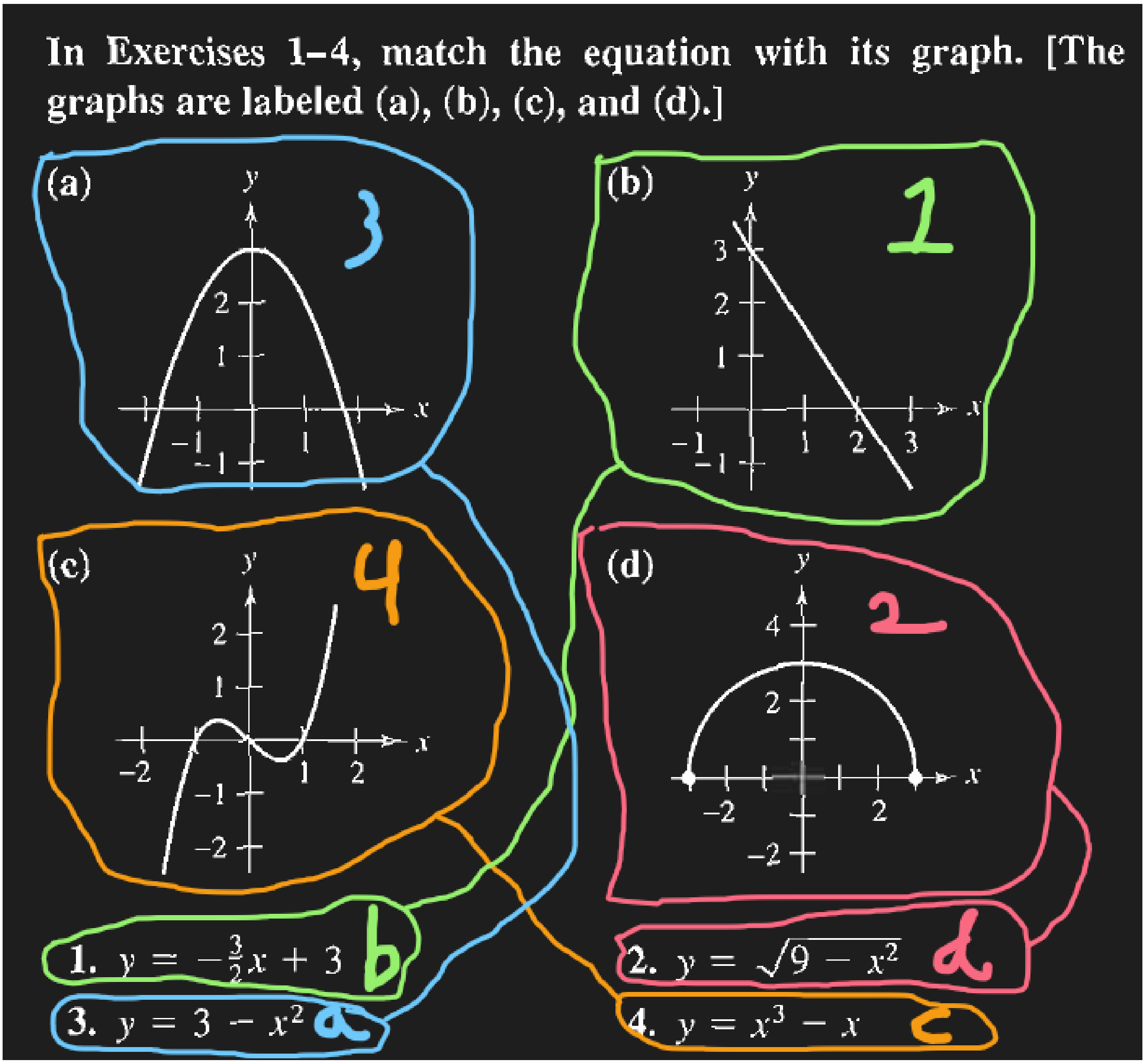
Chapter P.1 Homework

Homework for Chapter P.1:

Q2:

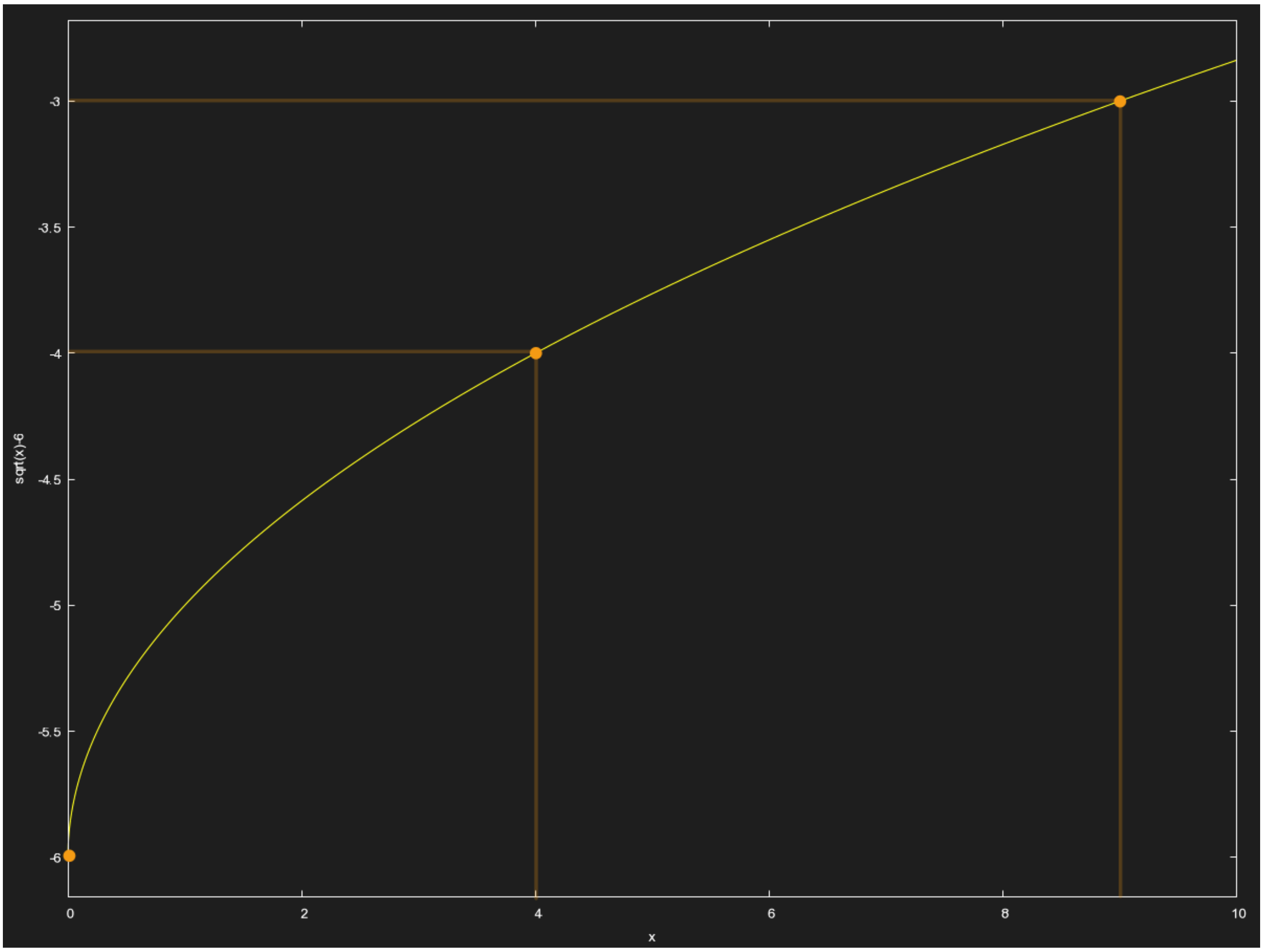


Q11:

> Sketch the graph of the equation $y = \sqrt{x} - 6$ by point plotting.

x	y
0	-6
4	-4

x	y
9	-3



Q24:

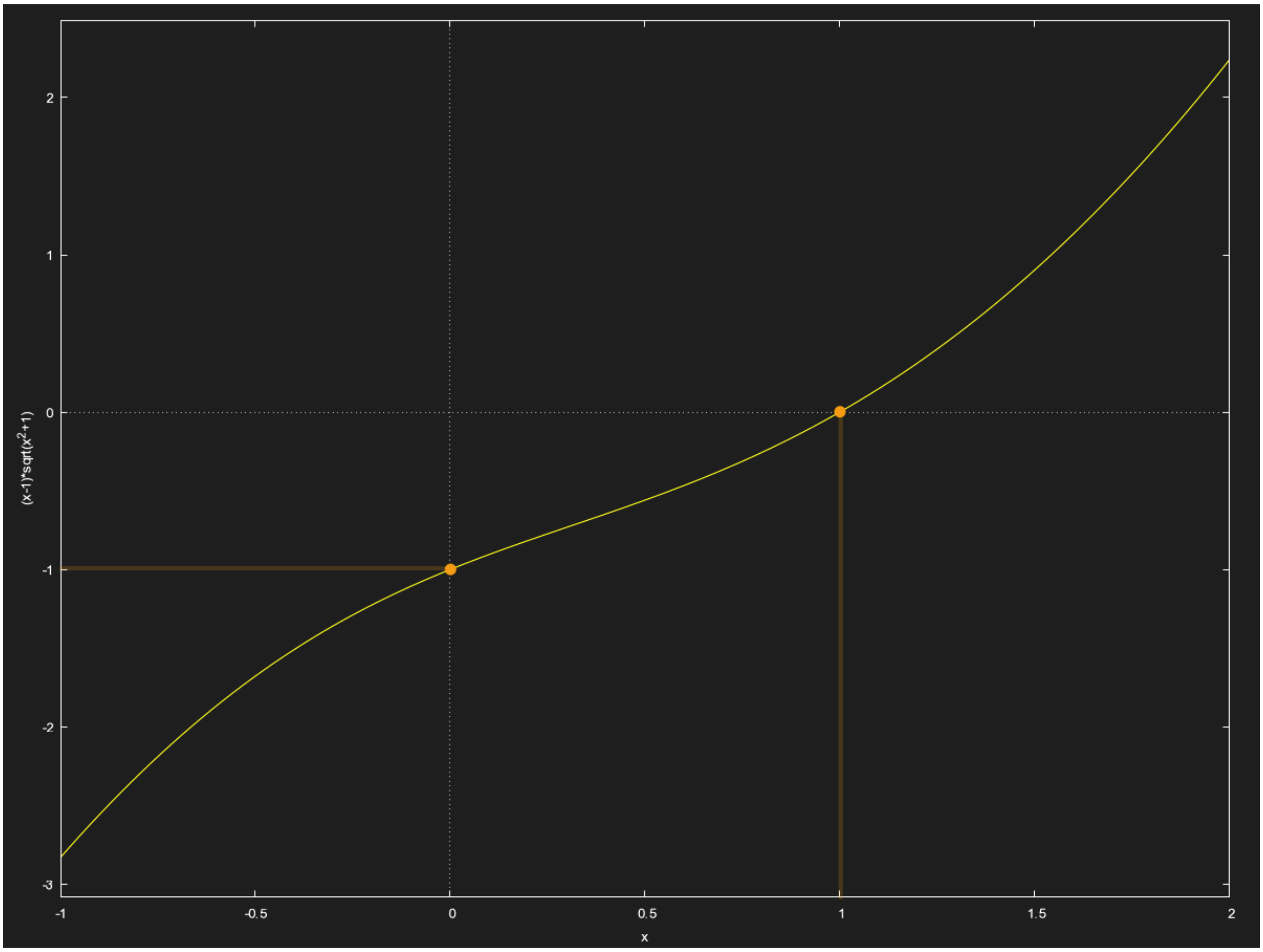
> Find any intercepts of the equation $y = (x-1)\sqrt{x^2+1}$

1. Evaluate at $x = 0$:
   $y = (0-1)\sqrt{0^2+1}$
   $y = -\sqrt{1} = -1$
   Therefore, the y intercept of the equation is $(0, -1)$.
2. Evaluate at $y = 0$:
   $(x-1)\sqrt{x^2+1} = 0$

$(x-1) = 0$	$\sqrt{x^2+1} = 0$
$x = 1$	$x^2 + 1 = 0$
	$x^2 = -1$
	$x = \sqrt{-1}(\text{undefined})$

Therefore, the x intercept of the equation is $(1, 0)$.

Here is a graph for reference:



Q28:

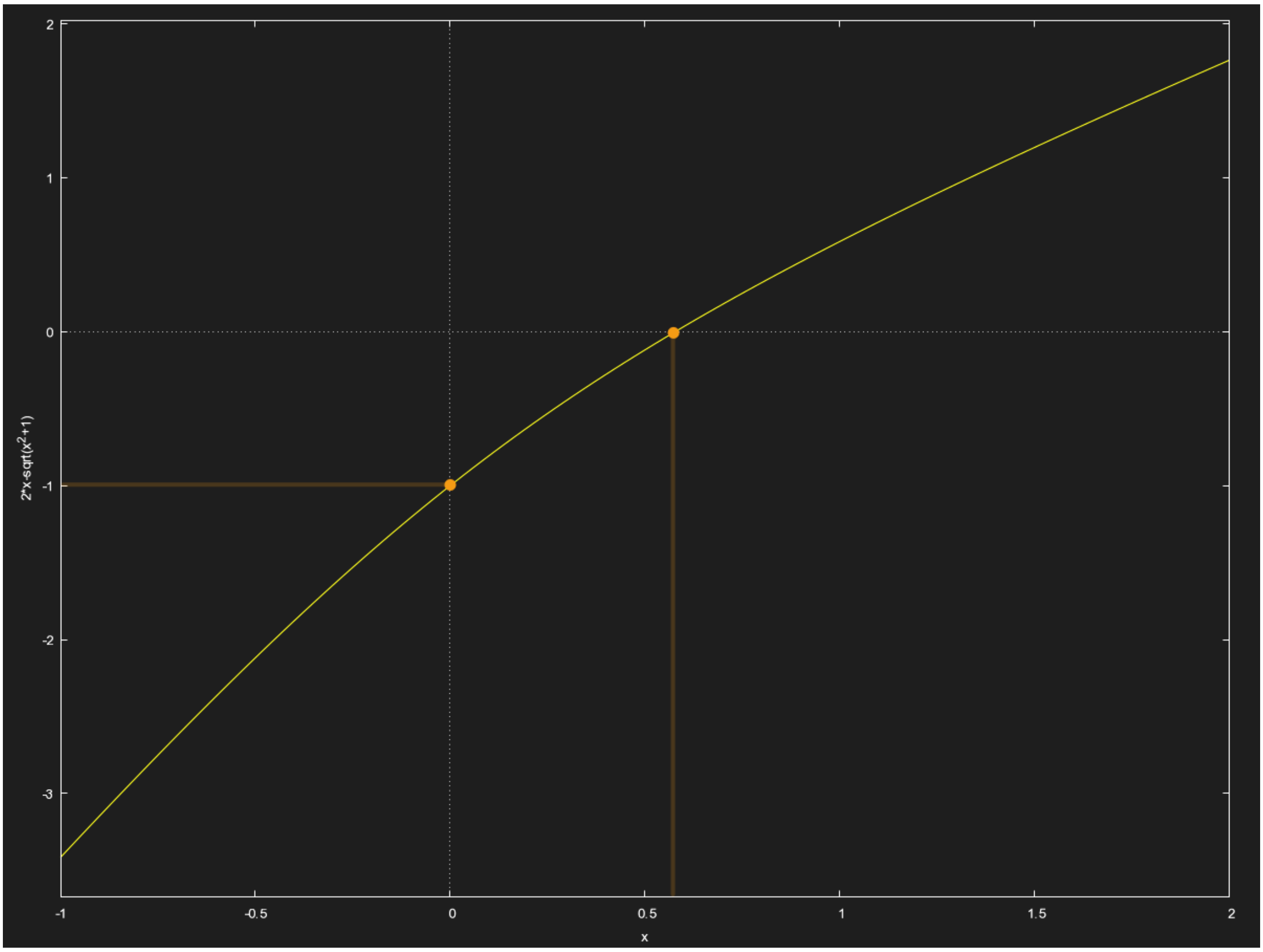
> Find any intercepts of the equation $y = 2x - \sqrt{x^2 + 1}$

1. Evaluate at $x = 0$:
   $y = 2(0) - \sqrt{0^2 + 1}$
   $y = -\sqrt{1} = -1$
   Therefore, the y intercept of the equation is $0, -1$
2. Evaluate at $y = 0$:
   $2x - \sqrt{x^2 + 1} = 0$
   $2x = \sqrt{x^2 + 1}$
   $(2x)^2 = \pm(x^2 + 1)$
   $4x^2 = \pm(x^2 + 1)$
   $4x^2 \pm (x^2 + 1) = 0$

$4x^2 + (x^2 + 1) = 0$	$4x^2 - (x^2 + 1) = 0$
$4x^2 + x^2 + 1 = 0$	$4x^2 - x^2 - 1 = 0$
$5x^2 + 1 = 0$	$3x^2 - 1 = 0$
$5x^2 = -1$	$3x^2 = 1$
$x = \sqrt{\frac{-1}{5}}(\text{undefined})$	$x = \sqrt{\frac{1}{3}} \approx 0.5773502691896257\ldots$

Therefore, the x intercept is $(\sqrt{\frac{1}{3}}, 0)$.

Here is a graph for reference:



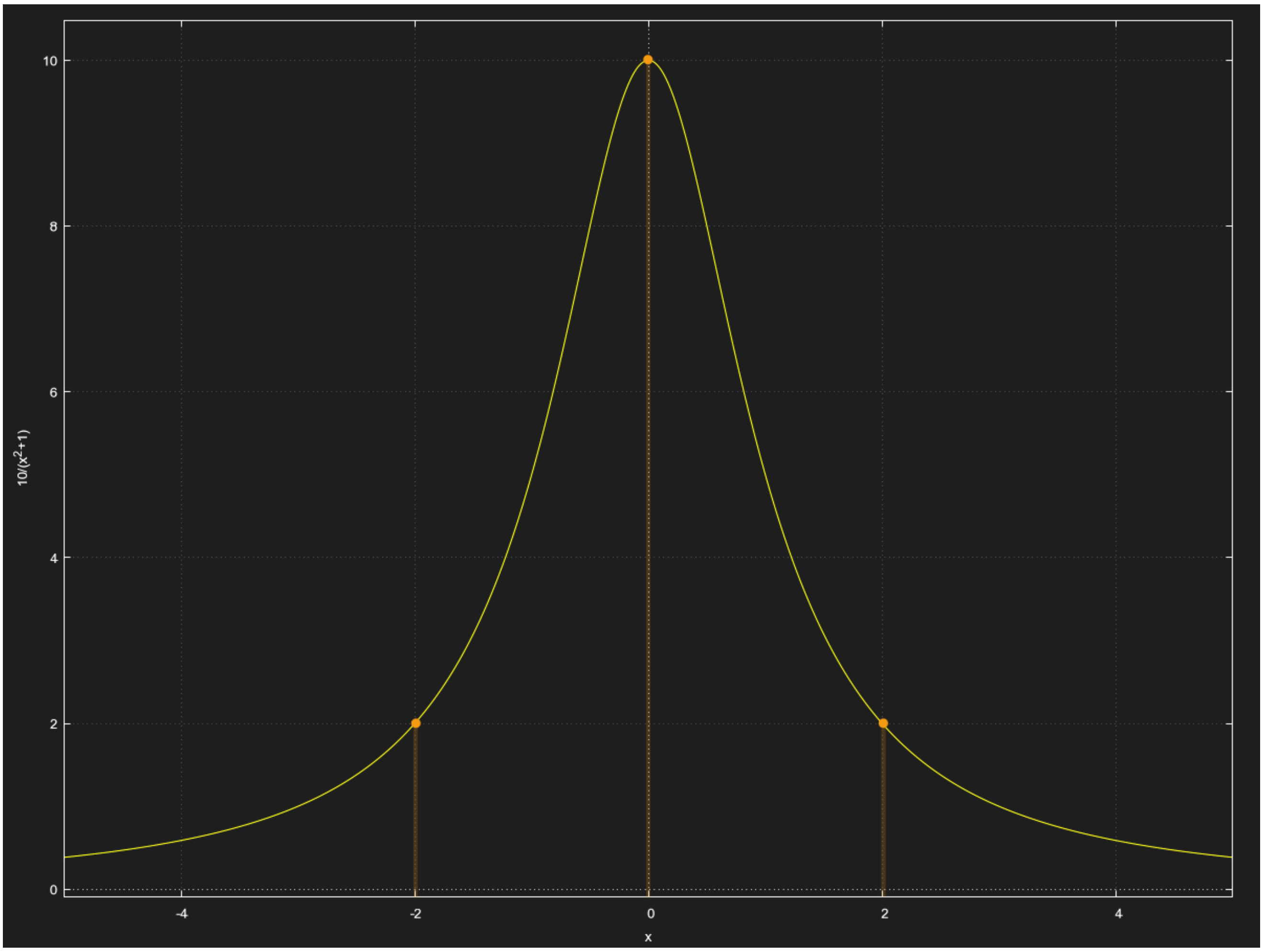
Q56:

> Sketch the graph of the equation $y = \frac{10}{x^2+1}$. Identify any intercepts and test for symmetry.

1. Function Properties:
   - Since the function contains no x variable in the numerator, $y \neq 0 \ \forall x \in \mathbb{R}$.
   - The domain of the equation is $\{x \in \mathbb{R}\}$, because $x^2 + 1 \neq 0 \ \forall x \in \mathbb{R}$.

2. Symmetries:
   - x axis symmetry test:
     $y = \frac{10}{x^2+1}, \quad \text{substitute } (-y) \text{ for y:}$
     $(-y) = \frac{10}{x^2+1}$
     $y = -\frac{10}{x^2+1}$
     $\{y = -\frac{10}{x^2+1}\} \neq \{y = \frac{10}{x^2+1}\}$
     Therefore, this equation is not symmetrical with respect to the x axis.
   - y axis symmetry test: $y = \frac{10}{x^2+1}, \quad \text{substitute } (-x) \text{ for x: } y = \frac{10}{(-x)^2+1}$
     $y = \frac{10}{x^2+1}$ $\{y = \frac{10}{x^2+1}\} = \{y = \frac{10}{x^2+1}\}$ Therefore, this equation is symmetrical with respect to the y axis.
   - In fact, it is unnecessary to test for symmetry with respect to the origin now. And this is a proof why:
     $\text{Let } f(x) = y$
     $f(x) = f(-x) \text{ as shown previously.}$
     $f(x) \neq -f(x) \text{ as shown previously.}$
     $f(-x) \stackrel{?}{=} -f(x)$
     $\text{Substitute } f(x) \text{ for } f(-x) \text{ since they are equal:}$
     $f(x) \stackrel{?}{=} -f(x)$
     $\text{Since it was shown that } f(x) \neq -f(x) \text{ and since } f(x) = f(-x),$
     $f(-x) \neq -f(x)$
     $\text{An equation is symmetric with respect to the origin if and only if } f(-x) = -f(x$
     $\text{Since } f(-x) \neq -f(x) \text{, the equation is not symmetric with respect to the origin.}$
3. Graph:

x	y
2	2
-2	2
3	1
-3	1



Q66:

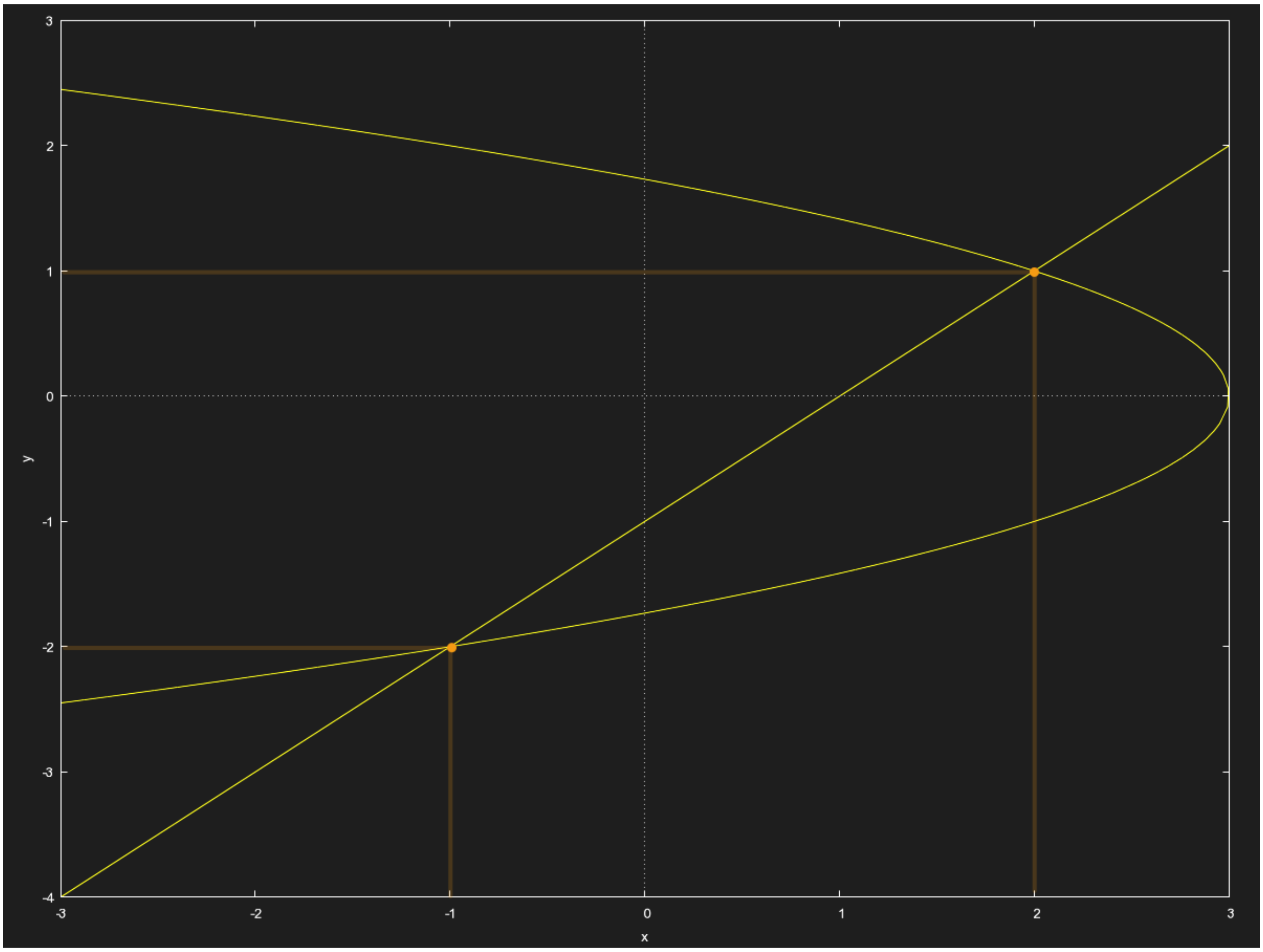
> Find the points of intersection of the following equations: $\begin{cases} x = 3 - y^2 \\ y = x - 1 \end{cases}$

1. Solve for y by elimination:
   $\begin{cases} x = 3 - y^2 \\ y = x - 1 \end{cases} \begin{cases} x - 3 = -y^2 \\ \quad y = x - 1 \end{cases}$
   $\begin{cases} y^2 = -x + 3 \\ \quad y = x - 1 \end{cases}$
   $y^2 + y = 2$
   $y^2 + y - 2 = 0$
   $(y - 1)(y + 2) = 0$
   $y = 1 \text{ and } y = -2$
2. Substitute y in the equations:
   $x = 3 - y^2, \quad \text{substitute } y = 1$
   $x = 3 - (1)^2 = 2$

   $x = 3 - y^2, \quad \text{substitute } y = -2$
   $x = 3 - (-2)^2 = 3 - 4 = -1$

Therefore, the points of intersection are $(2,1)$ and $(-2,-1)$.

Here is a graph for reference:



Q68:

> Find the points of intersection of the following equations: $\begin{cases} x^2+y^2=25 \\ -3x+y=15 \end{cases}$

1. Solve for y:

   $-3x+y=15$

   $y=3x+15$

2. Solve for x via substitution:

   $x^2+y^2=25, \quad \text{substitute } y=3x+15$

   $x^2+(3x+15)^2=25$

   $x^2+9x^2+(2)(15)(3)x+15^2=25$

   $10x^2+90x+225=25$

   $10x^2+90x+200=0$

   $x^2+9x+20=0$

   $x^2+4x+5x+20=0$

   $x(x+4)+5(x+4)=0$

$$(x+4)(x+5)=0$$
$$x=-4 \text{ and } x=-5$$

3. Back-substitute to solve for y:

$$y=3x+15, \quad \text{substitute } x=-4$$
$$y=3(-4)+15=-12+15=3$$

$$y=3x+15, \quad \text{substitute } x=-5$$
$$y=3(-5)+15=-15+15=0$$

Therefore, the points of intersection are $(-4,3)$ and $(-5,0)$.

Here is a graph for reference: