7. Exercises

7.1 Fundamental Equations of Groundwater Flow

Problem 1

In a laboratory Darcy's experiment is used to determine the permeability of a sand sample. The length of the sample is 20 cm and its cross-sectional area is 10 cm^2. The difference in head between the two ends is 25 cm, and the amount of water flowing through the sample in 5 min is measured to be 75 cm^3. Please calculate the value of the hydraulic conductivity (m/day).

Problem 2

For a certain type of sand the hydraulic conductivity has been measured to be K = 30 m/day, with water at a temperature of 20 ^oC (kinematical viscosity v = 10^{-6} m^2/s). What would be the value at a temperature of 5 ^oC (kinematical viscosity v = $1.5x10^{-6}$ m^2/s)?

Problem 3

For a certain type of sand the hydraulic conductivity has been measured to be k= 30 m/day. The porosity of the sand was n = 0.40. What would be the hydraulic conductivity if the sand were compacted (for instance by vibration) so that the porosity is reduced to 0.35, without crushing the particles?

7.2 Steady Groundwater Flow in Aquifers

Problem 1

Two rivers located 1000 m apart fully penetrate a confined aquifer (Figure 1). The confined aquifer consists of gravels in left part (500 m) and sand in right part (500 m). The parameters of the aquifer are:

Hydraulic conductivity: K_1 = 200 m/day; K_2 = 50 m/day;

Effective porosity: n_1 = 0.25; n_2 = 0.30;

Thickness of the aquifer: H = 10 m;

Water level in left river H_0 = 20 m;

Water level in right river H_L = 18 m;

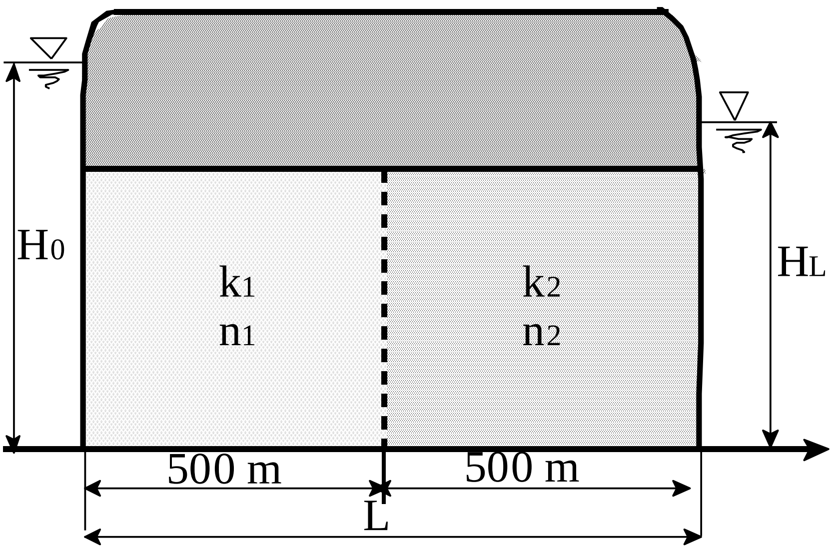

Figure 1 Groundwater flow in a confined aquifer

Please carry out the following tasks:

a) determine the distribution of groundwater head in the aquifer;

b) calculate the velocity of groundwater in the aquifer. Is the velocity same in the two parts of aquifer ?

c) calculate the unit width discharge to the right river;

d) calculate the travel time of a water particle travelling from left river to right river.

Problem 2

Two rivers located 1000 m apart fully penetrate a phreatic aquifer (Figure 2). The parameters of the aquifer are:

Hydraulic conductivity K = 15 m/day;

Uniform recharge w = 3.0 * 10^{-3} m/day;

Water level in left river h_0 = 20 m;

Water level in right river h_L = 18 m;

Porosity n=0.25.

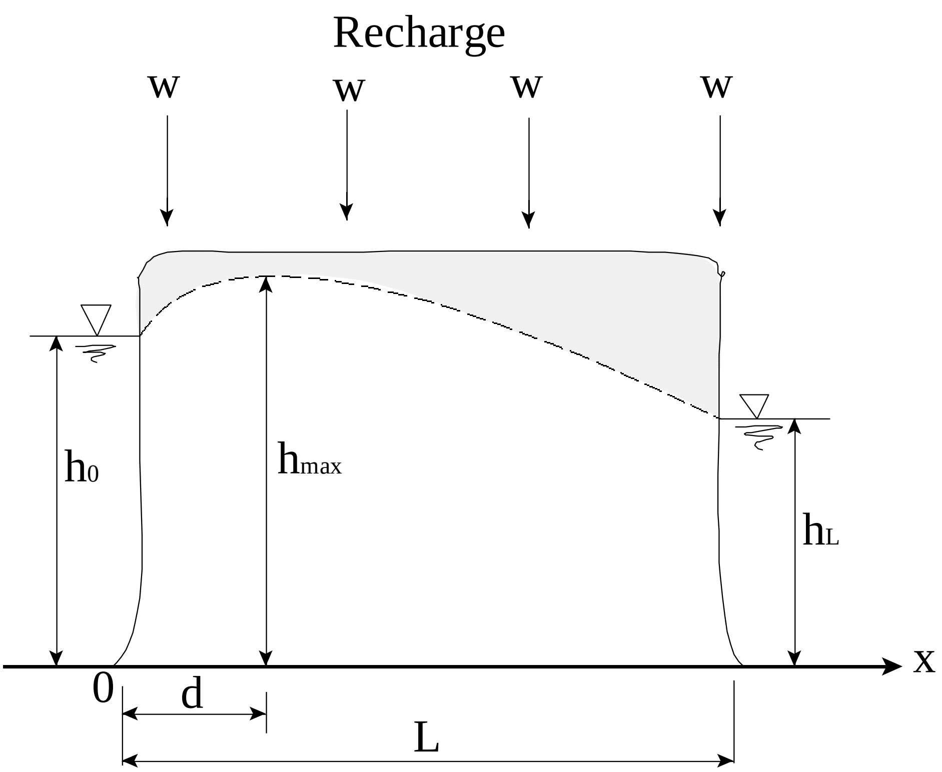

Figure 3.2 Groundwater flow in an unconfined aquifer

Please carry out the following tasks:

(a) determine the location (d) and height (h_{max}) of the water divide.

(b) calculate the unit width discharges of the aquifer into the left and right rivers.

(c) estimate the travel times from the left river to the right river when there is no recharge.

Problem 3

A dike (Figure 3) was built to separate a lake from low-lying land so that the land can be reclaimed for agricultural use. This low-lying land is called a polder in the Netherlands. The drainage has to be implemented to keep the required water level in the polder. The amount of drainage will depend on the seepage from the lake. It is assumed that the dike itself is completely impermeable, and that the soil consists of a permeable aquifer (semi-confined) with an overlying layer of low permeability (semi-permeable layer).

Data for the semi-confined aquifer are: hydraulic conductivity K = 10 m/d; thickness H = 14 m. Data for the semi-permeable layer are: hydraulic conductivity k' = 0.001 m/d; thickness d = 2m. Other data are: water level in the lake φ_1 = 20 m; water level in the polder φ_4 = 16 m; the width of the dike 2L = 4 m.

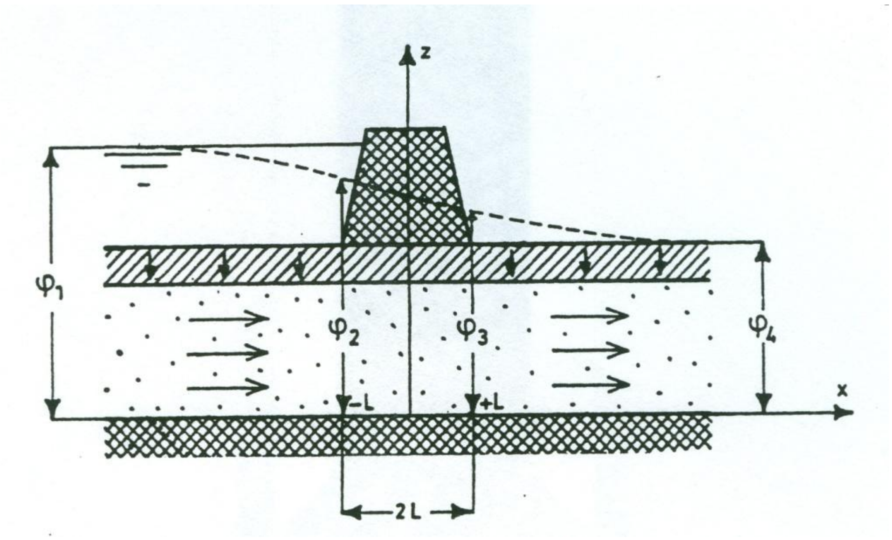

Figure 3.3 Groundwater flow under the dike

Please carry out the following tasks:

a) derive the formula of groundwater head and unit width discharge for the left part of the aquifer from negative infinite up to the left side of the dike ($-\infty < x < -L$).

b) derive the formula of groundwater head and unit width discharge for the middle part of the aquifer under the dike ($-L < x < +L$).

c) derive the formula of groundwater head and unit width discharge for the right part of the aquifer from the right side of the dike up to positive infinite ($+L < x < +\infty$).

d) derive the formula and calculate φ_2, φ_3 and the unit width discharge at x = + L (Q_0). Q_0 will be the seepage to the polder.

7.3 Steady Groundwater Flow to Wells

Problem 1

In the centre of a circular confined aquifer of thickness H = 20 m, a well is constructed to pump water with a constant rate Q_0 = 250 m^3/day. The aquifer consists of two regions with different hydraulic conductivity and effective porosity (see Figure). The values of parameters are:

K_1 = 50 m/day; n_1 = 0.25; K_2 = 25 m/day; n_2 = 0.3;

φ_0 = 25 m; R = 5,000 m; r_w = 0.5m; r_a = 50 m.

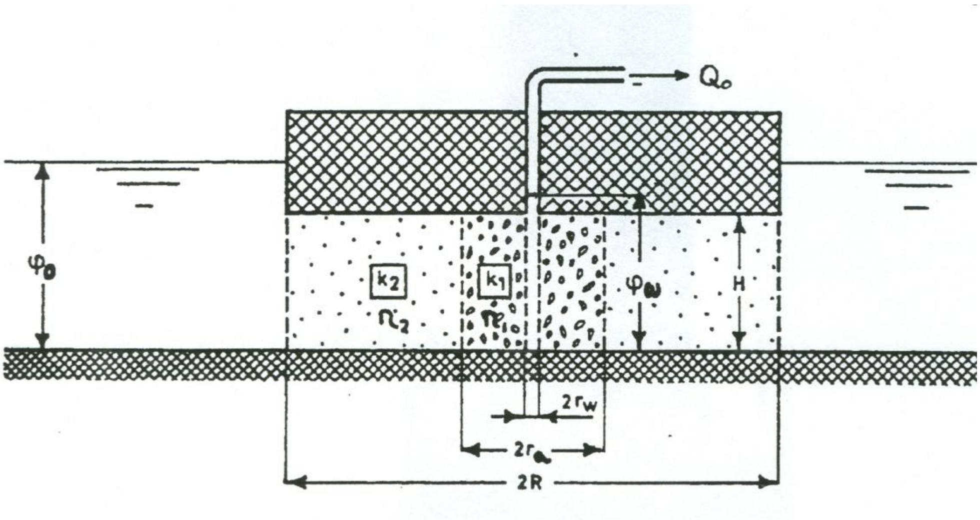

Figure 4.1 Groundwater flow to a well in a non-homogeneous confined aquifer

Please carry out the following tasks:

a) calculate the groundwater head and drawdown at $r = r_a$;

b) calculate the head and drawdown at the pumping well ($r = r_w$);

c) calculate groundwater velocity at $r = r_a$ and $r = r_w$;

d) calculate the total time of a water particle travelling from the boundary (lake) to the pumping well.

Problem 2

A well is constructed to pump water from a confined aquifer with a thickness of 20 m. Two observation wells, OW1 and OW2, are constructed at distance of 100 m and 500 m, respectively (Figure 2). With a pumping rate of 400 m^3/day, the steady drawdown is observed as 0.34 m in OW2 and 0.68 m in OW1.

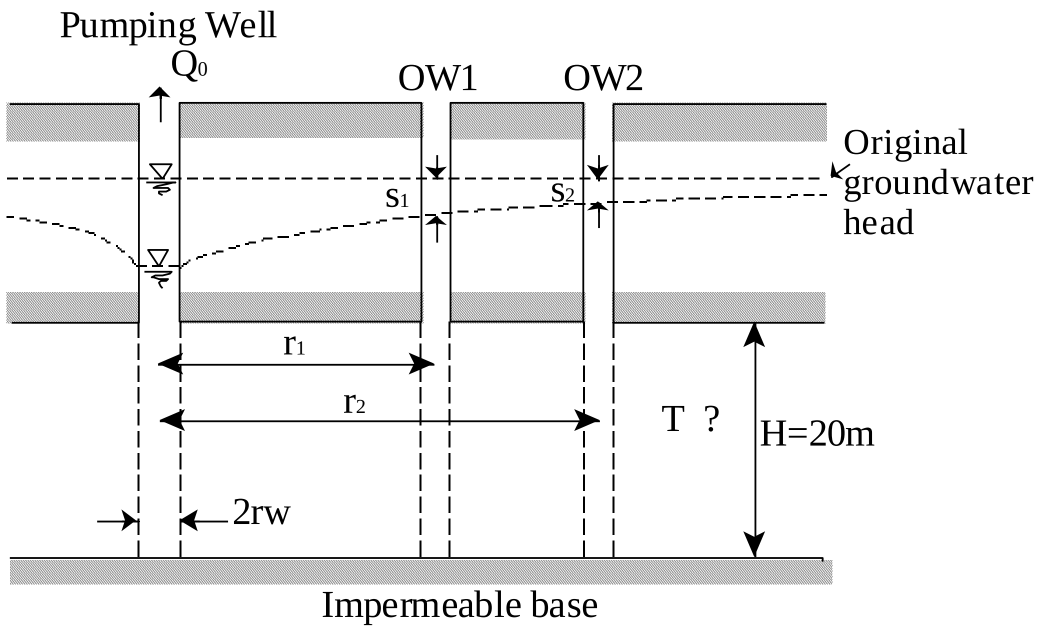

Figure 4.2 Pumping test in a confined aquifer

Please carry out the following tasks:

(a) determine the transmissivity and hydraulic conductivity of the aquifer from the measurements.

(b) determine the radius of influence (R) from the measurements.

(c) predict the drawdown at the pumping well (r_w = 0.1 m) when the pumping rate increases to 500 m^3/day.

Problem 3

A pumping test is conducted in an unconfined aquifer to determine the hydraulic conductivity and the proper pumping rate. The groundwater table before the pumping is 20 m above the impermeable bottom. The radius of the pumping well is 0.1 m and the distance between the pumping and observation well is 1000 m. With a pumping rate of 300m^3/day, the steady drawdown is observed as 1 m in the observation well OW1 and 3 m in the pumping well.

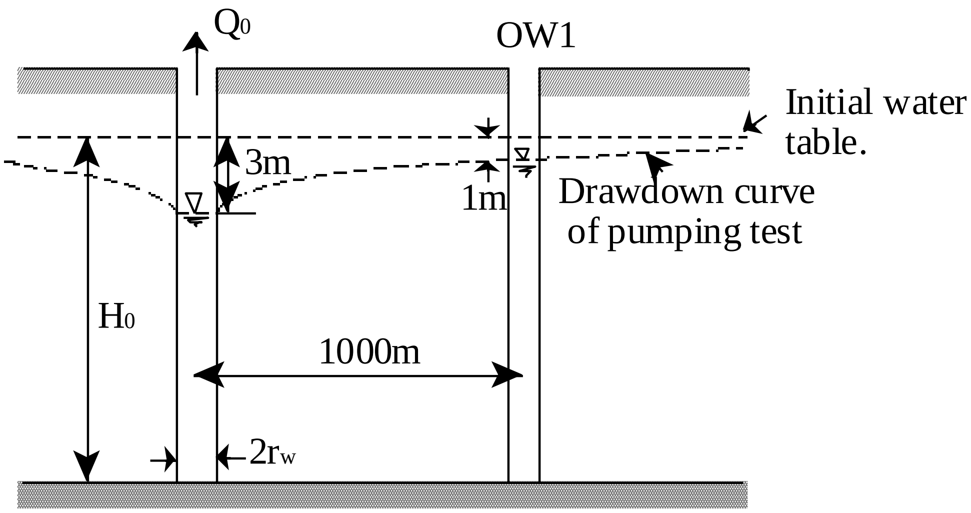

Figure 4.3 Pumping test in an unconfined aquifer

Please carry out the following tasks:

(a) determine the hydraulic conductivity of the aquifer from the pumping test data.

(b) determine the pumping rate if the maximum drawdown at the pumping well is allowed to be 6 m.

(c) predict the possible drawdown at the observation well with the new pumping rate.

Problem 4

A well with a radius of 0.1 m is constructed in a semi-confined aquifer to pump groundwater with a pumping rate of 300m^3/day. The thickness of the semi-confined aquifer is 20 m and hydraulic conductivity is 15 m/day. The thickness of the semi-permeable layer is 1 m and vertical hydraulic conductivity is 0.0001 m/day.

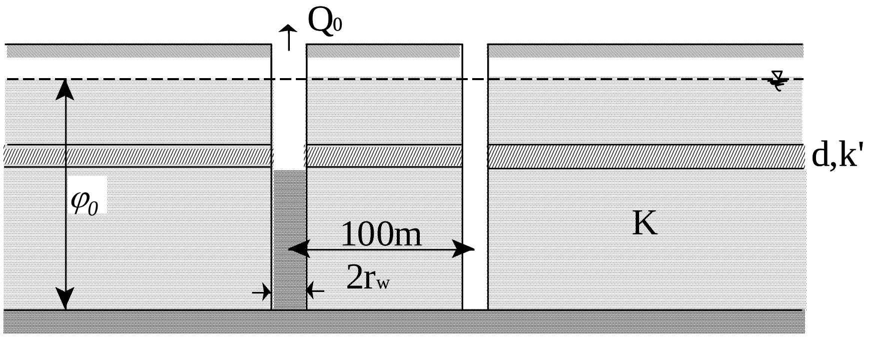

Figure 4.4 Groundwater flow to a well in a semi-confined aquifer

Please carry out the following tasks:

a) calculate the drawdown at the pumping well and at a distance of 100 m away from the pumping well with the exact solution;

b) assume that a confined aquifer of the same transmissivity has a circular recharge boundary with a radius which is equal to the practical radius of the influence (4λ), calculate the drawdown at the distance of 100 m away from the pumping well with the same pumping rate of 300 m^3/day. Compare the drawdown calculated in a).

7.4 Methods of superposition and image

Problem 1

Four wells of the same discharge Q_0 are operating in an unconfined aquifer in the corner points of a square with sides 2a. The aquifer is bounded externally by a circle of radius R, the centre of which coincides with the centre of the square. The radius R is so large compared to the dimension so that the individual wells can be central.

Please establish, by means of superposition, the formula for the height of the water table in (i) the centre of the square, and (ii) the mid-point of a side of the square, using the drawdown formula for the confined aquifer as the approximate solution for the unconfined aquifer.

This system is used to lower the water table to a depth of 4m below the original water table to permit the excavation of a building pit in the form of a square with sides of 40m. The original water table was at 10m above the impermeable base. What should be the discharge of each well in order to keep the bottom of the building pit dry, if k = 10 m/day and R = 2000m?

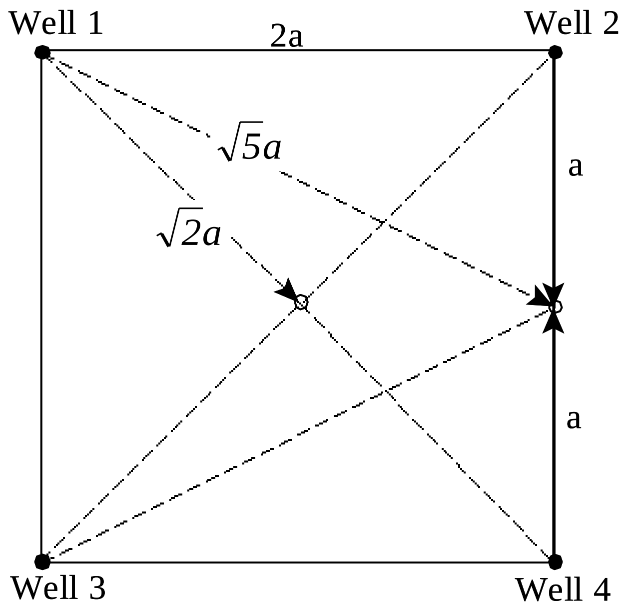

Figure 4.5 A system of 4 wells to lower water table for excavation

Problem 2

In a semi-confined aquifer, bounded by a straight impermeable boundary, a well is operating at a distance of 500m from the impermeable boundary. The radius of the well is r_w=0.50m, the transmissivity of the aquifer is T=200 m^2/day, the resistance of the confining layer is c=500 days, and the discharge of the well is Q_o=300 m^3/day.

Please calculate the drawdown in the well.

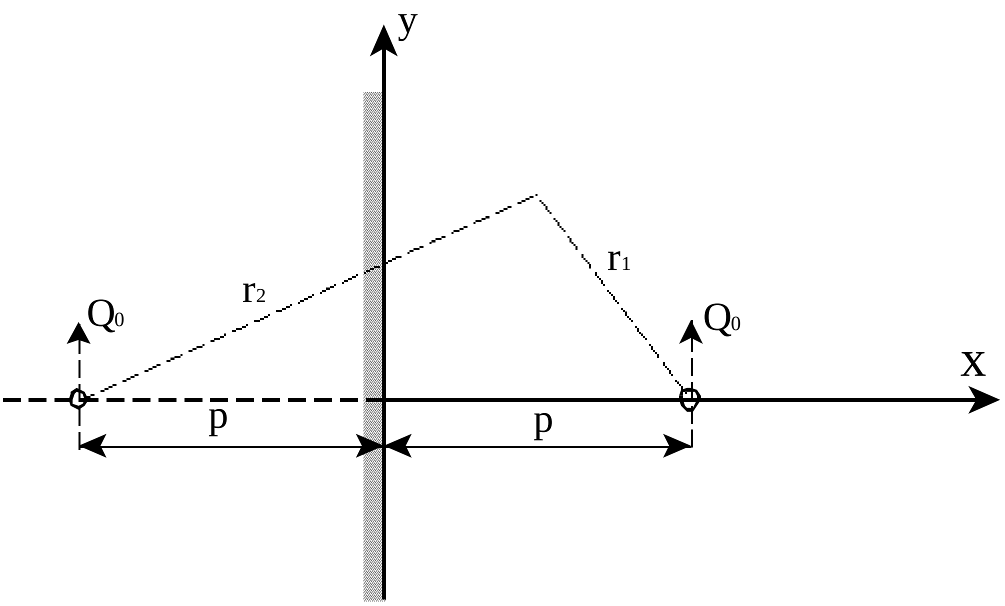

Figure 4.6 A pumping well near a straight impermeable boundary

Problem 3

A company wishes to extract water at a discharge of Q_o = 300 m^3/day from a completely confined aquifer, which is bounded by a straight canal. The transmissivity of the aquifer is 200 m^2/day. In order to obtain water of optimal quality it is desirable to construct the well as far from the canal as possible. On the other hand, however, local authorities require that at a distance of 400 m from the canal the ground water head in the aquifer may not be lowered by more than 0.10m. At what distance from the canal should the well be located.

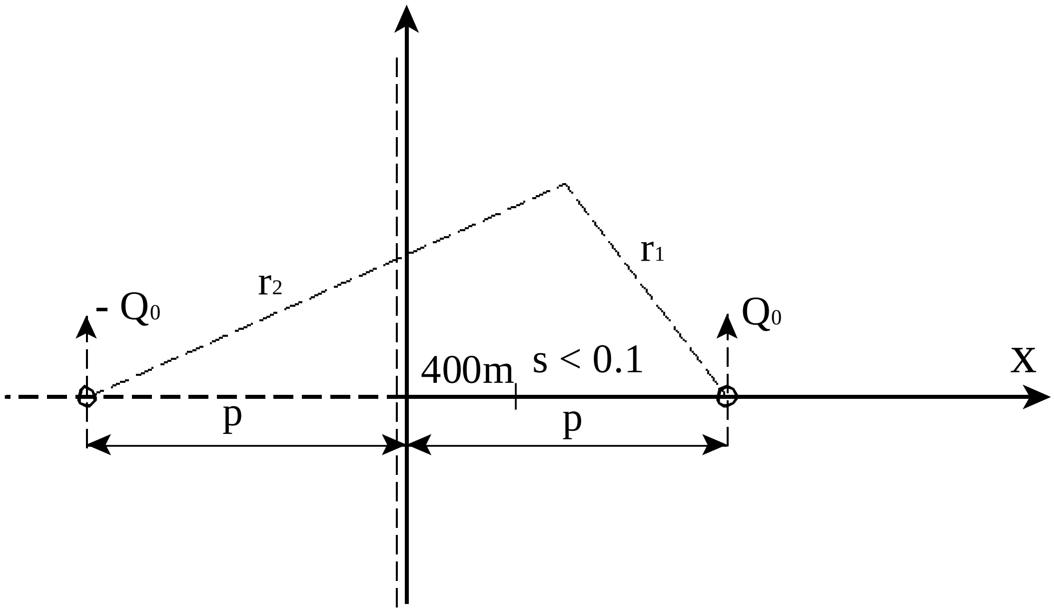

Figure 4.7 A pumping well near a straight canal