# AXIOMAS Y TEOREMAS MATD

Axiomas

15.1 $a+(b+c)=(a+b)+c$
15.2 $a\cdot(b\cdot c)=(a\cdot b)\cdot c$
15.3 $a+b=b+a$
15.4 $a\cdot b=b\cdot a$
15.5 $a+0=a$ (identi. +)
15.6 $a\cdot 1=a$ (identi. ·)
15.7 $a\cdot(b+c)=(a\cdot b)+(a\cdot c)$
15.8 $a+(-a)=0$ (inveso aditivo)
15.9 $c\neq 0 \Rightarrow (a\cdot c=b\cdot c)\Rightarrow a=b$

Teoremas

15.10 $a=b \Rightarrow a\cdot c=b\cdot c$
15.11 $a\cdot 0=0$ (anulador de ·)
15.12 $a+c=a \equiv c=0$ (iden. unica)
15.13 $a\neq 0 \Rightarrow (a\cdot c=a \equiv c=1)$
15.14 $a+b=0 \wedge a+c=0 \Rightarrow b=c$

Axioma sustracción
15.15 $a-b=a+(-b)$

Teoremas

15.16 $a+c=b+c \equiv a=b$
15.17 $a+b=0 \equiv a=-b$ (inverso aditivo)
15.18 $a=b \equiv -a=-b$
15.19 $-(-a)=a$ (doble inverso)
15.20 $-0=0$
15.21 $-(a+b)=(-a)+(-b)$
15.22 $-a=(-1)\cdot a$
15.23 $a\cdot(-b)=(-a)\cdot b=-(a\cdot b)$
15.24 $(-a)\cdot(-b)=a\cdot b$
15.25 $a-0=a$ (sustracción nula)
15.26 $(a-b)+(c-d)=(a+c)-(b+d)$
15.27 $(a-b)-(c-d)=(a+d)-(b+c)$
15.28 $(a-b)\cdot(c-d)=(a\cdot c+b\cdot d)-(a\cdot d+b\cdot c)$
15.29 $a-b=c-d \equiv a+d=b+c$

Axiomas
15.30 $\neg(a<a)$
15.31 $a<b \wedge b<c \Rightarrow a<c$
15.32 $a\neq b \Rightarrow a<b \vee b<a$
15.33 $a<b \Rightarrow a+c<b+c$
15.34 $a<b \wedge 0<c \Rightarrow a\cdot c<b\cdot c$

Def 15.1.
- Mayor: $a>b \equiv b<a$
- $a\leq b \equiv a=b \vee a<b$
- $a\geq b \equiv a=b \vee a>b$

Propied. Relacion de orden
15.35 $a<b \wedge c<d \Rightarrow a+c<b+d$
15.36 $a\leq b \wedge b<c \Rightarrow a<c$
15.37 $a<b \wedge b\leq c \Rightarrow a<c$
15.38 $a\leq b \wedge b\leq c \Rightarrow a\leq c$
15.39 $0<c \Rightarrow (a\cdot c<b\cdot c \Rightarrow a<b)$
15.40 $0\leq c \wedge a\leq b \Rightarrow a\cdot c\leq b\cdot c$
15.41 $(a<b \equiv a=b \equiv a>b) \wedge \neg(a<b \wedge a=b \wedge a>b)$
15.42 $a+c<b+c \Rightarrow a<b$
15.43 $a\leq b \wedge b\leq a \equiv a=b$
15.44 $a\leq a$
15.45 $a\leq b \equiv (\forall z : D : z\leq a \Rightarrow z\leq b)$
15.46 $a\leq b \equiv (\forall z : D : b\leq z \Rightarrow a\leq z)$
- Desigualdad indirecta.
15.47 $a=b \equiv (\forall z : D : z\leq a \equiv z\leq b)$
15.48 $a=b \equiv (\forall z : D : b\leq z \equiv a\leq z)$
- Igualdad indirecta.

Axiomas $\downarrow$ y $\uparrow$ 15.60
$(\forall z : D : z\leq a\downarrow b \equiv z\leq a \wedge z\leq b)$
$(\forall z : D : a\uparrow b\leq z \equiv a\leq z \wedge b\leq z)$

Propiedades $\downarrow$ y $\uparrow$
15.61 $a\downarrow b=b\downarrow a$ (simetria) $\downarrow$
15.62 $a\uparrow b=b\uparrow a$ (simetria) $\uparrow$
15.63 $a\downarrow(b\downarrow c)=(a\downarrow b)\downarrow c$
15.64 $a\uparrow(b\uparrow c)=(a\uparrow b)\uparrow c$
15.65 $a\downarrow a=a$ (idempotencia $\downarrow$)
15.66 $a\uparrow a=a$ (idempotencia $\uparrow$)

15.67 $a\leq b \equiv a\downarrow b=a$
15.68 $a\leq b \equiv a\uparrow b=b$
15.69 $a\downarrow b\leq a \wedge a\downarrow b\leq b$
15.70 $a\uparrow b\geq a \wedge a\uparrow b\geq b$
15.71 $a\downarrow(a\uparrow b)=a$
15.72 $a\uparrow(a\downarrow b)=a$
15.73 $a\downarrow b=a \vee a\downarrow b=b$
15.74 $a\uparrow b=a \vee a\uparrow b=b$

Regla de un punto

$(\exists x \mid x = E : P) \equiv P[x := E]$

Instanciación Testigo

← ↙ →

15.65 $a \downarrow b \leq c \equiv a \leq c \vee b \leq c$

15.66 $a \uparrow b \geqslant c \equiv a \geqslant c \vee b \geqslant c$

15.67 $a \uparrow b = c \equiv a \leq c \wedge b \leq c \wedge (\forall z \mid : a \leq z \wedge b \leq z \Rightarrow c \leq z)$

15.68 $a \downarrow b = c \equiv c \leq a \wedge c \leq b \wedge (\forall z \mid : z \leq a \wedge z \leq b \Rightarrow z \leq c)$

15.69 $a \downarrow b \leq a \uparrow b$

15.70 $a \leq b \Rightarrow a \downarrow c \leq b \downarrow c$

15.71 $a \leq b \Rightarrow a \uparrow c \leq b \uparrow c$

15.72 $-(a \downarrow b) = -a \uparrow -b$

15.73 $-(a \uparrow b) = -a \downarrow -b$

15.74 $a \uparrow (b \downarrow c) = (a \uparrow b) \downarrow (a \uparrow c)$

15.75 $a \downarrow (b \uparrow c) = (a \downarrow b) \uparrow (a \downarrow c)$

15.76 $c + (a \downarrow b) = c + a \downarrow c + b$

15.77 $c + (a \uparrow b) = c + a \uparrow c + b$

15.78 $c \geqslant 0 \Rightarrow c \cdot a \downarrow b = c \cdot a \downarrow c \cdot b$

12.79 $c \geqslant 0 \Rightarrow c \cdot a \uparrow b = c \cdot a \uparrow c \cdot b$

15.80 $c \leq 0 \Rightarrow c \cdot (a \downarrow b) = c \cdot a \uparrow c \cdot b$

15.81 $c \leq 0 \Rightarrow c \cdot (a \uparrow b) = c \cdot a \downarrow c \cdot b$

Axiomas 15.92. Piso y Techo

- $(\forall z \in \mathbb{Z} \mid : z \leq \lfloor x \rfloor \equiv z \leq x)$
- $(\forall z : z \mid : z \geqslant \lceil x \rceil \equiv z \geqslant x)$

## Propiedades

15.93 $\lfloor x \rfloor \leq x < \lfloor x \rfloor + 1$

15.94 $\lceil x \rceil - 1 < x \leq \lceil x \rceil$

15.95 $\lfloor K \rfloor = K = \lceil K \rceil$

15.96 $-\lfloor -x \rfloor = \lceil x \rceil$

15.97 $\lfloor x + n \rfloor = \lfloor x \rfloor + n$

Def. 15.2. $|a| \doteq a \uparrow -a$

## Teoremas valor absoluto

15.98 $|a| = |-a|$

15.99 $|a+b| \leq |a| + |b|$

15.100 $|\,|a|\,| = |a|$ idempotencia

15.101 $|a \cdot b| = |a| \cdot |b|$

15.102 $|a| \leq b \equiv -b \leq a \leq b$

15.103 $a \geqslant 0 \equiv |a| = a$

15.104 $a \leq a \equiv |a| = -a$

Divisibilidad $a \mid b \equiv (\exists z \mid : a \cdot z = b)$

16.1 $a \mid a$

16.2 $a \mid b \wedge b \mid c \rightarrow a \mid c$

16.3 $a \mid b \wedge b \mid a \rightarrow a = b \vee a = -b$

16.4 $a \mid 0$

16.5 $1 \mid a$

16.6 $0 \mid a \rightarrow a = 0$

16.7 $a \mid 1 \rightarrow a = 1 \vee a = -1$

16.8 $a > 1 \wedge a \mid b \rightarrow \neg(a \mid b+1)$

16.9 $a \mid b \rightarrow ac \mid bc$

16.10 $a \mid b = a \mid -b = -a \mid b$

16.11 $(a \mid |b| \equiv a \mid b) \wedge (|a| \mid b \equiv a \mid b)$

máximo común divisor

$(\forall z \mid : z \mid m \uparrow n \equiv z \mid m \wedge z \mid n)$

mínimo común múltiplo

$(\forall z \mid : m \downarrow n \mid z \equiv m \mid z \wedge n \mid z)$

Propiedades mcd mcm

| | |
|---|---|
| $a \downarrow b = b \downarrow a$ | $a \uparrow b \equiv b \uparrow a$ |
| $a \downarrow (b \downarrow c) = (a \downarrow b) \downarrow c$ | $a \uparrow (b \uparrow c) \equiv (a \uparrow b) \uparrow c$ |
| $a \downarrow a = \lvert a \rvert$ | $a \uparrow a = \lvert a \rvert$ |
| $a \downarrow (b \uparrow a) = \lvert a \rvert$ | $a \uparrow (b \downarrow a) = \lvert a \rvert$ |
| $1 \downarrow a = 1$ | $0 \downarrow a = \lvert a \rvert$ |
| $0 \uparrow a = 0$ | $1 \uparrow a = \lvert a \rvert$ |
| $a \cdot b \Rightarrow a \downarrow c \cdot b \downarrow c$ | |
| $a \cdot b \rightarrow a \uparrow c \cdot b \uparrow c$ | |
| $c \geqslant 0 \Rightarrow c \cdot (a \downarrow b) = c \cdot a \downarrow c \cdot b$ | |
| $c \geqslant 0 \Rightarrow c \cdot (a \uparrow b) = c \cdot a \uparrow c \cdot b$ | |
| $a \downarrow -a = \lvert a \rvert$ | $a \uparrow -a = \lvert a \rvert$ |
| $a \downarrow b = \lvert a \rvert \downarrow \lvert b \rvert$ | $a \uparrow b = \lvert a \rvert \uparrow \lvert b \rvert$ |
| $a \downarrow b = a \downarrow (b - a) = a \downarrow (b + a)$ | |
| $(a \downarrow b) \cdot (a \uparrow b) = \lvert a \cdot b \rvert$ | |
| $c \downarrow (a \uparrow b) = (c \downarrow a) \uparrow (c \downarrow b)$ | |
| $c \uparrow (a \downarrow b) = (c \uparrow a) \downarrow (c \uparrow b)$ | |

Teoremas monotonía

4.74 $A \Rightarrow B \Rightarrow A \vee C \Rightarrow B \vee C$

4.75 $A \Rightarrow B \Rightarrow A \vee C \Rightarrow B \vee C$

4.76 $\Leftarrow \Leftarrow \wedge \Rightarrow \wedge$

4.77 $\Leftarrow \Leftarrow \wedge \Leftarrow \wedge$

4.78 $(A \Rightarrow B) \wedge (C \Rightarrow D) \Rightarrow (A \vee C \Rightarrow B \vee D)$

4.79 $(A \Rightarrow B) \wedge (C \Rightarrow D) \Rightarrow (A \vee C \Leftarrow B \vee D)$

4.80 $(A \Rightarrow B) \wedge (C \Rightarrow D) \Rightarrow (A \wedge C \Rightarrow B \wedge D)$

4.81 $(A \Rightarrow B) \wedge (C \Rightarrow D) \Rightarrow (A \wedge C \Leftarrow B \wedge D)$

LINA BOTERO