Master in Computer Vision *Barcelona*

Module: 3D Vision
Lecture 2a: Planar transformations
Lecturer: Gloria Haro

# Planar transformations

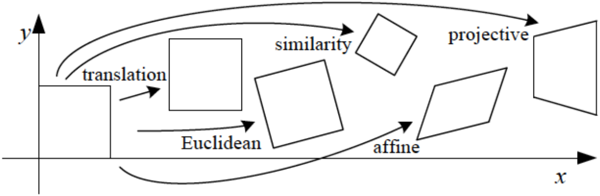



Image source: [Szeliski 2010]

# Planar transformations

a

b

c

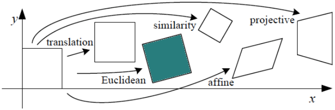
Similarity | Affine transformation | Projective transformation

Image source: [Hartley Zisserman 2004]

# Isometry

Let $\mathbf{x}$ and $\mathbf{x}'$ be homogeneous coordinates.

$$\mathbf{x}' = H_e\mathbf{x} = \begin{pmatrix} R & \vec{t} \\ \vec{0}^T & 1 \end{pmatrix}\mathbf{x}.$$




where $R$ is a rotation matrix (orthogonal matrix) and
$\vec{t}$ is a translation vector.

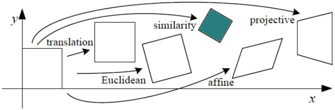
**Degrees of freedom:** 3
→ 1 for the rotation angle + 2 for the translation coefficients

**Invariants:** lengths, angles.

# Similarity

Let $\mathbf{x}$ and $\mathbf{x}'$ be homogeneous coordinates.

$$\mathbf{x}' = H_s\mathbf{x} = \begin{pmatrix} sR & \vec{t} \\ \vec{0}^T & 1 \end{pmatrix} \mathbf{x}.$$




where $s$ is an isotropic scaling factor,
$R$ is a rotation matrix (orthogonal matrix) and
$\vec{t}$ is a translation vector.

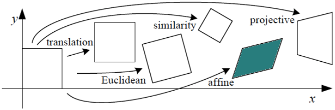
**Degrees of freedom:** 4
$\rightarrow$ 1 for the rotation angle + 2 for the translation coefficients
+ 1 for scaling factor

**Invariants:** angles, ratio of lenghts, ratio of two areas.

# Affine transformation

Let $\mathbf{x}$ and $\mathbf{x}'$ be homogeneous coordinates.

$$\mathbf{x}' = H_a \mathbf{x} = \begin{pmatrix} A & \vec{t} \\ \vec{0}^T & 1 \end{pmatrix} \mathbf{x}.$$




where $A$ is a non-singular $2 \times 2$ matrix ,
$\vec{t}$ is a translation vector.

If $\det(A) > 0$ (orientation preserving), the SVD decomposition of $A$ gives:

$$A = UDV^T = UV^T(VDV^T) = R_\theta R_{-\phi} D R_\phi$$

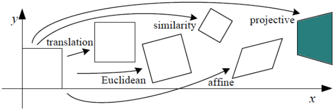
**Degrees of freedom:** 6
$\rightarrow$ 2 for the rotation angles $(\theta, \phi)$ + 2 for scaling factors
+ 2 for the translation coefficients

**Invariants:** parallel lines, ratios of parallel lenghts, ratio of two areas, line at infinity $\ell_\infty$.

# Projective transformation

Let $\mathbf{x}$ and $\mathbf{x}'$ be homogeneous coordinates.

$$\mathbf{x}' = H_p \mathbf{x} = \begin{pmatrix} A & \vec{t} \\ \vec{v}^T & v \end{pmatrix} \mathbf{x}.$$




where $H_p$ is called a 2D homography

$$H_p = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}$$

$H_p$ is a non-singular matrix $\rightarrow$ represents an invertible mapping

**Degrees of freedom:** 8
$\rightarrow 3 \times 3$ elements - 1 multiplicative factor

**Invariants:** concurrency, collinearity, order of contact, cross ratio

# Planar transformations

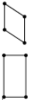
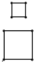
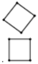
| Group | Matrix | Distortion | Invariant properties |
|---|---|---|---|
| Projective<br>8 dof | $\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$ |  | Concurrency, collinearity, **order of contact**: intersection (1 pt contact); tangency (2 pt contact); inflections (3 pt contact with line); tangent discontinuities and cusps. cross ratio (ratio of ratio of lengths). |
| Affine<br>6 dof | $\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$ |  | Parallelism, ratio of areas, ratio of lengths on collinear or parallel lines (e.g. midpoints), linear combinations of vectors (e.g. centroids). The line at infinity, $\mathbf{l}_\infty$. |
| Similarity<br>4 dof | $\begin{bmatrix} sr_{11} & sr_{12} & t_x \\ sr_{21} & sr_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$ |  | Ratio of lengths, angle. The circular points, **I**, **J** (see section 2.7.3). |
| Euclidean<br>3 dof | $\begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$ |  | Length, area |

Image source: [Hartley Zisserman 2004]

# Homographies

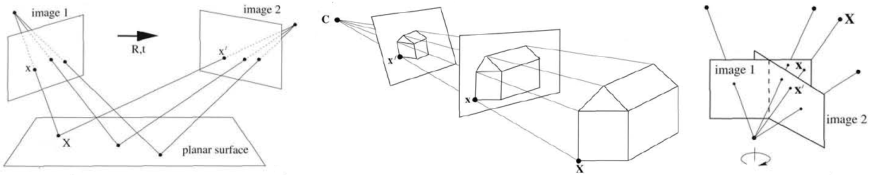
Projective transformation of points: $\mathbf{x}' = H\mathbf{x}$

Projective transformation of lines: $\ell' = H^{-T}\ell$

Projective transformation of conics: $C' = H^{-T}CH^{-1}$

Projective transformation of dual conics: $C^{*\prime} = HC^{*}H^{T}$

# Homographies




A homography relates two images:

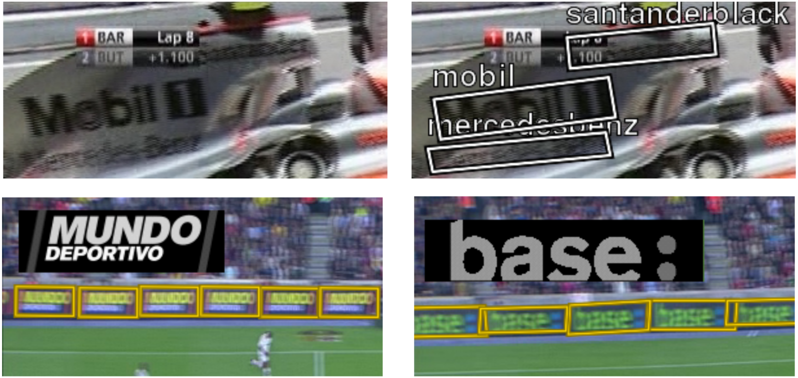
- of the same plane in the 3D scene;
- taken with a camera rotating about its centre;
- taken with the same static camera varying its focal length;
- the whole scene is far away from the camera.

Image source: [Hartley Zisserman 2004]

# Applications: Panorama / Image mosaic

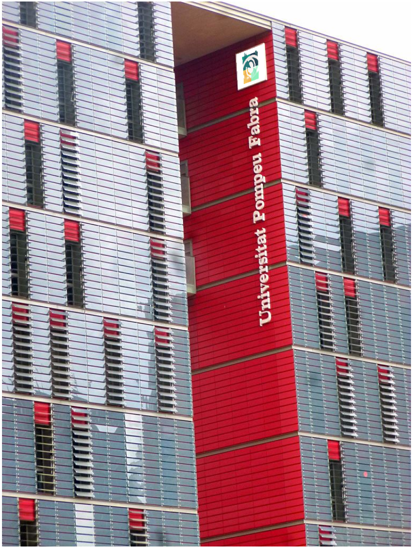
M. Brown and D. G. Lowe. Automatic Panoramic Image Stitching using Invariant Features. International Journal of Computer Vision, 2007.

# Applications: Logo detection

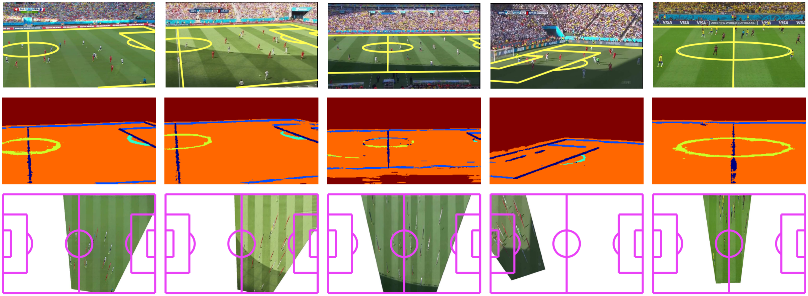



C. Constantinopoulos, E. Meinhardt-Llopis, Y. Liu, V. Caselles. A robust pipeline for logo detection, IEEE Int. Conf. on Multimedia and Expo, 2011.

# Applications: Logo insertion

# Applications: Sports field localization



N. Homayounfar, S. Fidler, R. Urtasun. Sports Field Localization via Deep Structured Models. In Proc. of Conference on Computer Vision and Pattern Recognition, 2017.

# Applications: Calibration and Augmented Reality


Image source: http://thinkmobiles.com

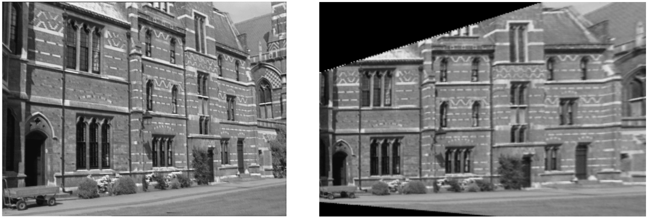
Z. Zhang. A Flexible New Technique for Camera Calibration. IEEE Transactions on Pattern Analysis and Machine Intelligence, 22 (11): 1330-1334, 2000.

# Applications: Multi-image fusion

G. Haro, A. Buades, J.M. Morel. Photographing Paintings by Image Fusion. SIAM Journal on Imaging Sciences, 2012.

# Applications: Image rectification

## Metric rectification

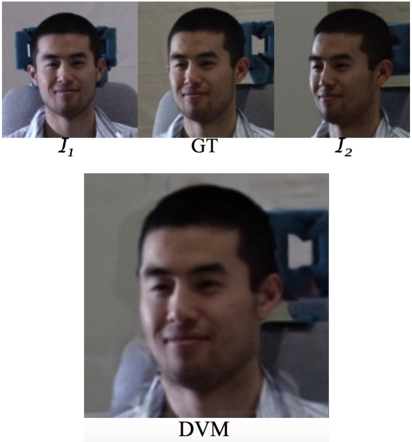


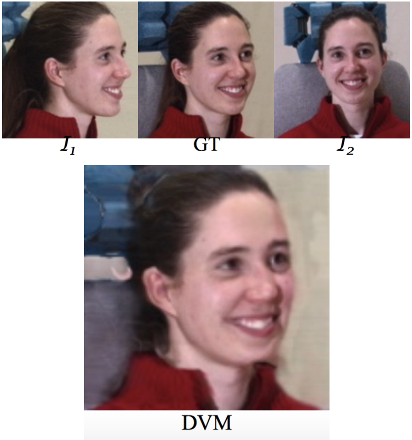
D.Liebowitz and A. Zisserman. Metric Rectification for Perspective Images of Planes. In Proc. of Computer Vision and Pattern Recognition, 1998.

## Stereo rectification

A. Fusiello and L. Irsara. Quasi-Euclidean uncalibrated epipolar rectification. In 19th International Conference on Pattern Recognition, 2008.

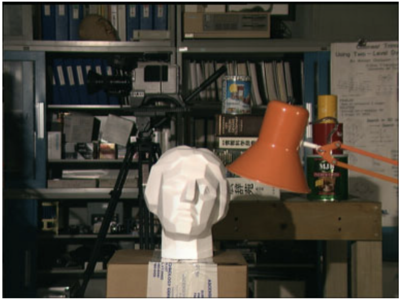
# Applications: New view synthesis




D. Ji, J. Kwon, M. Mcfarland, S. Savarese. Deep View Morphing, In Proc. of Conference on Computer Vision and Pattern Recognition, 2017.
S.M. Seitz and C.R. Dyer. View morphing. Proc. of the 23rd annual conference on Computer graphics and interactive techniques, 1996.

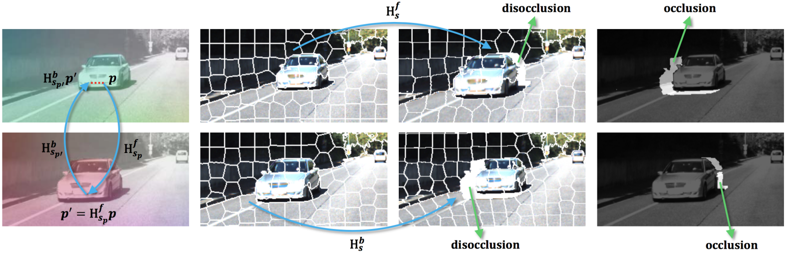
# Applications: Depth estimation



R. Collins. A Space-Sweep Approach to True Multi-Image Matching. In Proc. of Conference on Computer Vision and Pattern Recognition, 1996.

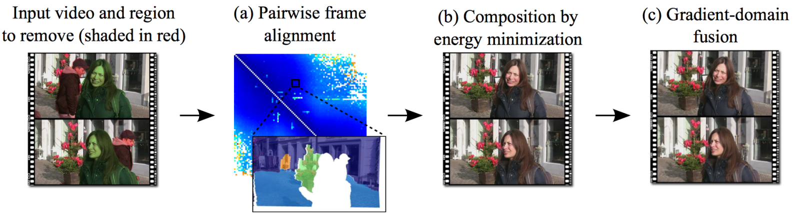
R. Yang and M. Pollefeys. Multi-resolution real-time stereo on commodity graphics hardware, In Proc. of Conference on Computer Vision and Pattern Recognition, 2003.

# Applications: Optical flow




J. Hur, S. Roth. MirrorFlow: Exploiting Symmetries in Joint Optical Flow and Occlusion Estimation. International Conference on Computer Vision, 2017.

# Applications: Video inpainting




M. Granados, K. I. Kim, J. Tompkin, J. Kautz, C. Theobalt. Background Inpainting for Videos with Dynamic Objects and a Free-moving Camera. In Proc. European Conference in Computer Vision, 2012.

# References

[Hartley and Zisserman 2004] R.I. Hartley and A. Zisserman, Multiple View Geometry in Computer Vision, Cambridge University Press, 2004.

[Szeliski 2010] R. Szeliski, Computer Vision: Algorithms and Applications, Springer, 2010.