Macroeconometrics Assignment 1

Empirical question: soft landing?

To answer this question, we encourage you to perform your analysis using Julia, and submit your replication code together with your answers to the questions below. You are free to use another programming language or software package of your choice, but if you choose to do so, you should provide a brief explanation why you did not use Julia. If you use a menu-driven software package, such as OxMetrics, please state in words the commands that you used in each step of your analysis.

There has been extensive coverage in the press about the possibility of a "soft landing" (avoidance of a recession) following recent disinflation exercises in various countries. We wish to investigate this by estimating the probability of a recession using VAR models. We will break up the problem in a number of steps.

1. Choose a country to study from the list provided in the sign-up spreadsheet on canvas, or using this link. Make sure your write down your name next to the country of your choice so nobody else chooses the same one!

2. Download and plot the data on real GDP, the corresponding deflator series and a measure of short term interest rates over the longest available sample period. [hint: try to get the data from FRED using MarketData.jl with the code provided in the sign-up spreadsheet.]

3. Perform any transformation of the variables (such as logarithms, first differences) you think are appropriate in order to estimate the dynamics of the series using a VAR. Justify your choice of transformations and your choice of estimation sample. Plot the series you have chosen to use in your VAR.

4. Estimate a VAR(4) in the variables you chose in part 3 and comment on the model's adequacy using the following diagnostics:

   (a) Compute and plot the correlogram of the residuals in each of the equations.

   (b) Perform a test of residual autocorrelation up to lag q (your choice). [Hint: estimate a VAR(q) in the estimated residuals from step 3 and test the null hypothesis that all the VAR coefficients are zero.]

5. Now reconsider the choice of $p = 4$ for the lag order of your VAR and justify any different choice you make for the ensuing analysis. [Hint: use information criteria for model selection.]

6. Let T denote the last period in your estimation sample. Report the minimum-MSFE 1- and 2-quarter ahead forecasts of real GDP growth and their 95% error bands (ignoring parameter uncertainty) using data up to T. Use the following definition of GDP growth: $400\times$(first difference of the natural logarithm of quarterly real GDP) [so a value of 1 corresponds to 1% annualized quarterly GDP growth].

7. A recession is defined as two consecutive quarters of negative real GDP growth. Assuming that the VAR errors are Gaussian and ignoring parameter uncertainty

   (a) derive a formula for the probability that the growth rate of GDP in periods $T+1$ and $T+2$ is negative conditional on the data up to time T;

   (b) report an estimate of that probability using the VAR model you chose to estimate in part 5.

8. Discuss any caveats/limitations of your analysis.

Theoretical question

1. Suppose that the conditions of Theorem 2.4.1 in the notes hold, with $\{y_t\}_{t\in\mathbb{Z}}$ being stationary (as in part (i) of that theorem). Show that

$$\hat{\Sigma}_T := \frac{1}{T}\sum_{t=1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_t^{\mathsf{T}} \xrightarrow{p} \Sigma_0$$

   where $\{\hat{\varepsilon}_t\}_{t=1}^{T}$ denote the OLS residuals

$$\hat{\varepsilon}_t := y_t - \sum_{i=1}^{p} \hat{\Phi}_i y_{t-i} = y_t - Z_{t-1}^{\mathsf{T}} \hat{\beta}_T = \varepsilon_t + Z_{t-1}^{\mathsf{T}}(\beta_0 - \hat{\beta}_T).$$

2. Suppose that $\{y_t\}_{t\geq 1}$ is generated by the stable VAR(1), with no intercept term,

$$y_t = \Phi y_{t-1} + \varepsilon_t$$

   where $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ is a stationary and ergodic martingale difference sequence (m.d.s.) with respect to $\{\mathcal{F}_t\}_{t\in\mathbb{Z}}$, to which y_t is also adapted (i.e. y_t is $\mathcal{F}_t$-measurable for each $t \in \mathbb{Z}$). In particular, we no longer require $\{\varepsilon_t\}$ to be i.i.d., so e.g. that $\mathbb{E}[\varepsilon_t \varepsilon_t^{\mathsf{T}} \mid \mathcal{F}_{t-1}] = \mathbb{E}\varepsilon_0 \varepsilon_0^{\mathsf{T}}$ no longer holds. Instead, we suppose that the conditional variances satisfy

$$\frac{1}{T}\sum_{t=1}^{T} \mathbb{E}[\varepsilon_t \varepsilon_t^{\mathsf{T}} \mid \mathcal{F}_{t-1}] \xrightarrow{p} \Sigma$$

   where Σ is a non-random $k \times k$ matrix. (Such a condition is often satisfied by models in which the errors have stationary conditional heteroskedasticity.) Also assume that there exists a $\delta > 0$ such that

$$\mathbb{E}[\|\varepsilon_t\|^{2+\delta} \mid \mathcal{F}_{t-1}] \leq C, \ t \in \mathbb{Z} \tag{1}$$

for some $C < \infty$. Let

$$y_t^* := \sum_{s=0}^{\infty} \Phi^s \varepsilon_{t-s},$$

which is itself therefore stationary and ergodic (you do not need to prove this), and suppose that $y_0 = y_0^*$, i.e. $\{y_t\}$ is given its stationary initialisation. Let $\hat{\phi}_T := \mathrm{vec}(\hat{\Phi})$ denote the equation-by-equation OLS estimator of $\phi_0 := \mathrm{vec}(\Phi_0)$ (with no intercept term). Show that, under each of conditions (a) and (b)

$$T^{1/2}(\hat{\phi}_T - \phi_0) \rightsquigarrow \mathrm{N}[0, \Gamma^{-1} \otimes \Sigma]$$

where $\Gamma := \mathbb{E} y_0 y_0^{\mathsf{T}}$.

3. Show that the conclusion of the previous question continues to hold if, instead of (1), we now assume that $\mathbb{E}\|\varepsilon_0\|^{4+\delta} < \infty$ for some $\delta > 0$ (note the '4' here.)