A PROOFS

A.1 Proof of Equivalence

We first provide the proof of Theorem 1.

Lemma 1. *Given m sets of discrete independent random variables that are uniformly distributed, i.e. $\bigcup_{i=1}^{m}\{x_{i,j}|x_{i,j} \sim U_i(0, N_i), j = 1, 2, ..., N_i\}$, we have*

$$\mathbb{E}_{x\sim U}(x) = \sum_{i=1}^{m} \beta_i \mathbb{E}_{x_i\sim U_i}(x_i), \tag{21}$$

where $\beta_i = N_i/\sum_{j=1}^{m} N_j$ and $U := U(0, \sum_{j=1}^{m} N_j)$ is a uniform distribution for all variables.

Proof. The expectation of all random variables is

$$\begin{aligned}
\mathbb{E}_{x\sim U}(x) &= \sum_{i=1}^{m}\sum_{j=1}^{N_i} \frac{1}{\sum_{k=1}^{m} N_k} x_{i,j} && (22)\\
&= \sum_{i=1}^{m}\sum_{j=1}^{N_i} \frac{N_i}{\sum_{k=1}^{m} N_k}\frac{1}{N_i} x_{i,j} && (23)\\
&= \sum_{i=1}^{m} \beta_i \sum_{j=1}^{N_i} \frac{1}{N_i} x_{i,j} && (24)\\
&= \sum_{i=1}^{m} \beta_i \mathbb{E}_{x_i\sim U_i}(x_i). && (25)
\end{aligned}$$

□

Theorem 1. *For old data $x_i \in X$, the NCE-II with the new data ΔX plus the InfoNCE only with X is equivalent to the one with all data X', i.e., $\mathcal{L}_i^{I,X'} = \mathcal{L}_i^{I,X} + \mathcal{L}_i^{II}$.*

Proof. For each input x_i, given the $\mathcal{L}_i^{I}$ in Eq.(1) and $\mathcal{L}_i^{II}$ in Eq.(8), we have

$$\mathcal{L}_i^{I,X'} = -\log \frac{f_i^+}{f^+ + K\mathbb{E}_{p_n^{X'}} f_i^-}, \tag{26}$$

for conciseness, we use f_i^+ for $f(x_i, x_i^+)$ and f_i^- for $f(x_i, x_i^-)$. Following Lemma 1, we have

$$\begin{aligned}
\mathcal{L}_i^{I,X'} &= -\log \frac{f_i^+}{f_i^+ + (1-\alpha)K\mathbb{E}_{p_n^X} f_i^- + \alpha K\mathbb{E}_{p_n^{\Delta X}} f_i^-} && (27)\\
&= -\log \frac{f_i^+}{f_i^+ + K\mathbb{E}_{p_n^X} f_i^- + \alpha K(\mathbb{E}_{p_n^{\Delta X}} f_i^- - \mathbb{E}_{p_n^X} f_i^-)} && (28)\\
&= -(\log \frac{f_i^+}{f_i^+ + K\mathbb{E}_{p_n^X} f_i^-} \\
&\quad + \log \frac{f_i^+ + K\mathbb{E}_{p_n^X} f_i^-}{f_i^+ + K\mathbb{E}_{p_n^X} f_i^- + \alpha K(\mathbb{E}_{p_n^{\Delta X}} f_i^- - \mathbb{E}_{p_n^X} f_i^-)}) && (29)\\
&= \mathcal{L}_i^{I,X} + \log(1 + \frac{\alpha K(\mathbb{E}_{p_n^{\Delta X}} f_i^- - \mathbb{E}_{p_n^X} f_i^-)}{f_i^+ + K\mathbb{E}_{p_n^X} f_i^-}) && (30)\\
&= \mathcal{L}_i^{I,X} + \log(1 + \alpha\frac{f_i^+ + K\mathbb{E}_{p_n^{\Delta X}} f_i^-}{f_i^+ + K\mathbb{E}_{p_n^X} f_i^-} - \alpha) && (31)\\
&= \mathcal{L}_i^{I,X} + \mathcal{L}_i^{II}. && (32)
\end{aligned}$$

Therefore, given an encoder trained by InfoNCE on the old data, the proposed NCE-II bridges the gap caused by the change of the noise distribution and is equivalent to retraining one. □

A.2 Proof of Bias Analysis

We next provide the proof of the bias analysis in Table 2.

Proof. For inference, the bias of old data is

$$\begin{aligned}
&\mathcal{L}_i^{I,X} - \mathcal{L}_i^{I,X'} \\
&= -\log \frac{f(x_i, x_i^+)}{f(x_i, x_i^+) + K\mathbb{E}_{x_i^-\sim p_n^X} f(x_i, x_i^-)} \\
&\quad + \log \frac{f(x_i, x_i^+)}{f(x_i, x_i^+) + K\mathbb{E}_{x_i^-\sim p_n^{X'}} f(x_i, x_i^-)} && (33)\\
&= \log f(x_i, x_i^+) + K\mathbb{E}_{x_i^-\sim p_n^X} f(x_i, x_i^-) \\
&\quad - \log f(x_i, x_i^+) + K\mathbb{E}_{x_i^-\sim p_n^{X'}} f(x_i, x_i^-) && (34)\\
&= \log \frac{f(x_i, x_i^+) + K\mathbb{E}_{x_i^-\sim p_n^X} f(x_i, x_i^-)}{f(x_i, x_i^+) + K\mathbb{E}_{x_i^-\sim p_n^{X'}} f(x_i, x_i^-)} && (35)\\
&= r_{i,X\to X'}. && (36)
\end{aligned}$$

And the bias of new data is self-evidence. For fine-tuning, the bias of old data is the same as inference and the bias of new data is

$$\begin{aligned}
&\mathcal{L}_i^{I,\Delta X} - \mathcal{L}_i^{I,X'} \\
&= -\log \frac{f(x_i, x_i^+)}{f(x_i, x_i^+) + K\mathbb{E}_{x_i^-\sim p_n^{\Delta X}} f(x_i, x_i^-)} \\
&\quad + \log \frac{f(x_i, x_i^+)}{f(x_i, x_i^+) + K\mathbb{E}_{x_i^-\sim p_n^{X'}} f(x_i, x_i^-)} && (37)\\
&= \log f(x_i, x_i^+) + K\mathbb{E}_{x_i^-\sim p_n^{\Delta X}} f(x_i, x_i^-) \\
&\quad - \log f(x_i, x_i^+) + K\mathbb{E}_{x_i^-\sim p_n^{X'}} f(x_i, x_i^-) && (38)\\
&= \log \frac{f(x_i, x_i^+) + K\mathbb{E}_{x_i^-\sim p_n^{\Delta X}} f(x_i, x_i^-)}{f(x_i, x_i^+) + K\mathbb{E}_{x_i^-\sim p_n^{X'}} f(x_i, x_i^-)} && (39)\\
&= r_{i,\Delta X\to X'}. && (40)
\end{aligned}$$

□

A.3 Proof of Bound

We next give the proof of Theorem 2.

Theorem 2. *The difference between the empirical risk of the method with the proposed NCE-II and retraining with InfoNCE in the entire training process is bounded by $\alpha\mathcal{R}^{old}$, where $\mathcal{R}^{old} = \frac{1}{N}\sum_{i=1}^{N}\mathcal{L}_i^{I,old}$ approaches to zero as $\mathcal{L}_i^{I,old}$ is minimized after previous training process on old data X and the growth ratio $\alpha \in [0, 1)$. Then we have $\alpha\mathcal{R}^{old} \to 0$.*

Proof. In the whole training process, the empirical risk of the proposed incremental method is

$$\begin{aligned}
\mathcal{R} &= \mathcal{R}^{old} + \mathcal{R}^{inc} && (41)\\
&= \frac{1}{N}\sum_{x_i\in X}\mathcal{L}_i^{I,X} + \frac{1}{N+\Delta N}\mathcal{L}. && (42)
\end{aligned}$$

Algorithm 1: Incremental Contrastive Learning

Input: old data X and new data ΔX, encoder $\phi(x;\theta)$ trained on X, number of negative samples K, growth rate α, DDPG networks $\mathcal{D}$ for X and $\mathcal{D}'$ for ΔX
Output: encoder $\phi(x;\theta)$

while *not converge* **do**
  foreach $x_i \in \Delta X$ **do**
    `// meta-train`
    let $\theta' = \theta$;
    foreach $x_j \in$ *sampled* $\{x_j\}_{j=1}^{\lceil(1-\alpha)/\alpha\rceil}$ *from* X **do**
      get positive x_j^+ with augmentation ;
      sample negatives $\{x_{j,k}^-\}_{k=1}^{K} \sim p_n^X, \{x_{j,k}^-\}_{k=K}^{2K} \sim p_n^{\Delta X}$;
      get the embeddings with $\phi(x;\theta')$;
      calculate $\mathcal{L}_j^{II} \leftarrow$ Eq.(8) ;
      extract state $s_j = \mathcal{F}(x_j, \theta')$;
      generate learning rate $lr_s \leftarrow$ Eq.(15) ;
      calculate new parameters $\theta' \leftarrow$ Eq.(11) ;
      get the reward $r_j \leftarrow$ Eq.(16) ;
      update $\mathcal{D} \leftarrow$ Eq.(17) ~ Eq.(19) ;
    end
    `// meta-test`
    get positive x_i^+ with augmentation ;
    sample negatives $\{x_{i,k}^-\}_{k=1}^{K} \sim p_n^{X'}$;
    get the embeddings with $\phi(x;\theta')$;
    calculate $\mathcal{L}_i^{I,X'} \leftarrow$ Eq.(1) ;
    extract state $s_i = \mathcal{F}(x_i, \theta')$;
    generate learning rate $lr_q \leftarrow$ Eq.(15) ;
    update parameters $\theta \leftarrow$ Eq.(11) ;
    get the reward $r_i \leftarrow$ Eq.(16) ;
    update $\mathcal{D}' \leftarrow$ Eq.(17) ~ Eq.(19) ;
  end
end

The empirical risk of retraining is

$$\mathcal{R}^{retrain} = \frac{1}{N+\Delta N}\sum_{x_i \in X'} \mathcal{L}_i^{I,X'} \tag{43}$$

$$= \frac{1}{N+\Delta N}\Big(\sum_{x_i \in X} \mathcal{L}_i^{I,X'} + \sum_{x_i \in \Delta X} \mathcal{L}_i^{I,X'}\Big) \tag{44}$$

$$= \frac{1}{N+\Delta N}\Big(\sum_{x_i \in X} \mathcal{L}_i^{I,X} + \sum_{x_i \in X} \mathcal{L}_i^{II} + \sum_{x_i \in \Delta X} \mathcal{L}_i^{I,X'}\Big). \tag{45}$$

Thus, the difference between the proposed method and retraining is

$$\Delta\mathcal{R} = \mathcal{R} - \mathcal{R}^{retrain} = \alpha\mathcal{R}^{old}. \tag{46}$$

Therefore, the bound is $\alpha\mathcal{R}^{old}$, where $\mathcal{R}^{old}$ approaches to zero since the origin training process on the old data X converges and is weighted by the growth ratio $\alpha \in [0, 1)$. That is, $\alpha\mathcal{R}^{old} \to 0$. □

B OVERALL ALGORITHM

The overall framework is in Figure 1, after training the encoder $\phi(\cdot;\theta)$ on the old data X with NCE-I, the ICL framework treats X as the support set and the new data ΔX as the query set. In the meta-train stage, the encoder is trained on a mini-batch of X with NCE-II and obtains the new parameters of the encoder θ'. The learning rate lr_s is learned by the DDPG networks D. In the meta-test stage, the parameters are optimized on ΔX by NCE-I with the learning rate lr_q generated from the other DDPG networks D'. For the detailed workflow, please refer to Algorithm 1.