FT $x(t)$ $X(f)$

$\delta(t-t_0)$ $e^{-j2\pi f t_0}$

$e^{j2\pi f_0 t}$ $\delta(f-f_0)$

$\cos(2\pi f_0 t)$ $\frac{1}{2}[\delta(f-f_0)+\delta(f+f_0)]$

$\sin(2\pi f_0 t)$ $\frac{1}{2j}[\delta(f-f_0)-\delta(f+f_0)]$

$\mathrm{sinc}(t)=\frac{\sin(\pi t)}{\pi t}$ $\Pi(f)$

AM

$u(t)=m(t)c(t)$

$=A_c m(t)\cos(2\pi f_c t+\phi_c)$

$u(f)=F\{u(t)\}$

$=\frac{A_c}{2}[X(f-f_c)+X(f+f_c)]$

demodulation

$r(t)=u(t)$

$r(t)C(t)$

$=\frac{1}{2}A_c m(t)[\cos(\phi_c-\phi)+\cos(2\pi 2f_c t+\phi_c+\phi)]$

$\rightarrow$ LPF phase-coherent

$=\frac{1}{2}A_c m(t)\cos(0)$ if $\phi_c=\phi$

BW requirement $f_m \ll f_c$

DSB-SC : $2f_m$

SSB : f_m (VSB是差一点的 SSB)

conventional AM (DSB)

$u(t)=A_c\,(1+m(t))\cos(2\pi f_c t+\phi_c)$

$|m(t)|<1 \Rightarrow 1+m(t)>0$

PM / FM

$u(t)=A_c\cos(\underbrace{2\pi f_c t+\phi}_{\theta(t)})$

$f_i(t)$ instantaneous freq

$f_i(t)=\frac{1}{2\pi}\frac{d}{dt}\theta(t)$

$f_i(t)=f_c+\frac{1}{2\pi}\frac{d}{dt}\phi(t)$

① PM: $\phi(t)=k_p m(t)$

② FM: $\frac{1}{2\pi}\frac{d}{dt}\phi(t)=k_f m(t)$

i.e. $\phi(t)=\begin{cases}k_p m(t)\\ k_f 2\pi\int m(t)dt\end{cases}$

$m(t)=a\cos(2\pi f_m t)$

$c(t)=A_c\cos(2\pi f_c t)$

$u(t)=\begin{cases}\text{PM: } A_c\cos[2\pi f_c t+k_p a\cos(2\pi f_m t)]\\ \text{FM: } A_c\cos(2\pi f_c t+\frac{k_f a}{f_m}\sin(2\pi f_m t)]\end{cases}$

modulation index

$\beta_p=k_p a$ general case $a=\max|m(t)|$

$\beta_f=\frac{k_f a}{f_m}$ $\Delta f=k_f a$

note:

$\cos(A+B)=\cos A\cos B-\sin A\sin B$

BW of PM/FM

Carson's rule $\rightarrow$ effective BW 98% of Power

β = modulation index

$B_c=2(\beta+1)f_m$

PCM

digitize analog signal (mapping)

- $q=2^n$ q: quantization level, n: n bits per sample
- $m(t)$ with BW $=w$
- $f_s=2w=\frac{1}{T_s}$ f_s: sampling freq

pulse/bit transmission rate $=2wn$

quantization interval $\Delta=\frac{b-a}{q}$

SQNR: $N_q=\frac{\Delta^2}{12}$ (quant. error)

$S_q=\frac{q^2-1}{12}\Delta^2$

$\text{SQNR}=\frac{S_q}{N_q}=q^2-1\approx q^2$ $(q\gg1)$

$=20\log q$ dB

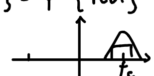
Key review for quiz 1.

1. sinc ⊛ sinc = sinc

LHS $=F^{-1}\{\text{rect}\times\text{rect}\}=F^{-1}\{\text{rect}\}$

2. $m(t)=\mathrm{sinc}+\mathrm{sinc}^2$

$M(f)=$ rect + triangle



3. $5\cos 1800\pi t+20\cos 2000\pi t+5\cos 2200\pi t$

$=10\cos(2000\pi t)[\cos(200\pi t)+2]$

$=20(1+\frac{1}{2}\cos 2\pi 100 t)\cos 2\pi 1000 t$

$\cos A\times\cos B=\frac{1}{2}[\cos(A-B)+\cos(A+B)]$

4. $\Delta=S_{max}-S_{min}$ (PCM find max error rate)

$E_{max}=\frac{\Delta}{2^{N+1}}\Big|_{max}\ni N=1$

Digital Modulation

ASK / FSK / PSK

idea: real energy signal is linear combination of N orthonormal basis set.

1. in constellation: avg power is the distance from origin.

2. P_e: prob of error

$\underset{Rx}{x(t)}=\underset{Tx}{s(t)}+\underset{\text{noise}}{n(t)}$

$n(t)$: power spectra density psd $=\frac{N_0}{2}$

$x(t)\Rightarrow$ power P_r

$s(t)\Rightarrow BW=2B$

$n(t)\Rightarrow$ noise power $=\frac{N_0}{2}\times2B=N_0B$

$\text{SNR}=\frac{P_r}{N_0B}$

Let E_s: energy per symbol

E_b: —— bit

T_s: symbol interval

T_b: bit ---

$\text{SNR}=\frac{P_r}{N_0B}=\frac{E_s}{T_sN_0B}=\frac{E_b}{T_bN_0B}$

$\gamma_s=\frac{E_s}{N_0}$ SNR / symbol

$\gamma_b=\frac{E_b}{N_0}$ SNR / bit ☆

2. Performance

①. minimum distance between points determines P_e

$d_{min}=\min_{i\neq j}\|s_i-s_j\|$

M points:

$P_e\leq(M-1)\,Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$

$Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^\infty\exp[\frac{-u^2}{2}]du.$

$Q(0)=\frac{1}{2}$, $Q(-\infty)=1$

② energy per symbol

E_s is the distance from origin

3. $P_b=Q(\sqrt{2\gamma_b})$ ☆

$P_s=P_b$ for BPSK

$P_s=1-(1-P_b)^2$

Gray code 16-QAM exp

0010	0110	1110	1010
0011	0111	1111	1011
0001	0101	1101	1001
0000	0100	1100	1000

Source coding (data compression)

1. fixed length code

   M symbols, $\bar{L}$: avg codeword length

   $\log_2 M \le \bar{L} \le \log_2 M + 1$

2. Property of codes

① singular / non-singular:

☆ each source symbol map to a different codeword

② Uniquely decodable (UD):

☆ only one way to decode a sequence of bits

③ prefix-free (PF):

no codeword is the prefix of another

√

☆: necessary √: want

PF ⇒ UD

3. Kraft's inequality

$$\sum_{i=1}^{k} 2^{-l_i} \le 1$$

KI ⟺ PF/UD existence

4. Huffman coding

   选prob最小的，两个合一

   重复这个过程.

   not unique solution

5. absolute min no. of bits necessary to represent a source? ⇒ entropy

$$H(X) = -\sum_{i=1}^{k} P_i \log_2 P_i \text{ bits}$$

$$H(X) \le \bar{L} \le H(X) + 1$$

Channel coding

add redundancy

1. n: blocklength

   k: dimension (no. of info bits)

   $R = \frac{k}{n}$ code rate

   tip: 2^k ⇒ 行数

   列数 ⇒ Blocklength

2. def: ① $d_H(x,y)$ = no. places differ

   $d_{min} = \min(d_H(c_i, c_j), i \ne j)$

   ② weight of code $w(v)$ = 有多少个1

3. given $d_{min} = d$ Ⓐ—○—○—Ⓑ

   can detect $d-1$ errors

   can correct $\lfloor \frac{d-1}{2} \rfloor$ errors

4. Code $C \sim (n, m, d)$

   n: blocklength m: m codewords

   d: dmin

   good code

   - $n\downarrow$ increase trans speed
   - $m\uparrow$ Large no. of input msgs
   - $d\uparrow$ can detect/correct more errors

   given n, d

   $M = A_2(n,d)$ $A_2(n,1) = 2^n$

   $A_2(n,n) = 2$

5. k bits → [encoder] → n

   k: dimension

   n: blocklength

   $n-k$: parity bits

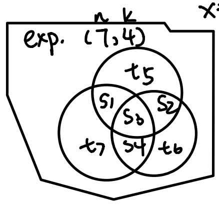
   rate: $R = \frac{k}{n}$

6. Linear:

   sum of any 2 codewords is a codeword.

   $\Rightarrow d_{min} = \min_{x \ne 0} \{w(x)\}$

   exp. $(\overset{n}{7}, \overset{k}{4})$



7. Generator matrix G.

   input m (size $1 \times k$)

   codeword x (size $1 \times n$)

   generator G (size $k \times n$)

   $x = mG$.

   exp.

$$x = \begin{pmatrix} 000 \\ 011 \\ 101 \\ 110 \end{pmatrix} \quad G = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$m = \begin{pmatrix} 00 \\ 01 \\ 10 \\ 11 \end{pmatrix}$$

8. parity check matrix H.

① H span the null-space of G (i.e. orthogonal vector spanned space),

   H (size $n-k \times n$)

② to find H:

   $GH^T = 0$ (H不可以是0)

③ consider

$$r = x + e$$

$$rH^T = (x+e)H^T = xH^T + eH^T$$

$$xH^T = 0$$

$$= eH^T \leftarrow \text{syndrome}$$

exp. $G = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$