CHAPTER THREE

GROUNDWATER FLOW TO WELLS

6.1   Introduction

Pumping Test is the examination of aquifer response, under controlled conditions, to the abstraction of water. Pumping test can be well test (determine well yield and well efficiency), aquifer test (determine aquifer parameters and examine water chemistry). Hydrogeologists try to determine the most reliable values for the hydraulic characteristics of the geological formations.

The objectives of the pumping test are:

  1   Determine well yield,
  2   Determine well efficiency,
  3   Determine aquifer parameters
  4   Examine water chemistry

General notes about pumping test:

  1   Pump testing is major investigative tool-but expensive.
  2   Proper planning, observations, interpretation essential!
  3   It is cheaper (much) if existing wells can be used.
  4   Pump testing also carried out in newly constructed wells, as a well test.

6.2   Definitions

Well yield: is a measure how much water can be withdrawn from the well over a period of time and measured in m3/hr or m3/day.

Specific capacity: is referring to whether the well will provide an adequate water supply. Specific capacity is calculated by dividing pumping rate over drawdown (Q/S).

Static water level: is the level of water in the well when no water is being taken out.

Dynamic Water level: is the level when water is being drawn from the well. The cone of depression occurs during pumping when water flows from all directions toward the pump.

Drawdown: the amount of water level decline in a well due to pumping. Usually measured relative to static (non-pumping) conditions, (see **Figure 6.1**).

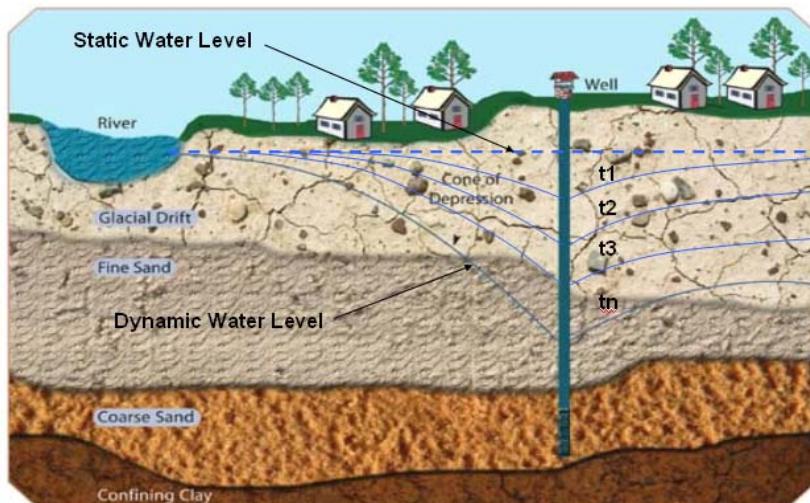Adapted from *Groundwater and Wells, Second Edition* by Fletcher G. Driscoll, Ph.D.

Figure 6.1 A cone of depression expanding beneath a riverbed creates a hydraulic gradient between the aquifer and river. The result in induced recharge to the aquifer from the river

6.3   Principles of Pumping Test

The principle of a pumping test involves applying a stress to an aquifer by extracting groundwater from a pumping well and measuring the aquifer response to that stress by monitoring drawdown as a function of time (see **Figures 6.2** and **6.3**) .

These measurements are then incorporated into an appropriate well-flow equation to calculate the hydraulic parameters of the aquifer.
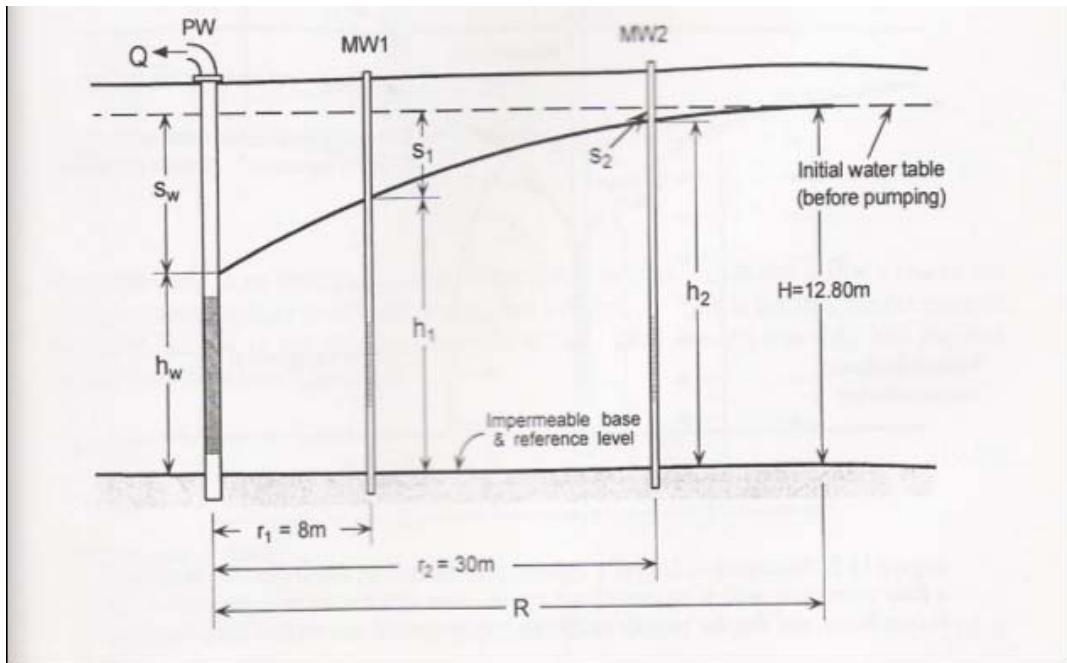

Figure 6.2   Pumping well with observation wells in unconfined aquifer

Figure 6.3   Pumping test in the field

6.3.1 Design of Pumping Tests

➢ **Parameters**

- Test well location, depth, capacity (unless existing well used).
- Observation well number, location, depth.
- Pump regime

➢ **General guidance**:

Confined aquifers: Transmissivity more important than storativity: observation wells not always needed (although accuracy lost without them!).

Unconfined aquifers: Storativity much larger, and has influence over transmissivity estimates: observation wells important as is larger test duration. Care needed if aquifer only partly screened.

6.3.2   The Importance of Pumping Tests

Pumping tests are carried out to determine:

1. How much groundwater can be extracted from a well based on long-term yield, and well efficiency?
2. The hydraulic properties of an aquifer or aquifers.
3. Spatial effects of pumping on the aquifer.
4. Determine the suitable depth of pump.
5. Information on water quality and its variability with time.

6.3.3   Design Considerations

There are several things should be considered before starting a pumping test:

1. Literature review for any previous reports, tests and documents that may include data or information regarding geologic and hydrogeologic systems or any conducted test for the proposed area.
2. Site reconnaissance to identify wells status and geologic features.
3. Pumping tests should be carried out within the range of proposed or designed rate (for new wells, it should be based on the results of Step Drawdown Test).
4. Avoid influences such as the pumping of nearby wells shortly before the test.
5. Determine the nearby wells that will be used during the test if it's likely they will be affected, this well depends on Radius of Influence. The following equation can be used to determine the radius of influence (R_0):

$$R_0 = \sqrt{\left(2.25 \times T \times \frac{t}{S}\right)} \qquad \text{(6.1)}$$

where, R_0 is the radius of influence (m)

  T is the aquifer transmissivity (m^2/day)
  t is time (day)
  S is the storativity

This equation can be applied for a pumping well in a confined aquifer.

6   Pumping tests should be carried out with open-end discharge pipe in order to avoid back flow phenomena (i.e. $P_p = P_{atm}$).

7   Make sure that the water discharged during the test does not interfere with shallow aquifer tests (Jericho Area).

8   Measure groundwater levels in both the pumping test well and nearby wells before 24 hours of start pumping.

9   Determine the reference point of water level measurement in the well.

10   Determine number, location and depth of observation wells (if any).

6.3.4   Methods of Measurement

The methods of measurement are:

1   Water level
           Dippers
           Water Level Records
           Data Loggers

2   Discharge
           Orifice Plate
           "V" Notch Weir
           Flow Meter
           Tank
           Orifice Bucket

The equipment required in measurement is:

Flow Meter: flow meter is recommended for most moderates to high flow-rate applications. Others means of gauging flow such as containers could be used for low- flow-rate applications (see **Figure 6.4**).

Figure 6.4   Measuring pumping rate by flow meter

Water level Indicator: To be used for measuring static and dynamic water levels such as *M-Scope or Data Logger*. Water level data should be recorded on aquifer test data sheet.

Figure 6.5   Measuring water level by M-scope

Stopwatch: The project team must have an accurate wristwatch or stop watch. All watches must be synchronized prior to starting pumping test.

Personal Requirements: Most of pumping tests will initially require a minimum of three qualified people. More staff is generally required for long-term constant rate tests with observation wells.

6.3.5   Measurement to be Taken

➢   Water levels measurements for pumping well could be taken as the following

Time since start of pumping (minutes)	Time intervals (minutes)
0 – 5	0.5
5 – 60	5
60 – 120	20
120 – shut down the pump	60

➢   Similarly, for observation wells, water level measurement can be taken as the following:

Time since start of pumping (minutes)	Time intervals (minutes)
0 – 5	0.5
5 – 15	1
15 – 50	5
50 – 100	10
100 – 300	30
300 – 2,880	60
2,880 – shut down the pump	480

Note: 480 min = (8 hours) and 2,880 min. = (48 hours)

➢   After the pump has been shut down, the water levels in the well will start to rise again. These rises can be measured in what is known as recovery test.

➢   If the pumping rate was not constant throughout the pumping test, recovery-test data are more reliable than drawdown data because the water table recovers at a constant rate.

> Measurements of recovery shall continue until the aquifer has recovered to within 95% of its pre-pumping water level.

> Amongst the arrangements to be made for pumping test is a **discharge rate control**. This must be kept constant throughout the test and measured at least once every hour, and any necessary adjustments shall be made to keep it constant.

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PERCUCTIONS

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1. If possible, stop abstractions 24 hours before test, and monitor recovery. If not possible, make sure wells pump at constant rate before and during test, and monitor the discharge, pumping water level for correction of test data where necessary.
2. Check all record water levels refer to same datum level,
3. Monitor possible influence of [air pressure, recharge, loading, earthquakes, etc.]. Possibly by monitoring similar outside influence of pumping. correct test data for these influences,
4. Interpretation of pump test data in aquifers with secondary permeability needs particular care.
5. Remember, water levels are very susceptible to minor variations in pumping rate.

6.3.6   Data to be Collected

> **Before Test**

1. **Site geology**: lithological logs for well and piezometers,
2. **Well construction data** and piezometers,
3. **Geometry of site**: layout, distances, features, potential boundaries …etc,
4. **Groundwater abstraction** in vicinity of site:
        Constant abstractions
        Suspended abstractions (time of suspension)

5. Pre-test groundwater levels.

> **During Test**

1. Time,
2. Discharge,
3. Groundwater level,
4. Temperature/quality of pumped water,
5. Air pressure,
6. Other abstractions in vicinity,
7. Rainfall, surface water levels, tides, etc..

6.3.7   Duration of Pumping Test

It's difficult to determine how many hours that pumping test required because period of pumping depends on the type and natural materials of the aquifer.   In general pumping test is still until pseudo-steady state flow is attained or low fluctuation in dynamic water is occur.

In some tests, steady state occurs a few hours after pumping, in others, they never occur. However, 24-72 hours testing is enough to produce diagnostic data and to enable the remaining wells for testing.

Tests taking longer than 24 hours may be required for large takes, such as community supplies, or situations where it may take longer to determine effects.

6.3.8   Pumping Regime

1 **Development**: Variable discharges and times, surging well for some hours to clean and develop well, develop and stabilize gravel pack.
2 **Recovery**
3 **Step Test:** Pumping well at incrementally increasing discharges, each step lasting and hour or so. To examine well efficiency and non-linear behavior.
4 **Recovery:** With observed water levels, period lasting long enough to stabilize after step test.
5 **Constant discharge test:** Main test discharge about 120% of target yield.
6 **Recovery:** Monitored until stable water level recovery ± 10 cm.

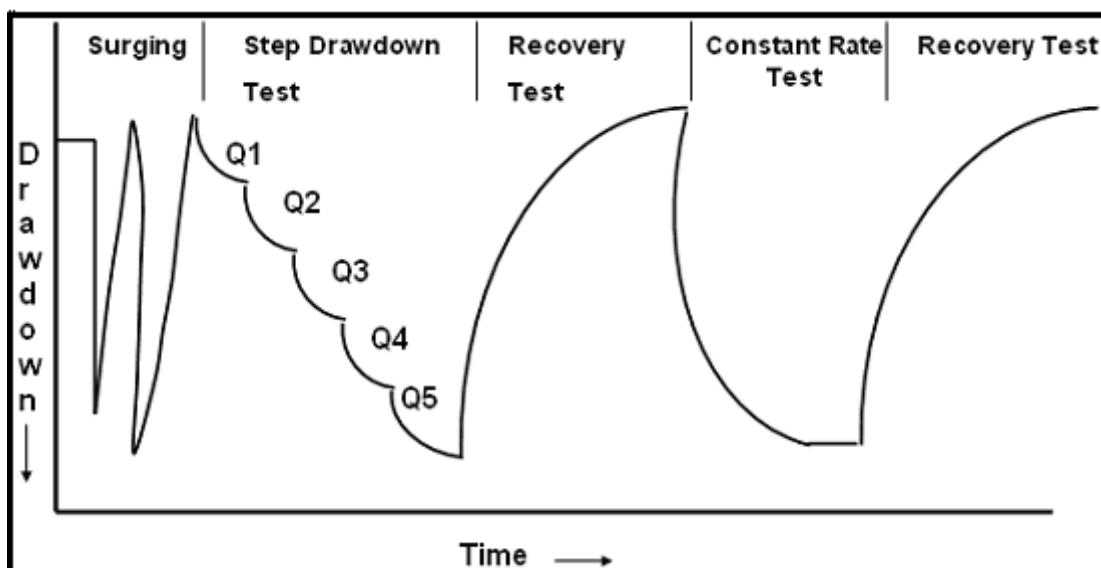**Figure 6.6** shows the sequence of types of pumping tests.

Figure 6.6   Well testing stages

Note: Data should be recorded in forms as shown in the following forms**:**

1   **Pumping Test Data Sheet,**
2   **Recovery Data Sheet**

PUMPING TEST DATA SHEET

Project _____
Name of abstraction well _____
Distance from observation well (m) _____
Well depth _____ Well diameter _____
Date of test: Start _____ Finished _____
Depth of pump _____ SWL _____
Remarks _____

Date	Actual time	Elapsed time "t" (min)	Depth to water table (m)	Drawdown (m)	Discharge Q (m³/hr)	Remark

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RECOVERY DATA SHEET

Project _____ Date _____ Sheet _____
Name of abstraction well _____
Distance from pumped well (m) _____
Discharge rate during pumping (m³/hr) _____ SWL _____
Remarks _____
_____

Actual time	Time (t) since pumping began (min)	Depth to water level (m)	Discharge rate (m³/hr)	Drawdown (m)	t/r^2

6.3.9 Location of Observation Wells

The following points should be taken into account while locating an observation well:

1   The distance from pumped well should be at Logarithmic Spacing,
2   Recommended steady drawdown should be ≥ 0.5 m (see **Figure 6.7**),
3   Not too close to pumping well: ≥ 5m or more,
4   Located on line parallel to any boundary,
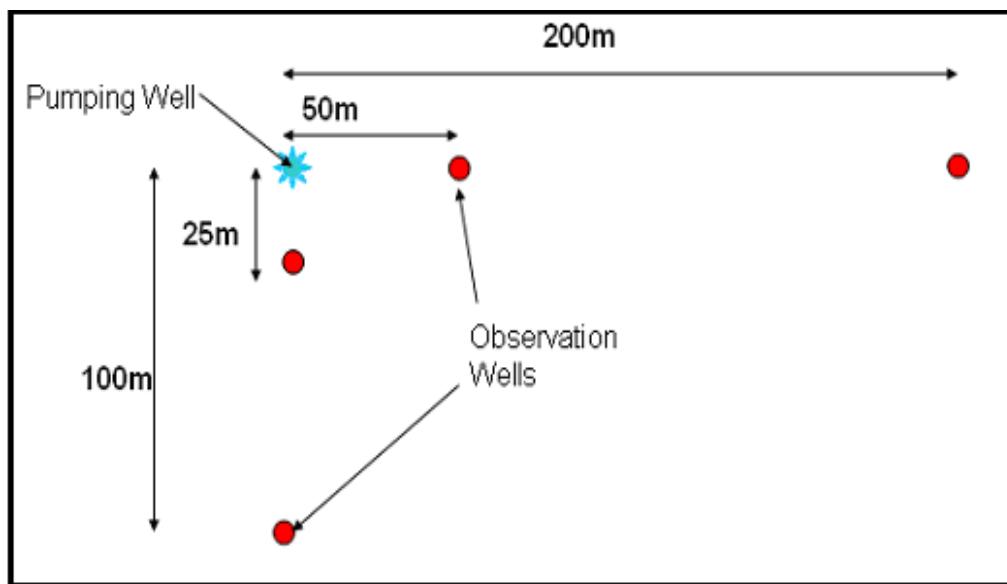5   Located on orthogonal line to identify boundary and any anisotropy.

Figure 6.7   Borehole array for a test well

6.3.10 Basic Assumptions

In this chapter we need to make assumptions about the hydraulic conditions in the aquifer and about the pumping and observation wells. In this section we will list the basic assumptions that apply to all of the situations described in the chapter. Each situation will also have additional assumptions:

1   The aquifer is bounded on the bottom by a confining layer.

2   All geological formations are horizontal and of infinite horizontal extent.

3   The potentiometric surface of the aquifer is horizontal prior to the start of the pumping.

4   The potentiometric surface of the aquifer is not changing with time prior to the start of the pumping.

5   All changes in the position of the potentiometric surface are due to the effect of the pumping well alone.

6   The aquifer is homogeneous and isotropic.

7   All flow is radial toward the well.

8   Groundwater flow is horizontal.

9   Darcy's law is valid.

10   Groundwater has a constant density and viscosity.

11   The pumping well and the observation wells are fully penetrating; i.e., they are screened over the entire thickness of the aquifer.

12   The pumping well has an infinitesimal diameter and is 100% efficient.

6.4 Using Pumping Tests to Estimate Hydraulic Conductivity (K), Transmissivity (T) and Drawdown (S_w).

(Steady Radial Flow to a Well)- (Equilibrium Radial Flow)

Hydraulic conductivity and transmissivity can be determined from steady state pumping tests.

6.4.1 Confined Aquifers – The Thiem Analysis

Assumptions

1. The aquifer is confined,
2. The aquifer has infinite aerial extent,
3. The aquifer is homogeneous, isotropic and of uniform thickness,
4. The piezometric surface is horizontal prior to pumping,
5. The aquifer is pumped at a constant discharge rate,
6. The well penetrates the full thickness of the aquifer and thus receives water by horizontal flow (see **Figure 6.8**)
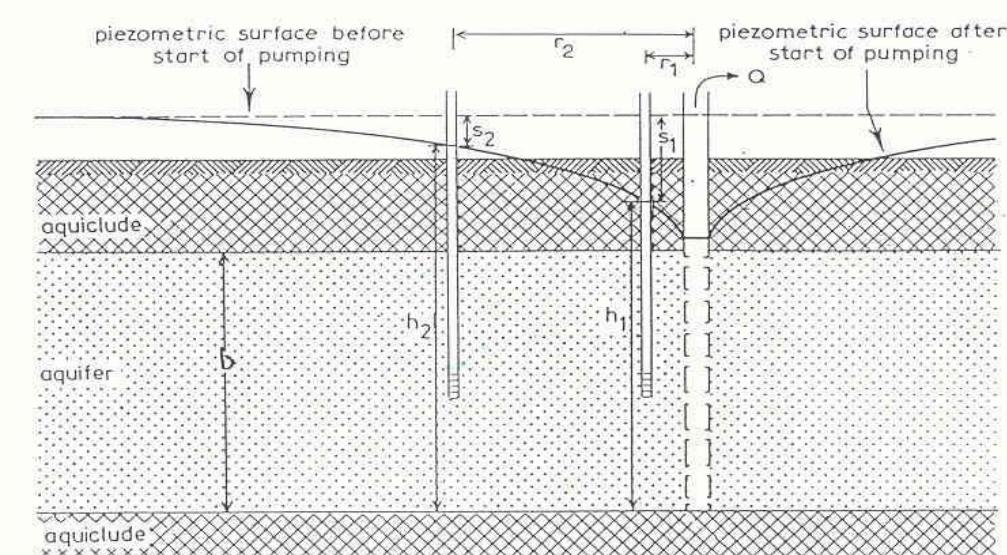
Figure 6.8 Cross-section of a pumped confined aquifer

Darcy's Law
$$q = - K \frac{\partial h}{\partial r}$$
(6.2)

Continuity
$$Q = - 2 \pi r b q$$
(6.3)

Eliminating q gives
$$Q = - 2 \pi r K b \frac{\partial h}{\partial r}$$
(6.4)

Rearranging and integration
$$\frac{Q}{2 \pi K b} \int_{r_1}^{r_2} \frac{1}{r} dr = \int_{h_1}^{h_2} dh$$
(6.5)

Which gives,
$$\frac{Q}{2 \pi K b} \ln\left(\frac{r_2}{r_1}\right) = h_2 - h_1$$
(6.6)

In terms of draw down (which is the measurement made in the field)

$$\frac{Q}{2\pi K b}\ln\left(\frac{r_2}{r_1}\right)=s_1-s_2 \qquad (6.7)$$

This is called the **Thiem Equation** and can be used to estimate the transmissivity

$$T=Kb=\frac{Q}{2\pi(s_1-s_2)}\ln\left(\frac{r_2}{r_1}\right) \qquad (6.8)$$

At least two piezometers should be used whenever possible (using the drawdown at just one piezometer and at the abstraction well leads to errors due to well losses at the abstraction well).

The previous equation can be integrated with the following boundary conditions:

1   At distance r_w (well radius) the head in a well is h_w,
2   At distance R from well (Radius of influence), the head is H (which is the undisturbed head and equal to initial head before pumping)
3   So, the equation can be written as:

$$s_w=H-h_w=\frac{Q}{2\pi T}\ln\left(\frac{R}{r_w}\right) \qquad (6.9)$$

6.4.2 Unconfined Aquifers

<u>**Assumptions**</u>

1   The aquifer is unconfined,
2   The aquifer has infinite aerial extent,
3   The aquifer is homogeneous, isotropic and of uniform thickness,
4   The water table is horizontal prior to pumping,
5   The aquifer is pumped at a constant discharge rate,
6   The well penetrates the full thickness of the aquifer and thus receives water from the entire saturated thickness of the aquifer (see **Figure 6.9**)
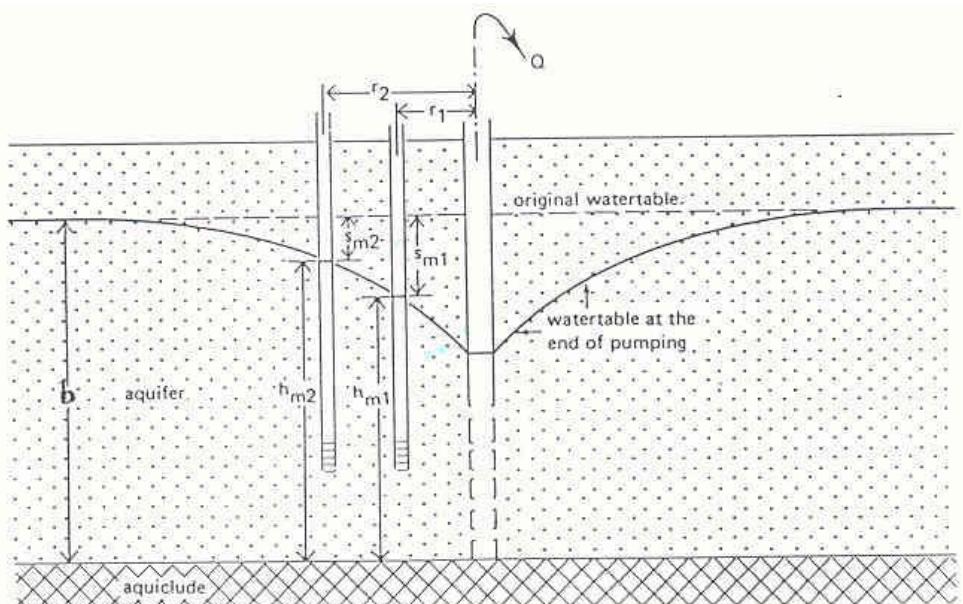

Figure 6.9   Cross-section of a pumped unconfined aquifer (steady-state flow)

| **Darcy's Law** | $q = -K\dfrac{\partial h}{\partial r}$ | **(6.10)** |

| **Continuity** | $Q = -2\pi r h q$ | **(6.11)** |

| Eliminating q gives | $Q = -2\pi r K h\dfrac{\partial h}{\partial r}$ | **(6.12)** |

| Rearranging and integration | $\dfrac{Q}{2\pi K}\displaystyle\int_{r_1}^{r_2}\dfrac{1}{r}\,dr = \int_{h_1}^{h_2} h\,dh$ | **(6.13)** |

| Which gives the **Dupuit Formula** | $\dfrac{Q}{2\pi K}\ln\left(\dfrac{r_2}{r_1}\right) = \dfrac{h_2^2 - h_1^2}{2}$ | **(6.14)** |

Based on the Dupuit and Forchheimer assumptions:

1   Flow lines are assumed to be horizontal and parallel to impermeable layer
2   The hydraulic gradient of flow is equal to the slope of water. (slope very small)

Since h=b-s, the discharge can be expressed in terms of drawdown as

$$\frac{Q}{2\pi K}\ln\left(\frac{r_2}{r_1}\right) = \left(s_1 - \frac{s_1^2}{2b}\right) - \left(s_2 - \frac{s_2^2}{2b}\right) \qquad \textbf{(6.15)}$$

Which is similar in form to the Thiem Equation.

➤   This equation fails to give an accurate description of the drawdown near the well where the strong curvature of the water table contradicts the initial assumptions.
➤   An approximate steady state flow condition in an unconfined aquifer will only be reached after long pumping time (see **Figure 6.10**)
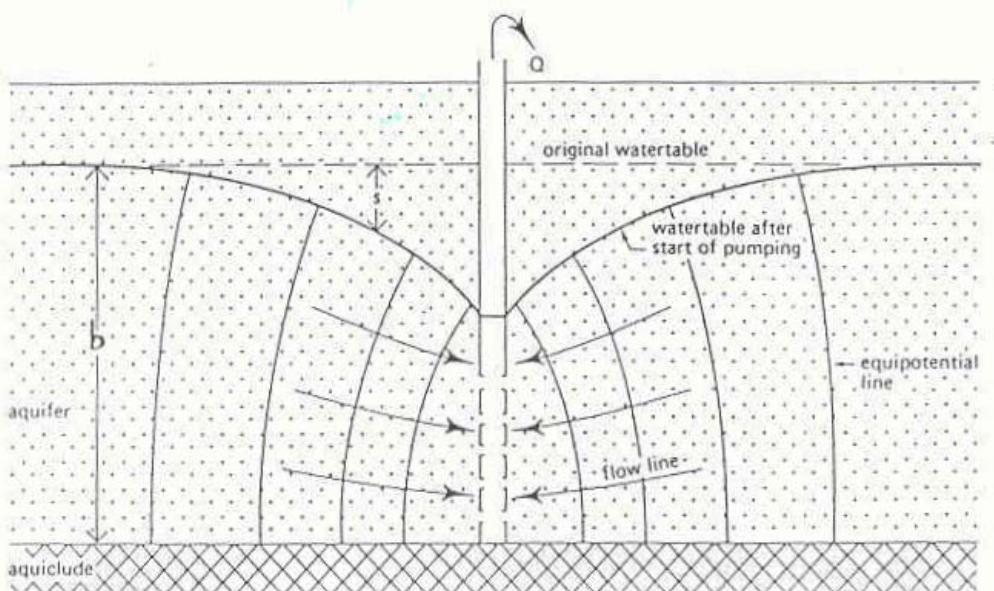

Figure 6.10   Cross-section of a pumped unconfined aquifer

6.5 Using Pumping Tests to Estimate Hydraulic Conductivity (K), Transmissivity (T), Storativity (S) and Drawdown (S_w).

Unsteady Radial Flow in a Confined Aquifer (Non-equilibrium Radial Flow)

When a well penetrating an extensive confined aquifer is pumped at a constant rate, the influence of the discharge extends outward with time. The rate of decline of head times the storage coefficient summed over the area of influence equals the discharge. Because the water must come from a reduction of storage within the aquifer, the head will continue to decline as long as the aquifer is effectively infinite; there for, unsteady, or transient or non-equilibrium flow exists. The rate of decline, however, decreases continuously as the area of influence expands.

Figure 6.11 shows a well fully penetrating a confined aquifer of thickness b. Let us consider flow through an annular cylinder of soil with radius r and thickness d, at a radial distance of r from the center of the well.
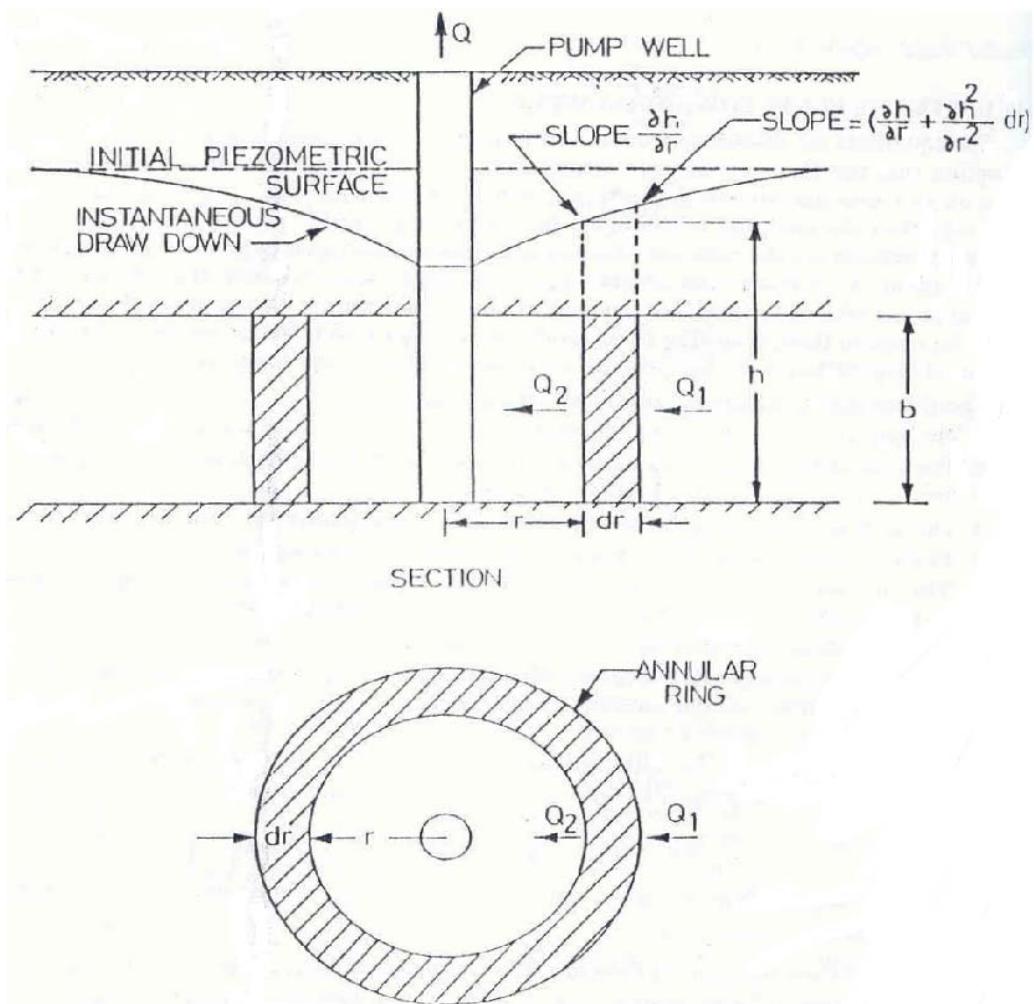

Figure 6.11

From the principle of continuity equation of flow, the difference of the rate of inflow and the rate of outflow from the annular cylinder is equal to the rate of change of volume of water within the annular space. Thus

$$Q_1 - Q_2 = \frac{\partial V}{\partial t} \tag{6.16}$$

where Q_1 is the rate of inflow, Q_2 is the rate of outflow and $\dfrac{\partial V}{\partial t}$ is the rate of change of volume (V) within the annular space.

The slope of the hydraulic gradient line (i.e. the piezometric surface) at the inner surface is $\dfrac{\partial h}{\partial t}$, where h is the height of piezometric surface above the impervious stratum. Therefore, the slope of the hydraulic gradient line at the outer surface is equal to $\dfrac{\partial h}{\partial r} + \dfrac{\partial^2 h}{\partial r^2} dr$, Now by Darcy's law.

$$Disch\arg e = K \times i \times (area\ of\ flow)$$

$$Therefore, \quad Inflow,\ Q_1 = K \times \left[\frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2} dr \right] \times 2\pi (r + dr) b \tag{6.17}$$

$$Outflow, Q_2 = K \times \frac{\partial h}{\partial r} \times (2\pi r) b$$

Now from the definition of storage coefficient (S), S is the volume of water released per unit surface area per unit change in head normal to the surface. Therefore,

$$Change\ in\ volume = \delta V = S\,(2\pi\,r)\,dr\,dh$$

$$Therefore, \quad \frac{\partial V}{\partial t} = S\,(2\pi\,r)\,dr\,\frac{dh}{dt} \tag{6.18}$$

where t is the time since the beginning of pumping.

Substituting **equations 6.17** and **6.18** in **equation 6.16**

$$K \times \left[\frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2} dr \right] \times 2\pi (r + dr) b \ - \ K \times \frac{\partial h}{\partial r} \times (2\pi r) b \ = \ S\,(2\pi\,r)\,dr\,\frac{\partial h}{\partial t}$$

$$Or, \tag{6.19}$$

$$Kb \left[\frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2} dr \right] \times 2\pi (r + dr) \ - \ Kb \times \frac{\partial h}{\partial r} \times (2\pi r) \ = \ S\,(2\pi\,r)\,dr\,\frac{\partial h}{\partial t}$$

Dividing by $Kb\,(2\pi\,r)\,dr$ throughout and neglecting the higher order terms,

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \times \frac{\partial h}{\partial r} = \frac{S}{Kb} \times \frac{\partial h}{\partial t} \tag{6.20}$$

Substituting the Transmissivity (T) for Kb,

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \times \frac{\partial h}{\partial r} = \frac{S}{T} \times \frac{\partial h}{\partial t} \tag{6.21}$$

Equation 6.21 is the basic equation of unsteady flow towards the well. In this equation, h is head, r is radial distance from the well, S is storage coefficient, T is transmissivity, and t is the time since the beginning of pumping.

6.5.1 Confined Aquifers – The Theis Method (Curve Matching Method)

Theis (1935) solved the non-equilibrium flow equations in radial coordinates based on the analogy between groundwater flow and heat condition. By assuming that the well is replaced by a mathematical sink of constant strength and imposing the boundary conditions $h = h_o \ \ for \, t = 0$, and $h \rightarrow h_o \ \ as \ r \rightarrow \infty \ \ for \ t \geq 0$, the solution

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-u}}{u} \, du \tag{6.22}$$

For this method the drawdown at a piezometer distance, **r** from the abstraction well is monitored over time.

Theis expressed the transient drawdown, s, as

$$s = \frac{Q}{4\pi T} \, W(u) \tag{6.23}$$

Where **W(u)** is the **well function** and **u** is given by

$$u = \frac{r^2 S}{4 T t} \tag{6.24}$$

Taking logarithms and rearranging these equations gives

$$\log s = \log\left(W(u)\right) + \log\left(\frac{Q}{4\pi T}\right) \tag{6.25}$$

and,
$$\log t = \log\left(\frac{1}{u}\right) + \log\left(\frac{r^2 S}{4T}\right) \tag{6.26}$$

Since the last term in each equation is constant, a graph of log s against log t should be the same shape as a graph of log $(W_{(u)})$ against log (1/u), but offset horizontally and vertically by the constants in the equation.

Assumptions

1. Prior to pumping, the potentiometric surface is approximately horizontal (No slope),
2. The aquifer is confined and has an "apparent" infinite extent,
3. The aquifer is homogeneous, isotropic, of uniform thickness over the area influenced by pumping,
4. The well is pumped at a constant rate,
5. The well is fully penetrating,
6. Water removed from storage is discharged instantaneously with decline in head,
7. The well diameter is small so that well storage is negligible.

The Data Required for the Theis Solution are

1. Drawdown vs. time data at an observation well,
2. Distance from the pumping well to the observation well,
3. Pumping rate of the well.

The procedure for finding parameters by Theis Method

1. On log-log paper, plot a graph of values of s_w against t measured during the pumping test,

2   Theoretical curve W(u) versus 1/u is plotted on a log-log paper. This can be done using tabulated values of the well function (**see Table 6.1**). ready printed type curves are also available (see **Figure 6.12**),

3   The field measurements are similarly plotted on a log-log plot with (t) along the x-axis and (s_w) along the y-axis (see **Figure 6.13**),

4   Keeping the axes correctly aligned, superimposed the type curve on the plot of the   data (i.e. The data analysis is done by matching the observed data to the type curve),

5   Select **any** convenient point on the graph paper (**a match point**) and read off the coordinates of the point on **both** sets of axes. This gives coordinates (1/u, W(u)) and (t, s_w) (see **Figures 6.14**),

6   Use the previous equations to determine T and S.

The points on the data plot corresponding to early times are the least reliable.

N.B.   The match point doesn't have to be on the type curve. In fact calculations are greatly simplified if the point is chosen where W(u) = 1 and 1/u=10.
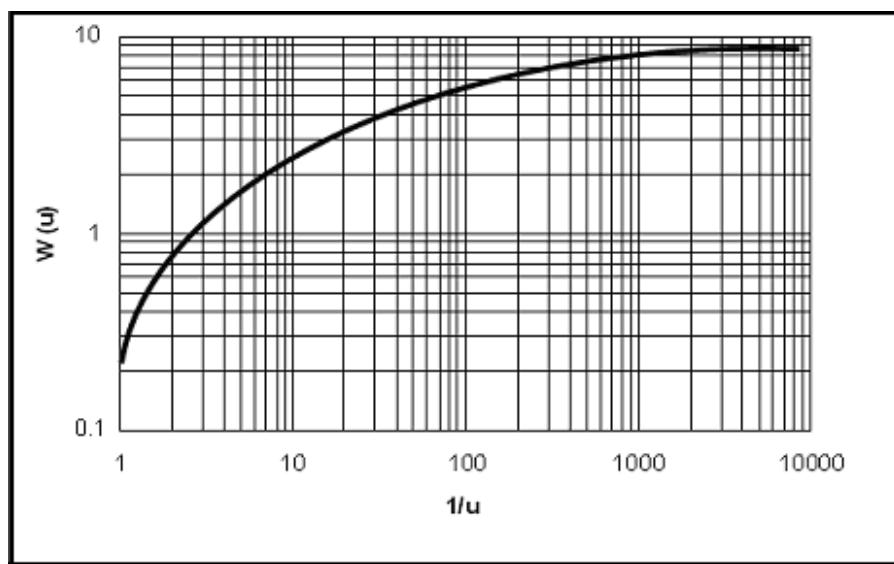

Figure 6.12   The non-equilibrium reverse type curve (Theis curve) for a fully confined aquifer
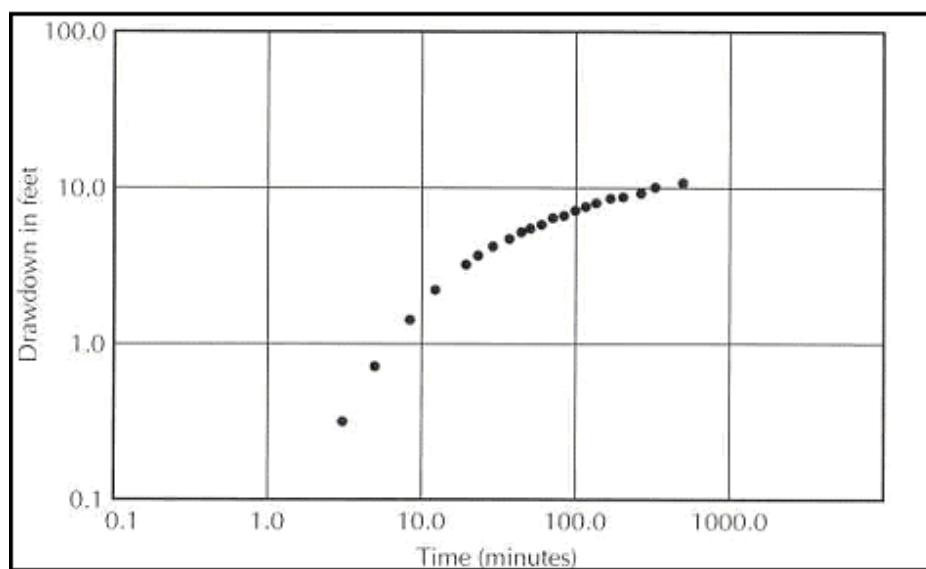

Figure 6.13   Field data plot on logarithmic paper for Theis curve-marching technique
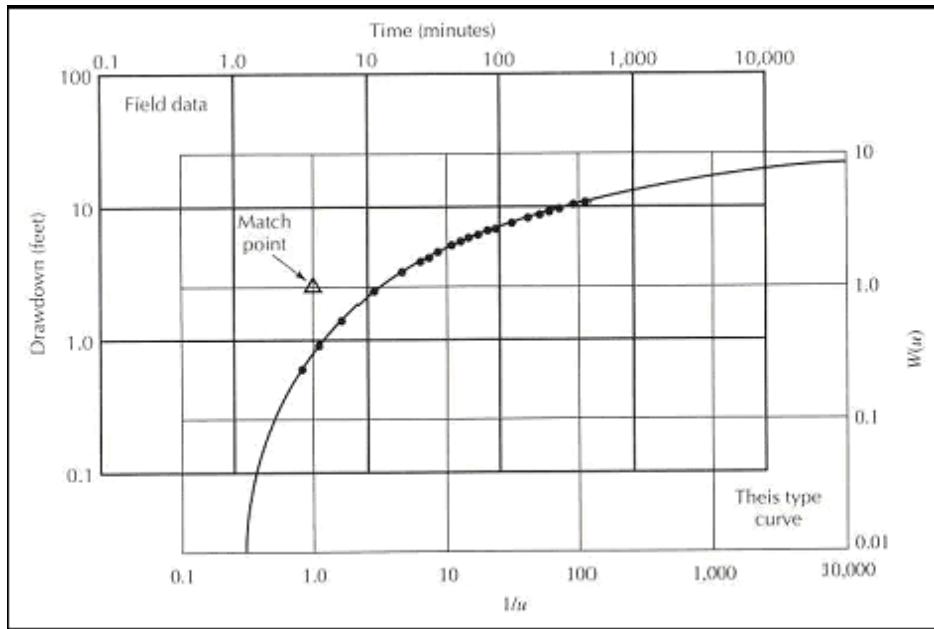
Figure 6.14   Match of field data plot to Theis Type curve

6.5.2 Confined Aquifers – Cooper-Jacob Method (Time-Drawdown)

The analysis presented here is of a pumping test in which drawdown at a piezometer distance, **r** from the abstraction well is monitored over time. This is also based upon the Theis analysis

$$s = \frac{Q}{4\pi T}\, W(u) = \frac{Q}{4\pi T}\left(-0.5772 - \ln u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \dots\right) \tag{6.27}$$

From the definition of u it can be seen that u decreases as the time of pumping increases and as the distance of the piezometer from the well decreases. So, for piezometers close to the pumping well after sufficiently long pumping times, the terms beyond $\ln u$ become negligible. Hence   for   small values of **u**, the drawdown can be approximated by:

$$s = \frac{Q}{4\pi T}\left(-0.5772 - \ln \frac{r^2 S}{4Tt}\right) \tag{6.28}$$

Changing to logarithms base 10 and rearranging produces

$$s = \frac{2.3Q}{4\pi T}\, \log \frac{2.25T}{r^2 S}t \tag{6.29}$$

and this is a straight line equation

$$s = \left(\frac{2.3Q}{4\pi T}\, \log \frac{2.25T}{r^2 S}\right) + \left(\frac{2.3Q}{4\pi T}\right)\log t$$

$$Y = \qquad C \qquad + \quad a \quad X$$

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Note: **Jacob method is valid** for **u** ≤ 0.05 or 0.01, **t** is large, **r** is small.

Value of "u"	0.01	0.03	0.05	0.1	0.15
Error (%)	negative	1	2	5	10

It follows that a plot of **s** against **log t** should be **a straight line** (see **Figure 6.15**). Extending this line to where it crosses that **t axis** (i.e. where **s is zero** and **t=t₀**) gives

$$\frac{2.25T}{r^2 S} t_o = 1 \qquad\qquad \textbf{(6.30)}$$

The gradient of the straight line (i.e. the increase per log cycle, Δs) is equal to

$$\Delta s = \frac{2.30\,Q}{4\,\pi\,T} \qquad\qquad \textbf{(6.31)}$$

At first T is calculated (**eq. 6.31**) then S can be calculated from **eq. 6.30** by using T and t_0
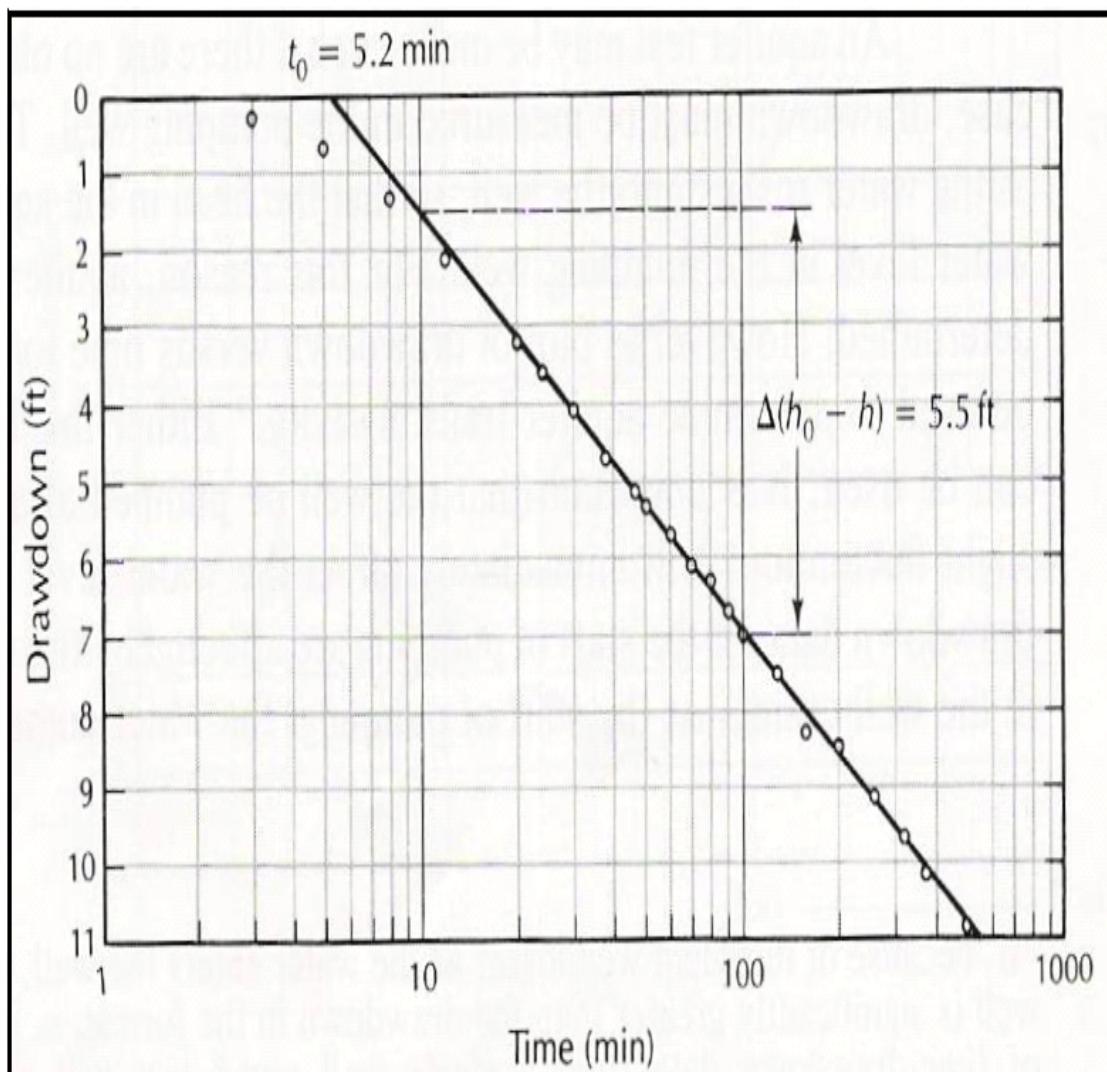

Figure 6.15 Jacob method of solution of pumping-test data for a fully confined aquifer. Drawdown is plotted as a function of time on semi-logarithmic paper

Table 6.1   Values of the function W(u) for various values of u

u	W(u)	u	W(u)	u	W(u)	u	W(u)
1×10^{-10}	22.45	7×10^{-8}	15.90	4×10^{-5}	9.55	1×10^{-2}	4.04
2	21.76	8	15.76	5	9.33	2	3.35
3	21.35	9	15.65	6	9.14	3	2.96
4	21.06	1×10^{-7}	15.54	7	8.99	4	2.68
5	20.84	2	14.85	8	8.86	5	2.47
6	20.66	3	14.44	9	8.74	6	2.30
7	20.50	4	14.15	1×10^{-4}	8.63	7	2.15
8	20.37	5	13.93	2	7.94	8	2.03
9	20.25	6	13.75	3	7.53	9	1.92
1×10^{-9}	20.15	7	13.60	4	7.25	1×10^{-1}	1.823
2	19.45	8	13.46	5	7.02	2	1.223
3	19.05	9	13.34	6	6.84	3	0.906
4	18.76	1×10^{-6}	13.24	7	6.69	4	0.702
5	18.54	2	12.55	8	6.55	5	0.560
6	18.35	3	12.14	9	6.44	6	0.454
7	18.20	4	11.85	1×10^{-3}	6.33	7	0.374
8	18.07	5	11.63	2	5.64	8	0.311
9	17.95	6	11.45	3	5.23	9	0.260
1×10^{-8}	17.84	7	11.29	4	4.95	1×10^{0}	0.219
2	17.15	8	11.16	5	4.73	2	0.049
3	16.74	9	11.04	6	4.54	3	0.013
4	16.46	1×10^{-5}	10.94	7	4.39	4	0.004
5	16.23	2	10.24	8	4.26	5	0.001
6	16.05	3	9.84	9	4.14		

Source: Adapted from L. K. Wenzel, *Methods for Determining Permeability of Water-Bearing Materials with Special Reference to Discharging Well Methods.* U.S. Geological Survey Water-Supply Paper 887, 1942.

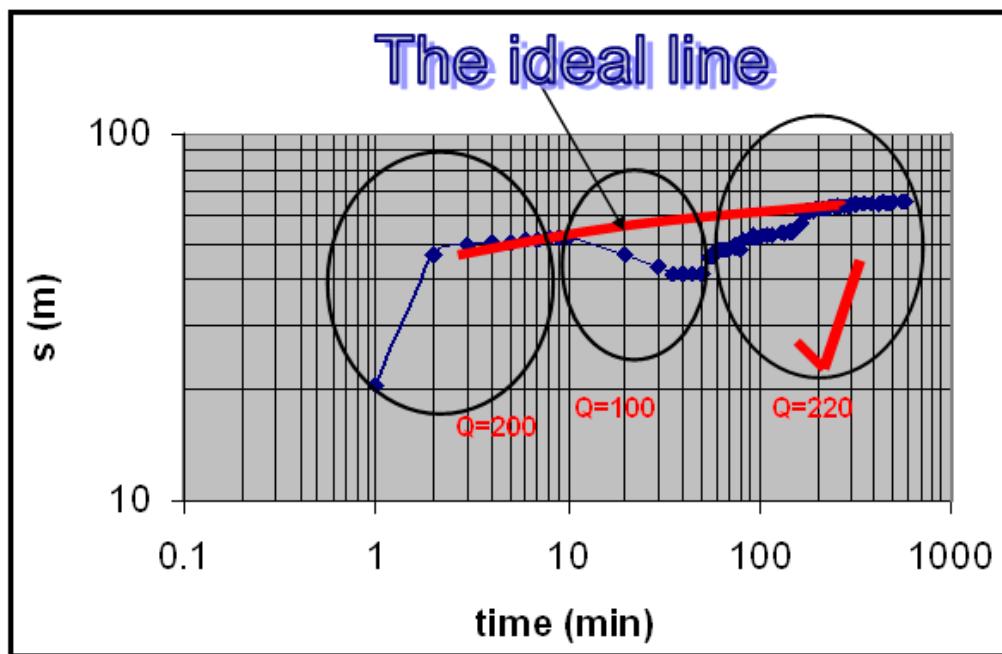Note: see **Figure 6.16** to see some problems that may occur during analysis

Figure 6.16   Problems that may encountered during analysis

6.5.3 Confined Aquifers – Cooper-Jacob Method (Distance-Drawdown)

If simultaneous observations are made of drawdown in three or more observation wells, the observation well distance is plotted along the logarithmic x-axes, and drawdown is plotted along the linear y-axes.

For the Distance-Drawdown method, transmissivity and storativity are calculated as follows:

$$T = \frac{2.30\,Q}{4\,\pi\,\Delta s} \quad \text{per one logarithmic cycle} \tag{6.32}$$

When $s_w = 0 \quad \rightarrow \quad \dfrac{2.25\,T\,t}{r^2 S} = 1 \tag{6.33}$

Therefore, $\quad S = \dfrac{2.25\,T\,t}{r_o^{\,2}} \tag{6.34}$

Where,

Δs is the change in drawdown over one logarithmic cycle, r_o is the distance defined by the intercept of the **straight-line** fit of the data and zero-drawdown axis, and **t** is the time to which the set of drawdown data correspond (see **Figure 6.17**).
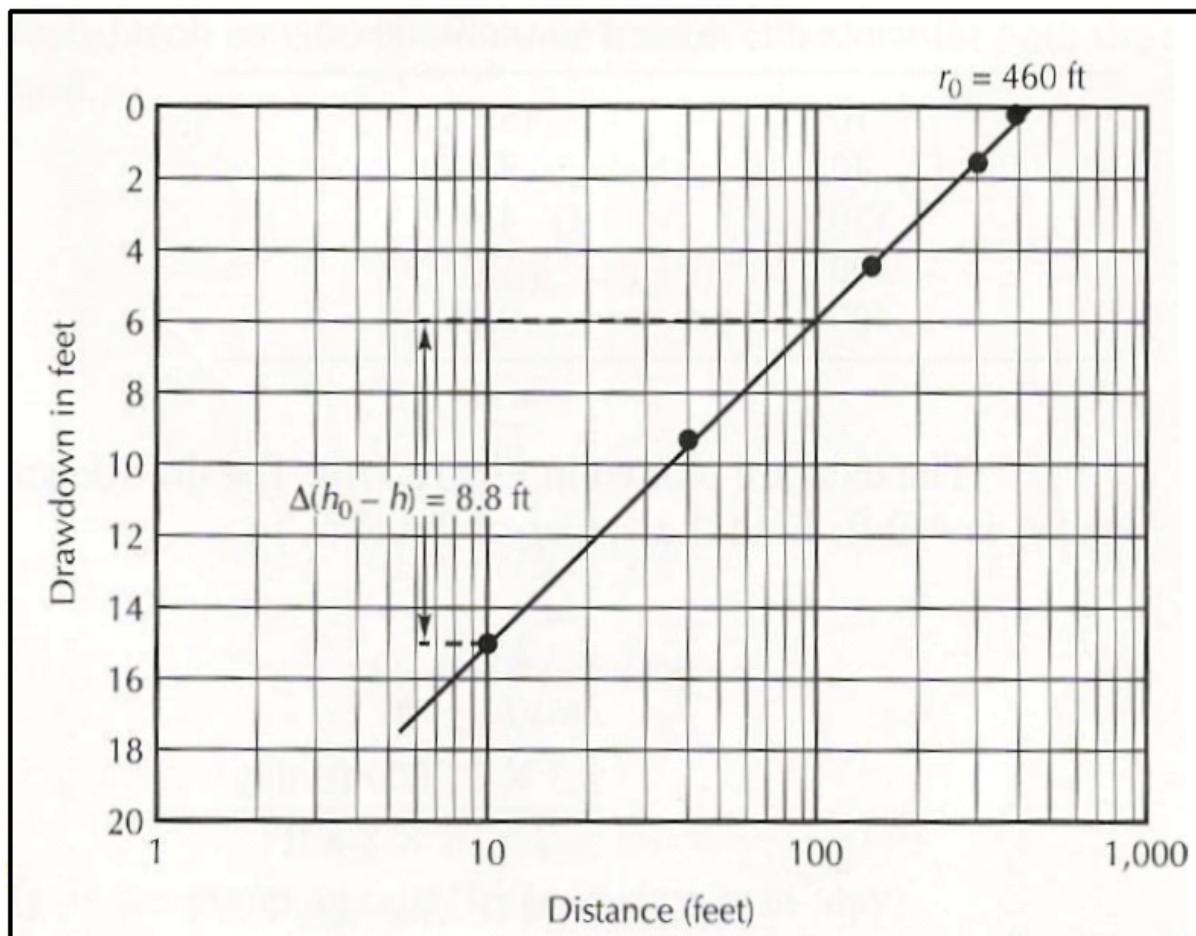

Figure 6.17   Straight line plot of Cooper-Jacob method (Disatnce-Drowdown, Confined)

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Unsteady Radial Flow in a Leaky Aquifer
(Non-equilibrium Radial Flow)

6.5.4 Leaky (Semi) Confined Aquifers – Hantush-Jacob Method and Walton Graphical Method

Leaky aquifer bounded to and bottom by less transmissive horizons, at least one of which allows some significant vertical water "leakage" into the aquifer.

Unsteady radial flow for leaky aquifer can be represented in the following equation:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \times \frac{\partial h}{\partial r} + \frac{e}{T} = \frac{S}{T} \times \frac{\partial h}{\partial t} \qquad\qquad \textbf{(6.35)}$$

Where,   r   is the radial distance from a pumping well (m)
                  e   is the rate of vertical leakage (m/day)

When a leaky aquifer, as shown in **Figure 6.18**, is pumped, water is withdrawn both from the aquifer and from the saturated portion of the overlying aquitard, or semipervious layer. Lowering the piezometric head in the aquifer by pumping creates a hydraulic gradient within the aquitard; consequently, groundwater migrates vertically downward into the aquifer. The quantity of water moving downward is proportional to the difference between the water table and the piezometric head.

Steady state flow is possible to a well in a leaky aquifer because of the recharge through the semipervious layer. The equilibrium will be established when the discharge rate of the pump equals the recharge rate of vertical flow into the aquifer, assuming the water table remains constant. Solutions for this special steady state situation are available, but a more general analysis for unsteady flow follows.
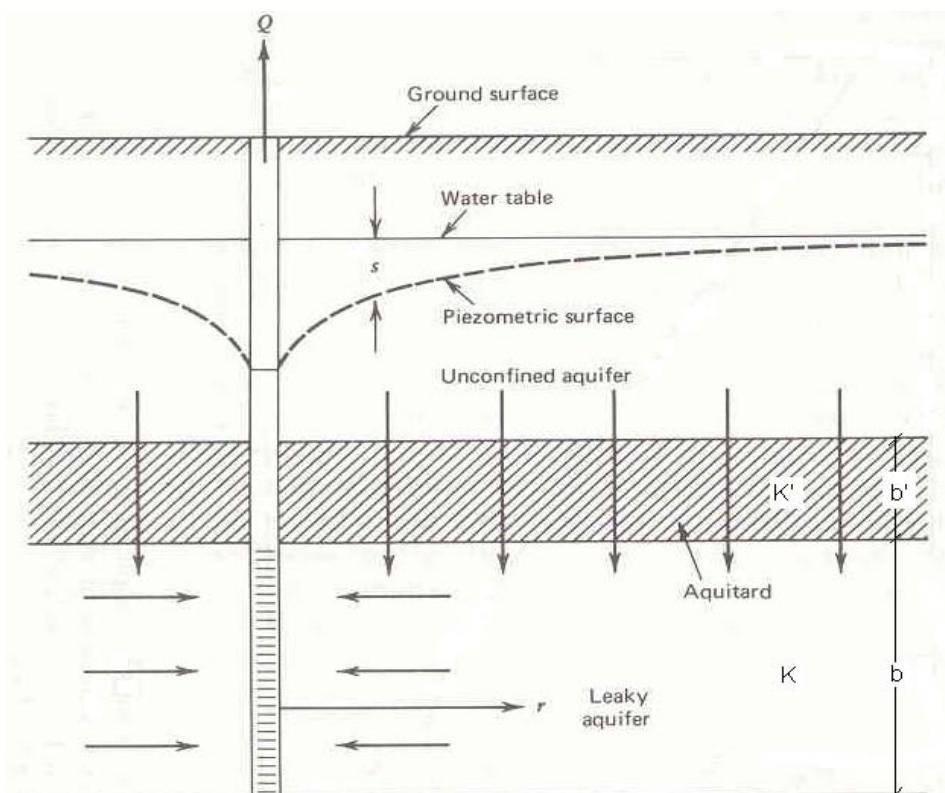

Figure 6.18   Well pumping from a leaky aquifer

Normal assumption leakage rate into aquifer = $K' \cdot \dfrac{\Delta H}{b'}$ and $\Delta H = s_w$, where d' is the thickness of the saturated semipervious layer.

Initially, pumped water from elastic storage in aquifer with increasing time, forces more water to come from induced "leakage" through aquitard. Water contributed from aquitard comes from, 1) storage in aquitard, 2) over/underlying aquifer.

The Hantush and Jacob solution has the following assumptions:

1. The aquifer is leaky and has an "apparent" infinite extent,
2. The aquifer and the confining layer are homogeneous, isotropic, and of uniform thickness, over the area influenced by pumping,
3. The potentiometric surface was horizontal prior to pumping,
4. The well is pumped at a constant rate,
5. The well is fully penetrating,
6. Water removed from storage is discharged instantaneously with decline in head,
7. The well diameter is small so that well storage is negligible,
8. Leakage through the aquitard layer is vertical.

The **Hantush and Jacob** (1955) solution for leaky aquifer presents the following equations (see Figure **6.18**):

$$s = \frac{Q}{4\pi T} W(u, \frac{r}{B}) \;\; \rightarrow \; T = \frac{Q}{4\pi s} W(u, \frac{r}{B}) \tag{6.36}$$

where,

$$u = \frac{r^2 S}{4T t} \;\; \rightarrow \; S = \frac{4T u t}{r^2} \tag{6.37}$$

where,

$W(u, \frac{r}{B})$: is the well function for leaky confined aquifer

B : is the leakage factor given as $B = \sqrt{Tb'/K'}$ (m)

where,

b' is thickness of the aquitard (m)

K' is hydraulic conductivity of the aquitard (m/day)

Walton Graphical Solution

1. Type curves $W(u, \frac{r}{B})$ *vs.* $\frac{1}{u}$ for various values of $\frac{1}{u}$ *and* $\frac{r}{B}$, see **Figure 6.19**.

2. Field data are plotted on drawdown (s_w) vs. time on full logarithmic scale.

3. Field data should match one of the type curves for $\frac{r}{B}$ (interpolation if between two lines)

4. From a match point, the following are known values $W(u, \frac{r}{B})$, $\frac{1}{u}$, t, s_w, *and* $\frac{r}{B}$.

5. Substitute in Hantush-Jacob equation:

$$T = \frac{Q}{4\pi s} W(u, \frac{r}{B})$$

$$S = \frac{4Tut}{r^2}$$

$$\frac{r}{B}(from\ match) = \frac{r}{\sqrt{Tb'\!/\!K'}}\ ,\ then$$

$$K' = \frac{Tb'(\frac{r}{B})^2}{r^2} \tag{6.38}$$

where,

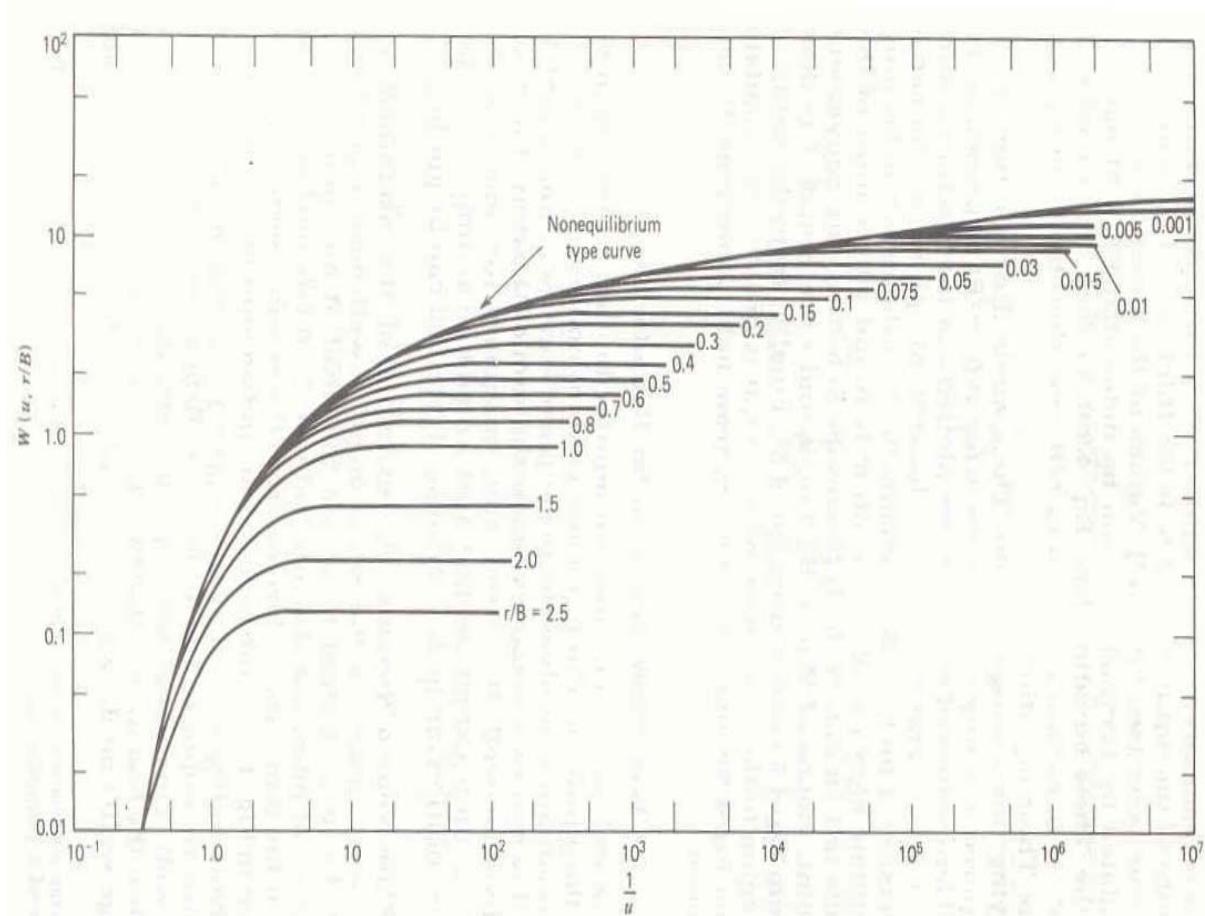|	
Q	is the pumping rate (m³/day)
t	is the time since pumping began (day)
r	is the distance from pumping well to observation well (m)
b′	is the thickness of aquitard (m)
K′	is the vertical hydraulic conductivity of confining bed (aquitard) (m/day)
B	is the leakage factor (m)

Figure 6.19   Log-log plot for Hantush method

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Unsteady Radial Flow in an Unconfined Aquifer (Non-equilibrium Radial Flow)

6.5.5 Unconfined– Neuman

The flow of water in an unconfined aquifer toward a pumping well is described by the following equation (Neuman & Witherspoon 1969)

$$K_r \frac{\partial^2 h}{\partial r^2} + \frac{K_r}{r} \times \frac{\partial h}{\partial r} + K_v \frac{\partial^2 h}{\partial z^2} = S_s \times \frac{\partial h}{\partial t} \qquad \textbf{(6.39)}$$

where,

h	is the saturated thickness of the aquifer (m)
r	is radial distance from the pumping well (m)
z	is elevation above the base of the aquifer (m)
S_s	is specific storage (1/m)
K_r	is radial hydraulic conductivity (m/day)
K_v	is vertical hydraulic conductivity (m/day)
T	is time (day)

A well pumping from a water-table aquifer extracts water by two mechanisms. (1) as with confined aquifer, the decline in pressure yields water because of the elastic storage of the aquifer storativity (S_s). (2) The declining water table also yields water as it drains under gravity from the sediments. Thus is termed specific yield (S_y).

There are three distinct phases of time-drawdown relations in water-table wells (see **Figure 6.20**). We will examine the response of any typical annular region of the aquifer located a constant distance from the pumping well:
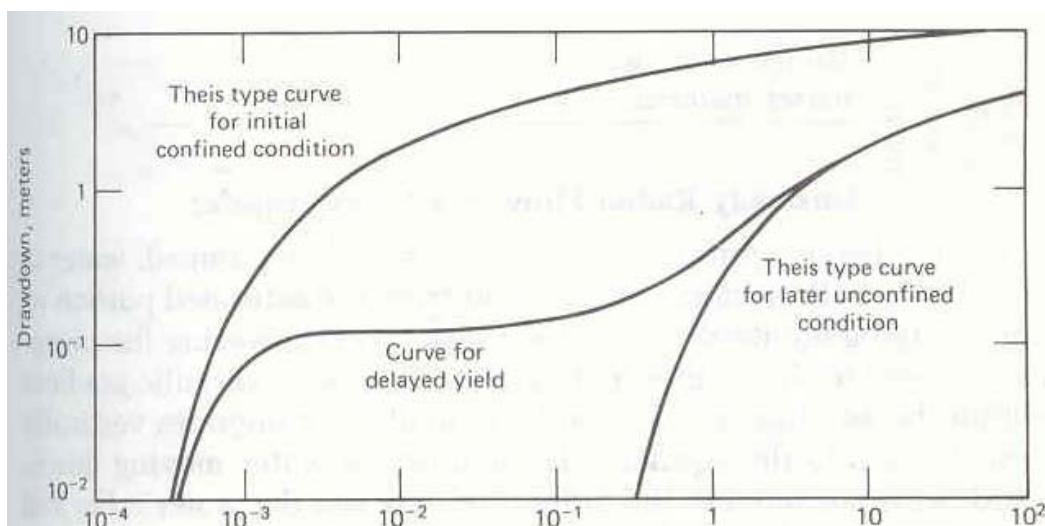

Figure 6.20   Type curves of drawdown versus time illustrating the effect of delayed yield for pumping tests in unconfined aquifers.

1   Some time after pumping has begun; the pressure in the annular region will drop. As the pressure first drops, the aquifer responds by contributing a small volume of water as a result of the expansion of water and compression of the aquifer. During this time, the aquifer behaves as an artesian aquifer, and the time-drawdown data follow the Theis non-equilibrium curve for S equal to the elastic storativity of the aquifer. Flow is horizontal during this period, as the water is being derived from the entire aquifer thickness.

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2   Following this initial phase, the water table begins to decline. Water is now being derived primarily from the gravity drainage of the aquifer, and there are both horizontal and vertical flow components. The drawdown-time relationship is a function of the ratio of horizontal-to-vertical conductivities of the aquifer, the distance to the pumping well, and the thickness of the aquifer.

3   As time progress, the rate of drawdown decreases and the contribution of the particular annular region to the overall well discharge diminishes. Flow is again essential horizontal, and the time-discharge data again follow a Theis type curve. The Theis curve now corresponds to one with a storativity equal to the specific yield to the elastic storage coefficient $\left(\dfrac{S_y}{S_s}\right)$. As the value of S_s approach zero. As S_y approach zero, the length of the first stage increases, so that if $S_y = 0$, the aquifer behaves as an artesian aquifer of storativity S_s.

In general all previous techniques of confined aquifer can be used for unconfined aquifer, BUT an adjustment should be done for drawdown as follow:

$$s' = s - \left(\frac{s^2}{2h}\right) \qquad \textbf{(6.40)}$$

Where,
   s'   is the adjusted drawdown (m)
   h   is the initial saturated thickness of aquifer (m)

Neuman (1972, 1973, 1974, and 1987) has published a solution to **Equation 6.39**. There are two parts to the solution, one for the time just after pumping has begun and the water is coming from specific storage and one for much later, when the water is coming from gravity drainage with the storativity equal to the specific yield.

Neuman's solution assumes the following, in addition to the basic assumptions:

1   The aquifer is unconfined.
2   The vadose zone has no influence on the drawdown.
3   Water initially pumped comes from the instantaneous release of water from elastic storage.
4   Eventually water comes from storage due to gravity drainage of interconnected pores.
5   The drawdown is negligible compared with the saturated aquifer thickness.
6   The specific yield is at least 10 times the elastic storativity.
7   The aquifer may be- but does not have to be- anisotropic with the radial hydraulic conductivity different than the vertical hydraulic conductivity.

With these assumptions Neuman's solution is:

$$s = \frac{Q}{4\pi T} W(u_A, u_B, \Gamma) \qquad \textbf{(6.41)}$$

Where,
   $W(u_A, u_B, \Gamma)$ is the well function of water-table aquifer, as tabulated in **Table 6.2**

For early time (early drawdown data)

$$s = \frac{Q}{4\pi T} W(u_A, \Gamma) \qquad and \qquad u_A = \frac{r^2 S_A}{4 K b t} \qquad \textbf{(6.42)}$$

For late time (late drawdown data)

$$s = \frac{Q}{4\pi T} W(u_B, \Gamma) \qquad and \qquad u_B = \frac{r^2 S_B}{4 K b t} \qquad \text{(6.43)}$$

$$\Gamma = \frac{r^2 K_v}{b^2 K_h} \qquad \text{(6.44)}$$

Where,

S	is the storativity (dimensionless)
S_y	is the specific yield (dimensionless)
r	radial distance from pumping well (m)
b	is the initial saturated thickness of aquifer (m)
K_v	is horizontal hydraulic conductivity (m/day)
K_h	is horizontal hydraulic conductivity (m/day)

The procedure for finding parameters by Penman Method

1 Two sets of type curves are used and plotted on log-log paper (Theoretical curve $s = \frac{Q}{4\pi T} W(u_A, u_B, \Gamma)$ versus $\frac{1}{u}$).

2 Superpose the early $(t-s)$ data on **Type-A curve**.

3 The data analysis is done by matching the observed data to the type curve.

4 From the match point of **Type-A curve**, determine the values for $W(u_A, \Gamma)$, $\frac{1}{u_A}$, s, t, and the value of Γ.

5 Use the previous equations to determine T and S

6 The latest $(s-t)$ data are then superposed on **Type-B Curve** for the Γ - values of previously matched **Type-A curve**, from the match point of **Type-B curve**, determine the values for $W(u_B, \Gamma)$, $\frac{1}{u_B}$, s, t (see **Figure 6.21**).

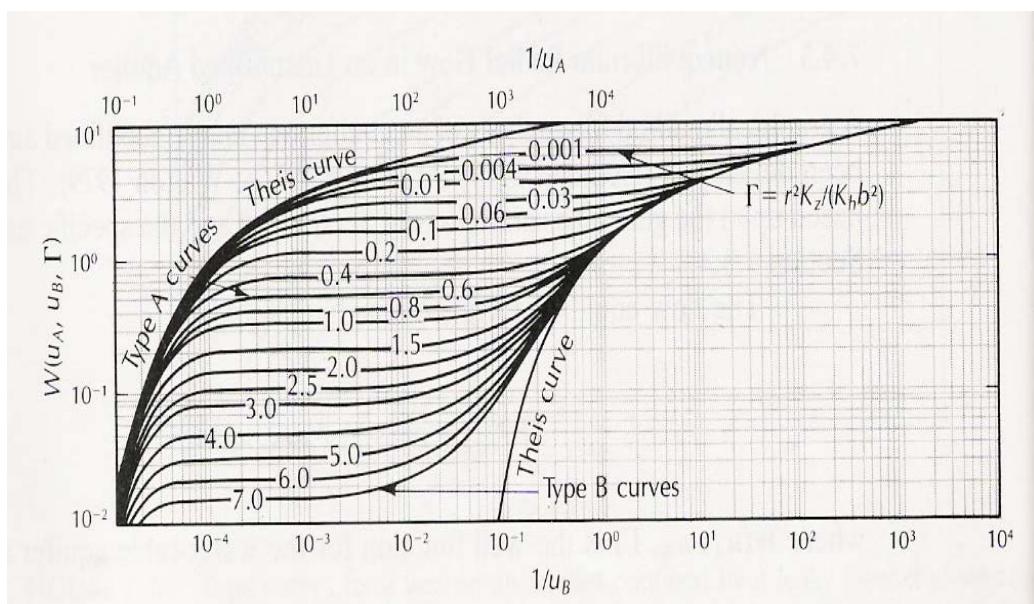7 By using the previous equations, the T and S can be determined.

Figure 6.21 Type curves for unconfined aquifers

Table 6.2

Values of the function $W(u_A, \Gamma)$ for water table aquifer

$1/u_A$	$\Gamma = 0.001$	$\Gamma = 0.01$	$\Gamma = 0.06$	$\Gamma = 0.2$	$\Gamma = 0.6$	$\Gamma = 1.0$	$\Gamma = 2.0$	$\Gamma = 4.0$	$\Gamma = 6.0$
4.0×10^{-1}	2.48×10^{-2}	2.41×10^{-2}	2.30×10^{-2}	2.14×10^{-2}	1.88×10^{-2}	1.70×10^{-2}	1.38×10^{-2}	9.33×10^{-3}	6.39×10^{-3}
8.0×10^{-1}	1.45×10^{-1}	1.40×10^{-1}	1.31×10^{-1}	1.19×10^{-1}	9.88×10^{-2}	8.49×10^{-2}	6.03×10^{-2}	3.17×10^{-2}	1.74×10^{-2}
1.4×10^{0}	3.58×10^{-1}	3.45×10^{-1}	3.18×10^{-1}	2.79×10^{-1}	2.17×10^{-1}	1.75×10^{-1}	1.07×10^{-1}	4.45×10^{-2}	2.10×10^{-2}
2.4×10^{0}	6.62×10^{-1}	6.33×10^{-1}	5.70×10^{-1}	4.83×10^{-1}	3.43×10^{-1}	2.56×10^{-1}	1.33×10^{-1}	4.76×10^{-2}	2.14×10^{-2}
4.0×10^{0}	1.02×10^{0}	9.63×10^{-1}	8.49×10^{-1}	6.88×10^{-1}	4.38×10^{-1}	3.00×10^{-1}	1.40×10^{-1}	4.78×10^{-2}	2.15×10^{-2}
8.0×10^{0}	1.57×10^{0}	1.46×10^{0}	1.23×10^{0}	9.18×10^{-1}	4.97×10^{-1}	3.17×10^{-1}	1.41×10^{-1}		
1.4×10^{1}	2.05×10^{0}	1.88×10^{0}	1.51×10^{0}	1.03×10^{0}	5.07×10^{-1}				
2.4×10^{1}	2.52×10^{0}	2.27×10^{0}	1.73×10^{0}	1.07×10^{0}					
4.0×10^{1}	2.97×10^{0}	2.61×10^{0}	1.85×10^{0}	1.08×10^{0}					
8.0×10^{1}	3.56×10^{0}	3.00×10^{0}	1.92×10^{0}						
1.4×10^{2}	4.01×10^{0}	3.23×10^{0}	1.93×10^{0}						
2.4×10^{2}	4.42×10^{0}	3.37×10^{0}	1.94×10^{0}						
4.0×10^{2}	4.77×10^{0}	3.43×10^{0}							
8.0×10^{2}	5.16×10^{0}	3.45×10^{0}							
1.4×10^{3}	5.40×10^{0}	3.46×10^{0}							
2.4×10^{3}	5.54×10^{0}								
4.0×10^{3}	5.59×10^{0}								
8.0×10^{3}	5.62×10^{0}								
1.4×10^{4}	5.62×10^{0}	3.46×10^{0}	1.94×10^{0}	1.08×10^{0}	5.07×10^{-1}	3.17×10^{-1}	1.41×10^{-1}	4.78×10^{-2}	2.15×10^{-2}

Values of the function $W(u_B, \Gamma)$ for water table aquifers

$1/u_B$	$\Gamma = 0.001$	$\Gamma = 0.01$	$\Gamma = 0.06$	$\Gamma = 0.2$	$\Gamma = 0.6$	$\Gamma = 1.0$	$\Gamma = 2.0$	$\Gamma = 4.0$	$\Gamma = 6.0$
4.0×10^{-4}	5.62×10^{0}	3.46×10^{0}	1.94×10^{0}	1.09×10^{0}	5.08×10^{-1}	3.18×10^{-1}	1.42×10^{-1}	4.79×10^{-2}	2.15×10^{-2}
8.0×10^{-4}								4.80×10^{-2}	2.16×10^{-2}
1.4×10^{-3}								4.81×10^{-2}	2.17×10^{-2}
2.4×10^{-3}								4.84×10^{-2}	2.19×10^{-2}
4.0×10^{-3}					5.08×10^{-1}	3.18×10^{-1}	1.42×10^{-1}	4.88×10^{-2}	2.21×10^{-2}
8.0×10^{-3}					5.09×10^{-1}	3.19×10^{-1}	1.43×10^{-1}	4.96×10^{-2}	2.28×10^{-2}
1.4×10^{-2}					5.10×10^{-1}	3.21×10^{-1}	1.45×10^{-1}	5.09×10^{-2}	2.39×10^{-2}
2.4×10^{-2}					5.12×10^{-1}	3.23×10^{-1}	1.47×10^{-1}	5.32×10^{-2}	2.57×10^{-2}
4.0×10^{-2}					5.16×10^{-1}	3.27×10^{-1}	1.52×10^{-1}	5.68×10^{-2}	2.86×10^{-2}
8.0×10^{-2}				1.09×10^{0}	5.24×10^{-1}	3.37×10^{-1}	1.62×10^{-1}	6.61×10^{-2}	3.62×10^{-2}
1.4×10^{-1}			1.94×10^{0}	1.10×10^{0}	5.37×10^{-1}	3.50×10^{-1}	1.78×10^{-1}	8.06×10^{-2}	4.86×10^{-2}
2.4×10^{-1}			1.95×10^{0}	1.11×10^{0}	5.57×10^{-1}	3.74×10^{-1}	2.05×10^{-1}	1.06×10^{-1}	7.14×10^{-2}
4.0×10^{-1}			1.96×10^{0}	1.13×10^{0}	5.89×10^{-1}	4.12×10^{-1}	2.48×10^{-1}	1.49×10^{-1}	1.13×10^{-1}
8.0×10^{-1}	5.62×10^{0}	3.46×10^{0}	1.98×10^{0}	1.18×10^{0}	6.67×10^{-1}	5.06×10^{-1}	3.57×10^{-1}	2.66×10^{-1}	2.31×10^{-1}
1.4×10^{0}	5.63×10^{0}	3.47×10^{0}	2.01×10^{0}	1.24×10^{0}	7.80×10^{-1}	6.42×10^{-1}	5.17×10^{-1}	4.45×10^{-1}	4.19×10^{-1}
2.4×10^{0}	5.63×10^{0}	3.49×10^{0}	2.06×10^{0}	1.35×10^{0}	9.54×10^{-1}	8.50×10^{-1}	7.63×10^{-1}	7.18×10^{-1}	7.03×10^{-1}
4.0×10^{0}	5.63×10^{0}	3.51×10^{0}	2.13×10^{0}	1.50×10^{0}	1.20×10^{0}	1.13×10^{0}	1.08×10^{0}	1.06×10^{0}	1.05×10^{0}
8.0×10^{0}	5.64×10^{0}	3.56×10^{0}	2.31×10^{0}	1.85×10^{0}	1.68×10^{0}	1.65×10^{0}	1.63×10^{0}	1.63×10^{0}	1.63×10^{0}
1.4×10^{1}	5.65×10^{0}	3.63×10^{0}	2.55×10^{0}	2.23×10^{0}	2.15×10^{0}	2.14×10^{0}	2.14×10^{0}	2.14×10^{0}	2.14×10^{0}
2.4×10^{1}	5.67×10^{0}	3.74×10^{0}	2.86×10^{0}	2.68×10^{0}	2.65×10^{0}	2.65×10^{0}	2.64×10^{0}	2.64×10^{0}	2.64×10^{0}
4.0×10^{1}	5.70×10^{0}	3.90×10^{0}	3.24×10^{0}	3.15×10^{0}	3.14×10^{0}	3.14×10^{0}	3.14×10^{0}	3.14×10^{0}	3.14×10^{0}
8.0×10^{1}	5.76×10^{0}	4.22×10^{0}	3.85×10^{0}	3.82×10^{0}	3.82×10^{0}	3.82×10^{0}	3.82×10^{0}	3.82×10^{0}	3.82×10^{0}
1.4×10^{2}	5.85×10^{0}	4.58×10^{0}	4.38×10^{0}	4.37×10^{0}	4.37×10^{0}	4.37×10^{0}	4.37×10^{0}	4.37×10^{0}	4.37×10^{0}
2.4×10^{2}	5.99×10^{0}	5.00×10^{0}	4.91×10^{0}	4.91×10^{0}	4.91×10^{0}	4.91×10^{0}	4.91×10^{0}	4.91×10^{0}	4.91×10^{0}
4.0×10^{2}	6.16×10^{0}	5.46×10^{0}	5.42×10^{0}	5.42×10^{0}	5.42×10^{0}	5.42×10^{0}	5.42×10^{0}	5.42×10^{0}	5.42×10^{0}
8.0×10^{2}	6.47×10^{0}	6.11×10^{0}	6.11×10^{0}	6.11×10^{0}	6.11×10^{0}	6.11×10^{0}	6.11×10^{0}	6.11×10^{0}	6.11×10^{0}
1.4×10^{3}	6.67×10^{0}	6.67×10^{0}	6.67×10^{0}	6.67×10^{0}	6.67×10^{0}	6.67×10^{0}	6.67×10^{0}	6.67×10^{0}	6.67×10^{0}
2.4×10^{3}	7.21×10^{0}	7.21×10^{0}	7.21×10^{0}	7.21×10^{0}	7.21×10^{0}	7.21×10^{0}	7.21×10^{0}	7.21×10^{0}	7.21×10^{0}
4.0×10^{3}	7.72×10^{0}	7.72×10^{0}	7.72×10^{0}	7.72×10^{0}	7.72×10^{0}	7.72×10^{0}	7.72×10^{0}	7.72×10^{0}	7.72×10^{0}
8.0×10^{3}	8.41×10^{0}	8.41×10^{0}	8.41×10^{0}	8.41×10^{0}	8.41×10^{0}	8.41×10^{0}	8.41×10^{0}	8.41×10^{0}	8.41×10^{0}
1.4×10^{4}	8.97×10^{0}	8.97×10^{0}	8.97×10^{0}	8.97×10^{0}	8.97×10^{0}	8.97×10^{0}	8.97×10^{0}	8.97×10^{0}	8.97×10^{0}
2.4×10^{4}	9.51×10^{0}	9.51×10^{0}	9.51×10^{0}	9.51×10^{0}	9.51×10^{0}	9.51×10^{0}	9.51×10^{0}	9.51×10^{0}	9.51×10^{0}
4.0×10^{4}	1.94×10^{1}	1.94×10^{1}	1.94×10^{1}	1.94×10^{1}	1.94×10^{1}	1.94×10^{1}	1.94×10^{1}	1.94×10^{1}	1.94×10^{1}

Data Plots Interpretation

When the field data curves of drawdown versus time are prepared we can match them with the theoretical curves of the main types of aquifer to know the type of aquifer (see **Figures 6.22** and **6.23**).
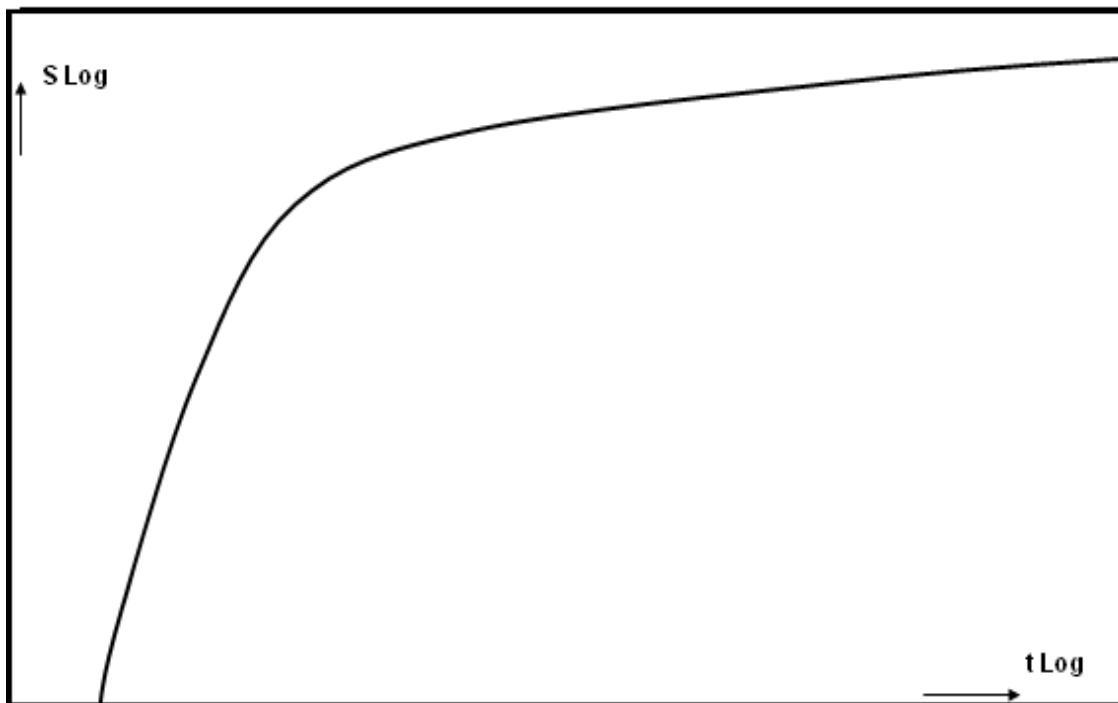

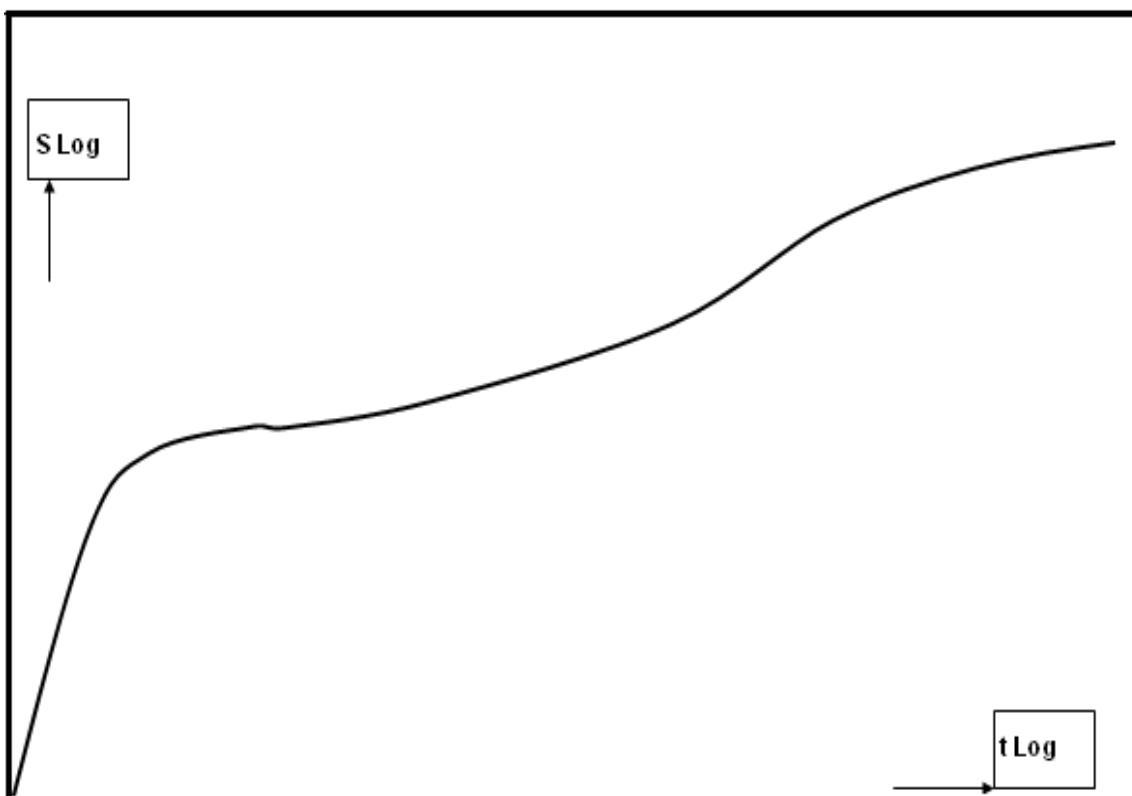**Figure 6.22**   Theoretical curve for confined aquifer

Figure 6.23   Theoretical curve for unconfined aquifer

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6.6   Recovery Test

At the end of a pumping test, when pumping is stopped, water levels in pumping and observation wells will begin to rise. This is referred to as the recovery of groundwater levels, while the measurements of drawdown below the original static water level (prior to pumping) during the recovery period are known as **residual drawdowns** ($s^{'}$). A schematic diagram of change in water level with time during and after pumping is shown in **Figure 6.24**.
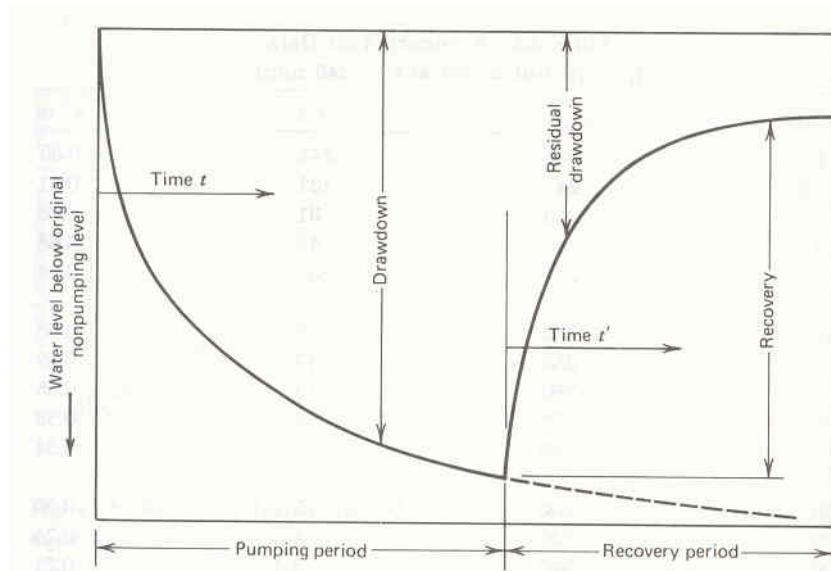
Figure 6.24   Drawdown and recovery curves in an observation well near a pumping well.

Note that: the resulting drawdown at any time after pumping stop is algebraic sum of drawdowns from well and buildup (negative drawdown) from imaginary recharge well (see **Figure 8.25**).
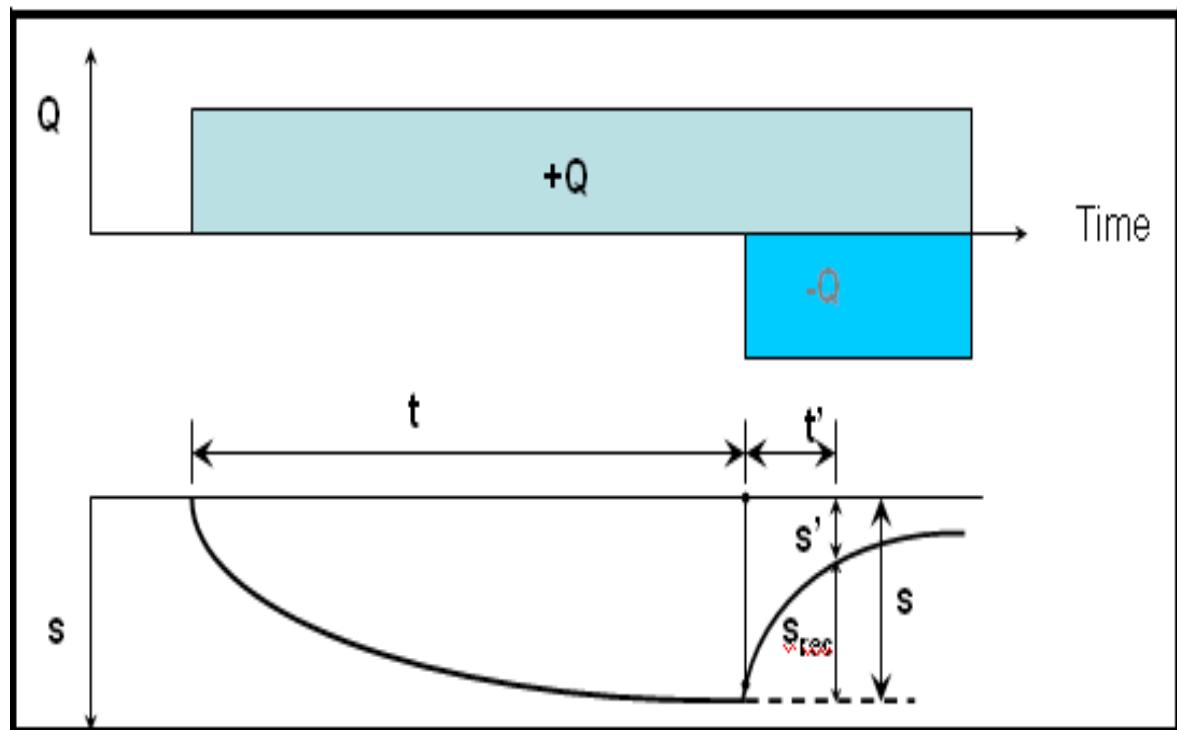
Figure 6.25 illustration of recovery test

It is a good practice to measure residential drawdowns because analysis of the data enable transmissivity to be calculated, thereby providing an independent check on pumping test results. Also, costs are nominal in relation to the conduct of a pumping test. Furthermore, the rate of change Q to the well during recovery is assumed constant and equal to the mean pumping rate, whereas pumping rates often vary and are difficult to control accurately in the field.

If a well is pumped for a known period of time and then shut down, the drawdown thereafter will be identically the same as if the discharge had been continued and a hypothetical recharge well with the same flow were superposed on the discharging well at the instant the discharge is shut down. From the principle of superposition and Theis showed that the residential drawdown s' can be given as:

$$s' = \frac{+Q}{4\pi T} W(u) + \frac{-Q}{4\pi T} W(u') \qquad \textbf{(6.45)}$$

$$s' = \frac{Q}{4\pi T} \left[W(u) - W(u') \right] \qquad \textbf{(6.46)}$$

Where

$$u = \frac{r^2 S}{4Tt}$$

$$u' = \frac{r^2 S}{4Tt'}$$

t is time since pumping started.

t' is time since pumping stopped.

For r small and t' large and u is less than 0.01, Theis equation can be simplified by Jacob and Cooper equation as:

$$s' = \frac{Q}{4\pi T} \left[\ln\left(\frac{2.25\,T\,t}{r^2 S} \right) - \ln\left(\frac{2.25\,T\,t'}{r^2 S} \right) \right] \qquad \textbf{(6.47)}$$

or

$$s' = \frac{Q}{4\pi T} \ln\left(\frac{t}{t'} \right) \qquad \textbf{(6.48)}$$

or

$$s' = \frac{2.303\,Q}{4\pi T} \log\left(\frac{t}{t'} \right) \qquad \textbf{(6.49)}$$

Thus, a plot of residual drawdown s' versus the logarithm of t/t' forms a straight line. The slope of the line equals (see **Figure 6.26**):

$$\Delta s' = \frac{2.303\,Q}{4\pi T} \qquad \textbf{(6.50)}$$

So that for $\Delta s'$, the residual drawdown per log cycle of t/t', the transmissivity becomes:

$$T = \frac{2.303\,Q}{4\pi \Delta s'} \qquad \textbf{(6.51)}$$

Note: No comparable value of S can be determined by this recovery test.
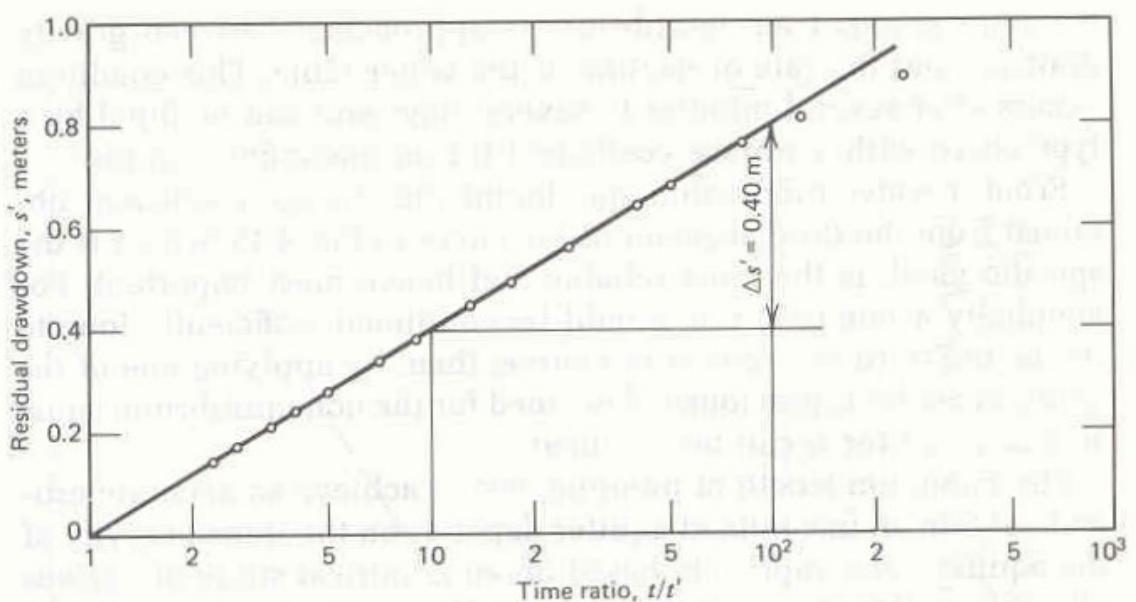
Figure 6.26   Recovery test method for solution of the non-equilibrium equation

6.7 Boundary Problems

Boundary conditions can be solved by 1) Superposition "for multiple well systems" and 2) "Image wells".

6.7.1 Multiple Well Systems

Where the cones of depression of two nearby pumping well over lap, one well is said to interfere with another because of the increased drawdown and pumping lift created. For a group of wells forming a well field, the drawdown can be determined at any point if the well discharges are known, or vice versa. From the principle of **superposition**, the drawdown at any point in the area of influence caused by the discharge of several wells is equal to the sum of the drawdowns caused by each well individually. Thus,

$$S_T = s_a + s_b + s_c + \ldots + s_n \tag{6.52}$$

where,

S_T is the total drawdown at a given point,

$s_a + s_b + s_c + \ldots + s_n$ are the drawdowns at the point caused by the discharge of wells a, b, c, …, n respectively.

The summation of drawdowns may be illustrated in a sample way by the well line of **Figure 6.27**; the individual and composite drawdown curves are given for $Q_1 = Q_2 = Q_3$. clearly, the number of wells and the geometry of the well field are important in determining drawdowns. Solutions of well discharge for equilibrium or non-equilibrium equation. Equations of well discharge for particular well patterns have been developed.

In general, wells in a well field designed for water supply should be spaced as far apart as possible so their areas of influence will produce a minimum of interference with each other. On the other hand economic factors such as cost of land or pipelines may lead to a least-cost well layout that includes some interference. For drainage wells designed to control water table elevations, it may be desirable to space wells so that interference increases the drainage effect.
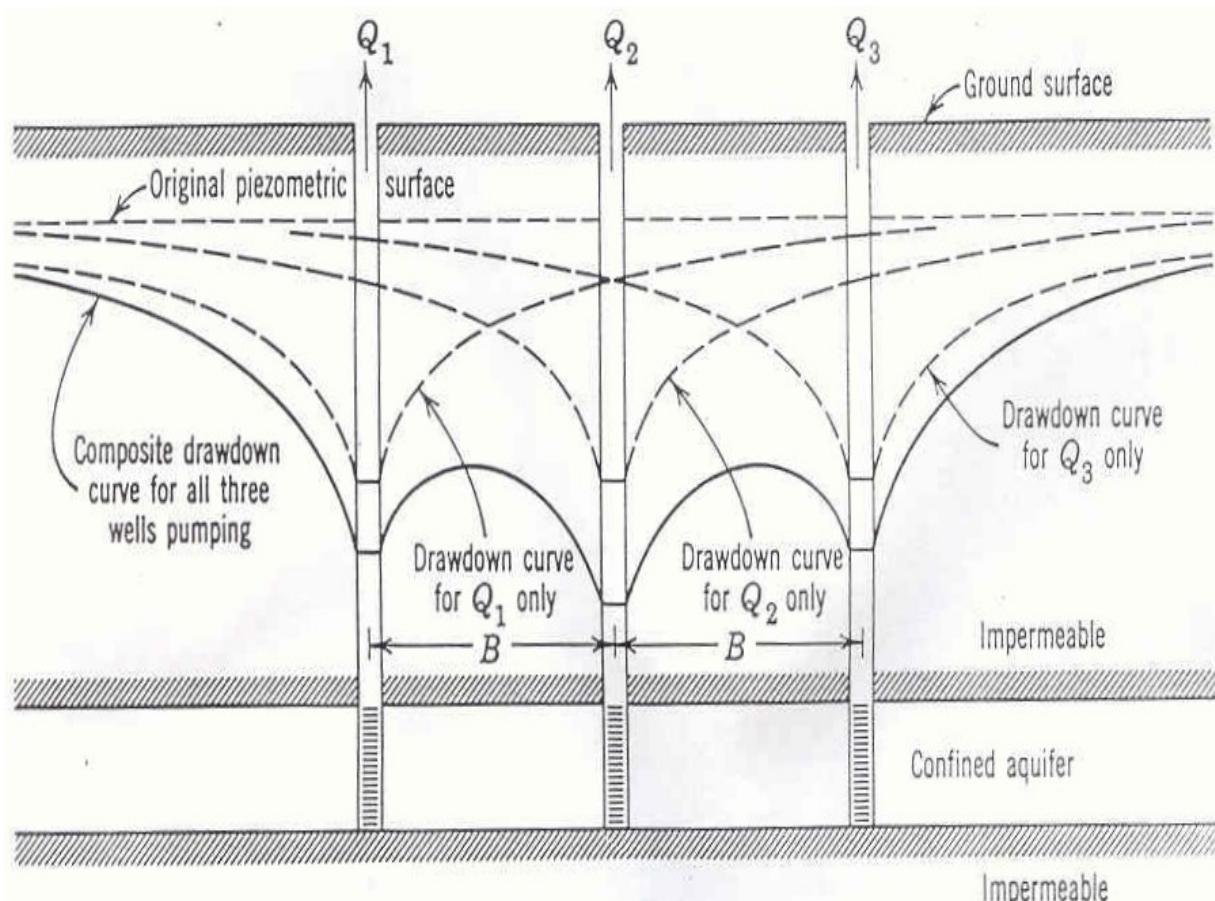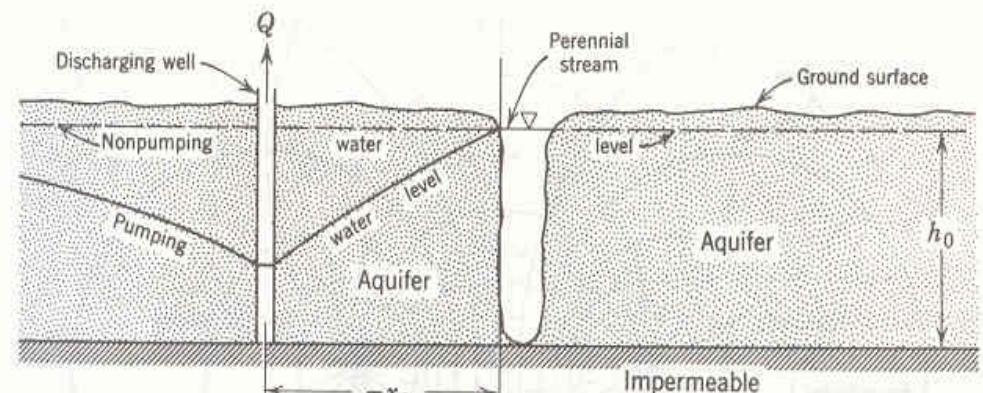
Figure 6.27   Individual and composite drawdown curves for three wells in a line.

6.7.2 Well Flow Near Aquifer Boundaries
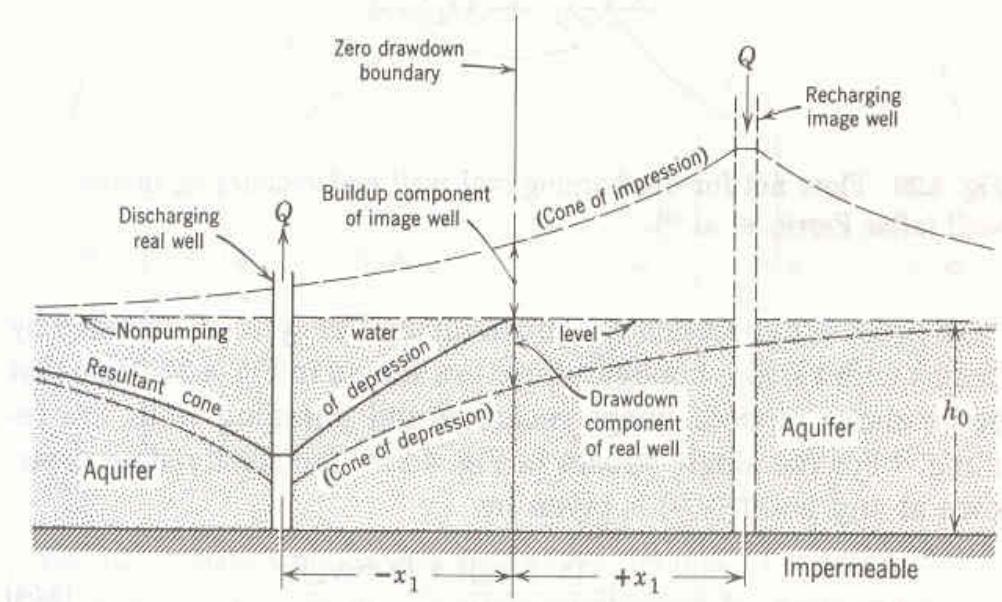
Where a well is pumped near an aquifer boundary, the assumption that the aquifer is of infinite areal extent no longer holds. Analysis of this of this situation involves the principle of **superposition** by which the draw down of two or more wells is the sum of the drawdowns of each individual well. By introducing imaginary (or image) wells, an aquifer of finite extent can be transformed into an infinite aquifer so that the solution methods previously described can be applied.

Well Flow near a Stream

An example of the usefulness of the method of images is the situation of a well near a perennial stream. It is desired to obtain the head at any point under the influence of pumping at a constant rate Q and to determine what faction of the pumping is derived from the stream. Sectional views are shown in **Figure 6.28** of the real system and an equivalent imaginary system. Note in Figure **6.28b** that the **imaginary recharge well** (a recharge well is a well through which water is added to an aquifer; hence, it is the reverse of a pumping well) has been placed directly opposite and at the same distance from the stream as the real well. This image well operates simultaneously and at the same rate as the real well so that the buildup (increase of head around a recharge well) and drawdown of head along the line of the stream exactly cancel. This furnishes a constant head along the stream, which is equivalent to the constant elevation of the stream forming the aquifer boundary. The resultant asymmetrical drawdown of the real well is given at any point by the algebraic sum of the drawdown of the real well and the buildup of the recharge well, as if these wells were located in an infinite aquifer.
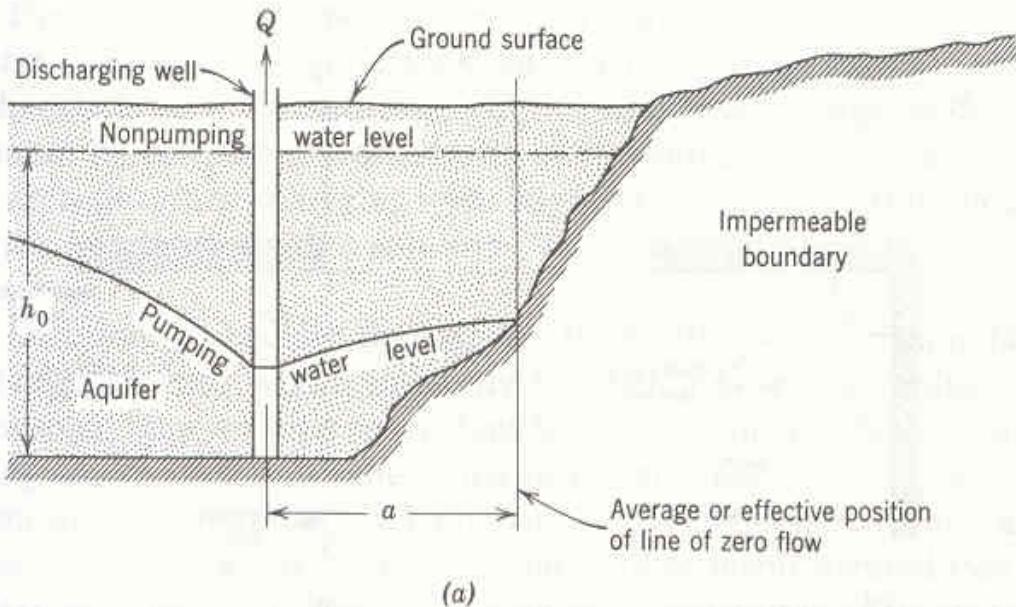
Figure 6.28 Sectional views. (a) Discharging well near a perennial stream. (b) Equivalent hydraulic system in an aquifer of infinite areal extent.

Well Flow near Other boundaries

In addition to the previous example, the method of images can be applied to a large number of groundwater boundary problems. As before, actual boundaries are replaced by an equivalent hydraulic system, which includes imaginary wells and permits solutions to be obtained from equations applicable only to extensive aquifers. Several boundary conditions to suggest the adaptability of the method are shown from **Figure 6.29 to Figure 6.37**. **Figure 6.29** shows a well pumping near an impermeable boundary. An image discharging well us placed opposite the pumping well with the same rate of discharge and at an equal distance from the boundary; therefore, along the boundary the wells offset one another, causing no flow across the boundary the wells offset one another, causing no flow across the boundary, which is the desired condition.
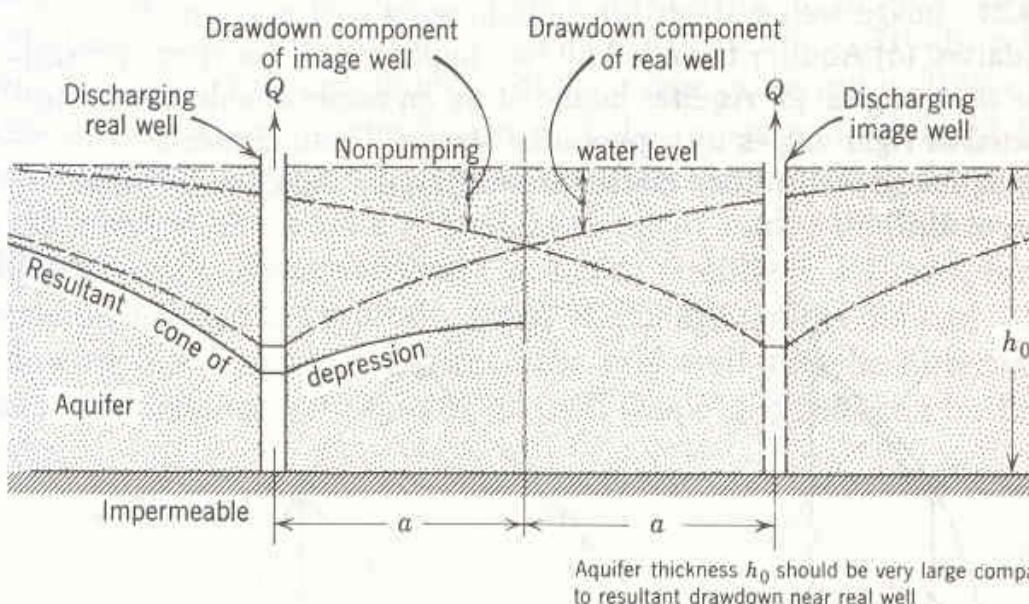
Figure 6.29 Sectional view (a) Discharging well near an impermeable boundary. (b) Equivalent hydraulic system in an aquifer of infinite areal extent.

Figure 6.30 shows a theoretical straight line plot of drawdown as a function of time on semi-log paper. The effect of recharge boundary is to retard the rate of drawdown. Change in drawdown can become zero if the well comes to be supplied entirely with recharges water. The effect of a barrier to flow in some region of the aquifer is to accelerate the drawdown rate. The water level declines faster than the theoretical straight line.

Figure 6.31a shows a discharging well in aquifer bounded on two sides by impermeable barriers. The image discharge wells I_1 and I_2 provide the required flow but, in addition, a third image well I_3 is necessary to balance drawdowns along the extensions of the boundaries. The resulting system of four discharging wells in an extensive aquifer represents hydraulically the flow system for the physical boundary conditions. **Figure 6.31b** presents the situation of a well near an impermeable boundary and a perennial stream. The image wells required follow from the previous illustration. As practice, see **Figures 6.32, 6.33, 6.34, 6.35** and **6.36** and try to figure them out.

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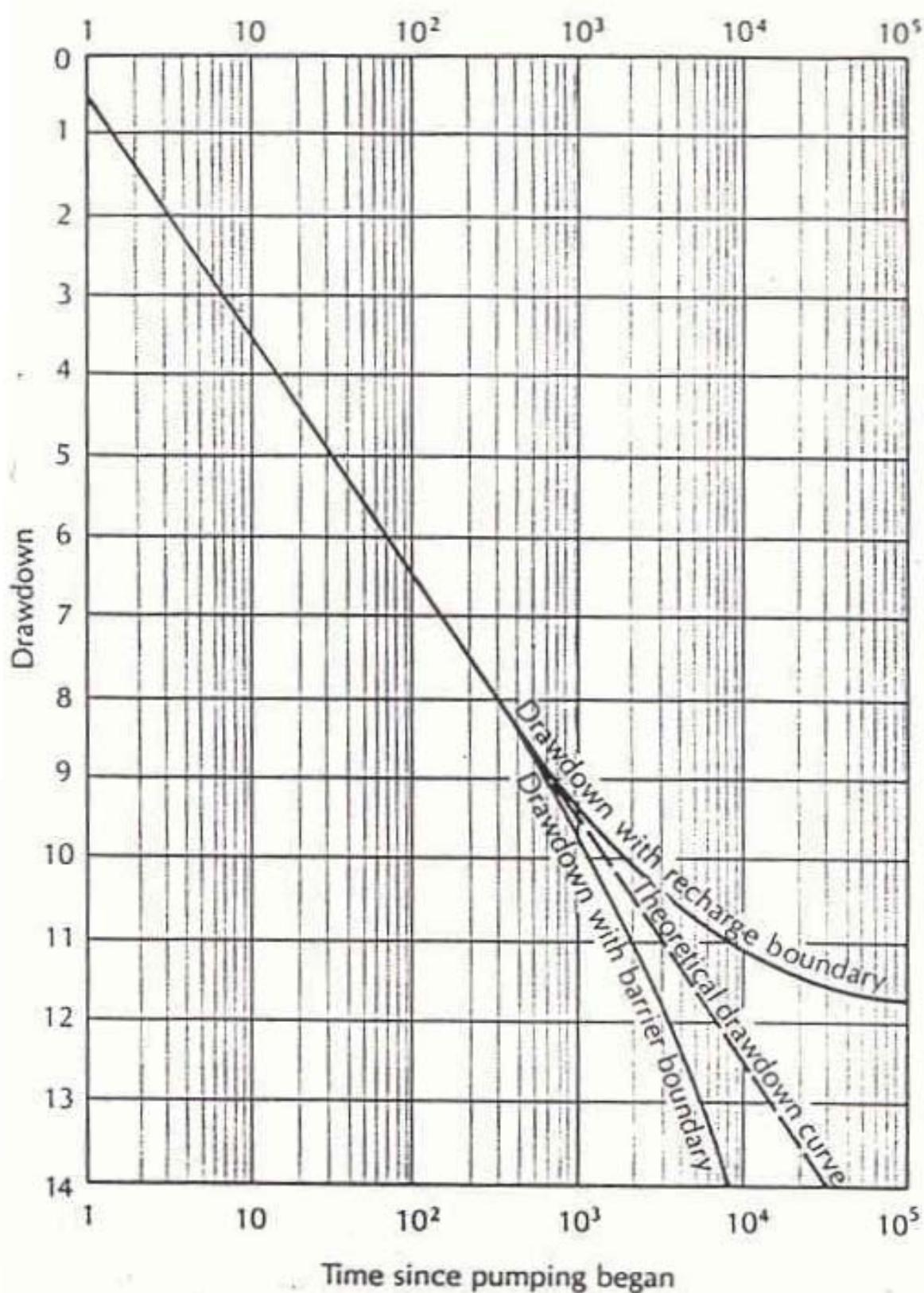
Figure 6.30   Impact or recharge and barrier boundaries on semi-logarithmic drawdown-time curve

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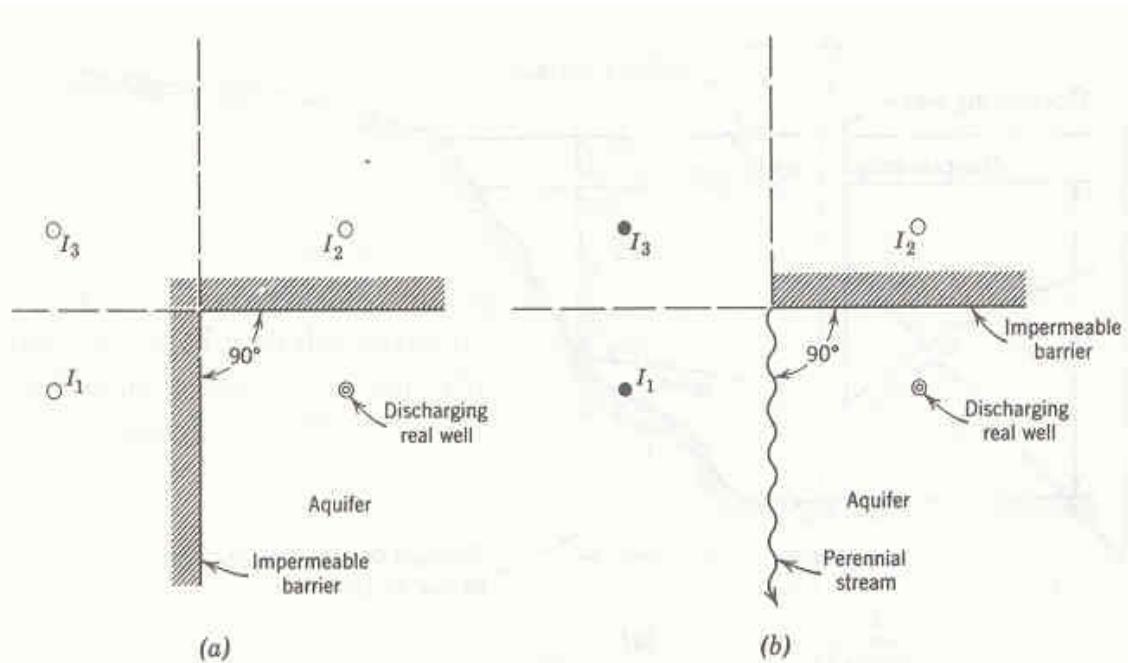
Figure 6.31 Image well systems for a discharging well near aquifer boundaries. **(a)** aquifer bounded by two impermeable barriers intersecting at a right angles.**(b)** Aquifer bounded by an impermeable barrier intersected at right angles by a perennial stream. Open circles are discharging image wells; filled circles are recharging image wells.
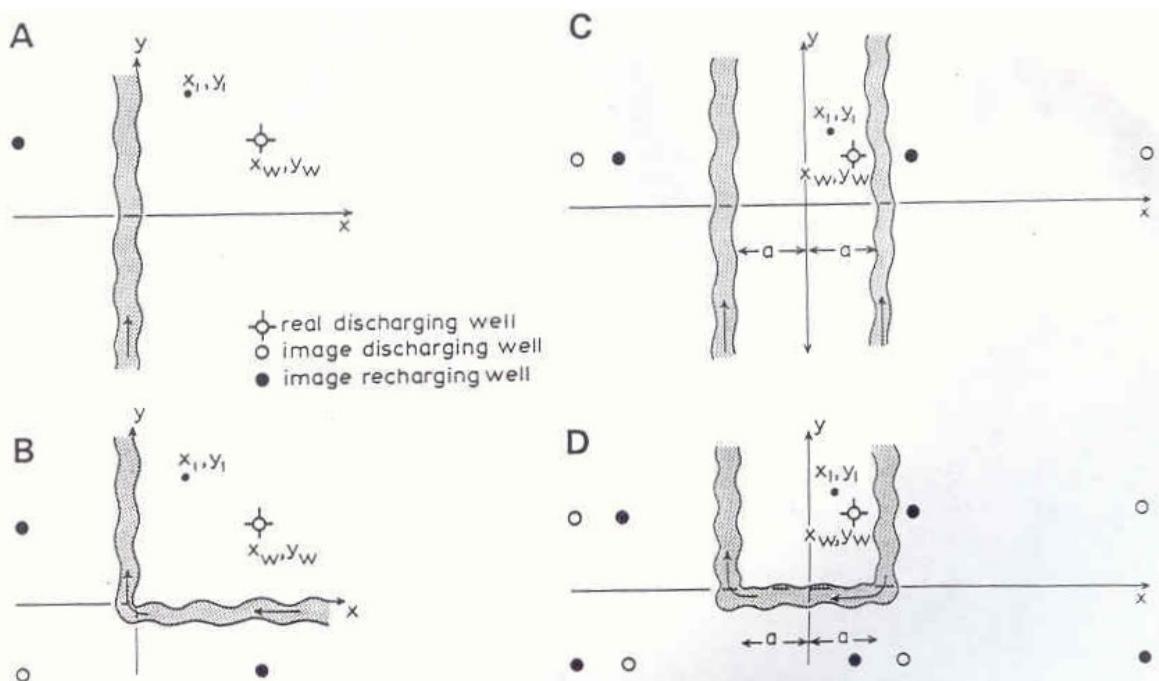
Figure 6.32 Image well systems for bounded aquifers.
       (A)   One straight recharge boundary
       (B)   Two straight recharge boundaries at right angles
       (C)   Two straight parallel recharge boundaries
       (D)   U-shape recharge boundary
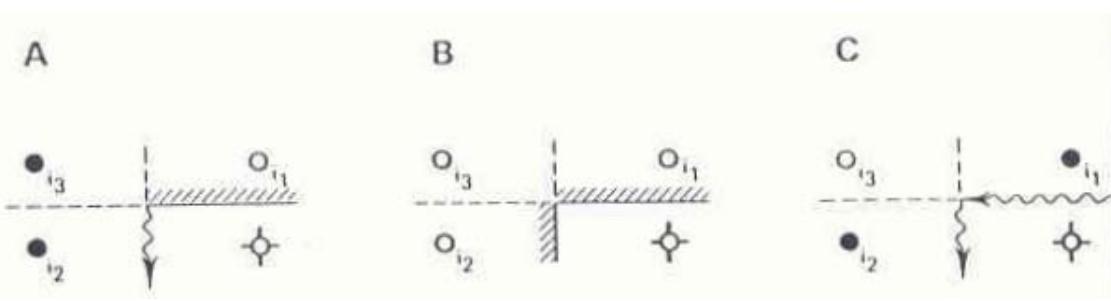
Figure 6.33   Two straight boundaries intersecting at right angles
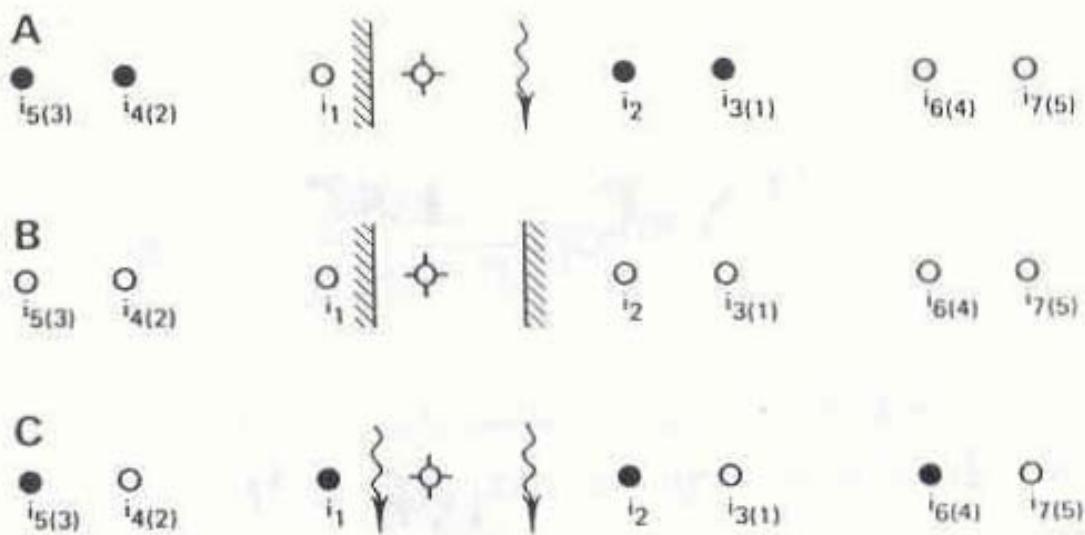
Figure 6.34   Two straight boundaries parallel boundaries
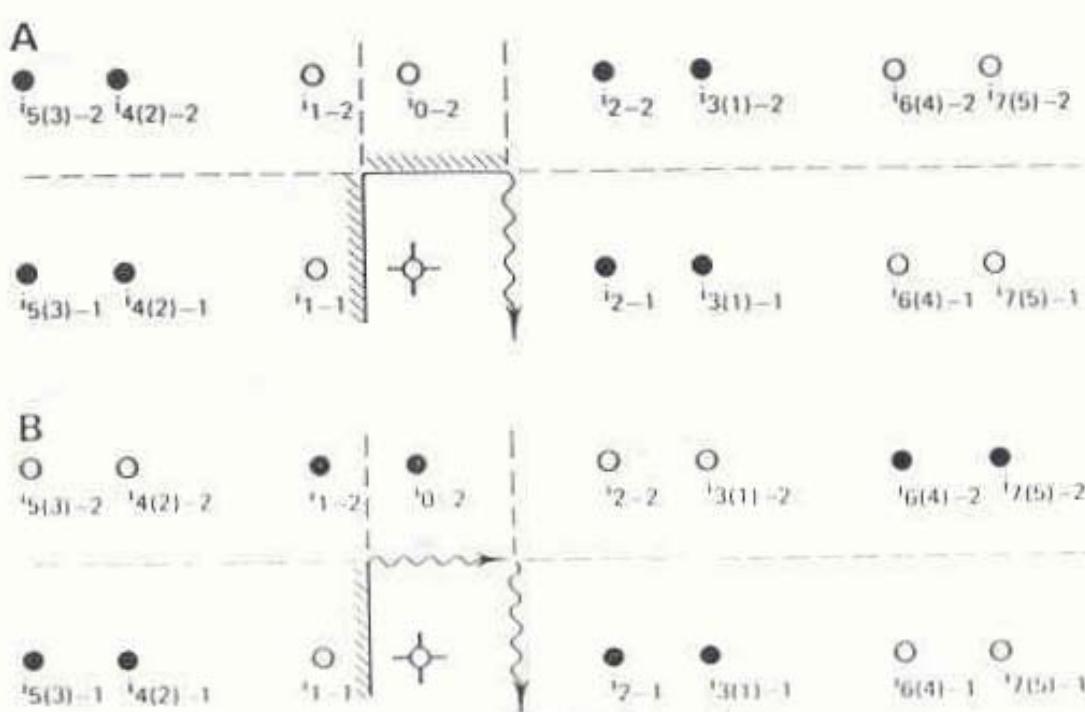
Figure 6.35   Two straight parallel boundaries intersected at right angles by a third boundary
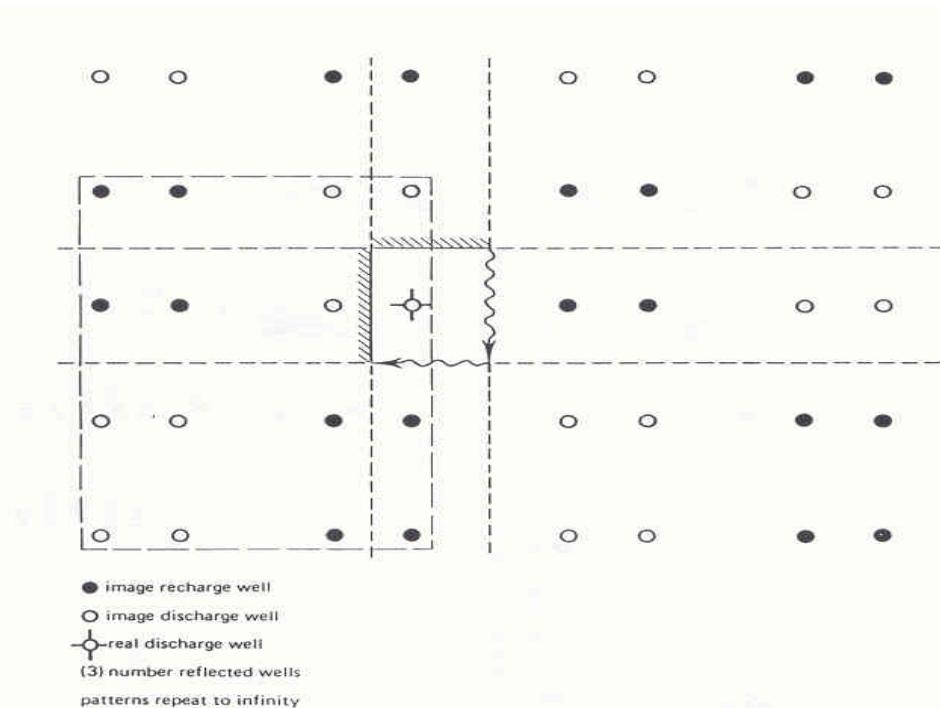
Figure 6.36 Four straight boundaries, i.e. two pairs of straight parallel boundaries intersecting at right angles

For a wedge-shaped aquifer, such as a valley bounded by two converting impermeable barriers, the drawdown at any location within the aquifer can be calculated by the same method of images. Consider the aquifer formed by two barriers intersecting at an angle of 45 degrees shown in **Figure 6.37**. Seven image pumping wells plus the single real well form a circle with its center the wedge apex; the radius equals the distance from the apex to the real pumping well. The drawdown at any point between the two barriers can then be calculated by summing the individual drawdowns. In general, it can be shown that the number of image wells n required for a wedge angle θ is given by

$$n = \frac{360^o}{\theta} - 1 \qquad \qquad \textbf{(6.53)}$$

where, θ is an aliquot part of 360 degrees.
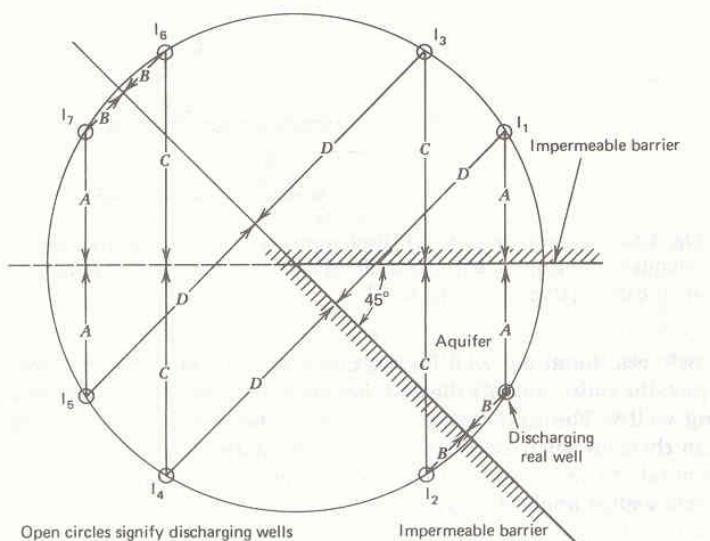
Figure 6.37 Image well system for a discharge well in an aquifer bounded by two impermeable barriers intersecting at an angle of 45 degrees

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6.8   Step Drawdown Test

6.8.1   What is a Step-Drawdown Test?

1   The borehole is pumped at a number of incremental rates, gradually increasing discharge, and drawdown is measured during each of these **steps** of pumping (see **Figure 6.38**).

2   It is usual to measure until drawdown begins to "stabilize" at each rate before proceeding to the next step (though in practice, none but a driller even thinks the level is stable- the driller reckons it's stable if it's dropping at less than a meter between each measurement!)

3   Step drawdown test developed to assess the Well Performance (Well losses due turbulent flow).

4   At least 5 pumping steps are needed, each step lasting from 1 to 2 hours.

5   Step drawdown test is used to determine the Optimum Pumping Rate.

6   Step drawdown test can be used to determine T and S from each step.
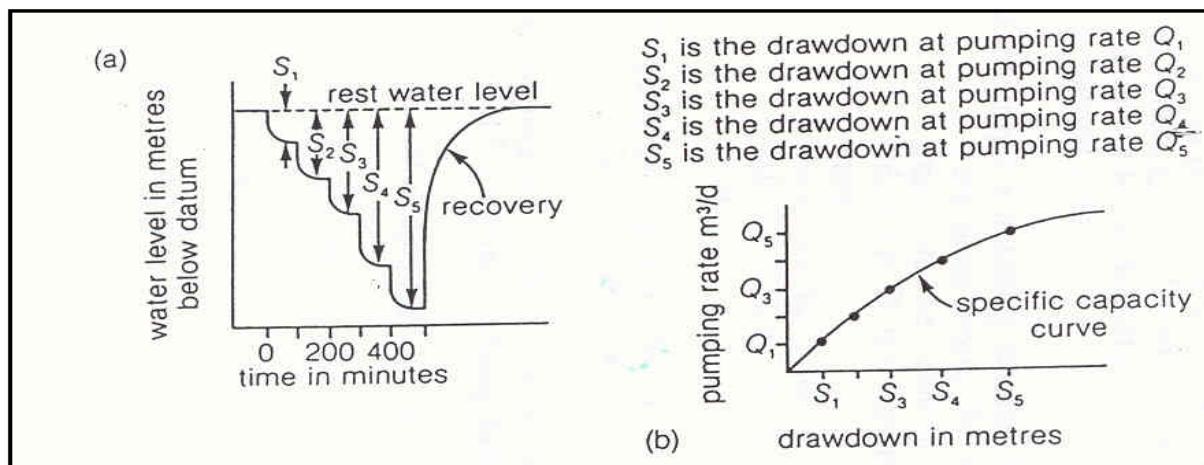
Figure 6.38   A step-drawdown test should be carried out at roughly equal pumping rates, with the borehole water-level monitored throughout. The drawdown is calculated by subtracting the stable pumping water level achieved at the end of each pumping rate, from the rest water level as shown in (a). A specific capacity curve (b) is constructed by plotting the pumping rate against drawdown. At higher rates of pumping, the amount of drawdown increases, indicating that the maximum yield of the well is being approached.

Specific Capacity (S_c)

The specific capacity S_c is the ratio of discharging (Q) to steady drawdown (s_w).

$$S_c = \frac{Q}{s_w}$$
(6.54)

The specific capacity can be calculated for each step, and (in theory) it should be roughly constant until a pumping rate beyond that sustainable by the borehole is attempted in a step.

The specific capacity is a valuable piece of information, as it can also be used as a means of estimating transmissivity (the two parameters share the dimensions of L^2/T)

However, it is important to realize that S_c is often a crude value, for "steady drawdown" is rarely attained in practice.

Using the non-equilibrium equation:

$$s_w = \frac{2.3Q}{4\pi T}\log\frac{2.25Tt}{r_w^2 S} + CQ^n \qquad (6.55)$$

So, the specific capacity:

$$\frac{Q}{s_w} = \frac{1}{\dfrac{2.3Q}{4\pi T}\log\dfrac{2.25Tt}{r_w^2 S} + CQ^{n-1}} \qquad (6.56)$$

This indicates that the specific capacity decreases with Q and t; the well data plotted in **Figure 6.39** demonstrate this effect. For a given discharge a well is often assumed to have a constant specific capacity. Although this is not strictly correct, it can be seen that the change with time is minor.

Any significant decline in the specific capacity of a well can be attributed either to a reduction in transmissivity due to a lowering of the groundwater level in an unconfined aquifer or to an increase in well loss associated with clogging or deterioration of the well screen.
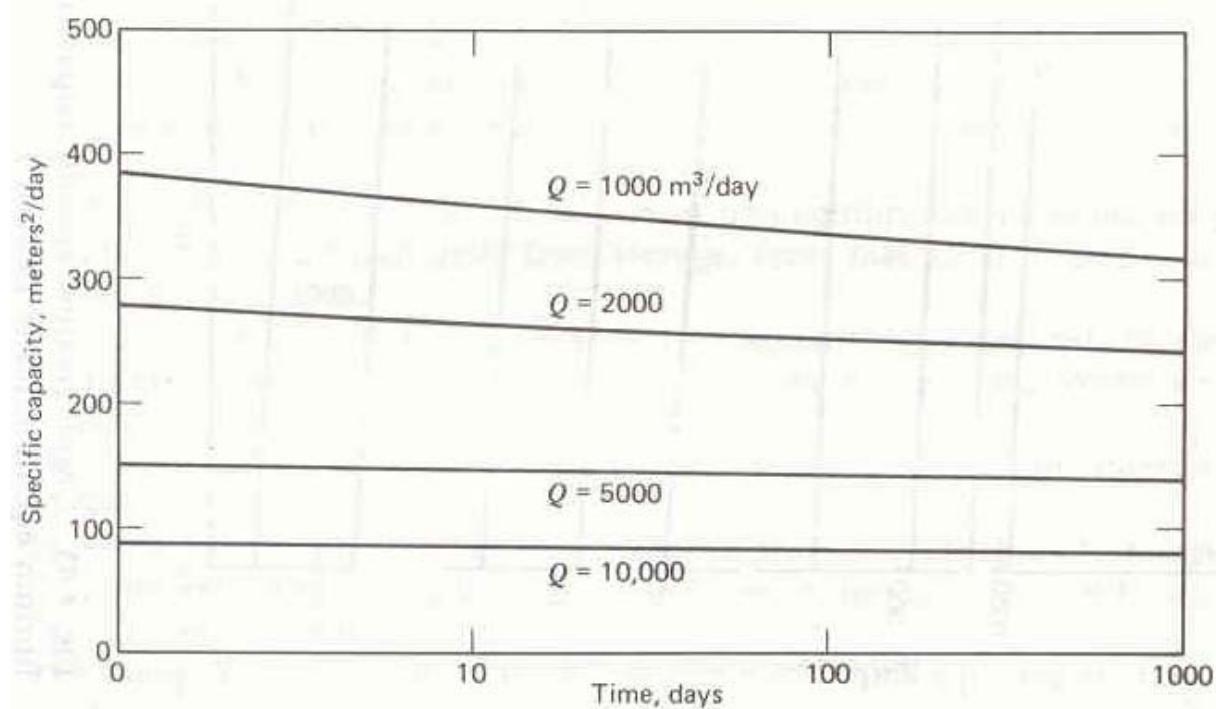
Figure 6.39   Variation in specific capacity of a pumping well with discharge and time

If a pumping well is assumed to be 100 percent efficient ($CQ^n = 0$), then the specific capacity from **equation 6.56** can be presented in the graphic form of **Figure 6.40**. Here specific capacity at the end of one day of pumping is plotted as a function of S, T, and a well diameter of 30 cm. This graph provides a convenient means for estimating T from existing pumping wells; any error in S has a small effect on T.
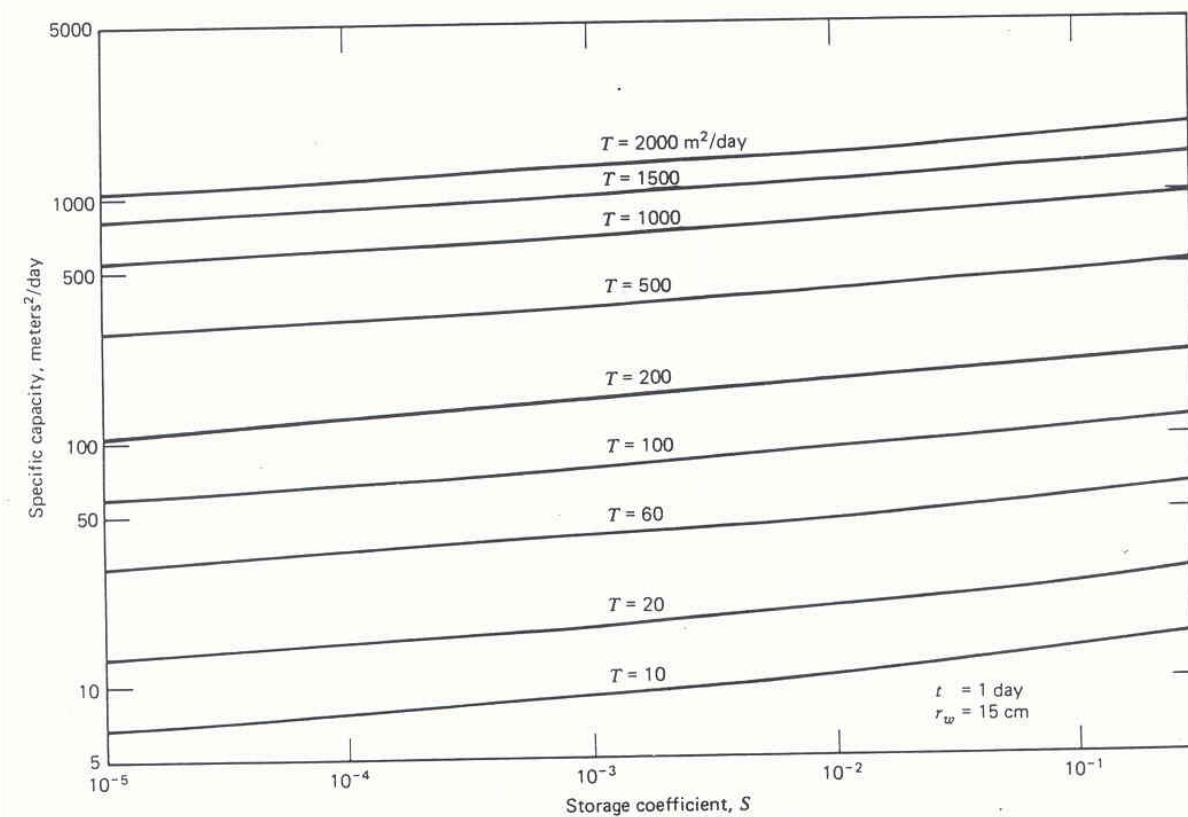
Figure 6.40 Graph relating specific capacity to transmissivity and storage coefficient from the non-equilibrium equation.

Use of Specific Capacity (S_c) Values

1   For Design Purposes

It is a simple matter to calculate drawdown associated with a particular design discharge using the specific capacity.

Example:

Suppose that from a step test we have determined the value of $S_c = 320$ m³/d/m of drawdown.

And the static water level (SWL) in the borehole lies at 5 m below ground level (m bgl), and we want at least 2 m of water in the hole above the pump during operation for safety reasons (you should never pump dry!). if the client insists on a yield of 2000 m³/d. How much rising main will need to purchase

Solution:

From **equation 6.54** $s_w = \dfrac{Q}{S_c} = \dfrac{2000}{S_c} = \dfrac{2000}{320} = 6.25\,m$ **.**

Therefore, steady pumping level will be at around 5 + 6.25 = 11.25 m below ground level. Allowing the 2 m extra above pump, the total depth to the pump (and, the length of rising main required) = 11.25 + 2 = 13.25 m. If the pipes come in 2 m lengths, so we'll need 7 lengths.

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2   To Estimate Transmissivity

Before a designed pumping test, a step-drawdown test is normally carried out to find out the yield and the maximum drawdown. This is done for:

   A   To determine Transmissivity from specific capacity data,
   B   To determine aquifer and well losses,
   C   To determine if the aquifer is dewatering or can be developed (see **Figure 6.41**),

If the well losses term is neglected

$$T = \left(\frac{Q}{s_w}\right)\frac{2.3}{4\pi}\log\frac{2.25\,T\,t}{r^2\,S}$$

(6.55)

Note that (T) appears in the arithmetic and logarithmic portions of the equation.
The above equation is difficult to solve because:

   A   It is not linear with respect to (T),
   B   (T) is dependent on (S) which is normally unknown,
   C   Well losses are neglected which is impractical,

Empirical Models

$$T = a\left(\frac{Q}{s_w}\right)^b \quad , \quad a = 15.3 \quad and \quad b = 0.67$$

(6.56)

A linear Model

$$T = a\left(\frac{Q}{s_w}\right) \quad , \quad a \approx 0-2$$

(6.57)
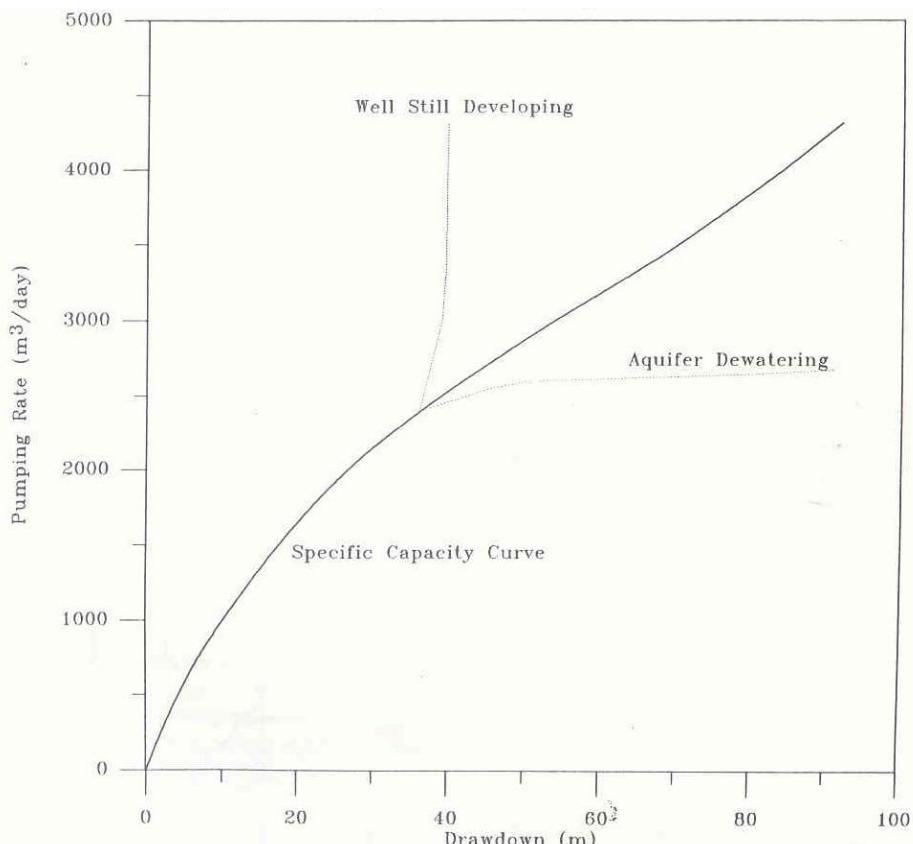
Figure 6.41   Specific capacity curve

Basic Operational Interpretation of Step-Drawdown Results

1   Plot Q (on the y-axes) against s_w (x-axes) to obtain a **specific capacity line** (see **Figure 6.38b**).

2   Inspect the plot and identify where (if at all) the line departs from a linear response, such that excessive increases in drawdown result from incremental increases in discharge. Mark the approximate point of departure from linearity.

3   Find the discharge value on the y-axis which corresponds to the departure point. This is taken to be the "maximum feasible yield" from the borehole. This limiting yield may reflect:

         (i)   the transmissivity of the aquifer and/or

         (ii)   the efficiency of the borehole as an engineered structure.

4   In designing your permanent pumping system for the borehole using this information, decide how best to achieve:

         (i)   required discharge

         (ii)   Lowest pumping head (to reduce costs associated with pumping from greater depths).

This is accomplished by simple calculations using the specific capacity value as shown before.

Well Loss and Well Efficiency

6.9.1 Well Loss

Well loss is the difference between the head in the aquifer immediately outside the well to the head inside the casing during pumping. Well losses usually divided into linear and non linear components (see **Figure 6.42**)

1   Linear components

         (i)   effects of head in mud-invaded zone

         (ii)   head loss in gravel pack (will be discussed in chapter 9)

         (iii)   head loss due to screen entry velocity (will be discussed in chapter 9)

2   Non-linear component: head loss due to turbulent flow in well casing.
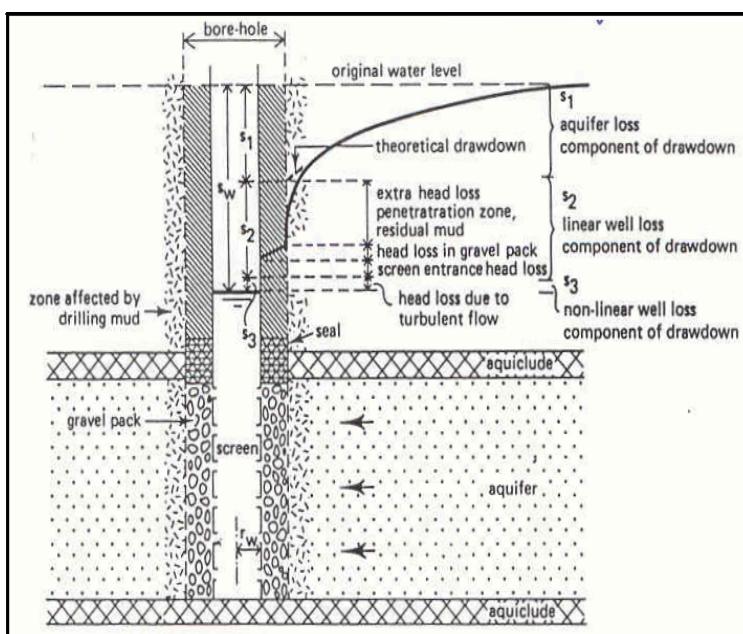
Figure 6.42   Variable head losses in a pumped well

It is assumed in some analysis that the non-linear well losses are negligible. However, according to Parsons:

(i)   Screen entrance losses are actually small in comparison with the turbulent upflow losses.

(ii)   Decrease in S_c will decrease in well diameter is only important if no gravel pack (or poor gravel pack less permeable than aquifer) is installed.

To summarize, the drawdown at a well includes not only that of the logarithmic drawdown curve at the well face, but also a well loss caused by flow through the well screen and flow inside of the well to the pump intake.

Because the well loss is associated with turbulent flow, it may be indicated as being proportional to an nth power of the discharge, as Q^n, where n is a constant greater than one. **Jacob** suggest that a value n=2 might be reasonably assumed.

Taking account of the well loss, the total draw town s_w at the well may be written for the steady-state confined case

$$s_w = \frac{Q}{2\pi T} \ln \frac{r_2}{r_1} + CQ^n \qquad \textbf{(6.58)}$$

where C is a constant governed by the radius, construction and condition of the well. For simply let

$$B = \frac{\ln (r_2/r_1)}{2\pi T} \qquad \textbf{(6.59)}$$

so that the total drawdown in a well can be represented by:

$$s_w = s_{aquifer\ losses} + s_{well\ losses} = BQ + CQ^2 \qquad \textbf{(6.60)}$$

Therefore, as shown in **Figure 6.43**, the total drawdown s_w consists of the formation loss BQ and the well loss CQ^2.
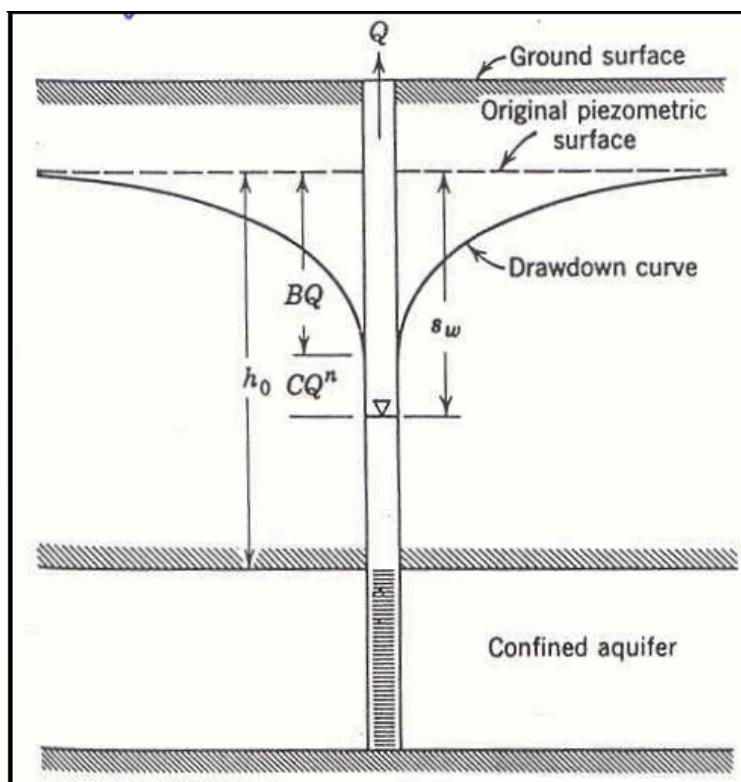
Figure 6.43   Relation of well loss CQ^n to draw-down for a well penetrating a confined aquifer

Consideration of **Equation 6.60** provides a useful insight to the relation between well discharge and well radius. From **equations 6.6** and **6.14** It can be seen that Q varies inversely with $(\ln \frac{r_2}{r_1})$, if all other variables are held constant. This is shows that the discharge varies only a small amount with well radius. For example, doubling a well radius increases the discharge only 10 percent. When the comparison is extended to include well loss, however, the effect is significant. Doubling the well radius doubles the intake area, reduces entrance velocities at almost half, and (if n=2) cuts the frictional loss to less than a third. For axial flow within the well, the area increases four times, reducing this loss an even greater extent.

It is apparent that the well loss can be a substantial fraction of total drawdown when pumping rates are large, as illustrated in **Figure 6.44**. With proper design and development of new wells (see Chapter Nine), well losses can be minimized. Clogging or deterioration of well screens can increase well losses in old wells. Based on field experience Walton suggested criteria for the well loss coefficient C in **equation 6.60**. These are presented in **Table 6.3** to aid in evaluating the condition of a well.
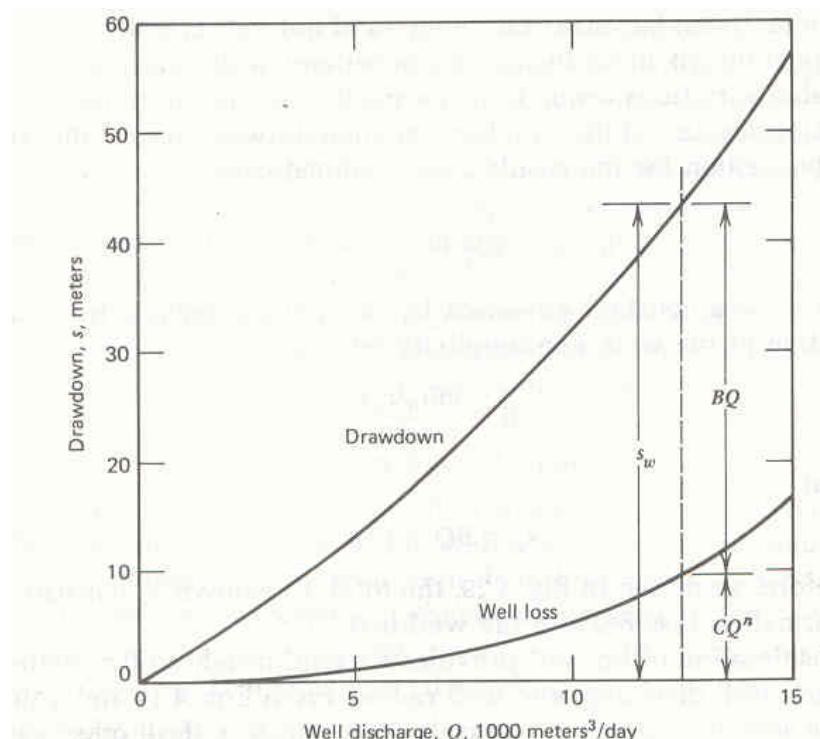
Figure 6.44   Variation of total drawdown s_w, aquifer loss BQ, and well loss CQ^n with well discharge.

6.9.2 Evaluation of Well Loss

To evaluate well loss a step-drawdown pumping test is required. This consists of pumping a well initially at a low rate until the drawdown within the well essentially stabilize. The discharge is then increased through a successive series of steps as shown by the time-drawdown data in **Figure 6.45a**. Incremental drawdowns Δs for each step are determined from approximately equal time intervals. The individual drawdown curves should be extrapolated with a slope proportional to the discharge in order to measure the incremental drawdowns.

From **equation 6.60** and by dividing the equation by Q yields:

$$\frac{s_w}{Q} = B + CQ \tag{6.61}$$

Therefore, by plotting s_w/Q versus Q (see **Figure 6.45b**) and fitting a straight line through the points, the well loss coefficient C is given by the slope of the line and the formation loss coefficient B by the intercept $Q=0$.
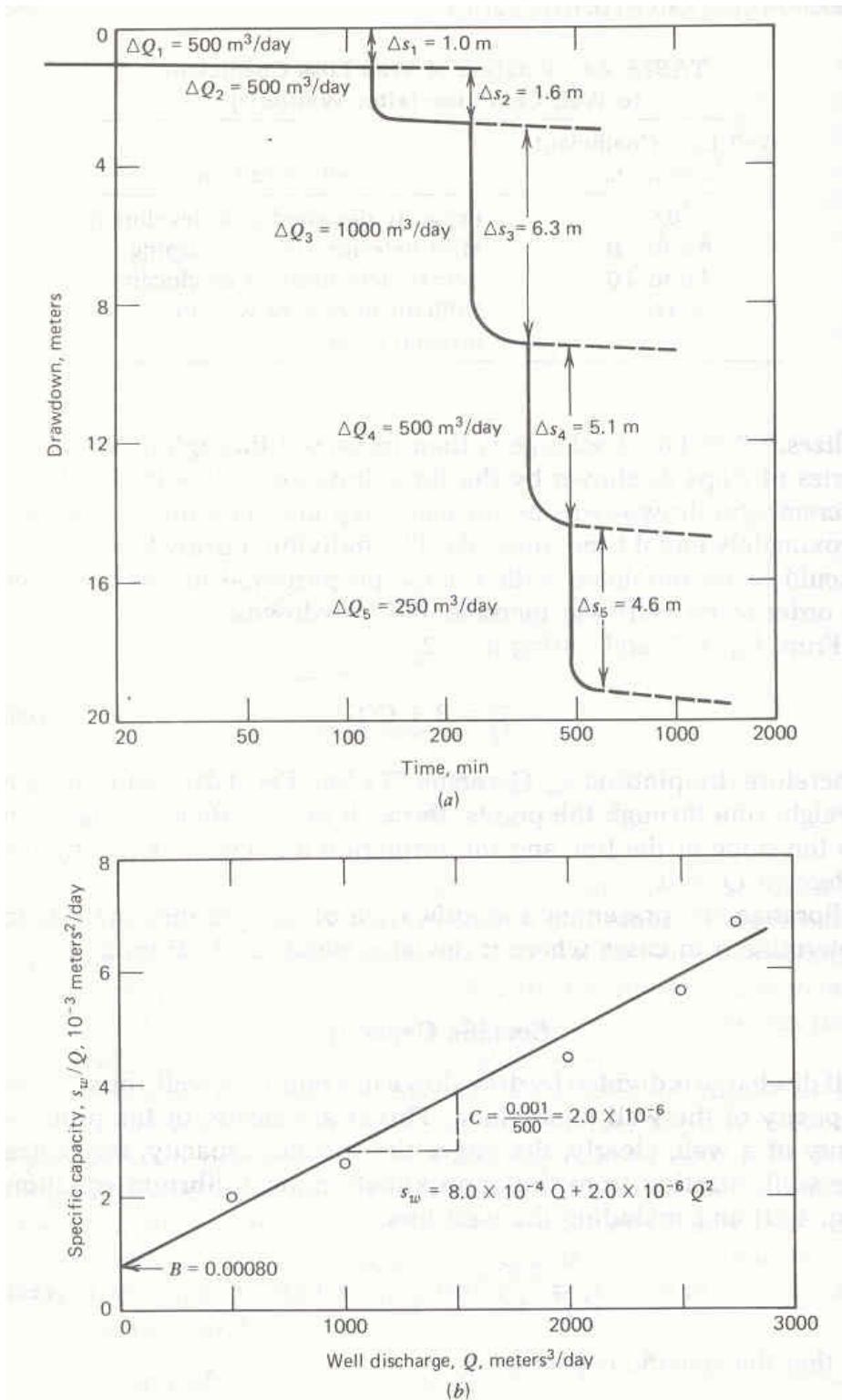

Figure 6.45 Step-drawdown pumping test analyses to evaluate well loss. (a) Time-drawdown data from step-drawdown pumping test. (b) Determination of B and C from graph s_w/Q versus Q.

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Table 6.3 Relation of well loss coefficient to well condition (after Walton)

Well Loss Coefficient (C) min²/m⁵	Well Condition
< 0.5	Properly designed and developed
0.5 to 1.0	Mild deterioration or clogging
1.0 to 4.0	Severe deterioration or clogging
> 4.0	Difficult to restore well to original capacity

Another parameter can computed from step-drawdown test:

$$L_p = \left(\frac{BQ}{\left(BQ + CQ^2\right)} \right) x100$$

(6.62)

Where,

L_p is the ratio of laminar head losses to the total head losses (this parameter can be considered also as well efficiency).

6.9.3 Well Efficiency

Well Efficiency is the ration between theoretical drawdown and the actual drawdown measured in the well expressed as (see **Figure 6.42**):

(i)   A well efficiency of 70% or more is usually acceptable.
(ii)   If a newly developed well has less than 65% efficiency, it should not be accepted.

A qualitative "Rule of Thumb" to recognize an inefficient well is: If the pump is shut off after 1 hour of pumping and 90% or more of the drawdown is recovered after 5 minutes, it can be concluded that the well is unacceptably inefficient.

Calculation of Well Efficiency

Figure 6.40 yields a theoretical specific capacity (Q/BQ) for known values of S and T in an aquifer. This computed specific capacity, when compared with one measured in the field (Q/s_w), defines the approximate efficiency of a well. Thus, for a specific duration of pumping, the well efficiency E_w is given as a percentage by:

$$E_w = 100 \frac{(Q/s_w)}{(Q/BQ)}$$

$$E_w = 100 \frac{BQ}{s_w}$$

(6.63)

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6.10 Optimum Pumping Rate

Determining the Optimum Pumping Rate is based mainly on the well losses and well efficiency, the procedure consists of the following steps:

1   For up to ten different Q, find s_w based on **equation 6.60**.

2   For the same pumping rates, find theoretical drawdown through the following equation:

$$s = \frac{Q}{2\pi T} \ln\left(\frac{R}{r_w}\right)$$ **(6.64)**

3   Calculate well efficiency for all pumping rates.

4   Demonstrate graph between efficiencies and pumping rates, and choose the Q value that correspond more than 65% efficiency or more (see **Figure 6.46**).
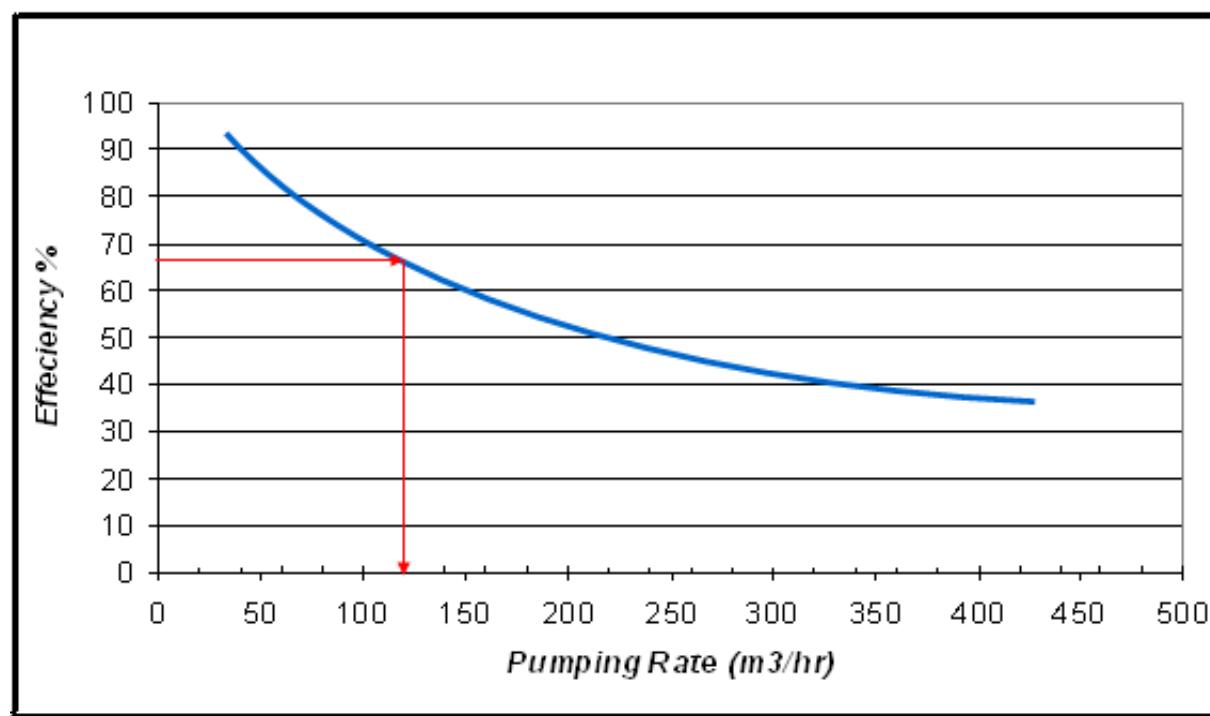
Figure 6.46   optimum pumping rates

6.11 Classification of Transmissivity

1   Values of transmissivity provide a basis for future groundwater exploration, development, abstraction and protection.

2   Quantitative transmissivity is normally used.

3   Classification of transmissivity magnitude.

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Table 6.4   Classification of Transmissivity

Magnitude (m²/day)	Class	Designation	Specific Capacity (m²/day)	Groundwater supply potential	Expected Q (m³/day) if s=5m
> 1000	I	Very high	> 864	Regional Importance	> 4320
100-1000	II	High	86.4 – 864	Lesser regional importance	432 – 4320
10-100	III	Intermediate	8.64 – 86.4	Local water supply	43.2 – 432
1-10	IV	Low	0.864 – 8.64	Private consumption	4.32 – 43.2
0.1-1	V	Very low	0.0864 – 0.864	Limited consumption	0.423 – 4.32
<0.1	VI	Imperceptible	< 0.0864	Very difficult to utilize for local water supply	< 0.432

Estimating Aquifer Storage Parameters From Known Lithology

1   The method has been developed by P.L. Younger of Newcastle University-Britain.

2   The method is simple and has been developed for areas devoid of pumping test data (especially when it is expensive to carry out pumping tests). This is the situation (normally) in the developing countries.

6.12.1   Estimating Specific Yield (S_y)

1   The porosity must be known.

2   To estimate S_y for a material of known effective porosity find the fraction of porosity accounted for by specific yield, and multiply the porosity by its fraction.

3   See the following example.

Estimating Specific Yield

Effective Porosity	Material	Fraction of porosity accounted for S_y	Specific Yield value
0.1	Limestone with intergranular macro-porosity	0.47	0.1 x 0.47 = 0.047
0.3	Fractured micro-porous rock	0.8	0.3 x 0.8 = 0.24

6.12.2   Estimating Specific Storage (S_s)

From **equation 1.26**

$$S_S = \rho_w \, g \left[\alpha + n\beta \right]$$

Substituting the following typical values

$$\rho_w = 1000 \; Kg/m^3$$
$$g = 9.81 \; m/s^2$$
$$\beta = 4.4 \times 10^{-10} \; ms^2/Kg$$

Then,

$$S_S = 9810\alpha + 4.4 \times 10^{-10} \, n$$

Values of Specific Storage Assuming Porosity Equal to 15 % (after Younger, 1993)

Typical Lithologies	Specific Storage (m^{-1})
Clay	9.81×10^{-3}
Silt, fine sand	9.82×10^{-4}
Medium sand, fine	9.87×10^{-5}
Coarse sand, medium gravel, highly fissured	1.05×10^{-5}
Coarse gravel, moderately fissured rock	1.63×10^{-6}
Unfissured rock	7.46×10^{-7}

To estimate Storativity for an aquifer of known lithology and thickness, select appropriate specific storage from the above table and multiply by the aquifer thickness.

6.13   Logan Approximation Method in Confined Aquifer

Assumptions and Applicability

1.   Steady state pumping.

2. $s_1 = 0$ at $r_1 = r_{max}$ (end of cone of depression), $s_2 = s_w$ at $r_2 = r_w$.

3.   Well losses are assumed minimal

4.   Log $(r_{max}/r_w) \approx 3.33$,   this approximation shouldn't generate significant errors.

5.   The above equation is derived for confined aquifers only and can be modified to be applicable to unconfined aquifers.

6.   Constant pumping rate.

7.   Water removed from storage, discharges instantaneously with decline in groundwater levels.

8.   Diameter of pumped well is small, i.e. , storage in the hole can be neglected,

Below is the derivation of Logan's rule from Thiem Equation,

$$s_2 - s_1 = \frac{Q}{2\pi T} \ln\left(\frac{r_1}{r_2}\right)$$

$$s_1 = 0 \quad at \quad r_1 = r_{max}$$

$$s_2 = s_w \quad at \quad r_2 = r_w$$

$$but, \ s_w = \frac{2.303 Q}{2\pi T} \log\left(\frac{r_{max}}{r_w}\right)$$

$$\log\left(\frac{r_{max}}{r_w}\right) \approx 3.33$$

$$\therefore s_w = \frac{(2.303)(3.33)Q}{2\pi T}$$

$$s_w = \frac{1.22 Q}{T} \tag{6.65}$$

Logan method in Unconfined Aquifer,

$$T = \frac{1.22 Q}{s'}$$

$$s' = s_w - \frac{s_w^2}{2H}$$

where,

s'   is the corrected drawdown.

H   is the initial thickness of the saturated aquifer.

Example 6.1: Steady State Flow

A 200 mm diameter well which fully penetrating a confined aquifer of thickness 25 m is pumped at a constant rate of 2000 m^3/day. The steady state drawdown in the well is 8 m and the drawdown in a piezometer 100 m from the well is 1.4 m.

a   Ignoring well losses, calculate the transmissivity of the aquifer, the hydraulic conductivity of the aquifer material and the radius of influence of the well.

b   Repeat the calculation for an unconfined aquifer with saturated thickness 25 m.

c   What would be the calculated values of T, K and R in (a) if the well is 80% efficient?

Answer 6.1

a   **In a confined aquifer**

$$h_1 - h_2 = \frac{Q}{2\pi T} \ln \frac{r_1}{r_2}$$

In terms of drawdowns $$s_2 - s_1 = \frac{Q}{2\pi T} \ln \frac{r_1}{r_2}$$

So, $$8 - 1.4 = \frac{2000}{2\pi T} \ln \frac{100}{0.1}$$

i.e. $$T = \frac{2000}{2\pi(8-1.4)} \ln \frac{100}{0.1}$$

$$= 333 \ m^2/day$$

Hydraulic conductivity $(K) = \dfrac{T}{b} = \dfrac{333}{25} = 13.2 \ m/day$

At radius of inf*luence* (R), *drawdown* $(s) = 0$.

So, $$8 - 0 = \frac{2000}{2\pi \times 333} \ln \frac{R}{0.1}$$

$$R = 0.1 e^{\frac{2\pi \times 333 \times 8}{2000}} = 433 m.$$

b   In an unconfined aquifer

$$h_1^2 - h_2^2 = \frac{Q}{\pi K} \ln \frac{r_1}{r_2}$$

where h_1 & h_2 are measured from the base of the aquifer.

So, $$(25 - 1.4)^2 - (25 - 8)^2 = \frac{2000}{\pi K} \ln\left(\frac{100}{0.1}\right)$$

So, $$K = \frac{2000}{\pi\left((25-1.4)^2 - (25-8)^2\right)} \ln\left(\frac{100}{0.1}\right) = 16.4 \ m/day$$

and $$T = K.h = 16.4 \times 25 = 410 \ m^2/day$$

At radius of inf luence (R), h = 25.

So, $$25^2 - (25 - 8)^2 = \frac{2000}{\pi \times 16.4} \ln\left(\frac{R}{0.1}\right)$$

$$R = 0.1 e^{\frac{\pi \times 16.4 \times \left(25^2 - (25-8)^2\right)}{2000}} = 574 \ m.$$

c   In an unconfined aquifer

Actual drawdown in formation at the well = 0.8 × 8 = 6.4 m.

Repeat the calculations in (a) with this value as the drawdown in the well gives:

$$T = 440 \ m^2/day$$
$$K = 17.6 \ m/day$$
$$R = 692 \ m$$

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Example 6.2: Groundwater Hydraulics

A   Explain the differences between the processes by which water is released from storage when groundwater is abstracted from a confined aquifer and an unconfined aquifer.

B   Steady-state to an abstraction well in an unconfined aquifer is given by the Dupuit formula

$$\frac{Q}{2\pi K} \ln\left(\frac{r_2}{r_1}\right) = \frac{h_2^2 - h_1^2}{2}$$

Define each term in this equation.

C   Show how the Dupuit formula can be re-arranged to be expressed in terms of corrected drawdown.

D   A well in an unconfined aquifer is pumped over a long period at a rate of 0.05 m³/s until approximate steady-state conditions are achieved. Two observation wells at distances of 25 m and 50 m from the abstraction well give observed drawdowns below the initial water table level of 15.3 m and 9.5 m respectively. If the saturated thickness of the aquifer prior to pumping is 80 m, determine the hydraulic conductivity of the aquifer material.

E   Explain why it is not recommended to use the water level in the abstraction well to help determine aquifer properties during a pumping test.

Answer 6.2
A
  1   In a confined aquifer, there are two mechanisms responsible for the release of water; compression of the rock matrix, and compaction of the water. The overburden on a volume of rock is supported in part by the structure of the rock, and in part by the pressure of the water in it. When water is pumped from the rock, the water pressure is lowered and so the support for the overburden is reduced. The consequence is that the rock matrix is compressed until a new equilibrium is established. The compression of the rock is achieved primarily by a rearrangement of the grains of rock that form the rock volume and this result in a reduction in the void space within the rock. During compression, some of the water was stored in these void spaces in then squeezed out of the rock and drawn into the well. Although the compressibility of water is low, if the pressures in the rock are high enough, the water in the rock will be measurably compressed. Reducing the pressure by pumping will result in the groundwater expanding, and being forced into and up the well. The release of water in a confined aquifer takes place over the entire aquifer thickness.

  2   In an unconfined aquifer, the main process responsible for the release of water is drainage of the matrix pores. The processes described for a confined aquifer still take place, but are less significant. When the water table falls, the region above it does not drain instantly, nor does it drain dry; some water is retained in small pores and on the rock surface by strong molecular forces. The release of water in an unconfined aquifer is a function only of the rock in the upper portion of the aquifer.

B

Q is the rate of abstraction from the well [L^3T^{-1}]

K is the hydraulic conductivity [$L\,T^{-1}$]

r_1 *and* r_2 are the distances from the abstraction well to observation wells [L]

h_1 *and* h_2 are the piezometric heads in the observation wells [L]

C

If the initial saturated thickness of the aquifer is b, the relationship between head (measured above the base of the aquifer) and the drawdown is $h = b - s$, and then the Dupuit equation can be expressed in terms of drawdown as

$$\frac{Q}{2\pi K}\ln\left(\frac{r_2}{r_1}\right) = \left(s_1 - \frac{s_1^2}{2b}\right) - \left(s_2 - \frac{s_2^2}{2b}\right)$$

Which has the same form as the Thiem equation, but with the drawdown replaced by the corrected drawdown, $s - \dfrac{s^2}{2b}$

D

$$Q = 0.05\ m^3/s$$
$$r_1 = 25\ m$$
$$r_2 = 50\ m$$
$$s_1 = 15.3\ m$$
$$s_2 = 9.5\ m$$
$$b = 80\ m$$

Rearrange Dupuit Formula to give

$$K = \frac{Q \times \ln\left(\dfrac{r_2}{r_1}\right)}{\pi\left(h_2^2 - h_1^2\right)}$$

Note that $h_1 = b - s_1$, $h_2 = b - s_2$, *and substitute values in the equation*

$$K = \frac{0.05 \times \ln(2)}{\pi\left(70.5^2 - 64.7^2\right)} = 1.41 \times 10^{-5}\ m/s$$
$$= 1.22\ m/day$$

E

The water level in a pumping well is usually a poor indicator of the piezometric or water level in the aquifer close to the well due to *well losses*. These are the energy losses that occur as water enters the well (*entry losses*) and as the water flows up the well (*pipe flow or upflow losses*)

Example 6.3: Groundwater Wells and Rivers, and Well Operations

a   Calculate the steady state drawdown at the pumping well which fully penetrates a confined aquifer if the pumping well is 50 m from a river which is in full hydraulic connection with the aquifer as shown in the **Figure 6.47** below.
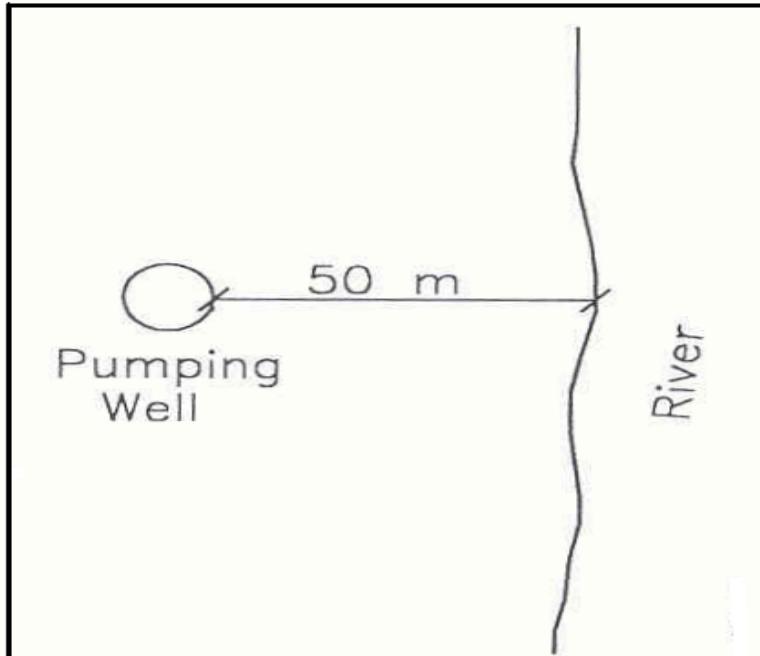

Figure 6.47

The followings are useful data that can be used in calculating (a):

Diameter of pumping well	=	0.2 m
Pumping rate	=	4000 m^3/day
Transmissivity	=	470 m^2/day
Type of utilized aquifer	=	confined
Distance between river and pumping well	=	50 m

<u>Hint:</u>   Apply Theim's equation, $s = \dfrac{Q}{2\pi T} \ln\left(\dfrac{r_1}{r_2}\right)$
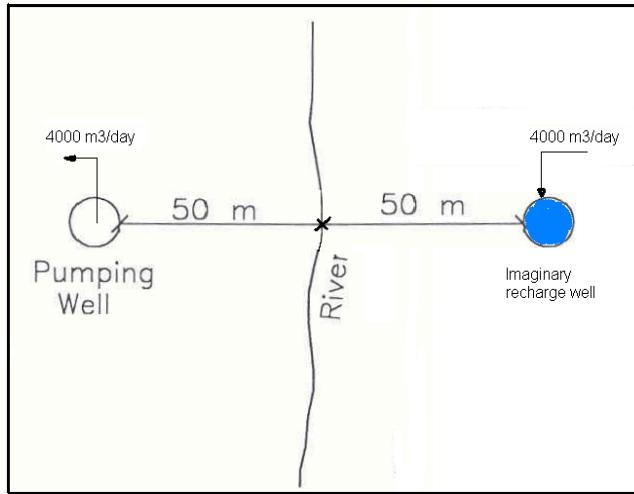
b   What would be the drawdown in a piezometer 100 m from the well and 25 m from the river?

c   Pumping tests in two groundwater wells 1 and 2 have yielded transmissivity values of 110 m^2/day and 95 m^2/day respectively. The effective saturated thickness of the aquifer in the area is 45 m. The local water authority wishes to use boreholes 1 and 2 for public supply abstractions at 3000 m^3/day each. Step-drawdown analyses have been shown that the two wells to have well loss coefficients of 3x10^{-7} day^2/m^5 and 1x10^{-6} day^2/m^5 respectively.

If the mean stage of the river is 65 m above sea level, **<u>what advice would you give the water authority on bringing the boreholes 1 and 2 into use?</u>** (Use Logan's equation)

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(a)

To estimate the effect of the river, introduce a recharge well as shown in the figure below.

Drawdown due to pumping well at the well itself is

$$s = \frac{Q}{2\pi T} \ln\left(\frac{r_1}{r_2}\right)$$

$$but,\ r_1 = r_0 \quad and \quad r_2 = r_w = \frac{diameter\ of\ the\ well}{2} = \frac{0.2}{2} = 0.1$$

$$\Rightarrow \qquad s_1 = \frac{Q}{2\pi T} \ln\left(\frac{r_0}{0.1}\right)$$

$$s_1 = \frac{Q}{2\pi(470)} \ln\left(\frac{r_0}{0.1}\right)$$

Drawdown due to image well is

$$s = -\frac{Q}{2\pi T} \ln\left(\frac{r_1}{r_2}\right)$$

$$but,\ r_1 = r_0 \quad and \quad r_2 = 100\ m$$
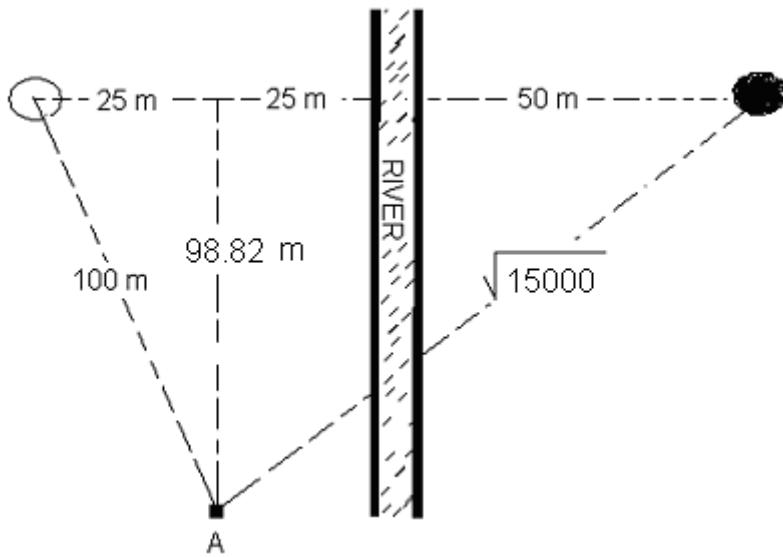
$$\Rightarrow \qquad s_2 = -\frac{Q}{2\pi(470)} \ln\left(\frac{r_0}{100}\right)$$

Total drawdown at a well = $s = s_1 + s_2$

$$= \frac{4000}{2\pi(470)} \ln\left(\frac{r_0}{0.1}\right) - \frac{4000}{2\pi(470)} \ln\left(\frac{r_0}{100}\right) = \frac{4000}{2\pi(470)} \ln\left[\frac{r_0}{0.1} \middle/ \frac{r_0}{100}\right] = \frac{4000}{2\pi(470)} \ln\left(\frac{100}{0.1}\right)$$

Total drawdown at a well= 9.36 m

(b)

Total drawdown at A;

Total drawdown at A = drawdown from real well − drawdown from imaginery well

$$s_A = \frac{Q}{2\pi T}\left(\ln\frac{R}{100} - \ln\frac{R}{\sqrt{15000}}\right)$$

$$= \frac{4000}{2\pi \times 470}\ln\left(\frac{\sqrt{15000}}{100}\right)$$

$$= 0.27\,m$$

(c)

Logan's Equation; $\quad s = \dfrac{1.22\,Q}{T}$

Saturated thickness = 45 m.

Total drawdown at well 1 = $\dfrac{1.22\times 3000}{110} + (3\times 10^{-7})(3000)^2 = 36\,m$

Total drawdown at well 2 = $\dfrac{1.22\times 3000}{95} + (1\times 10^{-6})(3000)^2 = 47.5\,m$

Operating well 2 will cause drawdown greater than saturated thickness.

We should advise the water authority to operate well 1 only at this pumping rate. However, well 2 can be operated at a lower pumping rate, i.e., 1500 m^3/day.

Example 6.4   Groundwater Wells and Springs

Given Data
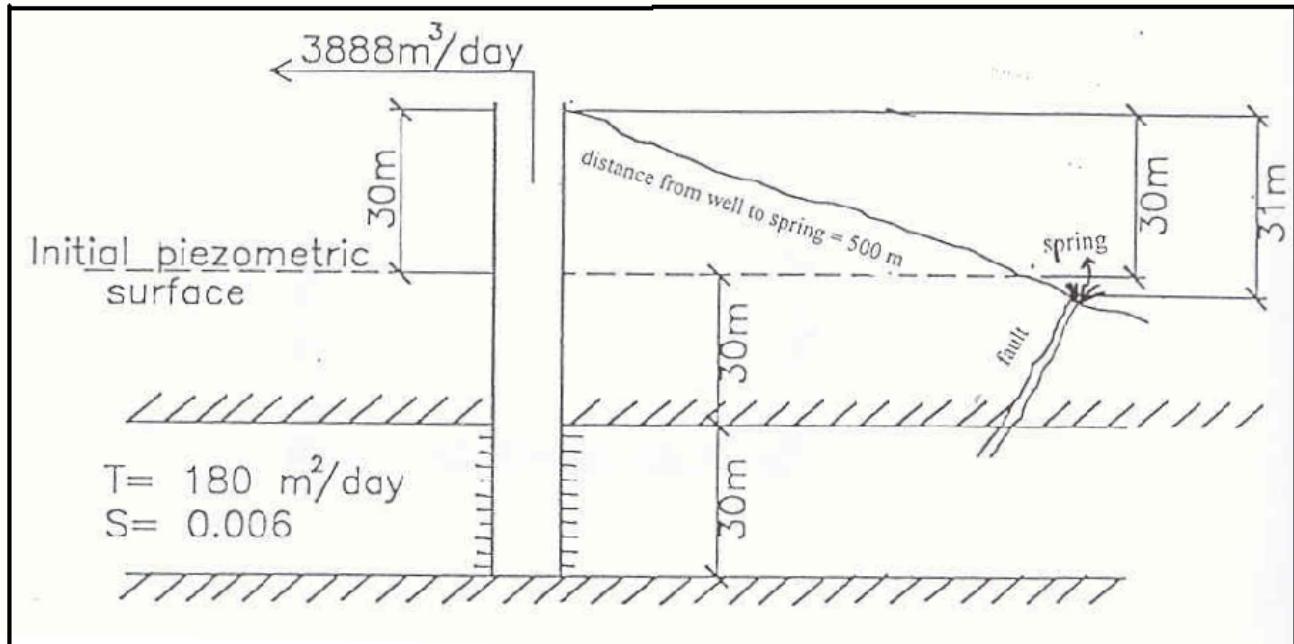
Figure 6.48

✓	Pumping rate	=	4000 m³/day
✓	Transmissivity	=	470 m²/day
✓	Storativity	=	0.006
✓	Type of utilized aquifer	=	confined
✓	Aquifer thickness	=	30 m
✓	Top of aquifer at the location of the well	=	60 m bgl
✓	Initial piezometric surface	=	30 m above the top of aquifer
✓	Distance between spring and pumping well	=	500 m

✓   The ground surface elevation is some 31 m lower than at the well. At this location there is an artesian spring supplied through a fracture from the aquifer.

Required

How long will it be before the spring ceases (stops) to flow if the pumping well is operating continuously at Q = 3888 m³/day?

Hint: 1.   Apply Theis equation $s = \dfrac{Q}{2\pi T}\left[-0.5772 - \ln u + u\right]$ and $u = \dfrac{r^2 S}{4T t}$

2.   You may need the following table:

u	0.1	0.2	0.3	0.4	0.5	0.6
W(u)	1.823	1.223	0.906	0.702	0.560	0.454

Answer 6.4

1   From the given data, it can be shown that the initial piezometric level is only 1 m above the location of the spring; therefore, the water level needs to drop only one meter in order to cause the spring to cease the flowing.

Hence, we need to find t for s = 1 m.

$$s = 1$$
$$\therefore Apply\ Theis\ equation\ yield$$

$$s = \frac{Q}{4\pi T} W(u)$$

$$1 = \frac{3888}{4\pi(180)} W(u) = 1.7188 \times W(u)$$

$$\therefore W(u) = \frac{1}{1.7188} = 0.5818$$

2   From interpolation of values in the table

$$\frac{0.4 - u}{0.4 - 0.5} = \frac{0.702 - 0.5818}{0.702 - 0.560}$$

$$\frac{0.4 - u}{-0.1} = 0.8465$$

$$\Rightarrow 0.4 - u = -0.08465$$

$$u = 0.4 + 0.08465$$

$$u = 0.485$$

3   Now,

$$u = \frac{r^2 S}{4T t} \quad \Rightarrow \quad t = \frac{S r^2}{4T t}$$

$$t = \frac{(0.006)(500)^2}{(4)(180)(0.485)} = 4.3\ days$$

The spring ceases (stops) to flow when the pumping well is operating continuously at Q = 3888 m³/day after **4.3 days**

Example 6.5 Pumping Test Analysis

PART A: Derivation of Jacob's Equation

The drawdown in a confined aquifer due to a constant pumping rate Q can be described by the equation:

$$s = \frac{Q}{4\pi T} W(u) \; , \quad where \quad u = \frac{r^2 S}{4Tt}$$

The well function can be expanded as a series
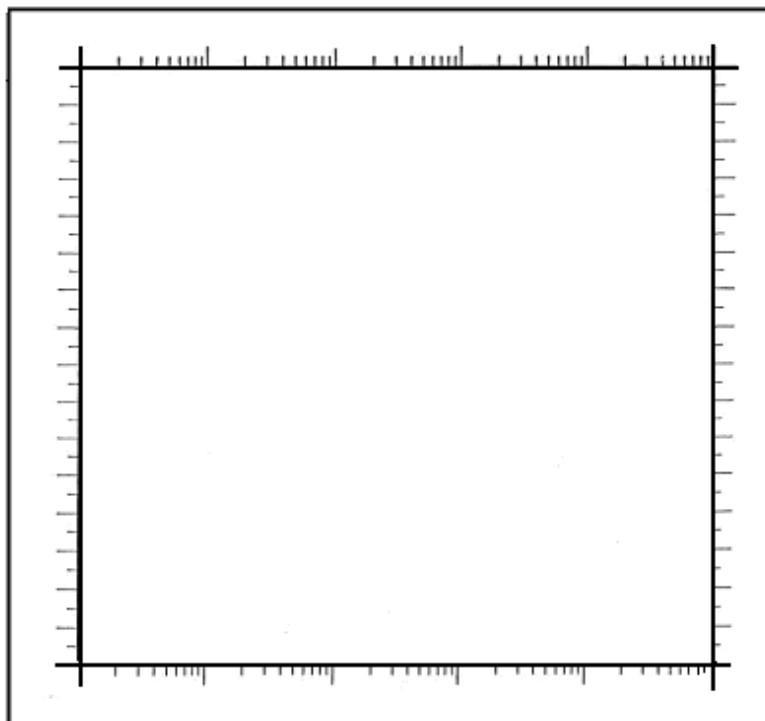
$$W(u) = -0.5772 - \ln u + u - \frac{u^2}{2.2!} \; ....$$

Show that for u < 0.01, the drawdown can be expressed to a good approximation by Jacob's equation:

$$s = \frac{2.3Q}{4\pi T} \log \frac{2.25Tt}{r^2 S}$$

PART B: Application of Jacob's Method

The following drawdown data were obtained from a pumping test performed on a confined sandy aquifer. **Determine the transmissivity and storage coefficient** using Jacob's equation above; if the pumping rate was constant at 200 liter/sec and the drawdown data were obtained from an observation well located 800 m from the abstraction well.

Time (minutes)	0	20	30	40	50	60	90	125	200	300	500
Water level (m above datum)	20	18.9	18.6	18.4	18.2	18.0	17.6	17.3	16.8	16.4	15.9

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Answer 6.5

A.

$$s = \frac{Q}{4\pi T} W(u), \qquad but \quad W(u) = -0.5772 - \ln u \qquad when \; u < 0.01$$

$$\therefore s = \frac{Q}{4\pi T}\left[-0.5772 - \ln u\right], \quad but \quad u = \frac{r^2 S}{4Tt}$$

$$\therefore s = \frac{Q}{4\pi T}\left[-0.5772 - \ln \frac{r^2 S}{4Tt}\right], \; but \; 0.5772 = \ln 1.78$$

$$\therefore s = \frac{Q}{4\pi T}\left[-\ln 1.78 - \ln \frac{r^2 S}{4Tt}\right] = \frac{Q}{4\pi T}\left[-1\left(\ln 1.78 + \ln \frac{r^2 S}{4Tt}\right)\right]$$
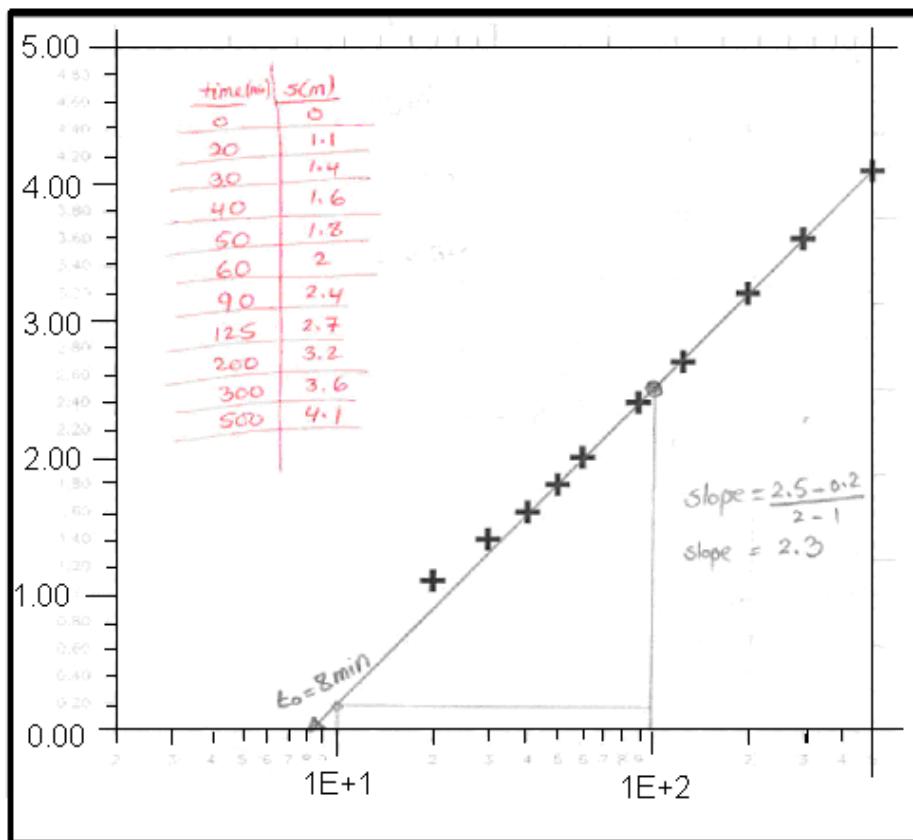
$$\therefore s = \frac{Q}{4\pi T}\left[-1 \ln \frac{1.78 \times r^2 S}{4Tt}\right] = \frac{Q}{4\pi T}\ln\left(\frac{4Tt}{1.78 r^2 S}\right)$$

but $\ln x = 2.3 \log x$

$$\therefore s = \frac{2.3Q}{4\pi T}\log \frac{4Tt}{1.78\, r^2 S}$$

$$\Rightarrow s = \frac{2.3Q}{4\pi T}\log \frac{2.25Tt}{r^2 S}$$

B.

As show in the previous figure the drawdown can be calculated by {20 – water level}

Time (minutes)	0	20	30	40	50	60	90	125	200	300	500
Water level (m above datum)	20	18.9	18.6	18.4	18.2	18.0	17.6	17.3	16.8	16.4	15.9
Drawdown	0	1.1	1.4	1.6	1.8	2.0	2.4	2.7	3.2	3.6	4.1

From the graph,

$$slope = \frac{2.3Q}{4\pi T} \Rightarrow T = \frac{2.3Q}{(4\pi)(slope)}, \quad but \ slope = 2.3$$

$$\therefore T = \frac{2.3 \ x \ 0.2 m^3/\sec \ x \ 60 \ x \ 60 \ x \ 24}{4 \ \pi \ x (2.3)} = 1375 \ m^2/day$$

$$From \ graph \ t_0 = 8 \min. = 5.56 \ x \ 10^{-3} \ day.$$

$$at \ t = t_0 \Rightarrow \log\left[\frac{2.25T \ t_0}{r^2 S}\right] = 0, \quad \therefore \frac{2.25T \ t_0}{r^2 S} = 1$$

$$\Rightarrow S = \frac{2.25T \ t_0}{r^2} = \frac{2.25 \ x \ 1375 \ x \ 5.56 \ x 10^{-3}}{(800)^2} = 2.7 \ x 10^{-5}$$

So,   T = 1375 m²/day

S = 2.7 × 10⁻⁵

Example 6.6   Pumping Test Analysis

Hantush's equation

$$s_w = BQ + CQ^2$$

Jacob's equations

$$s = \frac{2.303\,Q}{4\pi T}\log_{10}\left(\frac{2.25\,T\,t}{r^2\,S}\right)$$

$$T = \frac{2.303\,Q}{4\pi\,\Delta s}$$

$$S = \frac{2.25\,T\,t_0}{r^2}$$

Logan's equation

$$s_w = \frac{1.22\,Q}{T}$$

PART A: Step-drawdown Test Analysis

The principle results of a step-drawdown test undertaken

Discharge (m³/day)	End of Step Drawdown (meters)
500	1.40
1000	3.20
2000	6.60
3000	11.40

1   Use Hantush's graphical technique (use the graph paper attached, **Figure 6.49**) to determine <u>aquifer loss</u> and <u>well loss</u> coefficients.
2   Obtain an estimate of aquifer transmissivity.
3   Suggest a maximum reliable yield. (Use attached graph paper, **Figure 6.50**).
4   Estimate the well efficiency at Q = 3000 m³/day. What does this value tell you?

PART B: Constant Pumping Rate Test Analysis

Following a recovery period after the step test, a constant duration test was carried out at a discharge of 2000 m³/day with the data in Table 30.2 recorded in the pumping well itself knowing that its diameter = 0.2 m.

Time (min)	Drawdown (meters)	Time (min)	Drawdown (meters)
1	1.82	60	4.12
5	3.32	120	4.32
10	3.52	240	4.57
30	3.92		

1   Use the Jacob semi-equilibrium method to determine aquifer transmissivity and storativity (use the attached semi-log paper, **Figure 6.51**)
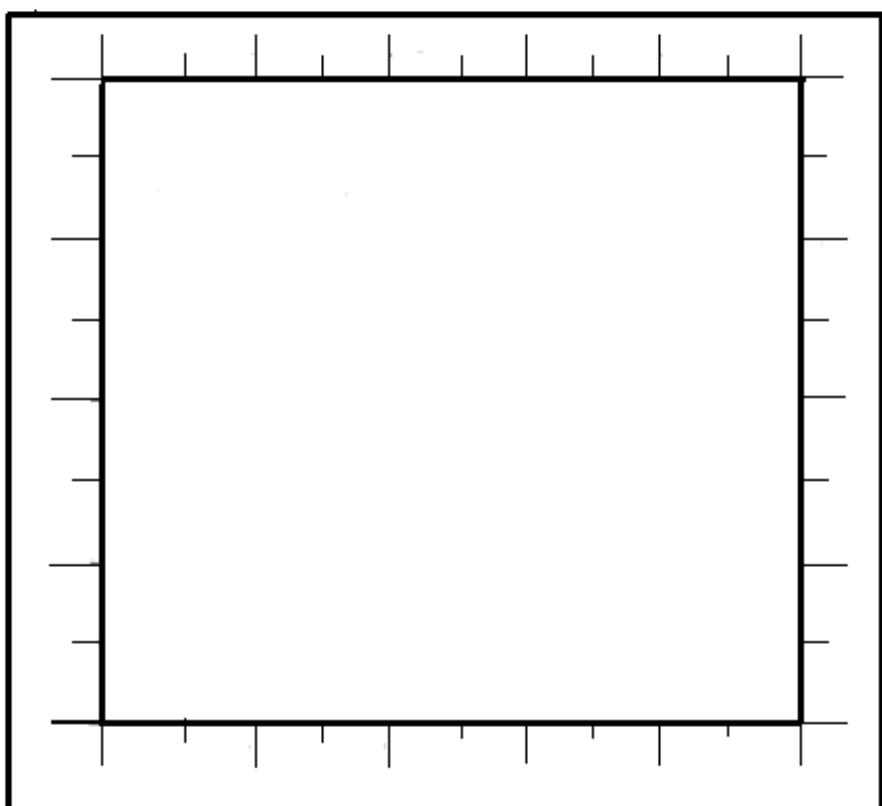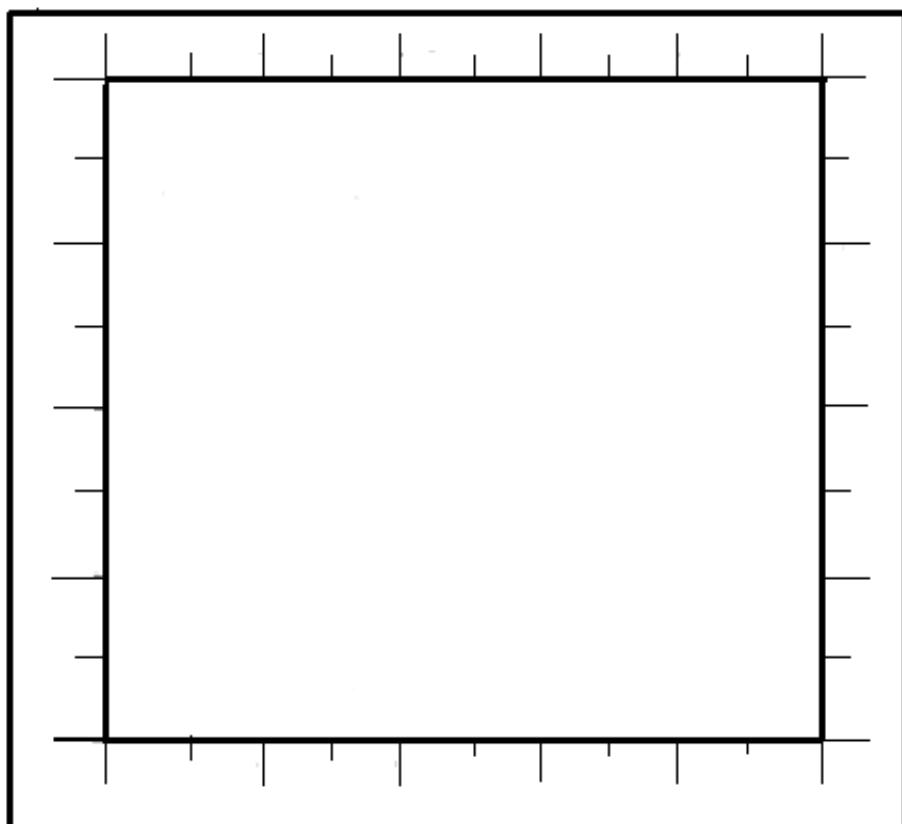2   The value of storativity you obtained is indeed not accurate. Why?
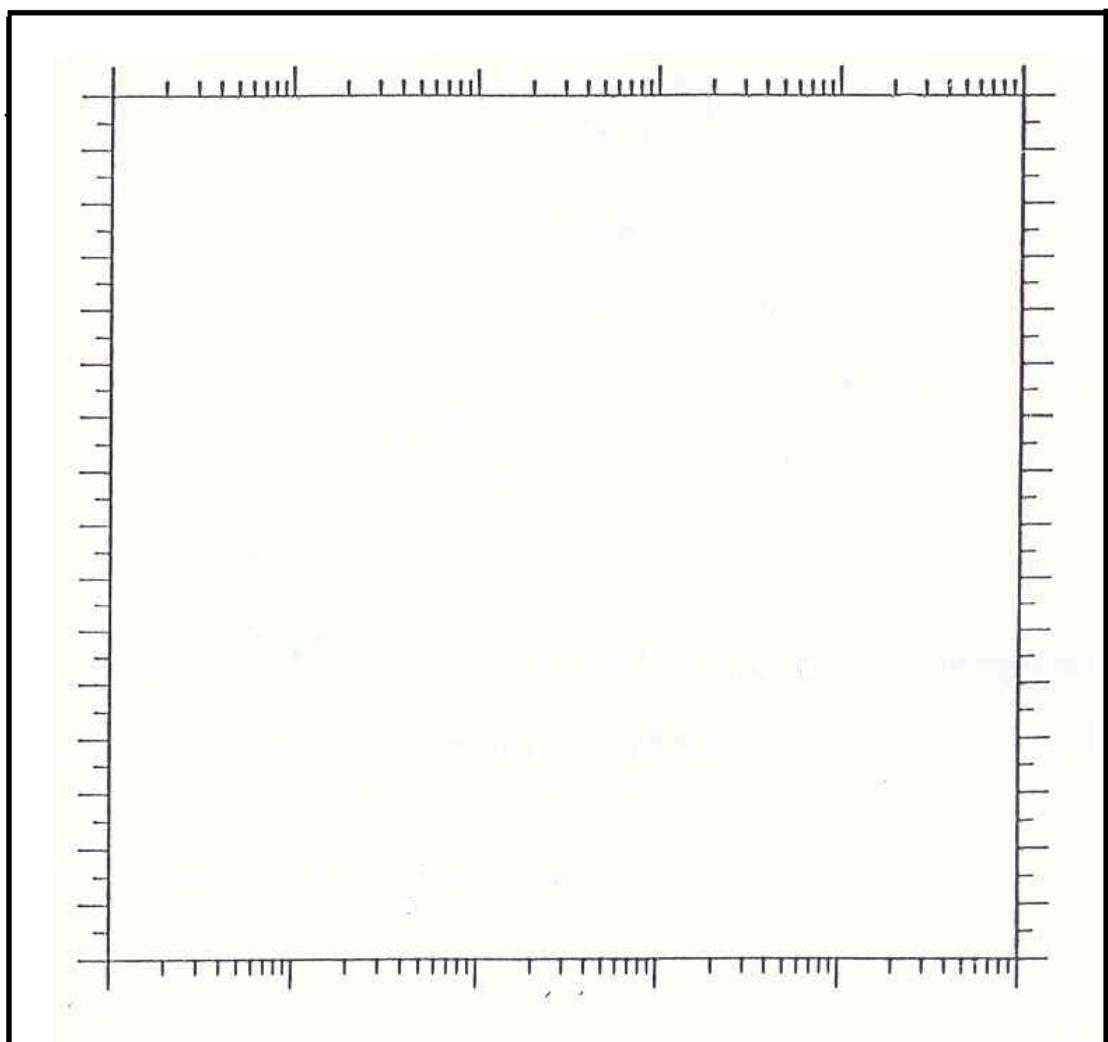
Figure 6.49

Figure 6.50

Figure 6.51

Answer 6.6

(A)

See Figure 6.49', $\{C= 3.56\times10^{-7}\ day^2/m^5\ /\ B= 2.7\times10^{-3}\ day/m^2\}$

Sw/Q = B + CQ

Slope = C = 0.000000356 (well loss coefficient)
Y-Intercept = B = 0.0027 (Aquifer loss coefficient)
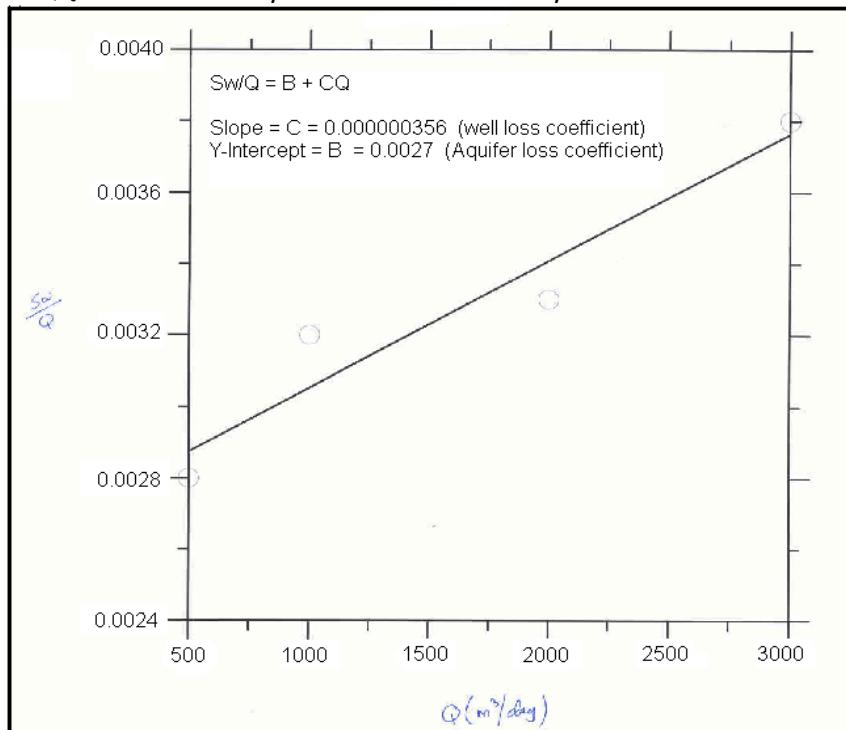
Figure 6.49'

$T = 1.22\ Q/s_w$, use the first step.   T = 500 / 1.4 = 357.14 m²/day

See Figure 6.50'

* No sign of aquifer mining.

* The maximum pumping rate is yet to
   be approached

For this particular test, the
maximum pumping rate
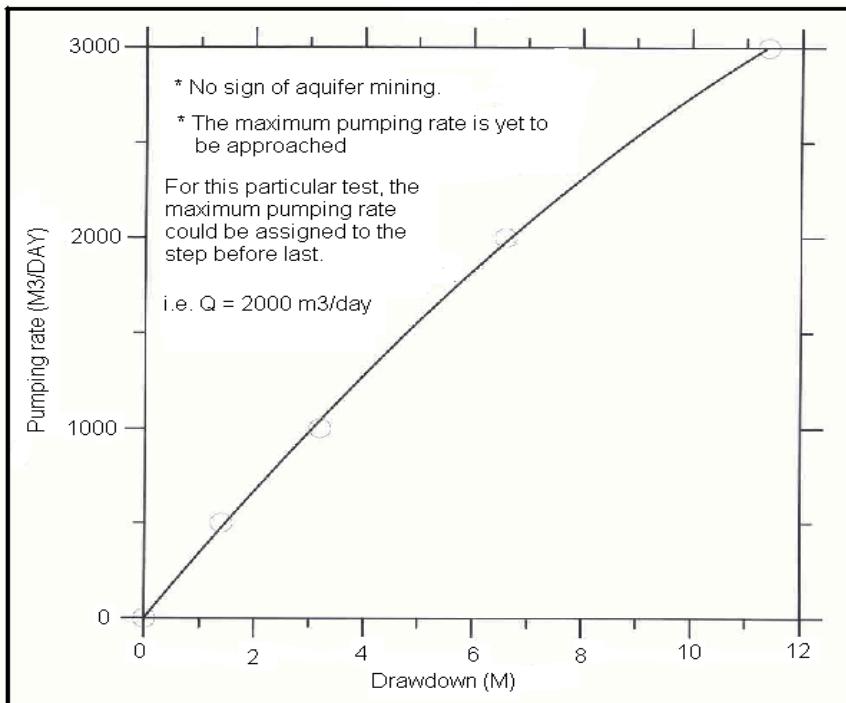could be assigned to the
step before last.

i.e. Q = 2000 m3/day

Figure 6.50'

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$$Efficiency = \frac{BQ}{drawdown}$$

$$B = 2.7 \, x10^{-3} \ day/m^2$$

$$BQ = 2.7 \, x10^{-3} \, x3000 = 8.1 \ m$$

$$E_w = \frac{8.1}{11.4} x100\% = 71\%$$

It tells me that the well does not require any development as $E_w = 71\% > 50\%$

(B)   Q = 2000 m³/day
C = 3.56 × 10⁻⁷ day²/m⁵
Well losses = CQ^2 = 3.56 × 10⁻⁷ × (2000)² = 1.42 m.

Then, the real drawdown in the formation at the well site is as follows:

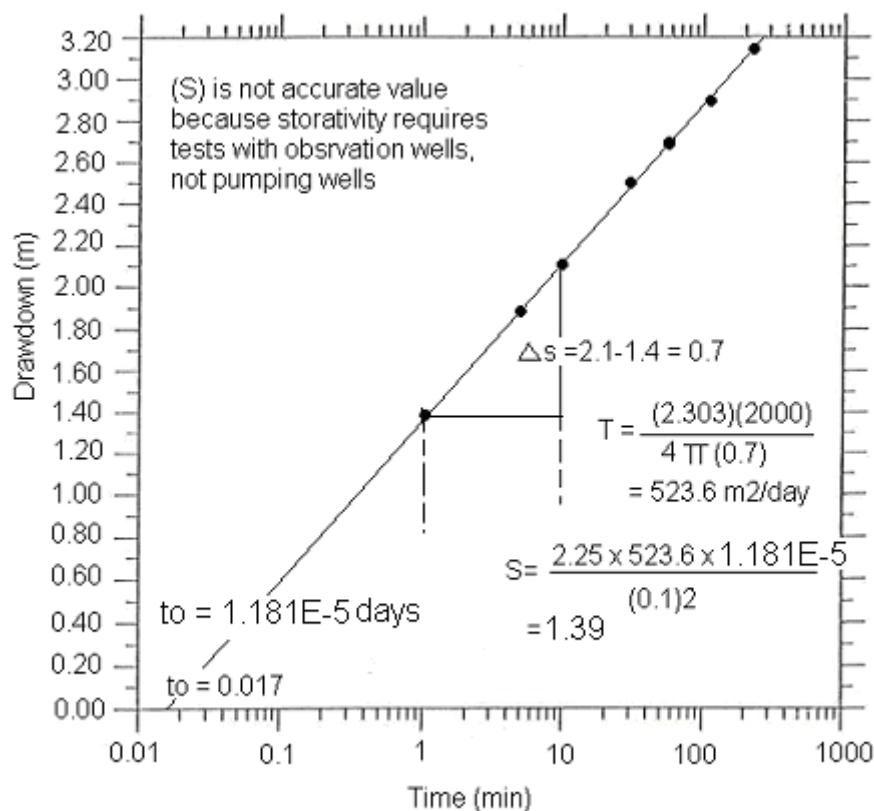| Time (min)	Real Drawdown ($s_w - CQ^2$)	Time (min)	Drawdown (meters)
1	1.82 – 1.42 = 1.4	60	4.12 – 1.42 = 2.7
5	3.32 – 1.42 = 1.9	120	4.32 – 1.42 = 2.9
10	3.52 – 1.42 = 2.1	240	4.57 – 1.42 = 3.15
30	3.92 – 1.42 = 2.5		

Figure 6.51'

Example 6.7   Jacob's Method of Pumping Test

The following are Jacob's Equations:

$$s = \frac{2.303Q}{4\pi T} \log \frac{2.25T\,t}{r^2 S}$$

$$S = \frac{2.25T\,t_0}{r^2}$$

i.   When may Jacob's Equation be applied to find values of Transmissivity (T) and Storativity (S) from pumping tests?

ii. A constant pumping rate test was carried out at a discharge of 2000 m³/day with the following data recorded in a piezometer located 10 m from the well.

Time (min)	1	5	10	30	60	120	240
Drawdown (m)	1.40	1.90	2.10	2.50	2.70	2.90	3.15

Use the Jacob semi-equilibrium method to determine aquifer T and S (use Figure 6.52).
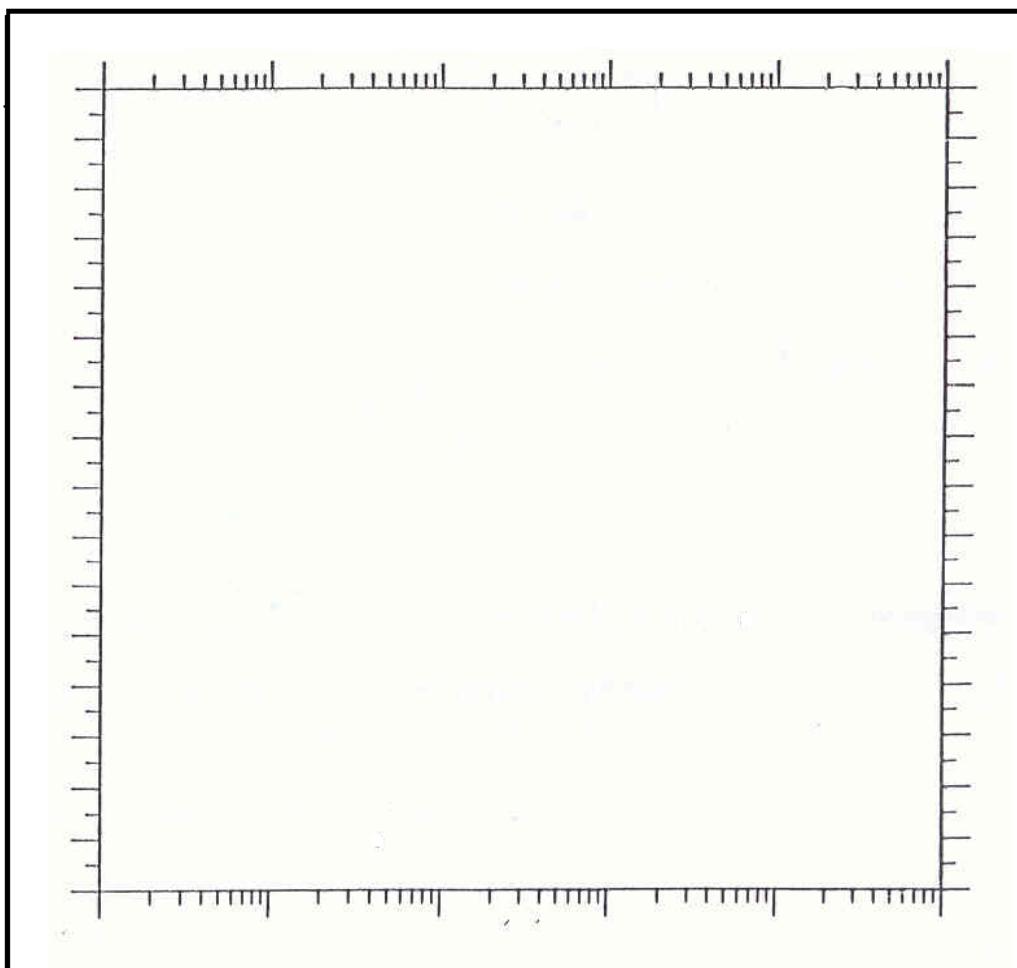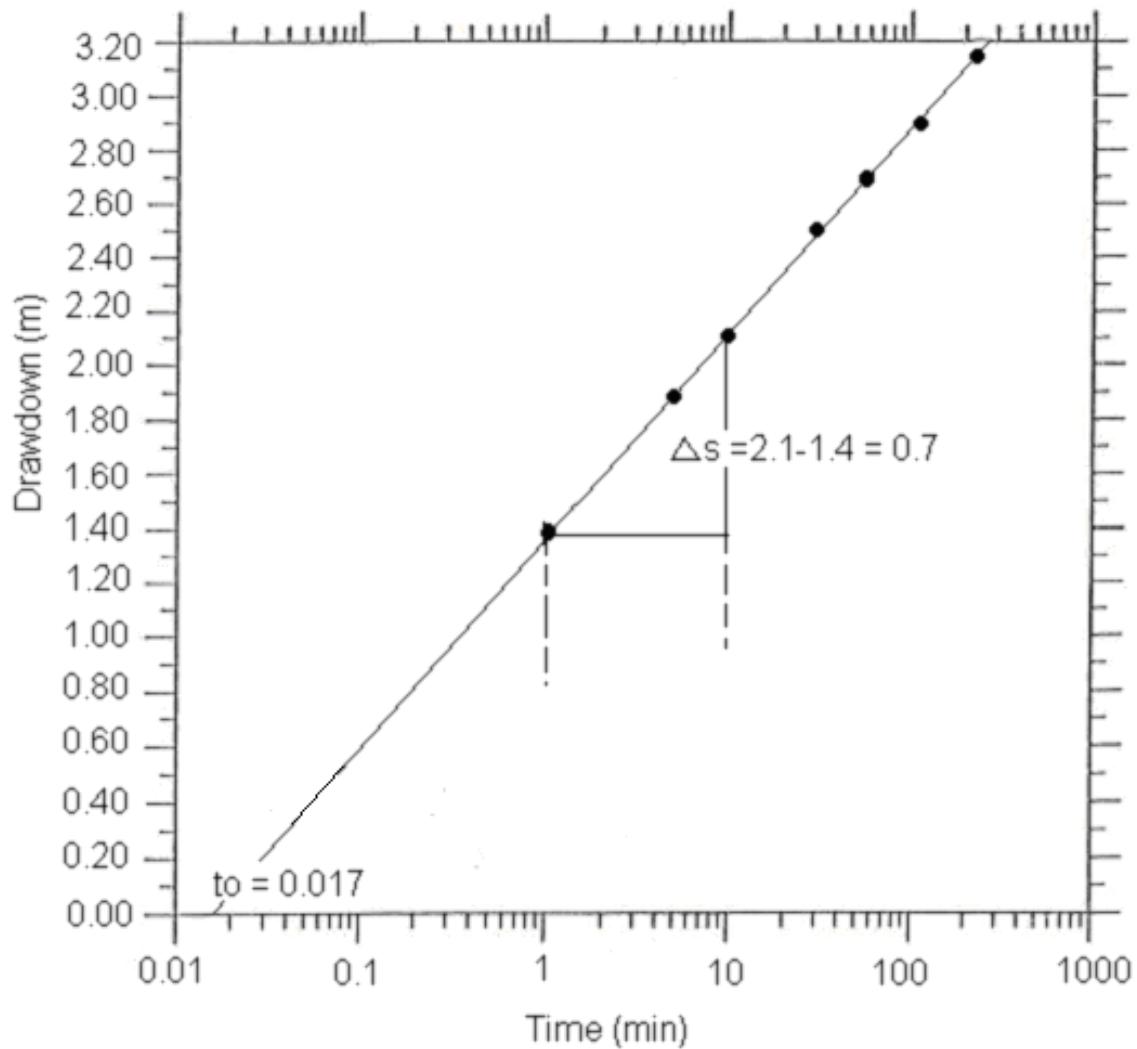

Figure 6.52

Answer 6.7

i.   u < 0.05   or u < 0.01, t is large, and r is small

ii.   see graph attached (semi-log)

$$T = \frac{2.303\,Q}{4\pi\,\Delta s} = \frac{(2.303)(2000)}{4\pi(0.7)} = 523.6\ m^2\,/\,day$$

From graph attached, t_0 = 1.7 x 10^{-2} min = 1.181 x 10^{-5} day

$$S = \frac{2.55\ x\ 523.6\ x\ 1.181 x 10^{-5}}{100} = 1.577\ x\ 10^{-4}$$

Example 6.8   Theis Method and Cooper-Jacob Method of Pumping Test

A well penetrating a confined aquifer is pumped at a uniform rate of 2500 m³/day. Drawdowns during the pumping period are measured in an observation well 60 m away; observation of t and s are listed in **Table 6.5.**

(i)   Determine the transmissivity and storativity using Theis method.

(ii)   Determine the transmissivity and storativity using Cooper-Jacob method.

Table 6.5   Pumping Test Data

t (min)	0	1.0	1.5	2.0	2.5	3.0	4	5	6	8	10	12	14
s (m)	0	0.20	0.27	0.30	0.34	0.37	0.41	0.45	0.48	0.53	0.57	0.60	0.63
t (min)	18	24	30	40	50	60	80	100	120	150	180	210	240
s (m)	0.67	0.72	0.76	0.81	0.85	0.90	0.93	0.96	1.00	1.04	1.07	1.10	1.17

Hint: The Following table is useful

Table 6.6

u	$W(u)$	u	$W(u)$	u	$W(u)$	u	$W(u)$
1×10^{-10}	22.45	7×10^{-8}	15.90	4×10^{-5}	9.55	1×10^{-2}	4.04
2	21.76	8	15.76	5	9.33	2	3.35
3	21.35	9	15.65	6	9.14	3	2.96
4	21.06	1×10^{-7}	15.54	7	8.99	4	2.68
5	20.84	2	14.85	8	8.86	5	2.47
6	20.66	3	14.44	9	8.74	6	2.30
7	20.50	4	14.15	1×10^{-4}	8.63	7	2.15
8	20.37	5	13.93	2	7.94	8	2.03
9	20.25	6	13.75	3	7.53	9	1.92
1×10^{-9}	20.15	7	13.60	4	7.25	1×10^{-1}	1.823
2	19.45	8	13.46	5	7.02	2	1.223
3	19.05	9	13.34	6	6.84	3	0.906
4	18.76	1×10^{-6}	13.24	7	6.69	4	0.702
5	18.54	2	12.55	8	6.55	5	0.560
6	18.35	3	12.14	9	6.44	6	0.454
7	18.20	4	11.85	1×10^{-3}	6.33	7	0.374
8	18.07	5	11.63	2	5.64	8	0.311
9	17.95	6	11.45	3	5.23	9	0.260
1×10^{-8}	17.84	7	11.29	4	4.95	1×10^{0}	0.219
2	17.15	8	11.16	5	4.73	2	0.049
3	16.74	9	11.04	6	4.54	3	0.013
4	16.46	1×10^{-5}	10.94	7	4.39	4	0.004
5	16.23	2	10.24	8	4.26	5	0.001
6	16.05	3	9.84	9	4.14		

Source: Adapted from L. K. Wenzel, *Methods for Determining Permeability of Water-Bearing Materials with Special Reference to Discharging Well Methods.* U.S. Geological Survey Water-Supply Paper 887, 1942.

Answer 6.8

(i)

First of all, values of r^2/t in m^2/min are computed see the following table. Then, Values of s and r^2/t are plotted on logarithmic paper. Values of W (u) and u from Table 8.6 are plotted on another sheet of logarithmic paper and a curve is drawn through the points. The two sheets are superposed and shifted with coordinate axes parallel until the observation points coincide with the curve, as shown in **Figure 6.53**. A convenient match point is selected with W (u) =1.00 and $u=1x10^{-2}$, so that s= 0.18 m and $r^2/t=150$ m^2/min = 216,000 m^2/day.

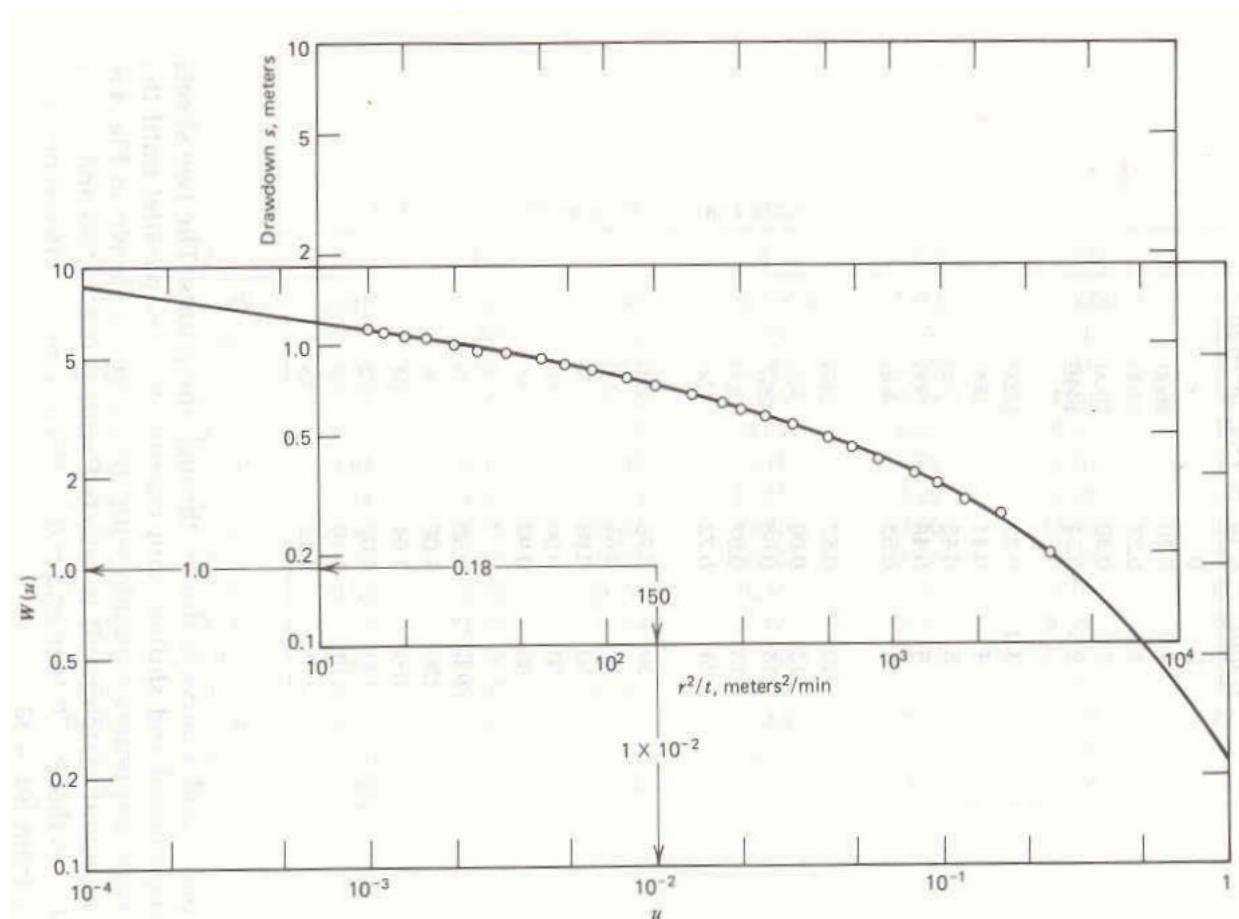| t (min)	0	1.0	1.5	2.0	2.5	3.0	4	5	6	8	10	12	14
s (m)	0	0.20	0.27	0.30	0.34	0.37	0.41	0.45	0.48	0.53	0.57	0.60	0.63
r^2/t m^2/min	∞	3600	2400	1800	1440	1200	900	720	600	450	360	300	257
t (min)	18	24	30	40	50	60	80	100	120	150	180	210	240
s (m)	0.67	0.72	0.76	0.81	0.85	0.90	0.93	0.96	1.00	1.04	1.07	1.10	1.17
r^2/t m^2/min	200	150	120	90	72	60	45	36	30	24	20	17	15

Figure 6.53   Theis method of superposition for solution of the nonequilibrium equation

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Thus,

$$T = \frac{Q}{4\pi s} W(u) = \frac{(2500) \times (1.00)}{4\pi (0.18)} = 1110 \ m^2/day$$

and,

$$S = \frac{4Tu}{r^2 \Big/ t} = \frac{4 \times (1110) \times (1 \times 10^{-2})}{216,000} = 0.000206$$

(ii)

From the pumping test data of **Table 6.5**, s and t are plotted on a semi-logarithmic paper as shown in **Figure 6.54**. A straight line is fitted through the points, and Δs=0.40m and t_0= 0.39 min = 2.70 x 10^{-4} day are read.
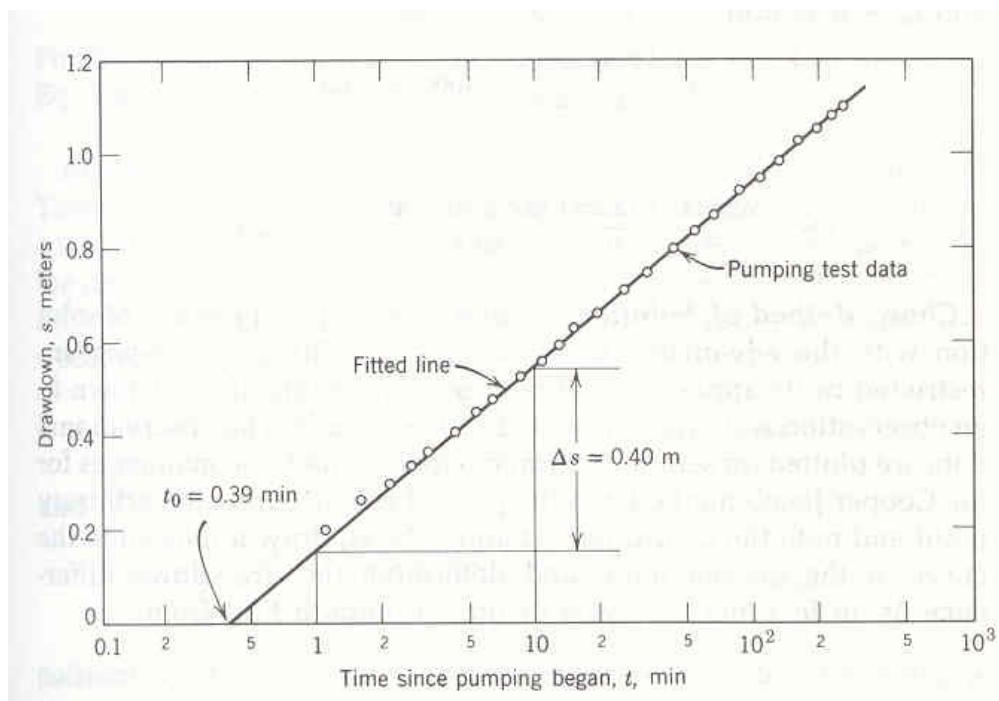
Figure 6.54   Cooper-Jacob method for solution of the nonequilibrium equation

Thus,

$$T = \frac{2.30 Q}{4\pi \Delta s} W(u) = \frac{2.30 \times (2500)}{4\pi (0.40)} = 1090 \ m^2/day$$

and,

$$S = \frac{2.25 T t_0}{r^2} = \frac{2.25 \times (1090) \times (2.70 \times 10^{-4})}{(60)^2} = 0.000184$$

Eexample 6.9 Recovery Test

A well pumping at a uniform rate of 2500 m³/day was shut down after 240 min; therefore, measurements of s' and t' tabulated in **Table 6.7** were made in an observation well. Calculate the transmissivity.
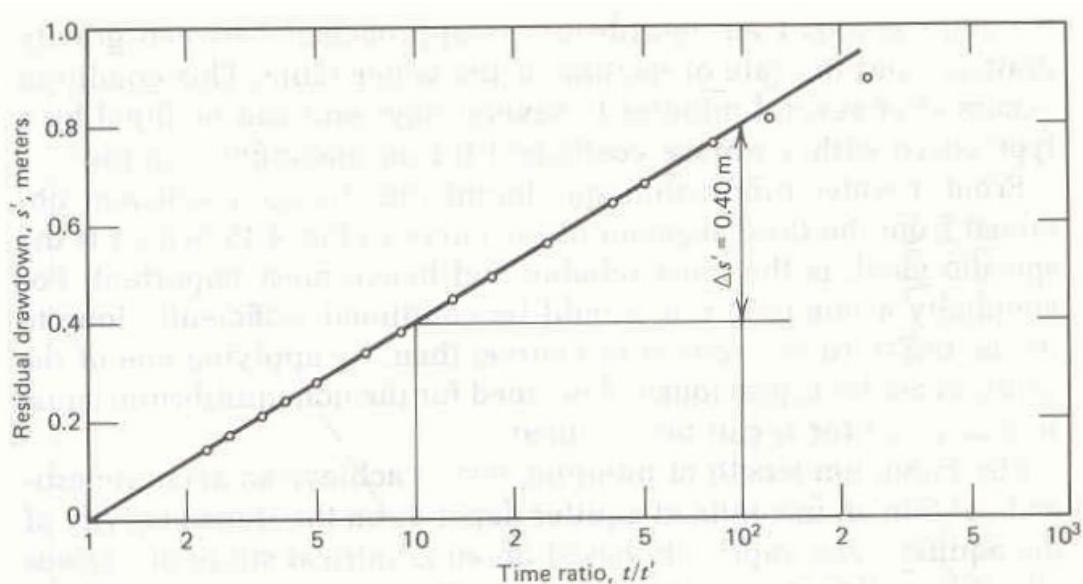
Table 6.7   Recovery Test Data (pump shut down at t=240 min)

t', (min)	t, (min)	t/t'	s', m
1.0	241	241	0.89
2.0	242	121	0.81
3.0	243	81	0.76
5	245	49	0.68
7	247	35	0.64
10	250	25	0.56
15	255	17	0.49
20	260	13	0.55
30	270	9	0.38
40	280	7	0.34
60	300	5	0.28
80	320	4	0.24
100	340	3.4	0.21
140	380	2.7	0.17
180	420	2.3	0.14

Answer 6.9

First, values of t/t' should be computed (they have been already computed in table 8.7), and then plotted versus s' on semi-logarithmic paper as shown below. A straight line is fitted through the points and Δs'=0.40 m is determined; then,

$$T = \frac{2.30\,Q}{4\pi\,s'} = \frac{2.30\times(2500)}{4\pi(0.40)} = 1140\ m^2/day$$

Example 6.10   Recovery Test

Figure 6.58 shows data from a recovery test for a fully penetrating well in a confined aquifer. Find the transmissivity of the aquifer from the data using Theis recovery equation (make your solution on Figure 8.58)

INFORMATION MAY BE REQUIRED TO SOLVE THE QUESTION

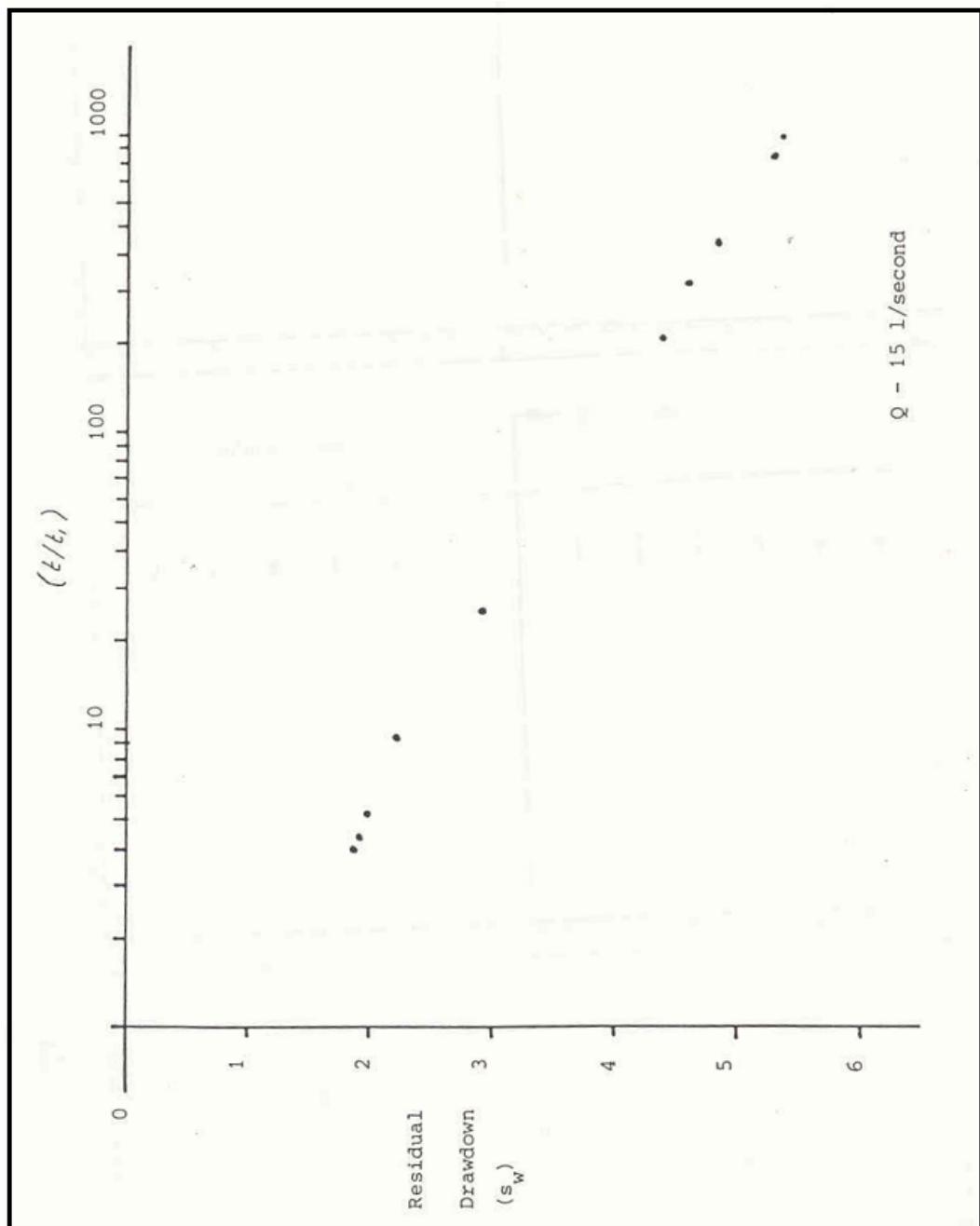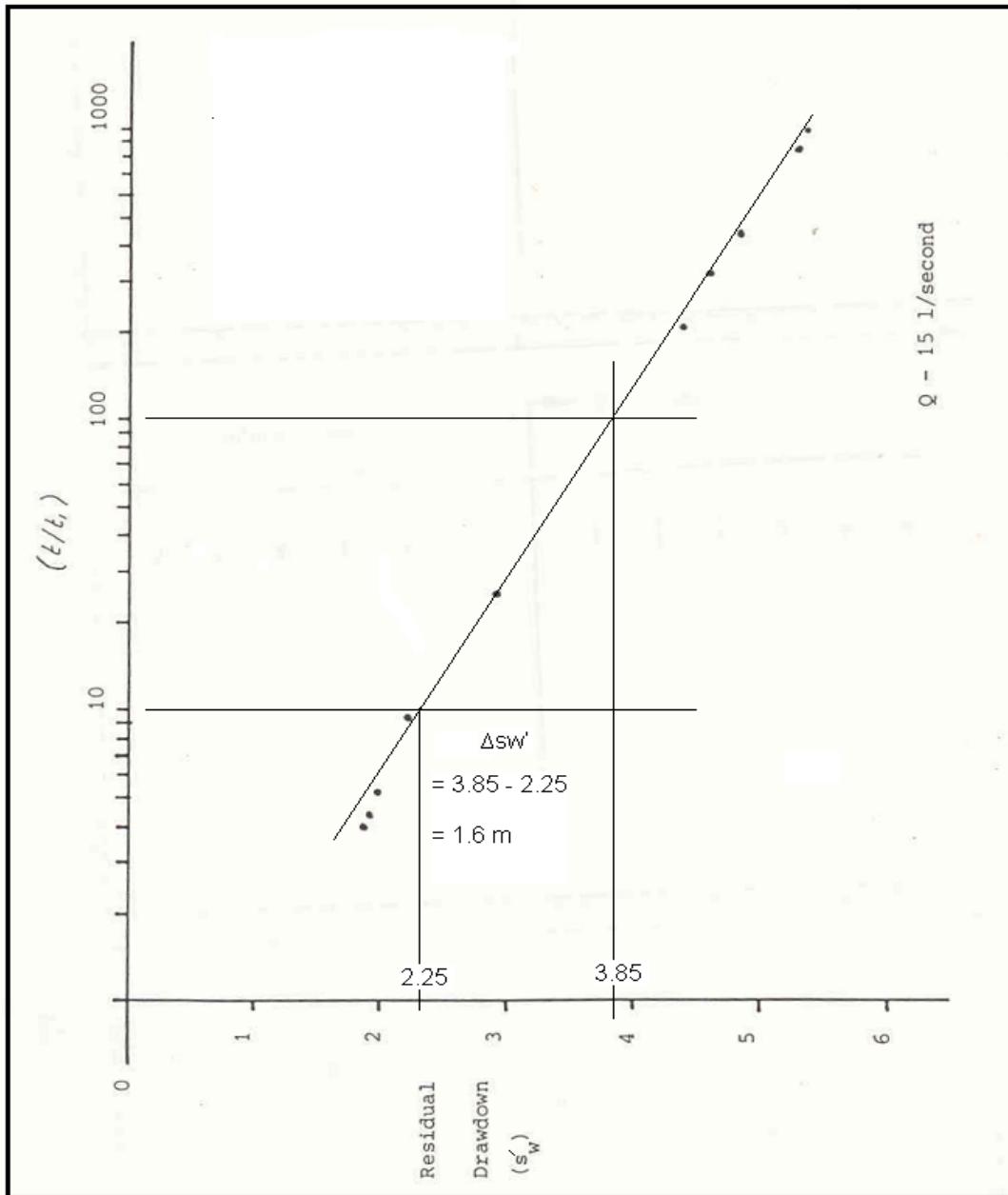$$T = \frac{Q}{2\pi(h_2 - h_1)} \ln\left(\frac{r_2}{r_1}\right) \qquad\qquad T = \frac{2.3Q}{4\pi\Delta s_w'}$$

Figure 6.58

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The graph shows Residual Drawdown (s'_w) on the horizontal axis and (t/t_r) on the vertical axis (logarithmic scale from 10 to 1000), with Q – 15 1/second.

$\Delta sw'$
= 3.85 - 2.25
= 1.6 m

$$Q = \frac{15}{1000} \, x\, 60\, x\, 60\, x\, 24$$

$$Q = 1296 \; m^3 / day$$

$$T = \frac{2.3Q}{4\pi \Delta s'_w}$$

$$= \frac{2.3\, x\, 1296}{4\pi\, x\, 1.6} = 148 \; m^2 / day$$

Example 6.11 Pumping Test Analysis for Deir Sharaf Well 2a (Nablus)

You have been asked to be a consultant for the Water Department of Nablus Municipality on Deir Sharaf Well Field. Your duties are to give instructions to the water department on the evaluation of the groundwater resources. Note that Nablus Municipality will undertake nothing more or less than you specify. Therefore, you must ensure that your suggestions and answers to all parts of this exercise are rational and workable. It is important to know that all the data you have for this exercise are the actual information, unless there is a specification of otherwise.

A number of pumping tests were carried out for DSW2a in order to determine the well performance and aquifer characteristics.

Figure 6.59 is the geological log of Deir Sharaf Well 2a.

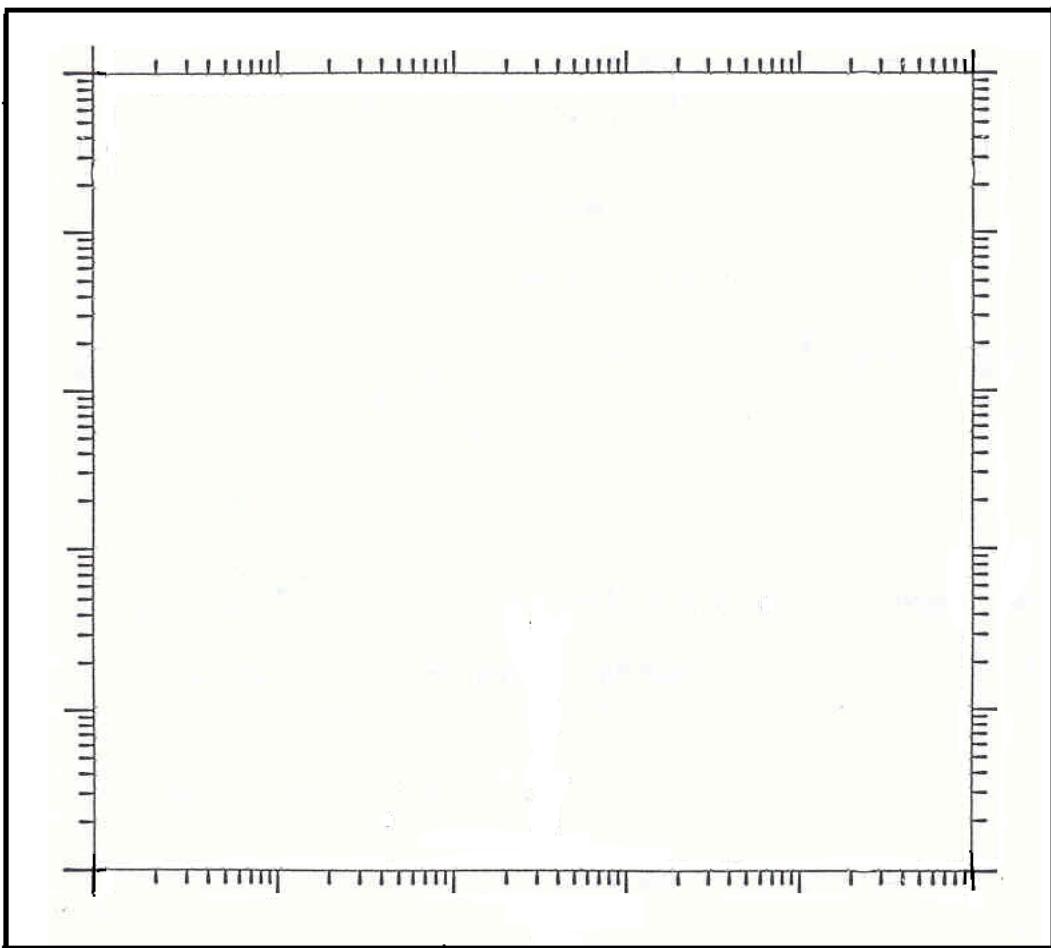| Depth [m bgl]	Formation	Geological Section	Lithological Description
0 – 6	Soil		
6 - 60	Bethlehem		yellowish, hard, crystalline limestone
60 – 255	Hebron	loss of circulation	gray, hard, crystalline dolomitic limestone
255 – 360	Yatta	loss of circulation	
360 – 580	Upper Beit Kahil	loss of circulation / loss of circulation	gray, crystalline, dolomitic limestone with some chert
580 - 670	Lower Beit Kahil		dark gray-brown, very hard dolomite

Figure 6.59   Deir Sharaf Well 2a Geological Log

PART A   Aquifer Type

Show that the Upper Beit Kahil formation is a confined aquifer. Use the following data of a constant pumping rate test:

Time (minutes)	Drawdown (m)
0.1	22.48
1.28	48.58
2.05	54.60
11.2	57.43
12.2	57.65
55.2	57.75
253.2	57.86

[NB: Log-log paper is attached]

300

Log- log paper

PART B   Aquifer Transmissivity

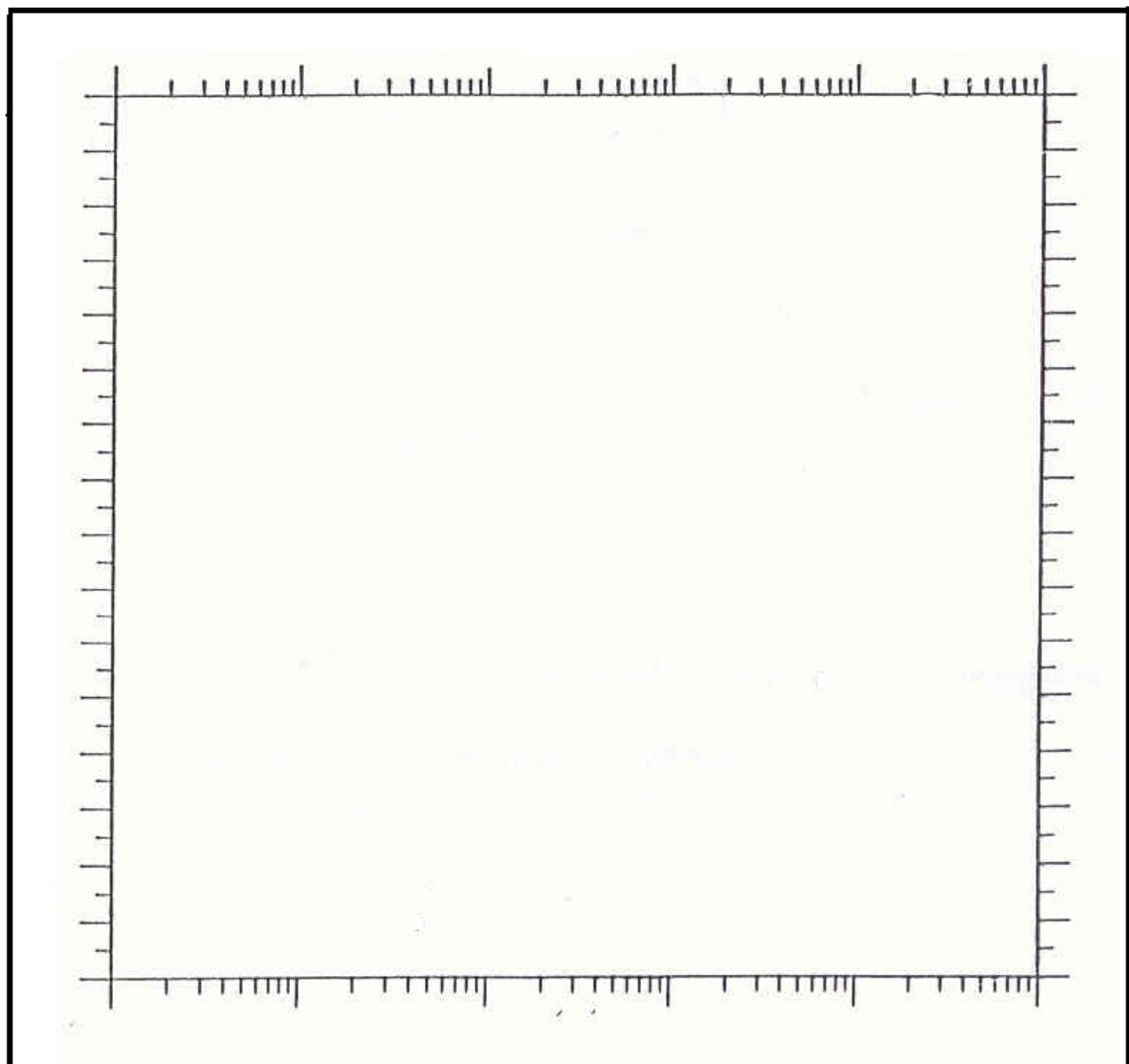1   Determine the Upper Beit Kahil transmissivity from the following recovery data.

Q = 150 m³/hr					
Pumping time = 610 minutes					
t' (min)	S' (m)	t' (min)	S' (m)	t' (min)	S' (m)
18	12.93	48	7.22	78	4.73
23	11.63	53	6.61	83	4.50
28	10.36	58	6.20	88	4.20
33	9.41	63	5.73	93	4.00
38	8.13	68	5.32	100	3.70
43	7.81	73	5.05	105	3.53

You will need the following equation: $T = \dfrac{2.3Q}{4\pi\,\Delta S}$

Where,

ΔS is residual drawdown per log cycle t/t'

t'   time since pumping stopped

t   total pumping time

S'   residual drawdown

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Semi-log paper

2   In the previous studies, it was reported that (**T**) of the Hebron Aquifer is about 300 m^2/day. **Why do you think the transmissivity you found for the Upper Beit Kahil is much smaller?**

PART C   **Step Drawdown Test**

A five-step test was carried out for DSW2a. The following data were recorded:

Step Number	Discharge (m³/day)	Accumulative Drawdown (m)
1	720	6.65
2	2040	27.41
3	3120	57.75
4	3360	65.73
5	4320	91.95

You will need the following equations in order to answer the questions of this section:

$$S = AQ + BQ^2$$

$$E_w = \frac{AQ}{S_{act}} x100\%$$

1   *Water Shortages and Pump Setting*

The Mayor of Nablus and the Water Department of the Municipality are putting much hope on this well in order to alleviate the water shortages of the Nablus area.

(i)   *Construct the Specific Capacity Curve* in order to answer Nablus Municipality whether the aquifer is developing or dewatering. *Do you think there is a chance for Nablus Municipality to increase the pumping rate?*

[NB: arithmetic paper is attached]

(ii)   You are informed that Nablus Municipality wants to increase the pumping rate of the well to nearly 250 m³/hr. Before doing that, they asked you to advise them on the feasibility of such increase in the yield. The current position of the pump intake is at 325 m bgl. *Discuss the feasibility of such increase with relation to the pump setting.*

Hint 1: Use Theis' equation to calculate the drawdown. Use the value of transmissivity that you calculated before, assume storativity= 3.586 x10^{-4}. Assume that the hydrogeological cycle is nine   months of continuous pumping throughout the year. The well is off operating for three months over winter. Given that the Static Water Level = 208 m bgl.

Thaïs's Equation:

$$s_w = \frac{Q}{4\pi T} W(u)$$
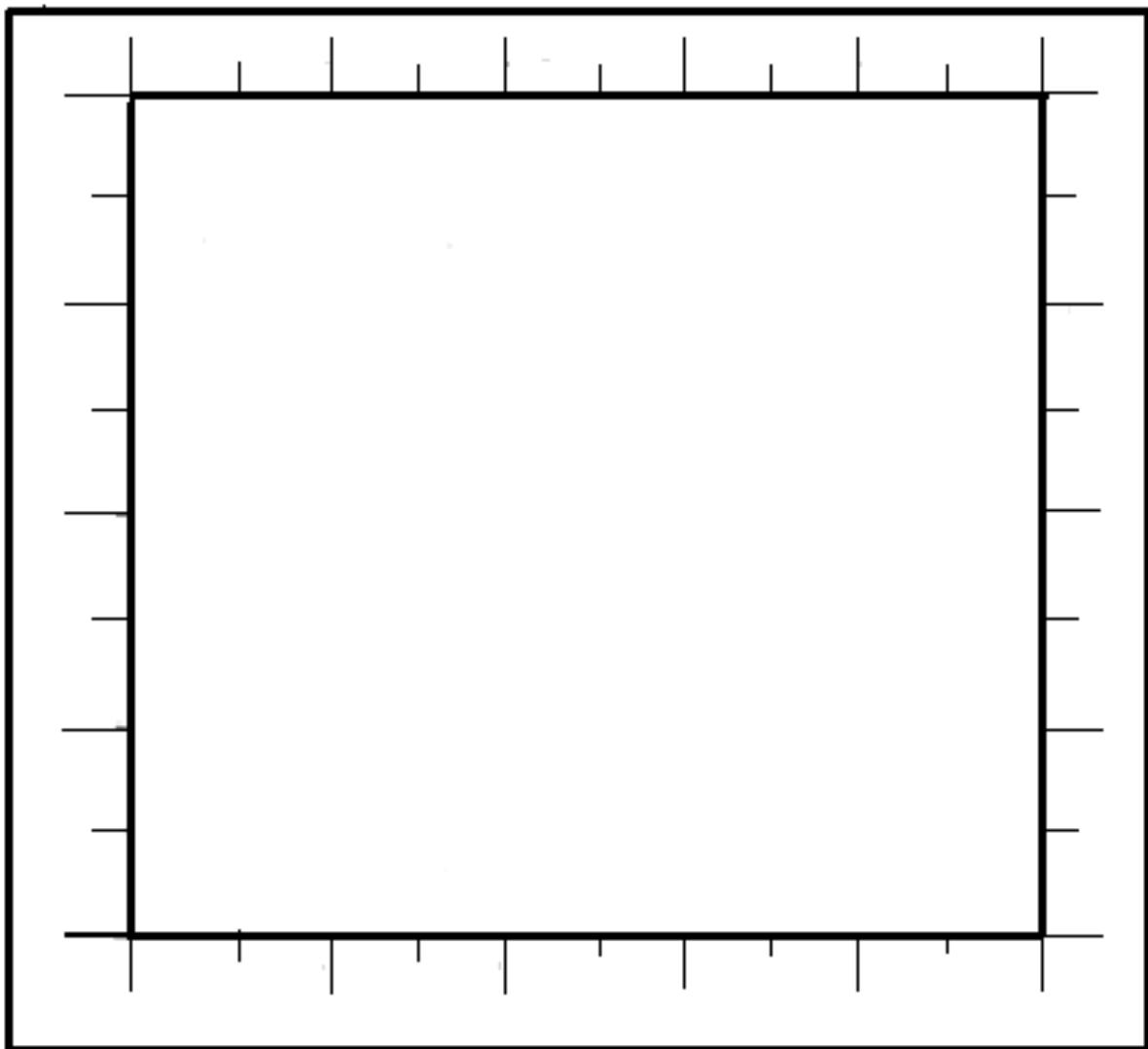
$$u = \frac{r^2 S}{4Tt}$$

$$W(u) = -0.5722 - \ln u + u - \frac{u^2}{2.2!}$$

Hint 2: Check the power required for the new pump setting. Chick the up-hole velocity for the new setting. Where do you suggest locating the new pump intake to meet the increase in the pumping rate?

The following information is for the current pump setting:

1   Pumping setting at 325 m bgl / Maximum diameter = 315 mm
2   Pump power = 400 hp
3   You may use the following equation $P = \dfrac{\rho g Q H}{746 \eta}$
4   Pump efficiency = 70%

(iii)   Assume that the dynamic water level dropped down below the top layer of the confined Upper Beit Kahil Aquifer. *How do you deal with this situation?*
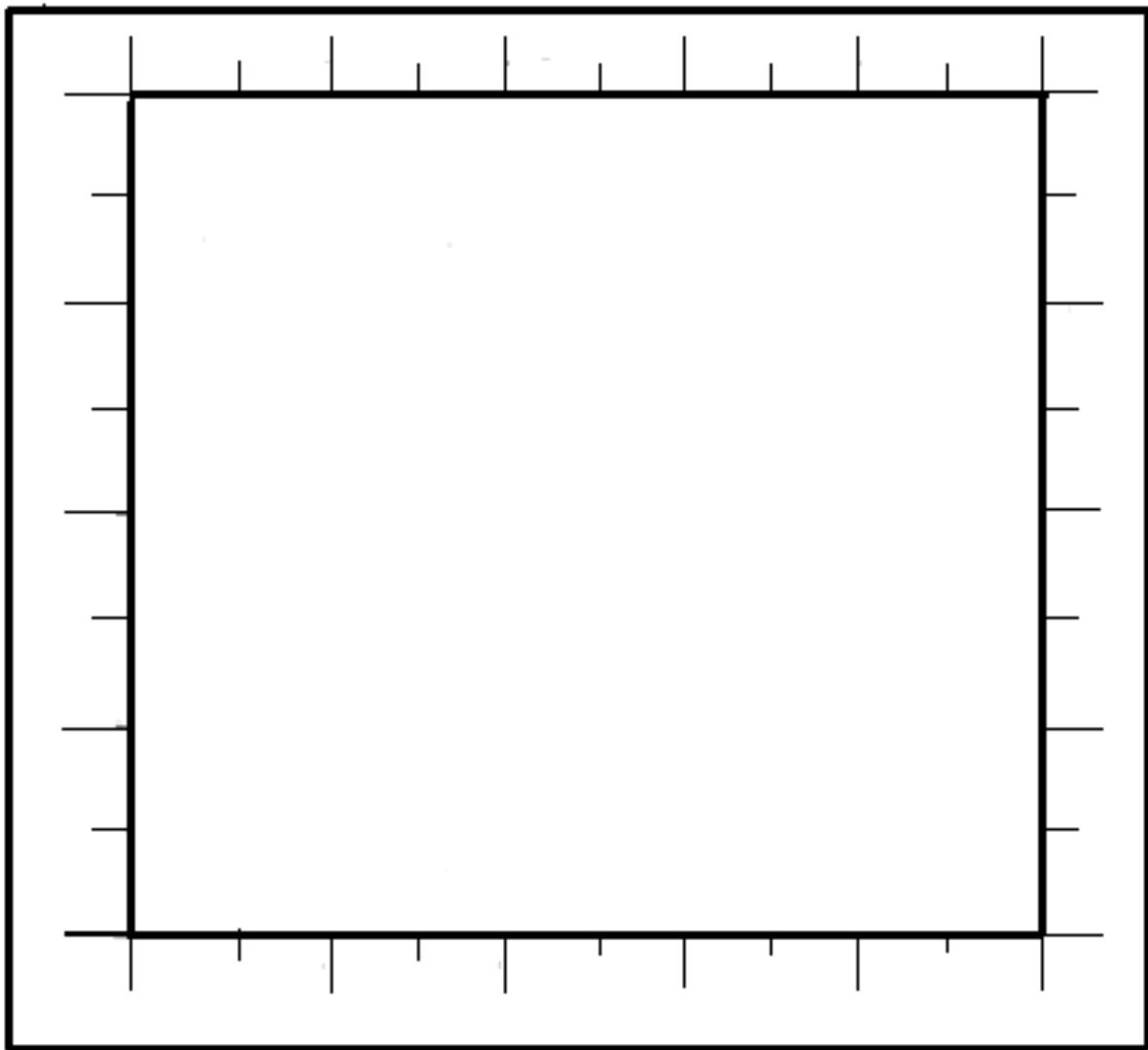
Arithmetic paper

2   *Well Performance*

(i)   Determine the well losses and aquifer losses coefficients using Hantusch-Bierschenk graphical method

[NB: arithmetic paper is attached]

Arithmetic paper
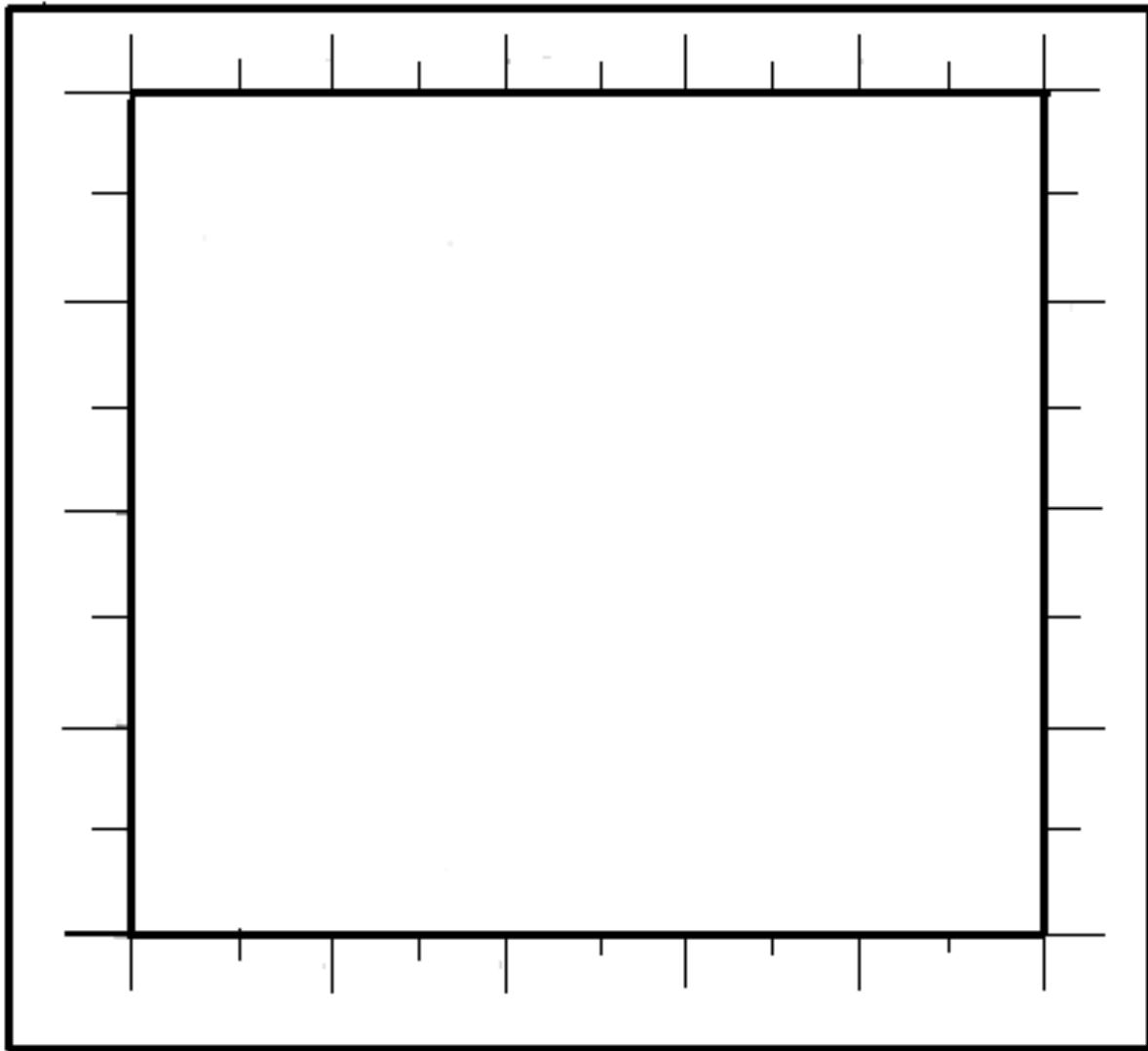
(ii)   *Determine the well efficiency.*

(iii)   Convert the value of well loss coefficient you obtained in min^2/m^5 and according to the following classification *determine the condition of DSW2a*:

Well loss coefficient (min^2/m^5)	Well Condition
< 0.5	Proper design
0.5 – 1.0	Mild deterioration
1.0 – 4.0	Sever deterioration
> 5.0	Immediate rehabilitation

3 Specific Capacity – Transmissivity Relationship

(i) *Use the value of Transmissivity you calculated earlier in order to develop a linear model relating the transmissivity and the specific capacity.*

(ii) *How do you relate your model with Logan's model?*

[NB: arithmetic paper is attached]

Arithmetic paper

4 Use Theim equation (equilibrium equation) to **calculate the radius of influence of the well. Do you think the heavy traffic of Nablus-Tulkarem road can affect the flow system in the Upper Beit Kahil Aquifer?**
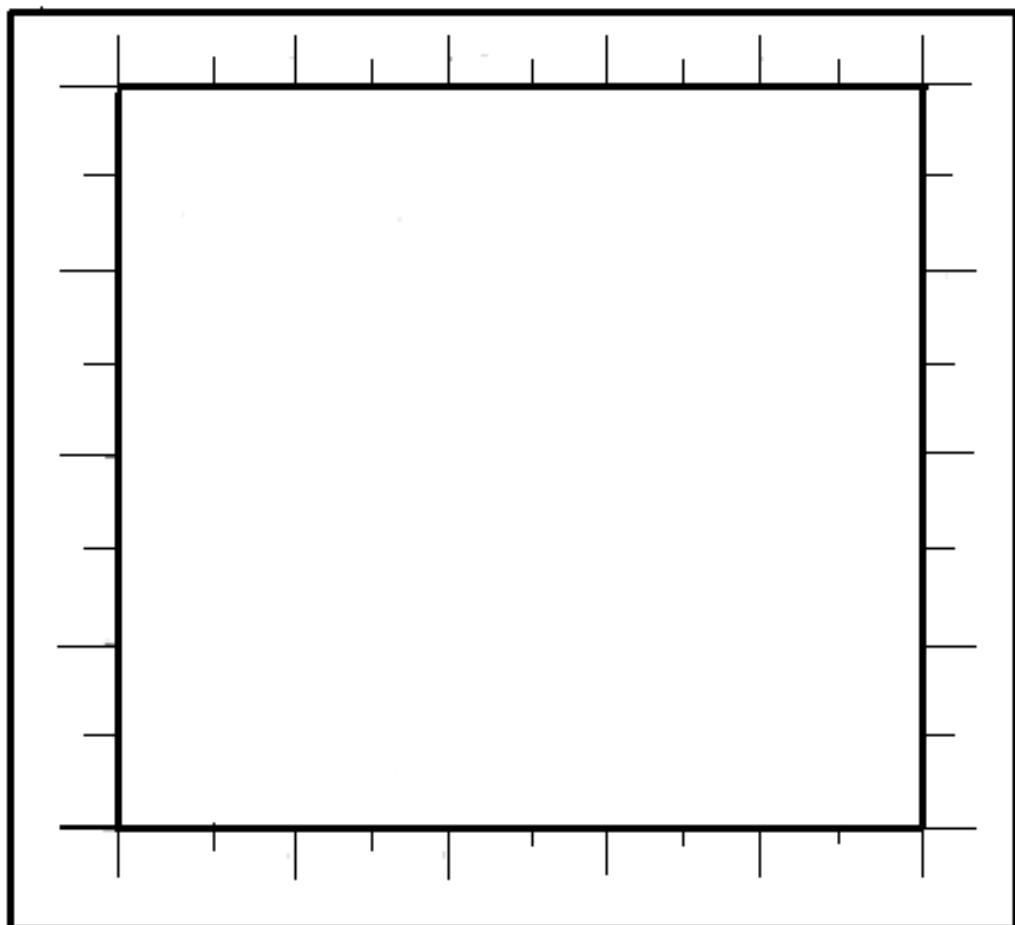
5   You are asked to give scientific reasoning for the following field observations:

The following table represents the drawdown data of the first step:

Time (min)	Drawdown (m)	Time (min)	Drawdown (m)
6	28.23	17	7.32
6.5	21.88	19	7.13
7	18.38	21	7.03
8.3	15.13	27	6.93
9.3	14.73	30	6.85
12	9.53	34	6.82
13	8.63	37	6.73
14	8.08	43	6.69
15	7.66	50	6.66
16	7.52	60	6.65

Plot the data on the arithmetic scales. The expected result is that the drawdown should have increased with time until it reached a sort of steady state at the end of the step. However, the curve you drew shows a recovery trend. Something went wrong in the procedure of carrying out this step. **Explain.**
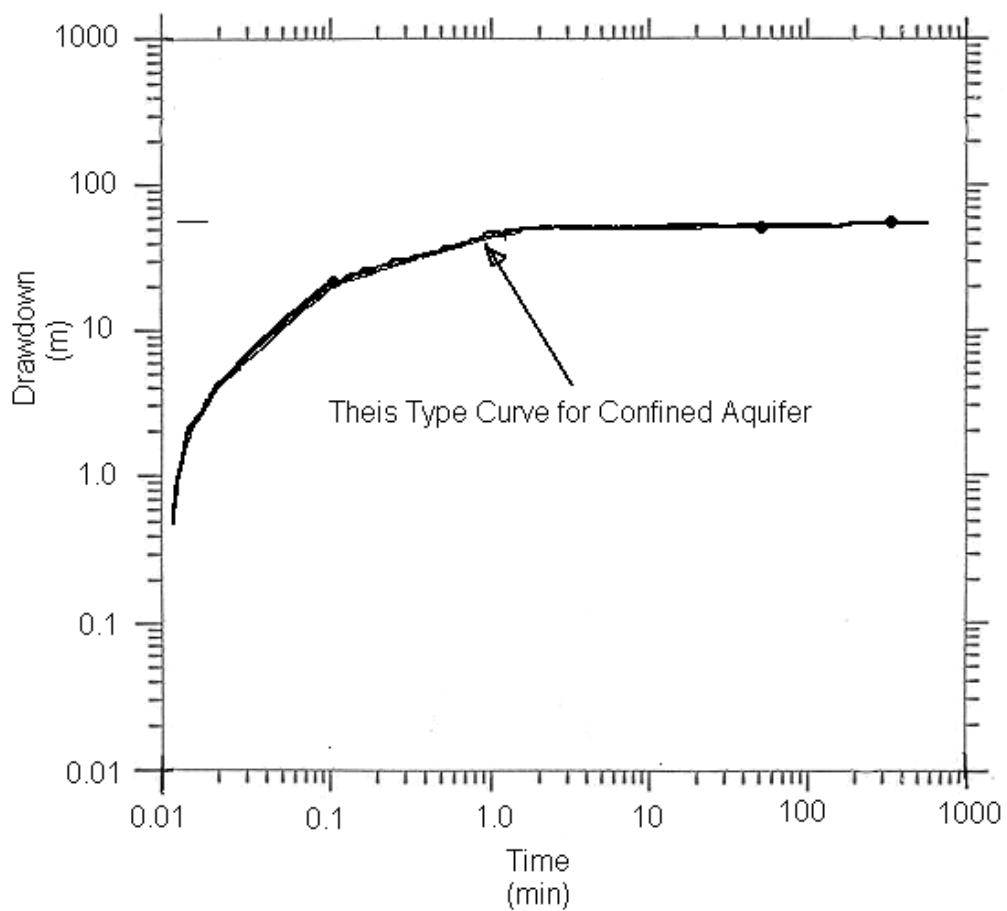
[NB: arithmetic paper is attached]

Arithmetic paper

Answer 6.11
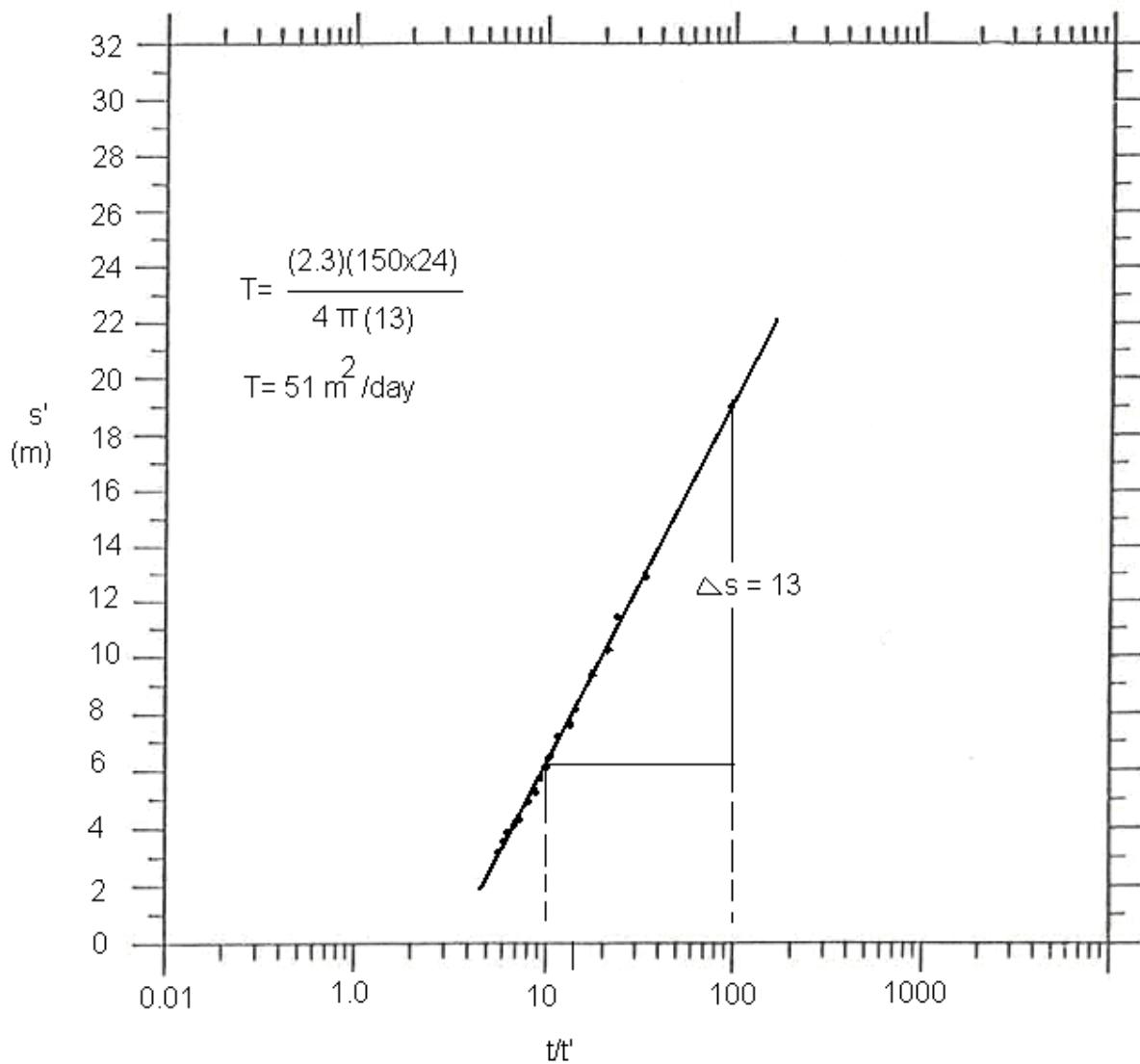
PART A

Theis Type Curve for Confined Aquifer

PART B

1

Pumping time = 610 minutes, so first we have to divide t/t' and then plot (t/t' versus s')

t' (min)	t/t'	s' (m)	t' (min)	t/t'	s' (m)	t' (min)	t/t'	s' (m)
18	33.9	12.93	48	12.7	7.22	78	7.8	4.73
23	26.5	11.63	53	11.5	6.61	83	7.4	4.50
28	21.8	10.36	58	10.5	6.20	88	6.9	4.20
33	18.5	9.41	63	9.7	5.73	93	6.6	4.00
38	16.1	8.13	68	9	5.32	100	6.1	3.70
43	14.2	7.81	73	8.4	5.05	105	5.8	3.53

$$T = \frac{(2.3)(150 \times 24)}{4\,\pi\,(13)}$$

$$T = 51 \text{ m}^2/\text{day}$$

$\triangle s = 13$
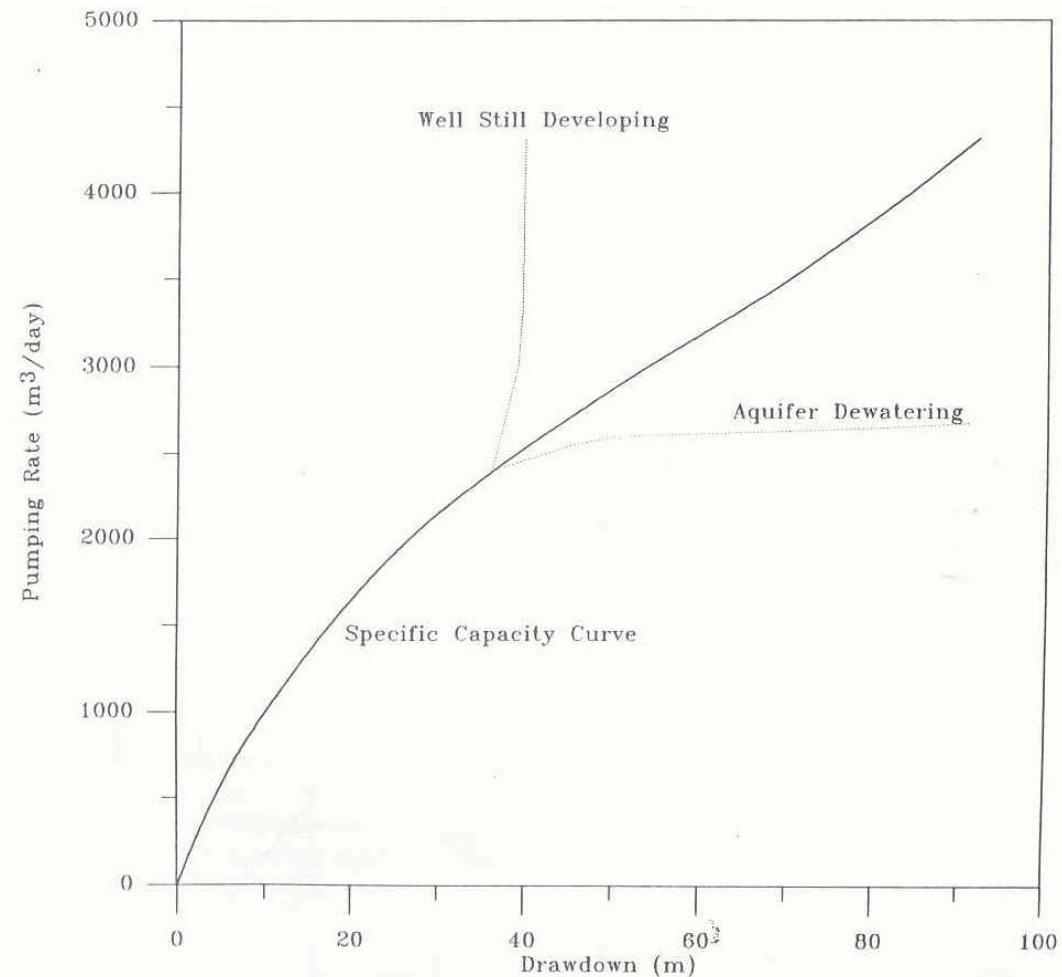
Solution (T) = 51 m²/day (as shown in the figure)

2 The transmissivity of the Upper Beit Kahil is much smaller that the transmissivity of Hebron Aquifer because the porosity (both primary and secondary) of the Upper Beit Kahil is much lower than that of Hebron Aquifer.

PART C

1   Water Shortages and Pump Setting
(i)

Step	Specific Capacity (m^3/day/m)	Changes in Specific Capacity
1	(720/6.65) = 108.3	
2	74.4	33.9
3	54	20.5
4	51.1	2.9
5	47	

The system doesn't show strong signs of dewatering. However, the specific capacity still getting down with increase of discharge;

For the first four steps a state of equilibrium (developing) was nearly achieved. After that (for the fifth step the rate of reduction of specific capacity increased.

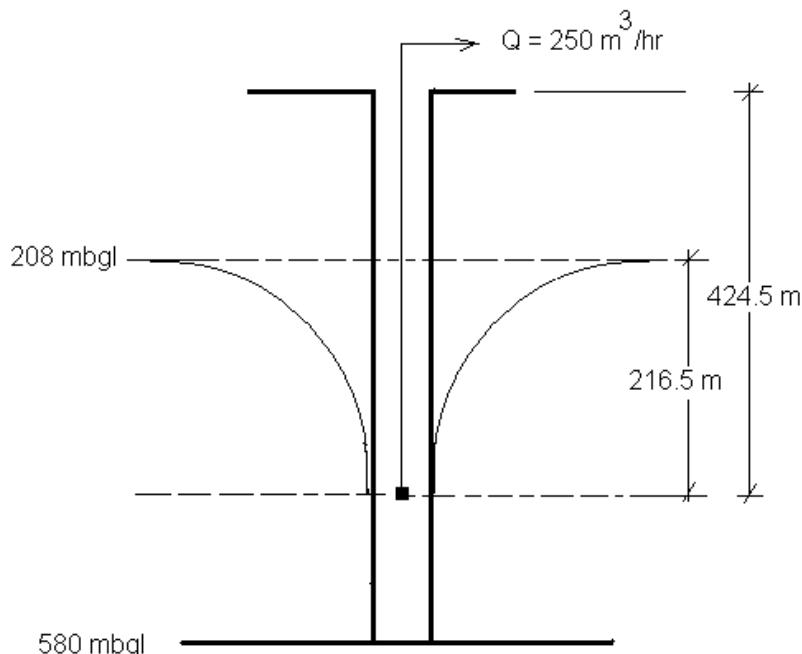 Yes, there is a chance for Nablus Municipality to increase the pumping rate.

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(ii)

$$T = 51 \ m^3 / day, \qquad S = 3.586 \times 10^{-4}, \qquad r_w = 0.315, \quad t = 9 \ months \times 30 = 270 \ days$$

$$u = \frac{(0.315)^2 \times 3.586 \times 10^{-4}}{4(51)(270)} = 5.14 \times 10^{-11}$$

$$W(u) = 23.12 \quad (from \ W(u) \ table)$$

$$s_w = \frac{(250)(24)}{4\pi(51)} \times 23.12 = 216.5 \ m$$

$$P = \frac{\rho \, g \, Q \, H}{746\eta} = \frac{1000 \times 9.81 \times 250 \times 424.5}{746 \times 0.7 \times 60 \times 60} = 554 \ hp > 400 \ hp \ (the \ current \ pump)$$

First, replace the current pump to a bigger one say 550 hp.
Second, pump intake should be at 425 m bgl + Safety factor (say 6 meters)

Now, check velocity in the rising pipes,
$$Q = VA$$

$$V = \frac{Q}{A} = \frac{250}{60 \times 60 \times \frac{\pi \times 0.315^2}{4}} = 0.89 \ m/\sec < 1.5 \ m/\sec \Rightarrow (O.K)$$
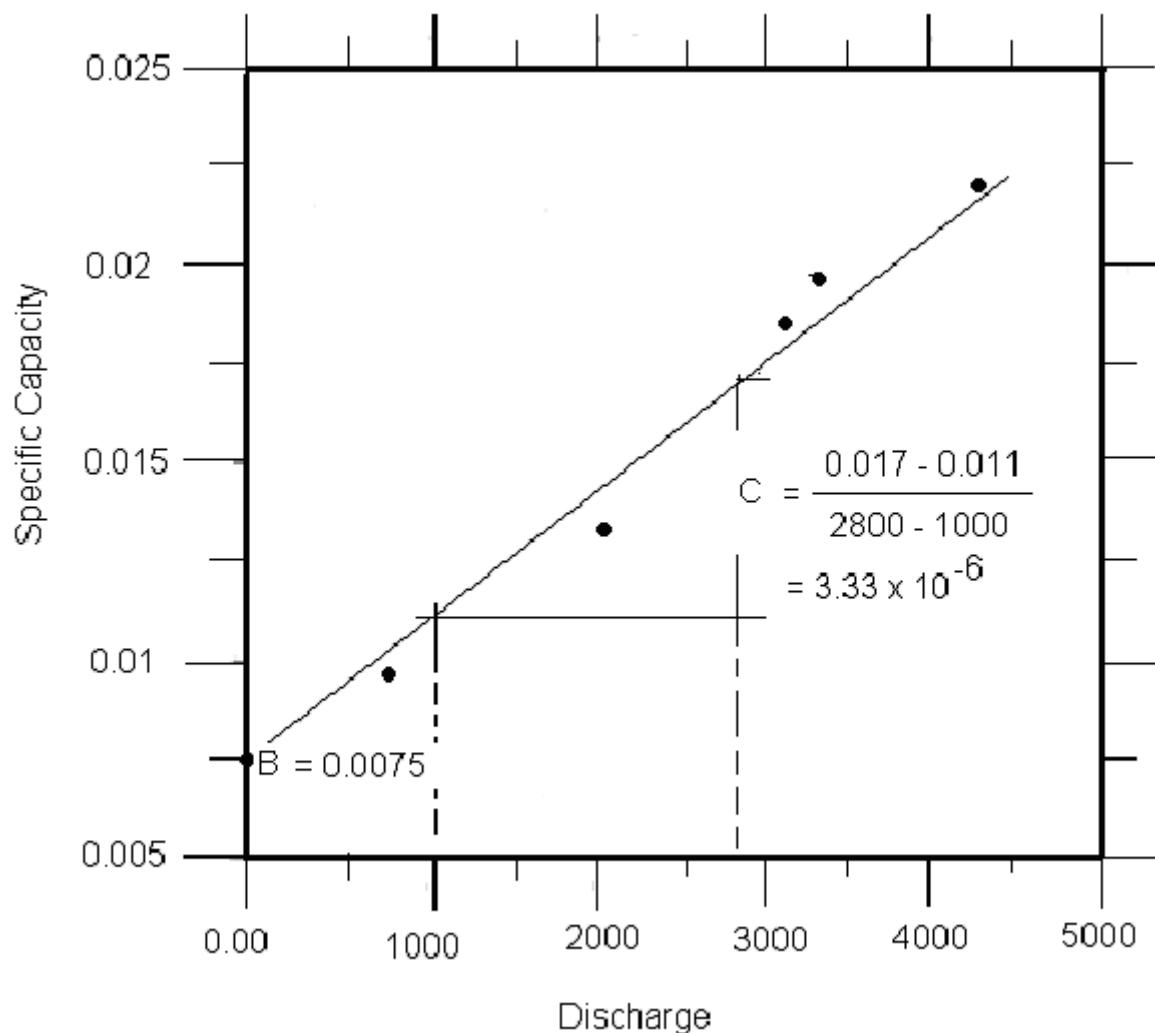
(iii)

If the water level dropped down below the top layer of the confined Upper Beit Kahil aquifer, then you can treat it as an unconfined aquifer.

2   Well Performance

(i)
First, we have to calculate the specific capacity for each step.

Discharge (m^3/day)	Drawdown (s_w) (meter)	Specific Capacity (Q/s_w) (m^3/day/m)
720	6.65	9.2×10^{-3}
2040	27.41	13.4×10^{-3}
3120	57.57	18.5×10^{-3}
3360	65.73	19.6×10^{-3}
4320	91.95	21.3×10^{-3}

From the graph,

Aquifer loss coefficient (B) = 7.5×10^{-3} day/m^2, and
Well loss coefficient (C) = 3.33×10^{-6} day^2/m^5

(ii)

Discharge (m³/day)	Drawdown (s_w) (meter)	Well Efficiency 100x (BQ/s_w)
720	6.65	81 %
2040	27.41	56 %
3120	57.57	41 %
3360	65.73	38 %
4320	91.95	35 %
AVERAGE Well Efficiency		50%

(iii)

Well loss coefficient = 3.33 x 10⁻⁶ (day²/m⁵) x (24x60)² = 6.9 min²/m⁵

C= 6.9 min²/m⁵ > 5, so **Immediate rehabilitation**, (from the given table)

3 **Specific Capacity- Transmissivity Relationship**

(i) Note that T=51 m²/day

$$\text{A linear model} = T = a \times \left(\frac{Q}{s_w} \right)$$

Step	Specific Capacity (m³/day/m)	a = (T/[Q/s_w])
1	108.3	0.47
2	74.4	0.69
3	54	0.94
4	51.1	1.00
5	47	1.09
Average value of a		**0.84**

At the fifth step **a= 1.09** which is the closest value to Logan's ($T = 1.22\left(\frac{Q}{s_w}\right)$), **a=1.22**

So, a liner model = $T = a \times \dfrac{Q}{s_w} = 0.84\left(\dfrac{Q}{s_w}\right)$

(ii) The relations between the prepared liner model and Logan's model:

1 Different coefficients 0.84 and 1.22
2 If we are closer to equilibrium, then we get closer coefficient to that of Logan's.

Notice 1.09 → 1.22 (*near equilbrium*)

0.47 → 1.22 (*far away from equilbrium*)

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4   Radius of Influence

$$T = \frac{2.303Q}{2\pi s_w} \log\left(\frac{R}{r_w}\right)$$

$$51 = \frac{2.303(4320)}{2\pi(91.95)} \log\left(\frac{R}{r_w}\right) \Rightarrow 51 = 17.22 \log\left(\frac{R}{r_w}\right)$$

$$\log\left(\frac{R}{r_w}\right) = 2.96 \qquad \Rightarrow \qquad \left(\frac{R}{r_w}\right) = 915$$

$$R = 915 \times 0.158 = 145 \; meters$$

** It sis not expected that traffic will affect deep confined aquifers even if the road is within the radius of influence.

5   Field observation and explanation

The technician set the pump at a higher rate than intended, when this was realised the pumping rate was adjusted to the intended one