Theory loss in purity vs. Logical rotation angle  $\beta$  cluster state with  $\phi = \frac{\pi}{4}$  NNN-XX rotation

