PHYS 141 Discussion Week 9 - Special Relativity

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Contents

1 Problems

1.1 Bomb Diffusal (Morin 11.6)

A train and a tunnel both have proper length L. The train, carrying a bomb at the front, moves towards the tunnel at speed v. The bomb is designed such that it explodes when the front of the train passes the far end of the tunnel. At the back of the tunnel is a deactivation sensor, which tells the bomb to disarm itself when the back of the train passes the near end of the tunnel. Does the bomb explode?

1.2 Train Lasers

A long train is moving at $0.6c$. Let L denote the (proper) length of the train. Alice and Bob are standing on the two ends of the train, with Alice at the back and Bob at the front. One-quarter of the way from the back of the train there is a target. Alice and Bob both fire their laser guns at the target, and their blasts hit the target at the same time.

(a) Consider the two events "Alice fires her gun" and "Bob fires his gun". How far apart are these two events in the train's frame of reference?

(b) In the frame of the train, who fired their gun first and by how much?

(c) In the frame of the ground, who fired their gun first and by how much?

(d) How far apart are the two events in the frame of the ground? Solve this using:

   (i) Length contraction and by studying how far the train travels in between the two events

   (ii) A lorentz transformation

   (iii) The spacetime invariant

   and verify the answers agree.

2 Solutions

2.1 Bomb Diffusal

Analyzing this problem from the two different frames seems to give contradictory answers. In the train frame, the tunnel appears contracted to L/γ and so the front of the train passes the far end first, causing the bomb to explode. In the tunnel frame, the train appears contracted to L/γ and so the back of the train appears to pass the near end first, causing the bomb to be defused. We appear to have a paradox.

The correct answer is that $\boxed{\text{the bomb explodes}}$. The second argument (in the tunnel frame) is incorrect in that it ignores the fact that the deactivation sensor must communicate (with communication speed bounded by the speed of light) with the bomb at the front of the train, and before it can do this, the front of the train passes the far end of the tunnel (causing it to explode, in agreement with the first argument in the train frame).

Let's suppose the speed of the signal is c and show that this is the case. The signal takes L/c time to reach the front of the train, and the front of the train takes time $L(1-1/\gamma)/v$ to reach the far end of the tunnel from the time when the back of the train crosses the rear end. For the bomb to not explode, we require that the former time is less than the latter, so:

$$L/c < L(1-1/\gamma)/v \implies \beta < 1-\sqrt{1-\beta^2} \implies \sqrt{1-\beta^2} < 1-\beta \implies \sqrt{1+\beta} < \sqrt{1-\beta} \tag{2.1}$$

which is never satisfied. Thus, as we claimed, the signal never reaches the front of the train in time and the bomb explodes.

2.2 Train Lasers

Throughout the primed frame is the train frame and the unprimed frame is the ground frame.

(a) In the train's frame, this is just $\boxed{\Delta x' = L}$.

(b) Since Bob is $L/2$ further from the target than Alice, for the lasers to hit the target at the same time, he fires first by $\frac{1}{2}\frac{L}{c}$, i.e. $\boxed{\Delta t' = -\frac{1}{2}\frac{L}{c}}$.

(c) Put Alice at $x'_A = 0$ and make $t'_A = 0$ when she fires. Then, Bob is at $x'_B = L$ and fires at $t'_B = -\frac{1}{2}\frac{L}{c}$. The γ factor is $\gamma = \frac{4}{5}$. Thus:

$$\Delta t = t_B - t_A = \gamma(\Delta t' + \frac{v\Delta x'}{c^2}) = \frac{5}{4}\left(-\frac{1}{2}\frac{L}{c} + \frac{3}{5}\frac{L}{c}\right) = \frac{1}{8}\frac{L}{c} \tag{2.2}$$

Thus Alice fires first by $\boxed{\Delta t = \frac{1}{8}\frac{L}{c}}$.

(d) (i) The train is length-contracted by $5/4$, so its length in the ground frame is $\frac{4}{5}L$. In the time between Alice and Bob firing, Bob moves away a distance $\frac{1}{8}\frac{L}{c}\cdot\frac{3}{5}c = \frac{3}{40}L$. Thus, the total distance between events is $\frac{4}{5}L + \frac{3}{40}L$ i.e. $\boxed{\Delta x = \frac{7}{8}L}$.

(ii) Using the Lorentz transformation:

$$\Delta x = \gamma(\Delta x' + v\Delta t') = \frac{5}{4}(L + \frac{3}{5}\cdot\frac{-1}{2}L) = \boxed{\frac{7}{8}L} \tag{2.3}$$

(iii) The spacetime invariant computed in the train frame is:

$$\Delta s'^2 = -c^2 \Delta t'^2 + \Delta x'^2 = c^2 \left(-\frac{1}{2}\frac{L}{c}\right)^2 + L^2 = \frac{3}{4}L^2 \tag{2.4}$$

This is equal to the spacetime invariant in the ground frame, so:

$$\Delta s'^2 = \Delta s^2 = -c^2 \Delta t^2 + \Delta x^2 \implies \Delta x = \sqrt{\Delta s'^2 + c^2 \Delta t^2} = \sqrt{\frac{3}{4}L^2 + c^2 \left(\frac{1}{8}\frac{L}{c}\right)^2} = \boxed{\frac{7}{8}L} \tag{2.5}$$

All three answers agree, as they should!