Calibrating a magnetometer sensor using multiple methods

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***Index Terms*—Magnetometer, calibration, compass, least squares, ellipsoid**

I. Introduction

MAGNETOMETERS are often used in attitude and heading reference systems (AHRS), mainly in order to compute the sensor heading without using an expensive dual GNSS antennas compass or an even more expensive fiber-optic gyroscope. However, magnetometers are very sensitive to magnetic interference in their environment and thus need to be calibrated after every change of environment.
According to Talat Ozyagcilar [2], there is two kind of magnetic interference: hard-iron and soft-iron. Hard-iron interference are generated by permanently magnetized ferromagnetic elements and are additive to the earth magnetic field. Soft-iron interference are distortions to the magnetic field by unmagnetized elements.
We will compare three calibration methods. Two of them are least squares based methods. One will permit to calibrate the sensor in an hard-iron only perturbed environment while the other will do the same in a both hard and soft-iron perturbed environment.The third method is a pseudo calibration, often used for inexpensive sensors, using only minimum and maximum computations on each axis of the magnetometer.

II. Soft and hard-iron calibration

A. Magnetic measurements model

According to [1], in an environment with both hard and soft iron perturbations, the measurement of the magnetic field $\mathbf{B_p}$ is:

$$\mathbf{B_p} = \mathbf{W}\mathbf{R_x}(\phi)\mathbf{R_y}(\theta)\mathbf{R_z}(\psi)B\begin{pmatrix}cos(\delta)\\0\\sin(\delta)\end{pmatrix} + \mathbf{V} \tag{1}$$

with $\mathbf{W}$ the soft-iron matrix, $\mathbf{V}$ the hard-iron offset, δ the magnetic inclination, $\mathbf{B}$ the magnetic magnitude and $\mathbf{R_x}(\phi)$ $\mathbf{R_y}(\theta)$ $\mathbf{R_z}(\psi)$ rotation matrix around the yaw (ψ), pitch (θ) and roll (ϕ) angles of the sensor.
For any rotation matrix $\mathbf{R}(\alpha)$:

$$\mathbf{R}^{\mathrm{T}}(\alpha) = \mathbf{R}^{-1}(\alpha) \implies \mathbf{R}^{\mathrm{T}}(\alpha)\mathbf{R}(\alpha) = \mathbf{I}$$

Using (1):

$$\begin{cases}\mathbf{W}^{-1}(\mathbf{B_p} - \mathbf{V}) = \mathbf{R_x}(\phi)\mathbf{R_y}(\theta)\mathbf{R_z}(\psi)B\begin{pmatrix}cos(\delta)\\0\\sin(\delta)\end{pmatrix}\\ (\mathbf{W}^{-1}(\mathbf{B_p} - \mathbf{V}))^{\mathrm{T}} = \left(\mathbf{R_x}(\phi)\mathbf{R_y}(\theta)\mathbf{R_z}(\psi)B\begin{pmatrix}cos(\delta)\\0\\sin(\delta)\end{pmatrix}\right)^{\mathrm{T}}\end{cases}$$

$$\begin{aligned}&\implies (\mathbf{W}^{-1}(\mathbf{B_p} - \mathbf{V}))^{\mathrm{T}}W^{-1}(\mathbf{B_p} - \mathbf{V}) = B^2\\ &\implies (\mathbf{B_p} - \mathbf{V})^{\mathrm{T}}\mathbf{A}(\mathbf{B_p} - \mathbf{V}) = B^2\\ &\text{with } \mathbf{A} = (\mathbf{W}^{-1})^{\mathrm{T}}(\mathbf{W}^{-1})\end{aligned} \tag{2}$$

According to the introduction of [3], equation (2) describe an ellipsoid of parameters $(\mathbf{A}, \mathbf{V})$.

B. Least squares fitting

The quadratic equation of an ellipsoid is [3]:

$$x^{\mathrm{T}}\mathbf{M}x + b^{\mathrm{T}}x + d = 0 \tag{3}$$

with $\mathbf{M} = \begin{pmatrix}m_1 & m_2 & m_3\\ m_2 & m_4 & m_5\\ m_3 & m_5 & m_6\end{pmatrix}$ $b = \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$ where $\mathbf{A}$ is a positive-definite matrix.

$$\begin{aligned}(3) \implies & m_1x_1^2 + m_4x_2^2 + m_6x_3^2 + m_2x_1x_2 + m_3x_1x_3 +\\ & m_5x_2x_3 + b_1x_1 + b_2x_2 + b_3x_3 + d = 0\end{aligned} \tag{4}$$

The equation (4) can easily be fitted using linear least squares.
With $\hat{\mathbf{M}}$, $\hat{b}$ and $\hat{d}$ the least squares estimate of $\mathbf{M}$, b and d, we can then calculate $\hat{\mathbf{A}}$ and $\hat{\mathbf{V}}$.
According to [3] (equation (5)),

$$\begin{cases}\hat{\mathbf{V}} = -\frac{1}{2}\hat{\mathbf{M}}^{-1}\hat{b}\\ \hat{\mathbf{A}} = \frac{1}{\hat{V}^{\mathrm{T}}\hat{\mathbf{M}}\hat{V} - \hat{d}}\end{cases}$$

In order to calibrate the sensor, we need to compute both $\hat{\mathbf{V}}$ and $\hat{\mathbf{W}}^{-1}$. Using equation (20) of [1], we know that $\hat{\mathbf{W}}^{-1} = A^{\frac{1}{2}}$. We could also use a Cholesky decomposition.
Finally, we can obtain calibrated magnetic measurements using the following equation:

$$\mathbf{B}_{cal} = \hat{\mathbf{W}}^{-1}(\mathbf{B}_{mes} - \hat{\mathbf{V}}) \tag{5}$$

III. Hard-iron only calibration

A. Magnetic measurements model

In this case, we assume that soft-iron interference are negligible compared to hard-iron. Thus, $\mathbf{W} = \mathbf{I}_3 \implies \mathbf{A} = \mathbf{I}_3$.
Equation (2) became:

$$(\mathbf{B_p} - \mathbf{V})^{\mathrm{T}}(\mathbf{B_p} - \mathbf{V}) = B^2 \tag{6}$$

B. Least squares fitting

Then, using equation (6) and equation (26) of [3], we can obtain the vector $\hat{\mathbf{V}}$ and $\hat{B}$ using linear least squares with the following equation:

$$\begin{pmatrix}\mathbf{B_p}_x & \mathbf{B_p}_y & \mathbf{B_p}_z & 1\end{pmatrix}\begin{pmatrix}\mathbf{V}_x \\ \mathbf{V}_y \\ \mathbf{V}_z \\ B^2-(\mathbf{V}_x^2+\mathbf{V}_y^2+\mathbf{V}_z^2)\end{pmatrix} = {\mathbf{B_p}_x}^2+{\mathbf{B_p}_y}^2+{\mathbf{B_p}_z}^2 \quad (7)$$

Finally, we can obtain calibrated magnetic measurements using the following equation:

$$\mathbf{B}_{cal} = \mathbf{B}_{mes} - \hat{\mathbf{V}} \quad (8)$$

IV. Pseudo-calibration

To pseudo calibrate magnetic measurements with a dataset of n samples of the form $B_m[i] = \begin{pmatrix}B_x[i] & B_y[i] & B_z[i]\end{pmatrix}^{\mathrm{T}}$:

- Compute x_{min}, x_{max}, y_{min}, y_{max}, z_{min} and z_{max} on the dataset,
- Compute x_{mean}, y_{mean} and z_{mean} on the dataset,
- For each measurement $B_m[i]$, $B_{cal}[i] = \begin{pmatrix}\frac{2(B_x[i]-x_{mean})}{x_{max}-x_{min}} \\ \frac{2(B_y[i]-y_{mean})}{y_{max}-y_{min}} \\ \frac{2(B_z[i]-z_{mean})}{z_{max}-z_{min}}\end{pmatrix}$

V. Results

To test these algorithms, we will use two sensors: a MPU-9250 (SparkFun "9DoF Razor IMU M0") and the magnetometers of an SBG Ellipse 2A. These tests use raw data from the sensors, and thus don't permit to compare or judge there performance as we did not use the manufacturer recommended way of calibrating the sensor.

All the calibrations have been done the same day in the same place. For each sensor, three calibrations have been performed: the first without an apparent perturbation, the second with a 0.8 kg piece of steel fixed to the sensor and the last with a smartphone attached to the sensor.

According to equation (1), after calibration, magnetic measurements should be in the following form:

$$\mathbf{B_{cal}} = \mathbf{R_x}(\phi)\mathbf{R_y}(\theta)\mathbf{R_z}(\psi)B\begin{pmatrix}cos(\delta) \\ 0 \\ sin(\delta)\end{pmatrix} \quad (9)$$

Thus each calibrated measurement must be on a sphere. After each calibration, we will measure the distance of each calibrated measurement to a sphere of radius $r=1$. We will then compute the mean and standard deviation of these distances to compare the performance of the different calibration methods. For each calibrations, the sensor has been rotated in all the directions at least two times.

No perturbation	MPU-9250		SBG Ellipse 2A	
	Mean	Std	Mean	Std
Soft and hard-iron calibration	0.0028	0.0236	0.0003	0.0108
Hard-iron only calibration	0.0004	0.0296	0.0002	0.0183
Pseudo-calibration	0.0266	0.029	0.0049	0.042

0.8kg piece of steel	MPU-9250		SBG Ellipse 2A	
	Mean	Std	Mean	Std
Soft and hard-iron calibration	0.0088	0.0462	0.0012	0.0219
Hard-iron only calibration	0.0079	0.1254	0.0251	0.2228
Pseudo-calibration	0.0469	0.155	0.0381	0.1226

Smartphone perturbation	MPU-9250		SBG Ellipse 2A	
	Mean	Std	Mean	Std
Soft and hard-iron calibration	0.0189	0.0717	0.0029	0.0288
Hard-iron only calibration	0.0016	0.0562	0.0006	0.0344
Pseudo-calibration	0.0431	0.0575	0.0124	0.0416

MPU-9250	Ellipsoid semi-axis	Hard-iron offset
No perturbation	$\begin{pmatrix}262 & 283 & 272\end{pmatrix}$	$\begin{pmatrix}-91.5 & 104 & -274\end{pmatrix}$
Piece of steel	$\begin{pmatrix}193 & 714 & 1362\end{pmatrix}$	$\begin{pmatrix}-130 & -177 & -175\end{pmatrix}$
Smartphone	$\begin{pmatrix}275 & 299 & 312\end{pmatrix}$	$\begin{pmatrix}-85.0 & 172 & -432\end{pmatrix}$

SBG Ellipse 2A	Ellipsoid semi-axis	Hard-iron offset
No perturbation	$\begin{pmatrix}1.09 & 1.02 & 1.05\end{pmatrix}$	$\begin{pmatrix}0.008 & -0.012 & 0.048\end{pmatrix}$
Piece of steel	$\begin{pmatrix}1.24 & 0.55 & 0.50\end{pmatrix}$	$\begin{pmatrix}0.11 & 0.0037 & 0.055\end{pmatrix}$
Smartphone	$\begin{pmatrix}0.93 & 0.85 & 0.86\end{pmatrix}$	$\begin{pmatrix}0.30 & 0.24 & 0.47\end{pmatrix}$

Ellipsoid semi-axis and hard-iron offset are obtained with the soft and hard-iron calibration.

The results in the "no perturbation" case shows that the both soft and the hard-iron calibration and hard-iron only one are almost equivalent. However, the pseudo-calibration gives worst results than the two others methods but the standard deviation stay in the same order of magnitude than the others methods. The "smartphone perturbation" gives the sames results. The hard-iron offset is strong in these cases compared to the differences between the ellipsoid semi-axis which represent the soft-iron interference.

In the "0.8kg piece of steel" case, both the pseudo-calibration and the hard-iron only one are really off compared to the soft and hard-iron one. This could be explained by the fact that the piece of steel seems to create an important soft-iron interference which could be observed in ellipsoid semi-axis.

VI. Conclusion

The soft and hard iron seems to be an efficient calibration method. The hard iron only method gives good performances in an magnetically undistorted by soft-iron interference environment. Compared to the soft and hard iron The pseudo-calibration is less efficient than the others methods and thus shall be only used in case we are not able to compute least squares algorithms.

Acknowledgment

The authors would like to thank Fabrice Le Bars for providing the sensors and Luc Jaulin for his help.

References

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