# A Simulation of a Simulation: Algorithms for Measurement-Based Quantum Computing Experiments

PHYS 449 - Oral Defense

Rio Weil

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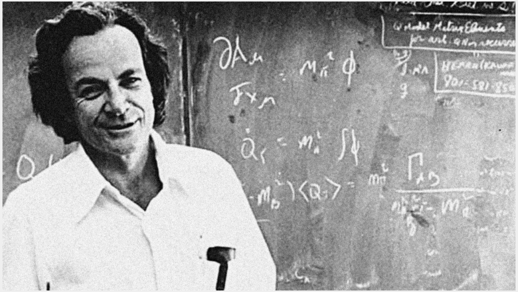
# Outline

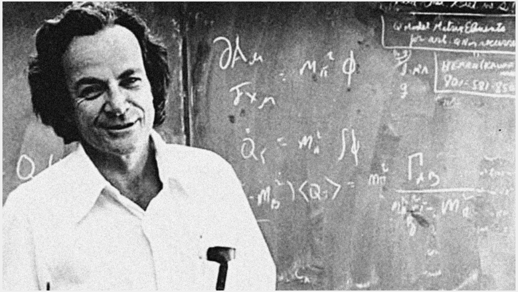
1. Motivation
2. Why Measurement-Based Quantum Computing?
3. MBQC Resource States and Computational Power
4. From Theory to Experiment
5. The Alluded Algorithms
6. Results
7. Making the Experiment "More Quantum"
8. Outlook & Conclusion



*"Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."*

- Feynman, 1981

# Motivation - What are Quantum Computers Good For?



*"Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."*

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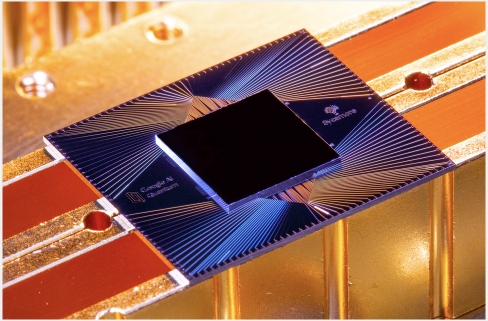
Also...

- Shor's Factoring Algorithm
- Grover's Search Algorithm

# Motivation - Outstanding Questions



1. What is the source of quantum advantage?

2. What can we do with NISQ (Noisy Intermediate-Scale Quantum) devices?

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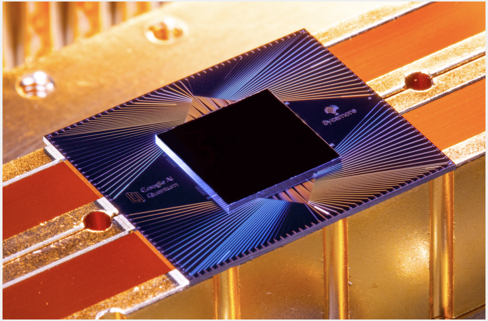
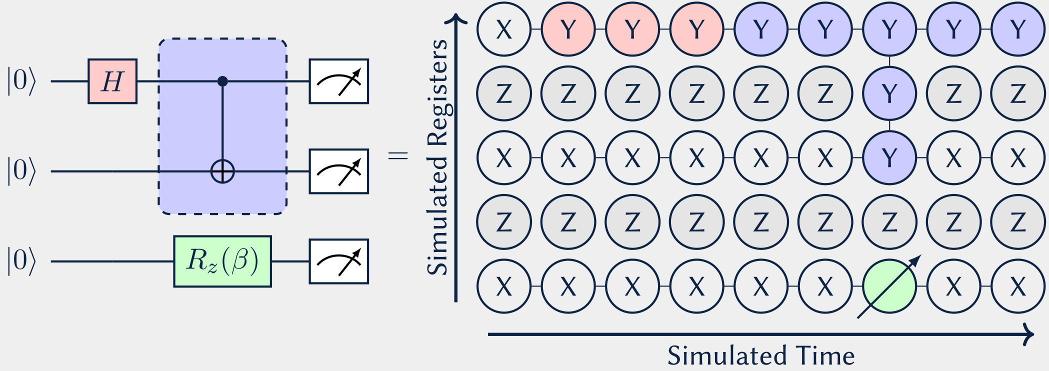
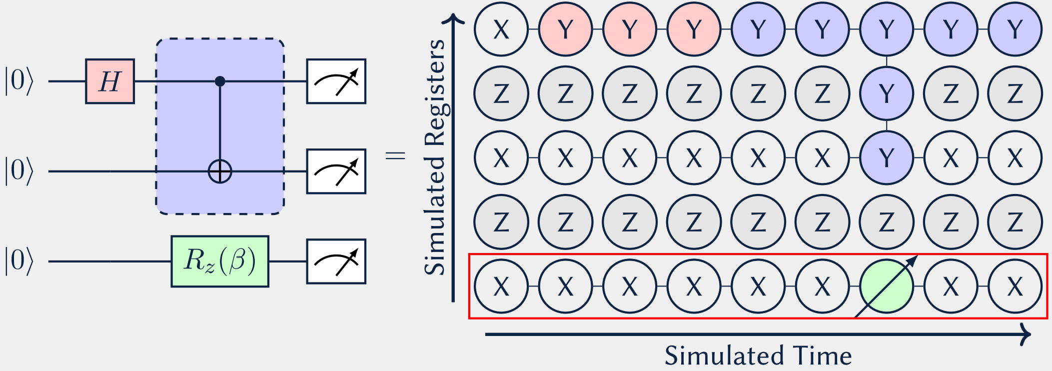


1. What is the source of quantum advantage?
   - Measurement-Based Quantum Computing - coming up!
2. What can we do with NISQ (Noisy Intermediate-Scale Quantum) devices?
   - Active research area... and this project!

# Why Measurement-Based Quantum Computing?




| | Gate Model | MBQC |
|---|---|---|
| Evolution Method | Unitary Gates | Single-Qubit measurements |
| “Power Source” | Intermediate Gates | Initial State |

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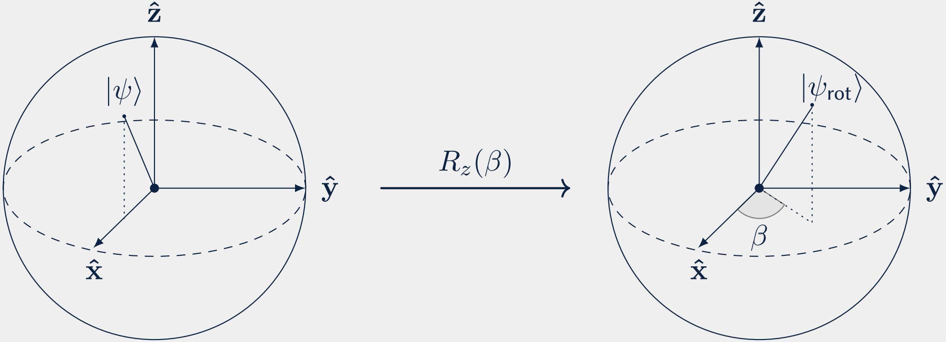
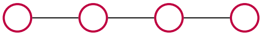



Ability to perform single qubit unitaries - rotations.

# 1-D Resource States

**Universal Resource: Cluster State $|C\rangle$**



Ground state of $H_{\text{cluster}}$

**Useless Resource: Product State $|+\rangle^{\otimes N}$**

Ground state of $H_{\text{product}}$

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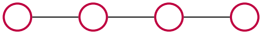


Ground state of $H_{\text{cluster}}$

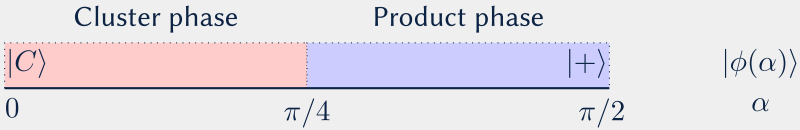
**Useless Resource: Product State $|+\rangle^{\otimes N}$**

Ground state of $H_{\text{product}}$

Question: Power of ground states $|\phi(\alpha)\rangle$ of:

$$H(\alpha) = \cos(\alpha) H_{\text{cluster}} + \sin(\alpha) H_{\text{product}}?$$

Answer (for infinite systems):



# 1D Resource States - Phase Diagram & Decoherence

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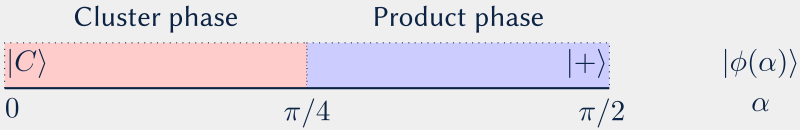


- The catch: Decoherence away from cluster state.

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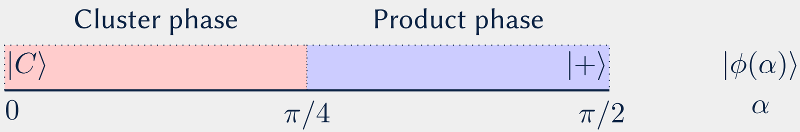
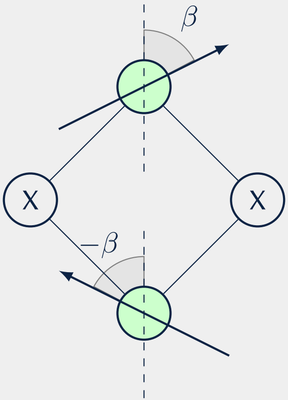
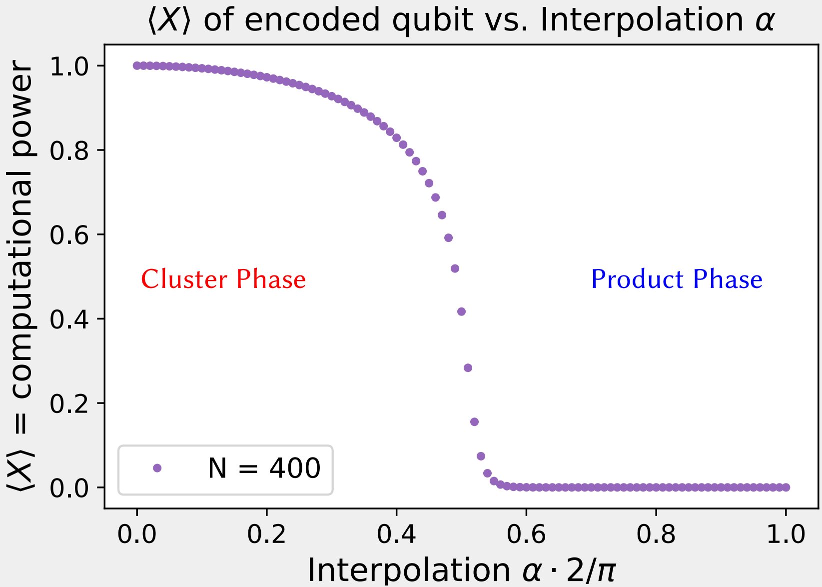



- The catch: Decoherence away from cluster state.
- Recently: New formalism for analyzing power in finite resource states.

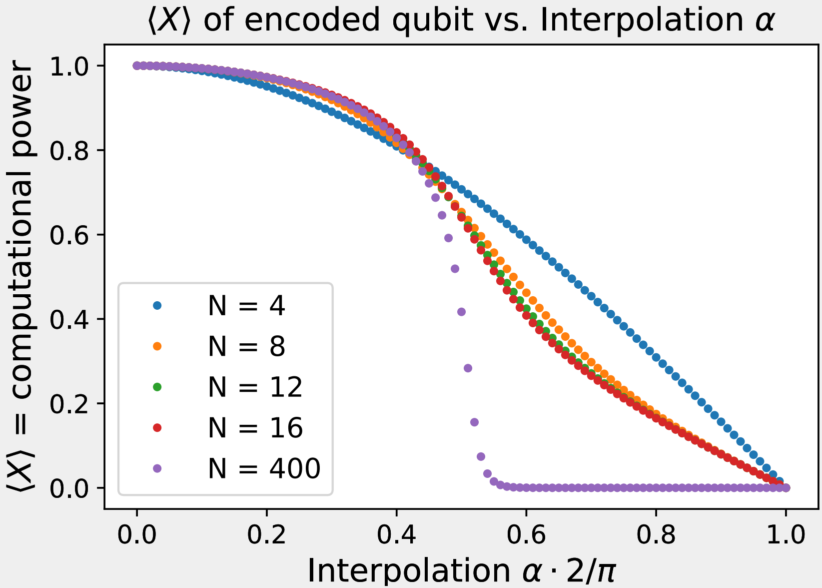


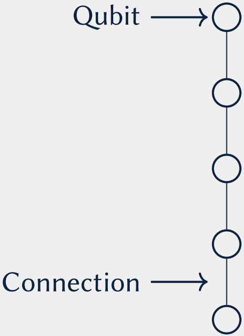

Demonstration: The rotation-counter rotation scheme

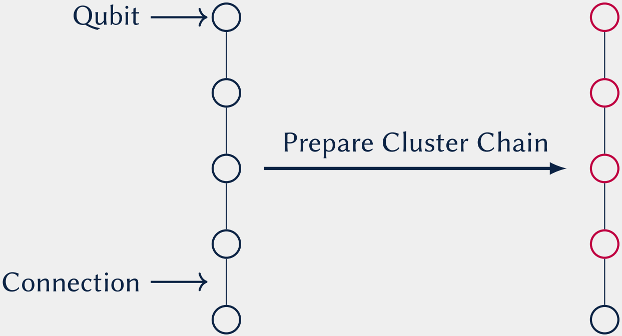
1. Prepare $|\phi(\alpha)\rangle$, and input $|+\rangle$.
2. Apply $\beta$ rotation, and $-\beta$ counterrotation via measurement.
3. Measure $\langle X\rangle$: computational power.

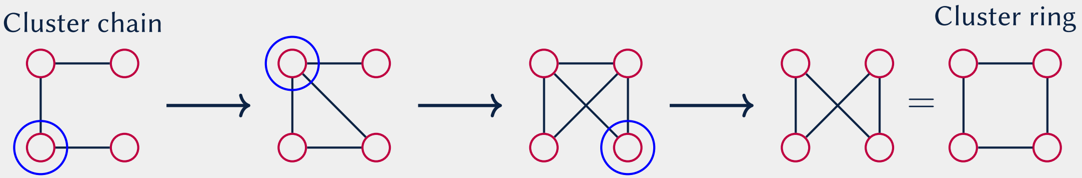


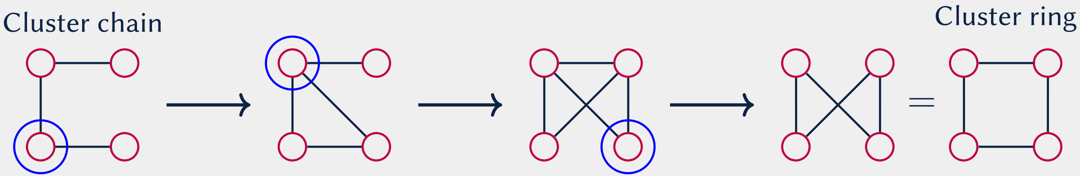
$\langle X \rangle$ of encoded qubit vs. Interpolation $\alpha$
$\langle X \rangle$ = computational power
1.0
0.8
0.6
0.4
0.2
0.0
Cluster Phase
Product Phase
N = 400
0.0
0.2
0.4
0.6
0.8
1.0
Interpolation $\alpha \cdot 2/\pi$


⟨X⟩ of encoded qubit vs. Interpolation α
⟨X⟩ = computational power
Interpolation α · 2/π
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
N = 4
N = 8
N = 12
N = 16
N = 400


Qubit
Connection


Qubit
Connection
Prepare Cluster Chain


Cluster chain
Cluster ring




Takeaway: Playing tricks to simulate a ring with a chain.

Recall $H(\alpha)$ and corresponding ground state $|\phi(\alpha)\rangle$:

$$H(\alpha) = \cos(\alpha) H_{\text{cluster}} + \sin(\alpha) H_{\text{product}}$$

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There exists unitary $U(\alpha)$ such that:

$$|\phi(\alpha)\rangle = U(\alpha) |C\rangle$$

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Further, we can exchange unitarity for a simpler representation:

$$U(\alpha) \cong T(\alpha) = \text{only } X\text{s and } I\text{s}$$

# From Theory to Experiment - Producing Ground States

Recall $H(\alpha)$ and corresponding ground state $|\phi(\alpha)\rangle$:

$$H(\alpha) = \cos(\alpha)H_{\text{cluster}} + \sin(\alpha)H_{\text{product}}$$

There exists unitary $U(\alpha)$ such that:

$$|\phi(\alpha)\rangle = U(\alpha)\,|C\rangle$$

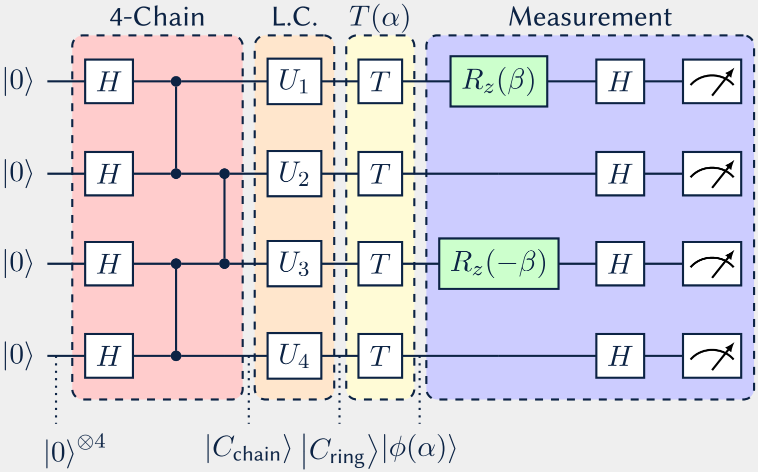
Further, we can exchange unitarity for a simpler representation:

$$U(\alpha) \cong T(\alpha) = \text{only } X\text{s and } I\text{s}$$

Finally, we can assume $T$ is local, so:

$$T(\alpha) = \bigotimes_{i=1}^{N}(aX_i + bI_i)$$

With these simplifications, $T(\alpha)$ can be found via classical optimization, for small rings.


4-Chain
L.C.
$T(\alpha)$
Measurement
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
$H$
$H$
$H$
$H$
$U_1$
$U_2$
$U_3$
$U_4$
$T$
$T$
$T$
$T$
$R_z(\beta)$
$R_z(-\beta)$
$H$
$H$
$H$
$H$
$|0\rangle^{\otimes 4}$
$|C_{\text{chain}}\rangle$
$|C_{\text{ring}}\rangle$
$|\phi(\alpha)\rangle$

Problem: $T(\alpha)$ is non-unitary in general. But quantum gates are unitary.

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- Solution: Redraw some brackets:

$$p(\mathbf{j}) = |\langle\mathbf{j}|\phi(\alpha)\rangle|^2 = |\langle\mathbf{j}|\,[T(\alpha)\,|C\rangle]|^2 = |[\langle\mathbf{j}|\,T(\alpha)]\,|C\rangle|^2 = \left|\left\langle T^\dagger(\alpha)\mathbf{j}\middle|C\right\rangle\right|^2$$

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- Conclusion: Don't implement $T(\alpha)$ at all. Instead, measure $T^\dagger(\alpha) |\mathbf{j}\rangle$ on $|C\rangle$ instead!

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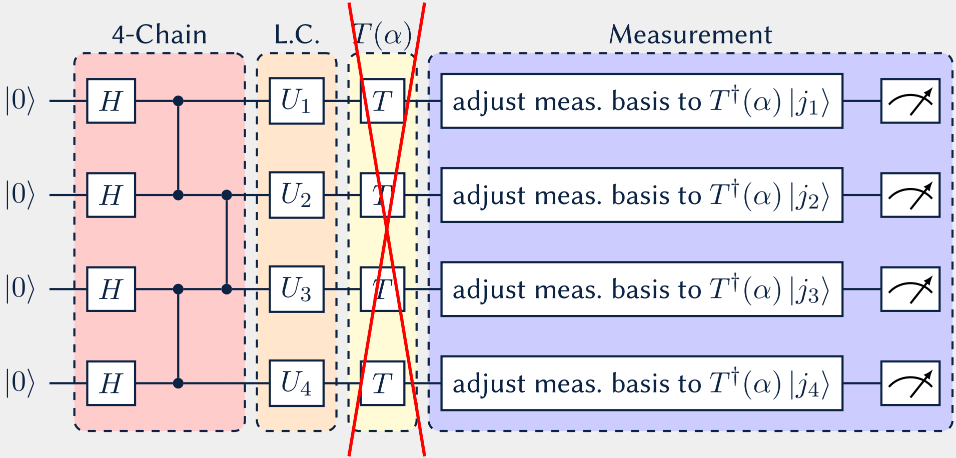
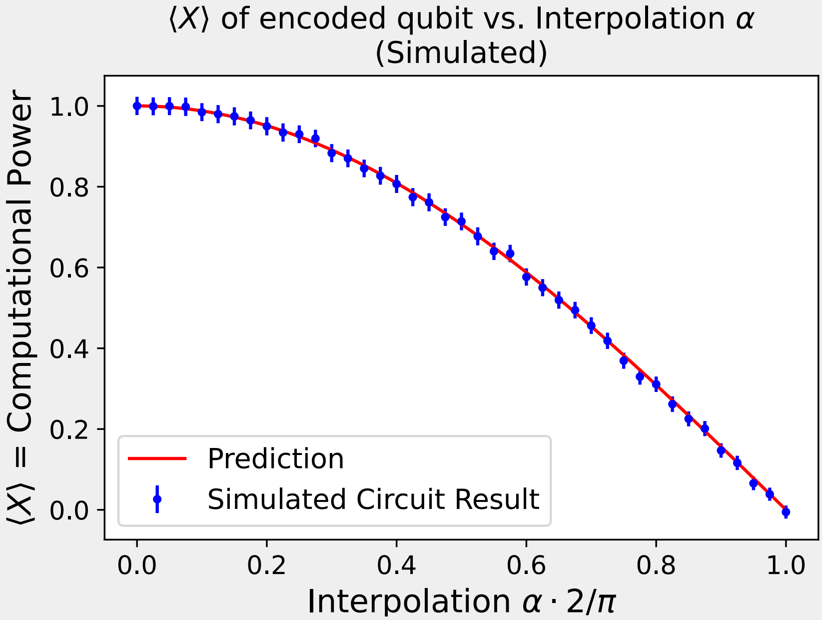
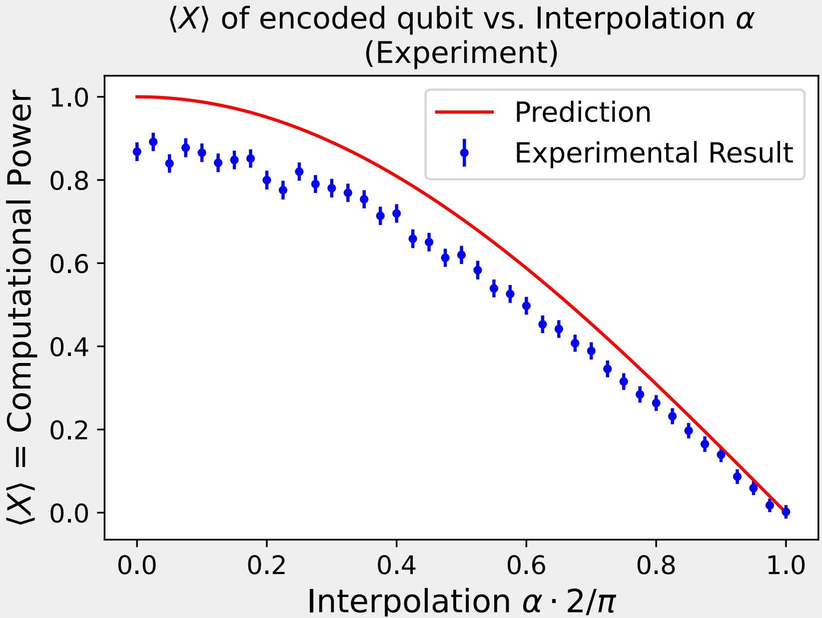
- Takeaway: Problem Decomposition.




The Task: Develop algorithm to make circuits and post-process measurement outcomes, obtain results *as if* $T(\alpha)$ had been implemented.



$\langle X \rangle$ of encoded qubit vs. Interpolation $\alpha$ (Simulated)
$\langle X \rangle$ = Computational Power
1.0
0.8
0.6
0.4
0.2
0.0
Prediction
Simulated Circuit Result
0.0
0.2
0.4
0.6
0.8
1.0
Interpolation $\alpha \cdot 2/\pi$



$\langle X \rangle$ of encoded qubit vs. Interpolation $\alpha$ (Experiment)
Prediction
Experimental Result
$\langle X \rangle$ = Computational Power
Interpolation $\alpha \cdot 2/\pi$
0.0
0.2
0.4
0.6
0.8
1.0

- Current setup: coefficients for $T(\alpha)$ found classically, and $T(\alpha)$ applied classically via post-processing.

# Making the Experiment "more quantum" - Using VQE

- Current setup: coefficients for $T(\alpha)$ found classically, and $T(\alpha)$ applied classically via post-processing.
- More quantum mechanical (and more generalizable): Find $T(\alpha)$ on a quantum computer. Use a Variational Quantum Eigensolver.

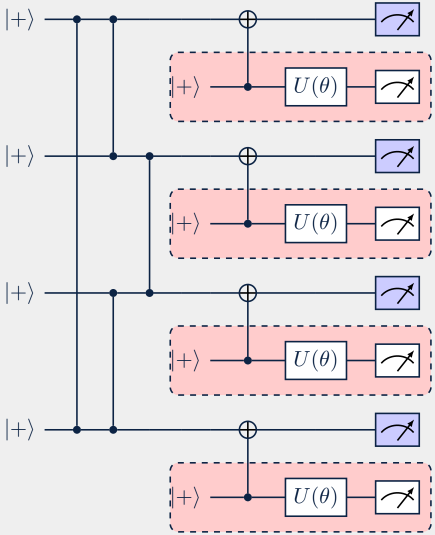
# Making the Experiment "more quantum" - Using VQE

- Current setup: coefficients for $T(\alpha)$ found classically, and $T(\alpha)$ applied classically via post-processing.
- More quantum mechanical (and more generalizable): Find $T(\alpha)$ on a quantum computer. Use a Variational Quantum Eigensolver.

$$\boxed{\begin{array}{c}\text{Prepare Ansatz}\\ |\psi(\theta)\rangle = T'(\theta)\,|C\rangle\end{array}} \longrightarrow \boxed{\begin{array}{c}\text{Measure } E(\alpha,\theta)\text{:}\\ \langle\psi(\theta)|\,H(\alpha)\,|\psi(\theta)\rangle\end{array}} \longrightarrow \boxed{\text{Update } \theta}$$

Repeat until $E(\alpha, \theta)$ minimized

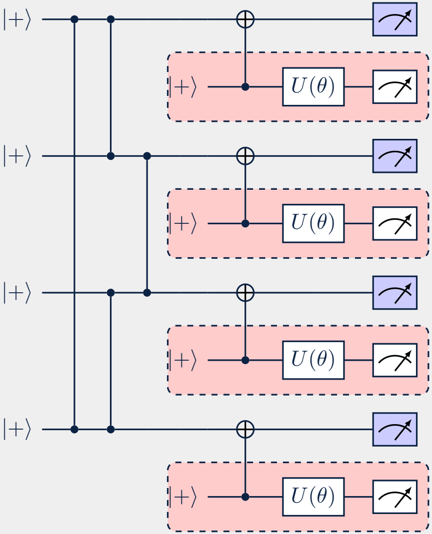
- Minimization (for a given $\alpha$) yields $\theta$ for which $T'(\theta) = T(\alpha)$.




Measure $E(\alpha, \theta)$

Prepare $T'(\theta)\,|C\rangle = |\psi(\theta)\rangle$

Large + Inefficient

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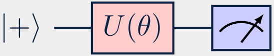
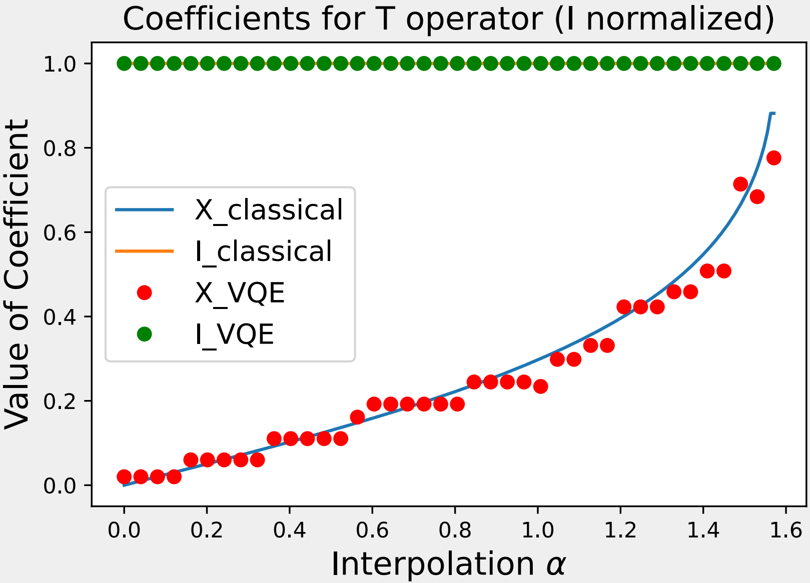
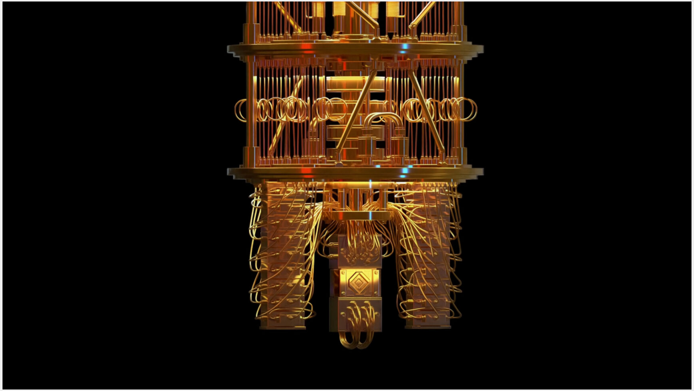
Large + Inefficient

$=$

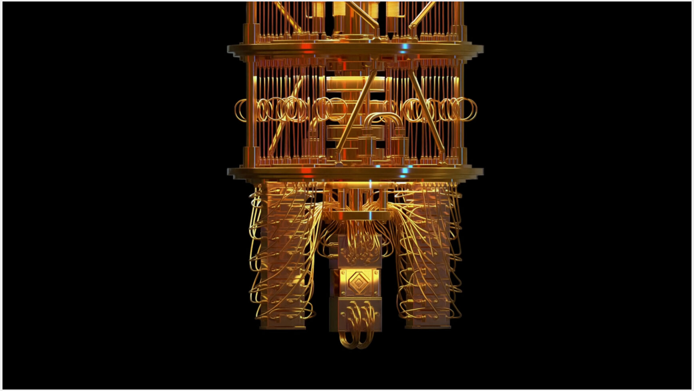



Small + Simple!


Coefficients for T operator (I normalized)
Value of Coefficient
Interpolation α
X_classical
I_classical
X_VQE
I_VQE
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6



- Use $T(\alpha)$ found on quantum computers to achieve a more quantum-mechanical demonstration.



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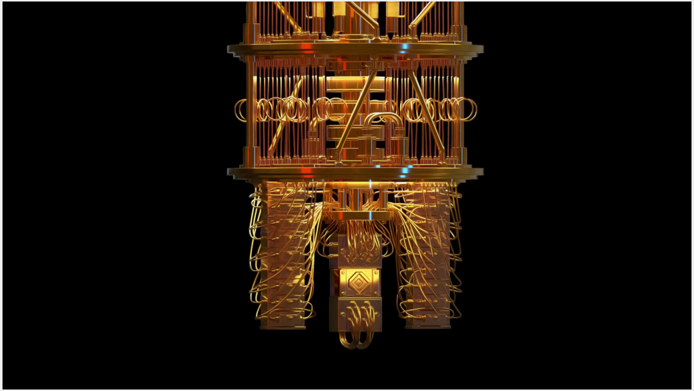
- Larger systems to demonstrate techniques to mitigate the observed decoherence.



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- Larger systems to demonstrate techniques to mitigate the observed decoherence.
- Impact: First experimental demonstration of robustness of quantum computational power. Understanding quantum advantage, and a use case for NISQ devices.