$$\alpha(\lambda)B(\lambda_1)\cdots B(\lambda_M)|0\rangle, \quad \alpha(\lambda_i)B(\lambda_1)\cdots \overbrace{B(\lambda)}^{i}\cdots B(\lambda_M)|0\rangle \tag{0.1}$$

$$\left[\frac{a(\lambda_1-\lambda)}{c(\lambda_1-\lambda)}B(\lambda_1)A(\lambda)-\frac{b(\lambda_1-\lambda)}{c(\lambda_1-\lambda)}B(\lambda)A(\lambda_1)\right]B(\lambda_2)\cdots B(\lambda_M)|0\rangle \tag{0.2}$$

$$-\frac{b(\lambda_1-\lambda)}{c(\lambda_1-\lambda)}B(\lambda)\left[\frac{a(\lambda_2-\lambda_1)}{c(\lambda_2-\lambda_1)}B(\lambda_2)A(\lambda_1)-\frac{b(\lambda_2-\lambda_1)}{c(\lambda_2-\lambda_1)}B(\lambda_1)A(\lambda_2)\right]B(\lambda_3)\cdots B(\lambda_M)|0\rangle$$

$$-\frac{b(\lambda_1-\lambda)}{c(\lambda_1-\lambda)}\left\{\prod_{i\neq 1}\frac{a(\lambda_i-\lambda_1)}{c(\lambda_i-\lambda_1)}\right\}\alpha(\lambda_1)B(\lambda)B(\lambda_2)\cdots B(\lambda_M)|0\rangle \tag{0.3}$$

$$-\frac{b(\lambda_j-\lambda)}{c(\lambda_j-\lambda)}\left\{\prod_{i\neq j}\frac{a(\lambda_i-\lambda_j)}{c(\lambda_i-\lambda_j)}\right\}\alpha(\lambda_j)B(\lambda_1)\cdots \overbrace{B(\lambda)}^{j}\cdots B(\lambda_M)|0\rangle \tag{0.4}$$

$$-\frac{b(\lambda-\lambda_j)}{c(\lambda-\lambda_j)}\left\{\prod_{i\neq j}\frac{a(\lambda_j-\lambda_i)}{c(\lambda_j-\lambda_i)}\right\}\delta(\lambda_j)B(\lambda_1)\cdots\overbrace{B(\lambda)}^{j}\cdots B(\lambda_M)|0\rangle \tag{0.5}$$

$$\alpha(\lambda_j)\frac{b(\lambda_j-\lambda)}{c(\lambda_j-\lambda)}\prod_{i\neq j}\frac{a(\lambda_i-\lambda_j)}{c(\lambda_i-\lambda_j)}+\delta(\lambda_j)\frac{b(\lambda-\lambda_j)}{c(\lambda-\lambda_j)}\prod_{i\neq j}\frac{a(\lambda_j-\lambda_i)}{c(\lambda_j-\lambda_i)}=0 \tag{0.6}$$

$$\frac{\alpha(\lambda_j)}{\delta(\lambda_j)}=\prod_{i\neq j}\frac{a(\lambda_j-\lambda_i)}{a(\lambda_i-\lambda_j)} \tag{0.7}$$

$$\left(\frac{a(\lambda_j-\eta)}{b(\lambda_j-\eta)}\right)^N=\prod_{i\neq j}\frac{a(\lambda_j-\lambda_i)}{a(\lambda_i-\lambda_j)} \tag{0.8}$$

$$\left(\frac{\lambda_j+\eta}{\lambda_j-\eta}\right)^N=(-1)^{M-1}\prod_{i\neq j}\frac{\lambda_j-\lambda_i+2\eta}{\lambda_j-\lambda_i-2\eta} \tag{0.9}$$

$$\Lambda(\lambda;\lambda_1,...,\lambda_M)=\alpha(\lambda)\prod_{i=1}^{M}\frac{a(\lambda_i-\lambda)}{c(\lambda_i-\lambda)}+\delta(\lambda)\prod_{i=1}^{M}\frac{a(\lambda-\lambda_i)}{c(\lambda-\lambda_i)} \tag{0.10}$$

$$L_n(\lambda)|\uparrow\rangle_n = \begin{pmatrix} a(\lambda)|\uparrow\rangle_n & c(\lambda)|\downarrow\rangle_n \\ 0 & b(\lambda)|\uparrow\rangle_n \end{pmatrix} \tag{0.11}$$

$$T(\lambda)|0\rangle = L_1(\lambda)\cdots L_N(\lambda)|0\rangle = \begin{pmatrix} a(\lambda)^N|0\rangle & * \\ 0 & b(\lambda)^N|0\rangle \end{pmatrix} \tag{0.12}$$

$$\alpha(\lambda) \equiv a(\lambda)^N, \quad \delta(\lambda) \equiv b(\lambda)^N \tag{0.13}$$

$$\tau(\lambda)|0\rangle = (A(\lambda) + D(\lambda))|0\rangle = (\alpha(\lambda) + \delta(\lambda))|0\rangle \tag{0.14}$$

$$\alpha(\lambda_1) = \delta(\lambda_1) \tag{0.15}$$

$$[A(\lambda) + D(\lambda)]B(\lambda_1)\cdots B(\lambda_M)|0\rangle \propto B(\lambda_1)\cdots B(\lambda_M)|0\rangle \tag{0.16}$$

$$L_{n,\alpha} = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & c & 0 \\ 0 & c & b & 0 \\ 0 & 0 & 0 & a \end{pmatrix} = \begin{pmatrix} a\pi_n^+ + b\pi_n^- & c\sigma_n^- \\ c\sigma_n^+ & b\pi_n^+ + a\pi_n^- \end{pmatrix} \tag{0.17}$$

$$= \frac{a+b}{2} 1_\alpha 1_n + \frac{a-b}{2} \sigma_\alpha^z \sigma_n^z + \frac{c}{2}(\sigma_\alpha^x \sigma_n^x + \sigma_\alpha^y \sigma_n^y) \tag{0.18}$$

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{0.19}$$

$$\pi^+ = \frac{1+\sigma^z}{2}, \quad \pi^- = \frac{1-\sigma^z}{2}. \tag{0.20}$$

$$\sigma^+ = \frac{\sigma^x + i\sigma^y}{2}, \quad \sigma^- = \frac{\sigma^x - i\sigma^y}{2}. \tag{0.21}$$

$$R(\lambda) = \begin{pmatrix} \sinh(\lambda + 2\eta) & 0 & 0 & 0 \\ 0 & e^{-\lambda}\sinh(2\eta) & \sinh\lambda & 0 \\ 0 & \sinh\lambda & e^{\lambda}\sinh(2\eta) & 0 \\ 0 & 0 & 0 & \sinh(\lambda+2\eta) \end{pmatrix} \tag{0.22}$$

$$R \equiv \lim_{\lambda\to\infty} 2e^{-\lambda-\eta} R(\lambda) = \begin{pmatrix} e^{\eta} & 0 & 0 & 0 \\ 0 & 0 & e^{-\eta} & 0 \\ 0 & e^{-\eta} & e^{\eta} - e^{-3\eta} & 0 \\ 0 & 0 & 0 & e^{\eta} \end{pmatrix} \tag{0.23}$$

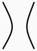
$$R = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & 0 & A^{-1} & 0 \\ 0 & A^{-1} & A - A^{-3} & 0 \\ 0 & 0 & 0 & A \end{pmatrix}, \quad A = e^{\eta} \tag{0.24}$$

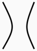
$$A = e^{\eta} \tag{0.25}$$

$$= \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & 0 & A^{-1} & 0 \\ 0 & A^{-1} & A - A^{-3} & 0 \\ 0 & 0 & 0 & A \end{pmatrix} \tag{0.26}$$

$$= 1 \tag{0.27}$$

$$= \begin{pmatrix} A^{-1} & 0 & 0 & 0 \\ 0 & A^{-1} - A^{3} & A & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & 0 & A^{-1} \end{pmatrix} \tag{0.28}$$

 $= A$  $+ A^{-1}$  (0.29)

 $= A$  $- A^2$  $= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -A^2 & 1 & 0 \\ 0 & 1 & -A^{-2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ (0.30)

 $= \mathrm{tr}$  $= -A^2 - A^{-2}$ (0.31)

 $= -A^3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (0.32)