

### MACM 316 – Assignment 5

a)

I) For  $n=100$ : this method takes roughly 0.004 seconds to compute the answer. For  $n=1000$ : this method takes roughly 0.005 seconds to compute the answer.

II) For  $n=100$ : this method takes roughly 0.004 seconds to compute the answer. For  $n=1000$ : this method takes roughly 0.022 seconds to compute the answer.

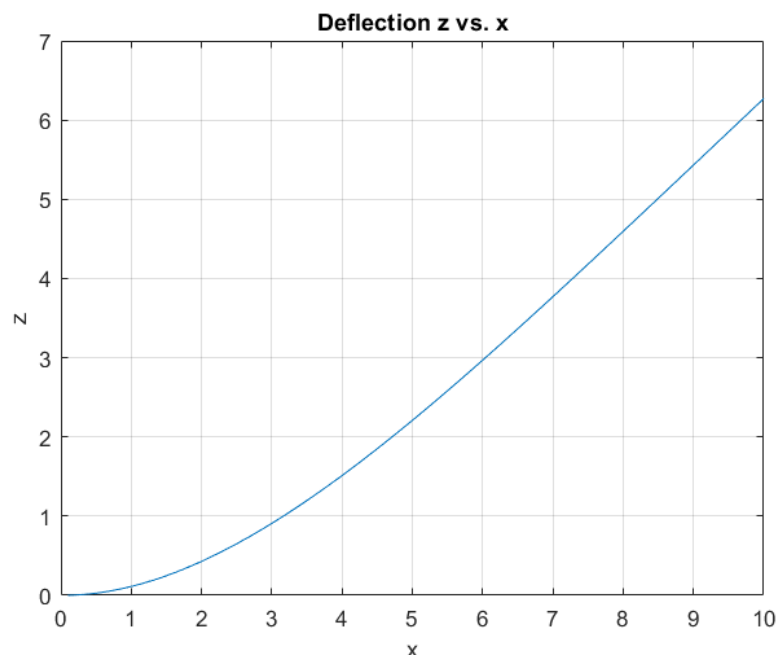
III) For  $n=100$ : this method takes roughly 0.46 seconds to compute the answer. For  $n=1000$ : this method takes roughly 91.3 seconds to compute the answer.

For  $n=100$ : I) yielded a norm of 32.097, II) yielded 32.097, and III) yielded 8.7871. For  $n=1000$ : I) yielded 100.2668, II) yielded 100.2670, and III) yielded 31.2192. The quickest would be method I) and judging from the norms, this method has a similar answer to II), which is just a bit slower. I) and II) are both good in terms of accuracy. I) is faster because it is calculating the matrix as a sparse matrix, saving time on the zeroes, whereas II) is dense. III) takes very long to compute, and the nature of Gauss-Seidel generates large fractions very quickly as iterations go on, resulting in large errors, hence the large difference in norms between this method and I) and II).

b)  $n=100$ : this method takes roughly 0.0056 seconds and has virtually the same 2-norm as I), which is 32.097, so this method is quite accurate.  $n=1000$ : the method takes about the same time and has the 2-norm 100.2699, which is nearly the same as part I), just off by  $\sim 10^{-4}$ . From this information, we can conclude this method is accurate and about as quick as part I.  $p(t) < 1$  in this case, but there could be error present in this calculation, so we are not completely sure if it will converge.

c)  $n=100$ : the condition number of  $A$  is  $2e8$ , whereas  $U$  is only  $2e4$ , and  $p(T) = 0.999999957539743$ .  $n=1000$ : the condition number of  $A$  is  $2e12$ , whereas  $U$  is only  $2e6$ , and  $p(T) = 0.99999999995867$ .  $A$ 's large condition number would yield greater errors, so the  $U$  method is more accurate than any of the methods from a).

d) For  $n=100$ :  $EI = 1.329$ . For  $n=1000$ :  $EI = 1.333$ . We can conclude that the bending stiffness is 1.333.



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% a)
format long
n = 100;
a = ones(n,1);
Asparse = spdiags([a -4*a 6*a -4*a a], -2:2, n, n);
Asparse(1,1) = 9;
Asparse(n,n) = 1;
Asparse(n,n-1) = -2;
Asparse(n-1,n) = -2;
Asparse(n-1,n-1) = 5;
L = 10;
F = .005;
h = L/n;
b = zeros(n,1);
for i = 1:n
    b(i) = F*h^4;
end
% I)
tic
A1 = Asparse\b;
toc
n1 = norm(A1);
Err1 = condest(Asparse)*(norm(b-Asparse*A1)/norm(b));
% II)
Adense = full(Asparse);
tic
A2 = Adense\b;
toc
n2 = norm(A2);
Err2 = condest(Asparse)*(norm(b-Asparse*A2)/norm(b));
% III)
z = ones(n,1);
tic
[A3, iter, T] = gs2(Asparse, b, z, 1e-8, 1e5)
toc
n3 = norm(A3);
% b)
U = spdiags([a, -2*a a], 0:2, n, n);
U(1,1) = 2;
full(U);
tic
z = U\b;
x1 = U'\z;
toc
n4 = norm(x1);
% c)
CAsparse = condest(Asparse);
CU = condest(U);
p = max(abs(eig(full(T))));
% d)
x = 0;
for j = 1:n
    x(j) = j*h;
end
EI = (F*L^4)./(6*max(x1))
plot(x, x1)
grid on
title("Deflection z vs. x")
xlabel("x")
ylabel("z")

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