Majorana フェルミオンによる 2 次元 Ising 模型の厳密解

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1. 転送行列

$$Z = \mathrm{Tr}(T^L) \tag{1}$$

$$T = V_1^{1/2} V_2 V_1^{1/2} \tag{2}$$

$$\langle \{\sigma\} | V_1 | \{\sigma'\} \rangle = \prod_i \delta_{\sigma_i \sigma_i'} \prod_i \mathrm{e}^{\beta \sigma_i \sigma_{i+1}} \tag{3}$$

$$\langle \{\sigma\} | V_2 | \{\sigma'\} \rangle = \prod_i \mathrm{e}^{\beta \sigma_i \sigma_i'} \tag{4}$$

$$V_1 = \prod_i \mathrm{e}^{\beta Z_i Z_{i+1}} = \prod_i (\cosh \beta + \sinh \beta Z_i Z_{i+1}) \tag{5}$$

$$V_2^{1/2} = \prod_i (\mathrm{e}^{\beta/2} + \mathrm{e}^{-\beta/2} X_i) \tag{6}$$

$$\sum_{q \in 2\pi \mathbb{Z}_L / L} \varepsilon_q \quad \sum_{q \in 2\pi (\mathbb{Z}_L + \frac{1}{2}) / L} \varepsilon_q \tag{7}$$

$$\sum_q \varepsilon_q = \sum_{q \in \frac{2\pi}{L} \mathbb{Z}_L} \sum_{x \in \mathbb{Z}} c_x \mathrm{e}^{\mathrm{i} q x} \tag{8}$$

$$\sum_q \varepsilon_q = \sum_{q \in \frac{2\pi}{L} (\mathbb{Z}_L + \frac{1}{2})} \sum_{x \in \mathbb{Z}} c_x \mathrm{e}^{\mathrm{i} q x} = \sum_{q \in \frac{2\pi}{L} \mathbb{Z}_L} \sum_{x \in \mathbb{Z}} c_x \mathrm{e}^{\mathrm{i} \pi x / L} \mathrm{e}^{\mathrm{i} q x} \tag{9}$$

$$L\sum_{n\in\mathbb{Z}}(1-(-1)^n)c_{Ln} = 2L\sum_{n\in 2\mathbb{Z}+1} c_{Ln} \tag{10}$$

$$\frac{1}{2}\left(\varepsilon\left(q+\frac{1}{2}\delta q\right)+\varepsilon\left(q-\frac{1}{2}\delta q\right)\right)-\varepsilon(q)=\frac{1}{8}\varepsilon''(q)\delta q^2+O(\delta q^4) \tag{11}$$

$$\sum_q \varepsilon''(q)\delta q^2 = O(\delta q^2) \tag{12}$$