AXIOMAS:

1. $(\phi \equiv (\psi \equiv \tau)) \equiv ((\phi \equiv \psi) \equiv \tau)$
2. $(\phi \equiv \psi) \equiv (\psi \equiv \phi)$
3. $(\phi \equiv true) \equiv \phi$
4. $(\phi \vee (\psi \vee \tau)) \equiv ((\phi \vee \psi) \vee \tau)$
5. $(\phi \vee \psi) \equiv (\psi \vee \phi)$
6. $(\phi \vee false) \equiv \phi$
7. $(\phi \vee \phi) \equiv \phi$
8. $(\phi \vee (\psi \equiv \tau)) \equiv ((\phi \vee \psi) \equiv (\phi \vee \tau))$
9. $(\neg\phi) \equiv (\phi \equiv false)$
10. $(\phi \not\equiv \psi) \equiv ((\neg\phi) \equiv \psi)$
11. $(\phi \wedge \psi) \equiv (\phi \equiv (\psi \equiv (\phi \vee \psi)))$
12. $(\phi \rightarrow \psi) \equiv ((\phi \vee \psi) \equiv \psi)$
13. $(\phi \leftarrow \psi) \equiv (\psi \rightarrow \phi)$

Bx_1: $(\forall x\, \phi) \equiv \phi$, si x no aparece libre en ϕ.
Bx_2: $\phi \vee (\forall x \psi) \equiv \forall x (\phi \vee \psi)$, ??.
Bx_3: $(\forall x\, \phi) \wedge (\forall x \psi) \equiv \forall x (\phi \wedge \psi)$.
Bx_4: $(\forall x\, \phi) \rightarrow \phi[x := \tau]$, ??.
Bx_5: $\exists x\, \phi \equiv \neg\forall x \neg\phi$.

- ECUANIMIDAD $\dfrac{\psi \quad (\psi \equiv \phi)}{\phi}$
- ECUANIMIDAD* $\dfrac{\psi \quad (\phi \equiv \psi)}{\phi}$
- LEIBNIZ $\dfrac{(\psi \equiv \tau)}{(\phi[p := \psi] \equiv \phi[p := \tau])}$
- LEIBNIZ* $\dfrac{(\tau \equiv \psi)}{(\phi[p := \psi] \equiv \phi[p := \tau])}$
- TRANSITIVIDAD $\dfrac{(\phi \equiv \psi) \quad (\psi \equiv \tau)}{(\phi \equiv \tau)}$
- IDENTIDAD $\dfrac{(\phi \equiv true)}{\phi}$
- IDENTIDAD $\dfrac{\phi}{(\phi \equiv true)}$
- CONMUTATIVIDAD $\dfrac{(\phi \equiv \psi)}{(\psi \equiv \phi)}$
- ASOCIATIVIDAD $\dfrac{((\phi \equiv \psi) \equiv \tau)}{(\phi \equiv (\psi \equiv \tau))}$
- CORTE $\dfrac{(\phi \vee \psi) \quad (\neg\phi \vee \tau)}{(\psi \vee \tau)}$
- SILOGISMO DISYUNTIVO $\dfrac{(\phi \vee \psi) \quad (\neg\phi)}{\psi}$
- MODUS PP. $\dfrac{(\phi \rightarrow \psi) \quad \phi}{\psi}$
- DEBILITAN. $\dfrac{\phi}{(\phi \vee \psi)}$
- MODUS TT $\dfrac{(\phi \rightarrow \psi) \quad (\neg\psi)}{(\neg\phi)}$
- MODUS TOLEN $\dfrac{(\phi \rightarrow \psi) \quad (\neg\psi)}{(\neg\phi)}$
- SIMPLIFICACION $\dfrac{(\phi \wedge \psi)}{\phi \quad \psi}$
- MODUS PONENS $\dfrac{\phi \quad (\phi \rightarrow \psi)}{\psi}$

Bx_6: $x = x$
Bx_7: $(x = t) \rightarrow (\phi \equiv \phi[x := t])$, t es libre para x en ϕ

TEOREMAS: TPS

4.6
1. $true$
2. $(\phi \equiv \phi) \equiv true$
3. $(\phi \equiv \phi)$

4.15
1. $(false \equiv (\neg true))$
2. $(\neg false) \equiv true$
3. $\neg false$
4. $(\neg(\phi \equiv \psi)) \equiv (\neg\phi) \equiv (\neg\psi)$
5. $((\neg\phi) \equiv \psi) \equiv (\phi \equiv (\neg\psi))$
6. $(\neg\neg\phi) \equiv \phi$
7. $(\phi \equiv \neg\phi) \equiv false$

4.16
1. $(\phi \not\equiv (\psi \not\equiv \tau)) \equiv ((\phi \not\equiv \psi) \not\equiv \tau)$
2. $(\phi \not\equiv \psi) \equiv (\psi \not\equiv \phi)$
3. $(\phi \not\equiv false) \equiv \phi$
5. $((\phi \not\equiv \psi) \not\equiv \psi) \equiv \phi$
4. $(\phi \not\equiv \phi) \equiv false$

4.19
1. $\phi \vee (\neg\phi)$
2. $(\phi \vee true) \equiv true$
3. $\phi \vee true$
4. $(\phi \vee \psi) \equiv ((\phi \vee (\neg\psi)) \equiv \phi)$

4.24
1. $(\phi \wedge (\psi \wedge \tau)) \equiv ((\phi \wedge \psi) \wedge \tau)$
2. $(\phi \wedge \psi) \equiv (\psi \wedge \phi)$
3. $(\phi \wedge True) \equiv \phi$
4. $(\phi \wedge false) \equiv false$
5. $(\phi \wedge \phi) \equiv \phi$

4.25
1. $(\phi \wedge \neg\phi) \equiv false$
2. $(\neg(\phi \wedge \psi)) \equiv (\neg\phi \vee \neg\psi)$
3. $(\neg(\phi \vee \psi)) \equiv (\neg\phi \wedge \neg\psi)$
4. $(\phi \wedge (\psi \equiv \tau)) \equiv ((\phi \wedge \psi) \equiv (\phi \wedge \tau) \equiv \phi)$
5. $(\phi \wedge (\psi \not\equiv \tau)) \equiv ((\phi \wedge \psi) \not\equiv (\phi \wedge \tau))$
6. $(\phi \wedge (\psi \vee \tau)) \equiv ((\phi \wedge \psi) \vee (\phi \wedge \tau))$
7. $(\phi \vee (\psi \wedge \tau)) \equiv ((\phi \vee \psi) \wedge (\phi \vee \tau))$

4.28
1. $(\phi \rightarrow \psi) \equiv (\neg\phi \vee \psi)$
2. $(\phi \rightarrow \psi) \equiv ((\phi \wedge \psi) \equiv \phi)$

4.29
1. $\phi \rightarrow True$
2. $false \rightarrow \phi$
3. $(True \rightarrow \phi) \equiv \phi$
4. $(\phi \rightarrow false) \equiv (\neg\phi)$

4.36
1. $((\phi \equiv \psi) \wedge (\psi \equiv \tau)) \rightarrow (\phi \equiv \tau)$
2. $((\phi \equiv \psi) \wedge (\psi \equiv \tau)) \rightarrow (\phi \rightarrow \tau)$
3. $((\phi \rightarrow \psi) \wedge (\psi \equiv \tau)) \rightarrow (\phi \rightarrow \tau)$

4.30
1. $(\phi \rightarrow (\psi \equiv \tau)) \equiv ((\phi \rightarrow \psi) \equiv (\phi \rightarrow \tau))$
2. $(\phi \rightarrow (\psi \vee \tau)) \equiv ((\phi \rightarrow \psi) \vee (\phi \rightarrow \tau))$
3. $(\phi \rightarrow (\psi \wedge \tau)) \equiv ((\phi \rightarrow \psi) \wedge (\phi \rightarrow \tau))$
4. $(\phi \rightarrow (\psi \rightarrow \tau)) \equiv ((\phi \rightarrow \psi) \rightarrow (\phi \rightarrow \tau))$
5. $(\phi \rightarrow (\psi \leftarrow \tau)) \equiv ((\phi \rightarrow \psi) \leftarrow (\phi \rightarrow \tau))$

4.31
1. $(\neg\phi \rightarrow \neg\psi) \equiv (\psi \rightarrow \phi)$
2. $(\neg(\phi \rightarrow \psi)) \equiv (\phi \wedge \neg\psi)$
3. $(\phi \equiv \psi) \equiv ((\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi))$
4. $(\phi \equiv \psi) \rightarrow (\phi \rightarrow \psi)$
5. $(\phi \rightarrow (\psi \rightarrow \tau)) \equiv ((\phi \wedge \psi) \rightarrow \tau)$
6. $(\phi \vee (\phi \rightarrow \psi))$
7. $(\phi \vee (\psi \rightarrow \phi)) \equiv (\psi \rightarrow \phi)$
8. $(\phi \wedge (\phi \rightarrow \psi)) \equiv (\phi \wedge \psi)$
9. $(\phi \wedge (\psi \rightarrow \phi)) \equiv \phi$

4.33
1. $\phi \rightarrow \phi$
2. $((\phi \rightarrow \psi) \wedge (\psi \rightarrow \tau)) \rightarrow (\phi \rightarrow \tau)$
3. $((\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)) \rightarrow (\phi \equiv \psi)$

4.35
1. $(\phi \rightarrow (\phi \vee \psi))$
2. $(\phi \wedge \psi) \rightarrow \phi$
3. $(\phi \wedge \psi) \rightarrow (\phi \vee \psi)$
4. $((\phi \vee \psi) \rightarrow (\phi \wedge \psi)) \equiv (\phi \equiv \psi)$
5. $((\phi \vee \psi) \rightarrow \tau) \equiv ((\phi \rightarrow \tau) \wedge (\psi \rightarrow \tau))$
6. $(\phi \rightarrow (\psi \wedge \tau)) \equiv ((\phi \rightarrow \psi) \wedge (\phi \rightarrow \tau))$

TEOREMAS [illegible] (L)

7.8.

1. $\forall x\ true \equiv true$
2. $\forall x\ false \equiv false$
3. $\forall x \forall x \phi \equiv \forall x \phi$

7.9

1. $(\forall x \mid \psi : \phi) \equiv (\forall x \mid : \neg\psi \vee \phi)$
2. $(\forall x \mid \psi \wedge t : \phi) \equiv (\forall x \mid : \psi \wedge t \to \phi)$
3. $(\forall x \mid \psi \wedge \tau : \phi) \equiv (\forall x \mid \psi : \tau \to \phi)$

7.10.

1. $\forall x (\psi \to \phi) \to (\forall x \psi \to \forall x \phi)$
2. $\forall x (\psi \equiv \phi) \to (\forall x \psi \equiv \forall x \phi)$
3. $(\forall x \mid t : \psi \to \phi) \to ((\forall x \mid \tau : \psi) \to (\forall x \mid \tau : \phi))$
4. $(\forall x \mid \tau : \psi \equiv \phi) \to ((\forall x \mid \tau : \psi) \equiv (\forall x \mid \tau : \phi))$

7.11

1. $\psi \wedge \forall x \phi \equiv \forall x (\psi \wedge \phi)$
2. $\psi \to \forall x \phi \equiv \forall x (\psi \to \phi)$

7.12.

1. $\forall x \mid false : \phi$
2. $(\forall x \mid \psi \vee \tau : \phi) \equiv (\forall x \mid \psi : \phi) \wedge (\forall x \mid \tau : \phi)$
3. $\forall x\ \phi \equiv \forall y (\phi [x := y])$
4. $\forall x \forall y \phi \equiv \forall y \forall x \phi$

7.14

1. $\exists x\ true \equiv true$
2. $\exists x\ false \equiv false.$
3. $\exists x \exists x \phi \equiv \exists x \phi$

7.16.

1. $\exists x (\psi \to \phi) \to (\exists x \psi \to \exists x \phi)$
2. $\exists x (\psi \equiv \phi) \to (\exists x \psi \equiv \exists x \phi)$
3. $(\exists x \mid \tau : \psi \to \phi) \to ((\exists x \mid \tau : \psi) \to (\exists x \mid \tau : \phi))$
4. $(\exists x \mid \tau : \psi \equiv \phi) \to ((\exists x \mid \tau : \psi) \equiv (\exists x \mid \tau : \phi))$

7.17.

1. $\psi \vee \exists x \phi \equiv \exists x (\psi \vee \phi)$
2. $\psi \to \exists x \phi \equiv \exists x (\psi \to \phi)$

7.18.

1. $(\exists x \mid False : \phi) \equiv false.$
2. $(\exists x \mid \psi \vee \tau : \phi) \equiv (\exists x \mid \psi : \phi) \vee (\exists x \mid \tau : \phi)$
3. $\exists x \phi \equiv \exists y (\phi [x := y])$, si y es libre para x en ϕ.
4. $\exists x \exists y \equiv \exists y \exists x \phi$

7.19

1. $\forall x \psi \to \phi \equiv \exists x (\psi \to \phi)$
2. $\exists x \psi \to \phi \equiv \forall x (\psi \to \phi)$

7.27.

1. $(\forall x \mid x = \tau : \phi) \equiv \phi [x := \tau].$
2. $(\exists x \mid x = \tau : \phi) \equiv \phi [x := \tau].$

7.15

1. $\phi [x := \tau] \to \exists x \phi$

metateorema 9.19.2 - Bx 3. 1.

metateorema 7.15 es el testigo. de divisibilidad.