# 1. Hypothesis Formulation: -

A company claims that their new energy drink increases focus and alertness. Formulate the null and alternative hypotheses for testing this claim.

To test this claim, we define the hypotheses as follows:

- **Null Hypothesis ($H_0$):** The new energy drink has no effect on focus and alertness.

  H0: $\mu=\mu0$

(Where $\mu0$ is the mean focus/alertness level without the drink.)

- **Alternative Hypothesis ($H_1$):** The new energy drink increases focus and alertness.

  H1: $\mu>\mu0$

This represents a one-tailed test since we are specifically testing for an increase in focus and alertness.

# 2. Significance Level Selection: -

A researcher is conducting a study on the effects of exercise on weight loss. What significance level should they choose for their hypothesis test and why?

A researcher studying the effects of exercise on weight loss needs to choose an appropriate significance level (α\alphaα) for their hypothesis test.

**Common Significance Levels:**

- **0.05 (5%)** – Standard in most scientific studies, balancing Type I and Type II errors.
- **0.01 (1%)** – Used when stronger evidence is required, reducing the chance of a false positive.
- **0.10 (10%)** – Used in exploratory research where some leniency is acceptable.

**Recommended Choice:**

- **$\alpha$=0.05 (5%)** is a reasonable choice because:
  - It balances the risk of **Type I error** (rejecting a true null hypothesis) and **Type II error** (failing to detect a real effect).
  - Weight loss studies often involve natural variations, so a moderate confidence level is appropriate.
  - It aligns with standard practices in medical and health-related research.

However, if the study has **severe consequences** (e.g., testing a weight-loss drug with potential side effects), a stricter **$\alpha$=0.01** may be preferred.

## 3. Interpreting p-values: -

In a study investigating the effectiveness of a new teaching method, the calculated p-value is 0.03. What does this p-value indicate about the null hypothesis?

The **p-value** represents the probability of obtaining the observed data (or more extreme results) **assuming the null hypothesis is true**. A **lower p-value** indicates stronger evidence **against** the null hypothesis.

**Decision Making:**

- If the significance level ($\alpha$) is **0.05** (common choice):
    - Since **0.03 < 0.05**, we **reject the null hypothesis** (H0).
    - This means there is **statistically significant evidence** that the new teaching method is effective.
- If the significance level ($\alpha$) is **0.01** (stricter test):
    - Since **0.03 > 0.01**, we **fail to reject the null hypothesis** (H).
    - This means we **don't have strong enough evidence** to conclude that the new method is effective.

**Conclusion:**

- If the test uses **$\alpha$=0.05**, the new teaching method **shows a significant effect**.
- If **$\alpha$=0.01**, more evidence is needed to confirm effectiveness.

## 4. Type I and Type II Errors: -

Describe a scenario in which a Type I error could occur in hypothesis testing. How does it differ from a Type II error?

**Scenario for Type I Error:**

Imagine a medical trial testing a new drug to lower blood pressure.

- **Null Hypothesis (H0)**: The new drug has no effect on blood pressure.
- **Alternative Hypothesis (H1)**: The new drug lowers blood pressure.

A **Type I error** occurs if the researchers **reject the null hypothesis when it is actually true**.

- This means they **conclude that the drug works when it actually does not**.
- As a result, patients might use an ineffective drug, leading to wasted resources and potential health risks.

| Error Type | Meaning | Consequence |
|---|---|---|
| **Type I Error (False Positive)** | Rejecting H0 when it is true | Believing an effect exists when it doesn't (e.g., approving an ineffective drug) |
| **Type II Error (False Negative)** | Failing to reject H0 when it is false | Missing a real effect (e.g., rejecting a drug that actually works) |

**How It Differs from a Type II Error:**

A **Type II error** occurs if the researchers **fail to reject the null hypothesis when it is actually false**.

- In this case, they **conclude that the drug does not work when it actually does**.
- This could mean missing out on a potentially beneficial treatment.

**Real-World Example:**

- **Type I Error:** A COVID-19 test incorrectly detects the virus in a healthy person.
- **Type II Error:** A COVID-19 test fails to detect the virus in an infected person.

## 5. Right-tailed Hypothesis Testing: -

A manufacturer claims that their new light bulb lasts, on average, more than 1000 hours. Conduct a right-tailed hypothesis test with a significance level of 0.05, given a sample mean of 1050 hours and a sample standard deviation of 50 hours.

**Step 1: Define Hypotheses**

The manufacturer claims that the new light bulb lasts more than 1000 hours on average.

- **Null Hypothesis (H0H_0H0)**: The average lifespan of the light bulb is **1000 hours**.

  H0: $\mu=1000$

- **Alternative Hypothesis (H1H_1H1)**: The average lifespan is **greater than** 1000 hours.

  H1: $\mu>1000$

This is a **right-tailed test** because we are testing for an **increase** in lifespan.

**Step 2: Given Data**

- Sample Mean ($\bar{x}$) = 1050 hours
- Population Mean ($\mu_0$) = 1000 hours
- Sample Standard Deviation (s) = 50 hours
- Sample Size (n) = Not Given (Assume n=30 for calculations)
- Significance Level ($\alpha$) = 0.05

**Step 3: Compute the Test Statistic (Z-score or T-score)**

Since the population standard deviation is unknown and n=30 (small sample), we use the **t-test** formula:

$$t=(\bar{x}-\mu_0)/(s/\sqrt{n})$$

Substituting the values:

$t = (1050-1000) / (50/\sqrt{30})$

Let's calculate this:

The calculated **t-score = 5.48** (approximately).

**Step 4: Determine the Critical Value**

For a **right-tailed test** at a **significance level of 0.05** with **df=n−1=29 degrees of freedom**, we look up the **critical t-value** from the t-distribution table.

Using statistical tables or a calculator, the **critical t-value** for $\mathbf{t_{0.05,29}}$ is approximately **1.699**.

**Step 5: Compare and Make a Decision**

- Since **t = 5.48** is **greater than** the critical value **1.699**, we **reject the null hypothesis ($H_0$)**.

**Step 6: Conclusion**

At a **5% significance level**, there is **strong evidence** to support the manufacturer's claim that the new light bulb lasts **more than 1000 hours on average**.

# 6. Two-Tailed Hypothesis Testing: -

A researcher wants to determine if there is a difference in mean exam scores between two groups of students. Formulate the null and alternative hypotheses for this study as a two-tailed test.

**Step 1: Define Hypotheses**

- **Null Hypothesis (H0)**: There is **no difference** in the mean exam scores between the two groups.

  H0: $\mu1=\mu2$

  (The means of both groups are equal.)

- **Alternative Hypothesis (H1)**: There **is a difference** in the mean exam scores between the two groups.

  H1: $\mu1\neq\mu2$

(The means are not equal—one group may perform better or worse than the other.)

**Why a Two-Tailed Test?**

- The researcher is **not** specifying whether one group scores **higher or lower** than the other, only that a difference exists.
- This requires checking for differences **in both directions**, making it a **two-tailed test**.

## 7. One-sample t-test: -

A manufacturer claims that the mean weight of their cereal boxes is 500 grams. A sample of 30 cereal boxes has a mean weight of 490 grams and a standard deviation of 20 grams. Conduct a one-sample t-test to determine if there is evidence to support the manufacturer's claim at a significance level of 0.05.

A manufacturer claims that the **mean weight** of their cereal boxes is **500 grams**. We conduct a **one-sample t-test** to check if there is evidence **against** this claim.

**Step 1: Define Hypotheses**

- **Null Hypothesis (H0)**: The mean weight of the cereal boxes is **500 grams**.

  H0: $\mu=500$
- **Alternative Hypothesis (H1)**: The mean weight of the cereal boxes is **not 500 grams** (it could be lower or higher).

  H1: $\mu\neq500$

  Since we are checking for **any difference** (not specifically greater or less), this is a **two-tailed test**.

**Step 2: Given Data**

- Sample Mean ($\bar{x}$) = **490 grams**
- Population Mean ($\mu_0$) = **500 grams**
- Sample Standard Deviation (s) = **20 grams**
- Sample Size (n) = **30**
- Significance Level ($\alpha$) = **0.05**

**Step 3: Compute the Test Statistic (t-score)**

The **t-test formula** is:

$$t=(\bar{x}-\mu_0)/(s/\sqrt{n})$$

Substituting the given values:

$$t = (490-500)/(20/\sqrt{30})$$

Let's calculate this:

The calculated **t-score = -2.74** (approximately).

**Step 4: Determine the Critical t-Value**

For a **two-tailed test** at a **significance level of 0.05** with **df=n−1=30−1=29** degrees of freedom, we find the critical t-value from statistical tables:

$t_{0.025,29} \approx \pm 2.045$

**Step 5: Compare and Make a Decision**

- The **calculated t-score** is **-2.74**, which is **less than -2.045** (falls in the rejection region).
- Since **−2.74<−2.045**, we **reject the null hypothesis (H0)**.

**Step 6: Conclusion**

At a **5% significance level**, there is **strong evidence** that the mean weight of cereal boxes is **significantly different from 500 grams**.

- Since the t-score is **negative**, it suggests that the cereal boxes **weigh less than 500 grams on average**.

This result indicates that the manufacturer's claim **may not be accurate**.

## 8. Two-sample t-test: -

A researcher wants to compare the mean reaction times of two different groups of participants in a driving simulation study. Group A has a mean reaction time of 0.6 seconds with a standard deviation of 0.1 seconds, while Group B has a mean reaction time of 0.55 seconds with a standard deviation of 0.08 seconds. Conduct a two-sample t-test to determine if there is a significant difference in mean reaction times between the groups at a significance level of 0.01.

A researcher is comparing the **mean reaction times** of two groups of participants in a **driving simulation study**. We conduct a **two-sample t-test** to determine if there is a significant difference.

**Step 1: Define Hypotheses**

- **Null Hypothesis (H0)**: There is **no significant difference** in the mean reaction times of Group A and Group B.

  $H0: \mu A = \mu B$

- **Alternative Hypothesis (H1)**: There **is a significant difference** in the mean reaction times.

  $H1: \mu A \neq \mu B$

Since we are checking for **any difference** (not specifically greater or less), this is a **two-tailed test**.

**Step 2: Given Data**

| Group | Mean Reaction Time ($\bar{x}$) | Standard Deviation (s) | Sample Size (n) |
|---|---|---|---|
| A | 0.6 sec | 0.1 sec | 30 |
| B | 0.55 sec | 0.08 sec | 30 |

- **Significance Level (α)** = **0.01**
- **Sample Sizes ($n_A$=30, $n_B$=30)**

**Step 3: Compute the Test Statistic (t-score)**

For a **two-sample t-test**, we use the formula:

$t = (\bar{x}_A - \bar{x}_B) / \sqrt{(s_A/n_A)^2 + (s_B/n_B)^2}$

Substituting the given values:

$t = (0.6 - 0.55) / \sqrt{(0.1/30)^2 + (0.08/30)^2}$

Let's calculate this:

The calculated **t-score = 2.14** (approximately).

**Step 4: Determine the Critical t-Value**

For a **two-tailed test** at a **significance level of 0.01** with **$df = n_A + n_B - 2 = 30 + 30 - 2 = 58$** degrees of freedom, we find the critical t-value from statistical tables:

$t_{0.005,58} \approx \pm 2.660$

**Step 5: Compare and Make a Decision**

- The **calculated t-score** is **2.14**, which is **less than 2.660** (it does not fall in the rejection region).
- Since **$|2.14| < 2.660$**, we **fail to reject the null hypothesis (H0)**.

**Step 6: Conclusion**

At a **1% significance level**, there is **not enough evidence** to conclude that the mean reaction times of Group A and Group B are significantly different.

- This means the observed difference **could be due to random variation** rather than an actual effect.

## 9. Process Control Example: -

A call center manager implements a new training program aimed at reducing call waiting times. The average waiting time before the training program was 4.5 minutes, and after the program, it is measured to be 4.0 minutes with a standard deviation of 0.8 minutes. Conduct a hypothesis test to determine if there is evidence that the training program has reduced waiting times, using a significance level of 0.05.

The call center manager wants to test if a **new training program** has reduced the average **call waiting time**.

- **Before the training**: Average waiting time = **4.5 minutes**
- **After the training**: Average waiting time = **4.0 minutes**
- **Standard Deviation (s)** = **0.8 minutes**
- **Sample Size (n)** = Not given (let's assume **n = 30** for calculations)
- **Significance Level (α\alphaα)** = **0.05**

**Step 1: Define Hypotheses**

- **Null Hypothesis (H0H_0H0)**: The training program has **no effect** on reducing the average waiting time (i.e., the mean waiting time remains the same as before).

  H0: $\mu_{afte}r \geq \mu_{before} = 4.5$

- **Alternative Hypothesis (H1H_1H1)**: The training program has **reduced** the average waiting time (i.e., the mean waiting time after training is **less than** before).

  H1: $\mu_{afte}r < \mu_{before} = 4.5$

  This is a **left-tailed test** since we're testing if the waiting time has decreased.

  **Step 2: Compute the Test Statistic (t-score)**

  For a **one-sample t-test**, we use the formula:

  $t = (\bar{x} - \mu_0) / (s/\sqrt{n})$

  Where:

- x = Sample mean (after training) = **4.0 minutes**
- $\mu 0$ = Population mean (before training) = **4.5 minutes**
- s = Sample standard deviation = **0.8 minutes**
- n = Sample size = **30**

Let's calculate the t-score:

$t = (4.0-4.5)/(0.8/\sqrt{30}) \approx -3.42$

**Step 4: Determine the Critical t-Value**

For a **left-tailed test** at a **significance level of 0.05** with **df=n−1=30−1=29** degrees of freedom, the critical t-value from statistical tables is approximately:

$t_{0.05,29} \approx -1.699$

**Step 5: Compare and Make a Decision**

- The **calculated t-score** is **-3.42**, which is **less than** the critical value **-1.699** (it falls in the rejection region).
- Since **-3.42 < -1.699**, we **reject the null hypothesis (H0)**.

**Step 6: Conclusion**

At a **5% significance level**, there is **strong evidence** that the training program **has reduced the average call waiting time**.

## 10. Interpreting Results: -

After conducting a hypothesis test, the calculated p-value is 0.02. What can you conclude about the null hypothesis based on this result, assuming a significance level of 0.05?

**Interpreting the p-value of 0.02**

The **p-value** represents the probability of observing the sample data (or more extreme results) assuming that the **null hypothesis** is true.

Given that the **calculated p-value is 0.02** and the **significance level (α) is 0.05**, here's how we can interpret the result:

- **If p-value < α** (0.05), we **reject the null hypothesis**.
- **If p-value ≥ α**, we **fail to reject the null hypothesis**.

**Conclusion:**

- Since **0.02 < 0.05**, we **reject the null hypothesis**.
- This means that there is **enough evidence** to support the **alternative hypothesis**, and we can conclude that the observed effect is statistically significant at the 5% significance level.

In simple terms, the result suggests that the data **provides sufficient evidence** to conclude that the null hypothesis is not true, and the effect or difference being tested is likely real rather than due to random chance.