Society of St. Francis Xavier, Pilar's
Fr. Conceicao Rodrigues College of Engineering
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NUMBER SYSTEM AND UNITS OF MEASUREMENT

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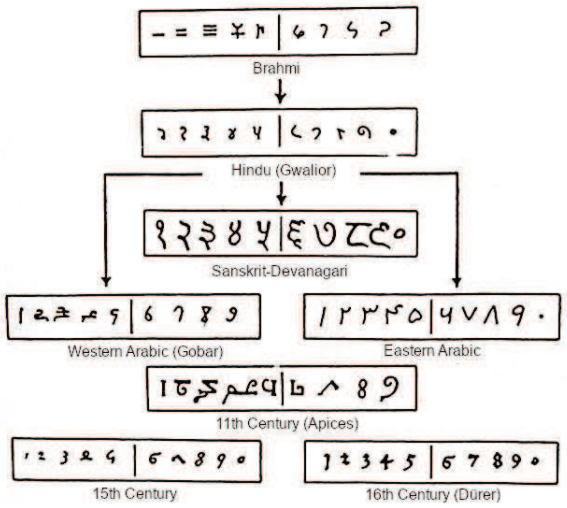
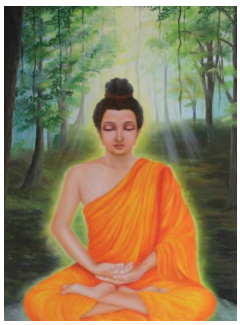


The greatest contribution of India to the development of Science and Technology lies in establishing a robust and a mature number system much before the dawn of the Common Era (CE). This was inherited up by the Arabic world during 8th Century CE and eventually passed on to the West by 11th Century CE.

Gautama Buddha on Number System



$(10^{(7 + 46*n)})*(10^2)^{23}$

When $n = 0$, the number is 10^{53} (tallakṣaṇa)

$n = 1$, the number is 10^{99} (dhvajāgravatī)

$n = 2$, the number is 10^{145} and so on.

Since Siddhārtha mentioned of eight more counting systems, at $n = 8$, the number is 10^{421}. Such large numbers are unheard of even today.

Unit of measure	*Relationship*
1 Yojana	4 Krośa from Magadha
1 Krośa	1,000 arcs
1 Arc	4 Cubits
1 Cubit	2 Spans
1 Span	12 Phalanges of fingers
1 Phalanx of a finger	7 Grains of Barley
1 Grain of Barley	7 Mustard seeds
1 Mustard seed	7 Poppy seeds
1 Poppy seed	7 Specks of dust stirred by a cow
1 Speck of dust stirred by a cow	7 Specks of dust stirred by a ram
1 Speck of dust stirred by a ram	7 Specks of dust stirred by a hare
1 Speck of dust stirred by a hare	7 Specks of dust stirred by the wind
1 Speck of dust stirred by the wind	7 Tiny Specks of dust
1 Tiny speck of dust	7 Particles of dust of first atoms (Paramāṇu)

Number System in India – Historical Evidence

Based on extensive research, **Ifrah** presented evidence from Europe and the Arabic world to show that modern number systems indeed originated in India. Using several references during 810 CE to 1814 CE, he presented several observations of the past writers on Indian Mathematics.

Number System in India – Historical Evidence

Pierre-Simon Laplace remarked,

"*The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. Its simplicity lies in the way it facilitated calculation and placed arithmetic foremost amongst useful inventions*".

Number System in India – Historical Evidence

The statue of *Al-Biruni* in *United Nations Office* in *Vienna*

In his book on India, composed around 1030 CE, Al-Biruni wrote,

"*Whilst we use letters for calculation according to their numerical value, the Indians do not use letters at all for arithmetic. And just as the shape of the letters that they use for writing is different in different regions of their country, so the numerical symbols vary*"

Number System in India – Historical Evidence

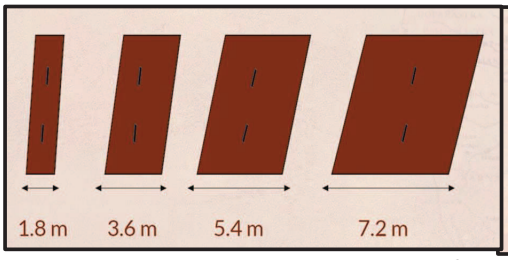
Bharukachcha
594 CE, place-value format

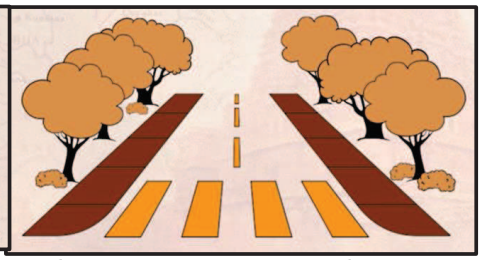
Historical Evidence:
- Gwalior
  - Samvat 933
  - The numbers 50 and 270 were recorded with a small circle appearing at the appropriate positional place for zero

Number System in India – Historical Evidence

The street widths in the Indus-Saraswati Civilization were highly standardized.

In Indian:
1, 2, 3 and 4
Dhanus



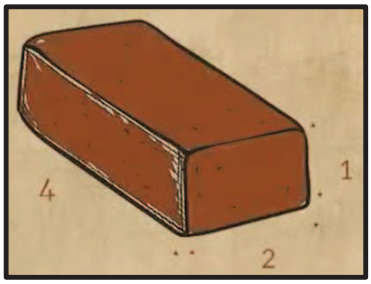


Kalibangam, a city in the Indus-Saraswati Civilization (in Rajasthan, India)

Number System in India – Historical Evidence

In the Arthaśāstra:
Two types of Dhanus as units for measuring lengths and distances:
Dhanus = 96 Aṅgulas;
Gārhapatya-dhanus = 108 Angulas
used for measurement of roads and distances.

In the Arthaśāstra:
- ✓ Excavations of Harappa, Mohenjo Daro, Dholavira and Lothal
- ✓ Several constructions were done using fired bricks of *standard dimension length:width:depth =4:2:1*
- ✓ Similar patterns for weights and measures were found.



Salient features of the Indian Numeral System

- ❖The concept of zero and its use beyond being a placeholder
- ❖Developing a robust place value system for the numerals
- ❖A decimal system that opened vast possibilities for arithmetic operations.
- ❖Unique methods to represent numbers
- ❖A legacy of using large numbers with unique number names for these large numbers

- ♦ The concept of zero was established during the period 500–300 BCE.
- ♦ Ancient Indians used a decimal system that allowed them to develop a method for handling large numbers.

- ♦ Brahmagupta developed a symbol for zero in 628 CE.
- ♦ There are references to large numbers in canonical works in Jainism.

Salient features of the Indian Numeral System

- The ***first nine digits*** have unique names ***(ekam, dve, trīṇi, catvāri, pañca, ṣaṭ, sapta, aṣṭa, nava)***.
- There are unique names for numbers from ***10 to 100*** in steps of ten ***(daśa, viṃśati, triṃśat, catvāriṃśat, pañcāśat, ṣaṣṭi, saptati, aśīti, navati, śata)***.
- Beyond 100, there has been the use of names for numbers, which extend up to very large numbers.

Salient features of the Indian Numeral System

The Concept of Zero and Its Importance

Usage of the word Śūnya
Piṅgala's Chandaḥ-śāstra

0

Symbol for Zero by Brahmagupta

Zero

- Symbol or Numeral
- Concept

Properties of Zero
Bhāskara II's Bīja-gaṇita

$0 + 0$
$0 - 0$

Salient features of the Indian Numeral System

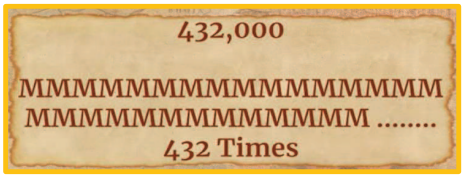
Place Value of Numerals

Place value of numerals is a concept in which a numeral or a symbol used has a unique meaning and value.

Illustration: Roman numerals system is based on addition (and sometimes subtraction) of seven different values; I (1), V (5), X (10), L (50), C (100), D (500), M (1000). All numbers (corresponding symbols) to be subtracted are to be written to the left of the symbol from which we subtract and additions to the right. For example, 397 is represented using notations for 100 + 100 + 100 + (100 – 10) + 5 + 1 + 1 = CCCXCVII.

Illustration: Roman System		
397	928	107
CCCXCVII	CMXXVIII	CVII
8	8	4

- CCCXCVII + CMXXVIII = ?
- CCCXCVII - CMXXVIII = ?
- CCCXCVII + CVII = ?
- CCCXCVII - CVII = ?
- CMXXVIII + CVII = ?
- CMXXVIII - CVII = ?



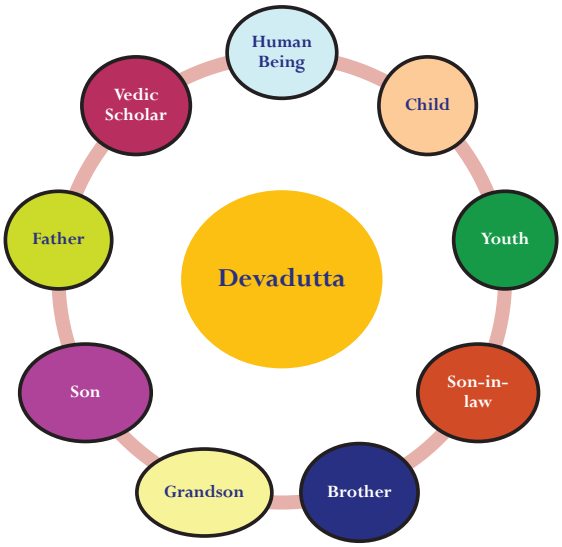
Salient features of the Indian Numeral System

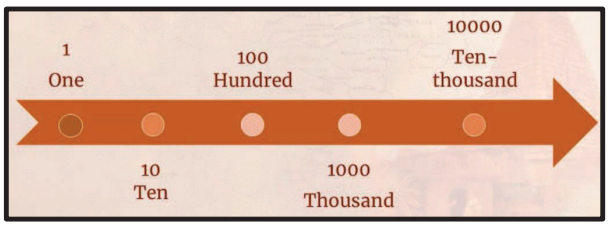
Place Value of Numerals

Sārīraka-bhāṣya of Śaṃkarācārya:

यदा एकोऽपि सन् देवदत्तः लोके स्वरूपं सम्बन्धिरूपं च अपेक्ष्य अनेकशब्दप्रत्ययभाग्भवति – मनुष्यः, ब्राह्मणः, श्रोत्रियः, वदान्यः, बालः, युवा, स्थविरः, पिता, पुत्रः, पौत्रः, भ्राता, जामाता इति ।
यथा च एकापि सती रेखा स्थानान्यत्वेन निविशमाना एक-दश-शत-सहस्रादि शब्दप्रत्ययभेदम् अनुभवति तथा सम्बन्धिनोरेव ...

"...an individual by the name Devadutta may be called differently as a father, son, son-in-law, brother, grandson, child, youth, etc., although the stroke is the same, yet by a change of values, one, ten hundred, thousand, etc. ..."





Salient features of the Indian Numeral System

Place Value of Numerals

Mahāvīrācārya's Gaṇita-sāra-saṅgraha (850 CE)

एकादिषडन्तानि क्रमेण हीनानि
(ekādi-ṣaḍ-antāni krameṇa hīnāni)

This means beginning with one and going up to 6 and then decreasing by one. The number depicted is '12345654321' which is the square of the number '111111111'.

Salient features of the Indian Numeral System

Decimal System

A logical extension of two basic ideas:
- Use of zero as a number and a placeholder
- Place value system with numerals from 0 to 9

Today's universal way of dealing with numbers is using 10 as the base.

- The Indian System of numerals employs the use of decimal system.
- Decimal number system originated in India much before the 12^{th} – 11^{th} century BCE

List of 33 inscriptions and grant plates

- These inscriptions range from 595 CE to 975 CE

Salient features of the Indian Numeral System

Bhāskarācārya's Līlāvatī on Decimal System and Place value

एक-दश-शत-सहस्र-अयुत-लक्ष-प्रयुत-कोटयः क्रमशः अर्बुदं-अब्जं-खर्व-निखर्व-महापद्म-शङ्कवस्तस्मात् ।
जलधिश्च-अन्त्यं-मध्यं-परार्धमिति दशगुणोत्तरं संज्ञा: संख्यायाः स्थानानां व्यवहारार्थं कृताः पूर्वैः ।।

eka-daśa-śata-sahasra-ayuta-lakṣa-prayuta-koṭayaḥ kramaśaḥ
arbudam-abjaṃ-kharva-nikharva-mahāpadma-śaṅkavastasmāt |
jaladhiśca-antyaṃ-madhyaṃ-parārdhamiti daśaguṇottaraṃ
saṃjñā: saṃkhyāyāḥ sthānānāṃ vyavahārārthaṃ kṛtāḥ pūrvaiḥ ||

एक (eka) – 1 (10^0)	प्रयुत (Prayuta) – 10^6 (Million)	महापद्म (Mahāpadma) – 10^{12} (Trillion)
दश (daśa) – 10^1	कोटि (Koṭi) – 10^7	शङ्ख (Śaṅka) – 10^{13}
शत (śata) – 10^2	अर्बुद (Arbuda) – 10^8	जलधि (Jaladhi) – 10^{14}
सहस्र (sahasra) – 10^3	अब्ज (Abja) – 10^9 (Billion)	अन्त्य (Antya) – 10^{15} (Zillion)
अयुत (Ayuta) – 10^4	खर्व (Kharva) – 10^{10}	मध्य (Madhya) – 10^{16}
लक्ष (Lakṣa) – 10^5	निखर्व (Nikharva) – 10^{11}	परार्ध (Parārdha) – 10^{17}

An interesting observation here is that he mentions this was already done by his ancestors for transacting!

Salient features of the Indian Numeral System

Large Numbers and Their Representation

The Vedic Corpus

- Ṛgveda names of numbers scattered through its ten mandalas
- Taittirīya Saṃhitā, Taittirīya and Bṛhadāraṇyaka Upaniṣads

Interest in Astronomy

- Several references in the Vedic and allied corpus
- Need for large numbers could have been on account of this?

Salient features of the Indian Numeral System

Large Numbers and Their Representation

Three categories of naming conventions were employed to develop the number names in Sanskrit:

Śūnya-0
Ekam-1
Dve-2
Trīṇi-3
Catvāri-4
Pañca-5
Ṣaṭ-6
Sapta-7
Aṣṭa-8
Nava-9

1. *All numbers in the unit digit from 0 to 9 had unique names*

Salient features of the Indian Numeral System

Large Numbers and Their Representation

Three categories of naming conventions were employed to develop the number names in Sanskrit:

2. ***All numbers in the range of 11 to 99 had an additive principle for naming.***

For example:

45 was 5 + 40 **(pañca-catvāriṃśat)**

18 was 8 + 10 **(aṣṭā-dasa)** and so on.

Optionally, the ***subtracting principle*** was employed

For example, 29 is 30 – 1: **ekona-triṃśat**.

Salient features of the Indian Numeral System

Large Numbers and Their Representation

Three categories of naming conventions were employed to develop the number names in Sanskrit:

3. *All numbers of higher powers of 10 starting from 10^2 were named using a multiplicative principle using the unit digits as factors for multiplication.*

For example:

8,000 was 8 × 1000 (aṣṭa-sahasram)

70,000 was 7 × 10,000 (sapta-ayuta)

Salient features of the Indian Numeral System

Large Numbers and Their Representation

Reference to Large Numbers in Ancient Texts

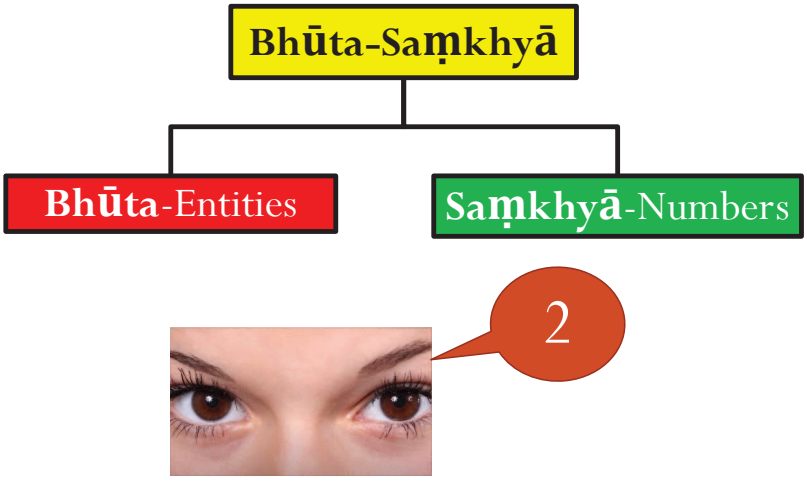
Sl No	Text	Context	Number Quoted	Remarks
1	Lalitavistāra-sūtra	Siddhārtha during Swayamvara	10^{421}	Source: Ifrah (2004)
2	Kāccāyana's Pāli Grammar		10^{140}	Source: Dutta and Singh (1962)
3	Rāmāyaṇam (Yuddha-kāṇḍa)	Size of Rāma's army	10^{62}	Referred to as Mahaugha
4	Jain canonical works	Śīrṣaprahelikā - An estimate of time	$(8,400,000)^{28}$	Source: Dutta and Singh (1962)
5	Anuyogadvāra-sūtra	Jain canonical text	10^{28}	Source: Divakaran (2018)
6	Taittirīya-upaniṣad - Brahmānandavalli	An inquiry into bliss	10^{21}	Deduced from the description
7	Līlāvatī	Description of place value number system	10^{17}	Referred to as Parārdha
8	Taittirīya-sāṃhitā - Book 7, Chapter 2	Count of oblations to Prajāpati	10^{13}	Source: Divākaran (2018)
9	Anuyogadvāra-sūtra – Jain Canonical work	Number of human beings in the world (100 BCE)	2^{96}	Source: Dutta and Singh (1962)

Unique Approaches to Represent Numbers

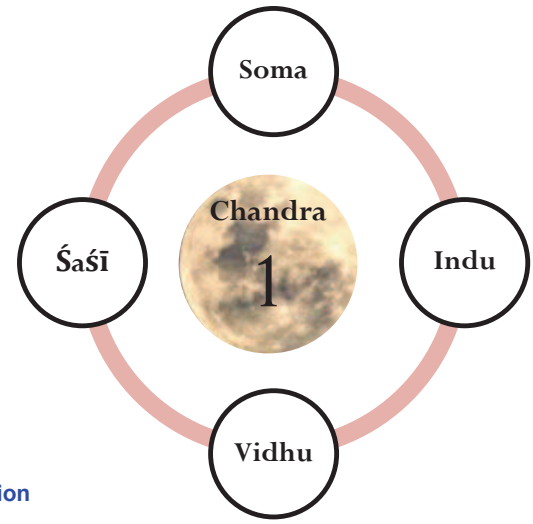
- Bhūta-Saṃkhyā system is a system of expressing numbers by means of words representing certain entities.
- Kaṭapayādi system employs a technique to convert the numerals to alphabets using certain rules.

Bhūta-Saṃkhyā system

Express numbers 0 to 9 using words representing certain entities



Usage of Synonyms



Bhūta-Saṃkhyā system

- Word name for the number itself – śūnya, eka, dvi, tri … nava
- Physical entities such as earth, moon, stars, mountain, fire, sky, direction
- Examples from the animal kingdom such as elephant, horse, snake
- Parts of the body – eyes, limbs, seven dhātus, etc.
- Names of Gods – Śiva, Viṣṇu, Indra, Manu, Agni, etc.
- Other concepts – such as seasons, month, days, five bhūtas

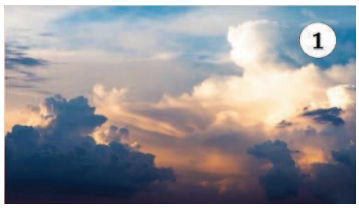
Śūnya-0
Ekam-1
Dve-2
Trīṇi-3
Catvāri-4
Pañca-5
Ṣaṭ-6
Sapta-7
Aṣṭa-8
Nava-9

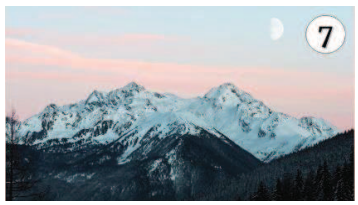
Bhūta-Saṃkhyā system

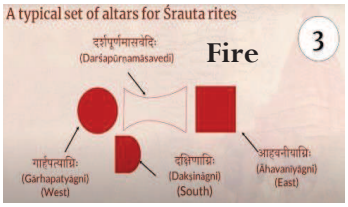
- **Word name for the number itself**

Śūnya-0
Ekam-1
Dve-2
Trīṇi-3
Catvāri-4
Pañca-5
Ṣaṭ-6
Sapta-7
Aṣṭa-8
Nava-9

- **Physical entities**

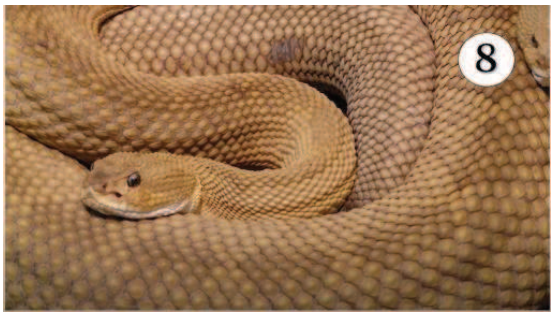




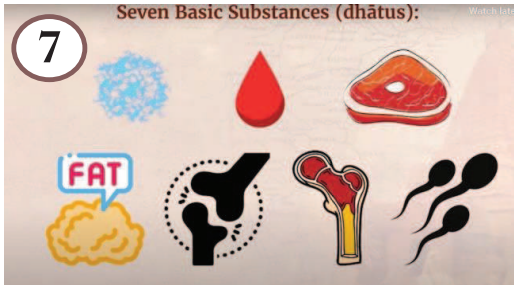


Bhūta-Saṃkhyā system

- Animal Kingdom

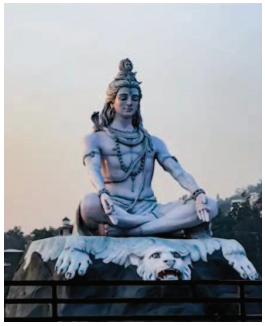


- Parts of the body



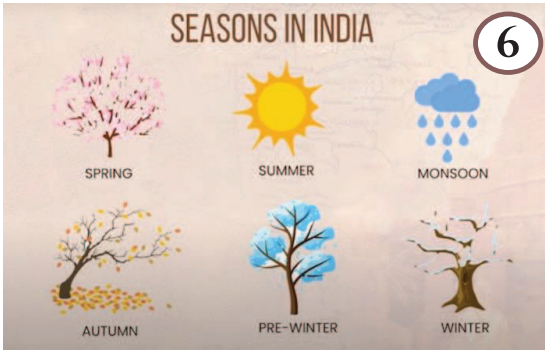
Bhūta-Saṃkhyā system

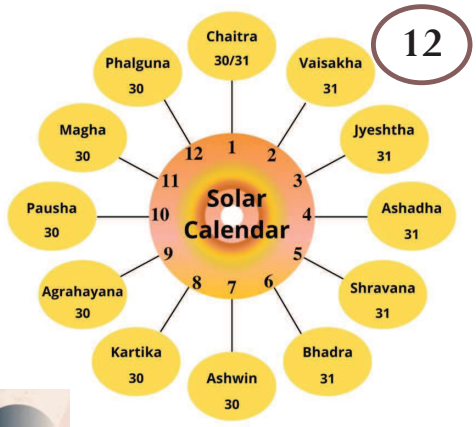
- Names of Gods

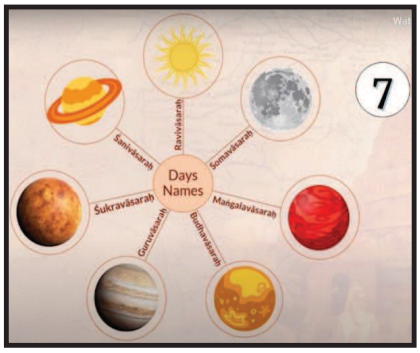


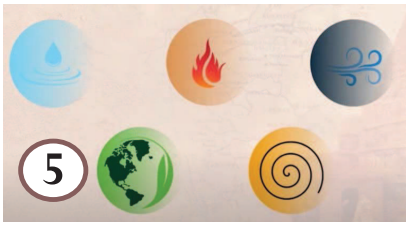
Bhūta-Saṃkhyā system

- Other concepts – such as seasons. month. days. five bhūtas









Bhūta-Saṃkhyā system

Word Numerals for Bhūta-saṃkhyā System

Number	Represented by (partial list only)*
0	Śūnya, Pūrṇa, Kha
1	Ādi, Candra, Prithivī, Eka
2	Aśvin, Pairs Of Limbs, Ayana, Dvandva, Dvi
3	Rāma, Guṇa, Loka, Kāla, Agni, Trinetra
4	Veda, Sṛuti, Yuga, Āśrama, Varṇa, Samudra, Kṛta
5	Bhūta, Śāstra, Bāṇa, Pāndava, Indriya
6	Aṅga, Ṛtu, Darśana, Ṣanmukha, Ṣaṭ
7	Ṛṣi, Adri, Svara, Dhātu, Chandas
8	Vasu, Bhujaṅga, Siddhi, Dik, Kuñjara, Nāga
9	Gṛha, Aṅka, Nanda
10	Dik, Aṇgulī, Avatāra, Rāvaṇaśira
11	Rudra
12	Āditya, Rāśi
13	Viśva, Kāma
14	Manu
15	Tithi, Dina

Let us split the words and associate the numbers to them.

वेदवेदाङ्कचन्द्राः (veda-vedāṅka-candrāḥ)
Veda (4); Veda (4); Aṅka (9); Candra (1).
Therefore, the number is 1944.

खाद्रिरामाग्नयः (khādri-rāmāgnayaḥ)
Kha (0); Adri (7); Rāma (3); Agni (3).
Therefore, the number is 3370.

भुजङ्गनन्दद्विनगाङ्गबाणषट्कृतेन्दव:(bhujaṅga-nanda-dvi-naga-aṅga-bāṇa-ṣaṭ-kṛta-indavaḥ)
bhujaṅga (8), nanda (9), dvi (2), naga (synonym of adri) (7), aṅga (6), bāṇa (5), ṣaṭ (6), kṛta (4), indu (1).
Therefore, the number is 146,567,298.

Bhūta-Saṃkhyā system

Rāma-candra-guṇa-nanda-ṛtu-pādāḥ
Number:
Rāma = 3
Candra = 1
Guṇa = 3
Nanda = 9
Ṛtu = 6
Pādāḥ = 2
Therefore, the number is 2,69,313.

Given number = 724,543
Guṇa = 3
Varṇa = 4
Bhūta = 5
Yuga = 4
Netra = 2
Dhātu = 7
Therefore, the bhūta-saṃkhyā representation could be:
Guṇa-varṇa-bhūta-yuga-netra-dhātu

2. The value of π

The ratio of the circumference of a circle to its diameter is a constant, denoted by π. Its value is given by Āryabhaṭa I in the following stanza:

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।
अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाहः ॥

"Add 4 to 100, multiply by 8 and add to 62,000; this is *approximately* the circumference of a circle whose diameter is 20,000."

– *ĀBh.* II, 10

This means a circle of diameter 20,000 units has its circumference *approximately* equal to (100 + 4) × 8 + 62,000 i.e., 62,832 so that we get

$$\pi = \frac{\text{Circumference}}{\text{Diameter}} = \frac{62,832}{20,000} = 3.1416.$$

It is remarkable that Āryabhaṭa I is the first Indian mathematician to have given the value of π which is correct to four decimal places. Even then, he mentions that this value of π is *approximate* ("āsanna").

Bhūta-saṃkhyā System – An example Mādhavācārya's approximation to π

विबुध-नेत्र-गजाहि-हुताशन
त्रि-गुण-वेद-भ-वारण-बाहवः ।
नवनिखर्वमिते वृतिविस्तरे
परिधिमानमिदं जगदुर्बुधाः ॥

vibudha-netra-gaja-ahi-hutāśana-triguṇa-veda-bha-vāraṇa-bāhavaḥ |
navanikharva-mite vṛtitivistare paridhimānam-idaṃ jagadurbudhāḥ ||

vibudha - Devas (33);
netra – Eyes (2);
gaja – Elephant (8);
ahi - Snake (8);
hutāśana – Agni (3);
tri – (3)
guṇa – (3);
veda – (4)
bha – stars (27);
vāraṇa – Elephant (8);
bāhu – Hands (2);

नवनिखर्वमिते वृतिविस्तरे
nava – (9)
nikharva – 10^{11}
This number is $9x10^{11}$

Taking this ratio will yield us the value of π

$$\pi = \frac{Circumerence}{Diameter} = \frac{2827433388233}{9X10^{11}} =$$

3.14159265359222

Therefore the number mentioned is 2,827,433,388,233

Kaṭapayādi system

Kaṭapayādi system employs certain rules to convert the numerals to alphabets:

1. The vowels when standing alone indicate the number zero
   1. In all other cases when they are conjoined with consonants, they merely facilitate pronunciation of the consonants
2. Each consonant is uniquely associated with a number from 0 to 9
   1. More than one consonant may be associated with each of the numerals
3. When more than one consonant is used in conjunction, only the terminal consonant preceding a vowel is to be considered
4. A standalone consonant will have to be ignored

ख = ख् + अ = Kh + a

For example, the consonants ka-ṭa-pa-ya was used to denote number 1. Therefore, the system was referred to as the Kaṭapayādi system. Similarly, kha-ṭha-pha-ra denotes number 2, and so on.

युक्ति
Yuk-t-i

Following verse found in Śaṅkaravarman's Sadratnamāla explains the mechanism of the system.

नञावचश्च शून्यानि संख्या: कटपयादय:।
मिश्रे तूपान्त्यहल् संख्या न च चिन्त्यो हलस्वर:॥
nanyāvachaścha śūnyāni sankhyāḥ kaṭapayādayaḥ
miśre tūpāntyahal sankhyā na cha chintyo halasvaraḥ

न, ञ तथा अ शून्य को निरूपित करते हैं। (स्वरों का मान शून्य है) शेष नौ अंक **क, ट, प और य** से आरम्भ होने वाले व्यंजन वर्णों द्वारा निरूपित होते हैं। किसी संयुक्त व्यंजन में केवल बाद वाला व्यंजन ही लिया जायेगा। बिना स्वर का व्यंजन छोड़ दिया जायेगा।

1	2	3	4	5	6	7	8	9	0
क	ख	ग	घ	ङ	च	छ	ज	झ	ञ
ट	ठ	ड	ढ	ण	त	थ	द	ध	न
प	फ	ब	भ	म	-	-	-	-	-
य	र	ल	व	श	ष	स	ह	-	-

- राम = 52
- महेन्द्र = 285
- कौटिल्य = 111

Kaṭapayādi system

Letter Numerals of the Kaṭapayādi System

1	2	3	4	5	6	7	8	9	0
क (ka)	ख (kha)	ग (ga)	घ (gha)	ङ (ṅa)	च (ca)	छ (cha)	ज (ja)	झ (jha)	ञ (ña)
ट (ṭa)	ठ (ṭha)	ड (ḍa)	ढ (ḍha)	ण (ṇa)	त (ta)	थ (tha)	द (da)	ध (dha)	न (na)
प (pa)	फ (pha)	ब (ba)	भ (bha)	म (ma)					
य (ya)	र (ra)	ल (la)	व (va)	श (śa)	ष (ṣa)	स (sa)	ह (ha)		

भवति (bhavati) Splitting them into separate letters **bha-va-ti.** After ignoring all the vowels and reading from the table the corresponding numbers, we get 4 – 4 – 6. Therefore, the number is 644.

1	2	3	4	5	6	7	8	9	0
क (ka)	ख (kha)	ग (ga)	घ (gha)	ङ (ṅa)	च (ca)	छ (cha)	ज (ja)	झ (jha)	ञ (ña)
ट (ṭa)	ठ (ṭha)	ड (ḍa)	ढ (ḍha)	ण (ṇa)	त (ta)	थ (tha)	द (da)	ध (dha)	न (na)
प (pa)	फ (pha)	ब (ba)	भ (bha)	म (ma)					
य (ya)	र (ra)	ल (la)	व (va)	श (śa)	ष (ṣa)	स (sa)	ह (ha)		

शक्त्यालोके (śaktyāloke – śa-ktyā-lo-ke) In this case, the second group has conjoined consonant 'k' followed by 't' and 'ya'. We ignore all the preceding ones and take only 'ya' for consideration. The resultant numbers for the string of these alphabets are '5 – 1 – 3 – 1'. Therefore, the number is 1315.

सर्वार्थशीलस्थिरः (sarvārthaśīlasthiraḥ – sa-rvā-rtha-śī-la-sthi-rah) As per the rules and the table, we have (7 – 4 – 7 – 5 – 3 – 7 – 2). Therefore, the number is 2,735,747.

आयुरारोग्यसौख्यम् (āyurārogyasaukhyam – ā-yu-rā-ro-gya-sau-khya-m) In this case, there is a standalone vowel (ā), which will indicate 0. There is a standalone consonant at the end (m), which will have to be ignored. The digits are 0 – 1 – 2 – 2 – 1 – 7 – 1. Therefore, the number is 1,712,210.

1	2	3	4	5	6	7	8	9	0
क (ka)	ख (kha)	ग (ga)	घ (gha)	ङ (ṅa)	च (ca)	छ (cha)	ज (ja)	झ (jha)	ञ (ña)
ट (ṭa)	ठ (ṭha)	ड (ḍa)	ढ (ḍha)	ण (ṇa)	त (ta)	थ (tha)	द (da)	ध (dha)	न (na)
प (pa)	फ (pha)	ब (ba)	भ (bha)	म (ma)					
य (ya)	र (ra)	ल (la)	व (va)	श (śa)	ष (ṣa)	स (sa)	ह (ha)		

स्फीतां लीलावतारैरिदमिह
कुरुतामायुरारोग्यसौख्यम् ॥
sphītāṁ līlāvatārairidamiha
kurutāmāyurārogyasaukhyam ॥

पञ्चभ्यः खलु शून्येभ्यः परं द्वे सप्त चाम्बरम् । एकम् त्रीणि च रूपं च …

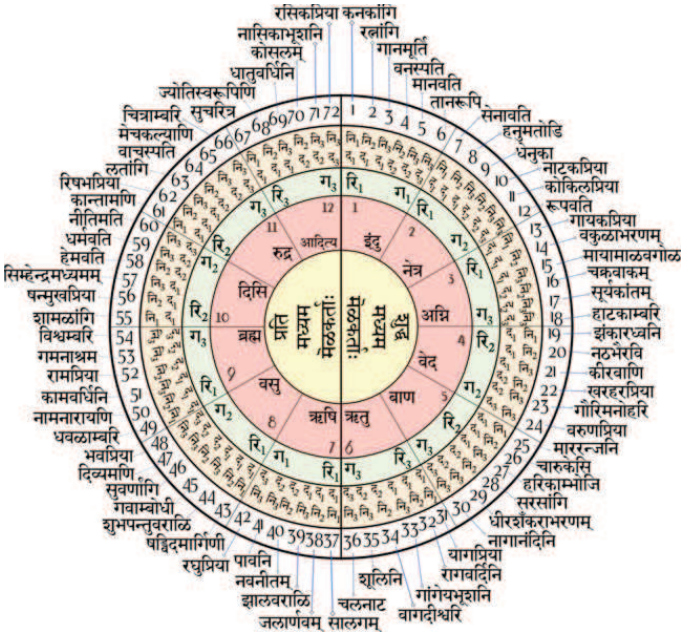
pañcabhyaḥ khalu śūnyebhyaḥ paraṃ dve sapta cāmbaram | ekam trīṇi ca rūpaṃ ca …

Bhūta-saṃkhyā System:

The number being described in the verse as five 'zeros', followed by 2, 7, and 0 and further by 1, 3, and 1. Therefore, the number specified here is **13,107,200,000**.

Kaṭapayādi System- Example

72 Melakartas in Carnatic Music



मेचकल्याणि – मे = **5**, च = **6**; Therefore **65.**

वागदीश्वरी – वा = **4**, ग = **3**; Therefore **34.**

गानमूर्ति – गा = **3**, न = **0**; Therefore **3.**

1	2	3	4	5	6	7	8	9	0
क (ka)	ख (kha)	ग (ga)	घ (gha)	ङ (ṅa)	च (ca)	छ (cha)	ज (ja)	झ (jha)	ञ (ña)
ट (ṭa)	ठ (ṭha)	ड (ḍa)	ढ (ḍha)	ण (ṇa)	त (ta)	थ (tha)	द (da)	ध (dha)	न (na)
प (pa)	फ (pha)	ब (ba)	भ (bha)	म (ma)					
य (ya)	र (ra)	ल (la)	व (va)	श (śa)	ष (ṣa)	स (sa)	ह (ha)		

Kaṭapayādi System- Example

72 Melakartas in Carnatic Music

'Mēḷakartā Rāgas'					
Shuddha Madhyama			Prati Madhyama		
No.	Raga	Scale	No.	Raga	Scale
1. *Indu Chakra*			7. *Rishi Chakra*		
1	*Kanakāngi*	S R₁ G₁ M₁ P D₁ N₁ Ṡ	37	*Sālagam*	S R₁ G₁ M₂ P D₁ N₁ Ṡ
2	*Ratnāngi*	S R₁ G₁ M₁ P D₁ N₂ Ṡ	38	*Jalārnavam*	S R₁ G₁ M₂ P D₁ N₂ Ṡ
3	*Gānamūrti*	S R₁ G₁ M₁ P D₁ N₃ Ṡ	39	*Jhālavarāḷi*	S R₁ G₁ M₂ P D₁ N₃ Ṡ
4	*Vanaspati*	S R₁ G₁ M₁ P D₂ N₂ Ṡ	40	*Navanītam*	S R₁ G₁ M₂ P D₂ N₂ Ṡ
5	*Mānavati*	S R₁ G₁ M₁ P D₂ N₃ Ṡ	41	*Pāvani*	S R₁ G₁ M₂ P D₂ N₃ Ṡ
6	*Tānarūpi*	S R₁ G₁ M₁ P D₃ N₃ Ṡ	42	*Raghupriyā*	S R₁ G₁ M₂ P D₃ N₃ Ṡ
2. *Netra Chakra*			8. *Vasu Chakra*		
7	*Senāvati*	S R₁ G₂ M₁ P D₁ N₁ Ṡ	43	*Gavāmbhodi*	S R₁ G₂ M₂ P D₁ N₁ Ṡ
8	*Hanumatodi*	S R₁ G₂ M₁ P D₁ N₂ Ṡ	44	*Bhavapriyā*	S R₁ G₂ M₂ P D₁ N₂ Ṡ
9	*Dhenukā*	S R₁ G₂ M₁ P D₁ N₃ Ṡ	45	*Śubhapantuvarāḷi*	S R₁ G₂ M₂ P D₁ N₃ Ṡ
10	*Nātakapriyā*	S R₁ G₂ M₁ P D₂ N₂ Ṡ	46	*Shaḍvidamārgini*	S R₁ G₂ M₂ P D₂ N₂ Ṡ
11	*Kokilapriyā*	S R₁ G₂ M₁ P D₂ N₃ Ṡ	47	*Suvarnāngi*	S R₁ G₂ M₂ P D₂ N₃ Ṡ
12	*Rūpavati*	S R₁ G₂ M₁ P D₃ N₃ Ṡ	48	*Divyamani*	S R₁ G₂ M₂ P D₃ N₃ Ṡ
3. *Agni Chakra*			9. *Brahma Chakra*		
13	*Gāyakapriyā*	S R₁ G₃ M₁ P D₁ N₁ Ṡ	49	*Dhavaḷāmbari*	S R₁ G₃ M₂ P D₁ N₁ Ṡ
14	*Vakuḷābharaṇam*	S R₁ G₃ M₁ P D₁ N₂ Ṡ	50	*Nāmanārāyaṇi*	S R₁ G₃ M₂ P D₁ N₂ Ṡ
15	*Māyāmāḷavagowla*	S R₁ G₃ M₁ P D₁ N₃ Ṡ	51	*Kāmavardhini*	S R₁ G₃ M₂ P D₁ N₃ Ṡ
16	*Chakravākam*	S R₁ G₃ M₁ P D₂ N₂ Ṡ	52	*Rāmapriyā*	S R₁ G₃ M₂ P D₂ N₂ Ṡ
17	*Sūryakāntam*	S R₁ G₃ M₁ P D₂ N₃ Ṡ	53	*Gamanāśrama*	S R₁ G₃ M₂ P D₂ N₃ Ṡ
18	*Hātakāmbari*	S R₁ G₃ M₁ P D₃ N₃ Ṡ	54	*Viśvambari*	S R₁ G₃ M₂ P D₃ N₃ Ṡ
4. *Veda Chakra*			10. *Disi Chakra*		
19	*Jhankāradhvani*	S R₂ G₂ M₁ P D₁ N₁ Ṡ	55	*Śāmaḷāngi*	S R₂ G₂ M₂ P D₁ N₁ Ṡ
20	*Naṭabhairavi*	S R₂ G₂ M₁ P D₁ N₂ Ṡ	56	*Śanmukhapriyā*	S R₂ G₂ M₂ P D₁ N₂ Ṡ
21	*Kīravāṇi*	S R₂ G₂ M₁ P D₁ N₃ Ṡ	57	*Simhendramadhyamam*	S R₂ G₂ M₂ P D₁ N₃ Ṡ
22	*Kharaharapriyā*	S R₂ G₂ M₁ P D₂ N₂ Ṡ	58	*Hemavati*	S R₂ G₂ M₂ P D₂ N₂ Ṡ
23	*Gourimanohari*	S R₂ G₂ M₁ P D₂ N₃ Ṡ	59	*Dharmavati*	S R₂ G₂ M₂ P D₂ N₃ Ṡ
24	*Varuṇapriyā*	S R₂ G₂ M₁ P D₃ N₃ Ṡ	60	*Nītimati*	S R₂ G₂ M₂ P D₃ N₃ Ṡ

5. *Bana Chakra*			11. *Rudra Chakra*		
25	*Māraranjani*	S R₂ G₃ M₁ P D₁ N₁ Ṡ	61	*Kāntāmaṇi*	S R₂ G₃ M₂ P D₁ N₁ Ṡ
26	*Chārukesi*	S R₂ G₃ M₁ P D₁ N₂ Ṡ	62	*Riśabhapriyā*	S R₂ G₃ M₂ P D₁ N₂ Ṡ
27	*Sarasāngi*	S R₂ G₃ M₁ P D₁ N₃ Ṡ	63	*Latāngi*	S R₂ G₃ M₂ P D₁ N₃ Ṡ
28	*Harikāmbhōji*	S R₂ G₃ M₁ P D₂ N₂ Ṡ	64	*Vāchaspati*	S R₂ G₃ M₂ P D₂ N₂ Ṡ
29	*Dhīraśankarābharaṇam*	S R₂ G₃ M₁ P D₂ N₃ Ṡ	65	*Mechakalyāni*	S R₂ G₃ M₂ P D₂ N₃ Ṡ
30	*Nāganandini*	S R₂ G₃ M₁ P D₃ N₃ Ṡ	66	*Chitrāmbari*	S R₂ G₃ M₂ P D₃ N₃ Ṡ
6. *Rutu Chakra*			12. *Aditya Chakra*		
31	*Yāgapriyā*	S R₃ G₃ M₁ P D₁ N₁ Ṡ	67	*Sucharitrā*	S R₃ G₃ M₂ P D₁ N₁ Ṡ
32	*Rāgavardhini*	S R₃ G₃ M₁ P D₁ N₂ Ṡ	68	*Jyoti svarupini*	S R₃ G₃ M₂ P D₁ N₂ Ṡ
33	*Gāngeyabhuśani*	S R₃ G₃ M₁ P D₁ N₃ Ṡ	69	*Dhāthuvardhani*	S R₃ G₃ M₂ P D₁ N₃ Ṡ
34	*Vāgadhīśvari*	S R₃ G₃ M₁ P D₂ N₂ Ṡ	70	*Nāsikābhūśani*	S R₃ G₃ M₂ P D₂ N₂ Ṡ
35	*Śūlini*	S R₃ G₃ M₁ P D₂ N₃ Ṡ	71	*Kōsalam*	S R₃ G₃ M₂ P D₂ N₃ Ṡ
36	*Chalanāṭa*	S R₃ G₃ M₁ P D₃ N₃ Ṡ	72	*Rasikapriyā*	S R₃ G₃ M₂ P D₃ N₃ Ṡ

Kaṭapayādi System- Example

Mathematics and astronomy

(स्याद्) भद्राम्बुधिसिद्धजन्मगणितश्रद्धा स्म यद् भूपगीः

(syād) bhadrāmbudhisiddhajanmagaṇitaśraddhā sma yad bhūpagīḥ

Splitting the consonants in the relevant phrase gives,

भ	द्	रा	म्	बु	द्	धि	सि	द्	ध	ज	न्	म	ग	णि	त	श्	र	द्	धा	स्	म	य	द्	भू	प	गी
bha	d	rā	m	bu	d	dhi	si	d	dha	ja	n	ma	ga	ṇi	ta	ś	ra	d	dhā	s	ma	ya	d	bhū	pa	gī
4	–	2	–	3	–	9	7	–	9	8	–	5	3	5	6	–	2	–	9	–	5	1	–	4	1	3

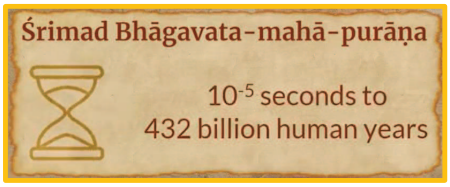
Reversing the digits to modern-day usage of descending order of decimal places, we get *314159265358979324* which is the value of pi (π) to 17 decimal places, except the last digit might be rounded off to 4.

गोपीभाग्यमधुव्रात-शृङ्गिशोदधिसन्धिग॥
खलजीवितखाताव गलहालारसंधर॥

This verse directly yields the decimal equivalent of pi divided by 10:
pi/10 = 0.31415926535897932384626433832792

Traditionally, the order of digits are reversed to form the number, in katapayadi system. This rule is violated in this sloka.

Measurements for time, distance, and weight



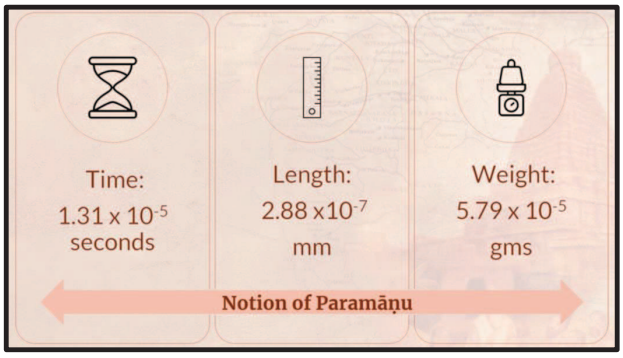
Notion of Paramāṇu

The measures for paramāṇu are as follows:

Paramāṇu (Length): 2.88×10^{-7} mm

Paramāṇu (Weight): 5.79×10^{-5} g

Paramāṇu (Time): 1.31×10^{-5} seconds



Ancient Indian Measures for Length

Unit	Multiplier of preceding unit*	No. of Paramāṇus	Length (in mm)
Paramāṇu-raja	1	1	2.8778×10^{-7}
Reṇu	7	7	2.0145×10^{-6}
Truṭi	7	49	1.4101×10^{-5}
Vāṭāyana-raja	7	343	9.8709×10^{-5}
Śaśa-raja	7	2,401	6.9096×10^{-4}
Edaka-raja	7	16,807	4.8367×10^{-3}
Go-raja	7	1,17,649	0.033857202
Likṣā-raja	7	8,23,543	0.237000411
Sarṣapa	7	57,64,801	1.65900288
Yava	7	4,03,53,607	11.61302016
Aṅguli-parva	7	28,24,75,249	81.29114114

Unit	Multiplier of preceding unit*	No. of Aṅgulas	Length (in metre)
Aṅgula	1	1	0.016764
Dhanurmuṣṭi	8	8	0.134112
Prājāpatya-hasta	3	24	0.402336
Dhanus	4	96	1.609344
Gārhapatya-dhanus	1.125	108	1.810512
Goruta	2000	216,000	3621.024
Yojana	4	864,000	14484.096

* The length of an Aṅgula is 16.764 mm as per Indus inch.

Measurement of Time – An Illustration from Purāṇa

द्वादशार्धपलोन्मानं चतुर्भिश्चतुरङ्गुलैः ।
स्वर्णमाषैः कृतच्छिद्रं यावत् प्रस्थजलप्लुतम् ॥
dvādaśārdha palonmānam
caturbhiś-caturangulaiḥ
svarṇamāshaiḥ kṛtac-chidram
yāvat prastha-jala-plutam

12/2=6 Palas

4 Palas

4 Angulas

Weight

Length

Take a copper pot weighing six palas (1 pala=48 gms) which can hold water of one prastha (1 prastha = 640gms; in the case of water it is 640 ml)

The vessel shall be bored at the bottom with a golden needle weighing four masas (1 masa =1 gm) and of length four angulas

Leave the pot in water and start a stopwatch

Wait until the vessel is filled fully with water and it just submerges in the water

Stop the watch and record the time. This elapsed time is Nadika.

Ancient Indian Measures for Time

Unit	Multiplier of preceding unit*	No. of Paramāṇus	Time (Seconds)
Paramāṇu	1	1	1.3133×10^{-5}
Aṇu	2	2	2.6266×10^{-5}
Trasreṇu	3	6	7.8797×10^{-5}
Truṭi	3	18	2.3639×10^{-4}
Vedha	100	1,800	2.3639×10^{-2}
Lava	3	5,400	7.0917×10^{-2}
Nimeṣa	3	16,200	0.212750617
Kṣaṇa	3	48,600	0.638251852
Kāṣṭha	5	2,43,000	3.191259259
Laghu	15	36,45,000	47.86888889
Nāḍikā	15	5,46,75,000	718.0333333
Muhūrta	2	10,93,50,000	1436.066667
Prahara	7.5	82,01,25,000	10770.51

* For example, 100 Truṭis make a Vedha

Unit	Multiplier of preceding unit*	No. of human years
Māsa	1	0.08333
Ṛtu	2	0.16667
Ayana	3	0.50000
Human Year	2	1
Human Life Span	100	100
Celestial Life Span	360	36,000
Mahā-yuga	12000	43,20,00,000
Kalpa	1000	4,32,00,00,00,000

* For example, 3 Ṛtus make an Ayana

Measures for Weight

Arthaśāstra provides vivid details on measures of weight

- Lower denominations of weights for precious metals
- Guidelines for the development of 16 types of balances
- Verification once in 3 months to ensure the balances are calibrated

An Illustration of a Balance (Mauryan Time)

Unit	Multiplier of preceding unit*	No. of Paramāṇus	Weight (Grams)
Paramāṇu	1	1	5.787×10^{-5}
Vaṁśī	30	30	1.736×10^{-3}
Sarṣapa	9	270	1.563×10^{-2}
Yava	8	2,160	0.125
Guñja	4	8,640	0.5
Māṣaka	6	51,840	3
Karṣa	4	2,07,360	12
Pala	4	8,29,440	48
Tulā	100	8,29,44,000	4,800
Bhāra	20	1,65,88,80,000	96,000

* For example, 100 Palas make a Tulā and 4 Yavas make a Guñja

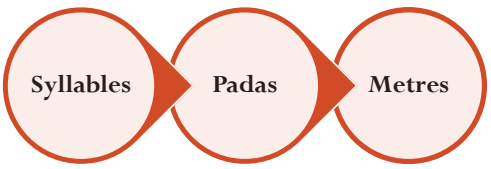
Piṅgala and the Binary system

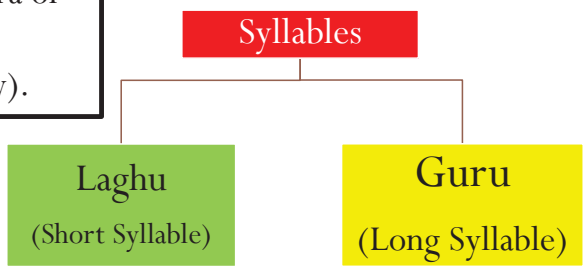
Piṅgala, who lived during 200–300 BCE developed **Chandaḥ-śāstra**, which dealt with the rules governing prosody.

Laghu (Short Syllable) – Any syllable with a short vowel

Determining **Guru** (Long Syllable):

1. Any syllable with a long vowel.
2. Any short syllable followed by conjunction of consonants.
3. Any short syllable followed by 'ṃ' known as anusvāra or visarga denoted by ':'.
4. The last syllable in the quarter of a meter (optionally).





Replacing Laghu with "1" & Guru with "0"

Identification of Laghu and Guru: Example

यदा यदा हि धर्मस्य ग्लानिर्भवति भारत ।
अभ्युत्थानमधर्मस्य तदात्मानं सृजाम्यहम् ॥
yadā yadā hi dharmasya glānirbhavati bhārata |
abhyutthānamadharmasya tadātmānaṃ sṛjāmyaham | |

Identification of the Laghu and Guru as per Rules of Chandaḥ Śāstra

ya	dā	ya	dā	hi	dha	rma	sya	glā	ni	rbha	va	ti	bhā	ra	ta
L	G	L	G	L	G	G	G	G	G	L	L	L	G	L	G

a	bhyu	tthā	na	ma	dha	rma	sya	ta	dā	tmā	naṃ	sṛ	jā	mya	ham
G	G	G	L	L	G	G	G	L	G	G	G	L	G	L	G

Eight "Ganas" Defined by Pingala

Sl. No.	*Gana Name*	*Binary Word**
1	'Ya' Gana	100
2	'Ma' Gana	000
3	'Ta' Gana	001
4	'Ra' Gana	010
5	'Ja' Gana	101
6	'Bha' Gana	011
7	'Na' Gana	111
8	'Sa' Gana	110

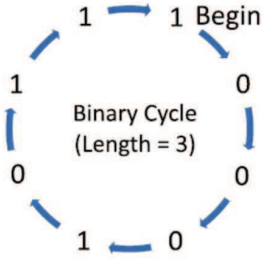
* 1 = Laghu; 0 = Guru

यमाता-राज-भान-सलगम्
yamātā-rāja-bhāna-salagam

Laghus and Gurus in the Phrase

ya	mā	tā	rā	ja	bhā	na	sa	la	gam
L	G	G	G	L	G	L	L	L	G
1	0	0	0	1	0	1	1	1	0

Binary Cycle of Length 3



LGG
GGG
GGL
GLG
LGL
GLL
LLL
LLG

ಆಭಾರ ಸ್ಯಶುಧನ್ಯವಾದ ತುಹಾಡಾ ਧੰਨਵਾਦ ನன்றிధన్యవాదాలు
ధన్యవాదనగ್ಗಿ धन्यवाद ಆಭಾರ ಸ್ಯಶುधन्यवाद तुहाडा ਧੰਨਵਾਦ
நன்றி ధన్యవాదాలు ధన్యవాద ನಗ್ಗಿಆಭಾರ ಸ್ಯಶುधन्यवाद तुहाडा
ਧੰਨਵਾਦ நன்றிధన్యవాదాలు ధన్యవాదనగ్గిఆభార ಸ್ಯಶು
धन्यवाद तुहाडा ਧੰਨਵਾਦ நன்றிధన్యవాదాలు ధన్యవాదనగ్గి
ఆభార ಸ್ಯಶುधन्यवाद तुहाडा ਧੰਨਵਾਦ धन्यवाद
நன்றிధన్యవాదాలు धन्यवाद ధన్యవాదనగ్గి ఆభార ಸ್ಯಶು
धन्यवाद तुहाडा ਧੰਨਵਾਦ நன்றிధన్యవాదాలు ధన్యవాదనగ్గి
धन्यवाद ధన్యవాదనగ్గి ఆభార ಸ್ಯಶುधन्यवाद तुहाडा ਧੰਨਵਾਦ
நன்றிధన్యవాదాలు ధన్యవాదనగ్గి ఆభార धन्यवाद तुहाडा
ਧੰਨਵਾਦ நன்றிధన్యవాదాలు धन्यवाद तुहाडा ਧੰਨਵਾਦ
நன்றிధన్యవాదాలు

धन्यवाद

Thank you