MATH 20C Notes - Week Six

C-Rin

October 2019

Introduction

Deep

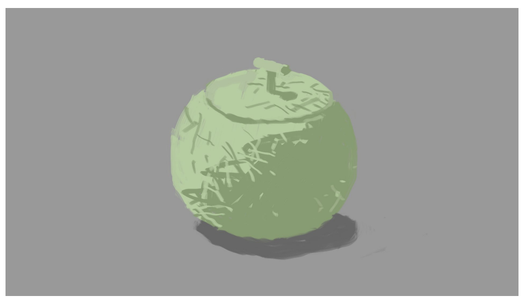

Figure 1: Cucumis melo

1 Integrating over Non-Rectangular Regions

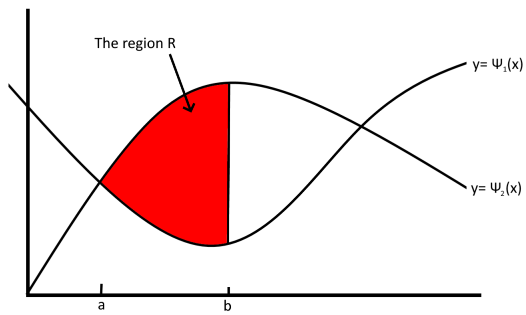

Figure 2:

$f(x,y)$ is a continuous function

$$\iint_R f(x,y)dydx = \int_b^a \int_{\Psi_2(x)}^{\Psi_2(x)} f(x,y)dydx$$

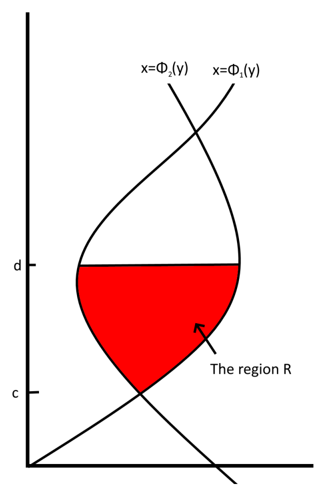

Figure 3:

$$\iint_R f(x,y)dydx = \int_c^d \int_{\Phi_1(x)}^{\Phi_2(y)} f(x,y)dxdy$$

2 Switching the Order of Integration

On a non-rectangular region, you have to be careful. There are two ways to

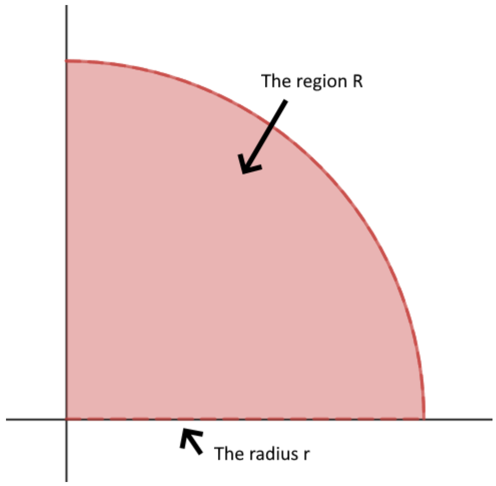

Figure 4: Quarter Circle of $y = \sqrt{r^2 - x^2}$

integrate $f(x, y)$ over R.

1. $\int_0^r \int_0^{\sqrt{r^2-x^2}} f(x, y)dydx$
2. $\int_0^r \int_0^{\sqrt{r^2-y^2}} f(x, y)dxdy$

To switch the order of integration,

1. Draw the region,
2. Figure out how to describe the other way

Example

Evaluate:

$$\int_0^r \int_0^{\sqrt{r^2-x^2}} \sqrt{r^2 - y^2}dydx$$

Switch the order

$$= \int_0^r \int_0^{\sqrt{r^2-y^2}} \sqrt{r^2 - y^2}dxdy$$

$$= \int_0^r \sqrt{r^2 - y^2} \cdot \int_0^{\sqrt{r^2-y^2}} 1dydx = \int_0^r \sqrt{r^2 - y^2} \cdot \sqrt{r^2 - y^2}dy$$

$$= \int_0^r (r^2 - y^2)dy$$

$$= r^3 - \frac{r^3}{3} = \frac{2}{3}r^3$$

Exercise

Evaluate $\int_0^1 \int_x^1 xydydx$

Switch the order and evaluate.

$$\int_0^1 \int_y^1 xydxdy = \int_0^1 \left[\frac{x^2y}{2}\right]_y^1 dy$$

$$= \int_0^1 (\frac{y}{2} - \frac{y^3}{2})dy = \left[\frac{y^2}{4} - \frac{y^4}{8}\right]_0^1 = \frac{1}{8}$$