Time Series

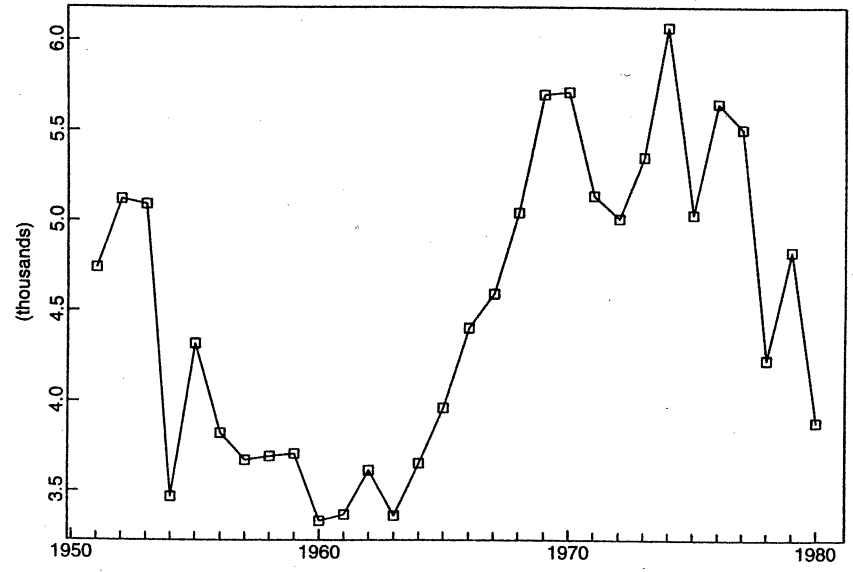
- A time series is a set of observations x_t, each one being recorded at a specific time t.
- A discrete-time time series is one in which the set T_0 of times at which the observations are taken is a discrete set.
- For example, if the observations are made at fixed time intervals, the time series is a discrete time series.
- A continuous time series is obtained when the observations are recorded continuously over some time interval.
- The measured variables may be continuous or discrete.
- As for example, the stock prices over time are continuous variables. However, number of accidents in a month in a pre-specified area is discrete.
- It is in fact possible to address the same question with a continuous or a discrete random variable. Suppose we are interested in the monthly wine sales of Australian red wine. We may measure it in litres, which is a continuous variable. On the other hand we may measure the sales in number of bottles which is a discrete variable.

Discrete-time and Continuous-time Time Series

- For a discrete-time time series the data is observed at discrete time points, like every minute, hourly, monthly, quarterly, yearly etc.
- These time points are pre-specified. They are not random.
- However, in many situations the time point at which events occur may themselves be random.
- There are several such examples. Arrival times of a Honda civic car the at science parking lot is random. Time at which certain components fail in a machine is random. Time at which a patient under treatment dies or decides to drop out of the treatment is random.
- Most models in practice are continuous-time models, which are easier to specify.
- Even though many events in real life occur "continuously" with time, they are either observed at discrete time points.
- In many cases even if the observation times can be considered continuous, during analysis the they are discretised.

Nature of Time Series

- There can be several kinds of data varying with time.
- The datasets that we would be concerned with in this module have one feature in common. That feature is "cyclicity".
- Time series data shows cycles of several lengths. They may be seasonal like hourly, quarterly, yearly or may be long range lasting several years.



Strikes in the U.S.A., 1951–1980.

Some Examples

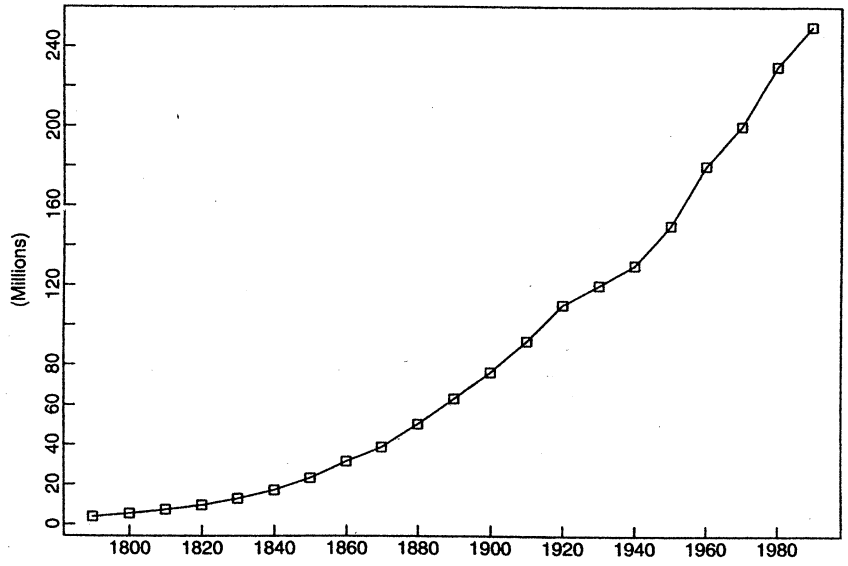
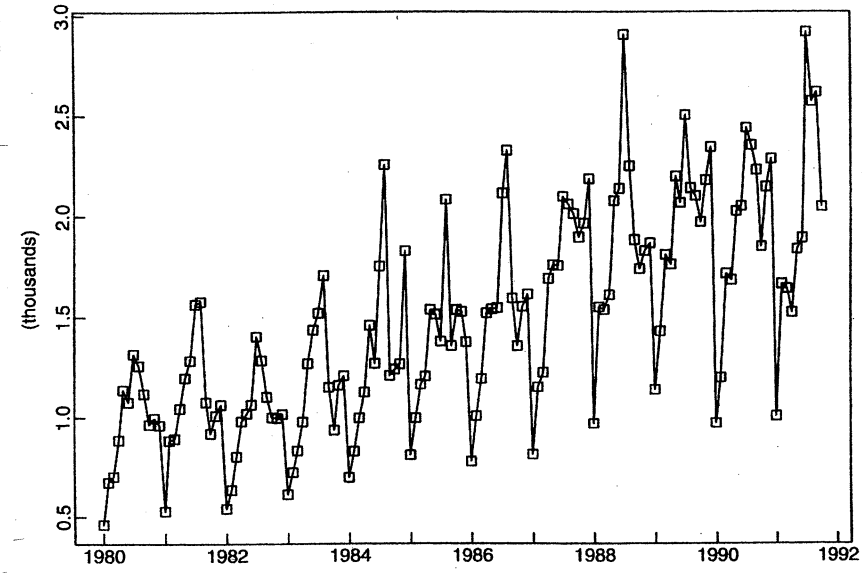


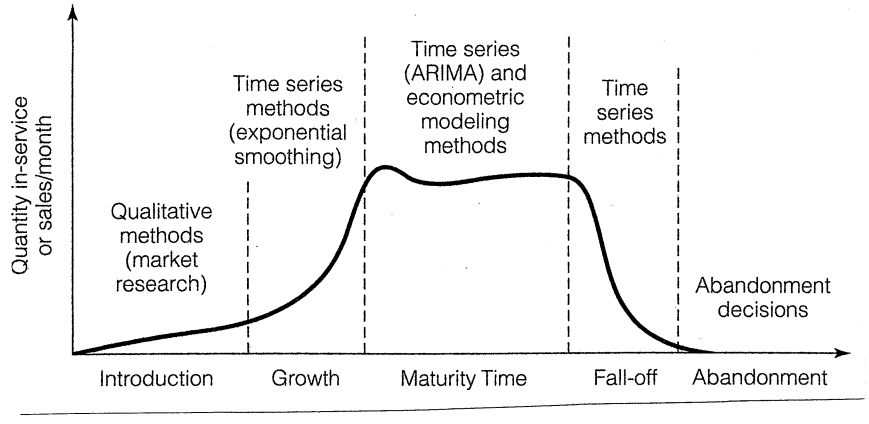
Figure 1-5
Population of the U.S.A. at ten-year intervals, 1790–1990.



The Australian red wine sales, Jan. '80 – Oct. '91.

A Life-Cycle Perspective

- The life cycle of a successful product goes through several phases.
- They are product development, product introduction, rapid growth, mature stage, fall-off and abandonment.
- Anyone remembers myspace or orkut?

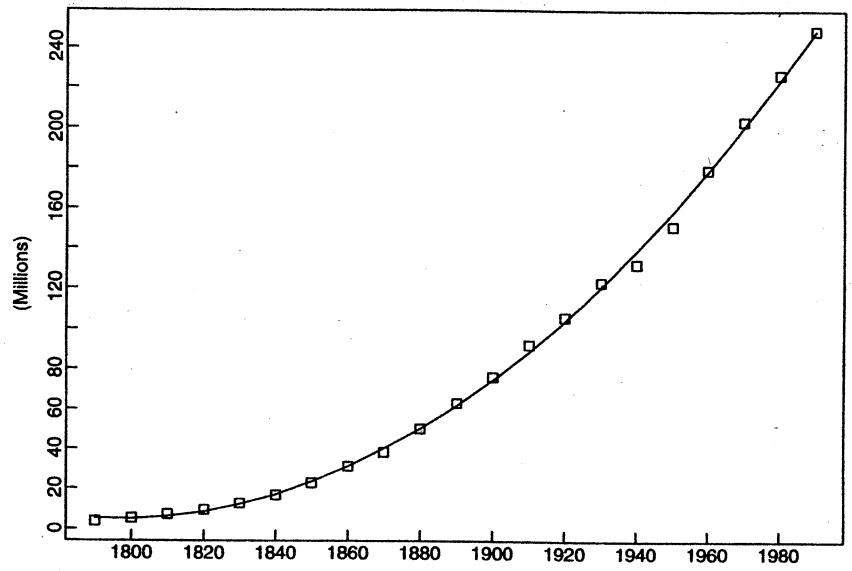


Basic Decomposition of a Time Series

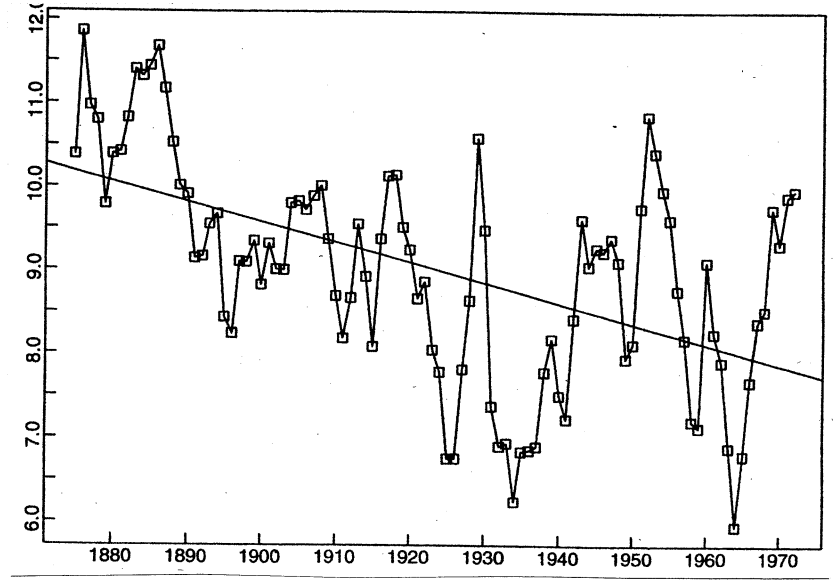
- A time series can be decomposed into four unobservable components.
- These four components are given by:
  1. Trend,
  2. cycle,
  3. seasonality,
  4. irregularity.
- These decompositions are empirical and an abstraction of reality.
- They provide a structured way to handle the time series data and find an appropriate model for them.
- Note that, there are several approaches to model a time series.
- Decomposition method is one of them.

Trend

- A trend is the basic tendency of a measured variable to grow or decline over a long period.
- Trend is assumed non-cyclic, at least within the time range of the data.
- Most of the cases a trend is represented by a polynomial in time.



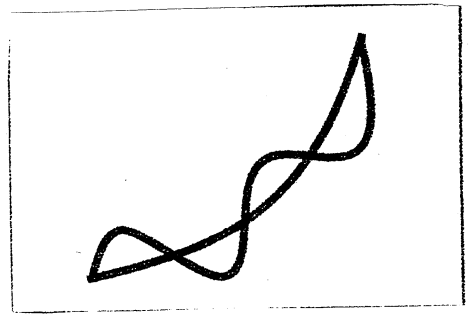
Population of the U.S.A. showing the quadratic trend fitted by least squares.

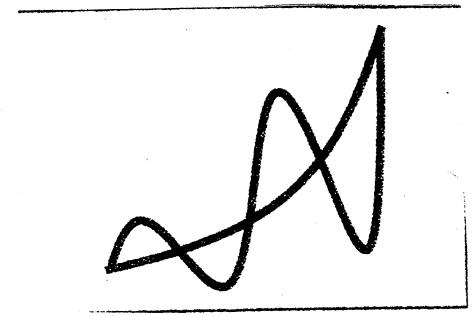


Level of Lake Huron 1875–1972 showing the line fitted by least squares.

Seasonality

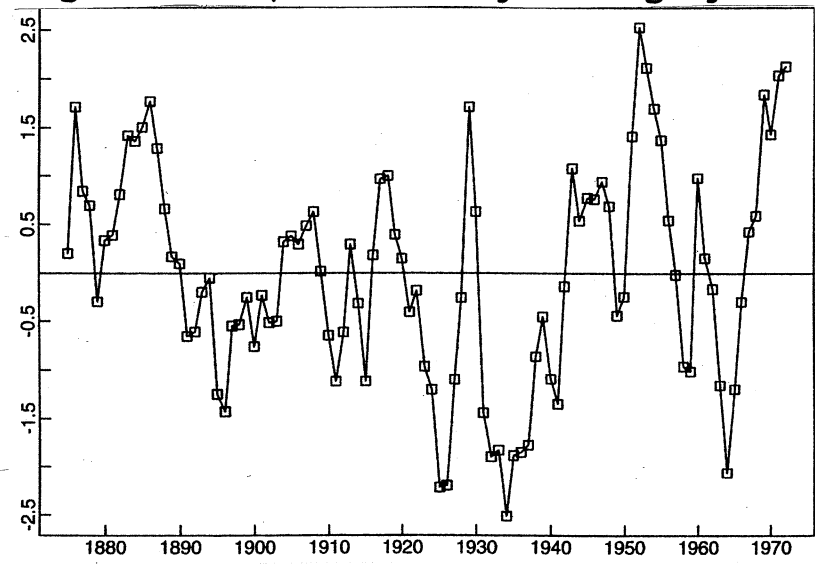
- Seasonality is a regularly recurring or systematic yearly variation in a time series.
- A seasonal effect is different from a cyclical effect. They differ in the length. The former has a short length usually less than a year, where as the latter can have a large length spanning several year even decades.
- If a trend is present, the seasonal effect can be additive or multiplicative.
- If the effect is additive, the amplitude of the cycles remain the same. On the other hand if seasonality is multiplicative, the amplitude of the cycles would increase or decrease with time.





Irregular Component

- The irregular component is the part of the data which cannot be explained by the other three components.
- Because of this irregular component the cycles differ with each other.
- An irregular component in a decomposition model is not like the error in a regression model.
- In the regression model the errors are assumed to be independent. However the irregular component may be highly correlated with each other.



Residuals from fitting a line to the Lake Huron data in Figure 1.9.

Notations

- We shall use the following notations throughout this module.
- The symbol t will represent time. Throughout we shall assume that we observe the data at regular intervals.
- The random variable observed at time t is denoted by X_t.
- Let m_t and s_t be the trend and the seasonal component at time t.
- Y_t denotes the irregular or the random noise component.
- The decomposition model (without the cyclical) component is represented by:

$$X_t = m_t + s_t + Y_t, \quad t = 1, 2, \ldots, T,$$

where $E[Y_t] = 0$, $s_{t+d} = s_t$ and $\sum_{j=1}^{d} s_j = 0$.

- The last two restrictions on the seasonal component assume that it has a period of length d.