Problem Set 1

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Q1.

The social planner's problem is:

$$\max_{\{\pi_t, x_t\}_{t=0}^{\infty}} -\frac{\omega}{2}\sum_{t=0}^{\infty}\beta^t\left(x_t^2+\lambda_h\cdot\pi_{h,t}^2\right) \tag{1}$$

subject to:

$$\pi_{h,t}=\kappa x_t+\beta E_t\left(\pi_{h,t+1}\right)+u_t \tag{2}$$

We can determine $\{x_t, \pi_{h,t}\}_{t=0}^{\infty}$ by maximising the loss function subject to the AS function. $\{i_t\}_{t=0}^{\infty}$ can be derived by substituting the target output gap and inflation into the AD equation to obtain the optimal policy plan.

The Langrangian is:

$$\mathcal{L}=-\frac{\omega}{2}\sum_{t=0}^{\infty}\beta^t\Bigg[\left(x_t^2+\lambda_h\cdot\pi_{h,t}^2\right)+2\mu_t\left(\kappa x_t+\beta\pi_{t+1}+u_t-\pi_t\right)\Bigg] \tag{3}$$

The FOCs with respect to x_t and π_t are, respectively:

$$x_t+k\cdot\mu_t=0 \tag{4}$$

$$\pi_{h,t}\cdot\lambda_h=\mu_t-\mu_{t-1} \tag{5}$$

We obtain the optimal policy by combining them:

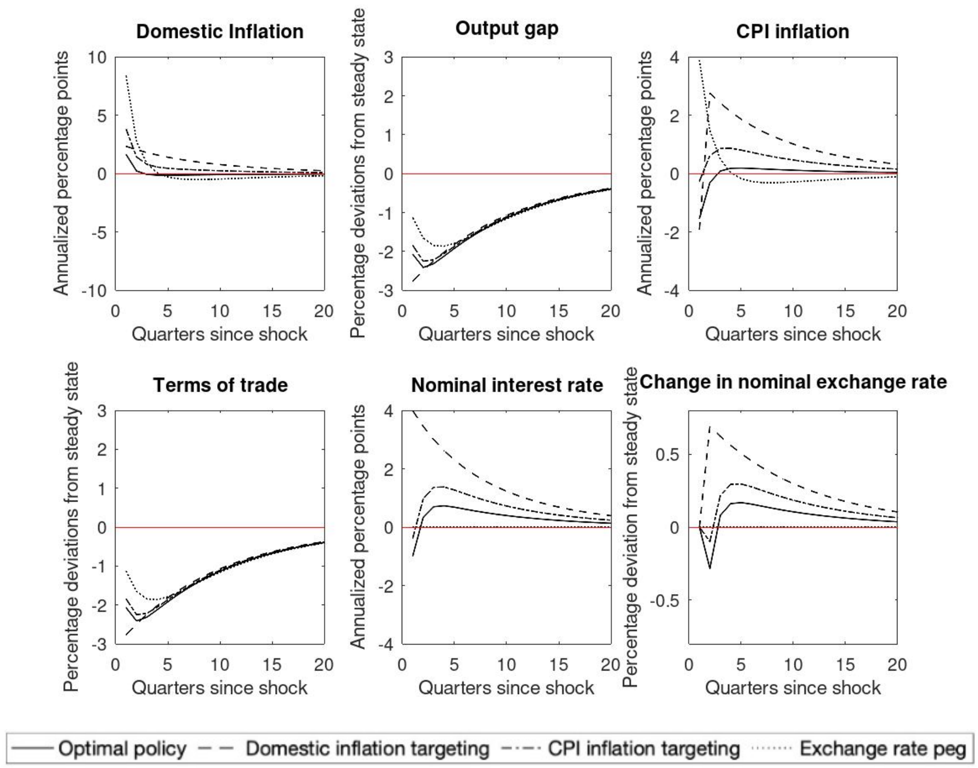
$$\kappa\lambda_\pi\pi_{h,t}=-(x_t-x_{t-1}) \tag{6}$$

By substituting $\kappa\equiv\kappa(1+\phi)$ into equation (6) and solving for $\pi_{h,t}$, the optimal policy expression simplifies to:

$$\pi_{h,t}=\frac{x_{t-1}-x_t}{\epsilon} \tag{7}$$

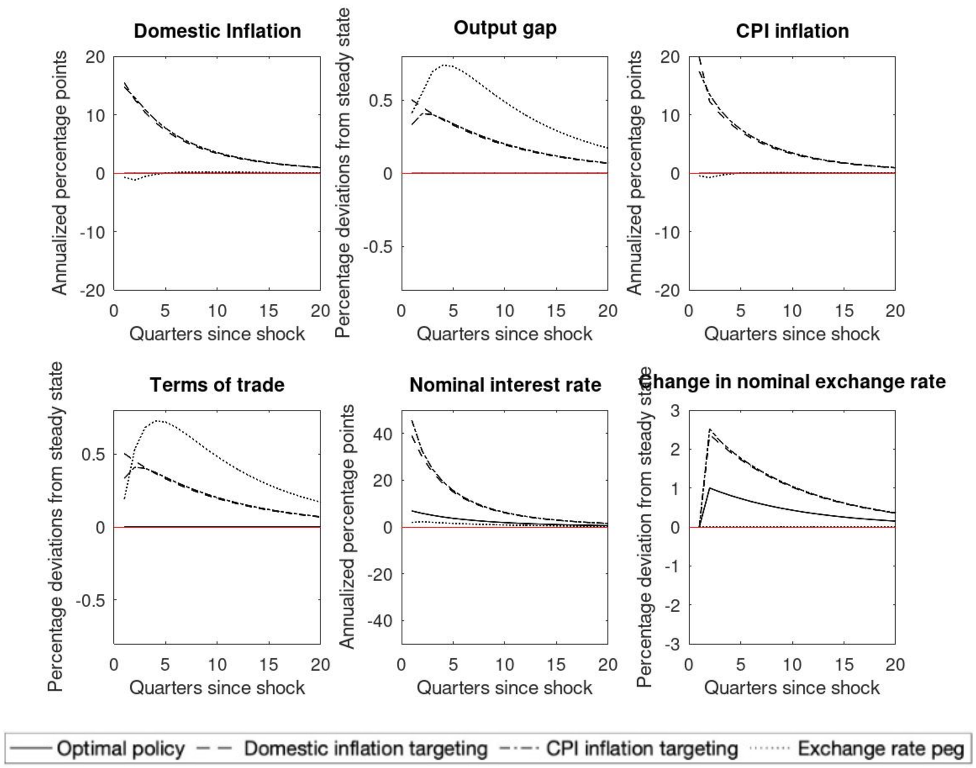
The optimal targeting rule sets the target inflation in the opposite direction to the change in the output gap.

Impulse response functions for a unit standard deviation cost-push shock (u_t)

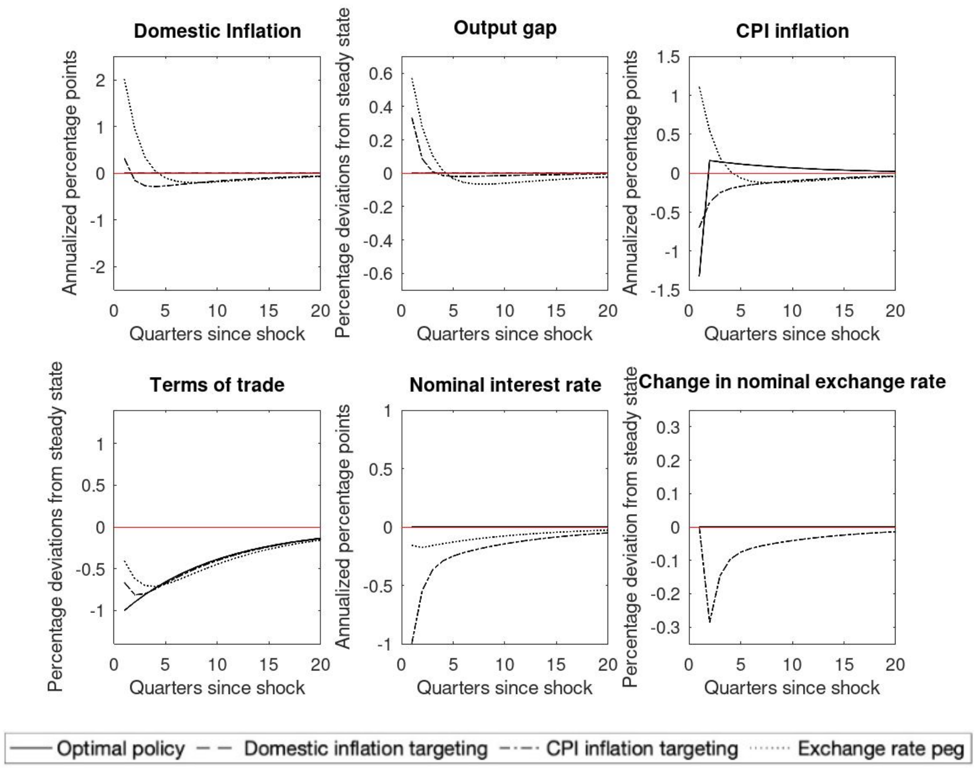


Q2.

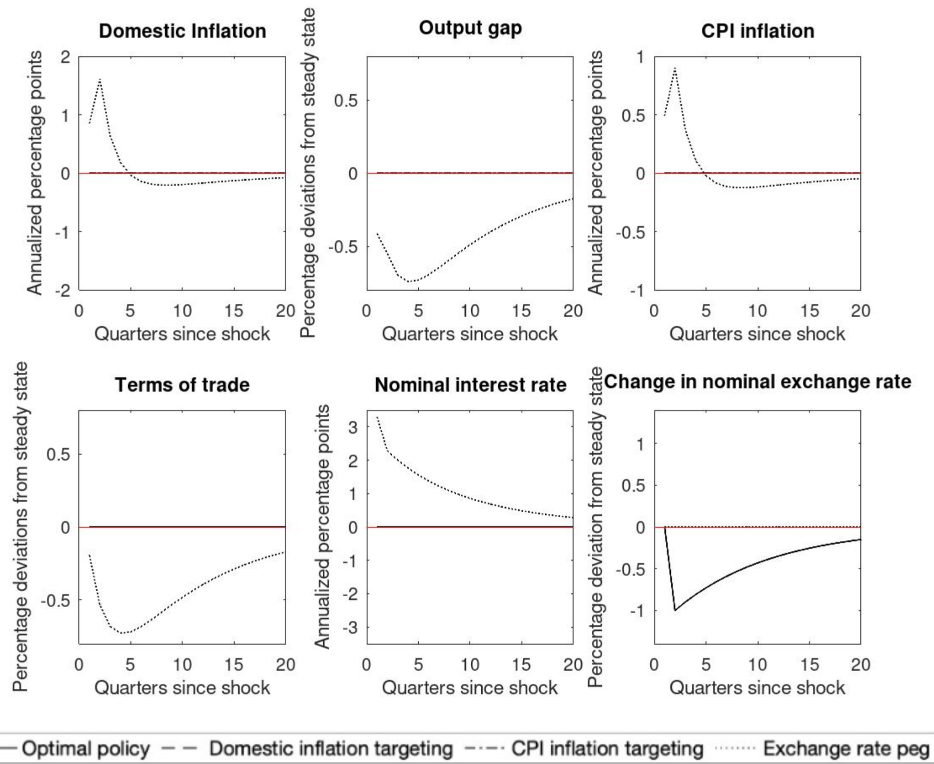
Impulse response functions for a unit standard deviation technology shock (r_t^n)



Impulse response functions for a unit standard deviation foreign output (z_t)



Impulse response functions for a unit standard deviation foreign interest shock (i_t^*)



We begin with the following system of equations:

$$\pi_{h,t} = \kappa x_t + \beta E_t(\pi_{h,t+1}) + u_t \tag{8}$$
$$x_t = x_{t+1} - (i_t - \pi_{h,t+1} - r_t^n) \tag{9}$$
$$\pi_t = \pi_{h,t} + \alpha(\tau_t - \tau_{t-1}) \tag{10}$$
$$x_t = z_t + \tau_t \tag{11}$$
$$i_t = i_t^* + \Delta e_t \tag{12}$$
$$\tag{13}$$

We can easily compute the steady state from the system above:

$$x_{ss} = 0$$
$$\tau_{ss} = 0$$
$$\pi_{ss} = 0$$
$$\pi_{h,ss} = 0$$
$$i_{ss} = 0$$
$$\Delta e_{ss} = 0$$

We now solve for $\{x_t, \tau_t, \pi_t, \pi_{h,t}, i_t, \Delta e_t\}_{t=0}^{\infty}$ in the above system of equations, using the steady-states as the initial values. To obtain the impulse response functions under optimal policy, add the optimal policy rule (7) to the system above, where $\{u_t, r_t^n, z_t, i_t^*\}_{t=0}^{\infty}$ are exogenous AR(1) processes:

$$u_t = \rho u_{t-1} + \epsilon_{u,t}$$
$$r_t = \rho r_{t-1} + \epsilon_{r,t}$$
$$z_t = \rho z_{t-1} + \epsilon_{z,t}$$
$$i_t^* = \rho i_{t-1}^* + \epsilon_{i^*,t}$$

with persistence $\rho = .9$ and $\epsilon_{i,t} \sim \mathcal{N}(0,1)$. The impulse response functions for $\{\pi_{h,t}, x_t, \pi_t, \tau_t, i_t, \Delta e_t\}_{t=0}^{20}$ are plotted in solid lines in figures **??** **??**.

Next, we solve for the endogenous variables under the domestic inflation Taylor rule:

$$i_t = \phi_\pi \pi_{H,t} \tag{14}$$

where $\phi_\pi = 1.5$. The impulse response functions are plotted in dashed lines. Finally, we solve for the endogenous variables under the CPI inflation Taylor rule:

$$i_t = \phi_\pi \pi_t \tag{15}$$

The impulse response functions are plotted in dashed-dotted lines.

In response to a cost-push shock, the central bank reacts by increasing nominal interest under domestic targeting. Under the optimal policy rule and CPI targeting by contrast, the central bank initially decreases interest to react to a decrease in CPI inflation caused by the decline in the terms of trade. By construction (see eq. 13), the change in the nominal exchange rate follows the same pattern as the movement in the nominal interest. Moreover, the increase in domestic inflation results in a decline in the term of trade, by definition. Through equation 12, this implies a decline in the output gap.

Domestic inflation targeting results in lower domestic inflation compared to CPI inflation targeting, which is unsurprising given that CPI inflation is a weighted average of domestic and inflation of imported goods. However, note that optimal policy results in lower domestic inflation than under an explicit domestic inflation targeting. This might be because the central bank chooses the smallest possible target inflation given an output gap under optimal policy, such that the welfare loss function is minimised. Under domestic inflation targeting on the other hand, the Taylor rule coefficient, $\phi_p i$, of 1.5 is chosen arbitrarily and may not result in the lowest possible inflation.

Q3.

Under all policies, the shock in the natural rate of interest results in an increase in the output gap through the AD equation (eq.(10)). The AS equation (eq. (9)) implies that domestic inflation increases drastically, in part through the expectation channel. The central bank reacts by increasing interest rate, although the increase in nominal interest is less drastic under the optimal policy rule. Terms of trade follows the path of the output gap through equation (12), while the nominal exchange rate follows the nominal interest rate through equation (13).

Under CPI targeting, the shock in the difference between foreign and domestic natural output results in a decrease in the terms of trade through equation (12) and consequently, an increase in the domestic inflation through equation (9). Since the central bank does not react to domestic inflation, the output gap also increases. Moreover, the effects of the terms of trade outweigh the effects of domestic inflation, such that CPI inflation decreases from equation (12), resulting in the central bank decreasing its policy rate. Nominal exchange rate as a result, decreases through the UIP. We do not observe similar phenomena under the other policy rules because the central bank immediately stabilises output and terms of trade absorbs most of the shock.

Under all policies, a shock in the foreign interest rate is fully absorbed by the nominal exchange rate through the UIP. All other variables remain at steady state.

Q4.

The Blanchard-Kahn condition does not hold under an exchange rate peg policy rule because the uncovered interest rate parity condition becomes:

$$i_t = i_t^* \tag{16}$$

where i_t is now an exogenous process. Therefore, π_t is unrestricted and the eigenvalues of this system will have an absolute value that is greater than one. This implies that inflation becomes explosive given a sunspot shock, such that the system does not converge to a unique solution.

There are two solutions to this problem. The first solution is to assume that the rest of the world fully stabilises inflation, such that $p_t^* = 0$. From the definition of terms of trade, we know that:

$$\tau_t = \frac{P_{h,t}^*}{P_t} \tag{17}$$

where $P_{h,t}$ is domestic price level and P_t^* is the price of imported goods. From this equation, we derive that:

$$\tau_t = \tau_{t-1} + \pi_t^* - \pi_{h,t} \tag{18}$$

From our assumption of constant price of imported goods, we can add the following equation to the system of equations:

$$\tau_t = \tau_{t-1} - \pi_{h,t} \tag{19}$$

which imposes an additional on $\pi_{h,t}$ through the terms of trade. The impulse response functions are shown in dotted lines in figures **??** to **??**.

The second solution is to add a risk premium to the uncovered interest rate parity condition to reflect risks due to inflation. The UIP states that through arbitrage, the returns on a domestic bond equals the returns earned through a carry trade selling the domestic bond and buying a foreign bond. In a world with inflation, the real returns on a domestic bond falls, while the returns through the carry trade is protected from inflation, assuming the future exchange rate adjusts to future inflation (i.e. the domestic currency depreciates). This results in higher returns on the carry trade and therefore, results in a risk premium on domestic returns such that:

$$i_t = i_t^* + f(E_t(\pi_{t+1})) \tag{20}$$

Empirically, risk premiums are estimated at around 5-12 percentage points, so re-writing the interest rate equation as:

$$i_t = i_t^* + 0.05 \cdot \mathbb{E}[\pi_{h,t+1}] \tag{21}$$

satisfies the Blanchard Khan conditions.

For a given cost-push shock, all variables under an exchange rate peg comove with the variables under a CPI Taylor rule because CPI targeting is an indirect form of exchange rate peg.

In reaction to shock in the difference between foreign output and the domestic natural output, terms of trade decreases while the output gap increases, similar to CPI targeting. However, unlike CPI inflation targeting, the effects of the domestic inflation outweigh the effects of terms of trade, such that CPI inflation increases. This difference is observed because from the UIP equation (eq. 13), the central bank cannot raise nominal interest in reaction to the increase in inflation under a strict peg, resulting in a large increase in domestic inflation. Consequently, the output gap also increases by more than under CPI inflation targeting through the expectation channel of the AD equation.

For a given shock in foreign interest, the domestic nominal interest increases given the perfect peg, resulting in the negative output gap. Terms of trade decreases as a result (see eq. 12). It is difficult to explain the increase in both domestic and CPI inflation, as we expected a decrease given a negative output gap and decrease in terms of trade.

Q5.

To compute the welfare losses, we take the output gap and inflation at each period and compute them into the welfare loss function. Since the impulse response functions eventually converge to steady-state, the loss function should also converge for a large enough time horizon. In this case, $T = 100$ was chosen for this computation. Hence, we compute:

$$\mathcal{L} = -\frac{\omega}{2} \sum_{t=0}^{20} \beta^t \left(x_t^2 + \lambda_h \pi_{h,t}^2 \right) \tag{22}$$

The results are shown in the table below.

Table 1: Welfare Impact of Shocks on Different Policies

Shocks	Optimal	Domestic Inflation	CPI	Peg ($\pi^* = 0$)	Peg with Risk
u shock	-47.288	-68.166	-54.251	-54.67	-69.468
r^n shock	0	-264.11	-273.8	-71.904	-9.486
z shock	0	0	-1.0111	0	-5.4733
i^* shock	0	0	0	-71.904	-9.486

The optimal policy produces the least welfare loss by construction. Given a cost-push shock, domestic inflation targeting performs poorly compared to CPI targeting or an interest rate peg because nominal interest may be over-reacting to inflation and decreasing the output gap more than necessary (note that the coefficient in the Taylor rule, ϕ_π, was chosen arbitrarily). Given a shock in r^n, the interest rate peg performs better than domestic inflation or CPI targeting because it manages to keep domestic inflation at a significantly lower level, which is again puzzling. For a shock in i_t^*, the currency peg performs poorly compared to all other policy because the exchange rate cannot absorb any of the foreign shocks.

Appendix A: Code

Main file

```
pkg load dataframe

%------------------------------------------------------------------
% 0. Housekeeping (close all graphic windows)
%------------------------------------------------------------------

clear all;
close all;


%------------------------------------------------------------------
% 1. Run Models
%------------------------------------------------------------------

dynare optimal;
dynare ditr;
dynare citr;
dynare peg1;
dynare peg2;

%------------------------------------------------------------------
% 2. Load Models
%------------------------------------------------------------------

optimal_results = load("optimal_results.mat");
ditr_results = load("ditr_results.mat");
citr_results = load ("citr_results.mat");
peg1_results = load ("peg1_results.mat");
peg2_results = load("peg2_results.mat");


%------------------------------------------------------------------
% 3. Define Functions
%------------------------------------------------------------------

% Function to setup axes for subplots
function setup_subplot_axes()
    xlims = xlim; % Get the current x-axis limits
    line(xlims, [0 0], 'Color', 'r', 'LineWidth',0.5); % Draw grey line at y=0 u
    max_ylim = max(abs(ylim)); % Get max absolute value from y-axis
    ylim([-max_ylim max_ylim]); % Set symmetric y limits around 0
```

```
    xlabel('Quarters since shock');
endfunction

function loss0 = calculate_welfare_loss(x, pih)
  alpha = .4;
  beta = .99;
  epsilon = 6;
  theta = .75;
  phi = 3;
  lambda = (1 - theta) * (1 - beta * theta) / theta;
  omega = (1 - alpha) * (1 + phi);
  lambda_pi = epsilon / (lambda * (1 + phi));
  loss = zeros(1, length(x));

  for t = 1:length(x)
    loss(t) = - omega / 2 * beta ^ t * (x(t)^2 + lambda_pi * pih(t) ^ 2);

  endfor

  loss0 = sum(loss);

endfunction
%---------------------------------------------------------------------------
% 3. Plot the Results
%---------------------------------------------------------------------------


% annualise inflation and convert to percentage points
optimal_pih_err_u = (exp(optimal_results.oo_.irfs.pih_err_u(1:20)) - 1) * 4;
ditr_pih_err_u = (exp(ditr_results.oo_.irfs.pih_err_u(1:20)) - 1) * 4;
citr_pih_err_u = (exp(citr_results.oo_.irfs.pih_err_u(1:20)) - 1) * 4;
peg2_pih_err_u = (exp(peg2_results.oo_.irfs.pih_err_u(1:20)) - 1) * 4;

optimal_pi_err_u = (exp(optimal_results.oo_.irfs.pi_err_u(1:20)) - 1) * 4;
ditr_pi_err_u = (exp(ditr_results.oo_.irfs.pi_err_u(1:20)) - 1) * 4;
citr_pi_err_u = (exp(citr_results.oo_.irfs.pi_err_u(1:20)) - 1) * 4;
peg2_pi_err_u = (exp(peg2_results.oo_.irfs.pi_err_u(1:20)) - 1) * 4;

optimal_i_err_u = (exp(optimal_results.oo_.irfs.i_err_u(1:20)) - 1) * 4;
ditr_i_err_u = (exp(ditr_results.oo_.irfs.i_err_u(1:20)) - 1) * 4;
citr_i_err_u = (exp(citr_results.oo_.irfs.i_err_u(1:20)) - 1) * 4;
peg2_i_err_u = (exp(peg2_results.oo_.irfs.i_err_u(1:20)) - 1) * 4;

% Plotting the cost-push shock Graph
figure('Position', [100, 100, 1500, 800]);
    % pih IRFs
```

```
subplot(2,3,1)
    optimal = plot(optimal_pih_err_u, 'Linewidth', 1, 'Color', 'k');
    hold on
    ditr = plot(ditr_pih_err_u, '--', 'LineWidth',1, 'Color', 'k');
    citr = plot(citr_pih_err_u, '-.', 'LineWidth',1, 'Color', 'k');
    peg2 = plot(peg2_pih_err_u, ':', 'LineWidth',1, 'Color', 'k');
    setup_subplot_axes();
    title('Domestic Inflation');
    ylabel('Annualized percentage points');
% x IRFs
subplot(2,3,2)
    plot(optimal_results.oo_.irfs.x_err_u(1:20), 'Linewidth', 1, 'Color', 'k
    hold on
    plot(ditr_results.oo_.irfs.x_err_u(1:20), '--', 'LineWidth',1, 'Color',
    plot(citr_results.oo_.irfs.x_err_u(1:20), '-.', 'LineWidth',1, 'Color',
    plot(peg2_results.oo_.irfs.x_err_u(1:20), ':', 'LineWidth',1, 'Color', '
    setup_subplot_axes();
    title('Output gap');
    ylabel('Percentage deviations from steady state');
% pi
  subplot(2,3,3)
    plot(optimal_pi_err_u, 'Linewidth', 1, 'Color', 'k');
    hold on
    plot(ditr_pi_err_u, '--', 'LineWidth',1, 'Color', 'k');
    plot(citr_pi_err_u, '-.', 'LineWidth',1, 'Color', 'k');
    plot(peg2_pi_err_u, ':', 'LineWidth',1, 'Color', 'k');
    setup_subplot_axes();
    title('CPI inflation');
    ylabel('Annualized percentage points');
% tau
subplot(2,3,4)
    plot(optimal_results.oo_.irfs.tau_err_u(1:20), 'Linewidth', 1, 'Color',
    hold on
    plot(ditr_results.oo_.irfs.tau_err_u(1:20), '--', 'LineWidth',1, 'Color'
    plot(citr_results.oo_.irfs.tau_err_u(1:20), '-.', 'LineWidth',1, 'Color'
    plot(peg2_results.oo_.irfs.tau_err_u(1:20), ':', 'LineWidth',1, 'Color',
    setup_subplot_axes();
    title('Terms of trade');
    ylabel('Percentage deviations from steady state');
% i
subplot(2,3,5)
    plot(optimal_i_err_u, 'Linewidth', 1, 'Color', 'k');
    hold on
    plot(ditr_i_err_u, '--', 'LineWidth',1, 'Color', 'k');
    plot(citr_i_err_u, '-.', 'LineWidth',1, 'Color', 'k');
    plot(peg2_i_err_u, ':', 'LineWidth',1, 'Color', 'k');
```

```
        setup_subplot_axes();
        title('Nominal interest rate');
        ylabel('Annualized percentage points');
    % delta_e
    subplot(2,3,6)
        plot(optimal_results.oo_.irfs.delta_e_err_u(1:20), 'Linewidth', 1, 'Colo
        hold on
        plot(ditr_results.oo_.irfs.delta_e_err_u(1:20), '--', 'LineWidth',1, 'Co
        plot(citr_results.oo_.irfs.delta_e_err_u(1:20), '-.', 'LineWidth',1, 'Co
        plot(peg2_results.oo_.irfs.delta_e_err_u(1:20), ':', 'LineWidth',1, 'Col
        setup_subplot_axes();
        title('Change in nominal exchange rate');
        ylabel('Percentage deviation from steady state');
        %{
    % Add a global legend using the plots from the first subplot
        lgd = legend([optimal, ditr, citr, peg2], {'Optimal policy', 'Domestic i
        set(lgd, 'location', 'south', 'orientation', 'horizontal', 'fontsize', 1
    % Manually adjust the position to be at the bottom center of the figure
        legendPosition = [0.25, 0.01, 0.5, 0.05];
        set(lgd, 'Position', legendPosition);
        %}
    % Saving the graph
    saveas(gcf, 'Cost-push shock.jpg');



% annualise inflation and convert to percentage points
%{
optimal_pih_err_r = (exp(optimal_results.oo_.irfs.pih_err_r(1:20)) - 1) * 4;
ditr_pih_err_r = (exp(ditr_results.oo_.irfs.pih_err_r(1:20)) - 1) * 4;
citr_pih_err_r = (exp(citr_results.oo_.irfs.pih_err_r(1:20)) - 1) * 4;
peg2_pih_err_r = (exp(peg2_results.oo_.irfs.pih_err_r(1:20)) - 1) * 4;
%}

optimal_pih_err_r = (exp(optimal_results.oo_.irfs.pih_err_r(1:20)) - 1) * 4;
ditr_pih_err_r = (exp(ditr_results.oo_.irfs.pih_err_r(1:20)) - 1) * 4;
citr_pih_err_r = (exp(citr_results.oo_.irfs.pih_err_r(1:20)) - 1) * 4;
peg2_pih_err_r = (exp(peg2_results.oo_.irfs.pih_err_r(1:20)) - 1) * 4;

optimal_pi_err_r = (exp(optimal_results.oo_.irfs.pi_err_r(1:20)) - 1) * 4;
ditr_pi_err_r = (exp(ditr_results.oo_.irfs.pi_err_r(1:20)) - 1) * 4;
citr_pi_err_r = (exp(citr_results.oo_.irfs.pi_err_r(1:20)) - 1) * 4;
peg2_pi_err_r = (exp(peg2_results.oo_.irfs.pi_err_r(1:20)) - 1) * 4;

optimal_i_err_r = (exp(optimal_results.oo_.irfs.i_err_r(1:20)) - 1) * 4;
ditr_i_err_r = (exp(ditr_results.oo_.irfs.i_err_r(1:20)) - 1) * 4;
```

```
citr_i_err_r = (exp(citr_results.oo_.irfs.i_err_r(1:20)) - 1) * 4;
peg2_i_err_r = (exp(peg2_results.oo_.irfs.i_err_r(1:20)) - 1) * 4;

% Plotting the technology shock graph
figure('Position', [100, 100, 1200, 800]);
    % pih IRFs
    subplot(2,3,1)
        optimal = plot(optimal_pih_err_r, 'Linewidth', 1, 'Color', 'k');
        hold on
        ditr = plot(ditr_pih_err_r, '--', 'LineWidth',1, 'Color', 'k');
        citr = plot(citr_pih_err_r, '-.', 'LineWidth',1, 'Color', 'k');
        peg2 = plot(peg2_pih_err_r, ':', 'LineWidth',1, 'Color', 'k');
        setup_subplot_axes();
        title('Domestic Inflation');
        ylabel('Annualized percentage points');
    % x IRFs
    subplot(2,3,2)
        plot(optimal_results.oo_.irfs.x_err_r(1:20), 'Linewidth', 1, 'Color', 'k
        hold on
        plot(ditr_results.oo_.irfs.x_err_r(1:20), '--', 'LineWidth',1, 'Color',
        plot(citr_results.oo_.irfs.x_err_r(1:20), '-.', 'LineWidth',1, 'Color',
        plot(peg2_results.oo_.irfs.x_err_r(1:20), ':', 'LineWidth',1, 'Color', '
        setup_subplot_axes();
        title('Output gap');
        ylabel('Percentage deviations from steady state');
    % pi
       subplot(2,3,3)
        plot(optimal_pi_err_r, 'Linewidth', 1, 'Color', 'k');
        hold on
        plot(ditr_pi_err_r, '--', 'LineWidth',1, 'Color', 'k');
        plot(citr_pi_err_r, '-.', 'LineWidth',1, 'Color', 'k');
        plot(peg2_pi_err_r, ':', 'LineWidth',1, 'Color', 'k');
        setup_subplot_axes();
        title('CPI inflation');
        ylabel('Annualized percentage points');
    % tau
    subplot(2,3,4)
        plot(optimal_results.oo_.irfs.tau_err_r(1:20), 'Linewidth', 1, 'Color',
        hold on
        plot(ditr_results.oo_.irfs.tau_err_r(1:20), '--', 'LineWidth',1, 'Color'
        plot(citr_results.oo_.irfs.tau_err_r(1:20), '-.', 'LineWidth',1, 'Color'
        plot(peg2_results.oo_.irfs.tau_err_r(1:20), ':', 'LineWidth',1, 'Color',
        setup_subplot_axes();
        title('Terms of trade');
        ylabel('Percentage deviations from steady state');
    % i
```

```
    subplot(2,3,5)
        plot(optimal_i_err_r, 'Linewidth', 1, 'Color', 'k');
        hold on
        plot(ditr_i_err_r, '--', 'LineWidth',1, 'Color', 'k');
        plot(citr_i_err_r, '-.', 'LineWidth',1, 'Color', 'k');
        plot(peg2_i_err_r, ':', 'LineWidth',1, 'Color', 'k');
        setup_subplot_axes();
        title('Nominal interest rate');
        ylabel('Annualized percentage points');
    % delta_e
    subplot(2,3,6)
        plot(optimal_results.oo_.irfs.delta_e_err_r(1:20), 'Linewidth', 1, 'Colo
        hold on
        plot(ditr_results.oo_.irfs.delta_e_err_r(1:20), '--', 'LineWidth',1, 'Co
        plot(citr_results.oo_.irfs.delta_e_err_r(1:20), '-.', 'LineWidth',1, 'Co
        plot(peg2_results.oo_.irfs.delta_e_err_r(1:20), ':', 'LineWidth',1, 'Col
        setup_subplot_axes();
        title('Change in nominal exchange rate');
        ylabel('Percentage deviation from steady state');
    %{
    % Add a global legend using the plots from the first subplot
        lgd = legend([optimal, ditr, citr, peg2], {'Optimal policy', 'Domestic i
        set(lgd, 'location', 'south', 'orientation', 'horizontal', 'fontsize', 1
    % Manually adjust the position to be at the bottom center of the figure
        legendPosition = [0.25, 0.01, 0.5, 0.05];
        set(lgd, 'Position', legendPosition);
        %}
% Saving the graph
saveas(gcf, 'Technology shock.jpg');



% annualise inflation and convert to percentage points
optimal_pih_err_z = (exp(optimal_results.oo_.irfs.pih_err_z(1:20)) - 1) * 4;
ditr_pih_err_z = (exp(ditr_results.oo_.irfs.pih_err_z(1:20)) - 1) * 4;
citr_pih_err_z = (exp(citr_results.oo_.irfs.pih_err_z(1:20)) - 1) * 4;
peg2_pih_err_z = (exp(peg2_results.oo_.irfs.pih_err_z(1:20)) - 1) * 4;

optimal_pi_err_z = (exp(optimal_results.oo_.irfs.pi_err_z(1:20)) - 1) * 4;
ditr_pi_err_z = (exp(ditr_results.oo_.irfs.pi_err_z(1:20)) - 1) * 4;
citr_pi_err_z = (exp(citr_results.oo_.irfs.pi_err_z(1:20)) - 1) * 4;
peg2_pi_err_z = (exp(peg2_results.oo_.irfs.pi_err_z(1:20)) - 1) * 4;

optimal_i_err_z = (exp(optimal_results.oo_.irfs.i_err_z(1:20)) - 1) * 4;
ditr_i_err_z = (exp(ditr_results.oo_.irfs.i_err_z(1:20)) - 1) * 4;
```

```
citr_i_err_z = (exp(citr_results.oo_.irfs.i_err_z(1:20)) - 1) * 4;
peg2_i_err_z = (exp(peg2_results.oo_.irfs.i_err_z(1:20)) - 1) * 4;

% Plotting the demand shock graph
figure('Position', [100, 100, 1200, 800]);
    % pih IRFs
    subplot(2,3,1)
        optimal = plot(optimal_pih_err_z, 'Linewidth', 1, 'Color', 'k');
        hold on
        ditr = plot(ditr_pih_err_z, '--', 'LineWidth',1, 'Color', 'k');
        citr = plot(citr_pih_err_z, '-.', 'LineWidth',1, 'Color', 'k');
        peg2 = plot(peg2_pih_err_z, ':', 'LineWidth',1, 'Color', 'k');
        setup_subplot_axes();
        title('Domestic Inflation');
        ylabel('Annualized percentage points');
    % x IRFs
    subplot(2,3,2)
        plot(optimal_results.oo_.irfs.x_err_z(1:20), 'Linewidth', 1, 'Color', 'k
        hold on
        plot(ditr_results.oo_.irfs.x_err_z(1:20), '--', 'LineWidth',1, 'Color',
        plot(citr_results.oo_.irfs.x_err_z(1:20), '-.', 'LineWidth',1, 'Color',
        plot(peg2_results.oo_.irfs.x_err_z(1:20), ':', 'LineWidth',1, 'Color', '
        setup_subplot_axes();
        title('Output gap');
        ylabel('Percentage deviations from steady state');
    % pi
       subplot(2,3,3)
        plot(optimal_pi_err_z, 'Linewidth', 1, 'Color', 'k');
        hold on
        plot(ditr_pi_err_z, '--', 'LineWidth',1, 'Color', 'k');
        plot(citr_pi_err_z, '-.', 'LineWidth',1, 'Color', 'k');
        plot(peg2_pi_err_z, ':', 'LineWidth',1, 'Color', 'k');
        setup_subplot_axes();
        title('CPI inflation');
        ylabel('Annualized percentage points');
    % tau
    subplot(2,3,4)
        plot(optimal_results.oo_.irfs.tau_err_z(1:20), 'Linewidth', 1, 'Color',
        hold on
        plot(ditr_results.oo_.irfs.tau_err_z(1:20), '--', 'LineWidth',1, 'Color'
        plot(citr_results.oo_.irfs.tau_err_z(1:20), '-.', 'LineWidth',1, 'Color'
        plot(peg2_results.oo_.irfs.tau_err_z(1:20), ':', 'LineWidth',1, 'Color',
        setup_subplot_axes();
        title('Terms of trade');
        ylabel('Percentage deviations from steady state');
    % i
```

```
    subplot(2,3,5)
        plot(optimal_i_err_z, 'Linewidth', 1, 'Color', 'k');
        hold on
        plot(ditr_i_err_z, '--', 'LineWidth',1, 'Color', 'k');
        plot(citr_i_err_z, '-.', 'LineWidth',1, 'Color', 'k');
        plot(peg2_i_err_z, ':', 'LineWidth',1, 'Color', 'k');
        setup_subplot_axes();
        title('Nominal interest rate');
        ylabel('Annualized percentage points');
    % delta_e
    subplot(2,3,6)
        plot(optimal_results.oo_.irfs.delta_e_err_z(1:20), 'Linewidth', 1, 'Colo
        hold on
        plot(ditr_results.oo_.irfs.delta_e_err_z(1:20), '--', 'LineWidth',1, 'Co
        plot(citr_results.oo_.irfs.delta_e_err_z(1:20), '-.', 'LineWidth',1, 'Co
        plot(peg2_results.oo_.irfs.delta_e_err_z(1:20), ':', 'LineWidth',1, 'Col
        setup_subplot_axes();
        title('Change in nominal exchange rate');
        ylabel('Percentage deviation from steady state');
    %{
    % Add a global legend using the plots from the first subplot
        lgd = legend([optimal, ditr, citr, peg2], {'Optimal policy', 'Domestic i
        set(lgd, 'location', 'south', 'orientation', 'horizontal', 'fontsize', 1
    % Manually adjust the position to be at the bottom center of the figure
        legendPosition = [0.25, 0.01, 0.5, 0.05];
        set(lgd, 'Position', legendPosition);
        %}
% Saving the graph
saveas(gcf, 'Demand shock.jpg');




% annualise inflation and convert to percentage points
optimal_pih_err_istar = (exp(optimal_results.oo_.irfs.pih_err_istar(1:20)) - 1)
ditr_pih_err_istar = (exp(ditr_results.oo_.irfs.pih_err_istar(1:20)) - 1) * 4;
citr_pih_err_istar = (exp(citr_results.oo_.irfs.pih_err_istar(1:20)) - 1) * 4;
peg2_pih_err_istar = (exp(peg2_results.oo_.irfs.pih_err_istar(1:20)) - 1) * 4;

optimal_pi_err_istar = (exp(optimal_results.oo_.irfs.pi_err_istar(1:20)) - 1) *
ditr_pi_err_istar = (exp(ditr_results.oo_.irfs.pi_err_istar(1:20)) - 1) * 4;
citr_pi_err_istar = (exp(citr_results.oo_.irfs.pi_err_istar(1:20)) - 1) * 4;
peg2_pi_err_istar = (exp(peg2_results.oo_.irfs.pi_err_istar(1:20)) - 1) * 4;

optimal_i_err_istar = (exp(optimal_results.oo_.irfs.i_err_istar(1:20)) - 1) * 4;
```

```
ditr_i_err_istar = (exp(ditr_results.oo_.irfs.i_err_istar(1:20)) - 1) * 4;
citr_i_err_istar = (exp(citr_results.oo_.irfs.i_err_istar(1:20)) - 1) * 4;
peg2_i_err_istar = (exp(peg2_results.oo_.irfs.i_err_istar(1:20)) - 1) * 4;

% Plotting the foreign shock graph
figure('Position', [100, 100, 1200, 800]);
    % pih IRFs
    subplot(2,3,1)
        optimal = plot(optimal_pih_err_istar, 'Linewidth', 1, 'Color', 'k');
        hold on
        ditr = plot(ditr_pih_err_istar, '--', 'LineWidth',1, 'Color', 'k');
        citr = plot(citr_pih_err_istar, '-.', 'LineWidth',1, 'Color', 'k');
        peg2 = plot(peg2_pih_err_istar, ':', 'LineWidth',1, 'Color', 'k');
        setup_subplot_axes();
        title('Domestic Inflation');
        ylabel('Annualized percentage points');
    % x IRFs
    subplot(2,3,2)
        plot(optimal_results.oo_.irfs.x_err_istar(1:20), 'Linewidth', 1, 'Color'
        hold on
        plot(ditr_results.oo_.irfs.x_err_istar(1:20), '--', 'LineWidth',1, 'Colo
        plot(citr_results.oo_.irfs.x_err_istar(1:20), '-.', 'LineWidth',1, 'Colo
        plot(peg2_results.oo_.irfs.x_err_istar(1:20), ':', 'LineWidth',1, 'Color
        setup_subplot_axes();
        title('Output gap');
        ylabel('Percentage deviations from steady state');
    % pi
       subplot(2,3,3)
        plot(optimal_pi_err_istar, 'Linewidth', 1, 'Color', 'k');
        hold on
        plot(ditr_pi_err_istar, '--', 'LineWidth',1, 'Color', 'k');
        plot(citr_pi_err_istar, '-.', 'LineWidth',1, 'Color', 'k');
        plot(peg2_pi_err_istar, ':', 'LineWidth',1, 'Color', 'k');
        setup_subplot_axes();
        title('CPI inflation');
        ylabel('Annualized percentage points');
    % tau
    subplot(2,3,4)
        plot(optimal_results.oo_.irfs.tau_err_istar(1:20), 'Linewidth', 1, 'Colo
        hold on
        plot(ditr_results.oo_.irfs.tau_err_istar(1:20), '--', 'LineWidth',1, 'Co
        plot(citr_results.oo_.irfs.tau_err_istar(1:20), '-.', 'LineWidth',1, 'Co
        plot(peg2_results.oo_.irfs.tau_err_istar(1:20), ':', 'LineWidth',1, 'Col
        setup_subplot_axes();
        title('Terms of trade');
        ylabel('Percentage deviations from steady state');
```

```
    % i
    subplot(2,3,5)
        plot(optimal_i_err_istar, 'Linewidth', 1, 'Color', 'k');
        hold on
        plot(ditr_i_err_istar, '--', 'LineWidth',1, 'Color', 'k');
        plot(citr_i_err_istar, '-.', 'LineWidth',1, 'Color', 'k');
        plot(peg2_i_err_istar, ':', 'LineWidth',1, 'Color', 'k');
        setup_subplot_axes();
        title('Nominal interest rate');
        ylabel('Annualized percentage points');
    % delta_e
    subplot(2,3,6)
        plot(optimal_results.oo_.irfs.delta_e_err_istar(1:20), 'Linewidth', 1, '
        hold on
        plot(ditr_results.oo_.irfs.delta_e_err_istar(1:20), '--', 'LineWidth',1,
        plot(citr_results.oo_.irfs.delta_e_err_istar(1:20), '-.', 'LineWidth',1,
        plot(peg2_results.oo_.irfs.delta_e_err_istar(1:20), ':', 'LineWidth',1,
        setup_subplot_axes();
        title('Change in nominal exchange rate');
        ylabel('Percentage deviation from steady state');
    %{
    % Add a global legend using the plots from the first subplot
        lgd = legend([optimal, ditr, citr, peg2], {'Optimal policy', 'Domestic i
        set(lgd, 'location', 'south', 'orientation', 'horizontal', 'fontsize', 1
    % Manually adjust the position to be at the bottom center of the figure
        legendPosition = [0.25, 0.01, 0.5, 0.05];
        set(lgd, 'Position', legendPosition);
        %}
% Saving the graph
saveas(gcf, 'Foreign shock.jpg');


%------------------------------------------------------------------------
% 3. Welfare Loss Calculation
%------------------------------------------------------------------------

% cost-push shock
loss_optimal_u = calculate_welfare_loss(optimal_results.oo_.irfs.x_err_u, optima
loss_ditr_u = calculate_welfare_loss(ditr_results.oo_.irfs.x_err_u, ditr_results
loss_citr_u = calculate_welfare_loss(citr_results.oo_.irfs.x_err_u, citr_results
loss_peg1_u = calculate_welfare_loss(peg1_results.oo_.irfs.x_err_u, peg1_results
loss_peg2_u = calculate_welfare_loss(peg2_results.oo_.irfs.x_err_u, peg2_results

% shock to r
loss_optimal_r = calculate_welfare_loss(optimal_results.oo_.irfs.x_err_r, optima
loss_ditr_r = calculate_welfare_loss(ditr_results.oo_.irfs.x_err_r, ditr_results
```

```
loss_citr_r = calculate_welfare_loss(citr_results.oo_.irfs.x_err_r, citr_results
loss_peg1_r = calculate_welfare_loss(peg1_results.oo_.irfs.x_err_r, peg1_results
loss_peg2_r = calculate_welfare_loss(peg2_results.oo_.irfs.x_err_r, peg2_results

% shock to z
loss_optimal_z = calculate_welfare_loss(optimal_results.oo_.irfs.x_err_z, optima
loss_ditr_z = calculate_welfare_loss(ditr_results.oo_.irfs.x_err_z, ditr_results
loss_citr_z = calculate_welfare_loss(citr_results.oo_.irfs.x_err_z, citr_results
loss_peg1_z = calculate_welfare_loss(peg1_results.oo_.irfs.x_err_z, peg1_results
loss_peg2_z = calculate_welfare_loss(peg2_results.oo_.irfs.x_err_z, peg2_results

% shock to i
loss_optimal_istar = calculate_welfare_loss(optimal_results.oo_.irfs.x_err_istar
loss_ditr_istar = calculate_welfare_loss(ditr_results.oo_.irfs.x_err_istar, ditr
loss_citr_istar = calculate_welfare_loss(citr_results.oo_.irfs.x_err_istar, citr
loss_peg1_istar = calculate_welfare_loss(peg1_results.oo_.irfs.x_err_istar, peg1
loss_peg2_istar = calculate_welfare_loss(peg2_results.oo_.irfs.x_err_istar, peg2

% create table

loss_table = {"Shocks", "Optimal Policy", "Domestic Inflation Targeting", "CPI T
              "Cost-push shock (u)", loss_optimal_u, loss_ditr_u, loss_citr_u, l
              "Technology shock (r^n)", loss_optimal_r, loss_ditr_r, loss_citr_r
              "Demand shock (z)", loss_optimal_z, loss_ditr_z, loss_citr_z, loss
              "Foreign shock (i*)", loss_optimal_istar, loss_ditr_istar, loss_ci
              };

dataframe(loss_table);
```

Optimal policy mod file

```
%----------------------------------------------------------------
% 0. Housekeeping (close all graphic windows)
%----------------------------------------------------------------

close all;

%----------------------------------------------------------------
% 1. Defining variables
%----------------------------------------------------------------

var x z i istar r delta_e pi pih tau u;

varexo err_u err_r err_z err_istar;

parameters alpha beta epsilon theta kappa lambda phi rho;

%----------------------------------------------------------------
% 2. Calibration
%----------------------------------------------------------------

alpha = .4;
beta = .99;
epsilon = 6;
theta = .75;
phi = 3;
rho = .9;
lambda = (1 - theta) * (1 - beta * theta) / theta;
kappa = lambda * (1 + phi);


%----------------------------------------------------------------
% 3. Model
%----------------------------------------------------------------

model;

  pih = kappa * x + beta * pih(+1) + u; % AS function
  x = -(i - pih(+1) - r) + x(+1); % AD function
  pi = pih + alpha * (tau - tau(-1)); % domestic inflation & term of trade
  x = z + tau; % domestic output gap
  i(-1) = istar(-1) + delta_e; % uncovered interest rate parity
  pih = - (x - x(-1)) / epsilon; % optimal policy rule
```

```
  istar = rho *  istar(-1) + err_istar; % shocks, AR(1)
  r = rho * r(-1) + err_r;
  u = rho * u(-1) + err_u;
  z = rho * z(-1) + err_z;

end;


initval;

  x = 0;
  pi = 0;
  pih = 0;
  tau = 0;
  i = 0;
  delta_e = 0;

  istar = 0;
  r = 0;
  u = 0;
  z = 0;


end;


shocks;

  var err_u;
  stderr 1;
  var err_r;
  stderr 1;
  var err_z;
  stderr 1;
  var err_istar;
  stderr 1;

  end;

1;


stoch_simul(order=1, irf=100, irf_plot_threshold=0) x pih pi tau i delta_e;


%-----------------------------------------------------------------------
```

```
% 4. Save the Results
%----------------------------------------------------------------------------

optimal_results = oo_;
save optimal_results;
```

Domestic inflation mod file

```
%----------------------------------------------------------------
% 0. Housekeeping (close all graphic windows)
%----------------------------------------------------------------

close all;

%----------------------------------------------------------------
% 1. Defining variables
%----------------------------------------------------------------

var x z i istar r delta_e pi pih tau u;

varexo err_u err_r err_z err_istar;

parameters alpha beta epsilon theta kappa lambda phi psi_pi rho;

%----------------------------------------------------------------
% 2. Calibration
%----------------------------------------------------------------

alpha = .4;
beta = .99;
epsilon = 6;
theta = .75;
phi = 3;
psi_pi = 1.5;
rho = .9;
lambda = (1 - theta) * (1 - beta * theta) / theta;
kappa = lambda * (1 + phi);

%----------------------------------------------------------------
% 3. Model
%----------------------------------------------------------------

model;

  pih = kappa * x + beta * pih(+1) + u; % AS function
  x = -(i - pih(+1) - r) + x(+1); % AD function
  pi = pih + alpha * (tau - tau(-1)); % domestic inflation & term of trade
  x = z + tau; % domestic output gap
  i(-1) = istar(-1) + delta_e; % uncovered interest rate parity
  i = psi_pi * pih; % monetary policy rule
```

```
  istar = rho *  istar(-1) + err_istar; % shocks, AR(1)
  r = rho * r(-1) + err_r;
  u = rho * u(-1) + err_u;
  z = rho * z(-1) + err_z;

end;


initval;

  x = 0;
  pi = 0;
  pih = 0;
  i = 0;
  tau = 0;
  delta_e = 0;

  istar = 0;
  r = 0;
  u = 0;
  z = 0;

end;


shocks;

  var err_u;
  stderr 1;
  var err_r;
  stderr 1;
  var err_z;
  stderr 1;
  var err_istar;
  stderr 1;

end;


stoch_simul(order=1, irf=100, irf_plot_threshold=0) x pih pi tau i delta_e;


%----------------------------------------------------------------
% 4. Save the Results
%----------------------------------------------------------------
```

```
ditr_results = oo_;
save ditr_results;
```

CPI mod file

```
%----------------------------------------------------------------
% 0. Housekeeping (close all graphic windows)
%----------------------------------------------------------------

close all;

%----------------------------------------------------------------
% 1. Defining variables
%----------------------------------------------------------------

var x z i istar r delta_e pi pih tau u;

varexo err_u err_r err_z err_istar;

parameters alpha beta epsilon theta kappa lambda phi psi_pi rho;

%----------------------------------------------------------------
% 2. Calibration
%----------------------------------------------------------------

alpha = .4;
beta = .99;
epsilon = 6;
theta = .75;
phi = 3;
psi_pi = 1.5;
rho = .9;
lambda = (1 - theta) * (1 - beta * theta) / theta;
kappa = lambda * (1 + phi);

%----------------------------------------------------------------
% 3. Model
%----------------------------------------------------------------

model;

  pih = kappa * x + beta * pih(+1) + u; % AS function
  x = -(i - pih(+1) - r) + x(+1); % AD function
  pi = pih + alpha * (tau - tau(-1)); % domestic inflation & term of trade
  x = z + tau; % domestic output gap
  i(-1) = istar(-1) + delta_e; % uncovered interest rate parity
  i = psi_pi * pi; % monetary policy rule
  istar = rho *  istar(-1) + err_istar; % shocks, AR(1)
```

```
  r = rho * r(-1) + err_r;
  u = rho * u(-1) + err_u;
  z = rho * z(-1) + err_z;

end;


initval;

  x = 0;
  pi = 0;
  pih = 0;
  i = 0;
  tau = 0;
  delta_e = 0;

  istar = 0;
  r = 0;
  u = 0;
  z = 0;

end;


shocks;

  var err_u;
  stderr 1;
  var err_r;
  stderr 1;
  var err_z;
  stderr 1;
  var err_istar;
  stderr 1;

end;


stoch_simul(order=1, irf=100, irf_plot_threshold=0) x pih pi tau i delta_e;


%----------------------------------------------------------------
% 4. Save the Results
%----------------------------------------------------------------

citr_results = oo_;
```

```
save citr_results;
```

Peg with $i^* = 0$ mod file

```
%----------------------------------------------------------------------------
% 0. Housekeeping (close all graphic windows)
%----------------------------------------------------------------------------

close all;

%----------------------------------------------------------------------------
% 1. Defining variables
%----------------------------------------------------------------------------

var x z i istar r delta_e pi pih tau u;

varexo err_u err_r err_z err_istar;

parameters alpha beta epsilon theta kappa lambda phi rho sigma;

%----------------------------------------------------------------------------
% 2. Calibration
%----------------------------------------------------------------------------

alpha = .4;
beta = .99;
epsilon = 6;
theta = .75;
phi = 3;
rho = .9;
sigma = 0.01;
lambda = (1 - theta) * (1 - beta * theta) / theta;
kappa = lambda * (1 + phi);

%----------------------------------------------------------------------------
% 3. Model
%----------------------------------------------------------------------------

model;

  pih = kappa * x + beta * pih(+1) + u; % AS function
  x = -(i - pih(+1) - r) + x(+1); % AD function
  pi = pih + alpha * (tau - tau(-1)); % domestic inflation & term of trade
  x = z + tau; % domestic output gap
  i(-1) = istar(-1) + delta_e + sigma * pih; % uncovered interest rate parity
  delta_e = 0;
  istar = rho *  istar(-1) + err_istar; % shocks, AR(1)
```

```
  r = rho * r(-1) + err_r;
  u = rho * u(-1) + err_u;
  z = rho * z(-1) + err_z;

end;


initval;

  x = 0;
  pi = 0;
  pih = 0;
  i = 0;
  tau = 0;
  delta_e = 0;

  istar = 0;
  r = 0;
  u = 0;
  z = 0;

end;


shocks;

  var err_u;
  stderr 1;
  var err_r;
  stderr 1;
  var err_z;
  stderr 1;
  var err_istar;
  stderr 1;

end;


stoch_simul(order=1, irf=100, irf_plot_threshold=0) x pih pi tau i delta_e;


%----------------------------------------------------------------
% 4. Save the Results
%----------------------------------------------------------------

peg_results = oo_;
```

```
save peg_results;
```

Peg with risk premium mod file

```
%------------------------------------------------------------
% Close all graphic windows
%------------------------------------------------------------

close all;

% ------------------------------------------------------------
% Declaring variables
% ------------------------------------------------------------

var x tau pi pih i istar r u z delta_e pistar y yn;

varexo err_r err_u err_z err_istar;

parameters alpha beta epsilon theta phi lambda kappa ro;

alpha = 0.4;
beta = 0.99;
epsilon = 6;
theta = 0.75;
phi = 3;
lamba = ((1 - theta) * (1 - beta * theta)) / theta;
kappa = lamba * (1 + phi);
ro = 0.9;

% ------------------------------------------------------------
% MODEL
% ------------------------------------------------------------

model;

    x = y - yn;
    pih = kappa * x + beta * pih(+1) + u;
    pi = pih + alpha * (tau - tau(-1));
    x = x(+1) - (i - pih(+1) - r);
    x = z + tau;
    i = istar;
    tau = tau(-1) - pih + pistar;

    delta_e = 0;
    pistar = 0;

    r = 0.9 * r(-1) + err_r;
    u = 0.9 * u(-1) + err_u;
```

```
    z = 0.9 * z(-1) + err_z;
    istar = 0.9 * istar(-1) + err_istar;

end;

initval;
    x = 0;
    tau = 0;
    pi = 0;
    pih = 0;
    i = 0;
    istar = 0;
    r = 0;
    u = 0;
    z = 0;
end;


shocks;

    var err_u;
    stderr 1;
    var err_r;
    stderr 1;
    var err_z;
    stderr 1;
    var err_istar;
    stderr 1;

end;

stoch_simul(order = 1, irf=100, irf_plot_threshold = 0) pih, x, pi, tau, i, delt
```