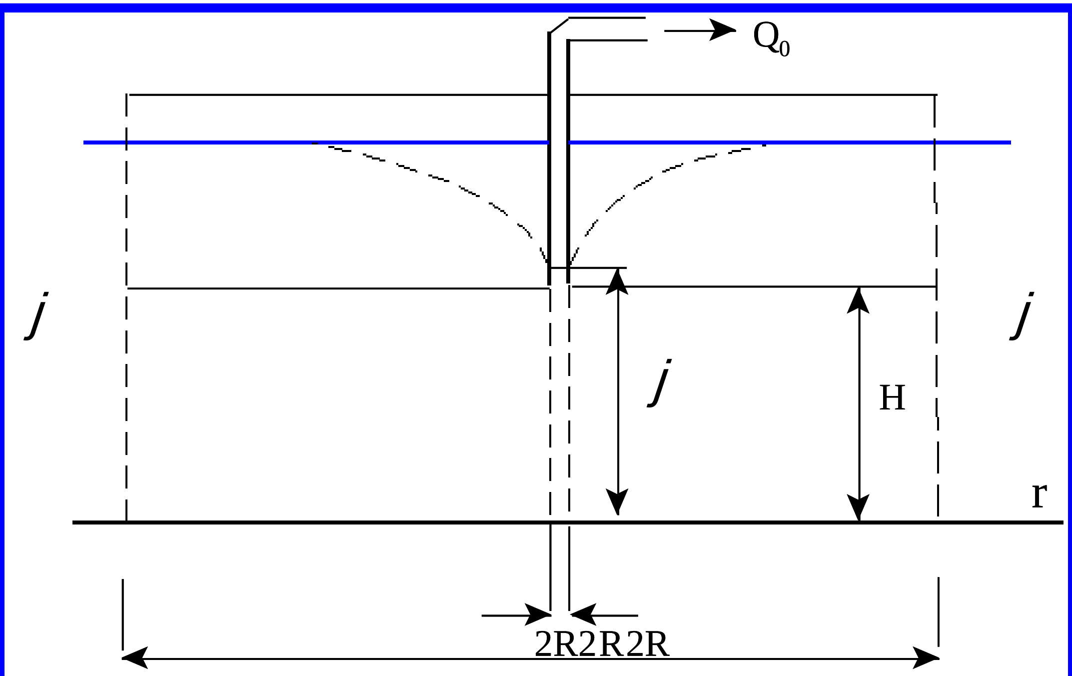
Steady groundwater flow to wells

- A well in a confined aquifer
- A well in an unconfined aquifer
- A well in a semi-confined aquifer

- Conceptual hydrogeological model
- Mathematical model
- Analytical solution
- Analysis of the solution
- Application to pumping test

Steady flow to a well in a confined aquifer

- Conceptual hydrogeological model
  - A confined aquifer is located in a circular island surrounded by a lake with constant level;
  - The aquifer is homogeneous and isotropic;
  - In the center of the island a pumping well fully penetrates the whole thickness of aquifer operating with a constant pumping rate;
  - Before pumping, groundwater head is in the same level in the lake.



Questions:

1. Drop of groundwater head?
2. Where does water come from?
3. Mean residence time?
4. Protection area?

Steady flow to a well in a confined aquifer

Example:

Q_0=5000 m^3/d

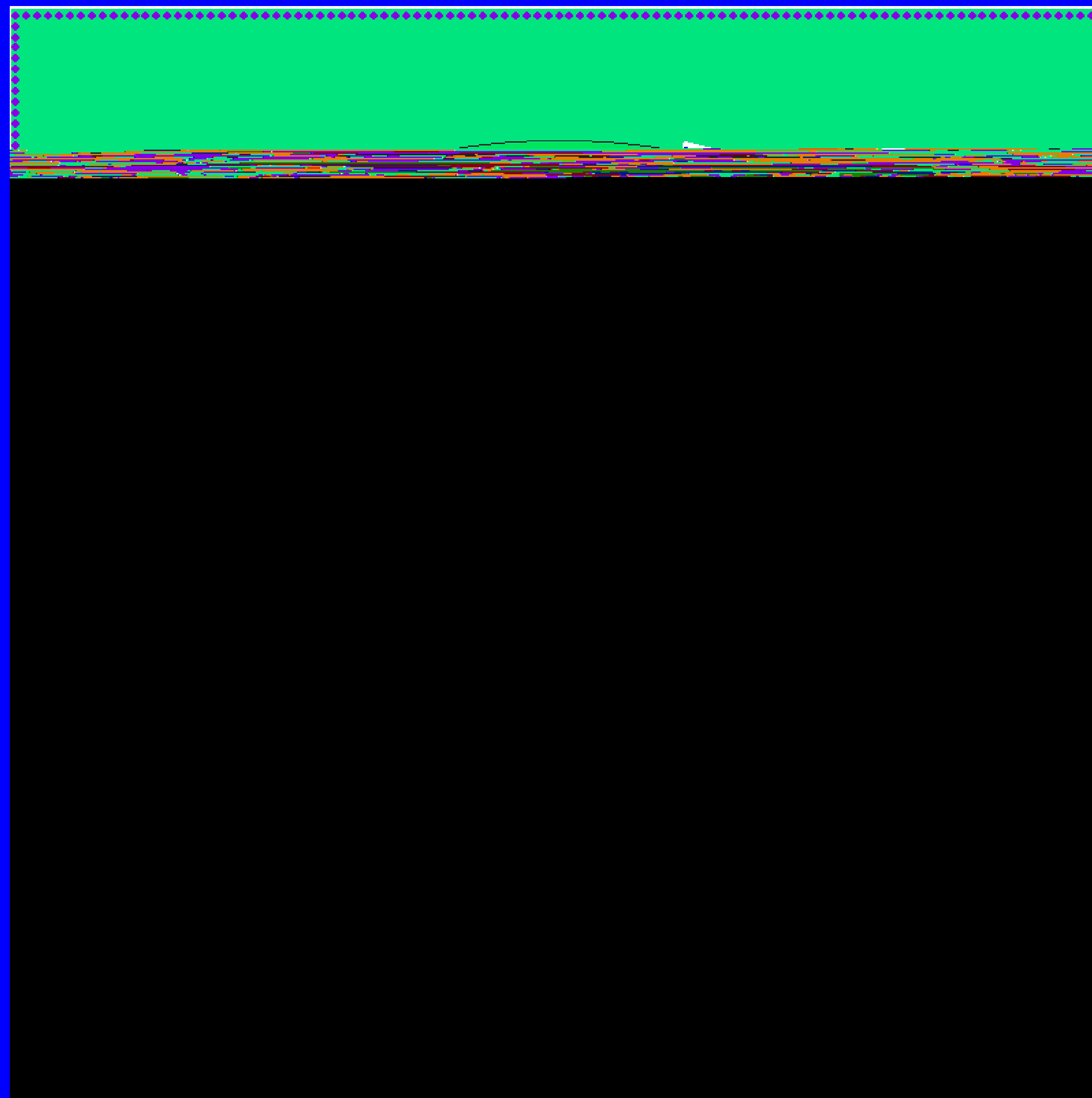
r_w=0.2m

H=50 m

K=50 m/d

R=2500 m

s_w=3.0 m



Steady flow to a well in a confined aquifer

- Source of water to the well

Specific discharge:

$$q_r = -K\frac{d\varphi}{dr} = -\frac{Q_0}{2\pi H}\cdot\frac{1}{r}$$

Total discharge:

$$Q_r = 2\pi r H q_r = 2\pi r\ H\ (-\frac{Q_0}{2\pi H}\frac{1}{r}) = -Q_0$$

The water to the well comes from the inflow of the boundary

Steady flow to a well in a confined aquifer

Velocity:

$$v_r = \frac{q_r}{n_e} = -\frac{Q_0}{2\pi n_e H}\frac{1}{r}$$

Example:

Q_0=5000 m^3/d

r_w=0.2m

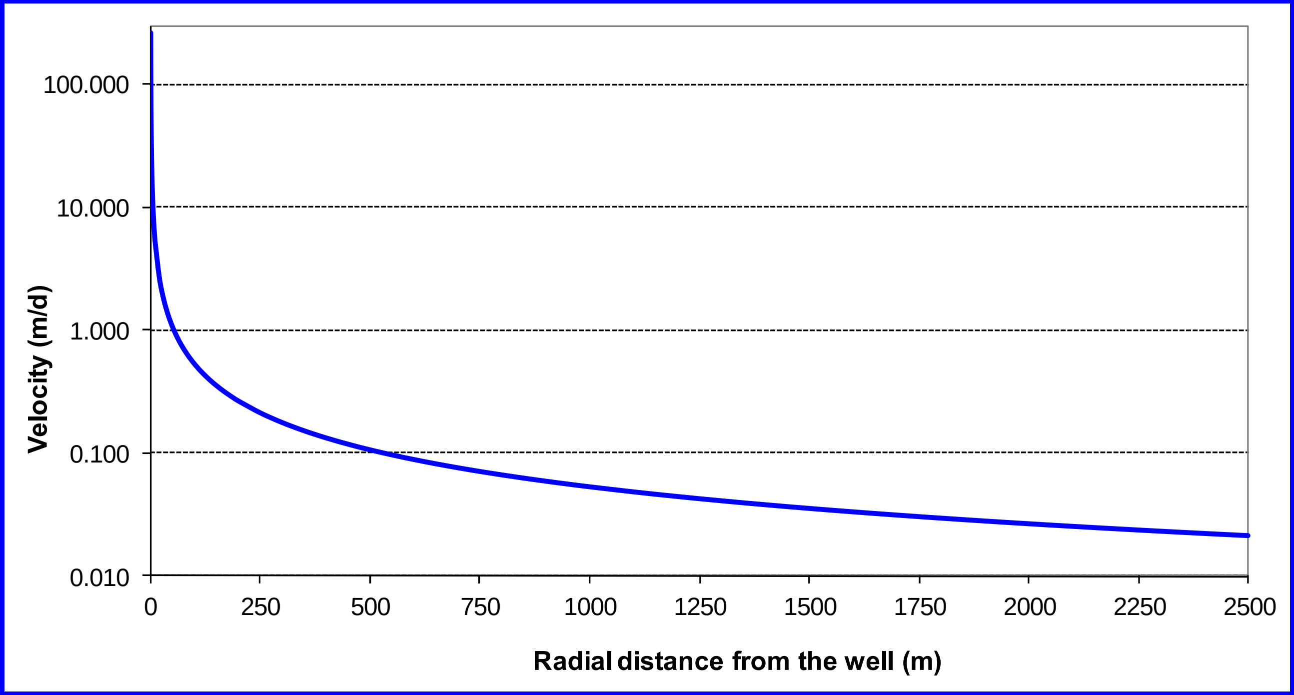
H=50 m

K=50 m/d

R=2500 m

n_e=0.3

v_w=265 m/d



Steady flow to a well in a confined aquifer

Travel time:

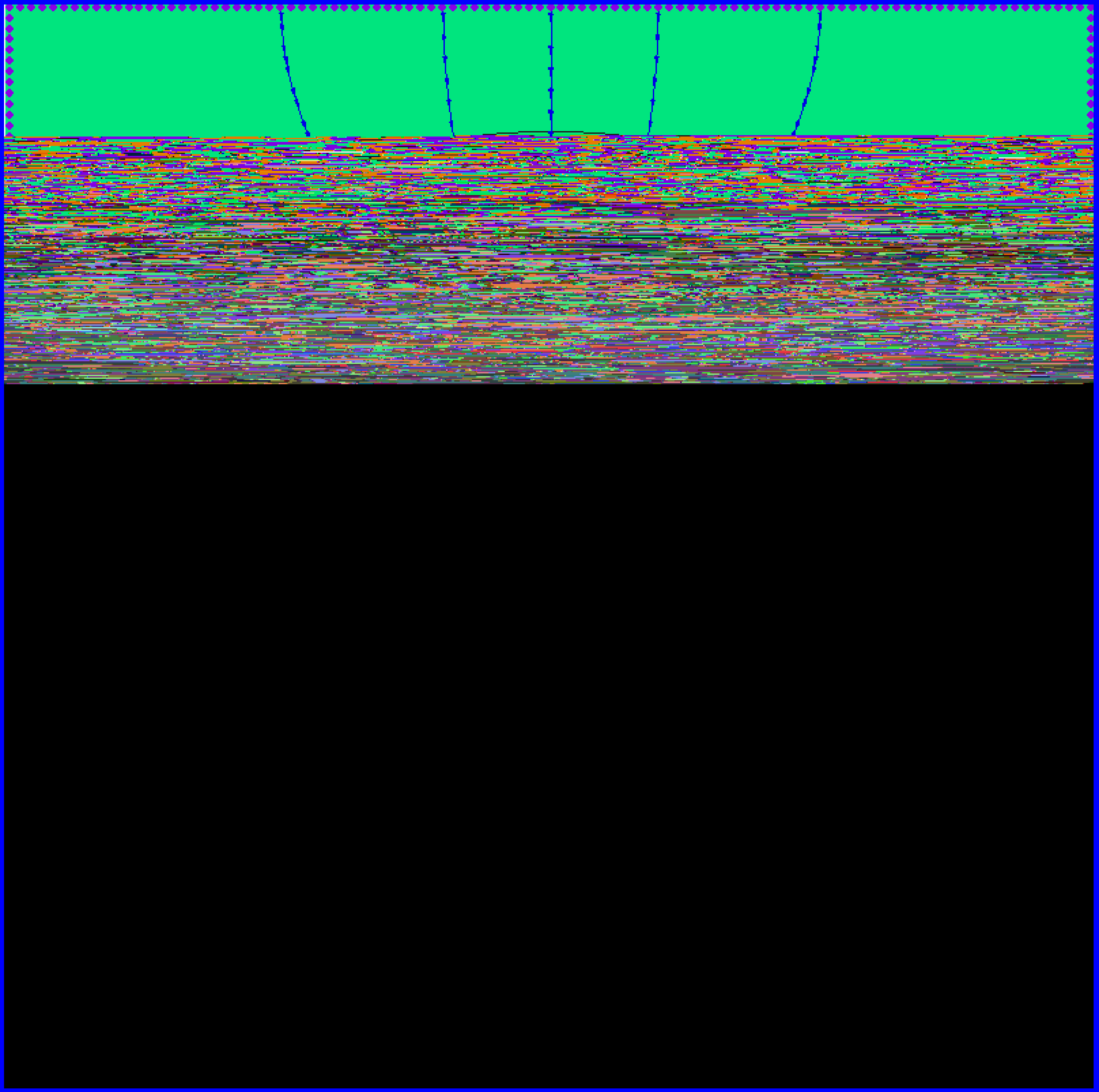
$$\frac{}{} \qquad \frac{}{Q} \qquad r\,dr = \frac{n\;\;H}{Q}\left(R \;-\; r\;\right)$$

Groundwater mean residence time

Example:
Q_0=5000 m^3/d
r_w=0.2m
H=50 m
K=50 m/d
R=2500 m
n_e=0.3
t_R=161 years

$$\frac{}{} \qquad \frac{}{Q} \qquad r\,dr = \frac{n\;\;H}{Q}\left(R \;-\; r\;\right)$$

Steady flow to a well in a confined aquifer



Model generated flowlines:

Q_0=5000 m^3/d

r_w=0.2m

H=50 m

K=50 m/d

R=2500 m

n_e=0.3

Travel time between two marks is 10 years

Steady flow to a well in a confined aquifer

Well protection zones:

- Wellhead protection zone
- 50-60 days for pathogenic germs;
- 10 to 25 years for more persistent pollutants.

Travel time of the protection zone:

$$\tilde{n} \frac{\quad}{\quad} \qquad \frac{p}{\qquad} \tilde{n} \qquad \frac{p}{\qquad} \qquad {}^{2} \qquad {}^{2}$$

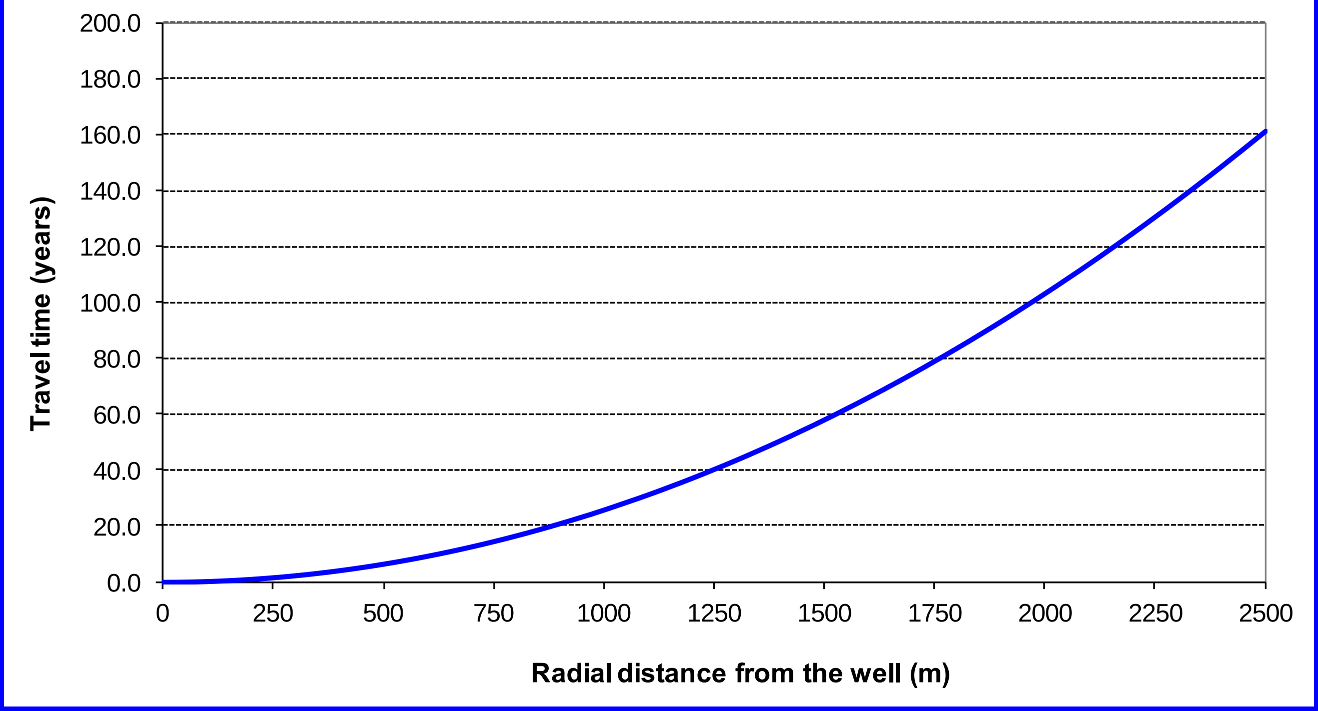
Radius of the protection area:

$$= \sqrt{\frac{\qquad}{p}}$$

Steady flow to a well in a confined aquifer

Protection zone	Radius of the protection area (m)
60 days	88
10 years	622
25 years	984

Water comes from the inflow boundary at a radial distance of 2500m!



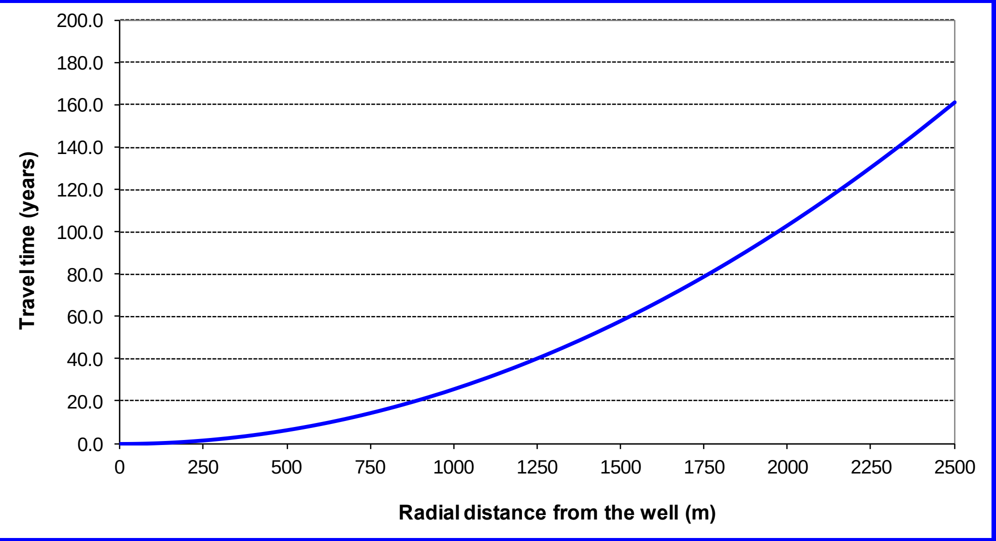
Steady flow to a well in a confined aquifer

For a confined aquifer, the idea of the protection area defined by the travel time is not relevant since there might be no direct pollution from land surface. The protection of the inflow source is more important. For the example case, the residence time is 161 years, so that pollutants from the inflow boundary will on average take 161 years to arrive at the pumping well. Systematic monitoring of the inflow source water will provide more useful information for the protection of the well.

Assume:

Q_0=5000 m^3/d, r_w=0.2m, H=50 m, K=50 m/d, R=2500 m, n_e=0.3.

Mean residence time Ǘ= 161 years



Steady flow to a well in a confined aquifer

- Application to pumping test

Steady state head in the observation well 1:

$$\varphi_1 = \varphi_0 - \frac{Q_0}{2\pi T}\ln\left(\frac{R}{r_1}\right)$$

Steady state head in the observation well 2:

$$\frac{R}{r})$$

Transmissivity:

)

$$\frac{R}{r})$$

Steady flow to a well in a confined aquifer

Example:

Q_0=5000 m^3/d

r_w=0.2m

H=50 m

K=50 m/d

R=2500 m

n=0.3

Pumping test:

Q_0=5000 m^3/d

r_1=50m, s_1=1.24m

r_2=100m, s_2=1.02m

H=50 m

Result:

T=2507 m^2/d

K=50 m/d

R=2487 m

Steady flow to a well in a confined aquifer

What to learn in this chapter:

- What are the key assumptions of the aquifer system?
- What is the essential water balance equation?
- Which factors determine the cone of depression?
- Where does water come from to the pumping well?
- How to delineate the well protection zone?
- How to design a pumping test to calculate hydraulic conductivity?

Steady flow to a well in a confined aquifer

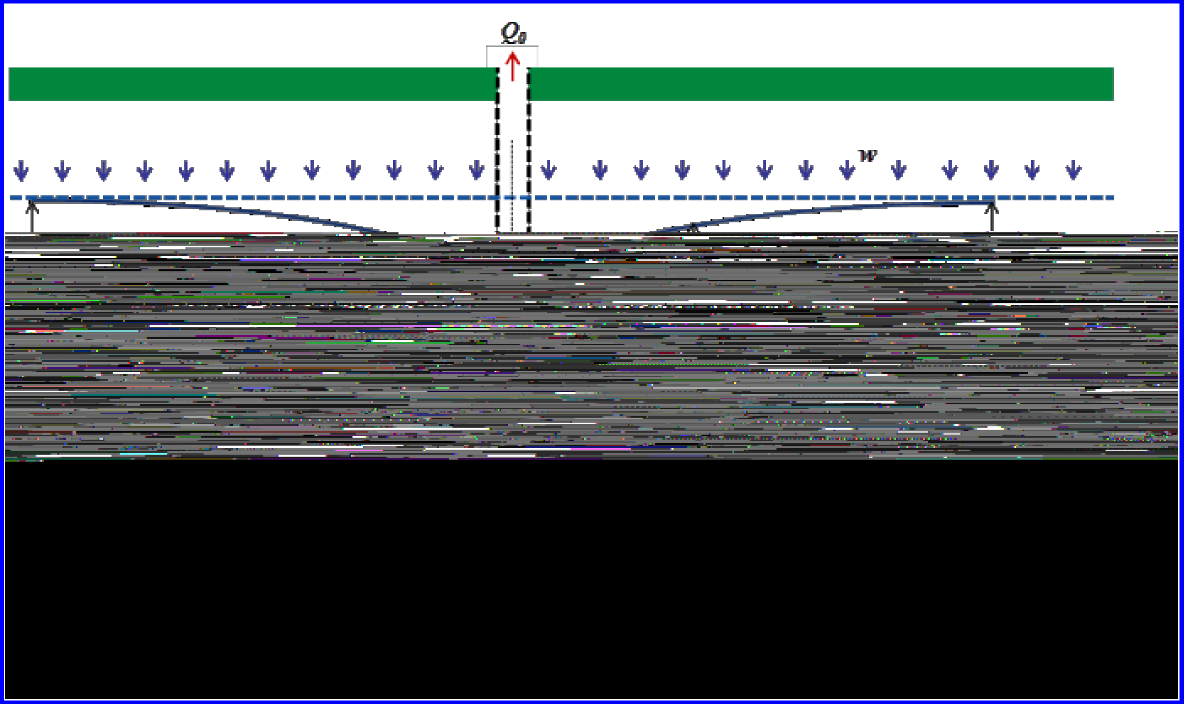
Assignments:

Å Summarize ñWhat to learnò

Å Solve problems 1 and 2 in Exercise 7.3

Source of water, arrival time and protection zones of a pumping well in an unconfined aquifer

- Conceptual hydrogeological model
  - The unconfined aquifer is homogeneous and isotropic;
  - The aquifer receives the uniform recharge (   ) from the top;
  - A pumping well fully penetrates the aquifer with a constant pumping rate;
  - At the radius of the influence (R), groundwater level is kept constant at H_0.



Questions:
1. Drop of groundwater level?
2. Where does water come from?
3. Arrival time?
4. Protection area?

Area (Pi*R^2) * Recharge (w)

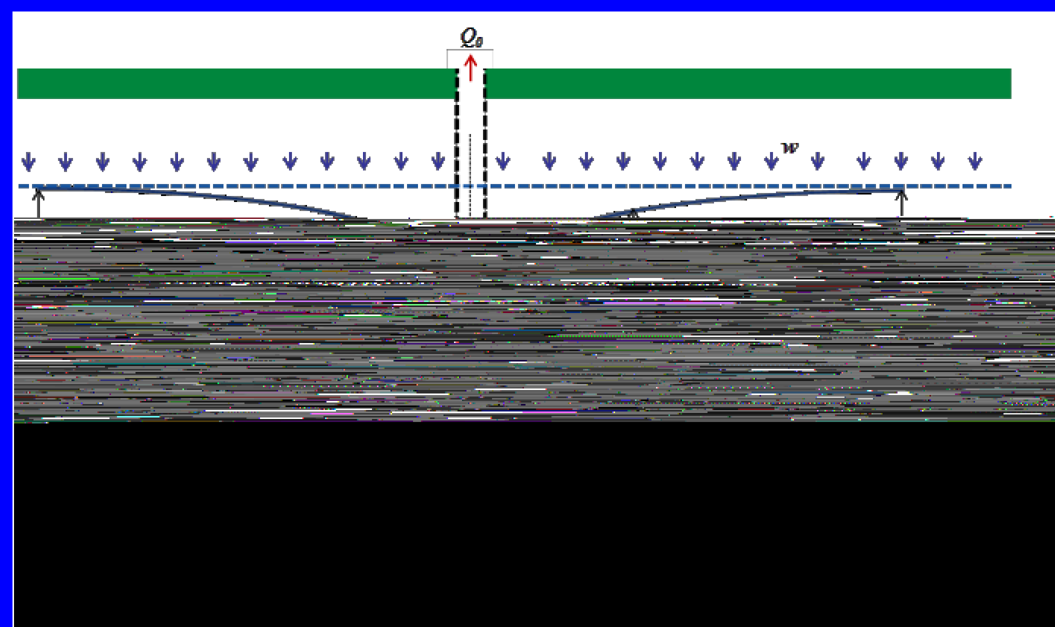
Source of water, arrival time and protection zones of a pumping well in an unconfined aquifer

- Mathematical model

Governing equation:

Boundary conditions:

$$h|_{r=R} = H_0$$



Source of water, arrival time and protection zones of a pumping well in an unconfined aquifer

Drawdown distribution

Questions:

(1) Is drawdown becoming smaller or larger when there is no recharge?

(2) Is drawdown smaller or larger comparing to the confined aquifer?

Example:

Q_0=5000 m^3/d

r_w=0.1m

H_0=50 m

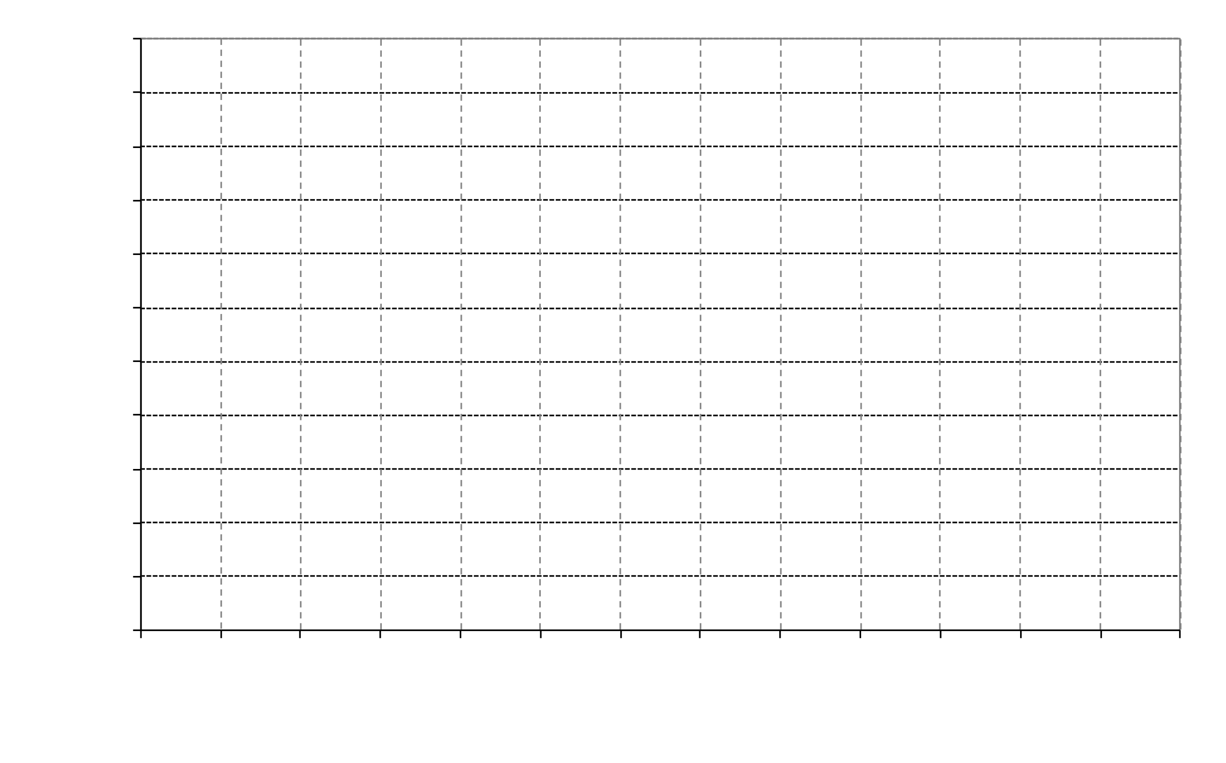
K=50 m/d

w=0.001m/d

R=1261.5 m,

Maximum Drawdown

s_{max}=2.93m



- Cone of depression simulated with a numerical model
- Q_0 = 5000 m^3/d, w= 0.001 m/d, H0 =50 m, K = 50 m/d, n_e = 0.3.
- These values results in a radius of the influence R = 1261.5 m. The numerical model consisted of a circular unconfined aquifer with a radius of 1265 m. The model area is discretized by 253 rows and 253 columns with a uniform cell size of 10 m by 10m. The center cell where the pumping well is located was

Source of water, arrival time and protection zones of a pumping well in an unconfined aquifer

Source of water to the well

p 2

Example:

Q_0=5000 m^3/d

r_w=0.1m

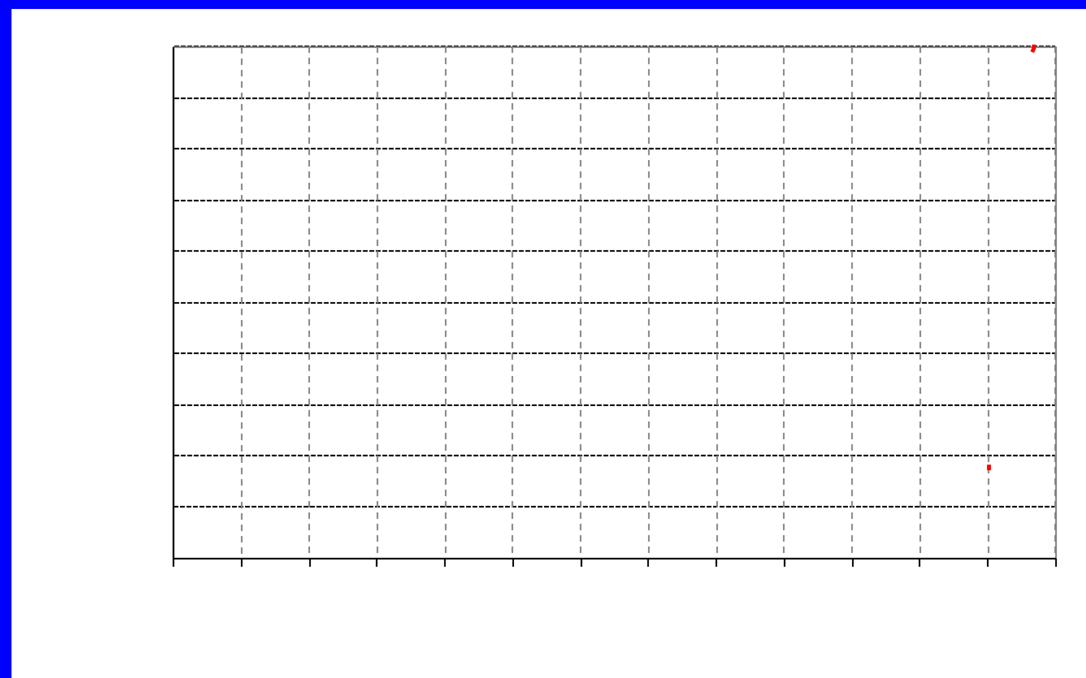
H_0=50 m

K=50 m/d

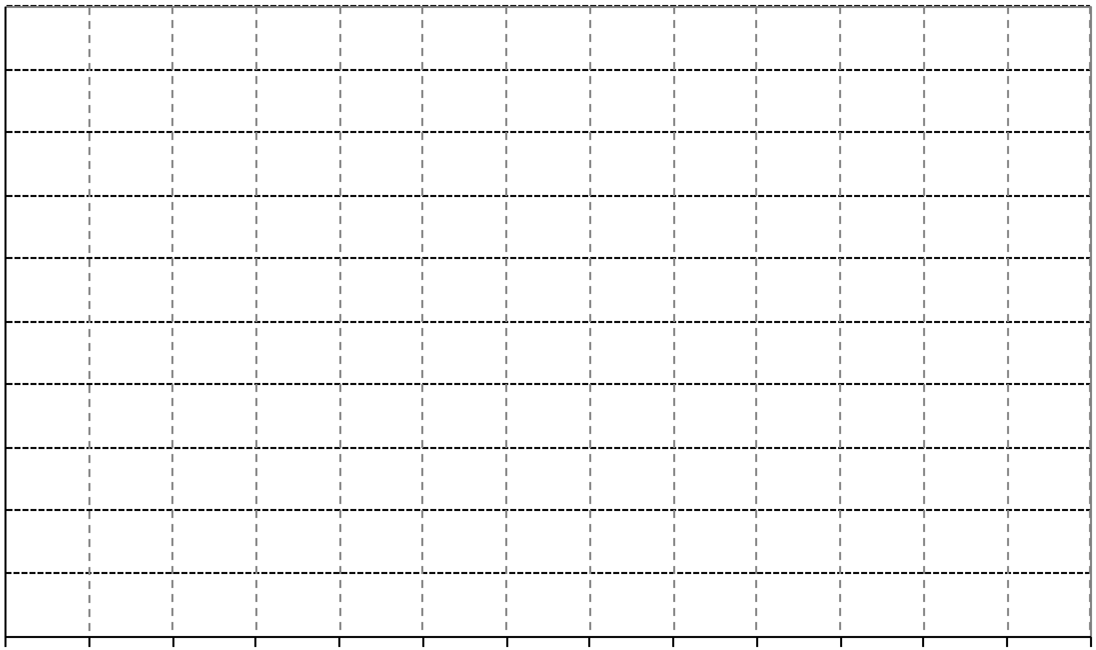
w=0.001m/d

R=1261.5 m,

The total recharge is 5000 m^3/d.



More recharge water comes from larger distance where the recharge area is larger.



Source of water, arrival time and protection zones of a pumping well in an unconfined aquifer

Å Travel time

Approximate solution when h=H_0 (Zhou and Haitjema, 2011)

$$\tilde{}(\ ) \quad t \quad \ln\left(\frac{{}_0 - p^{2}}{{}_0 - p^{2}}\right) = t \quad \ln\left(\frac{1 - p^{2} \ / \ {}_0}{1 - p^{2} \ / \ {}_0}\right)$$

$$t = \frac{p^{2} \ {}_0}{p^{2}} = \frac{{}_0}{}$$

Approximate mean residence time

$$h = \frac{p^{2}}{{}_0}$$

The ratio of the recharge water within radius r to the total well discharge

$$\tilde{}(\ ) \quad t \quad \ln\left(\frac{1}{1 - h}\right)$$

Source of water, arrival time and protection zones of a pumping well in an unconfined aquifer

Arrival time to the well

Example:

Q_0=5000 m^3/d

r_w=0.1m

H_0=50 m

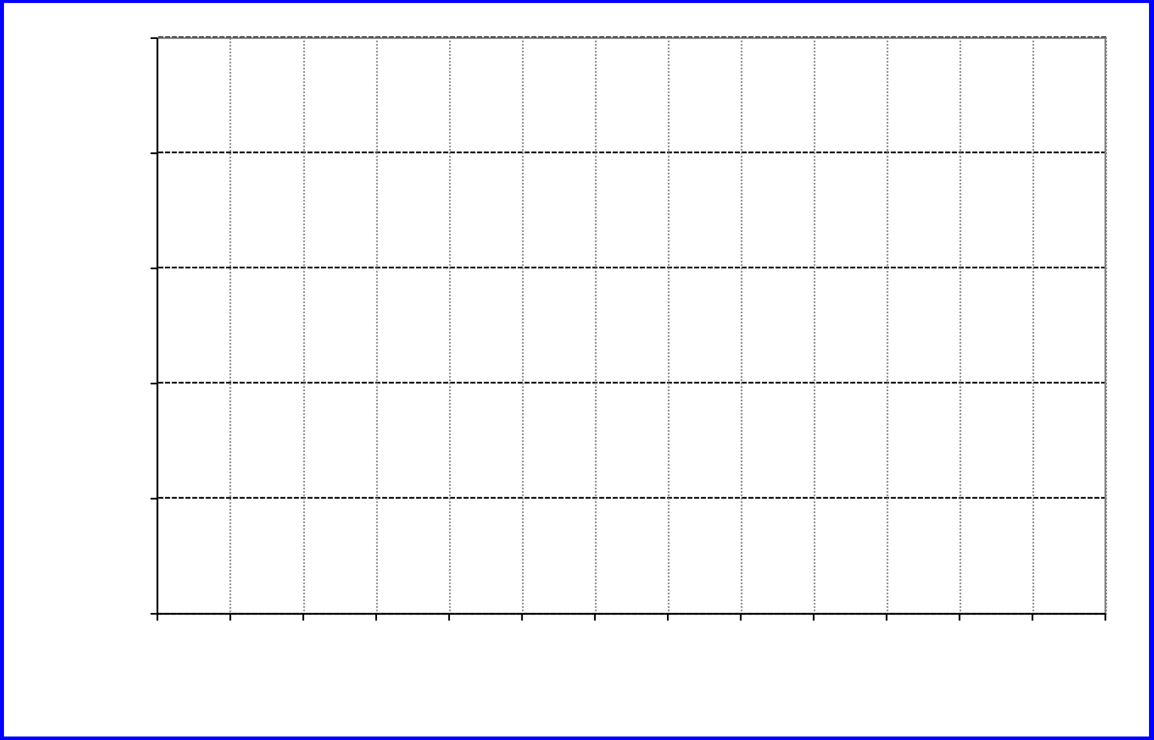
K=50 m/d

w=0.001m/d

R=1261.5 m,

n_e=0.3

The maximum travel time is around 284 years.



Travel time increases with the increase of distance from the well

Source of water, arrival time and protection zones of a pumping well in an unconfined aquifer

Mean residence time

Example:

Q_0=5000 m^3/d

r_w=0.1m

H_0=50 m

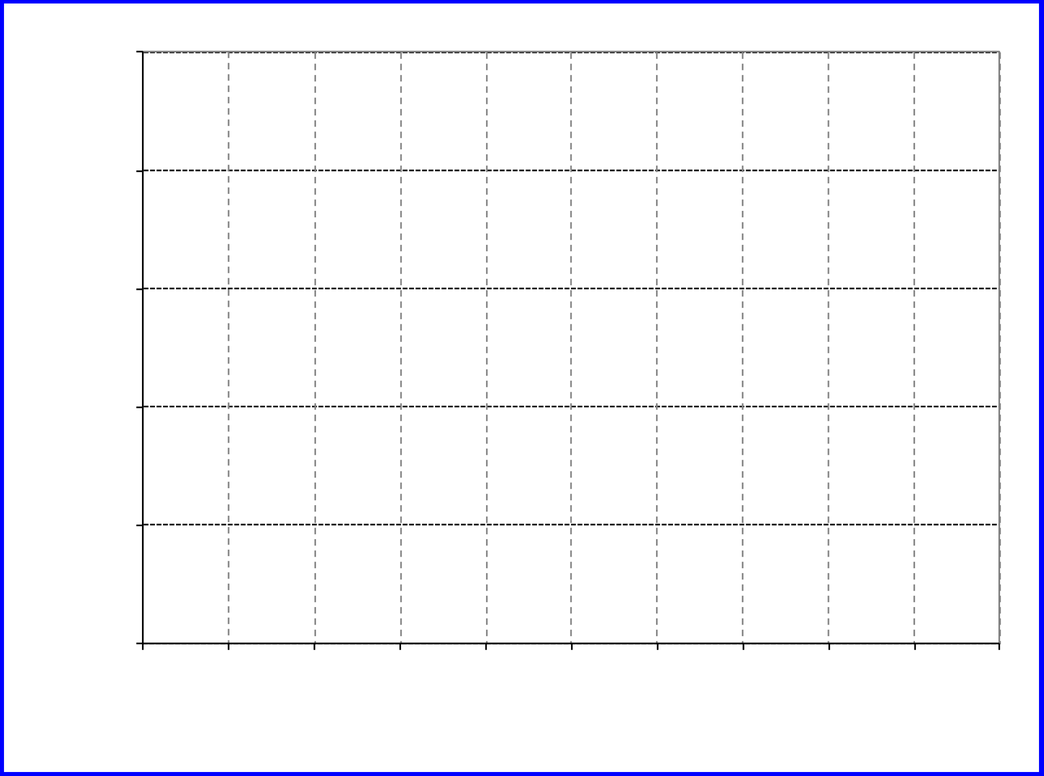
K=50 m/d

w=0.001m/d

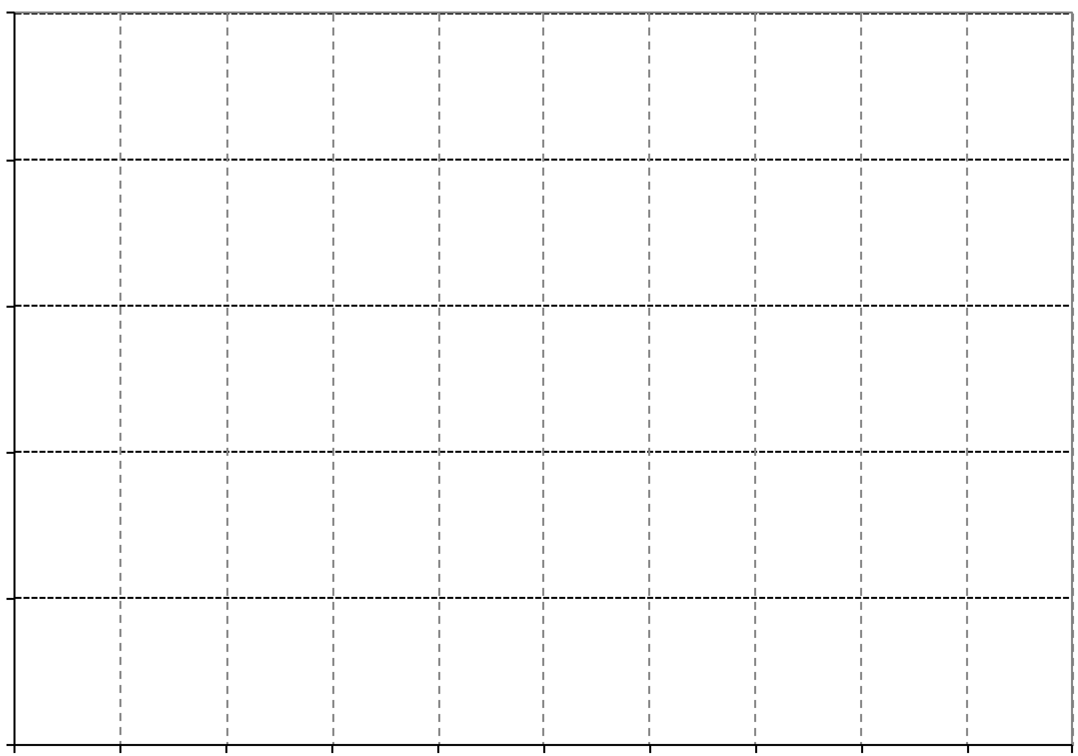
R=1261.5 m,

n_e=0.3

The mean residence time is 41 years

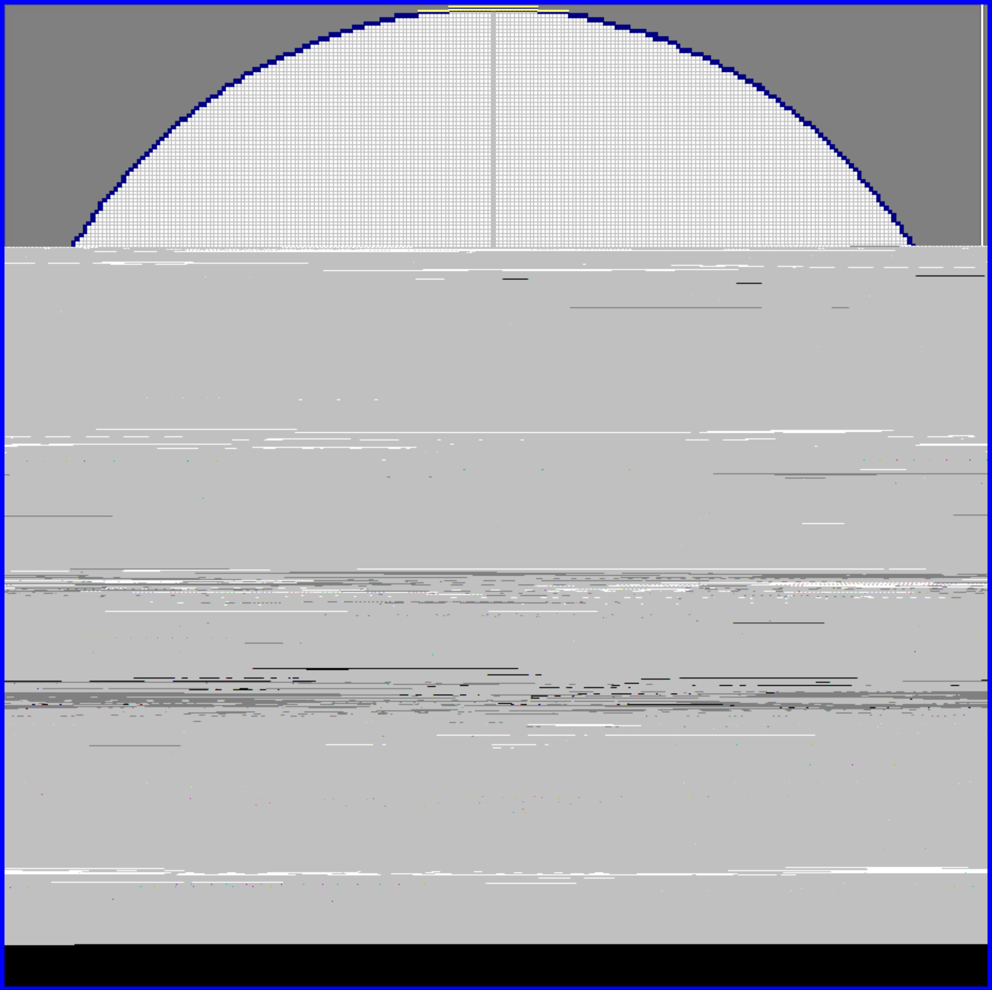


Recharge rate weighed residence time is 41 years.



Source of water, arrival time and protection zones of a pumping well in an unconfined aquifer

Å 10 years well protection zone simulated with numerical models



Example:

Q_0=5000 m^3/d

r_w=0.1m

H_0=50 m

K=50 m/d

w=0.001m/d

R=1261.5 m,

n_e=0.3

The mean residence time is 41 years

Source of water, arrival time and protection zones of a pumping well in an unconfined aquifer

Å Well protection zones

Using percentage of recharge water to the well

Arrival time (years)	**Radius of the protection area (m)**	**Percentage of the recharge to pumping rate (%)**
95	1197	90
123	1230	95
191	1255	99
284	1261	99.9

For the safety of the water supply, the whole catchment area of the well should be protected. It means that the radius of the protection area should be the radius of the influence of the well.

Example:

Q_0=5000 m^3/d

r_w=0.1m

H_0=50 m

K=50 m/d

w=0.001m/d

R=1261.5 m,

n_e=0.3

The mean residence time is 41 years

Source of water, arrival time and protection zones of a pumping well in an unconfined aquifer

- Application to pumping test

Steady state level in the observation well 1:

$$h_1^2 = H_0^2 - \frac{Q_0}{pk}\ln(\frac{R}{r_1})$$

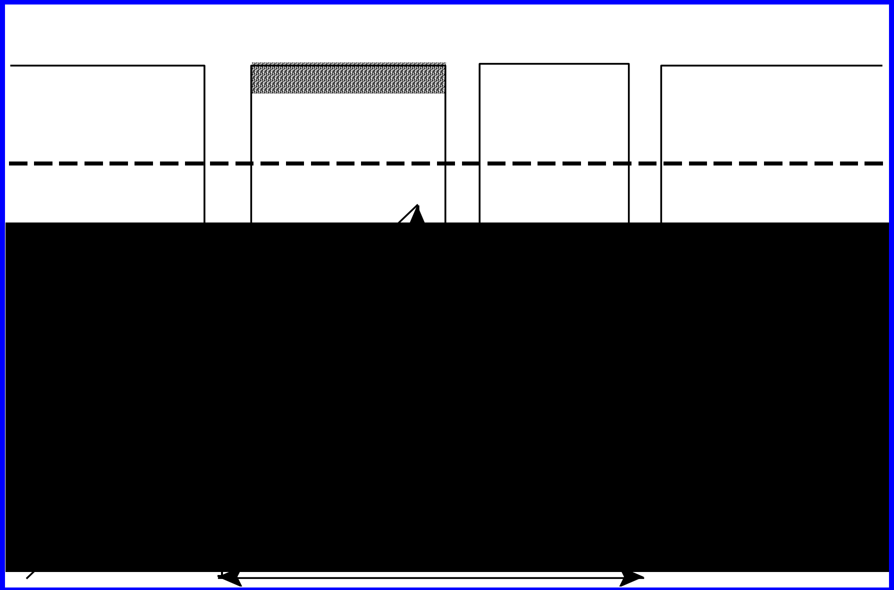
Steady state level in the observation well 2:

$$h_2^2 = H_0^2 - \frac{Q_0}{pk}\ln(\frac{R}{r_2})$$

Hydraulic conductivity:

$$K = \frac{Q_0}{p(h_2^2 - h_1^2)}\ln(\frac{r_2}{r_1})$$

$$K = \frac{Q_0}{p(s_1 - s_2)(2H_0 - s_1 - s_2)}\ln(\frac{r_2}{r_1})$$



Source of water, arrival time and protection zones of a pumping well in an unconfined aquifer

- Application to pumping test

Example:

Q_0=5000 m^3/d

r_w=0.1m

H_0=50 m

K=50 m/d

R=2500 m

n=0.3

Pumping test:

Q_0=5000 m^3/d

r_1=50m, s_1=1.26m

r_2=100m, s_2=1.03m

H_0=50 m

Result:

K=49 m/d

R=2343 m

Source of water, arrival time and protection zones of a pumping well in an unconfined aquifer

What to learn in this chapter:

Å What are the key assumptions of the aquifer system?

Å What is the essential water balance equation?

Å Which factors determine the cone of depression?

Å Where does water come from to the pumping well?

Å How to delineate the well protection zone?

Å How to design a pumping test to calculate hydraulic conductivity?

Source of water, arrival time and protection zones of a pumping well in an unconfined aquifer

Assignments:

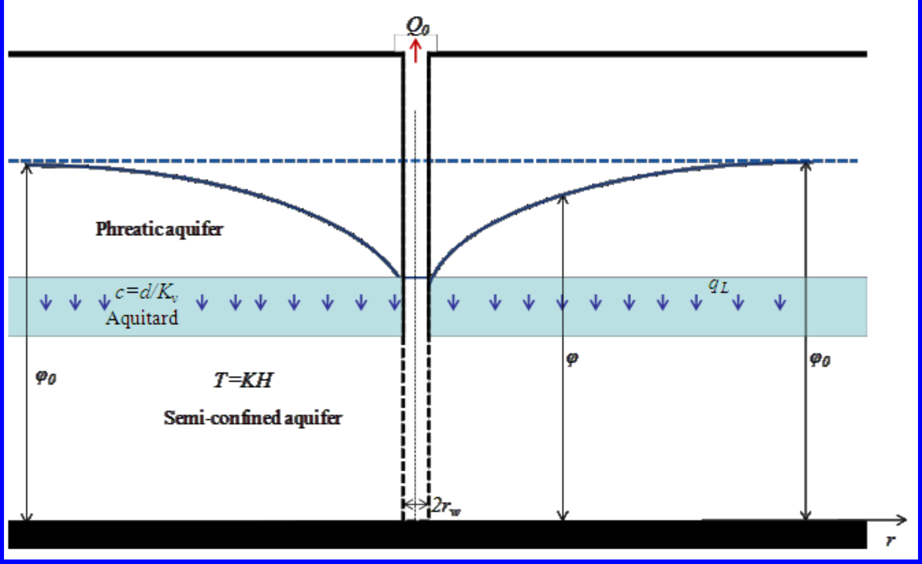
Å Summarize ñWhat to learnò

Å Solve problem 3 in Exercise 7.3

Å Read paper: ñApproximate Solutions for Radial Travel Timeò and prepare one question or one comment to be discussed in next class (Monday, January 4)

Source of water, arrival time and protection zones of a pumping well in a leaky aquifer

- Conceptual hydrogeological model
  - The semi-confined aquifer is homogeneous and isotropic with uniform thickness;
  - The semi-permeable layer is homogeneous with uniform thickness;
  - A pumping well fully penetrates the semi-confined aquifer with a constant pumping rate;
  - Before the pumping, groundwater head in the semi-confined aquifer is the same as the water table in the phreatic aquifer;
  - The aquifer system extends to infinite laterally.



Questions:
1. Drop of groundwater head?
2. Where does water come from?
3. Arrival time ?
4. Protection areas?

Source of water, arrival time and protection zones of a pumping well in a leaky aquifer

Mathematical model

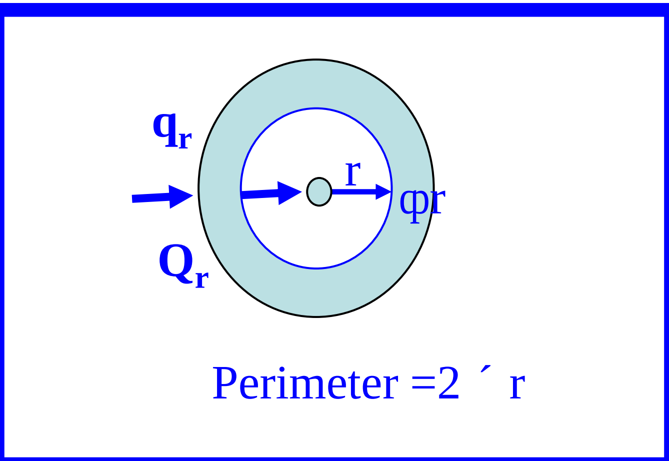
Water balance equation:

$$Q_r + Q_L = (Q_r + \frac{dQ_r}{dr} Dr)$$

$$Q_r = -2prHK \frac{dj}{dr}$$

$$Q_L = 2prDr \frac{j_0 - j}{c}$$

$$-2pT[r \frac{d^2 j}{dr^2} + \frac{dj}{dr}] Dr - 2prDr \frac{j_0 - j}{c} = 0$$



Source of water, arrival time and protection zones of a pumping well in a leaky aquifer

Mathematical model

Governing equation:

Boundary conditions:

$$-2\,S T[r \frac{d^2 M}{dr^2} + \frac{d M}{dr}]\,'r - 2\,S r\,'r\,\frac{M_0 - M}{c} = 0$$

Source of water, arrival time and protection zones of a pumping well in a leaky aquifer

- Analytical solution

Specific solution:

$$s = c_2 K_0\left(\frac{r}{\lambda}\right)$$

From inner boundary:

$$c_2 = \frac{Q_0}{2\pi T}\,\frac{1}{(r_w/\lambda)K_1(r_w/\lambda)}$$

Final solution:

$$s = \frac{Q_0}{2\pi T}\,\frac{K_0(r/\lambda)}{(r_w/\lambda)K_1(r_w/\lambda)}$$

Source of water, arrival time and protection zones of a pumping well in a leaky aquifer

Drawdown distribution

Example:

Q_0=5000 m^3/d;

r_w=0.2 m;

H=50 m;

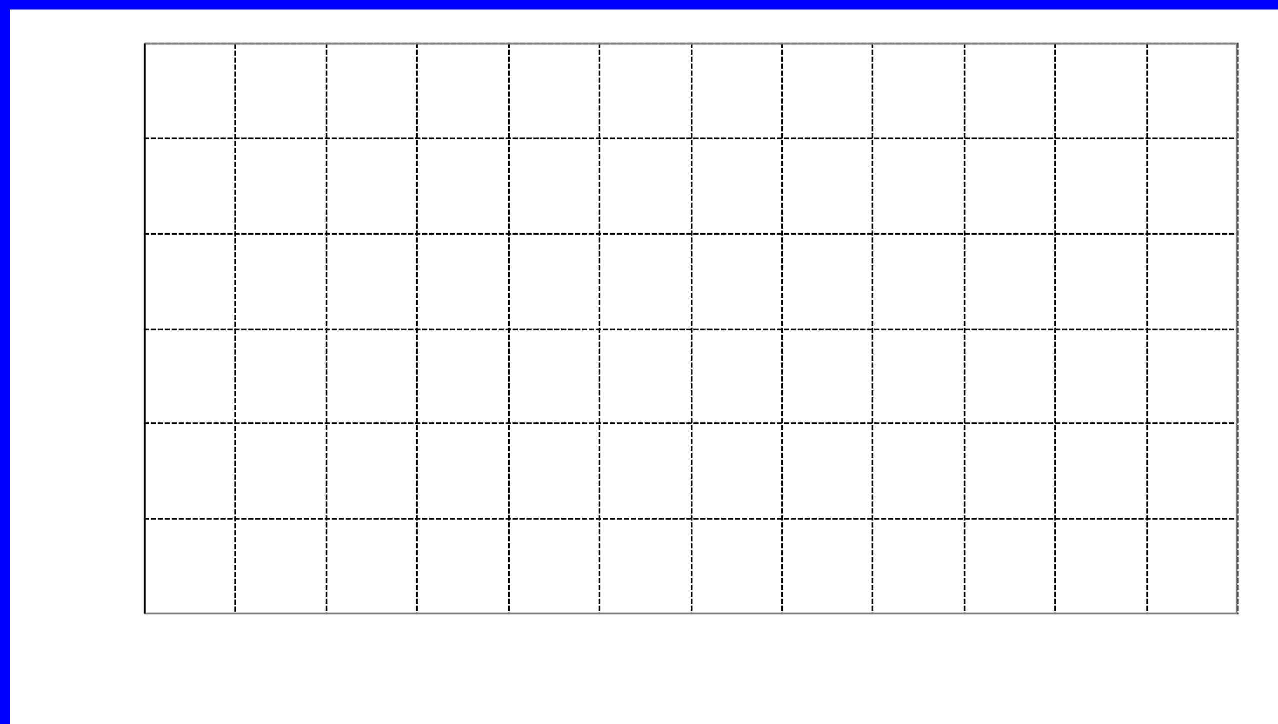
K=50 m/d;

c=100 d;

ϰ=500m;

n_e=0.3.

s_{max}=2.53 m



Source of water, arrival time and protection zones of a pumping well in a leaky aquifer

- Sources of water to the well

Analytical solution by Zhan and Bian (2006), Zhou (2011)

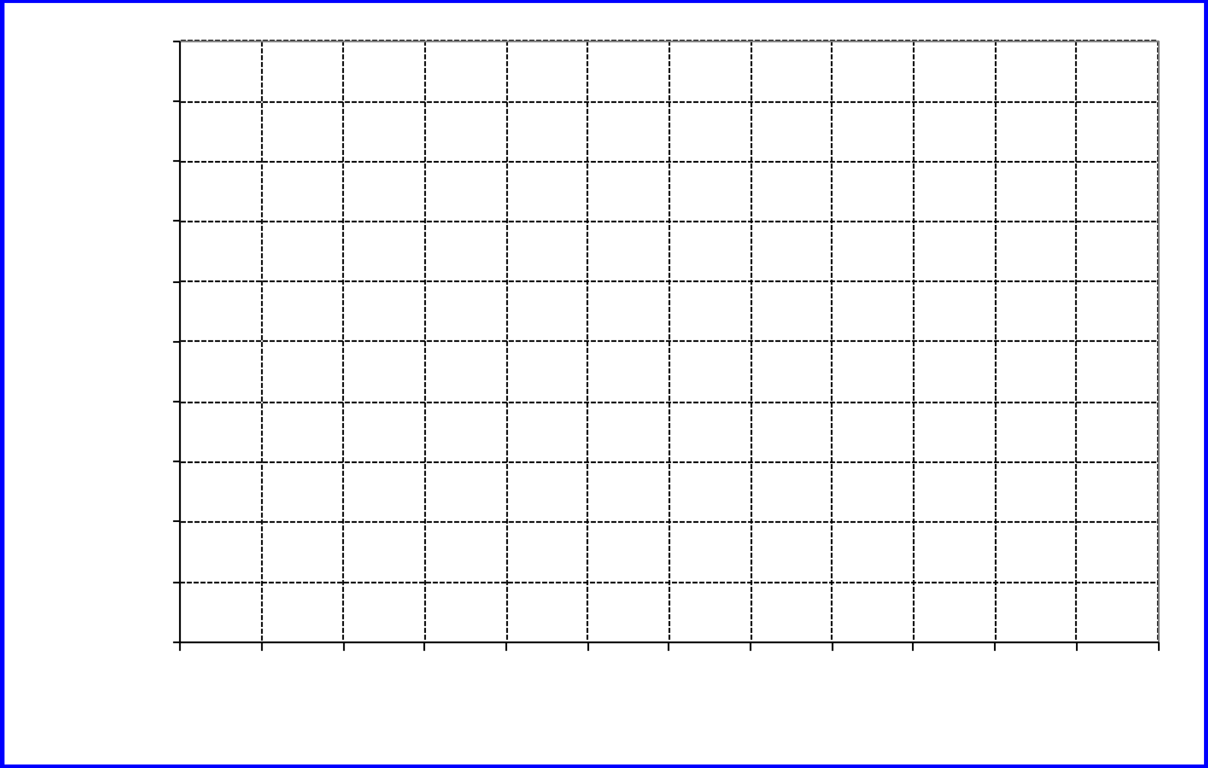
Example:
$Q_0=5000\ m^3/d$
$r_w=0.2$ m
$H=50$ m
$K=50$ m/d
$c=100$ d
$n_e=0.3$
$\lambda=500$ m

Total leakage = pumping rate

Source of water, arrival time and protection zones of a pumping well in a leaky aquifer

- Sources of water to the well

Example:
Q_0=5000 m^3/d
r_w=0.2 m
H=50 m
K=50 m/d
c=100 d
n_e=0.3
λ=500 m

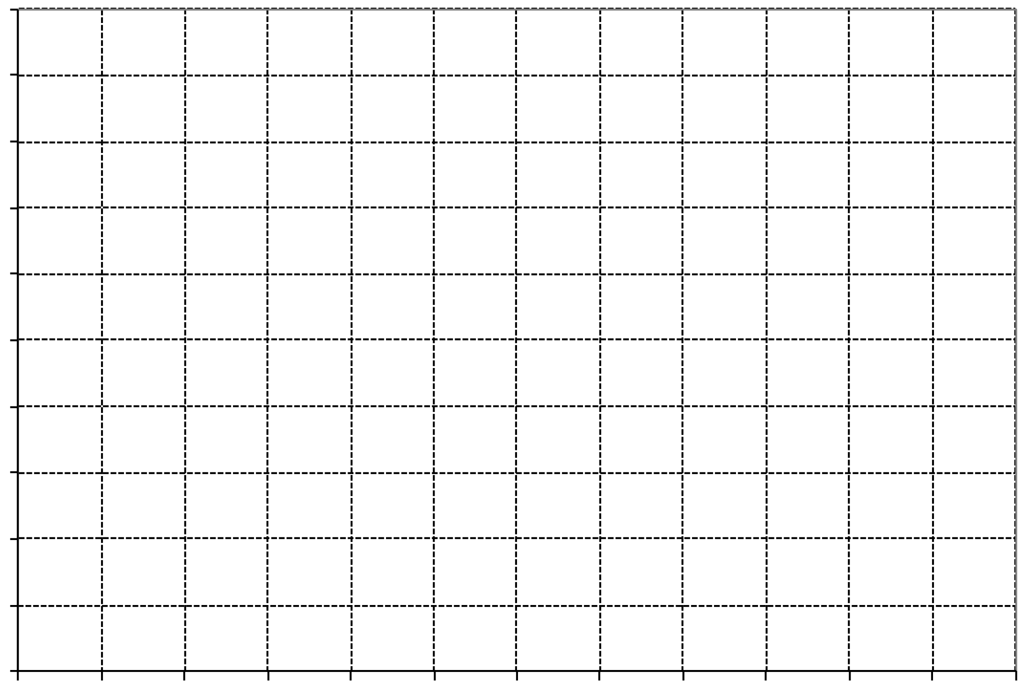


Large leakage rate at short distance

Source of water, arrival time and protection zones of a pumping well in a leaky aquifer

- Sources of water to the well

Example:
Q_0=5000 m^3/d
r_w=0.2 m
H=50 m
K=50 m/d
c=100 d
n_e=0.3
λ=500 m



$R_{99\%Q0}$=2885m; $R_{95\%Q0}$=2000m; $R_{90\%Q0}$=1610m

Source of water, arrival time and protection zones of a pumping well in a leaky aquifer

- Arrival time

Example:
Q_0=5000 m^3/d
r_w=0.2 m
H=50 m
K=50 m/d
c=100 d
n_e=0.3
λ=500 m

Arrival time = 865 years at $R_{95\%Q0}$=2000m

Source of water, arrival time and protection zones of a pumping well in a leaky aquifer

Å Mean residence time

Example:

Q_0=5000 m^3/d

r_w=0.2 m

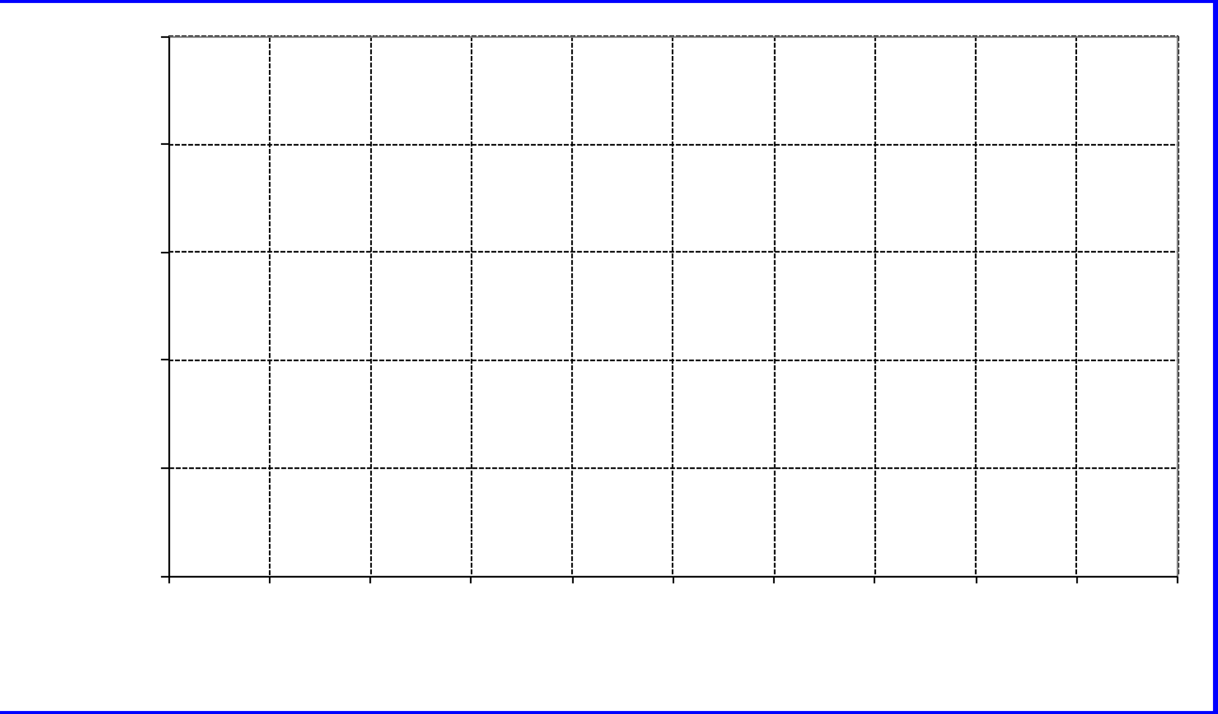
H=50 m

K=50 m/d

c=100 d

n_e=0.3

ϰ=500m



42% of leakage water with travel time <10 years

79% of leakage water with travel time <100 years

5% of leakage water with travel time >1000 years

Conclusions

- The delineation of well protection zones based on the travel time criteria is fundamentally flawed and exposes majority of sources of water at risk of pollution. Examples show that a 25 years protection zone can only protect less than 45% of the source of water to the well in an unconfined aquifer, and 61% the leakage water to the well in a semi-confined aquifer. The well protection area should be delineated based on the sources of the water to the well, regardless of the travel time. To safeguard drinking water supply, majority sources of water to the well should be protected.

Conclusions

- For a confined aquifer, the idea of the protection area defined by the travel time is not relevant since there might be no direct pollution from land surface. The protection of the inflow source is more important. For the example case, the residence time is 161 years, so that pollutants from the inflow boundary will on average take 161 years to arrive at the pumping well. Systematic monitoring of the inflow source water will provide more useful information for the protection of the well.

Conclusions

- In an unconfined aquifer with the uniform recharge, majority of sources of water to the well is derived from recharge water at far distance with longer arrival times. All recharge water within the radius of the influence of the well should be protected. The radius of the influence of the well is determined by the recharge rate and pumping rate. The mean residence time can be calculated as the recharge rate weighted average of residence times and used as a timescale for the