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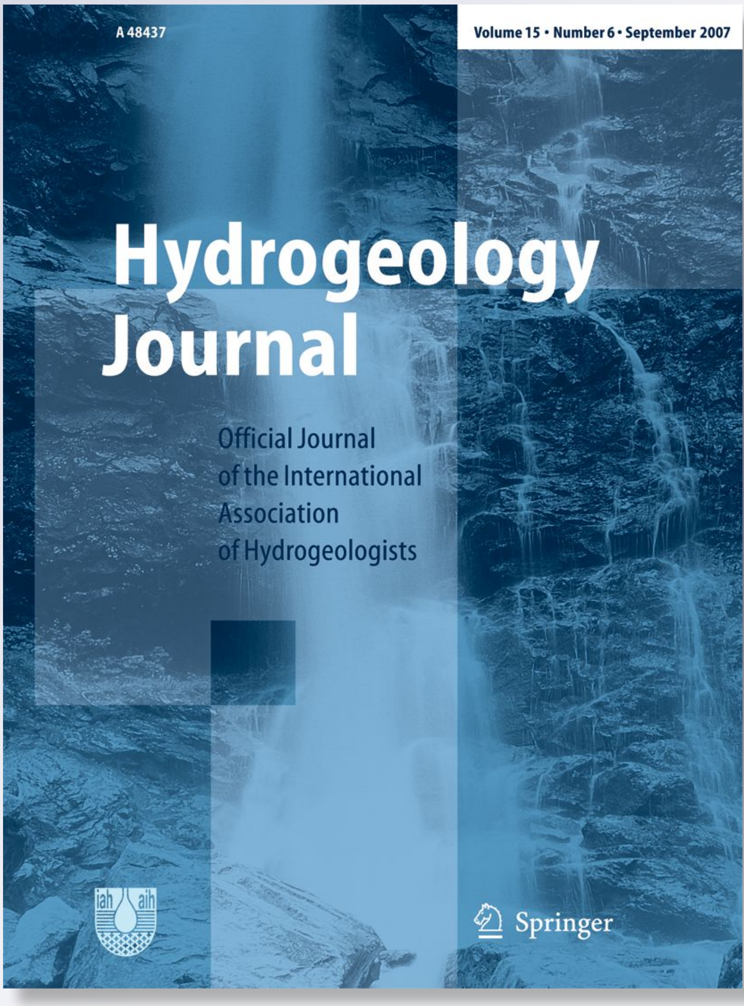

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Abstract This paper presents new findings in interpreting analytical solutions of steady radial flow to a well in a semi-confined aquifer (overlain by a phreatic aquifer and aquitard), and demonstrates that 95% of pumped water is derived from leakage water within a radius of 4 times the leakage factor. The travel times of the leakage water from the radii of influence to the well are usually much longer than those derived from the travel time criteria currently used to delineate the well protection areas. The delineation of well protection zones based on the travel time criteria will not properly protect the source of water to the well. Therefore, the percentage of leakage water to the well is used as a new criterion to define the well protection areas. Within each well protection area, the mean residence time is used as an indicator of the renewable period of the aquifer system. Leakage-rate weighted residence times are used to calculate the mean residence time. For the safety and sustainability of drinking water supplies, groundwater in the phreatic aquifer within the radius of influence should be protected.

Keywords Analytical solutions · Groundwater hydraulics · Semi-confined aquifer · Source of water · Protection area

Introduction

Steady radial flow in a semi-confined (leaky) aquifer was first approached in Kooper (1914) and solved comprehensively by De Glee (1930) in The Netherlands (as reported by De Vries 2006). The same solution was published much later in Jacob (1946). Since then, the solution has been extended to transient radial flow in a leaky aquifer (Hantush and Jacob 1955), to elastic storage release from an aquitard (Hantush 1960; Neuman and Witherspoon 1969), and to multiple layered aquifer systems (Hemker 1984, 1985; Hunt 1985; Maas 1986, 1987a, b; Hemker and Maas 1987; Wu 1987; Cheng and Morohunfola 1993). Some of these solutions have been used to analyse pumping test data to determine aquifer parameters (Hantush 1956; Vandenberg 1977; Motz 1990, 1991; Kruseman and de Ridder 1994).

The problems of aquifer depletion and groundwater pollution have created a demand for methods of identifying sources of the water to the well, determining the mean residence time of groundwater in aquifer-well systems, predicting the arrival time of contaminant plumes to wells, and delineating well protection zones. Recently, analytical and semi-analytical solutions have been derived to compute leakage induced by pumping in a semi-confined aquifer (Butler and Tsou 2003; Zhan and Bian 2006). Analytical solutions to compute the residence time of leakage water under natural flow in a bounded semi-confined aquifer were derived by Braunsfurth and Schneider (2008). However, there have been no systematic analyses of the sources of the water to the well, mean residence time, or their influence on well protection area delineation in semi-confined aquifers.

For the wellhead protection program, the US Environmental Protection Agency (EPA) mandates the use of travel time criteria as the basis to delineate the protection zones around a drinking water production well. The US EPA (1987) suggests a series of delineation methods, ranging from a simple arbitrary fixed-radius method to a complex numerical groundwater model. The calculated fixed-radius (commonly called volumetric or cylinder) method is widely used, especially when the protection zones for thousands of rural drinking water wells are to be defined. However, the US EPA (1994) recognizes that the volumetric method is not appropriate for unconfined aquifers where there are recharge and overestimates the protection area for semi-confined aquifers with vertical leakage. Numerical groundwater models perform better when strong regional groundwater flow is present (Landmeyer 1994). Bogue (1994) evaluated several wellhead protection methods and found that while numerical groundwater models are more accurate, analytical models may still be adequate. In practice, the lack of reliable field data and the cost of computer modelling lead to the choice of simple analytical models for delineating wellhead protection areas.

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Y. Zhou (✉)
UNESCO-IHE Institute for Water Education,
P.O. Box 3015, 2601 DA Delft, The Netherlands
e-mail: y.zhou@unesco-ihe.org

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This paper analyzes the sources of water supplying a well in a conceptual semi-confined aquifer. An approximate solution is introduced to compute travel times of the leakage water to the well. The paper demonstrates that the travel time criterion is not adequate to protect the sources of water to a well in a semi-confined aquifer and proposes to use the percentage of the leakage water to the well as an additional criterion to define the wellhead protection area.

Conceptual hydrogeological model and analytical solution

The semi-confined aquifer is assumed to be homogeneous and isotropic with constant hydraulic conductivity (K) and uniform thickness (H). The aquitard is homogeneous with constant vertical hydraulic conductivity (K_V) and uniform thickness (d). A pumping well fully penetrates the semi-confined aquifer with a constant pumping rate (Q_0). Before the pumping, the groundwater head in the semi-confined aquifer is assumed to be the same as the water table in the phreatic aquifer (φ_0). After pumping, groundwater from the phreatic aquifer leaks into the semi-confined aquifer. Flow is assumed to be vertical in the aquitard and in a horizontal radial direction towards the pumping well in the semi-confined aquifer. The aquifer system laterally extends to infinity. It further assumes that no regional flow is present. Figure 1 shows the cross-section through the well of the conceptual aquifer system.

Under the aforementioned assumptions, the analytical solution in terms of drawdown ($s=\varphi_0-\varphi$) has been found to be (Hantush 1964; Verruijt 1970):

$$s = \frac{Q_0}{2\pi T}\frac{K_0(r/\lambda)}{(r_w/\lambda)K_1(r_w/\lambda)} \tag{1}$$

where: $K_0(x)$ is the second kind of modified Bessel function of order zero, and $K_1(x)$ is the second kind of modified Bessel function of order one, r is the radial distance and r_w is the radius of the well, λ is the leakage factor. $\lambda^2=Tc$, where T is the transmissivity and c is the resistance of the aquitard (thickness divided by the vertical hydraulic conductivity). When $r_w/\lambda<<1$, $(r_w/\lambda)K_1(r_w/\lambda)=1$, Eq. (1) becomes:

$$s = \frac{Q_0}{2\pi T}K_0(r/\lambda) \tag{2}$$

which is the solution found by Jacob in 1946. It shows that the drawdown follows a modified Bessel function of the radial distance scaled by the leakage factor. It also depends on the aquifer transmissivity (T) and pumping rate (Q_0). To visualise the drawdown distribution as a function of radial distance, a simple case is proposed with the following parameter values: Q_0=5,000 m^3 d^{-1}, well radius r_w=0.2 m, and hydraulic conductivity 50 m d^{-1} and thickness 50 m for the semi-confined aquifer. The vertical hydraulic conductivity and thickness are 0.1 m d^{-1} and 10 m, respectively for the aquitard. These will give values of T=2,500 m^2 d^{-1} and λ= 500 m for the aquifer. Figure 2 shows the drawdown distribution as a function of the radial distance scaled by the leakage factor. The maximum drawdown at the well is s_w= 2.53 m. The drawdown decreases with increasing radial distance. When the radial distance is around 4 times the leakage factor, the drawdown is practically zero. The leakage factor, defined for a semi-confined aquifer with a dimension of length, determines the zone of influence of a well. In practice, the leakage factor can be determined by the pumping test (Hantush 1956, Walton 1962).

Sources of water supplying the well

Under equilibrium conditions, the pumped water in a semi-confined aquifer comes from leakage through the aquitard. The actual source of the water is the water in the phreatic aquifer. The amount of the leakage can be calculated with the Darcy equation. The cumulative leakage (Q_L) through the aquitard can be calculated by

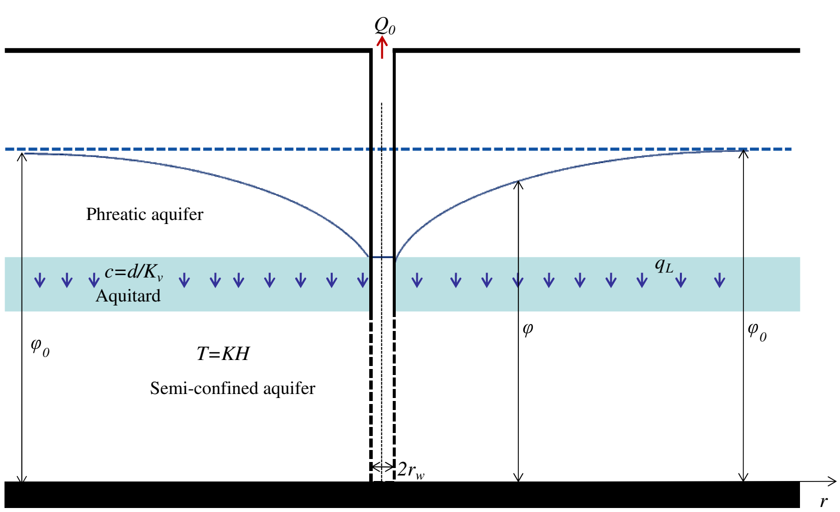

Fig. 1 A pumping well in a semi-confined aquifer

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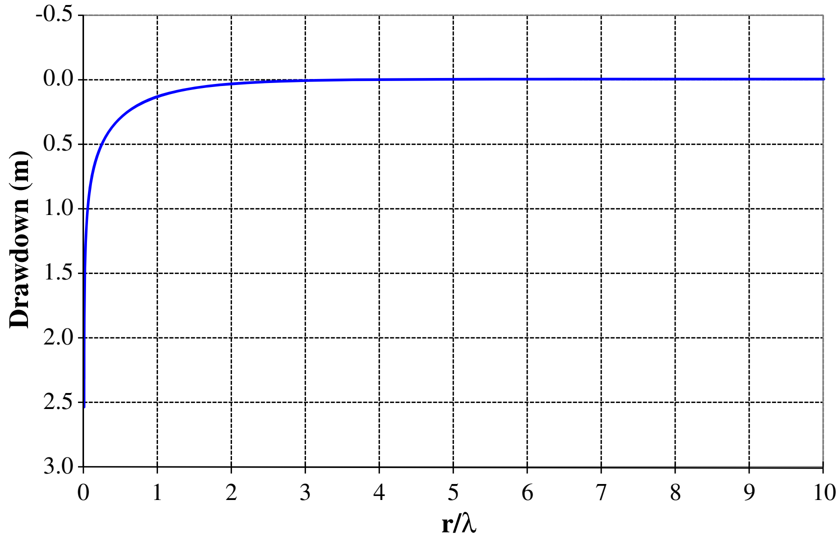

Fig. 2 Drawdown as a function of radial distance from the well (normalized by the leakage factor)

performing the integration from the well radius r_w to any radial distance r, as:

$$Q_L = \int_{r_w}^{r} 2\pi r q_L dr = \int_{r_w}^{r} \frac{Q_0}{\lambda^2} K_0\left(\frac{r}{\lambda}\right) r dr \tag{3}$$

Where q_L is the specific leakage (m d^{-1}). An analytical solution of Eq. 3 has been found by Zhan and Bian (Eq. 16 in Zhan and Bian 2006), and it can be rearranged as:

$$\frac{Q_L}{Q_0} = 1 - \frac{r}{\lambda} K_1\left(\frac{r}{\lambda}\right) \tag{4}$$

Here, λ is the leakage factor with a dimension of length. However, λ was defined as a leakage parameter (dimensionless) in Zhan and Bian (2006). Equation 4 shows that the percentage of leakage water to the well is only a function of the radial distance scaled by the leakage factor. When the radial distance approaches infinity, the total leakage (Q_L) equals the pumping rate (Q_0). When the radial distance equals 4 times the leakage factor (r=4 λ), Eq. 4 yields a value of 0.95. Therefore, 95% of the leakage water to a well will always come from within a radial distance of 4 times the leakage factor regardless of the actual pumping rate. Only 5% of the leakage water is derived from distances beyond 4 times the leakage factor. Therefore, the radial distance of 4 times the leakage factor can be defined as the practical radius of influence (or the catchment area) of a well in a semi-confined aquifer. For the example case, the practical radius of influence is 2,000 m.

Equation 3 can also be solved using numerical integration by a Riemann sum (Anton 1999). Figure 3 shows the percentage of leakage water to the well ($100 \times Q_L/Q_0$) as a function of the radial distance divided by the leakage factor. The Riemann sum calculated sufficiently accurate leakage rate compared to the analytical solution. The relative error is less than 0.04%. Notably, Fig. 3 is a generic plot of Eq. 4 independent of

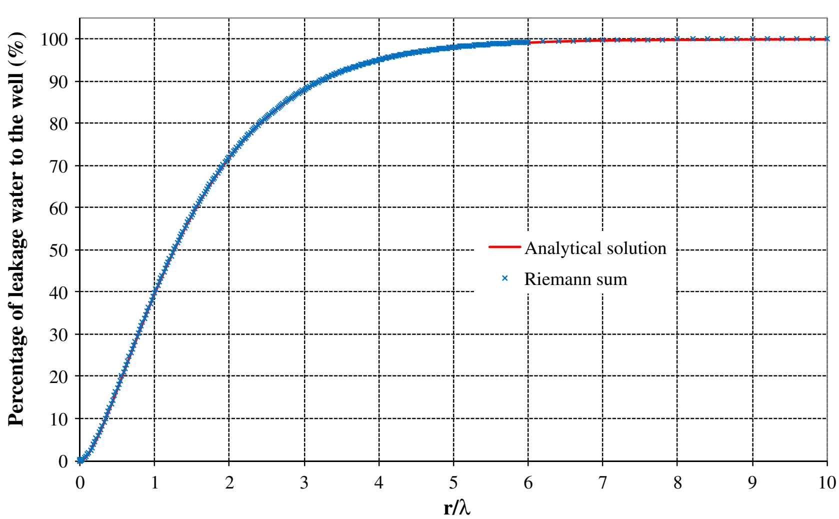

Fig. 3 Percentage of leakage water to the well as a function of radial distance (normalized by the leakage factor)

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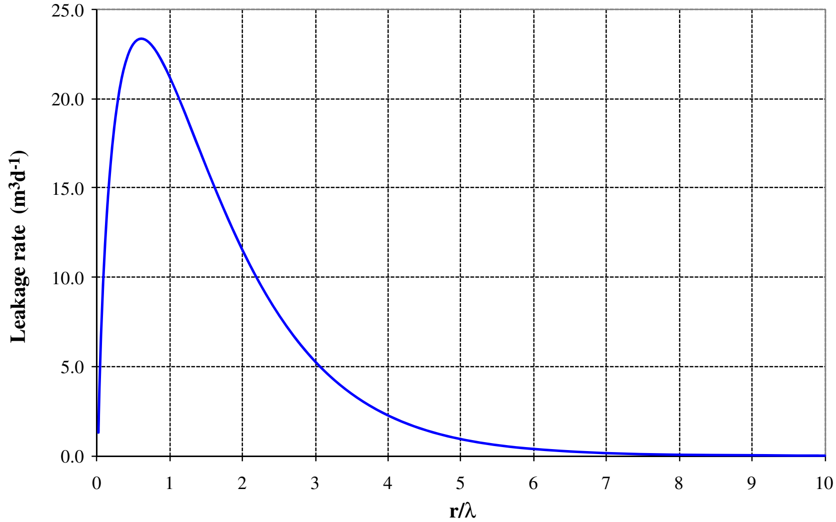

Fig. 4 Leakage rate as a function of radial distance (normalized by the leakage factor)

actual pumping rate and site-specific parameters, which is valid for all homogeneous semi-confined aquifer systems. Clearly, the increase of the leakage becomes very slow beyond the practical radius of influence (4 λ).

Figure 4 shows the distribution of the leakage rate as a function of the radial distance. The leakage rate follows a skewed distribution to the right. The maximum leakage rate occurs at a radial distance smaller than one leakage factor.

Arrival time and mean residence time

The groundwater velocity (v_r) in a semi-confined aquifer can be calculated with the Darcy equation as:

$$v_\mathrm{r} = \frac{q_\mathrm{r}}{n_\mathrm{e}} = -\frac{Q_0}{2\pi r_\mathrm{w} H n_\mathrm{e}} \frac{K_1(r/\lambda)}{K_1(r_\mathrm{w}/\lambda)} \tag{5}$$

where n_e is the effective porosity of the semi-confined aquifer and was taken as 0.3 in the example case, and q_r is the specific discharge and is derived from Eq. 1 using Darcy law. The travel time of the leakage water from any radial distance r to the well in the semi-confined aquifer can be calculated as:

$$t = \int_{\mathrm{r}}^{\mathrm{r_w}} \frac{dr}{v_\mathrm{r}} = \int_{\mathrm{r_w}}^{\mathrm{r}} \frac{2\pi r_\mathrm{w} H n_\mathrm{e}}{Q_0} \frac{K_1(r_\mathrm{w}/\lambda)}{K_1(r/\lambda)} dr \tag{6}$$

An explicit formula for the travel time is not available. Instead, approximate travel time was calculated by the numerical integration of the Riemann sum. Simpson et al. (2003) compared the analytical solution of travel time with numerical integration by a Riemann sum for steady radial flow in an unconfined aquifer. They showed that the numerical integration produced identical travel times and is easier to implement in a worksheet. Figure 5 shows the

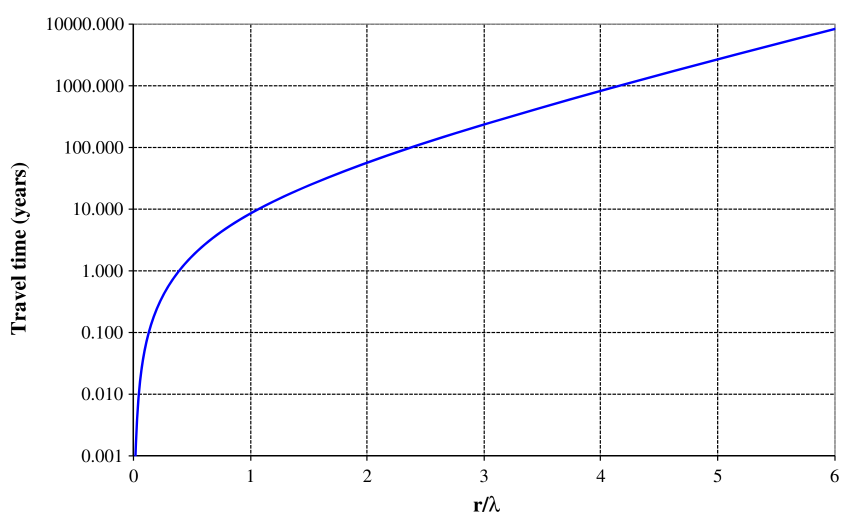

Fig. 5 Arrival time of leakage water at the well as a function of radial distance (normalized by the leakage factor)

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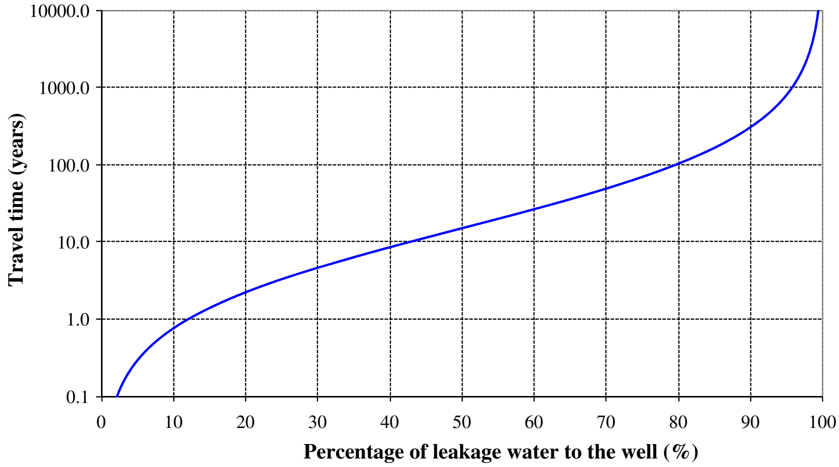

Fig. 6 Relation between arrival time and percentage of leakage water to the well

travel-time distribution for the example case. The arrival time of leakage water to the well ranges from a few days in the vicinity of the well to 857 years at the practical radius of influence of the well (2,000 m). The travel time of the leakage water increases drastically beyond the practical radius of influence, indicating the stagnant nature of leakage water at large radial distances.

Figure 6 shows the relation between the arrival time and the percentage of leakage water to the well and reveals that the pumped water is a mixture of leakage water with various residence times. Around 12% of leakage water to the well has a residence time less than 1 year; 42.3% of leakage water to the well has a residence time less than 10 years; about 20% of leakage water to the well has a residence time longer than 100 years. The mean residence time of all leakage water to the well can be used as an indicator of the renewable period of a semi-confined aquifer system. The mean residence time of a semi-confined aquifer system should be calculated as the leakage-rate weighted average of the residence times of the leakage water. The mean residence time within the practical radius of influence of the example case was calculated to be 64 years, which indicates that a long period of time is required to remediate polluted groundwater with the pump-treatment method once an aquifer is polluted. It should be pointed out that the conventional method of calculating the mean residence time, by dividing the storage by the flow rate (Kazemi et al 2006), yields a larger mean residence time. For example, a mean residence time of 108 years was calculated by dividing the storage by the total leakage rate within the radius of influence. The reason is that the storage divided by the total leakage rate gives an arithmetic mean of residence times of leakage water, which overestimates the mean residence time since the leakage rate distribution is skewed to the right.

Well protection zones

To protect sources of water to the well, well protection zones are designated (US EPA 1987; Chave et al 2006). Well protection zones have usually been delineated using travel-time criteria. In the Netherlands, three protection zones are defined in a porous-medium aquifer to protect drinking-water sources (Van Waegeningh 1981): an inner protection zone with a travel time of 60 days, an outer protection zone with travel times from 10 to 25 years, and the remaining catchment area, in which soil and groundwater protection measures are in place. The radius of the protection area can be calculated according to travel-time criteria, and some typical results are listed in Table 1, showing that a 25-year protection zone has a radius of 750 m but only protects 58.4% of leakage water to the well. Furthermore, the calculation of the travel time did not consider the travel time of water moving through the phreatic aquifer and aquitard. When these travel times are included, a 25-year protection zone would result in an even smaller radius, thus protecting even less of the source water for the well. It is clear that the traditional method of delineating well-protection zones based on the travel-time criterion is inadequate to protect drinking-water sources.

The objective of the protection area in a semi-confined aquifer should be to protect the leakage water for the well. Because an infinite protection area is out of the question from a management point of view, the percentage of leakage water to the well can be used as a new criterion to define well-protection areas. Table 1 shows some examples of the percentage of leakage water to the well and the corresponding protection area radius. Figure 7 plots the well protection area versus the percentage of leakage water to the well. The higher the percentage of leakage water to be protected, the larger the protection area that is

Table 1 Radius of protection area as the percentage of leakage water to the well calculated for the proposed example

Protection zone	Radius of the protection area (m)	Percentage of leakage water to the well (%)	Leakage-rate weighted mean residence time
60 days	80	3.15	29 days
10 years	530	42.30	3.4 years
25 years	750	58.40	7 years
325 years	1,610	90.00	39 years
857 years	2,000	95.00	64 years
6654 years	2,885	99.00	151 years

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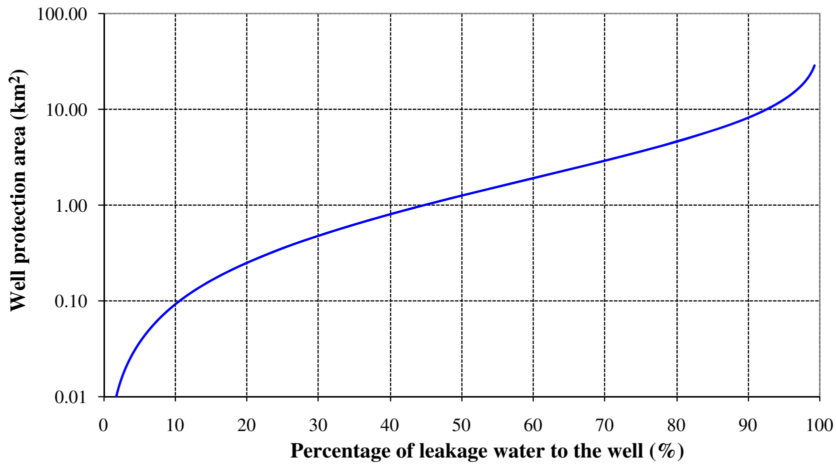

Fig. 7 Relation between the well protection area and percentage of leakage water to the well

required. For example, a protection area of around 8 km^2 is required to protect 90% of the leakage water to the well. The protection area for 95% of the leakage water to the well is around 13 km^2. The protection area increases drastically beyond 95% of the leakage water to the well. The protection area will increase to 26 km^2 when 99% of the leakage water to the well is to be protected. Therefore, it is logical to use the practical radius of influence (4 times the leakage factor) as the well protection zone. In practice, water supply companies can make a compromise between the safety of the water supply and the cost of well protection using Fig. 7.

Notably, the protection area for 95% of leakage water to the well will always be the area within the practical radius of influence (4 times leakage factor) regardless of the actual pumping rate or site-specific aquifer parameters. However, the size of the protection area and the maximum travel time do depend on site-specific aquifer parameters and pumping rate. Table 1 lists the values for the example case. Equations 3 and 6 should be used to calculate the percentage of leakage water to the well and corresponding travel times for other homogeneous semi-confined aquifers. For aquifer systems with complex geometries, heterogeneous media and boundary conditions, numerical groundwater flow models combined with particle tracking methods should be used to delineate the source area, to calculate the percentage of the leakage water to the well and the corresponding travel times. The model results can be used to construct Figs. 6 and 7 to determine the well-protection area.

Conclusions and discussion

Well water in a semi-confined aquifer comes from leakage of the phreatic aquifer water through the aquitard. The majority of the leakage water comes from within a limited radial distance. A value of 95% of the leakage water to a well comes from within a radius of 4 times the leakage factor. This radius can be called the practical radius of influence of a well in a semi-confined aquifer system. The leakage factor is a very important characteristic length of semi-confined aquifer systems that determines the practical radius of influence of a well. The travel time of leakage water from the practical radius of influence to the well is much longer than the travel-time criterion presently used to delineate well-protection areas. For example, a well-protection area based on a 25-year travel time criterion only protects less than 60% of leakage water to the well in the example case. The objective of a well-protection area in a semi-confined aquifer should be to protect the leakage water for the well. Therefore, it is more appropriate to use the percentage of leakage water to the well as the criterion to define the well-protection area. The well protection area for 95% of leakage water to the well is the area within the practical radius of influence of the well (4 times the leakage factor). The protection area increases drastically with further increases in the percentage of leakage water to the well beyond 95%. Thus, the use of the practical radius of the influence of a well to define the well protection area is a good compromise for the safety of the drinking-water supply against the cost of well protection.

The mean residence time is an indicator of the renewable period of the aquifer system. The mean residence time indicates the time required to remediate groundwater pollution in the aquifer when using the pump-treatment method. It also indicates that pumped water is a mixture of water with various residence times, and likely, different qualities. A long mean residence time may indicate stable quality of the pumped water with a higher percentage of groundwater of longer residence time. The mean residence time should be calculated as the leakage-rate weighted residence time of leakage water to the well.

Although the concept and approach were illustrated with the analytical solution of the steady radial flow to a well in a homogeneous and isotropic semi-confined aquifer without the presence of regional flow, the principles presented in this paper can be applied to aquifer systems with complex geometries, hydrogeological characteristics and boundaries. In these cases, numerical groundwater flow modelling combined with particle tracking methodology should be used to delineate the source areas and to calculate the

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percentage of the sources of water to the well and their corresponding travel times. The results of the numerical model can be used to construct curves of the percentage of the sources of water to the well versus travel times and thereby delineate well protection areas. These curves can be used by water-supply companies to make informed decisions, balancing the safety of the drinking-water supply with the cost of well protection.

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