# MATH 320/321 (Real Analysis) Notes

Rio Weil

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# 1 The Real and Complex Number Systems

## 1.1 The Naturals, Integers, and Rationals

We begin by a review of number systems which are already familiar.

The **Naturals**, denoted by $\mathbb{N}$, is the set $\{1, 2, 3, \ldots\}$. For $x, y, \in \mathbb{N}$, we have that $x + y \in \mathbb{N}$ and $xy \in \mathbb{N}$, so the naturals are closed under addition and multiplication. However, we note that it is not closed under subtraction; take for example $2 - 4 = -2 \notin \mathbb{N}$.

The **Integers**, denoted by $\mathbb{Z}$, is the set $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. It is closed under addition, multiplication, and subtraction. However, it is not closed under division; for example, $1/2 \notin \mathbb{Z}$.

The **Rationals**, denoted by $\mathbb{Q}$, does not have as obvious of a denumeration. A first attempt would be $\left\{\frac{m}{n} : m \in \mathbb{Z}, n \in \mathbb{N}\right\}$, where $\frac{m_1}{n_1}$ and $\frac{m_2}{n_2}$ are identified if $m_1 n_2 = m_2 n_1$. This is a good definition if we already have the same rigorous idea of what a rational number is in our mind; i.e. it works because we have a shared preconceived understanding of a rational number.

If this is not the case, it may help to define the rational numbers more rigorously/formally (even if the definition may be slightly harder to parse). As a second attempt at a definition, we can say that $\mathbb{Q}$ is the set of ordered pairs $\big\{(m, n) : m \in \mathbb{Z}, n \in \mathbb{N}\big\}$. However, this is not quite enough as we need a notion of equivalence between two rational numbers (e.g. $(1, 2) = (2, 4)$). Hence, a complete and rigorous definition would be $\mathbb{Q} = \big\{(m, n) : m \in \mathbb{Z}, n \in \mathbb{N}\big\} / \sim$ where $(m_1, n_1) \sim (m_2, n_2)$ if $m_1 n_2 = m_2 n_1$.

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