ground water

Methods Note/

Approximate Solutions for Radial Travel Time and Capture Zone in Unconfined Aquifers

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Abstract

Radial time-of-travel (TOT) capture zones have been evaluated for unconfined aquifers with and without recharge. The solutions of travel time for unconfined aquifers are rather complex and have been replaced with much simpler approximate solutions without significant loss of accuracy in most practical cases. The current "volumetric method" for calculating the radius of a TOT capture zone assumes no recharge and a constant aquifer thickness. It was found that for unconfined aquifers without recharge, the volumetric method leads to a smaller and less protective wellhead protection zone when ignoring drawdowns. However, if the saturated thickness near the well is used in the volumetric method a larger more protective TOT capture zone is obtained. The same is true when the volumetric method is used in the presence of recharge. However, for that case it leads to unreasonableness over the prediction of a TOT capture zone of 5 years or more.

Introduction

Threats of contamination of groundwater supply wells triggered the development of methods to calculate groundwater travel times. Groundwater travel time is useful to predict the arrival time of contaminants to water supply wells and to delineate the wellhead protection area. For practical applications, numerical groundwater flow modeling combined with particle tracking is often used to compute travel times (Shafer 1987; Pollock 1988). Simpson et al. (2003) presented an analytic solution for these travel times for the idealized case of steady radial flow to a well in an unconfined aquifer without recharge. The solution is not explicit in terms of the travel time and is not simple to evaluate as it includes the calculation of the imaginary error function. Chapuis and Chesnaux (2006a, 2006b) presented an analytical solution for the same case in a different form, but it is essentially the same solution found by Simpson et al. (2003). Chapuis (2011) presented an approximate solution of travel time to a well for the case of unconfined flow with recharge, but his solution (equation 20) is exactly the same equation 14 in Chapuis and Chesnaux (2006a), which is an approximate equation for travel time in unconfined aquifer *without* recharge. No analytical solution is available for travel time to a well in unconfined aquifers with recharge.

The calculated fixed radius method (also called volumetric or cylinder method, U.S. EPA 1987) is widely used to delineate the wellhead protection area for small drinking water supply wells because it is simple and easy to use. The volumetric method requires a minimum of field data; only the aquifer thickness and porosity and the pumping rate of the well. It can be evaluated with a pocket calculator. Ceric and Haitjema (2005) provided guidelines for the applicability of the volumetric method when ambient flow is present. The volumetric method is developed for aquifers of constant saturated thickness and without recharge. The applicability of the volumetric method under unconfined flow conditions (variable saturated thickness) and in the presence of recharge has not been documented.

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In this paper, we present approximate solutions for travel time to a well in unconfined aquifers with and without recharge. For the case of no recharge, the solution of Simpson et al. (2003) is simplified to obtain an approximate solution. For the case of uniform recharge, an approximate solution is derived by assuming a constant saturated aquifer thickness. Comparison with a numerical evaluation of the solution by Simpson et al. (2003) indicates that large relative differences in travel time occur near the well, but travel times are very small near the well. The approximate solution, therefore, is sufficiently accurate overall, and has the advantage of providing an explicit analysis of influences of parameters on travel time and fast assessment of the wellhead protection area. We also investigated the effect of varying aquifer thickness and recharge on the delineation of time-of-travel (TOT) capture zones.

Approximate Solution of Travel Time in the Case of No Recharge

For a homogeneous and isotropic unconfined aquifer assuming that the flow is radial and that the head at a radial distance R from the well is given to be H_0, a steady-state solution of groundwater level response to a fully penetrating well with constant pumping rate in the center of the aquifer was found by Dupuit (1863):

$$h^2 = H_0^2 - \frac{Q_0}{\pi K} \ln\left(\frac{R}{r}\right) \tag{1}$$

where h is the groundwater level at radial distance r, H_0 the groundwater level at a constant head boundary of radius R, K the hydraulic conductivity, and Q_0 the (constant) pumping rate. The reference level for the heads is the impermeable bottom so that H_0 is also the initial saturated thickness, and h is the saturated thickness under the steady pumping state.

By applying Darcy's law to Equation 1, the velocity of radial groundwater flow can be expressed as follows:

$$v(r) = -\frac{Q_0}{2\pi r h n_\mathrm{e}} \tag{2}$$

where n_e is the effective porosity. The travel time of a water particle from a radial distance r to the well can be calculated as follows:

$$t = \int_r^{r_\mathrm{w}} \frac{\mathrm{d}\rho}{v(\rho)} = \frac{2\pi n_\mathrm{e}}{Q_0} \int_{r_\mathrm{w}}^{r} \rho h(\rho)\mathrm{d}\rho \tag{3}$$

where r_w is the radius of the well.

Simpson et al. (2003) derived an analytical solution to Equation 3 using integration by parts. The resulting equation 9 in the study by Simpson et al. (2003) consists of two parts. The first part can be transformed to

$$t_1(r) = \frac{\pi n_\mathrm{e}}{Q_0}\left[r^2\sqrt{h_\mathrm{w}^2 + \frac{Q_0}{\pi K}\ln\left(\frac{r}{r_\mathrm{w}}\right)} - r_\mathrm{w}^2 h_\mathrm{w}\right]. \tag{4a}$$

The second part can be transformed to

$$t_2(r) = \frac{\pi n_\mathrm{e} r_\mathrm{w}^2}{4}\sqrt{\frac{2}{K Q_0}} \exp\left(-\frac{2\pi K}{Q_0} h_\mathrm{w}^2\right) \times [\mathit{erfi}(g(r)) - \mathit{erfi}(g(r_\mathrm{w}))] \tag{4b}$$

where erfi(x) is the imaginary error function, and

$$g(r) = \sqrt{\frac{2\pi K}{Q_0}}\sqrt{h_\mathrm{w}^2 + \frac{Q_0}{\pi K}\ln\left(\frac{r}{r_\mathrm{w}}\right)}. \tag{4c}$$

The total travel time is calculated by

$$t(r) = t_1(r) - t_2(r) \tag{5}$$

Equations 4 and 5 were implemented in a spreadsheet, while Equation 3 was also numerically integrated by use of the trapezoidal rule for comparison. Two test cases were evaluated for different ratios of the drawdown ($s_\mathrm{w} = H_0 - h_\mathrm{w}$) at the well to the initial saturated thickness (H_0) of the aquifer: the first test case has a ratio of 6% and the second of 50%. For both cases, $t_2(r)$ is very small (less than a day) and can be neglected. Furthermore, the second term in $t_1(r)$ can also be neglected. So, a simplified formula for $t(r)$ can be derived as follows:

$$\tilde{t}(r) = \tau_0\left(\frac{r}{R}\right)^2\sqrt{1 - \frac{Q_0}{\pi H_0^2 K}\ln\left(\frac{1}{r/R}\right)} \tag{6}$$

where

$$\tau_0 = \frac{\pi R^2 H_0 n_\mathrm{e}}{Q_0} \tag{7}$$

The parameter τ_0 is the mean residence time (storage divided by discharge of the well) of the unconfined aquifer with a constant thickness under conditions of no recharge.

In Figure 1, we show the relative error in travel times (defined as percentage of exact travel time minus approximate travel time divided by the exact travel time) obtained from the simplified formula (Equation 6) compared with the exact solution (Equations 4 and 5). Large errors occur only near the well, but reduce rapidly to less than 0.15% at a small radial distance less than 1% of R. As expected, the relative error is larger for the larger ratio (50%) of the drawdown to the initial saturated thickness than for the smaller ratio (6%).

In Figure 2, we plot travel time curves calculated by the exact solution and the simplified solution for the second test case where the drawdown at the well is 50% of the initial saturated aquifer thickness. The values of parameters used are: $Q_0 = 5000$ m^3/d, $r_\mathrm{w} = 0.1$ m, $H_0 = 20$ m, $K = 50$ m/d, $n_\mathrm{e} = 0.3$, and $R = 1261.5$ m. We find from Equation 7 that $\tau_0 = 6000$ d (16.4 years). It is observed from Figure 2 that even for the case of 50% drawdowns the simplified solution (7) with (8) is adequate. The radial distance shown on the horizontal axis in Figure 2 is the radius of the circular TOT capture

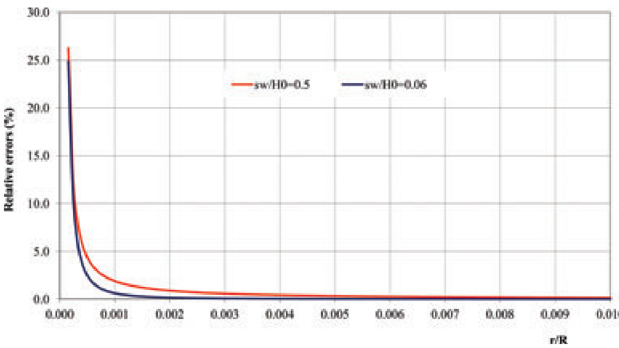

Figure 1. Relative errors of calculated travel time by simplified formula compared with exact solution.

zone concentric about the well with an associated travel time shown on the vertical axis. This is also true for the remaining figures in this paper.

Approximate Solution of Travel Time in the Case of Uniform Recharge

The analytical solution of the groundwater level for steady radial flow to a well in an unconfined aquifer with uniform recharge was given by Bouwer (1978) as follows:

$$h^2 = H_0^2 - \frac{Q_0}{\pi K}\ln\left(\frac{R}{r}\right) + \frac{w}{2K}(R^2 - r^2) \tag{8}$$

where w is the uniform recharge, and R is selected at the water divide of the catchment area of the well, so that:

$$R = \sqrt{\frac{Q_0}{\pi w}}. \tag{9}$$

For a uniform recharge $w = 0.001$ m/d and $Q_0 = 5000$ m^3/d, R is calculated to be 1261.5 m.

The groundwater velocity can be calculated by dividing the total radial flow (Q_r) at radial distance r with the cross-sectional area (A_r) of water flow. The total radial flow is as follows:

$$Q_r = -Q_0 + \pi r^2 w. \tag{10}$$

The cross-sectional area of water flow is:

$$A_r = 2\pi r h n_e. \tag{11}$$

The velocity follows from Equation 10, and Equation 11 is given as

$$v(r) = -\frac{Q_0 - \pi r^2 w}{2\pi r h n_e}. \tag{12}$$

The travel time of the recharge water at radial distance r from the well can be calculated as

$$t = \int_r^{r_w} \frac{d\rho}{v(\rho)} = \int_{r_w}^{r} \frac{2\pi n_e \rho h(\rho)}{Q_0 - \pi\rho^2 w} d\rho. \tag{13}$$

Analytical evaluation of Equation 13 is hampered by the fact that the saturated aquifer thickness h is a complicated function of r (Equation 8). By replacing h with the initial saturated aquifer thickness H_0 in Equation 13, an approximate solution is obtained as follows:

$$\tilde{t}(r) = \tau_w \ln\left(\frac{Q_0 - \pi r_w^2 w}{Q_0 - \pi r^2 w}\right) = \tau_w \ln\left(\frac{1 - \pi r_w^2 w/Q_0}{1 - \pi r^2 w/Q_0}\right) \tag{14}$$

where

$$\tau_w = \frac{\pi R^2 H_0 n_e}{\pi R^2 w} = \frac{H_0 n_e}{w} \tag{15}$$

is the mean residence time (storage divided by total recharge) of an aquifer with uniform recharge and constant thickness. Haitjema (1995) found the same formula for the mean residence time of a semi-confined aquifer with uniform recharge.

In Equation 14, the term in the numerator of the logarithm function, $\pi r_w^2 w/Q_0$, is very small and can be neglected. The term in the denominator, $\pi r^2 w/Q_0$, represents the ratio of the recharge water within radius r to the total well discharge and is denoted as η. The travel time Equation 14 becomes

$$\tilde{t}(r) = \tau_w \ln\left(\frac{1}{1-\eta}\right) \tag{16}$$

where

$$\eta = \frac{\pi r^2 w}{Q_0}. \tag{17}$$

To verify the accuracy of the approximate solution (16) we compared it with a numerical integration (Simpson's rule) of Equation 13. Since the travel time becomes infinite at $r = R$, the numerical integration was calculated up to $r = 1261$ m, within which the total recharge is about

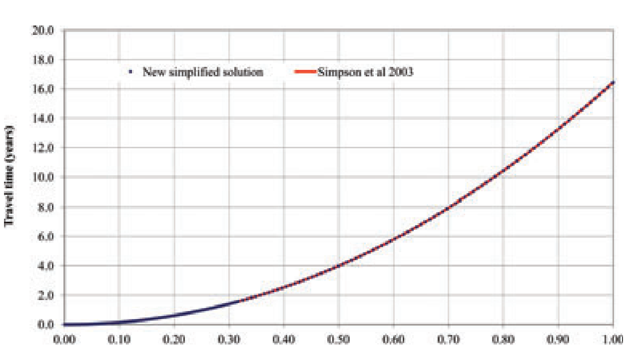

Figure 2. Comparison of travel time curves calculated by the exact solution and the simplified solution when the drawdown at the well is 50% of the initial saturated aquifer thickness.

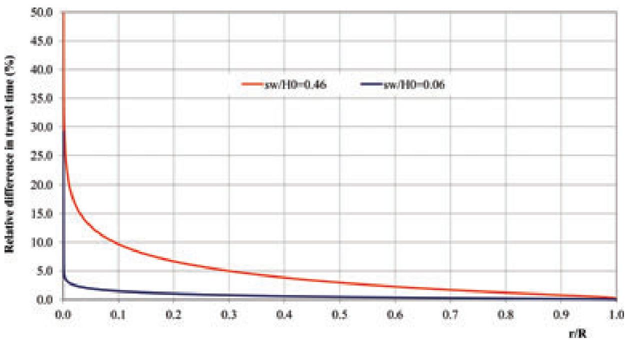

Figure 3. Relative differences of calculated travel times by approximate solution (16) and exact solution (13).

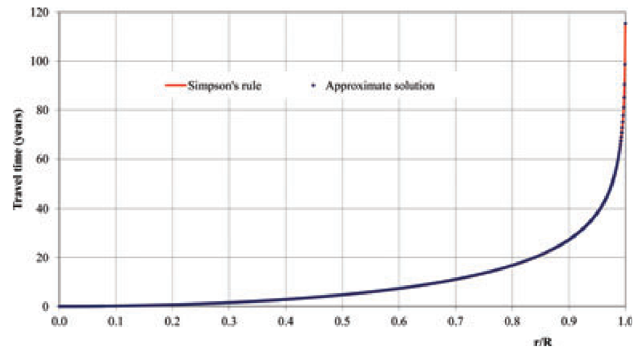

Figure 4. Comparison of travel time curves calculated by approximate solution (16) and a numerical integration of Equation 13 when the drawdown at the well is 46% of the initial saturated aquifer thickness.

99.9% of the pumping rate. Two cases were evaluated: the drawdown at the well is 6% of the initial saturated thickness ($H_0 = 50$ m) for the first case and 46% ($H_0 = 20$ m) for the second case. For the first case, the approximate solution yields slightly larger travel times near to the well than the numerical integration of the exact solution (13), but the relative difference becomes less than 5% at a radial distance less than 4% of R (Figure 3). For the second case, the relative difference between Equations 13 and 16 is much larger near the well and reduces to less than 5% at a radial distance of around 30% of R (Figure 3). However, the large relative differences near the well will not result in large differences in total travel times since the travel times near the well are very small (Figure 4).

It can be seen from Figure 4 that travel time increases drastically at longer distance, especially near the water divide. The travel time of recharge water near the water divide (at $r = 1261$ m) to the well was calculated to be 115.3 and 115.4 years by the exact and approximate formulation, respectively. The Simpson's rule and approximate solution are very close for computing the maximum travel time. The mean residence time is estimated with Equation 15 to be 6000 d (16.4 years), which might be overestimated by using a constant thickness. The mean residence time for the numerical solution is difficult to calculate because of the fact that the travel time approaches infinity near the water divide. Our calculation of the mean travel time from Simpson's rule did not include infinity, of course, and came out a little lower: 5951 d (16.3 years). A numerical integration of the total unconfined aquifer storage divided by the total recharge yields a mean residence time of 5939 d (16.3 years). Equation 15 appears sufficiently accurate to estimate the mean residence time even for the case where the drawdown at the well is 46% of the initial saturated aquifer thickness.

Implications for the Wellhead Protection Area

The volumetric method calculates a fixed radius (r_p) of the wellhead protection area (U.S. EPA 1987, 2004) using a volumetric flow equation:

$$r_p = \sqrt{\frac{Q_0 t_p}{\pi H n_e}} \tag{18}$$

where H is the aquifer thickness and t_p the TOT threshold. The typical threshold values of travel times used in the world are 5, 10, and 25 years (Van Waegeningh 1981).

In Figure 5, we present the relationships between capture zone radius and associated TOT for the cases of unconfined flow with and without recharge presented in this paper and for the volumetric method. We used the same pumping rate and aquifer parameters as before: $Q_0 = 5000$ m^3/d, $r_w = 0.1$ m, $H_0 = 20$ m, $K = 50$ m/d, $n_e = 0.3$, $R = 1261.5$ m, w $= 0.001$ m/d. For the volumetric method we selected both the initial saturated thickness H_0 and the saturated thickness near the well h_w as the constant aquifer thickness H in Equation 18. It is seen that for $H = h_w$ the radius of the capture zone is dramatically overestimated. The effect of recharge also leads to a smaller capture zone radius; see Figure 5. Consequently, ignoring recharge by using the volumetric method overestimates the capture zone size, which is more protective. However, for travel times beyond, say 5 years the overestimation of the radius

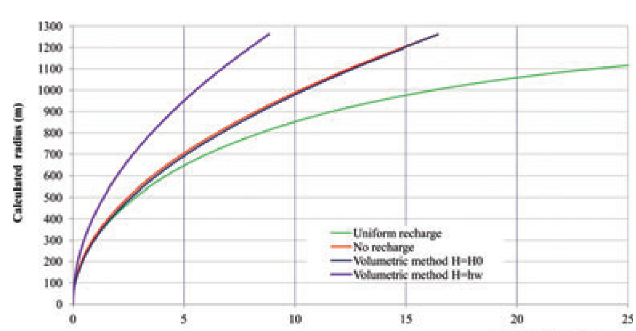

Figure 5. Calculated radius by the volumetric method and travel time equations for unconfined aquifers with and without recharge.

increases dramatically and the use of the volumetric method seems inappropriate.

Conclusions

We analyzed both the impact of recharge due to precipitation and the impact of a varying aquifer thickness in unconfined aquifers on the size of a radial TOT capture zone. For the case of unconfined aquifers without recharge, the relationship between the travel time and associated TOT capture zone radius is complicated by the presence of imaginary error functions. A much simpler approximate formulation for this case is nearly indistinguishable from the exact solution, except for very small travel times. We found that the radius of TOT capture zones in unconfined aquifers without recharge is slightly larger than when the volumetric method is used with the initial saturated aquifer thickness. However, the volumetric method calculates a much larger radius of TOT capture zones when the saturated aquifer thickness near the well is used.

We also compared the TOT capture zone for the unconfined aquifers to recharge. Here too we offer an approximate solution to the exact formulation of the problem, which appears nearly indistinguishable from the exact formulation. In our example, we found that both the volumetric method and the unconfined aquifer without recharge calculate larger radius of TOT capture zones for normally used travel time criteria (5, 10, and 25 years). Hence, ignoring recharge leads to a conservative (more protective) wellhead protection zone design.

In all of our analyses we considered radial flow, thus ignoring the effect of any ambient groundwater flow. The effect of ambient flow on the TOT capture zone for unconfined aquifers without recharge is expected to be similar as for the volumetric method (Ceric and Haitjema 2005). Generally, radial symmetry of the flow field is lost for larger travel times (larger distances from the well), see Ceric and Haitjema (2005). It is seen in Figure 5 that beyond a travel time of 5 years the volumetric method increasingly over predicts the TOT capture zone radius because it ignores recharge. As the presence of recharge is the norm, rather than the exception, the use of the volumetric method should probably be limited to relatively small travel times: 5 years or less. This is also consistent with the criteria developed by Ceric and Haitjema (2005) to maintain (approximate) radial symmetry of the TOT capture zone in the presence of ambient flow. If ambient flow is (nearly) absent and radial wellhead protection zones are to be developed for larger travel times, for instance 10 or 25 years, recharge should not be ignored (Figure 5). For that case we recommend using Equations 16 and 17.

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