Example: 7.64 : A steel plant is capable of producing x tonnes per day of a low-grade steel and y tonnes per day of a high-grade steel where $y = \dfrac{40-5x}{10-x}$, If the fixed market of low grade steel is half of a high-grade steel, ... maximum recipe.

Soln:-

Price per tonne of low-grade steel $\overset{(x)}{=} \dfrac{P}{2}$.

Price per tonne of high grade steel $\overset{(y)}{=}$ P.

$$f(x) = \dfrac{xP}{2} + yP$$

$$f(x) = \dfrac{Px}{2} + P\left(\dfrac{40-5x}{10-x}\right) \dfrac{U}{V}$$

$$f'(x) = \dfrac{P}{2} + P\left[\dfrac{(10-x)(-5)-(40-5x)(-1)}{(10-x)^2}\right]$$

$$= \dfrac{P}{2} + P\left[\dfrac{-50+5x+40-5x}{(10-x)^2}\right]$$

$$= \dfrac{P}{2} + P\left[\dfrac{-10}{(10-x)^2}\right] \quad\text{———①}$$

$$f'(x) = 0 \Rightarrow \dfrac{P}{2} = \dfrac{10P}{(10-x)^2}$$

$$\Rightarrow (10-x)^2 = 20.$$