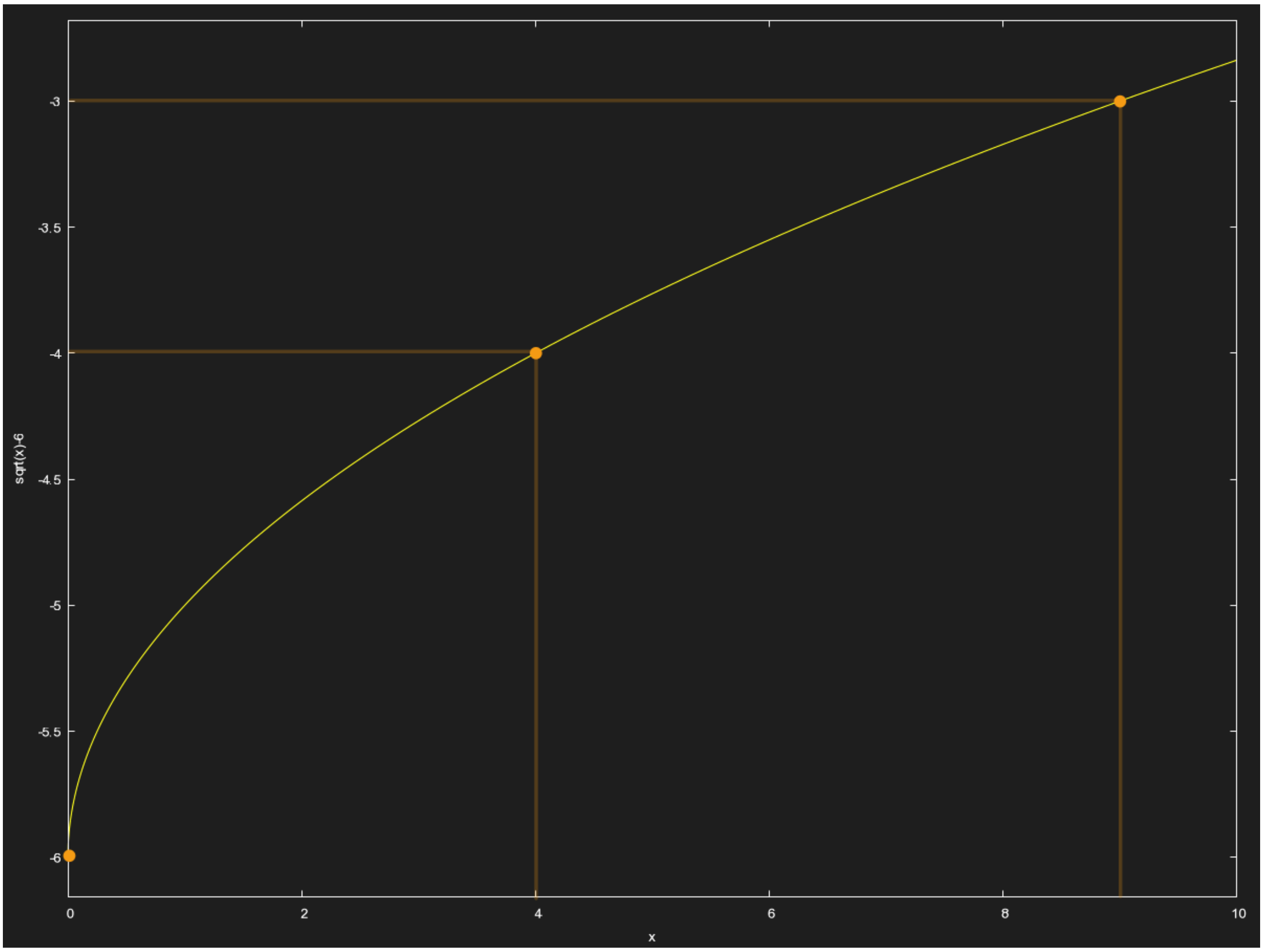
Q2:



Q11:

> Sketch the graph of the equation $y = \sqrt{x} - 6$ by point plotting.

x	y
0	-6
4	-4
9	-3



Q24:

> Find any intercepts of the equation $y = (x-1)\sqrt{x^2+1}$

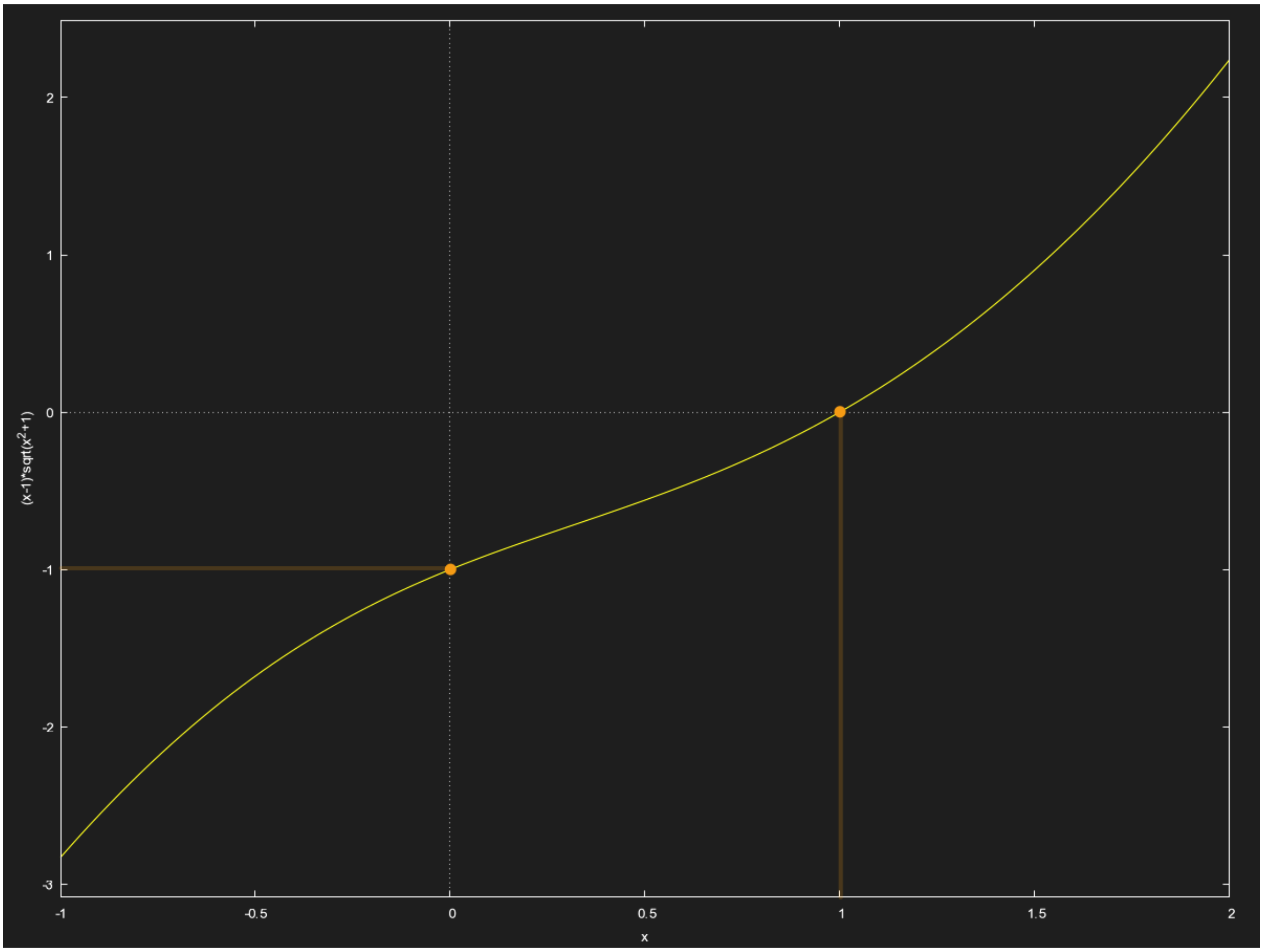
1. Evaluate at $x = 0$:
   $y = (0-1)\sqrt{0^2+1}$
   $y = -\sqrt{1} = -1$
   Therefore, the y intercept of the equation is $(0, -1)$.
2. Evaluate at $y = 0$:
   $(x-1)\sqrt{x^2+1} = 0$

$(x-1) = 0$	$\sqrt{x^2+1} = 0$
$x = 1$	$x^2+1 = 0$

$(x-1)=0$	$\sqrt{x^2+1}=0$
	$x^2=-1$
	$x=\sqrt{-1}(\text{undefined})$

Therefore, the x intercept of the equation is $(1, 0)$.

Here is a graph for reference:



Q28:

> Find any intercepts of the equation $y = 2x - \sqrt{x^2+1}$

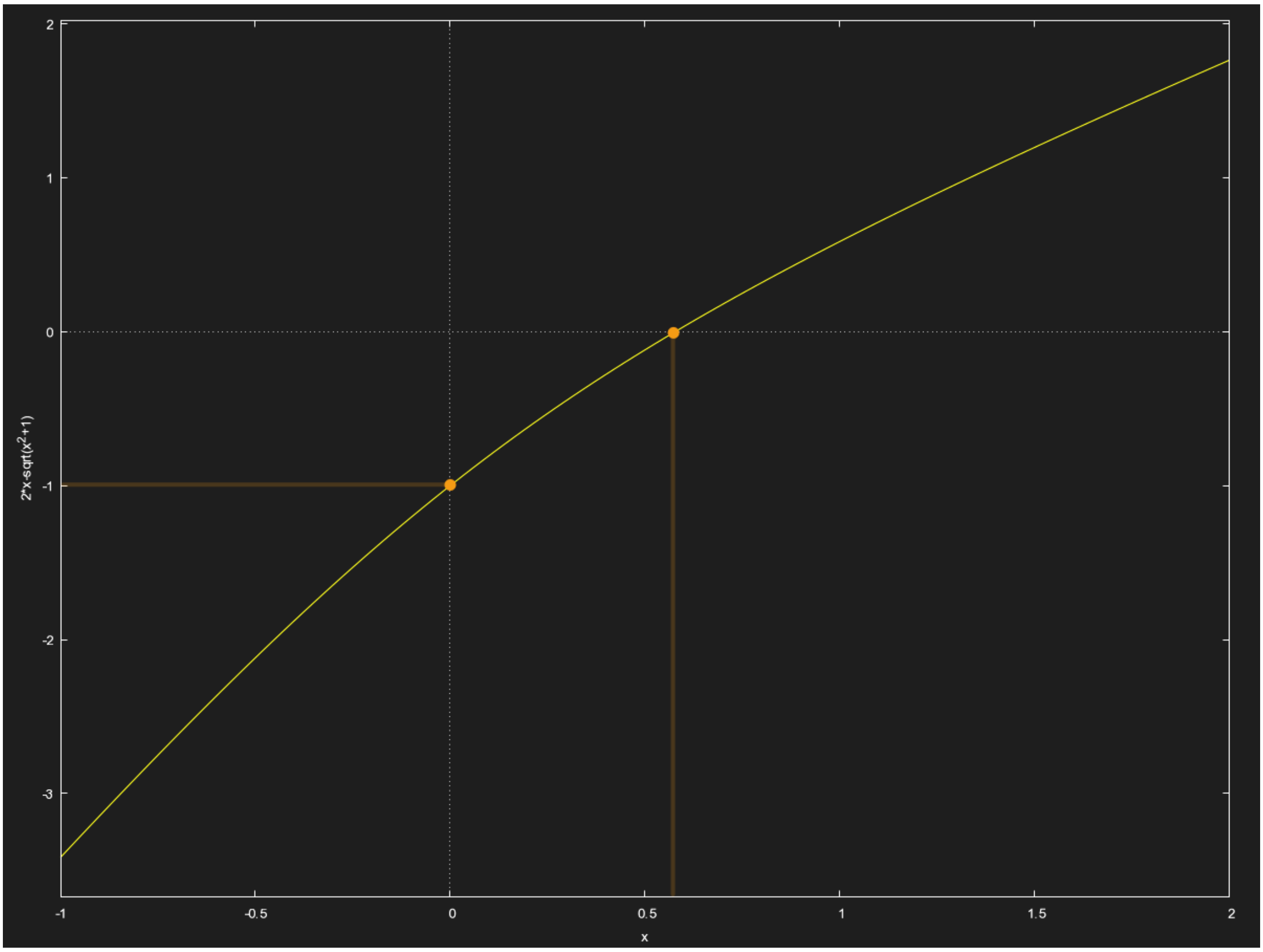
1. Evaluate at $x = 0$:
   $y = 2(0) - \sqrt{0^2+1}$
   $y = -\sqrt{1} = -1$
   Therefore, the y intercept of the equation is $0, -1$
2. Evaluate at $y = 0$:
   $2x - \sqrt{x^2+1} = 0$
   $2x = \sqrt{x^2+1}$
   $(2x)^2 = \pm(x^2+1)$

$$4x^2 = \pm(x^2+1)$$
$$4x^2 \pm (x^2+1) = 0$$

$4x^2 + (x^2+1) = 0$	$4x^2 - (x^2+1) = 0$
$4x^2 + x^2 + 1 = 0$	$4x^2 - x^2 - 1 = 0$
$5x^2 + 1 = 0$	$3x^2 - 1 = 0$
$5x^2 = -1$	$3x^2 = 1$
$x = \sqrt{\frac{-1}{5}}(\text{undefined})$	$x = \sqrt{\frac{1}{3}} \approx 0.5773502691896257\ldots$

Therefore, the x intercept is $(\sqrt{\frac{1}{3}}, 0)$.

Here is a graph for reference:



Q56:

> Sketch the graph of the equation $y = \frac{10}{x^2+1}$. Identify any intercepts and test for symmetry.

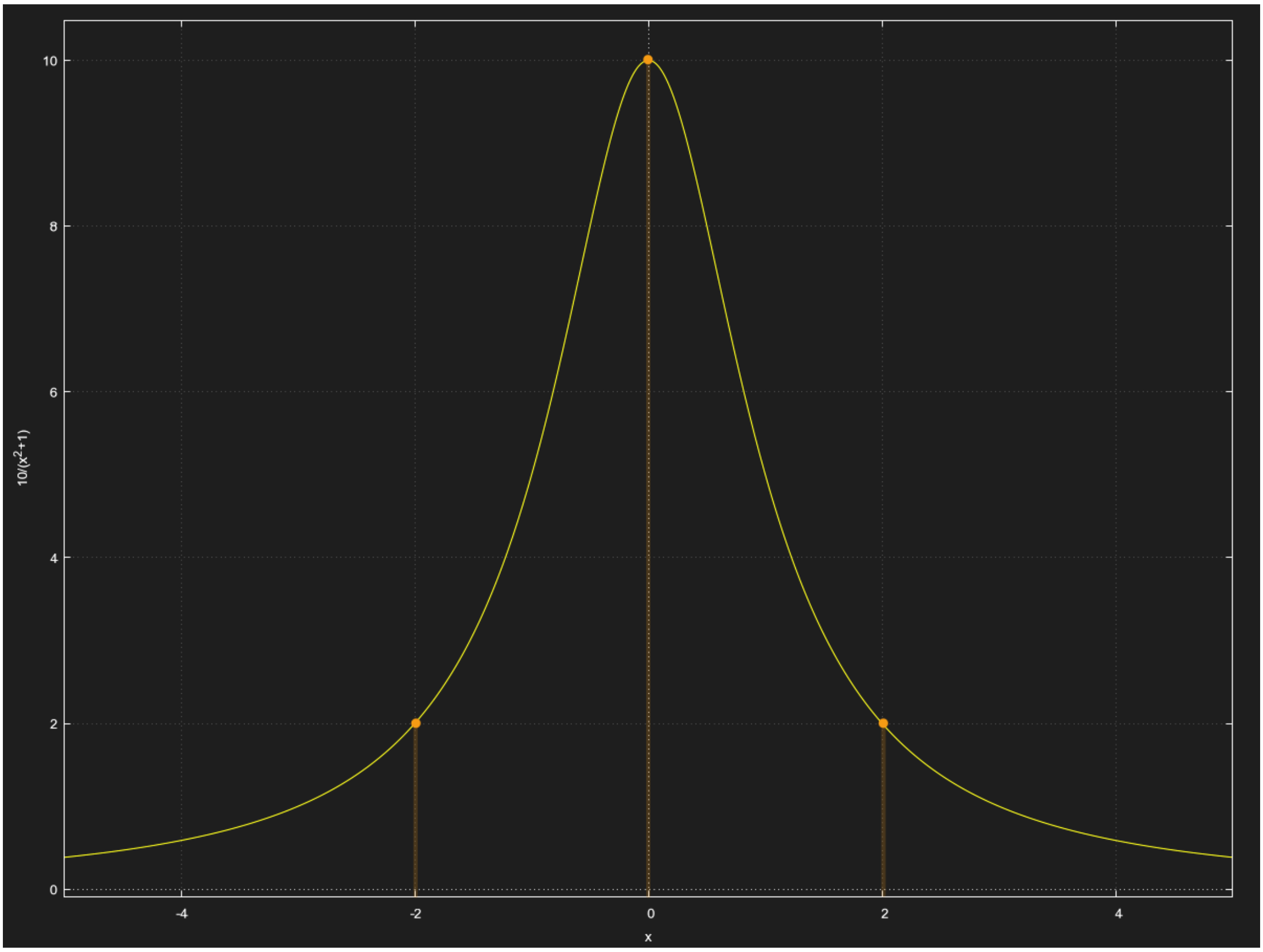
1. Function Properties:

- Since the function contains no x variable in the numerator, $y \neq 0 \; \forall x \in \mathbb{R}$.
- The domain of the equation is $\{x \in \mathbb{R}\}$, because $x^2 + 1 \neq 0 \; \forall x \in \mathbb{R}$.

2. Symmetries:
   - x axis symmetry test:
     $y = \frac{10}{x^2+1}, \quad \text{substitute } (-y) \text{ for y:}$
     $(-y) = \frac{10}{x^2+1}$
     $y = -\frac{10}{x^2+1}$
     $\{y = -\frac{10}{x^2+1}\} \neq \{y = \frac{10}{x^2+1}\}$
     Therefore, this equation is not symmetrical with respect to the x axis.
   - y axis symmetry test:
     $y = \frac{10}{x^2+1}, \quad \text{substitute } (-x) \text{ for x:}$
     $y = \frac{10}{(-x)^2+1}$
     $y = \frac{10}{x^2+1}$
     $\{y = \frac{10}{x^2+1}\} = \{y = \frac{10}{x^2+1}\}$
     Therefore, this equation is symmetrical with respect to the y axis.
   - In fact, it is unnecessary to test for symmetry with respect to the origin now. And this is a proof why:
     $\text{Let } f(x) = y$
     $f(x) = f(-x) \text{ as shown previously.}$
     $f(x) \neq -f(x) \text{ as shown previously.}$
     $f(-x) \stackrel{?}{=} -f(x)$
     $\text{Substitute } f(x) \text{ for } f(-x) \text{ since they are equal:}$
     $f(x) \stackrel{?}{=} -f(x)$
     $\text{Since it was shown that } f(x) \neq -f(x) \text{ and since } f(x) = f(-x),$
     $f(-x) \neq -f(x)$
     $\text{Since } f(-x) \neq -f(x), \text{ the equation is not symmetric with respect to the origin.}$

3. Graph:

x	y
2	2
-2	2
3	1
-3	1



Q66:

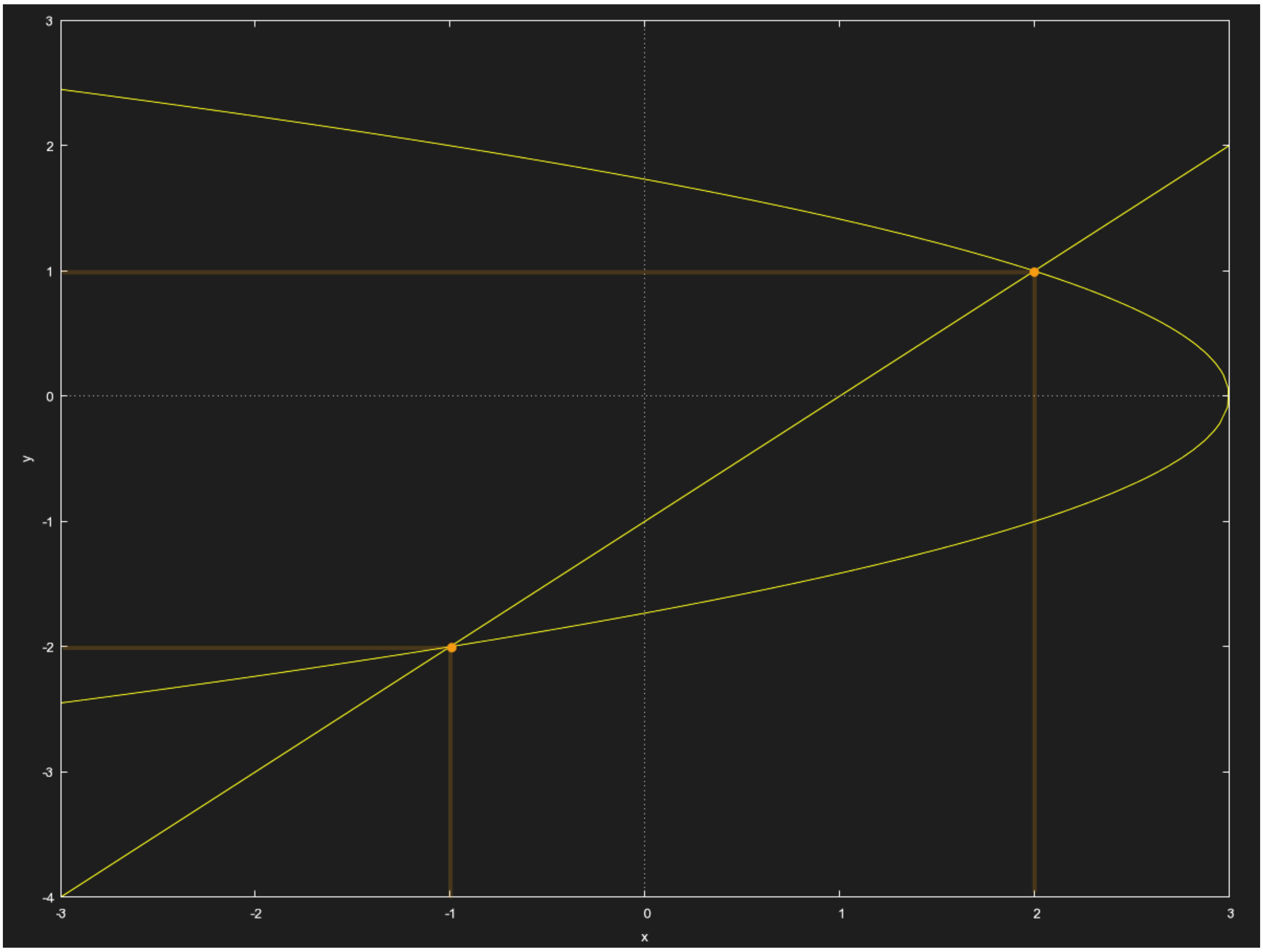
> Find the points of intersection of the following equations: $\begin{cases} x = 3 - y^2 \\ y = x - 1 \end{cases}$

1. Solve for y by elimination:
   $\begin{cases} x = 3 - y^2 \\ y = x - 1 \end{cases}$
   $\begin{cases} x - 3 = -y^2 \\ y = x - 1 \end{cases}$
   $\begin{cases} y^2 = -x + 3 \\ y = x - 1 \end{cases}$
   $y^2 + y = 2$
   $y^2 + y - 2 = 0$
   $(y - 1)(y + 2) = 0$
   $y = 1 \text{ and } y = -2$
2. Substitute y in the equations:
   $x = 3 - y^2, \quad \text{substitute } y = 1$
   $x = 3 - (1)^2 = 2$

$$x = 3 - y^2, \quad \text{substitute } y = -2$$
$$x = 3 - (-2)^2 = 3 - 4 = -1$$

Therefore, the points of intersection are $(2, 1)$ and $(-2, -1)$.

Here is a graph for reference:



Q68:

> Find the points of intersection of the following equations: $\begin{cases} x^2 + y^2 = 25 \\ -3x + y = 15 \end{cases}$

1. Solve for y:
   $$-3x + y = 15$$
   $$y = 3x + 15$$
2. Solve for x via substitution:
   $$x^2 + y^2 = 25, \quad \text{substitute } y = 3x + 15$$
   $$x^2 + (3x + 15)^2 = 25$$
   $$x^2 + 9x^2 + (2)(15)(3)x + 15^2 = 25$$
   $$10x^2 + 90x + 225 = 25$$
   $$10x^2 + 90x + 200 = 0$$

$$x^2 + 9x + 20 = 0$$
$$x^2 + 4x + 5x + 20 = 0$$
$$x(x + 4) + 5(x + 4) = 0$$
$$(x + 4)(x + 5) = 0$$
$$x = -4 \text{ and } x = -5$$

3. Back-substitute to solve for y:

$$y = 3x + 15, \quad \text{substitute } x = -4$$
$$y = 3(-4) + 15 = -12 + 15 = 3$$

$$y = 3x + 15, \quad \text{substitute } x = -5$$
$$y = 3(-5) + 15 = -15 + 15 = 0$$

Therefore, the points of intersection are $(-4, 3)$ and $(-5, 0)$.

Here is a graph for reference: