Macroeconometrics Assignment 2

Theoretical question

1. Suppose that $\{x_t\}$ is a weakly stationary process generated by the stable VAR(p),

$$x_t = \sum_{i=1}^{p} \Phi_i x_{t-i} + u_t$$

where $\{u_t\}$ is a weakly stationary white noise (i.e. not necessarily i.i.d.) process with $\mathbb{E}u_t = 0$ and $\mathbb{E}u_t u_t^\mathsf{T} = \Sigma$. Let $\varphi := (\{\Phi_i\}_{i=1}^p, \Sigma)$ denote the model parameters, and $\mathbf{\Gamma}_x := \{\Gamma_x(h)\}_{h\in\mathbb{Z}}$ the ACF. Let $\mathcal{P}$ collect all the φ for which the VAR parametrised by φ is stable, with nonsingular error variance matrix. Show that the mapping $\varphi \mapsto \mathbf{\Gamma}_x$ is injective (i.e. one-to-one) on $\mathcal{P}$.

2. Consider the structural VAR(p)

$$\Psi_0 y_t = \sum_{i=1}^{p} \Psi_i y_{t-i} + \eta_t,$$

where $\{y_t\}$ takes values in $\mathbb{R}^k$; suppose that the VAR is stable. Let

$$\mathrm{SIRF}_h := \partial_{\eta_t} y_{t+h}$$

denote the structural impulse response at horizon h, and $\mathrm{CIRF}_\infty := \sum_{h=0}^{\infty} \mathrm{SIRF}_h$.

(a) Show that $\sum_{h=0}^{\infty} \mathrm{SIRF}_h$ is convergent (assuming the VAR is stable).

Let $[A]_{ij}$ denote the (i, j) element of the matrix A. Show that restrictions of the form

(b) $[\Psi_\ell]_{ij} = 0$

(c) $[\mathrm{SIRF}_h]_{ij} = 0$; and

(d) $[\sum_{h=0}^{\infty} \mathrm{SIRF}_h]_{ij} = 0$

can be rendered in the form $R_m f(\Theta_0) e_m = 0$, where

$$f(\Theta_0 P) = f(\Theta_0) P$$

for any $k \times k$ orthogonal matrix P, and $\Theta_0 = \Psi_0^{-1}$.

3. Suppose that $\{y_t\}$ is a scalar process such that

$$y_t = u_t - u_{t-1},$$

where $\{u_t\}$ is a mean zero, stationary scalar white noise process (with $\mathbb{E}u_t^2 \neq 0$). Show that $\{u_t\}$ is fundamental for $\{y_t\}$, by considering the $\mathcal{L}^2$ limit of

$$\sum_{i=0}^{n}(1 - i/n)y_{t-i}$$

as $n \to \infty$. What can you say about the relationship between the invertibility of an MA process and fundamentalness, on the basis of this result?

Empirical question: high frequency identification

A number of authors have used high frequency changes in Federal funds futures, typically on the day (or within minutes) of a FOMC announcement (or policy event) as instruments for the unsystematic component (shock) of monetary policy (mps). A prominent example is Gertler and Karadi (2015) "Monetary Policy Surprises, Credit Costs, and Economic Activity" AEJ Macro. In this exercise, you are asked to replicate their approach.

Please include replication code for your analysis with your submission.

1. Download the data used in Gertler and Karadi (2015) from the AEA website using this link (or otherwise). You should find monthly time series for the log of industrial production (logip), log of the consumer price index (logcpi), one and two-year government bond yields (gs1 and gs2), and a measure of excess bond premium (ebp). Plot these series over the common available sample, and comment on their time-series properties (e.g., stationarity)

2. Estimate a 4-variable VAR(p) in logip, logcpi, one of gs1 and gs2, and ebp. The exact VAR specification is entirely up to you, such as whether to include the variables in levels, first or second differences, whether to include deterministic trends or seasonal dummies, and the choice of p. Please provide a justification for your chosen VAR specification. [Gertler and Karadi (2015) VARs in levels.]

3. Choose a particular short-run identification scheme to get the IRF to a mps (e.g., Cholesky with a particular order). Plot the IRF of all four variables to a 25 basis points increase in the monetary policy instrument due to a 25bp positive mps (the "unit effect" normalization) over horizons 0 to 48 months. (you should have four plots)

4. Add 90% confidence bands around the IRF estimates in the previous part using either the delta method or a bootstrap of your choice. Comment on the results.

5. Now use the surprise change in the 3-month Federal Funds futures rate around FOMC announcements (the series ff4_tc in the spreadsheet) as an external instrument to identify the mps and comment on your results. You may find it useful to follow these steps:

   (a) Re-estimate your reduced form VAR by OLS and store the residuals, $\hat{\varepsilon}_t$.

(b) Regress each of the residuals $\hat{\varepsilon}_{j,t}$, $j = 1, \ldots, 4$ on ff4_tc and collect all the coefficients in a 4×1 vector $\hat{b}$, say. (Note that this sample is shorter than the one you used to estimate your VAR).

(c) Let $i \in \{1, \ldots, 4\}$ denote the index in $\hat{b}$ of the coefficient corresponding to the residual of your measure of interest rate (either gs1 or gs2). Compute $\hat{\Theta}_{0,i} = \hat{b}/\hat{b}_i$ (a 4-vector with 1 in position i).

(d) Compute the structural IRF to the mps and and corresponding bootstrap confidence bands.