Society of St. Francis Xavier, Pilar’s

Fr. Conceicao Rodrigues College of Engineering

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Bandra (West) Mumbai -400 050

SCIENCE, ENGINEERING AND TECHNOLOGY IN IKS:

योजनानं सहस्रे द्वे द्वे शते द्वे च योजने ।
एकेन निमिषार्धेन क्रममाण नमोऽस्तुते ॥

Speed of Light

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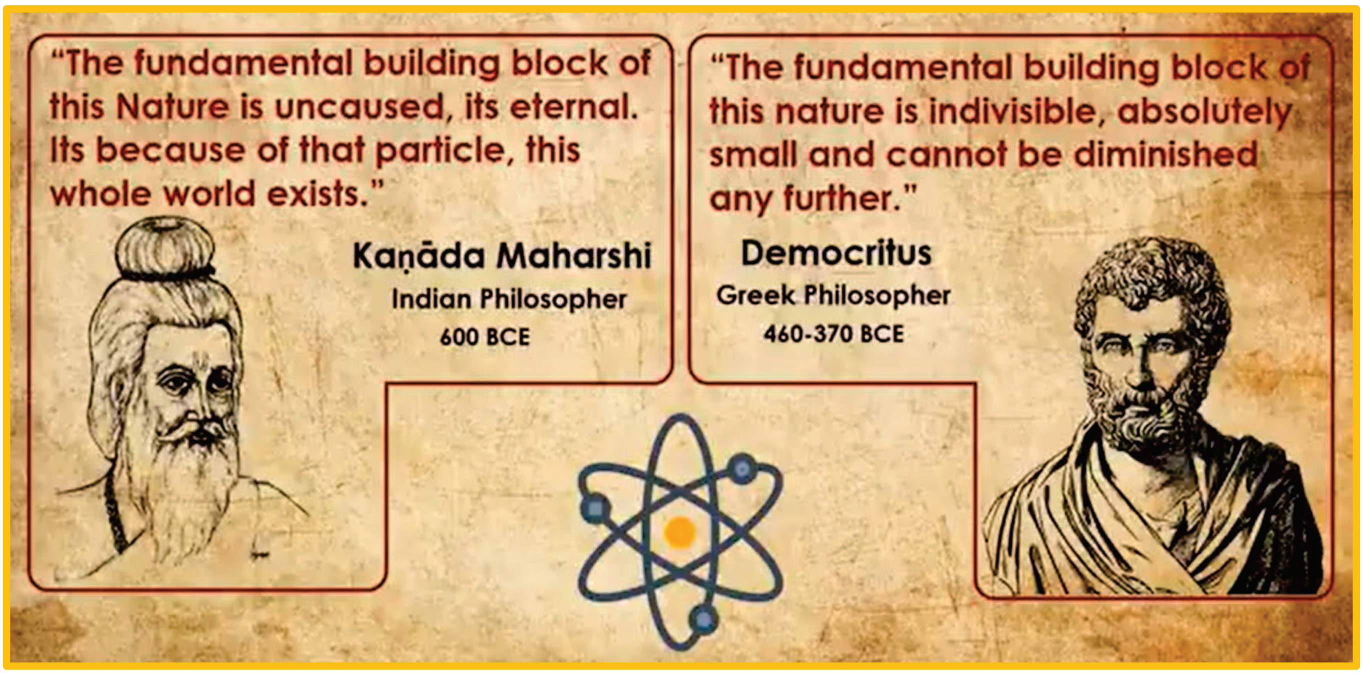


वैशेषिका दर्शनम
Chapter 1
The Background & Context
Chapter 2
About Vaiseshika Sutras
Chapter 3
The Scientific Concepts


1
2
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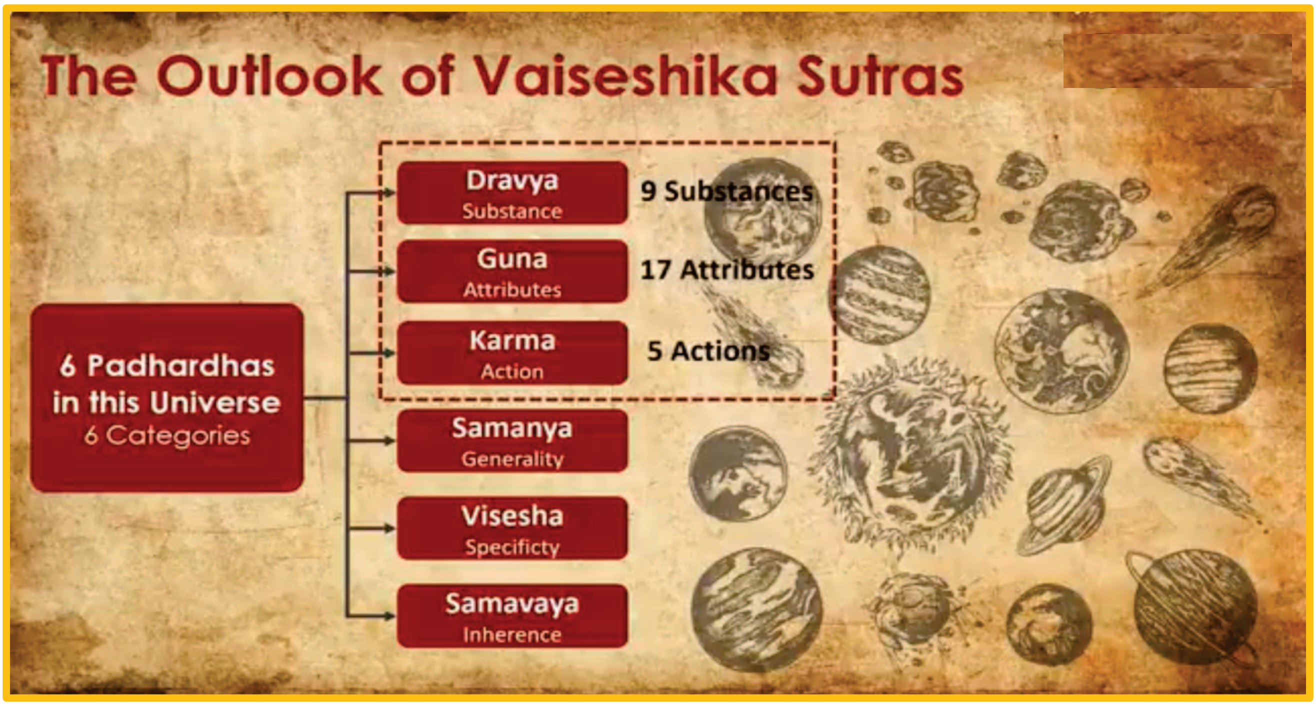
6
7
The Vaiseshika Sutras
Chapter 7 (53 Sutras)
• Nithya (Eternal) & Anithya (Transient)
• Structure of an Atom (Anu)
• Anuttva (Atomicity) & Mahattva (Magnitude)
• Akasha (Space), Kaala (Time) & Aatma (Self)



Moulding Engineers Who Can Build the Nation

Gravity & Atomic Theory in Vaiseshika Sutras

Dravya (Substance)	Guna (Attributes)		Karma (Action)
1.Pruthvi (Earth)	1. Rupa (Colour)	10. Paratava (Priority)	1. Utkshepana (Throw up)
2.Apas (Water)	2. Rasa (Taste)	11. Apartava (Non Priority)	2. Avakshepana (Throw down)
3.Tejas (Fire)	3. Gandha (Odour)	12. Buddhi (Intellect)	3. Akunchana (Contraction)
4.Vayu (Air)	4. Sparsha (Touch)	13. Sukha (Pleasure)	4. Prasarana (Expansion)
5.Aakasha (Space)	5. Samkhya (Number)	14. Dukha (Pain)	5. Gamana (Movement)
6.Kaala (Time)	6. Parimana (Dimension)	15. Ichha (Desire)	**Gravitational Force**
7.Dikh (Direction)	7. Samyoga (Conjunction)	16. Dvesha (Aversion)	
8.Atma (Self)	8. Vibhaga (Disjunction)	17. Prayatna (Will)	
9.Manas (Mind)	9. Prithaktava (Distinctiveness)		
Atomic Theory			

Gravity & Laws of Motion

Concept of Gravity & Laws of Motion are well detailed in Vaiseshika Sutras by Maharshi Kanada.

Gravity & Laws of Motion

संयोगाभावे गुरुत्वात् पतनम् ॥

On disjunction, an object falls down due to its own Mass (Gurutva).

- Vaiseshika Sutras 5.1.7

गुरुत्वप्रयत्नसंयोगानामुत्क्षेपणम् ॥

A upward throw of an object is resultant of Upward thrust and Downward Gravitational pull.

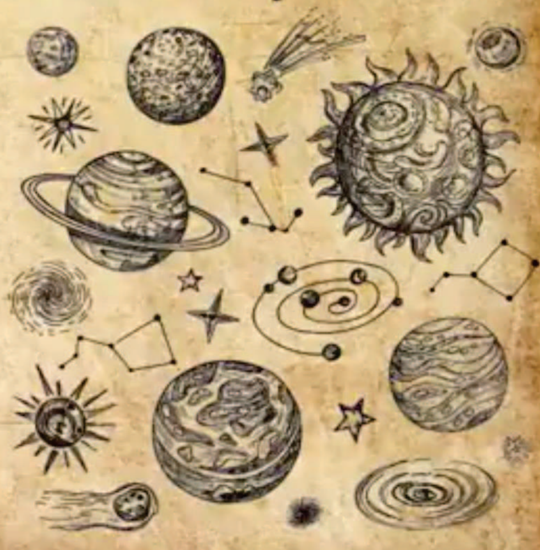
- Vaiseshika Sutras 1.1.29

Vaiseshika Sutras on Atomic Theory

Definition of Dravyas

पृथिव्यापस्तेजो वायुराकाशं कालो
दिगात्मा मन इति द्रव्याणि ॥ १ । १ । ५ ॥

- Pruthvi = Earth
- Apas = Water
- Tejas = Fire
- Vayu = Air
- Kala = Time
- Dikh = Space
- Atma = Self
- Manas = Mind



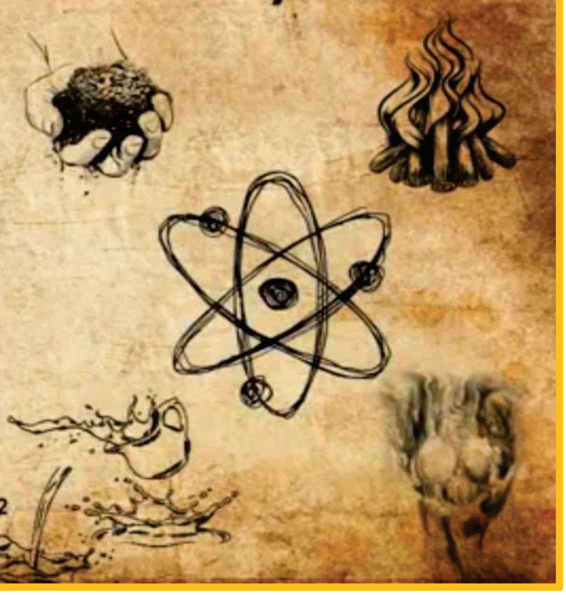
Vaiseshika Sutras on Atomic Theory

Definition of Atom

सदकारणवन्नित्यम् ॥
तस्य कार्य्यं लिङ्गम् ॥

"The fundamental existence of this nature is uncaused, its eternal. The Atom is the proof of this argument."

- Vaiseshika Sutras 4.1.1 & 2



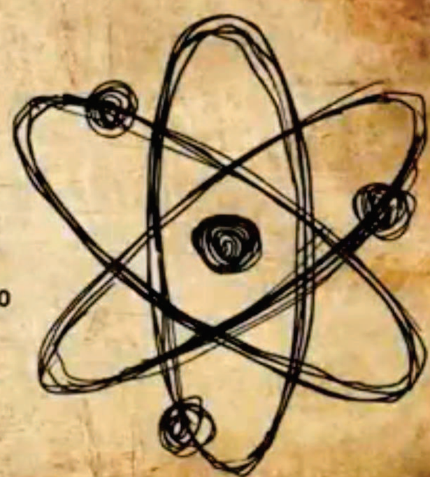
Vaiseshika Sutras on Atomic Theory

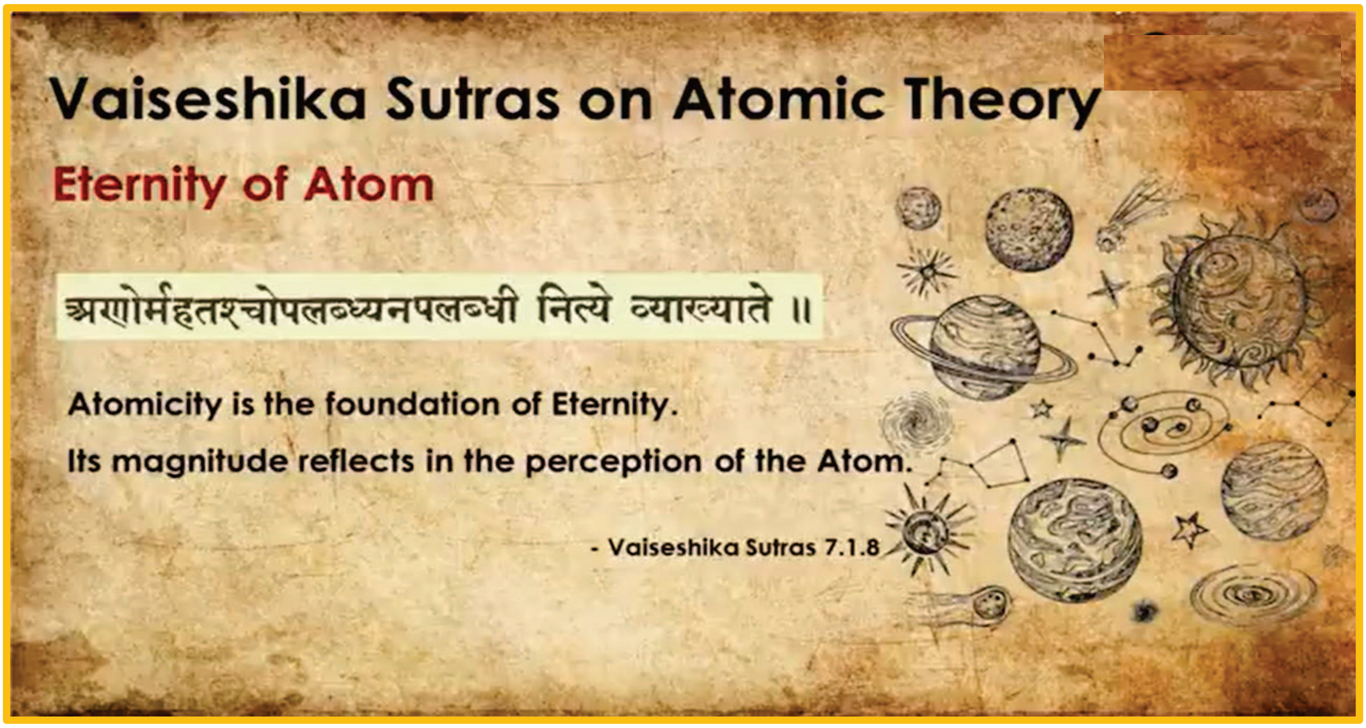
Structure of Atom

नित्यं परिमण्डलम् ॥ ७ । १ । २० ॥

That which is Eternal (Atom or Anu) is Spherical.

- Vaiseshika Sutras 7.1.20





The Rishi Kanada's Vaisheshika Sutra

First Sutra

वेगः निमित्तविशेषात कर्मणो जायते | [**Vegah Nimitta Visheshat Karmano Jayate].**

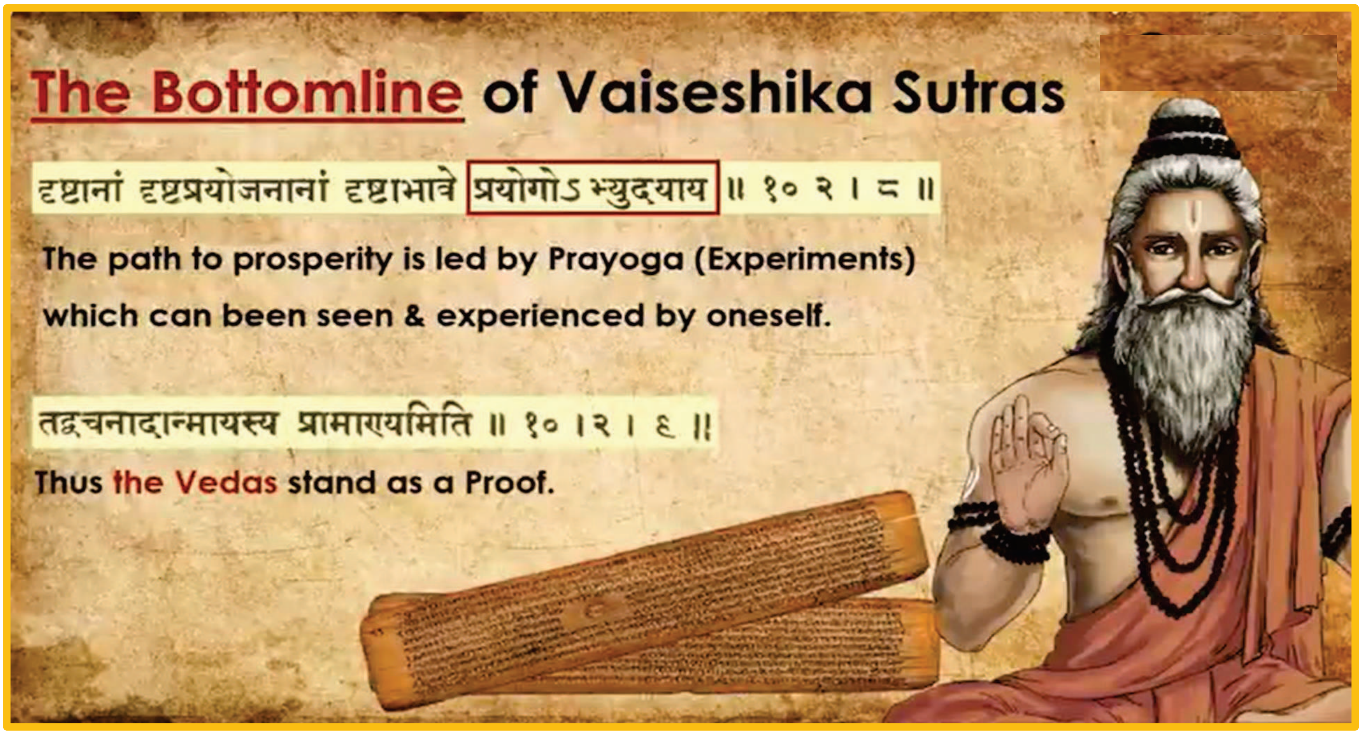
Second Sutra

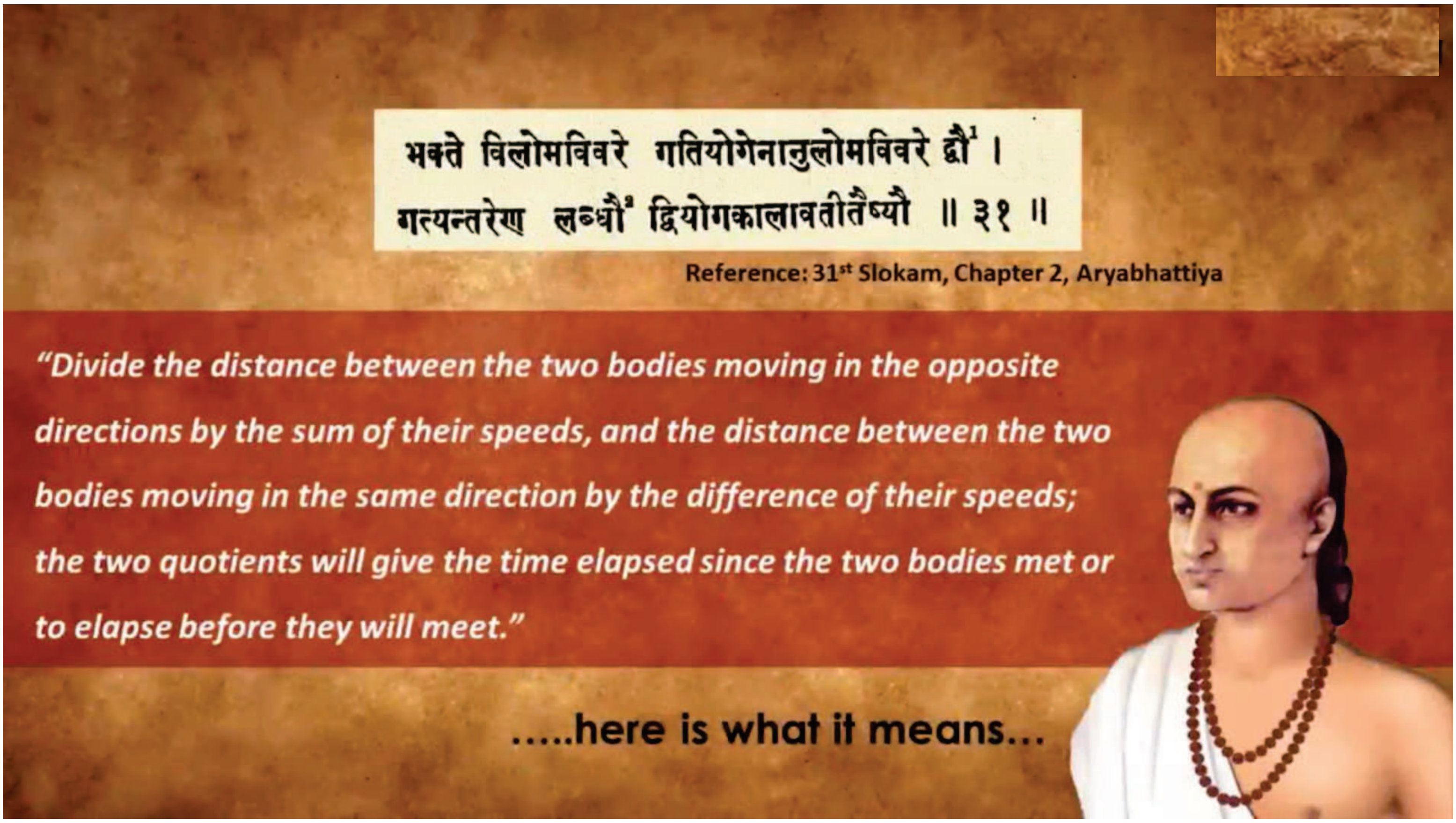
वेगः निमित्तापेक्षात कर्मणो जायते नियतदिक क्रियाप्रबन्धहेतु | [Vegah Nimitta Pekshat Karmano Jayate Niyatdik Kriya Prabandha Hetu].

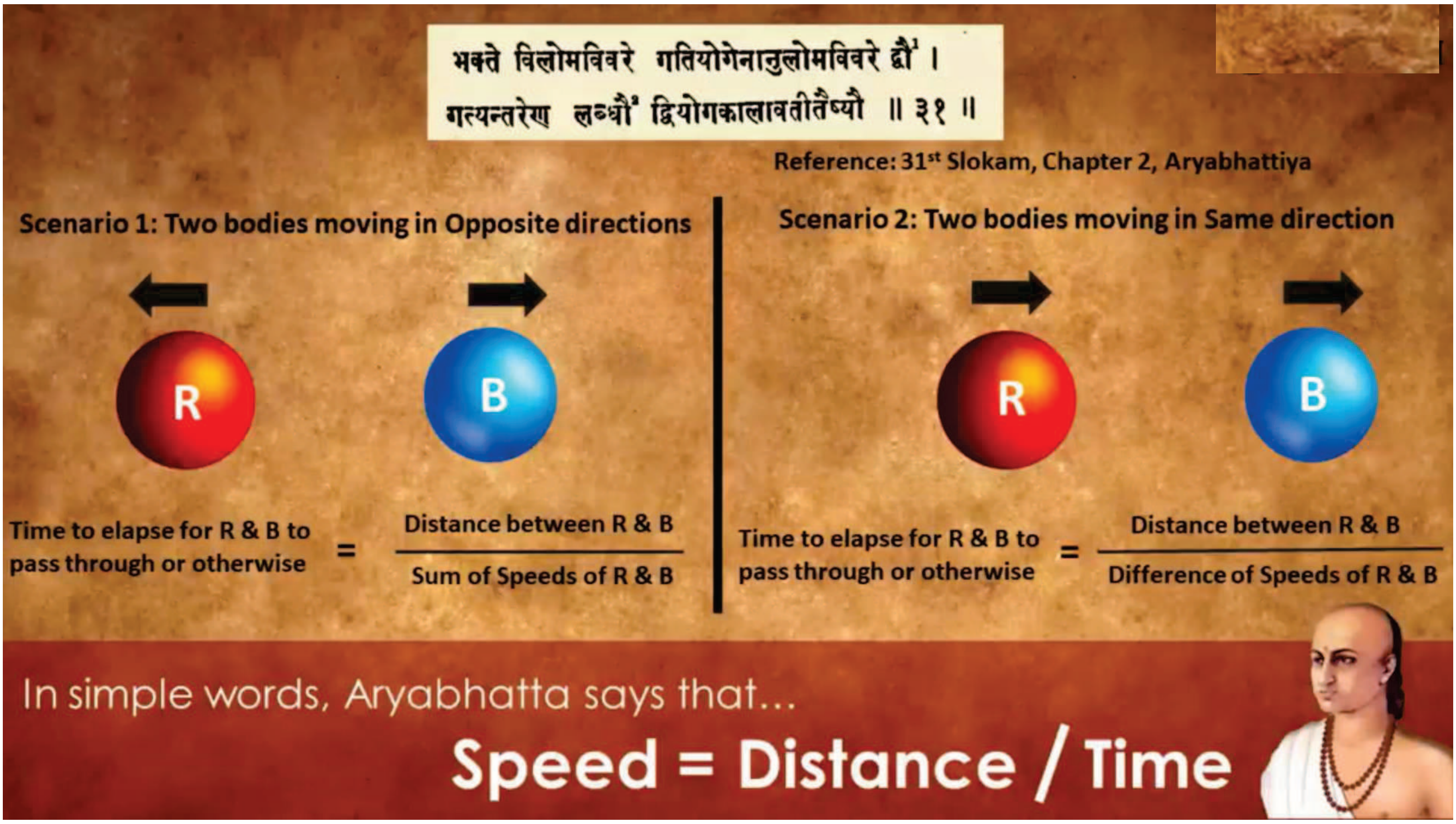
Third Sutra

वेगः संयोगविशेषविरोधी | **[Vegah Sanyog Vishesh Virodhi.]**

Comparison between the laws of motion of Sir Isaac Newton and the Sutra of Rishi Kanada		
Indicator	**Newton's Law**	**Rishi Kanada's Sutra**
Time of Invention	1687 AD	600 BCE
First Law	Every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force.	"Vegah Nimitta Visheshat Karmano Jayate". It means "Change of motion is due to impressed force".
Comparison	There is no intrinsically difference between the Newton's law of motion and the Kanada' Sutra.	
Second Law	Newton's second law states that the rate of change of momentum of a body is directly proportional to the force applied, and this change in momentum takes place in the direction of the applied force.	"Vegah Nimitta Pekshat Karmano Jayate Niyatdik Kriya Prabandha Hetu". It means that Change of motion is proportional to the impressed force and is in the direction of the force.
Comparison	Both the laws are bearing same meaning.	
Third Law	To every action there is always equal but opposed reaction.	" Vegah Sanyog Vishesh Virodhi." It means that action and reaction are equal and opposite.
Comparison	Both the laws are same and identical.	
Overall Explanations	Sir Isaac Newton published these laws in his book 'Philosophica Naturalis Principia Mathematica' on July 5, 1687 while the exact time of Rishi Kanada' Sutra is not known. From the ancient religious book/epics it is known to us that the time period of Rishi Kanada is 600 BCE. The invention of the Sutra by Rishi Kanada was before the time of innovation from "ZERO (invented by Aryabhatta)" to "INFINITY (invented by Bhaskaracharya)". So far as I understand, on account of scarcity of digits Rishi Kanada could not formulate his Sutra. On the contrary, the time period of Sir Isaac Newton was so far modern and at this time many things were either invented or discovered. As a result he could formulate his laws very easily in scientific way. Now question arises how the Newton's law and Sutra of Rishi Kanada are more or less same.	




भक्ते विलोमविवरे गतियोगेनानुलोमविवरे द्वौ[1] ।
गत्यन्तरेण लब्धौ[2] द्वियोगकालावतीतैष्यौ ॥ ३१ ॥
Reference: 31st Slokam, Chapter 2, Aryabhattiya
"Divide the distance between the two bodies moving in the opposite directions by the sum of their speeds, and the distance between the two bodies moving in the same direction by the difference of their speeds; the two quotients will give the time elapsed since the two bodies met or to elapse before they will meet."
.....here is what it means...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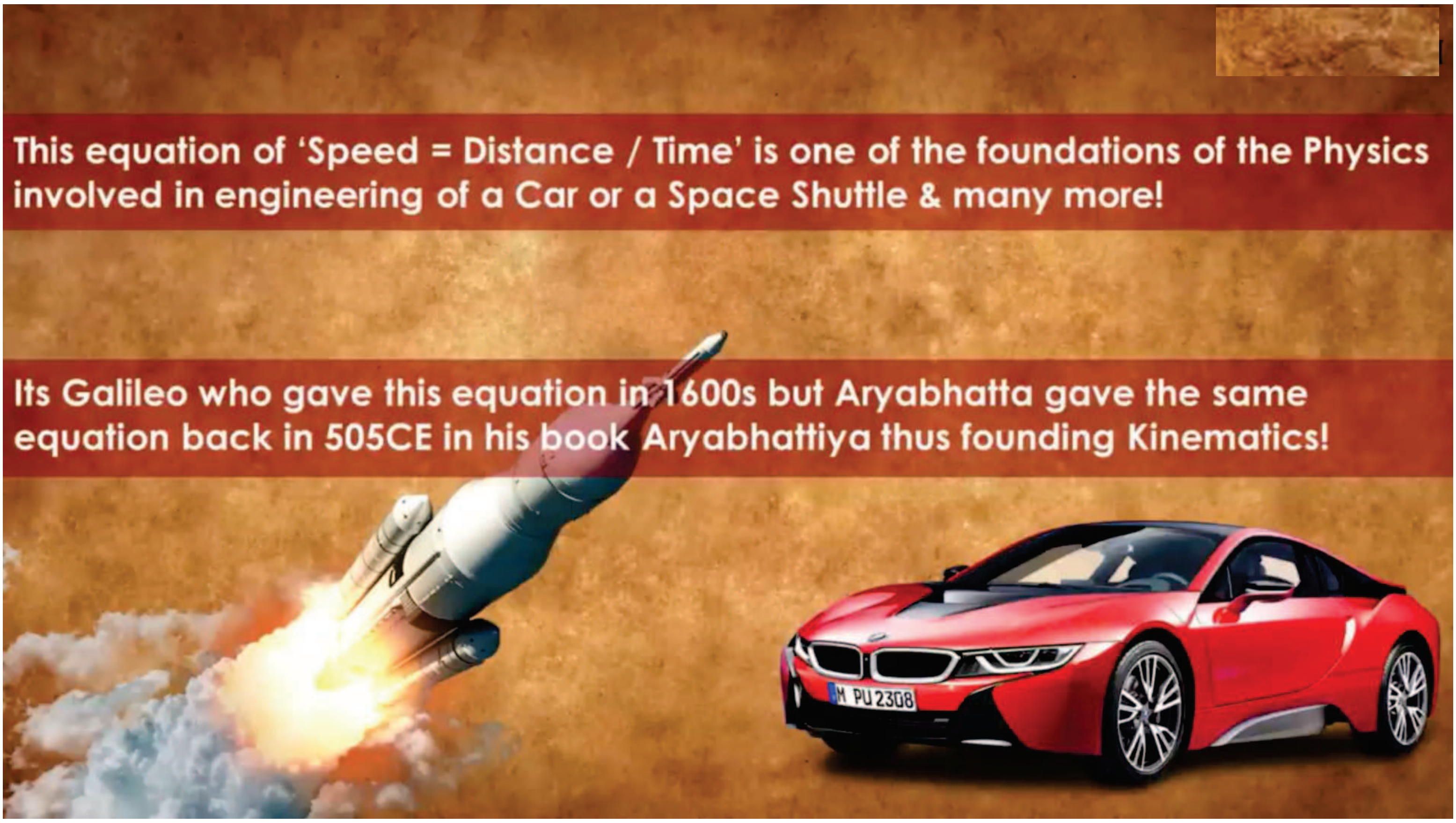
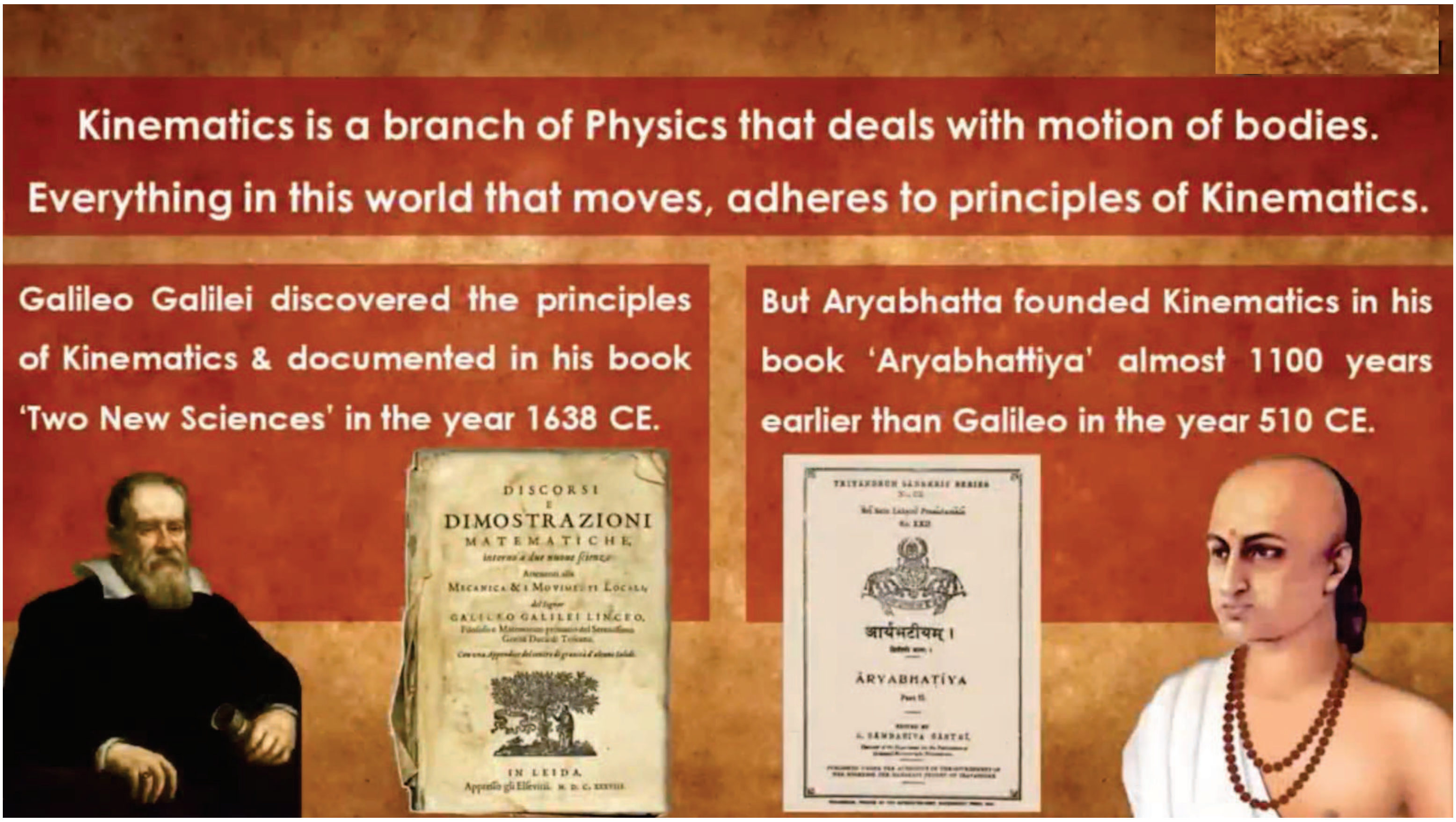
गत्यन्तरेण लब्धौ[2] द्वियोगकालावतीतैष्यौ ॥ ३१ ॥
Reference: 31st Slokam, Chapter 2, Aryabhattiya
Scenario 1: Two bodies moving in Opposite directions
R
B
Time to elapse for R & B to pass through or otherwise = Distance between R & B / Sum of Speeds of R & B
Scenario 2: Two bodies moving in Same direction
R
B
Time to elapse for R & B to pass through or otherwise = Distance between R & B / Difference of Speeds of R & B
In simple words, Aryabhatta says that...
Speed = Distance / Time


This equation of 'Speed = Distance / Time' is one of the foundations of the Physics involved in engineering of a Car or a Space Shuttle & many more!
Its Galileo who gave this equation in 1600s but Aryabhatta gave the same equation back in 505CE in his book Aryabhattiya thus founding Kinematics!
M PU 2308


Kinematics is a branch of Physics that deals with motion of bodies.
Everything in this world that moves, adheres to principles of Kinematics.
Galileo Galilei discovered the principles of Kinematics & documented in his book 'Two New Sciences' in the year 1638 CE.
But Aryabhatta founded Kinematics in his book 'Aryabhattiya' almost 1100 years earlier than Galileo in the year 510 CE.
DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE
IN LEIDA
आर्यभटीयम् ।
ĀRYABHAṬĪYA

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SCIENCE, ENGINEERING AND TECHNOLOGY IN IKS: MATHEMATICS

मखि भखि फखि धखि णखि ञखि ङखि हस्झ स्ककि किष्ग श्घकि किघ्व ।
घ्लकि किग्र हक्य धकि किच स्ग झश ङ्व क्ल प्त फ छ कला-अर्ध-ज्यास् ॥

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Vedic Mathematics

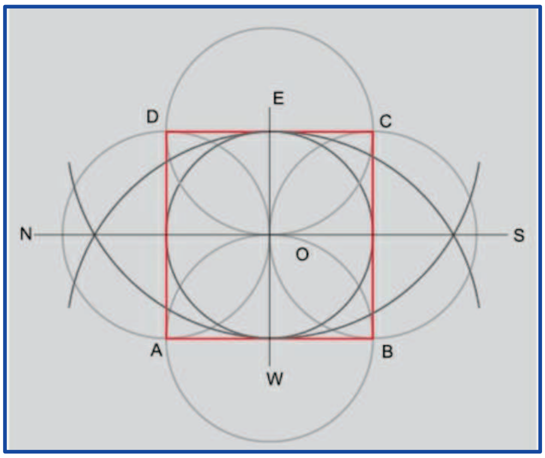
Example: How to predict a person's Date Of Birth?

a) Take the number of the month in which you were born (January is 1, February is 2 and so on…..)
(b) Double the number
(c) Add 5 to it
(d) Multiply it by 5
(e) Put a zero behind the answer
(f) Add date of birth (If they are born on 5th January then add 5)

(a) Take the month number	=	6
(b) Double the answer	=	12
(c) Add 5 to it	=	17
(d) Multiply it by 5	=	85
(e) Put a zero behind the answer	=	850
(f) Add the date of birth	=	850 + 26 = 876

	8	76
-	2	50
	6	26
	(month)	(date)

Sulba (means rope) Sutra: Set of rules with Rope

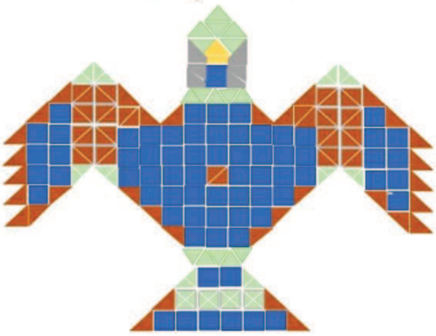


Procedure for construction of square using circles

Geometry is an ancient Science in India. Just with a pole anchored on the ground and a thread attached to it, Indians were able to generate complex geometrical shapes. What we see here is a procedure for construction of a square mentioned in Baudhāyana-śulba-sūtra, an ancient mathematical text taught in the Department of Mathematics in some Universities in the West as 'Rope Geometry'.

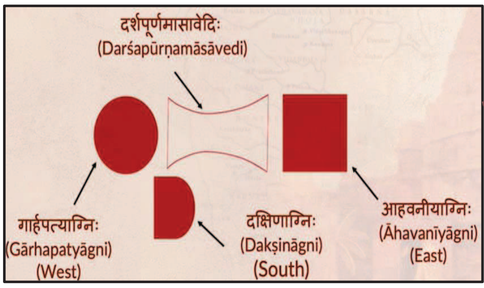
Ancient Indians: Tryst with Mathematics

The Śyena-citi



Type and the Number of Bricks Used in Falcon Shape Altar

	B1	B2	B3	B4	B5	
Head	1		6	6	1	14
Body	30	6	10			46
Wings	30	62	16			108
Tail	8	4	20			32
Total	69	72	52	6	6	200



- ✓ There are more than 70 types of altars e.g. Four different types of altars shown.
- ✓ Area of 'Garhapatyagni' should be same as 'Ahavaniyagni)

Ancient Indians: Tryst with Mathematics

- Indian Mathematics dealt with almost all areas of modern mathematics for more than 1000 years.
- Mathematical concepts found in Vedic texts, Buddhist and Jain works suggest that this culture is several thousand years old.

Vedāṅga Jyotiṣa:

- ✓ Mathematical concepts
- ✓ Measurements
- ✓ Predictions of the movement of celestial bodies in the sky

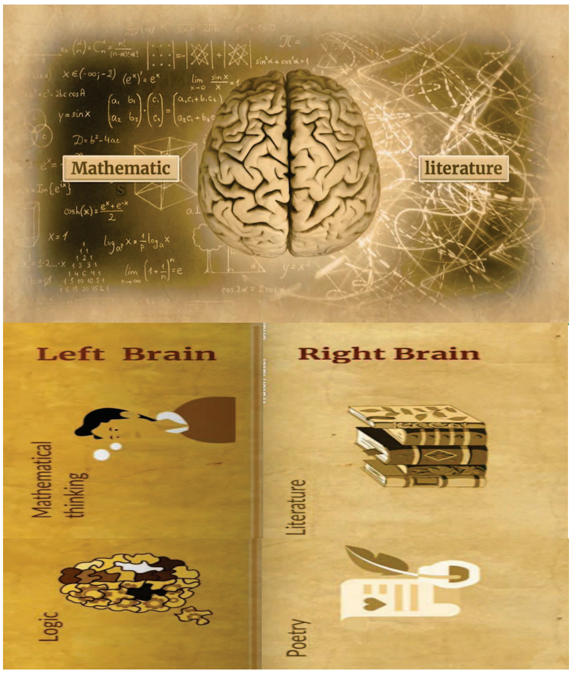
Kalpa:

- ✓ Geometry to construct Vedic altars

Buddhist and Jain work:

- ✓ Mathematical concepts

Unique aspects of Indian Mathematics



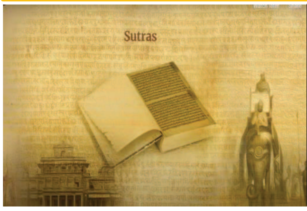
❖ Indian Mathematics (referred as **Ganita**) is a ***seamless blend*** of poetry, literature, logic, and mathematical thinking weaved into a single work. It has been integral part of Indians from very ancient times.

Unique aspects of Indian Mathematics

Spread across 'Akhand Bharat'

'Was a way of life'

Mathematics:
- ✓ Temple inscriptions
- ✓ Literature
- ✓ Discussion on religion or spirituality



The **use of sūtras and pithy** verses is characteristic in the Indian Mathematical tradition to convey complex ideas and concepts.

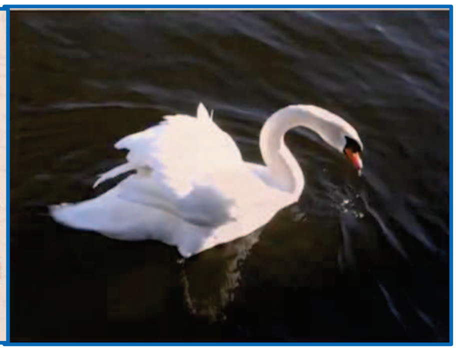
Constructive Approach ⟵ Procedure to solve s problem rather than merely looking for proofs of existence of solution

Unique aspects of Indian Mathematics

Try to solve?

Question:

Out of the swans in a certain lake, ten times the square root of their number went away to Manasa Sarovara when rains started, and one eighth the number went away to the forest Sthala Padmini. Three pairs of swans remained in the tank, engaged in water sports. What is the total number of swans?



Quiz on Mathematics during Swayamvara of Prince Siddartha

Place Value of Numerals
Sārīraka-bhāṣya of Śaṃkarācārya:

यदा एकोऽपि सन् देवदत्तः लोके स्वरूपं सम्बन्धिरूपं च अपेक्ष्य अनेकशब्दप्रत्ययभाग्भवति – मनुष्यः, ब्राह्मणः, श्रोत्रियः, वदान्यः, बालः, युवा, स्थविरः, पिता, पुत्रः, पौत्रः, भ्राता, जामाता इति ।
यथा च एकापि सती रेखा स्थानान्यत्वेन निविशमाना एक-दश-शत-सहस्रादि शब्दप्रत्ययभेदम् अनुभवति तथा सम्बन्धिनोरेव ...

"...an individual by the name Devadutta may be called differently as a father, son, son-in-law, brother, grandson, child, youth, etc., just as, although the stroke is the same, yet by a change of place it acquires values, one, ten hundred, thousand, etc. ...".

Indian Mathematicians and their Contributions

Vedic Texts

3000 BCE or earlier

एका च मे तिस्रश्च मे
पञ्च च मे सप्त च मे नव
च म एकादश च मे
त्रयोदश च मे पञ्चदश च ....
ekā ca me tisraśca me
pañca ca me sapta ca me
Nava ca ma ekādaśa ca
me trayodaśa ca me
pañcadaśa ca ....

Mathematical knowledge

सहस्रशीर्षा पुरुषः
सहस्राक्षः सहस्रपात्
Sahasrseershā
purushah
sahasrākshah
sahasrapāt

Number system

Pythagorean type triplets

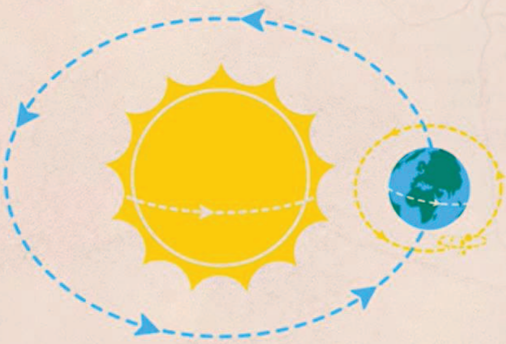
ॐ पूर्णमदः पूर्णमिदं पूर्णात्पूर्णमुदच्यते ।
पूर्णस्य पूर्णमादाय पूर्णमेवावशिष्यते ॥
oṁ pūrṇamadaḥ pūrṇamidaṁ
pūrṇātpūrṇamudacyate ।
pūrṇasya pūrṇamādāya
pūrṇamevāvaśiṣyate ॥

Concept of infinity

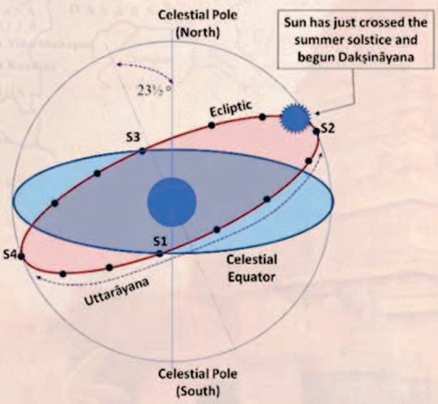
Indian Mathematicians and their Contributions

Lagada: Vedāṅga Jyotiṣa

~ 1300 BCE



Astronomical concepts;
A model for
sun-moon movement in time



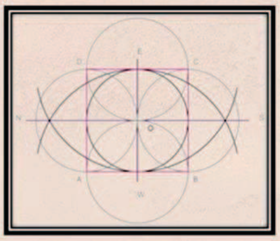
Equinoxes & Solstices

Indian Mathematicians and their Contributions

Śulba Sūtras:

(Baudhāyana, Āpasthamba, Kātyāyana and Mānava Śulba sūtras)

800 - 600 BCE



Earliest Texts of Geometry

प्रमाणं तृतीयेन वर्धयेत्
तच्चतुर्थेन
आत्मचतुस्त्रिंशेनोनेन सविशेषः
| 2.12
pramāṇaṃ tṛtīyena
vardhayet taccaturthena
ātmacatustriṃśenonena
saviśeṣaḥ | 2.12

Approximate value of the √ 2

Value of π= 3.08888
Geometrical Method

Value of π

मण्डलं चतुरस्रं चिकीर्षन्विष्कम्भमष्टौ
भागान्कृत्वा भागमेकोनत्रिंशधा
विभाज्याष्टाविंशतिभागानुद्धरेत् भागस्य च
षष्ठमष्टमभागोनम् ||1.59
maṇḍalaṁ caturasraṁ
cikīrṣanviṣkambhamaṣṭau
bhāgānkṛtvā bhāgamekonatriṁśadhā
vibhājyāṣṭāviṁśatibhāgānuddharet
bhāgasya ca
ṣaṣṭhamaṣṭamabhāgonam

Procedures for constructing and transforming

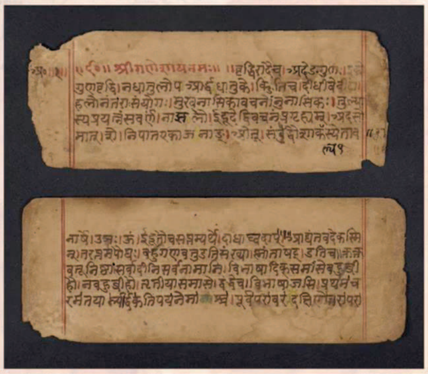
Indian Mathematicians and their Contributions

Mathematics
Contributions of Ancient Indians
(500 BCE to 500 CE)

Pāṇini:
Aṣṭādhyāyī

500 BCE; Śalalatura (in current Afghanistan)

- ✓ Algorithmic approaches
- ✓ Originator of the Backus–Naur Form (BNF)
- ✓ Context-sensitive rules
- ✓ Arrays
- ✓ Inheritance
- ✓ Polymorphism



Indian Mathematicians and their Contributions

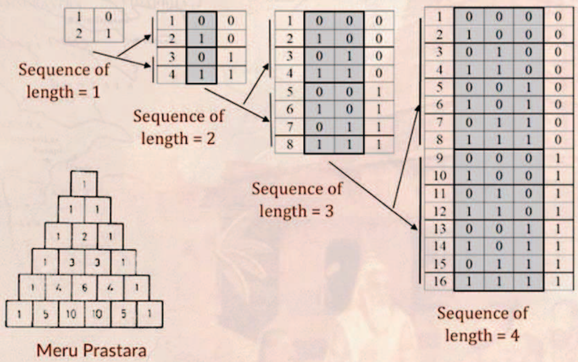
Mathematics
Contributions of Ancient Indians
(500 BCE to 500 CE)

Piṅgala:

Chandaḥ-Śāstra

300 BCE

- ✓ Binary sequences
- ✓ Conversion of Binary to Decimal system and vice versa
- ✓ 'Meru Prastara' (Pascal's triangle)
- ✓ Optimal Algorithms to calculate powers
- ✓ Zero as a Symbol



Indian Mathematicians and their Contributions

Mathematics
Contributions of Ancient Indians
(500 BCE to 500 CE)

Bauddha Mathematical works:

About 500 BCE to 500 CE

- ✓ Multi-valued logic
- ✓ Discussion of indeterminate and infinite numbers

$\infty - \infty =$ Indeterminate

Indian Mathematicians and their Contributions

Mathematics
Contributions of Ancient Indians
(200 BCE to 565 CE)

Jaina Mathematical works:

200 BCE to 300 CE

Works:

- ✓ Sūrya-Prajñapti
- ✓ Jambūdvīpa-prajñapti
- ✓ Bhagavatī-Sūtra
- ✓ Sthānāṅga-Sūtra
- ✓ Uttarādhyāyana-Sūtra
- ✓ Tiloyapannati
- ✓ Anuyoga-Dvāra-Sūtra

Concepts:

- ✓ Concept of logarithms
- ✓ Large numbers
- ✓ Algorithms for raising a number by a power
- ✓ The arc of a circle
- ✓ Combinatorics
- ✓ Mensuration
- ✓ Decimal system
- ✓ Approximation of π

Indian Mathematicians and their Contributions

Mathematics
Contributions of Ancient Indians
(200 BCE to 565 CE)

Āryabhaṭa:

Āryabhaṭīyam

476-550 CE; Kusumapura, near Pataliputra, Bihar

- ✓ Algorithm for square root
- ✓ Cube root
- ✓ Place value system
- ✓ Sine table; geometry
- ✓ Quadratic equations
- ✓ Linear indeterminate equations
- ✓ Sums of squares and cubes of numbers; planetary astronomy
- ✓ Plane and spherical trigonometry

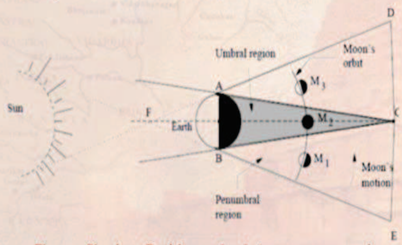


Figure: Shadow Problem - An Astronomy example

Indian Mathematicians and their Contributions

Varāha Mihira:
Bṛhat Samhitā, Bṛhat Jātaka, Pañca Siddhāntikā

482-565 CE, Ujjain, Madhya Pradesh

- ✓ Summary of five ancient siddhāntas
- ✓ Sine table
- ✓ Trigonometric identities
- ✓ Combinatorics
- ✓ Magic squares

https://commons.wikimedia.org/w/index.php?title=File:Varahamihira%27s_Brihajjataka,_Sanskrit,_Nepalaksara_script,_manuscript_copied_in_1399_CE,_Kathmandu.jpg&oldid=735928653

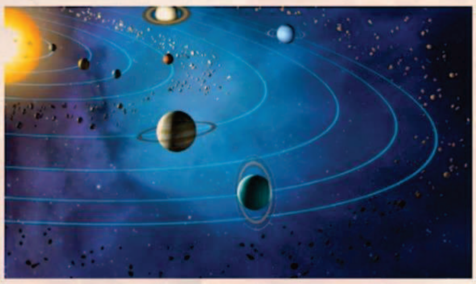
Indian Mathematicians and their Contributions

Mathematics
Contributions of Ancient Indians
(~ 600 CE)

Bhāskara I:
Commentary on Āryabhaṭīya, Laghu Bhāskarīyam and Mahā Bhāskarīyam

600-680 CE; Vallabhi region, Saurashtra, Gujarat

- ✓ Expanded Āryabhaṭa's work on Integer solutions for indeterminate equations
- ✓ Approximate formula for the sine function
- ✓ Planetary Astronomy



Indian Mathematicians and their Contributions

Brahmagupta:
Brahmasphuṭa Siddhānta; Khandakādhyāya

598-668 CE; Bhillamala in Rajasthan

- ✓ Rules of arithmetic operations with zero and negative numbers
- ✓ Algebra (Bījagaṇita)
- ✓ Linear and quadratic indeterminate equations
- ✓ Pythagorean triplets
- ✓ Formula for the diagonals and area of a cyclic quadrilateral
- ✓ Notion of arithmetic mean

Vrihanka:
Vṛttajātisamuccaya (in Prākṛt)
~ 600 CE

- ✓ Fibonacci numbers
- ✓ Moric metres

Indian Mathematicians and their Contributions

Mathematics
Contributions of Ancient Indians
(~ 870 CE)

Śrīdharācārya:
Triṣatikā and Pāṭīgaṇita

870-930 CE; Bhūrisriṣṭi or Bhurshut village, Hugli, West Bengal

- ✓ Arithmetic, Algebra, and Commercial Mathematics
- ✓ Approximation of the square root of a non-square number
- ✓ Quadratic equations
- ✓ Practical applications of algebra

Indian Mathematicians and their Contributions

Mathematics
Contributions of Ancient Indians
(~ 870 CE)

Mahāvīrācārya:
Gaṇita-Sāra-Saṅgraha

800-870 CE; Gulbarga Karnataka

- ✓ Arithmetic-geometry-algebra
- ✓ Continuing the ancient jaina mathematics tradition
- ✓ Permutations and combinations
- ✓ Arithmetic and geometric series
- ✓ The sum of squares and cubes of numbers in arithmetic progression

Indian Mathematicians and their Contributions

Mathematics
Contributions of Ancient Indians
(800 CE to 1500 CE)

Jayadeva:

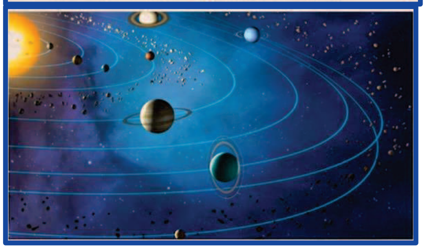
10th Century CE or earlier

Cakravāla (cyclic) method for the solution of the second-order indeterminate equation

Śripati:
Gaṇita Tilaka, Siddhānta Śekhara, Dhikoṭidakaraṇa etc.

1019 - 1066 CE; Rohiṇīkhaṇḍa, Maharashtra

- ✓ Planetary Astronomy



Indian Mathematicians and their Contributions

Bhāskarācārya (Bhaskara-II):

Līlāvatī on arithmetic and geometry;
Bījagaṇita on algebra;
Siddhānta-Śiromaṇi on astronomy;
Vāsanābhāṣya on Siddhānta-Śiromaṇi.

1114 - 1185 CE; Hailed from Bijjadavīda

- ✓ Upapatti (demonstration or proof);
- ✓ Addition formula for sine function
- ✓ Permutations, and combinations;
- ✓ Solution of indeterminate equations,
- ✓ Ideas of calculus,
- ✓ Including mean value theorem,
- ✓ Planetary astronomy;
- ✓ Construction of several instruments;

Indian Mathematicians and their Contributions

Nārāyaṇa Paṇḍita:

Gaṇita Kaumudī– a treatise on arithmetic and Bījaganita Vātāmśa – a treatise on algebra

1325 - 1400 CE

- ✓ Advanced textbooks taking forward the works of Bhāskarācārya
- ✓ Further properties of cyclical quadrilaterals
- ✓ Summation and repeated summations of arithmetic series
- ✓ Theory and construction of Magic squares
- ✓ Further developments in combinatorics

Indian Mathematicians and their Contributions

Mādhava of Saṅgamagrāma:

1340 - 1425 CE; Sangama Grama, Kerala

- ✓ Founder of Kerala School of Mathematics
- ✓ A pioneer in the development of calculus
- ✓ Infinite series and approximations for π
- ✓ Infinite series and approximations for cosine and sine functions

Indian Mathematicians and their Contributions

Parameśvara:

Dṛggaṇita, Siddhāntadīpikā;

Commentaries on Āryabhaṭīyam, Mahābhāskarīya Laghubhāskarīya, Līlāvatī, and Sūryasiddhānta

1360 - 1460 CE; Alathiyur, (near Tirur), Kerala

- ✓ Properties of Cyclic quadrilateral
- ✓ Iterative techniques

Indian Mathematicians and their Contributions

Nīlakanṭha Somayājī:

Tantra-Saṅgraha;

Āryabhaṭīya-Bhāṣya, Siddhānta-Darpaṇa,

1444 - 1544 CE; Near Tirur, Kerala

- ✓ Irrationality of π
- ✓ Basic ideas of calculus
- ✓ Revised planetary theory
- ✓ Exact results in spherical astronomy

$$\int x^n\,dx = (x^{n+1}) / (n+1) + C$$

Indian Mathematicians and their Contributions

Jyeṣṭhadeva:

Yukti-Bhāṣā

1500 - 1575 CE; Kerala

- ✓ Hailed as the first textbook of Calculus
- ✓ Detailed explanations and proofs of the infinite series given by Mādhava

Śaṅkaravāriyar:

Kriyākramakarī commentary on Līlāvātī and commetary on Tantrasaṅgraha

1500-1569 CE; Kerala

- ✓ Explanations and Proofs of the results and procedures given in Līlāvātī

Indian Mathematicians and their Contributions

Ganeśa Daivajña:

Buddhi vilāsinī (commentary on Līlāvatī);

1504 CE; Nandi Grama, Nadod, Gujarat

✓ Explanations and Proofs of the results and procedures given in Līlāvātī

Kṛiṣṇa Daivajña:

Bījapallva- Commentary on Bījagaṇita of Bhāskarācārya

1600 CE Delhi

✓ Explanations and proofs of results and procedures given in Bījagaṇita

Indian Mathematicians and their Contributions

Munīśvara:

Siddhānta-Sārvabhauma,
commentary on Līlāvatī; Pātisāra;

17th Century CE; Varanasi

✓ Explanations and Proofs of the results and procedures given in Līlāvātī;

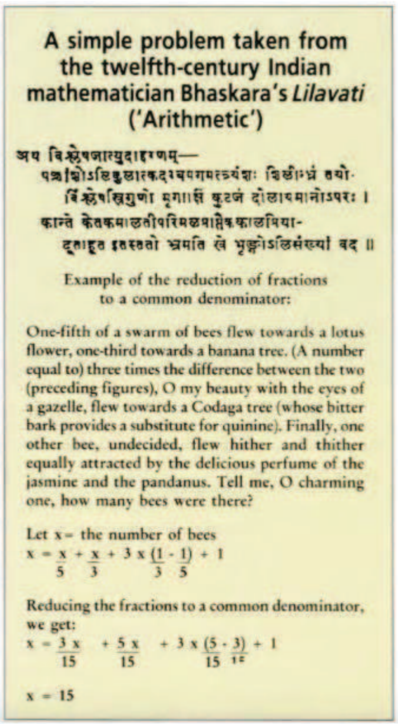
✓ Trigonometric identities

Ancient Indian Mathematicians and Their Salient Contributions

Sl. No.	Details of the Work/Mathematician	Period, Location	Salient Contributions
1	**Vedic Texts**	3000 BCE or earlier	The earliest recorded mathematical knowledge, number system, Pythagorean type triplets; Decimal system of naming numbers, the concept of infinity;
2	**Lagadha** – Vedāṅga-jyotiṣa	~ 1300 BCE	Astronomical concepts; a mathematical model for sun-moon movement in time; equinoxes and solstices;
3	**Śulba-sūtras** (Baudhāyana, Āpasthamba, Kātyāyana and Mānava Śulba-sūtras)	800–600 BCE	Earliest Texts of Geometry; Approximate value of the square root of 2, and π. Exact procedures for the construction and transformations of squares, rectangles, trapezia, etc.
4	**Pāṇini** – *Aṣṭādhyāyī*	500 BCE; Śalātura (in Khyber province in Pakistan)	Algorithmic approaches; Originator of the Backus–Naur Form (BNF), used in programming languages today, Context-sensitive rules, Arrays, inheritance, polymorphism;
5	**Piṅgala** – *Chandaḥ-śāstra*	300 BCE	Binary sequences; Conversion of Binary to Decimal system and vice versa; 'Meru Prastara' (Pascal's triangle); Optimal Algorithms to calculate powers; Zero as a Symbol;
6	**Bauddha Mathematical Works**	about 500 BCE to 500 CE	Multi-valued logic, Discussion of indeterminate and infinite numbers;
7	**Jaina Mathematical Works** – *Sūrya-Prajñapti, Jambūdvīpa-prajñapti, Bhagavatī-sūtra, Sthānāṅga-sūtra, Uttarādhyayana-sūtra, Tiloyapannati, Anuyoga-dvāra-sūtra*	200 BCE to 300 CE	Concept of logarithms, large numbers; algorithms for raising a number by a power; the arc of a circle; combinatorics; mensuration; Decimal system; Approximation of π;
8	**Āryabhaṭa** – *Āryabhaṭīyam*	476–550 CE; Kusumapura, near Pataliputra, Bihar	Concise verses; Algorithm for square root, cube root, Place value system; Sine table; geometry; quadratic equations; Linear indeterminate equations; Sums of squares and cubes of numbers; Planetary astronomy; Plane and spherical trigonometry;
9	**Varāha Mihira** – *Bṛhat Samhitā, Bṛhat-jātaka, Pañca-siddhāntikā*	482–565 CE, Ujjain, Madhya Pradesh	Summary of five ancient siddhāntas; Sine table, trigonometric identities; ($\sin^2 + \cos^2$); combinatorics; Magic squares;
10	**Bhāskara I** – *Commentary on Āryabhaṭīya, Laghu-bhāskarīyam and Mahā-bhāskarīyam*	600–680 CE; Vallabhi region, Saurashtra, Gujarat	Expanded Āryabhaṭa's work on Integer solution for indeterminate equations; Approximate formula for the sine function, Planetary Astronomy;
11	**Brahmagupta** – *Brahmasphuṭa-siddhānta; Khaṇḍakhādyaka*	598–668 CE; Bhillamala in Rajasthan	Rules of arithmetic operations with zero and negative numbers, Algebra (Bījagaṇita); ḷinear and quadratic indeterminate equations; Pythagorean triplets, Formula for the diagonals and area of a cyclic quadrilateral; notion of arithmetic mean.
12	**Virahāṅka** – *Vṛttajātisamuccaya (in Prākṛt)*	~600 CE	Fibonacci numbers; Moric metres.

Sl. No.	Details of the Work/Mathematician	Period, Location	Salient Contributions
13	**Śrīdharācārya** – *Triśatikā and Pāṭīgaṇita*	870–930 CE; Bhūrisṛṣṭi or Bhurshut village, Hugli, West Bengal	Arithmetic, Algebra, and Commercial Mathematics; Approximation of square root of a non-square number; Quadratic equations; Practical applications of algebra;
14	**Mahāvīrācārya** – *Gaṇita-sāra-saṅgraha*	800–870 CE; Gulbarga Karnataka;	A comprehensive, exclusive textbook on mathematics covering arithmetic-geometry- algebra. Continuing the ancient Jaina mathematics tradition; permutations and combinations; arithmetic and geometric series; the sum of squares and cubes of numbers in arithmetic progression;
15	**Jayadeva**	10th Century CE or earlier	Cakravāla (cyclic) method for solution of the second-order indeterminate equation.
16	**Shrīpati** – *Gaṇita-tilaka, Siddhānta-śekhara, Dhikoṭidākaraṇa, etc.*	1019–1066 CE; Rohiṇīkhaṇḍa, Maharashtra	Planetary Astronomy
17	**Bhāskarācārya (Bhaskara-II)** – *Līlāvatī on arithmetic and geometry; Bījagaṇita on algebra; Siddhānta-śiromaṇi on astronomy; Vāsanābhāṣya on Siddhānta-śiromaṇi.*	1114–1185 CE; Hailed from Bijjadavīda	Canonical textbooks used all over India, Detailed explanations including Upapatti (demonstration or proof); addition formula for sine function. Surds; permutations, and combinations; Solution of indeterminate equations, Ideas of calculus, including mean value theorem, planetary astronomy; construction of several instruments;
18	**Nārāyaṇa Paṇḍita** – *Gaṇita-kaumudī – a treatise on arithmetic and Bījaganita-aātāmśa – a treatise on algebra.*	1325–1400 CE;	Advanced textbooks taking forward the works of Bhāskarācārya, further properties of cyclical quadrilaterals, summation and repeated summations of arithmetic series, theory and construction of Magic squares, further developments in combinatorics.

19	**Mādhava of Saṅgamagrāma**	1340–1425 CE; Sangama Grama, in Kerala	Founder of Kerala School of Mathematics – a pioneer in the development of calculus; Infinite series and approximations for π, Infinite series and approximations for cosine and sine functions
20	**Parameśvara, –** *Dṛggaṇita, Siddhāntadīpikā;* Commentaries on Āryabhaṭīyam, Mahā-bhāskarīya; Laghu-bhāskarīya, Līlāvatī, and Sūryasiddhānta	1360–1460 CE; Alathiyur, (near Tirur), Kerala	Properties of Cyclic quadrilateral; iterative techniques.
21	**Nīlakaṇṭha Somayājī,** *Tantra-saṅgraha; Āryabhaṭīya-bhāṣya, Siddhānta-darpaṇa*	1444–1544 CE; Near Tirur, Kerala	Irrationality of π, basic ideas of calculus; revised planetary theory, which is a close approximation to Kepler's model; Exact results in spherical astronomy
22	**Jyeṣṭhadeva** – *Yukti-bhāṣā*	1500–1575 CE; Kerala	Hailed as the first textbook of Calculus; detailed explanations and proofs of the infinite series given by Mādhava
23	**Śaṅkaravāriyar** – *Kriyākramakarī commentary on Līlāvatī and commentary on Tantra-saṅgraha*	1500–1569 CE Kerala	Explanations and Proofs of the results and procedures given in Līlāvatī
24	**Gaṇeśa Daivajña** – *Buddhi-vilāsinī (commentary on Līlāvatī);*	1504 CE; Nandi Grama, Nadod, Gujarat	Explanations and Proofs of the results and procedures given in Līlāvatī
25	**Kṛṣṇa Daivajña** – *Bījapallva- Commentary on Bījagaṇita of Bhāskarācārya*	1600 CE Delhi	Explanations and Proofs of results and procedures given in Bījagaṇita
26	**Munīśvara** – *Siddhānta-sārvabhauma, commentary on Līlāvatī; Pātisāra;*	17th Century CE; Varanasi	Explanations and Proofs of the results and procedures given in Līlāvatī; trigonometric identities
27	**Kamalākara** – *Siddhānta-tattva-viveka*	1616–1700 CE; Varanasi, Uttara Pradesh	Addition and subtraction theorems for the sine and the cosine; Sines and cosines of double, triple, etc., angles.


A simple problem taken from the twelfth-century Indian mathematician Bhaskara's *Lilavati* ('Arithmetic')

अथ विश्लेषजात्युदाहरणम्—
पञ्चांशोऽलिकुलात्कदम्बमगमत्त्र्यंशः शिलीन्ध्रं तयो-
र्विश्लेषस्त्रिगुणो मृगाक्षि कुटजं दोलायमानोऽपरः ।
कान्ते केतकमालतीपरिमलप्राप्तैककालप्रिया-
दूताहूत इतस्ततो भ्रमति खे भृङ्गोऽलिसंख्यां वद ॥

Example of the reduction of fractions to a common denominator:

One-fifth of a swarm of bees flew towards a lotus flower, one-third towards a banana tree. (A number equal to) three times the difference between the two (preceding figures), O my beauty with the eyes of a gazelle, flew towards a Codaga tree (whose bitter bark provides a substitute for quinine). Finally, one other bee, undecided, flew hither and thither equally attracted by the delicious perfume of the jasmine and the pandanus. Tell me, O charming one, how many bees were there?

Let x = the number of bees

$$x = \frac{x}{5} + \frac{x}{3} + 3 \times \left(\frac{1}{3} - \frac{1}{5}\right) + 1$$

Reducing the fractions to a common denominator, we get:

$$x = \frac{3x}{15} + \frac{5x}{15} + 3 \times \frac{(5 - 3)}{15} + 1$$

$x = 15$

A problem from the Lilavati by Bhaskaracharya. Written in the 12th century. This appeared on page 18 of The Mathematical Mystery Tour by UNESCO in 1989

Arithmetic

Square of a Number

अन्त्यपदस्य वर्गं कृत्वा द्विगुणं तदेव चान्त्यपदम् ।
शेषपदैराहन्यात् उत्सार्योत्सार्य वर्गविधौ ।।

antyapadasya vargaṃ kṛtvā dviguṇaṃ tadeva cāntyapadam |
śeṣapadairāhanyāt utsāryotsārya vargavidhau ||

The steps of the algorithm as provided by the above verse can be enumerated as follows:
Step 1: अन्त्यपदस्य वर्गं कृत्वा (antyapadasya vargaṃ kṛtvā).
Square the last digit (most significant digit) first. Place it in a new row (two places to the right of digits in the previous row).
Step 2: द्विगुणं तदेव चान्त्यपदम् (dviguṇaṃ tadeva cāntyapadam).
Multiply the last digit with two and each of the remaining digits to the right and place them to the right in the same row.
Step 3: शेषपदैराहन्यात् उत्सार्योत्सार्य (śeṣapadairāhanyāt utsāryotsārya).
Remove the current most significant digit. If there are no more digits to operate go to step 5.
Step 4: The next digit becomes the last digit now. Go to step 1.
Step 5: Perform the final addition to get the square of the number.

It may be noted that this ancient rule for squaring, uses $\frac{n(n-1)}{2}$ multiplications for squaring an n-digit number.

Square of a Number

अन्त्यपदस्य वर्गं कृत्वा द्विगुणं तदेव चान्त्यपदम् ।
शेषपदैराहन्यात् उत्सार्योत्सार्य वर्गविधौ ।।
antyapadasya vargaṃ kṛtvā dviguṇaṃ tadeva cāntyapadam |
śeṣapadairāhanyāt utsāryotsārya vargavidhau ||

Example: Find the square of the number 1638

The steps of the algorithm as provided by the above verse can be enumerated as follows:

Step 1: अन्त्यपदस्य वर्गं कृत्वा (antyapadasya vargaṃ kṛtvā).

Square the last digit (most significant digit) first. Place it in a new row (two places to the right of digits in the previous row).

Final Digit Position	7	6	5	4	3	2	1
Row 1: Last digit = 1 Multiplication with other digits Digit '1' is removed	$\boxed{(1^2) = 1}$	(2*1*6) = 12	(2*1*3) = 6	(2*1*8) = 16			

Step 2: द्विगुणं तदेव चान्त्यपदम् (dviguṇaṃ tadeva cāntyapadam).

Multiply the last digit with two and each of the remaining digits to the right and place them to the right in the same row.

Final Digit Position	7	6	5	4	3	2	1
Row 1: Last digit = 1 Multiplication with other digits Digit '1' is removed	$(1^2) = 1$	(2*1*6) = 12	(2*1*3) = 6	(2*1*8) = 16			

Square of a Number

अन्त्यपदस्य वर्गं कृत्वा द्विगुणं तदेव चान्त्यपदम् ।
शेषपदैराहन्यात् उत्सार्योत्सार्य वर्गविधौ ।।
antyapadasya vargaṃ kṛtvā dviguṇaṃ tadeva cāntyapadam |
śeṣapadairāhanyāt utsāryotsārya vargavidhau | |

Example: Find the square of the number 1638

Step 3: शेषपदैराहन्यात् उत्सार्योत्सार्य (śeṣapadairāhanyāt utsāryotsārya).

Remove the current most significant digit. If there are no more digits to operate go to step 5.

Final Digit Position	7	6	5	4	3	2	1
Row 1: Last digit = 1 Multiplication with other digits Digit '1' is removed	$(1^2) = 1$	$(2*1*6) = 12$	$(2*1*3) = 6$	$(2*1*8) = 16$			

Step 4: The next digit becomes the last digit now. Go to step 1.

Final Digit Position	7	6	5	4	3	2	1
Row 1: Last digit = 1 Multiplication with other digits Digit '1' is removed	$(1^2) = 1$	$(2*1*6) = 12$	$(2*1*3) = 6$	$(2*1*8) = 16$			
Row 2: Last digit = 6 Multiplication with other digits Digit '6' is removed			$(6^2) = 36$	$(2*6*3) = 36$	$(2*6*8) = 96$		

Example: Find the square of the number 1638

Final Digit Position	7	6	5	4	3	2	1
Row 1: Last digit = 1 Multiplication with other digits Digit '1' is removed	$(1^2) = 1$	$(2*1*6) = 12$	$(2*1*3) = 6$	$(2*1*8) = 16$			
Row 2: Last digit = 6 Multiplication with other digits Digit '6' is removed			$(6^2) = 36$	$(2*6*3) = 36$	$(2*6*8) = 96$		
Row 3: Last digit = 3 Multiplication with other digits Digit '3' is removed					$(3^2) = 9$	$(2*3*8) = 48$	
Row 4: Last digit = 8 Digit '8' is removed No more digits available							$(8^2) = 64$

Step 5: Perform the final addition to get the square of the number

Final Addition of the numbers

Digit Position						
7	6	5	4	3	2	1
1						
1	2					
		6				
		1	6			
	3	6				
		3	6			
			9	6		
				9		
				4	8	
					6	4
2	6	8	3	0	4	4

The answer is 26,83,044

Square Root

- Āryabhaṭa presents an algorithm for determining the square root of a number.
- The methodology of finding the square root revolves around the concept of splitting the digits into pairs, starting from the least significant digit, the **odd place** being designated as **varga (V),** and the **even place** as **avarga (A).**
- For example, if we take two numbers 55225 and 205209 the split will look as follows:

V	A	V	A	V	A	V	A	V	A	V
5	5	2	2	5	2	0	5	2	0	9

Āryabhaṭa then provides a simple recursive algorithm that starts from the most significant varga place and progressively introduces one digit at a time until all digits are exhausted.

भागं हरेत् अवर्गात् नित्यं द्विगुणेन वर्गमूलेन ।
वर्गाद्वर्गे शुद्धे लब्धं स्थानान्तरे मूलम् ।।

bhāgaṃ haret avargāt nityaṃ dviguṇena vargamūlena |
vargādvarge śuddhe labdhaṃ sthānāntare mūlam ||

Find Square Root of number 19881

The above verse has indicated the following steps to the algorithm:

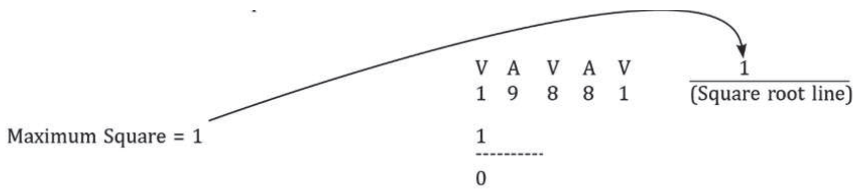
1. Designate the varga and avarga digits starting from right to left (least significant to most significant)

V	A	V	A	V
1	9	8	8	1

2. Take the first varga sthāna at the leftmost (most significant) along with an avarga digit at its left (if any).

V	A	V	A	V
1	9	8	8	1

3. Remove (subtract) the maximum possible square from this number, and the square root of the square that we can remove will be added to the square root line (this is a place where we are accumulating the answer as we perform the operation).

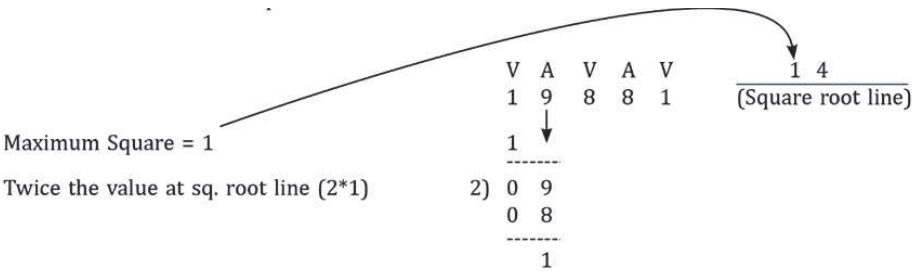


Find Square Root of number 19881

4. Along with the remainder of the previous operation, bring the next digit down. The next digit is avarga digit.

भागं हरेत् अवर्गात् नित्यं द्विगुणेन वर्गमूलेन
(bhāgaṃ haret avargāt nityaṃ dviguṇena vargamūlena)

Whenever we operate at avarga digit, we need to divide the number by two times the current value of the square root that we have stored in the square root line. The quotient obtained in this division will be added to the square root line.

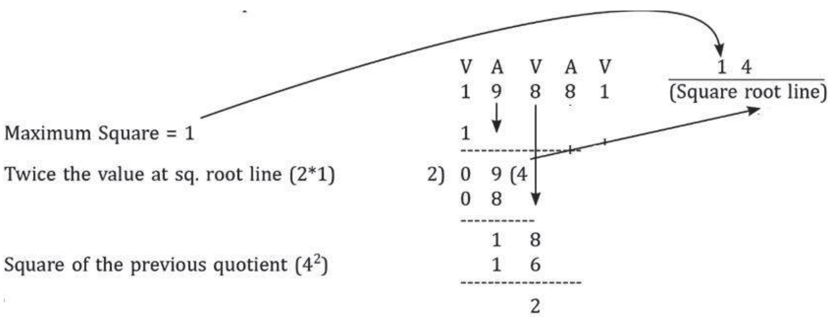


Find Square Root of number 19881

5. Along with the remainder of the previous operation, we will bring the next digit down. The next digit is varga digit.

वर्गाद्वर्गे शुद्धे (vargādvarge śuddhe)

Whenever we operate at a varga digit, we need to remove the square of the quotient obtained in the previous step.



Find Square Root of number 19881

6. If some more digits are remaining go to step 4, else go to step 7.

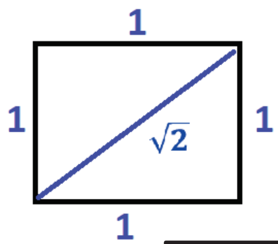
7. लब्धं स्थानान्तरे मूलम् (labdhaṃ sthānāntare mūlam).

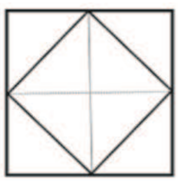
The final result in the square root line is the answer.

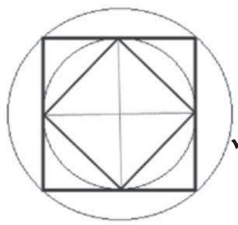
```
                                                V  A  V  A  V          1 4 1
                                                1  9  8  8  1     (Square root line)
Maximum Square = 1                              1
                                                ----------------
Twice the value at sq. root line (2*1)      2)  0  9 (4
                                                0  8
                                                ---------
                                                   1  8
Square of the previous quotient (4^2)              1  6
                                                ------------
Twice the value at sq. root line (2*14)    28)        2  8 (1
                                                      2  8
                                                ----------------
                                                         0  1
Square of the previous quotient (1^2)                       1
                                                ----------------
                                                            0
                                                ----------------
The final answer = 141.
```


Bodhayana Sulba Sutra (BSS 2.12) to find $\sqrt{2}$

Samasya dvikaraṇī







प्रमाणं तृतीयेन वर्धयेत् तच्चतुर्थेन आत्मचतुस्त्रिंशेनोनेन सविशेषः ।
pramāṇaṃ tṛtīyena vardhayet taccaturthena ātmacatustriṃśenonena saviśeṣaḥ |

Dvikarani – Diagonal (dividing the square into two), or Root of Two
Pramanam – Unit measure;
tṛtīyena vardhayet – increased by a 3rd (meaning add 1/3 to 1)
Tat caturtena (vardhayet) – that itself increased by a 4th, Atma – itself; (meaning add ¼ of this (1/4)*(1/3)
Caturtrimsah – is in excess by 34th part i.e. take out 1/34
Savisesah- Denotes it is a special case (meaning approximate number)

- ✓ Possibly in his quest to construct circular altars, he constructed 2 circles circumscribing the 2 squares as shown.
- ✓ Now, just as the areas of the squares, he realised that the inner circle should be exactly half of the bigger circle in area.
- ✓ By the same logic, just as the perimeters of the 2 squares, the perimeter of the outer circle should also be √2 times the perimeter of the inner circle.

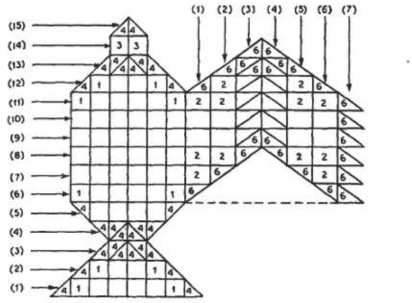
The value of $\sqrt{2} \approx 1 + \frac{1}{3} + \frac{1}{3\times 4}\left(1 - \frac{1}{34}\right)$

$$= \frac{577}{408} \approx 1.4142156863$$

Bodhayana Sulba Sutra…..continued

Number and types of bricks used in different parts of the fire-altar—first layer

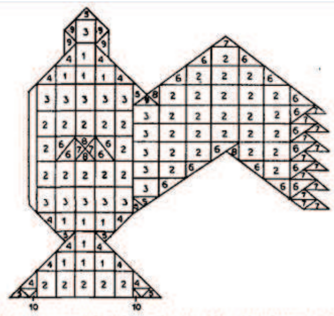
Parts of the *citi*	Brick types					Total
	B_1	B_2	B_3	B_4	B_6	
Head			2	2		4
Body	44			16		60
Wings		40			76	116
Tail	10			10		20
Total	54	40	2	28	76	200



Arrangement of bricks in the syenacit—second type, first layer.

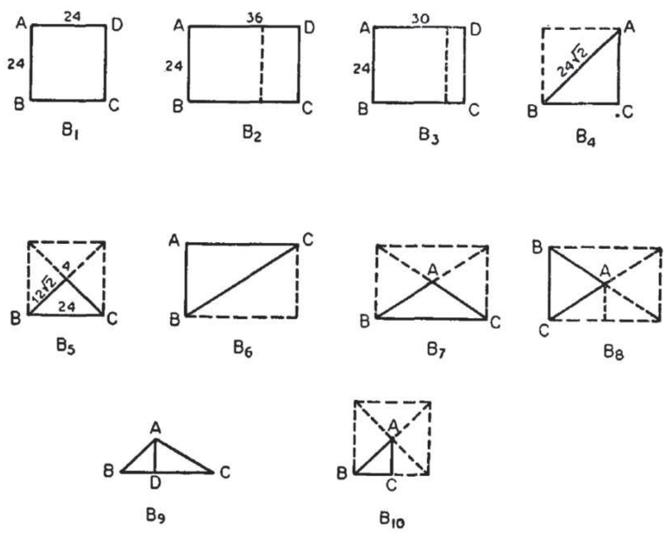
Arrangement of bricks in different parts of the fire-altar—second layer.

Parts of the *citi*	Brick type										Total
	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	
Head (including portion of body at junction, 12, 11)	1		1	2	1				4		9
Body (excluding portions at junctions with head, tail and wings, 4-10)	6	12	10	4		4	2	2			40
Wings (including junction with body and *patras*)		50	10		4	30	26	4	4		128
Tail (including junction with body, 1-3)	4	5		6	6					2	23
Total :	11	67	21	12	11	34	28	6	8	2	200



Arrangement of bricks in the *śyenacit*—second type, second layer.

Bodhayana Sulba Sutra…..continued



The ten brick types used in this construction (dimensions in *angulas*). Note that B4 to B10 are triangle bricks. In many cases we see that one or more smaller brick types combine to equal the area of a larger brick.

Cube Root (Ghana-mūla)

If a number has 'n' digits, the number of digits of the cube of that number will be > 3n – 2 and < 3n.

- ✓ An algorithm for computation of cube root was given for the first time by Āryabhaṭa.
- ✓ Brāhma-sphuṭa-Siddhānta, gives a good description of calculations with positive and negative numbers, Zero, fractions and surds.

Bakshali Method

Bakshali manuscripts (estimated to be written sometime during 300–600 CE), discusses several unique mathematical issues, including that of finding the square root of an imperfect square. Any imperfect square N may be expressed as $\sqrt{A^2+b}$. According to Bakshali Manuscript, the square root of the number N may be expressed as:

$$\sqrt{N}=\sqrt{A^2+b}\approx A+\frac{b}{2A}-\frac{\left(\frac{b}{2A}\right)^2}{2\left(A+\frac{b}{2A}\right)}$$

Approximations to the Square Root of a Non-square Number

Śrīdhara (850 CE) in his Triśatikā, has explained how Āryabhaṭa method can be used to get better approximations to the square root of a non-square number. For instance, if D is a non-square number, we can use the expression below to calculate $\sqrt{D}$ to any desired accuracy:

$$\sqrt{D}=\frac{\sqrt{D*10^{2n}}}{10^n}$$

Arithmetic Progression

Camaka-praśna (Taittirīya-saṃhitā 4.5.11)

एका च मे तिस्रश्च मे पञ्च च मे सप्त च मे नव च म एकादश च मे त्रयोदश च मे पञ्चदश च मे सप्तदश च मे नवदश च म एकविँशतिश्च मे त्रयोविँशतिश्च मे पञ्चविँशतिश्च मे सप्तविँशतिश्च मे नवविँशतिश्च म एकत्रिँशच्च मे त्रयस्त्रिँशच्च मे

ekā ca me tisraśca me pañca ca me sapta ca me nava ca ma ekādaśa ca me trayodaśa ca me pañcadaśa ca me saptadaśa ca me navadaśa ca ma ekaviँśatiśca me trayoviँśatiśca me pañcaviँśatiśca me saptaviँśatiśca me navaviँśatiśca ma ekatriँśacca me trayastriँśacca me

Camaka Prasna which is part of Gayatri Samhita gives arithmetic progression of **odd numbers starting from 1 to 33**.

Camaka-praśna (Taittirīya-saṃhitā 4.5.11)

चतस्रश्च मेऽष्टौ च मे द्वादश च मे षोडश च मे विँशतिश्च मे चतुर्विँशतिश्च मेऽष्टा विँशतिश्च मे द्वात्रिँशच्च मे षटत्रिँशच्च मे चत्वारिँशच्च मे चतुश्चत्वारिँशच्च मेऽष्टाचत्वारिँशच्च मे ...

catasraśca me'ṣṭau ca me dvādaśa ca me ṣoḍaśa ca me viँśatiśca me caturviँśatiśca me'ṣṭāviँśatiśca me dvaātriँśacca me ṣaṭtriँśacca me catvariँśacca me catuścatvāriँśacca me'ṣṭācatvāriँśacca me ...

Camaka Prasna which is part of Gayatri Samhita gives arithmetic progression of **even numbers starting from 4 to 48.**

Vājasaneyī-saṃhitā:

- Yugma ("even") series: 4, 8, 12, 16, 48
- Ayugma ("odd") series: 1, 3, 5, 7..... 31

The Pañcavimśa-brāhmaṇa has geometric series:

- 12, 24, 48, 96, 196608, ..., 393216

Āryabhaṭa I (499 CE), and Brahmagupta (628 CE)

- Cases of the sums of the sums.
- The squares and the cubes of the natural numbers.

Mahāvīra (850 CE)

- Rule for the summation of an interesting geometric series.

Nārāyaṇa (1356 CE)

- Method for repeated summation of partial series.

Arithmetic Series and Progressions

The term **upaciti or citi** is used to indicate a series in general.

For example, the series 1 + 2 + 3 + . . . + n, which starts with '1' and has a common difference of '1' is called **एकोत्तराद्युपचिति:** (ekottarādi-upacitiḥ).

Let us consider the case of the following the series of natural numbers up to n terms:

1, (1 + 2), (1 + 2 + 3), . . ., (1 + 2 + 3 + . . . + n).

The term **चितिघन:**(citighanaḥ) is used to denote the sum of this series.

Arithmetic Series and Progressions

Āryabhaṭa gives the formula for computing the sum in the following verse:

> एकोत्तराद्युपचितेर्गच्छाद्यकोत्तरत्रिसंवर्गः ।
> षड्भक्तः स चितिघनः सैकपदघनो विमूलो वा ॥ २१ ॥
> ekottarādy-upaciter-gacchādyakottara-trisaṃvargaḥ |
> ṣaḍbhaktaḥ sa citighanaḥ saikapada-ghano vimūlo vā || Gaṇita-pāda 21 ||

Of the series **(upaciti)** which begins with the term '1' and has a common difference '1', **(ekottarādi),** take three terms in continuation, of which the first is equal to the given number of terms, and find their continued product **(gacchādyakottara-trisaṃvargaḥ).**

That (product) divided by 6 gives the **citighana (ṣaḍbhaktaḥ sa citighanaḥ)**.

Alternatively **(vā),** it can be obtained by the number of terms plus one subtracted from the cube of that **(saikapada-ghano vimūlo)**, divided by 6 **(ṣaḍbhaktaḥ)**.

This can be expressed in the following notation:

$$1+(1+2)+(1+2+3)+\cdots+(1+2+3+\cdots+n)=\frac{n(n+1)(n+2)}{6} \text{ or } \frac{(n+1)^3-(n+1)}{6}$$

Sum of the Series of Squares and Sum of Series of Cubes

$$\sum N^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2$$

Āryabhaṭa designated ΣN2 as वर्गचितिघनः (varga-citighanaḥ)

$$\sum N^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3$$

ΣN3 as घनचितिघनः (ghana-citighanaḥ).

सैक-सगच्छ-पदानां क्रमात् त्रिसंवर्गितस्य षष्ठोंऽशः ।
वर्गचितिघनः स भवेत् चितिवर्गो घनचितिघनश्च ॥ २२ ॥
saika-sagaccha-padānāṃ kramāt trisaṃvargitasya ṣaṣṭhoṃ'śaḥ |
varga-citighanaḥ sa bhavet , citivargo ghana-citighanaśca | | Gaṇita-pāda 22 | |

Sum of the Series of Squares

The product of the three quantities (**trisaṃvargitasya**),

1. The number of terms (n)
2. The number of terms plus one (n + 1), and
3. The same increased by the number of terms (n + 1 + n) (**saika-sagaccha-padānāṃ kramāt**),

When divided by 6 (**ṣaṣṭhoṃ'śaḥ**)
Gives the sum of the series of squares of natural numbers (**varga-citighanaḥ sa bhavet**).

$$\sum N^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of the Series of Cubes

The square of the sum of the series of natural numbers (**citivargo ghana**) gives the sum of the series of cubes of natural numbers (**ghana-citighanaśca**).

$$\sum N^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Repeated Summation of Series (Vārasaṅkalita)

- Saṅkalita (sum of a series) for the first n natural numbers.
- A more general case of the repeated summation of such series was provided by Nārāyaṇa.
- Vāra means repeated (or again).
- Therefore, this method of repeated summation of a series can be designated as vārasaṅkalita.
- Let the symbol n_{V_1} denote the arithmetic series of the first 'n' natural numbers. This can be expressed as: nV1 = 1 + 2 + 3 + … + n.
- Similarly, let n_{V_2} denote the series of the partial sums of the series nV1.
- If we use 'r' to denote the number of terms up to which we want to sum up, then we can create partial sums of n_{V_1}.

For n = 1, 1_{V_1} = 1; n = 2, 2_{V_1} = 1 + 2; n = 3, 3_{V_1} = 1 + 2 + 3; and so on.

Using this we can write the second order sum of partial series as:

$$^{n}V_2 = \sum_{r=1}^{r=n} {}^{r}V_1$$

$$^{n}V_2 = 1 + (1 + 2) + (1 + 2 + 3) + \ldots + (1 + 2 + 3 + \ldots + n)$$

Similarly one can write the other sums of the partial series for the higher order in the following fashion: $^{n}V_3 = \sum_{r=1}^{r=n} {}^{r}V_2$

In general, the mth order sum of partial series of number can be represented as: $^{n}V_m = \sum_{r=1}^{r=n} {}^{r}V_{m-1}$

Nārāyaṇa provided a formula for the computation of the *m*th order *vāra-saṅkalita*. It denotes in this case the operation of forming a new series by taking the sums of the previous series. He provided an expression to calculate the sum using the following verse:

एकाधिकवारमिताः पदादिरूपोत्तरा पृथक् तेंऽशाः ।
एकाद्येकचयहरास्तद्घातो वारसङ्कलितम् ॥
ekādhika-vāramitāḥ padādi-rūpottarā pṛthak teṃ'śā: |
ekādy-ekacayaharās-tadghāto vārasaṅkalitam ||

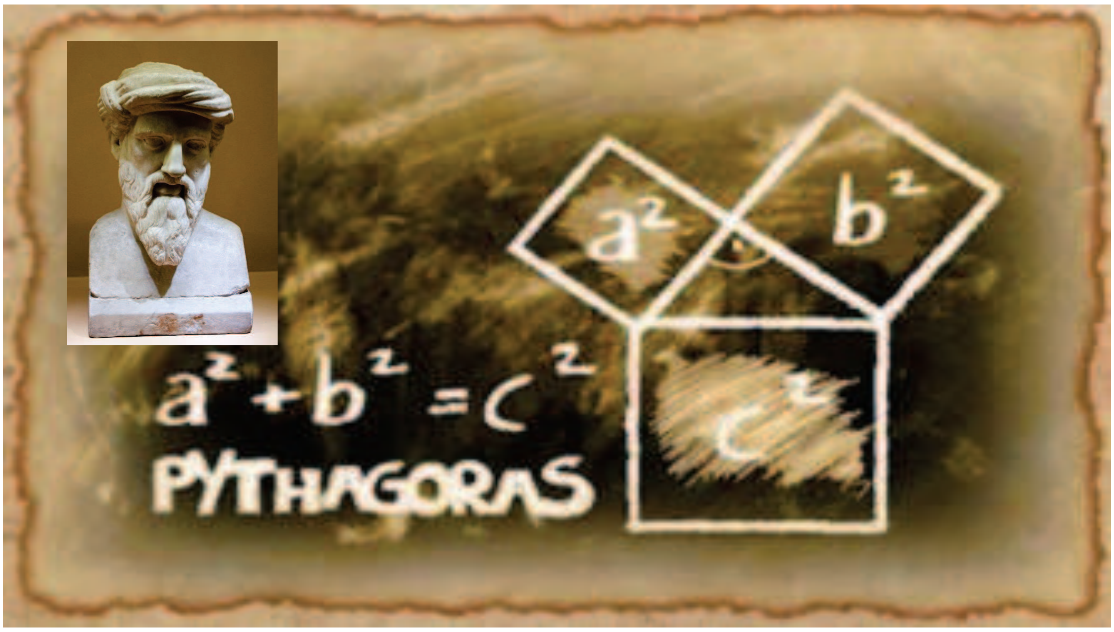
- Āryabhaṭīyam gives a good indication of the knowledge of algorithmic approach and use of recursive algorithms for problem-solving.
- The birth of Indian geometry could be traced to the Vedic time.

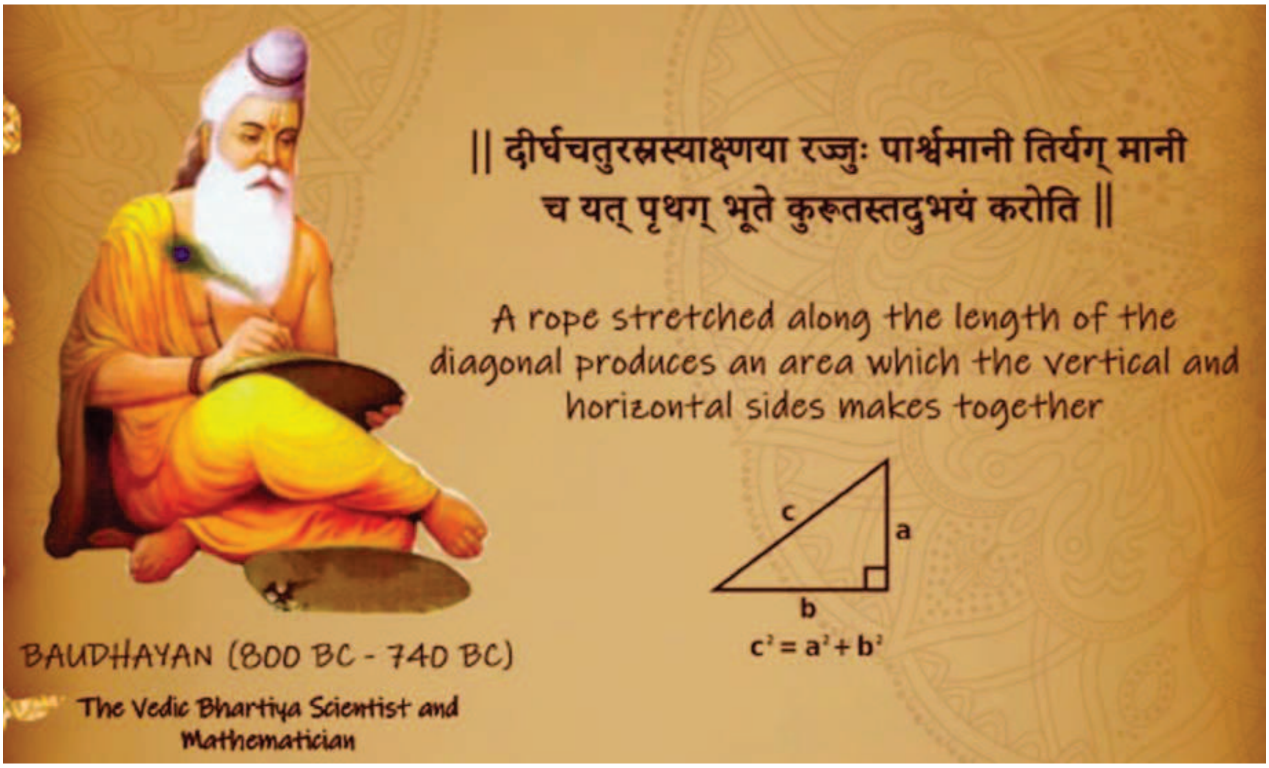
The terms of the sequence beginning with the pada (number of terms, i.e. *n*) and increasing by 1 (पदादिरूपोत्तराः) taken up to *(m+1)* times (*ekādhika-vāramitāḥ*) are successively the numerators (*pṛthak teṃ'śā:*) and the terms of the sequence beginning with unity and increasing by 1 (*ekādy-ekacayaharāḥ*) are respectively the denominators. The continued products of these (fractions) (*tadghātaḥ*) gives the vāra-saṅkalita (*vāra-saṅkalitam*). According to the above, since *n* is the number of terms and *m* the order, we get the following sequence of fractions:

$$\frac{n}{1}, \frac{n+1}{2}, \frac{n+2}{3}, \ldots, \frac{n+m}{m+1}$$

and the sum of the series is the product of this sequence of fractions, given by: $^{n}V_m = \frac{n(n+1)(n+2)\ldots(n+m)}{1.2.3\ldots(m+1)}$

Geometry


a^2
b^2
c^2
$a^2+b^2=c^2$
PYTHAGORAS


|| दीर्घचतुरस्रस्याक्ष्णया रज्जुः पार्श्वमानी तिर्यग् मानी
च यत् पृथग् भूते कुरुतस्तदुभयं करोति ||
A rope stretched along the length of the diagonal produces an area which the vertical and horizontal sides makes together
c
a
b
$c^2 = a^2 + b^2$
BAUDHAYAN (800 BC - 740 BC)
The Vedic Bhartiya Scientist and Mathematician

Geometry

Property of Right-Angled Triangle in Śulba-sūtras

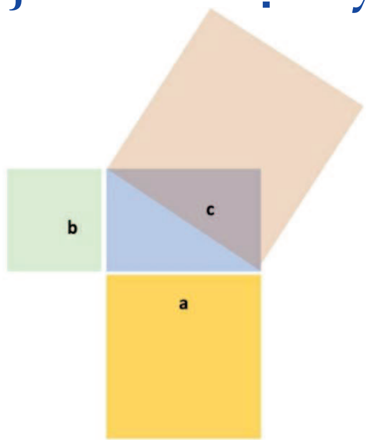
- Pythagoras theorem is actually found in the Śulba-sūtras.
- Theorem was known to Indians 1000 years before Pythagoas.

Bhuja-koti-karṇa-nyāya

Baudhāyana-śulba-sūtra:

"*The sum of the areas of the squares formed by the length and breadth of a rectangle equals the area produced by the diagonal of the rectangle*".

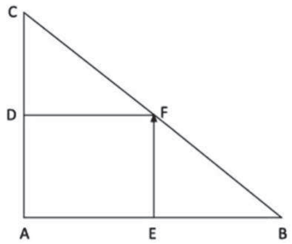
Let us consider a rectangle of length a and breadth b.
Let the diagonal of the rectangle be c.
According to the Baudhāyana formula, $a^2 + b^2 = c^2$.



Baudhāyana Formula for Right-angled Triangle

The shadow problem

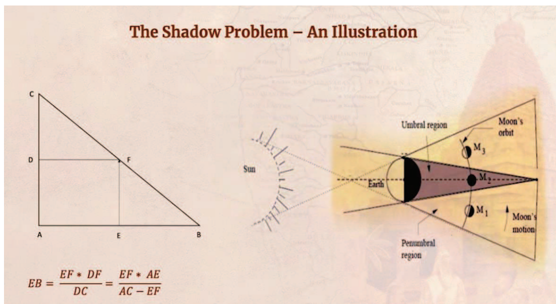
शङ्कुगुणं शङ्कुभुजाविवरं शङ्कुभुजयोर्विशेषहृतम् ।
यल्लब्धं सा छाया ज्ञेया शङ्कोः स्वमूलाद्धि ॥
śaṅku-guṇaṃ śaṅku-bhujāvivaraṃ śaṅku-bhujayor-viśeṣahṛtam |
yal-labdhaṃ sā chāyā jñeyā śaṅkoḥ sva-mūlāddhi ||

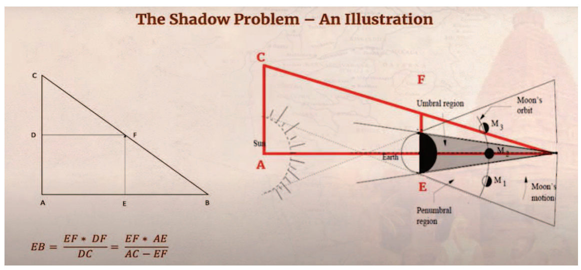


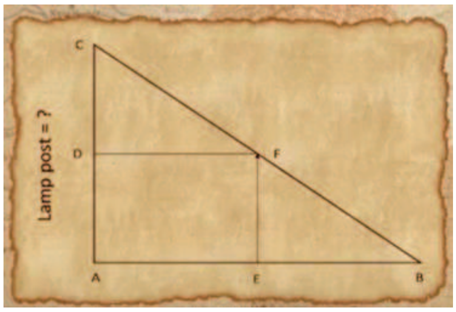
Triangles FEB and CDF are similar.

Therefore, $EB = \frac{EF * DF}{DC} = \frac{EF * AE}{AC - EF}$

EF is the half diameter of the earth (śaṅku)
AC is the half diameter of the sun (lamp post)
AE is the distance between sun and earth
EB is the earth's shadow (to be determined)





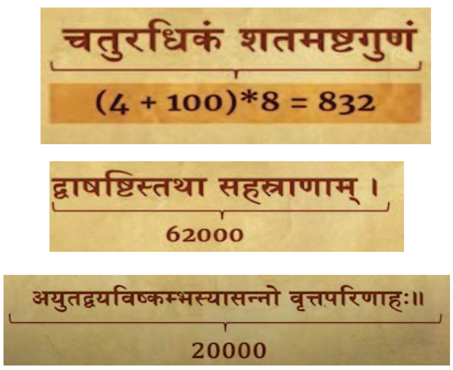


π

Āryabhaṭīyam (Gaṇitapāda, 2.10):

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।
अयुतद्वयविष्कम्भस्य आसन्नो वृत्तपरिणाहः ॥

caturadhikaṃ śatam-aṣṭaguṇaṃ dvāṣaṣṭis-tathā sahasrāṇām |
ayuta-dvaya-viṣkambhasya āsanno vṛttapariṇāhaḥ ||



This verse compute value of π to be:

$$\pi = \frac{(100+4)\times 8 + 62000}{20000} = \frac{62832}{20000} = 3.1416$$

π

Bhāskarācārya in his work Līlāvatī, (verse 199), derives the value of π using a different approach and arrives at the same value. According to this,

$$\pi = \frac{3927}{1250} = 3.1416$$

Several Infinite Series for π

Discovered by : Mādhava (14th century CE)

Re-discovered in Europe by:

- ✓ James Gregory (1671)
- ✓ Gottfried Leibniz (1674)
- ✓ Abraham Sharp (1699)

π

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

$$\frac{\pi}{\sqrt{12}} = 1 - \frac{1}{3.3} + \frac{1}{3^2 \times 5} - \frac{1}{3^3 \times 7} + \cdots$$

$$\frac{\pi}{4} = \frac{3}{4} + \frac{1}{(3^3 - 3)} - \frac{1}{(5^3 - 5)} + \frac{1}{(7^3 - 7)} - \cdots$$

$$\frac{\pi}{16} = \frac{1}{1^5 + 4.1} - \frac{1}{3^5 + 4.3} + \frac{1}{5^5 + 4.5} - \cdots$$

Bhūta-saṃkhyā system – An example
Mādhavācārya's approximation to π

Apart from giving an exact infinite series for π, Mādhava also gave a technique finding better approximations for π by using suitable end correction terms in these series.

In this way Mādhava estimated the value of π accurate to eleven decimal places.

Mādhava's verse uses the bhūta-saṃkhyā system to describe a large number:

विबुध-नेत्र-गज-अहि-हुताशन-त्रि-गुण-वेद-भवारण-बाहवः ।
नव-निखर्व-मिते वृत्तिविस्तरे परिधिमानमिदं जगदुर्बुधाः ॥

vibudha-netra-gaja-ahi-hutāśana-tri-guṇa-veda-bhavāraṇa-bāhavaḥ |
nava-nikharva-mite vṛttivistare paridhimānam-idaṃ jagadurbudhāḥ ||

In this verse, vibudha means devas, who are 33 in number. Using the bhūta-saṃkhyā system, we can compute π as follows:

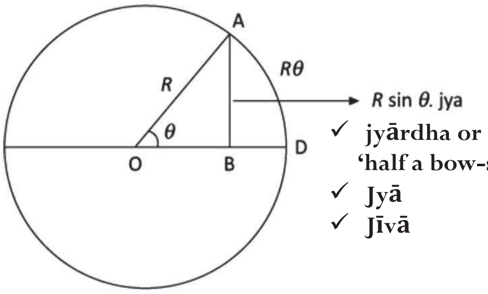
$$\pi = \frac{2827433388233}{9*10^{11}} = 3.141592653592\ldots$$

History of Approximations to π by Indian Mathematicians

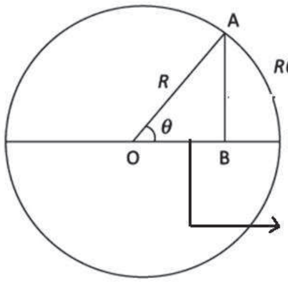
	Value of π	Accuracy (Decimal places)	Method
Śulba-sūtras (around 800 BCE)	3.08888	1	Geometrical
Jaina texts (500 BCE)	$\sqrt{10} = 3.1623$	1	Geometrical
Āryabhaṭa (४९९ CE)	$\frac{62832}{20000} = 3.1416$	4	Polygon doubling ($4.2^8 = 1024$ sides)
Bhāskarācārya (Līlāvatī)	3927/1250 = 3.1416	4	Polygon doubling
Mādhava (1375 CE)	$\frac{2827433388233}{9*10^{11}} = 3.141592653592\ldots$	11	Infinite series with end corrections
Ramanujan (1914 CE)		17 million	Modular equation

Trigonometry

Trigonometry is called *jyotpatti*, the science of computation of chords in Indian mathematics.



- ✓ jyārdha or 'half a bow-string'
- ✓ Jyā
- ✓ Jīvā



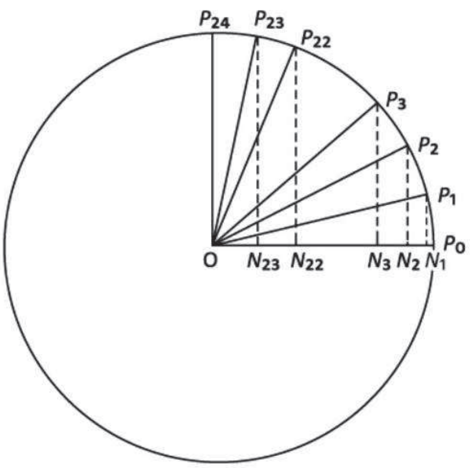
R cos θ

- ✓ Koṭijyā
- ✓ Kojyā
- ✓ Cojyā

DA = Rθ, is an arc.
AB = R sin θ,
OB = R cos θ

R is taken to be , $\frac{21600}{2\pi} = 3438$ where the circumference of the circle is 21,600 units (number of 'minutes' in a radian).

Computation of the 'R sines'



Arcs of Quadrant of a Circle

- Let this quadrant be divided into 24 parts (P0, P1, P2, …, P24).
- Each arc measures $\frac{90}{24} = 3^0 45'$
- We are interested in the chord lengths PiNi, which is R sin iθ.
- If we see the triangle P_2N_2O, the chord length P_2N_2 is the sine of the right-angle triangle of angle θ_2.
- For any chord in between these 24 chords, 1st order or 2nd order interpolation is used to get the intermediate values.

Āryabhaṭa has provided two methods to derive the sine tables.

1. geometric method
2. analytical method (which resembles the discrete version of the harmonic equation as we know today.)

Āryabhaṭa's Formula for R sine Differences
(*Gaṇita-pāda verse 12*)

प्रथमाच्चापज्यार्धाद्यैरूनं खण्डितं द्वितीयार्धम् ।
तत्प्रथमज्यार्धांशैस्तैस्तैरूनानि शेषाणि ॥

prathamāccāpajyārdhādyairūnaṃ khaṇḍitaṃ dvitīyārdham |
tatprathamajyārdhāṃśaistaistairūnāni śeṣāṇi ||

"*The first R sine divided by itself and then diminished by the quotient will give the second difference. For computing any other difference. The sum of all the preceding differences is divided by the first R sine and the quotient is subtracted from the preceding difference. Thus, all the remaining differences (can be calculated).*"

Let $R_1, R_2, R_3, \ldots, R_{24}$, denote the 24 R sines and δ_1 ($=R_1$), $\delta_2, \delta_3, \delta_4, \ldots, \delta_{24}$ denote the 24 R sine-differences. Then, according to the above verse,

$$\delta_2 = R_1 - \frac{R_1}{R_1}$$

$$\delta_{n+1} = \delta_n - \frac{\delta_1 + \delta_2 + \delta_3 + \cdots + \delta_n}{R_1} = \delta_n - \frac{R_n}{R_1}$$

Nīlakaṇṭha's Formula for R sine Differences

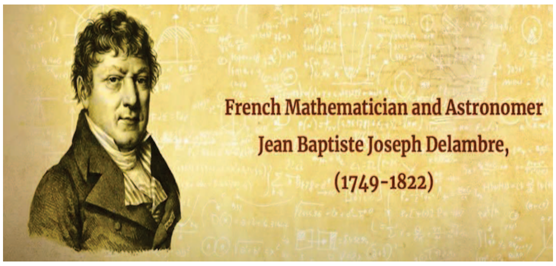
Nīlakaṇṭha (1500 CE) provided a much accurate estimate of R sine differences by providing a correction to the formula.

According to him, "*the first R sine divided by itself and then diminished by the quotient gives the second R sine difference. To obtain the other R sine differences, divide the preceding R sine by the first R sine and multiply the quotient by the difference between the first and second R sine differences and subtract the resulting product form the preceding R sine difference*".

It is stated mathematically below: $\delta_{n+1} = \delta_n - \left(\frac{R_n}{R_1}\right)(\delta_1 - \delta_2)$

This is the same as the relation:

$$2 \sin nx - \sin\{(n+1)x\} - \sin\{(n-1)x\} = (\sin nx/\sin x)(2 \sin x - \sin 2x)$$



"the method is curious: it indicates a method of calculating the table of sines using their second differences . . . , the differential process has not up to now been employed except for Briggs, who himself did not know that the constant factor was the square of the chord. Here then is a method, the Indians possessed, and which is found neither among Greeks not amongst the Arabs".

Āryabhaṭa's sine difference table

मखि भकि फखि धखि णखि ञखि ङखि हस्झ स्ककि किष्ग श्घकि किघ्व ।
घ्लकि किग्र हक्य धकि किच स्ग झश ङ्व क्ल प्त फ छ कलार्धज्याः ॥

makhi bhaki phakhi dhakhi ṇakhi ñakhi ṅakhi hasjha skaki kiṣga śghaki kighva |
ghlaki kigra hakya dhaki kica sga jhaśa ṅva kla pta pha cha kalārdhajyāḥ ||

Rsine differences: 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, 7.

Algebra

The ancient Indian name for the science of algebra is **bīja-gaṇita.**
Bīja means 'element' or 'analysis' and gaṇita 'the science of calculation.'

Āryabhaṭa in his work Āryabhaṭīyam (Gaṇitapāda, Verse 24), discusses an interesting case of how to determine the two numbers whose product and difference are known. The procedure explained in the verse can be algebraically stated as follows.

Let us consider two numbers x and y. Let $x - y = a$ and $xy = b$.

Then as per the verse, x and y can be determined as follows:

$$x = \frac{\sqrt{4b+a^2}+a}{2};\ y = \frac{\sqrt{4b+a^2}-a}{2}$$

Binary Mathematics and Combinatorial Problems in Chandaḥ-śāstra of Piṅgala (300 BCE)

Syllables

- Laghu (Short Syllable)
- Guru (Long Syllable)

Syllabic metre (varṇa-vṛtta) is characterized as a sequence of L and G

Replacing Laghu with "1" & Guru with "0"

Piṅgala in his work on Chandaḥ-śāstra dating back to the 2nd century BCE defined eight groups of binary numbers each of word length 3 (equivalent to what is now known in computer science as De Bruijn sequence).

Binary Mathematics and Combinatorial Problems in Chandaḥ-śāstra of Piṅgala (300 BCE)

Prastāra: A procedure by which all possible metrical patterns or, equivalently, binary sequences of a given length are generated sequentially as an array (prastāra).

Saṃkhyā: The process of finding the total number of binary sequences (rows) in the prastāra (array).

Naṣṭa: For a given row number in an array, the process of identifying the corresponding binary sequence directly.

Uddiṣṭa: Given a binary sequence, the process of identifying the corresponding row number in the array (prastāra), directly.

Lagakriyā: The process of finding the number of binary sequences in the array with a given number of '1's or '0's.

Adhvayoga: The process of finding the space occupied by the array (prastāra) (to determine the floor area needed).

Prastāra:

A procedure by which all possible metrical patterns or, equivalently, binary sequences of a given length are generated sequentially as an array (prastāra)

1	0
2	1

1	0	0
2	1	0
3	0	1
4	1	1

1	0	0	0
2	1	0	0
3	0	1	0
4	1	1	0
5	0	0	1
6	1	0	1
7	0	1	1
8	1	1	1

1	0	0	0	0
2	1	0	0	0
3	0	1	0	0
4	1	1	0	0
5	0	0	1	0
6	1	0	1	0
7	0	1	1	0
8	1	1	1	0
9	0	0	0	1
10	1	0	0	1
11	0	1	0	1
12	1	1	0	1
13	0	0	1	1
14	1	0	1	1
15	0	1	1	1
16	1	1	1	1

Saṅkhyā:

The process of finding the total number of binary sequences(rows) in the prastāra (array)

1	0	0	0	0
2	1	0	0	0
3	0	1	0	0
4	1	1	0	0
5	0	0	1	0
6	1	0	1	0
7	0	1	1	0
8	1	1	1	0
9	0	0	0	1
10	1	0	0	1
11	0	1	0	1
12	1	1	0	1
13	0	0	1	1
14	1	0	1	1
15	0	1	1	1
16	1	1	1	1

Naṣṭa :

For a given row number in an array, the process of identifying the corresponding binary sequence directly

1	0	
2	1	

1	0	0
2	1	0
3	0	1
4	1	1

1	0	0	0
2	1	0	0
3	0	1	0
4	1	1	0
5	0	0	1
6	1	0	1
7	0	1	1
8	1	1	1

1	0	0	0	0
2	1	0	0	0
3	0	1	0	0
4	1	1	0	0
5	0	0	1	0
6	1	0	1	0
7	0	1	1	0
8	1	1	1	0
9	0	0	0	1
10	1	0	0	1
11	0	1	0	1
12	1	1	0	1
13	0	0	1	1
14	1	0	1	1
15	0	1	1	1
16	1	1	1	1

Uddiṣṭa:

Given a binary sequence, the process of identifying the corresponding row number in the array (prastāra), directly

1 0 0 0

Lagakriyā:

The process of finding the number of binary sequences in the array with a given number of "1"s or "0"s

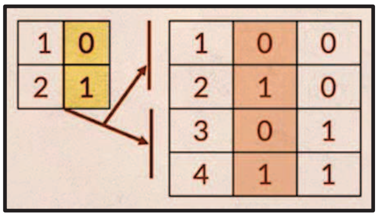
Adhvayoga:

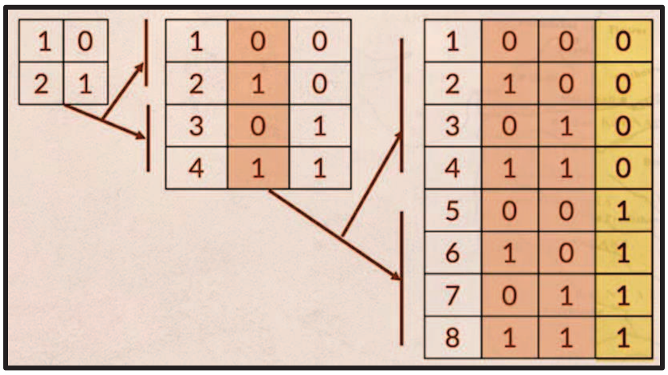
The process of finding the space occupied by the array (prastāra)

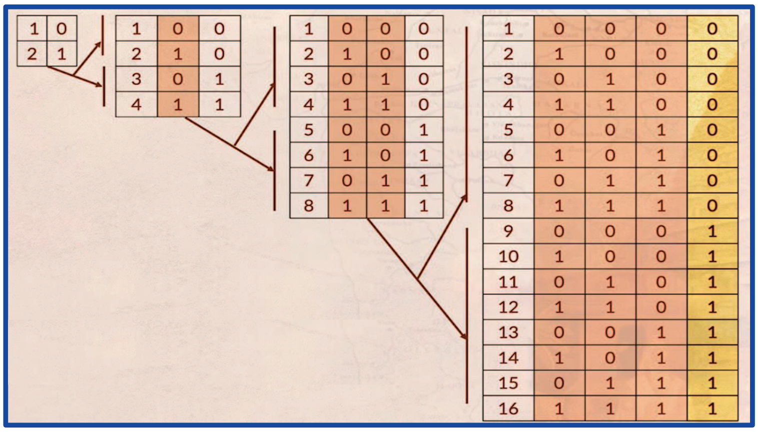
Progressive Generation of Binary Tables of Increasing Length

(Prastaras of Piṅgala)

The existing array is replicated at every iteration, followed by adding one more column to the replicated array.

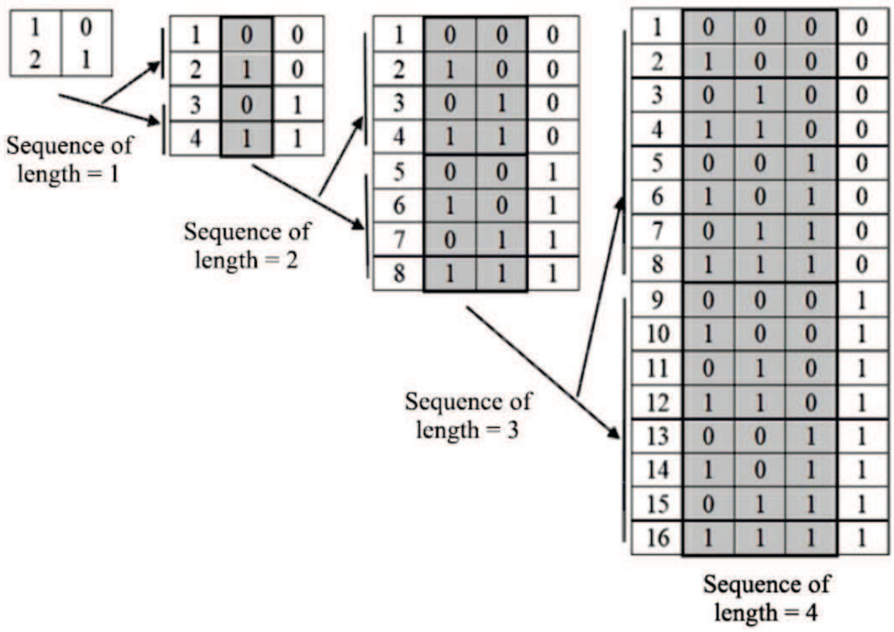






Progressive Generation of Binary Tables of Increasing Length

(Prastaras of Piṅgala)



Total number of rows in a array

(Saṃkhyā)

The number of binary sequences of length n is 2^n. This is called the Saṃkhyā or the total number of rows in the array (prastāra).

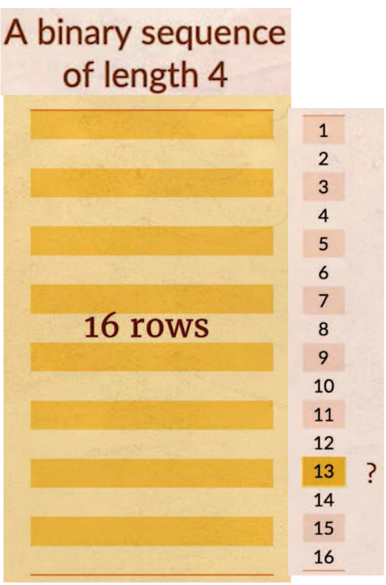
Row-number of a binary sequence =
Mirror image of the binary sequence (viewed as a binary number) +1

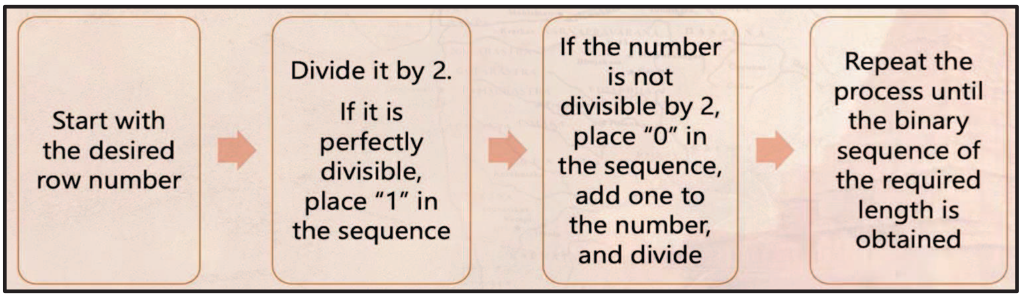
For example:

The fifth row in the array of sequences of length 3 is 001.
The mirror image of this is 100.
This, when viewed as binary number, is: $22.1 + 2.0 + 1.0 = 4$;
and $4 + 1 = 5$.

Naṣṭa:

Piṅgala's Algorithm: Binary sequence associated with a row-number





Binary sequence of Row 13 – "0011"

Uddiṣṭa:

Piṅgala's Algorithm:
Row-number associated with a Binary sequence

Start with number
1
Currnet number=1

Scanning the binary sequence from the right
Locate the first "1" in the sequence. Multiply the current number by 2.

If "1" is encountered, double the current number. If "0" is encountered double it and subtract 1 from it.
Move the next number to the left

Example: What is row number associated with binary sequence 0111 ?

Array of Length	**0**	**1**	**1**	**1**
First Number				1 X 2 = 2
Second Number			2 X 2 = 4	
Third Number		4 X 2 = 8		
Last Number	8 X 2 =16 16 – 1=**15**			

The row position is 15

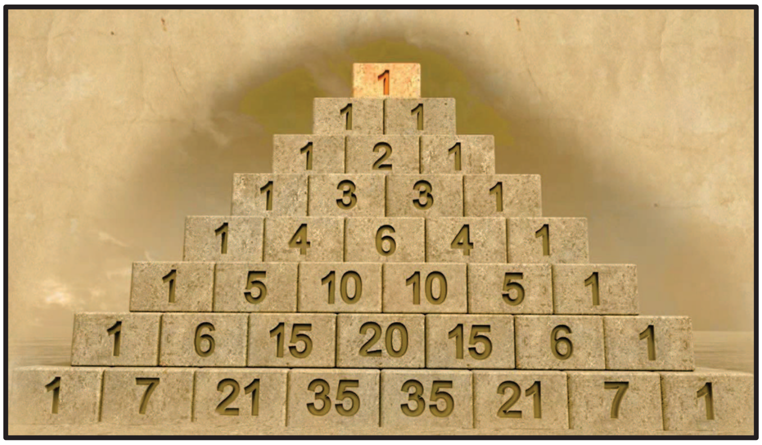
Combinatorial Problem

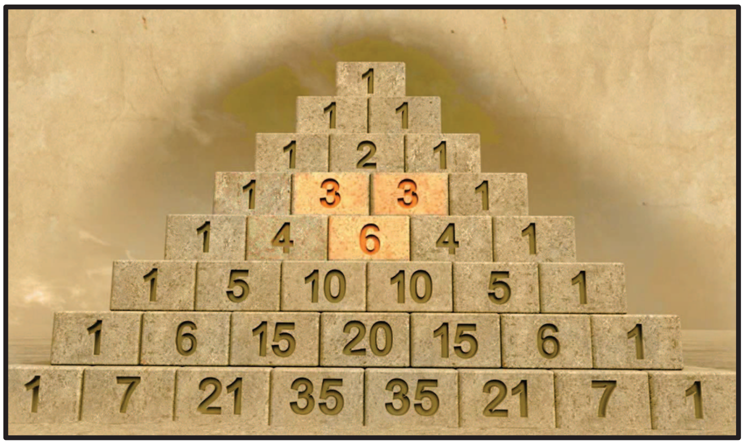
Another interesting problem is to *find out how many binary sequences of length 'n' there are, that contain 'r' number of '1's.*
Essentially, this boils down to the combinatorial problem of choosing 'r out of n', which leads to the binomial co-efficient n_{C_r}.
Piṅgala refers to this problem as **lagakriyā**. The procedure is best explained by the "Varṇa-Meru" of Piṅgala.

Varṇa Meru (due to Piṅgala)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

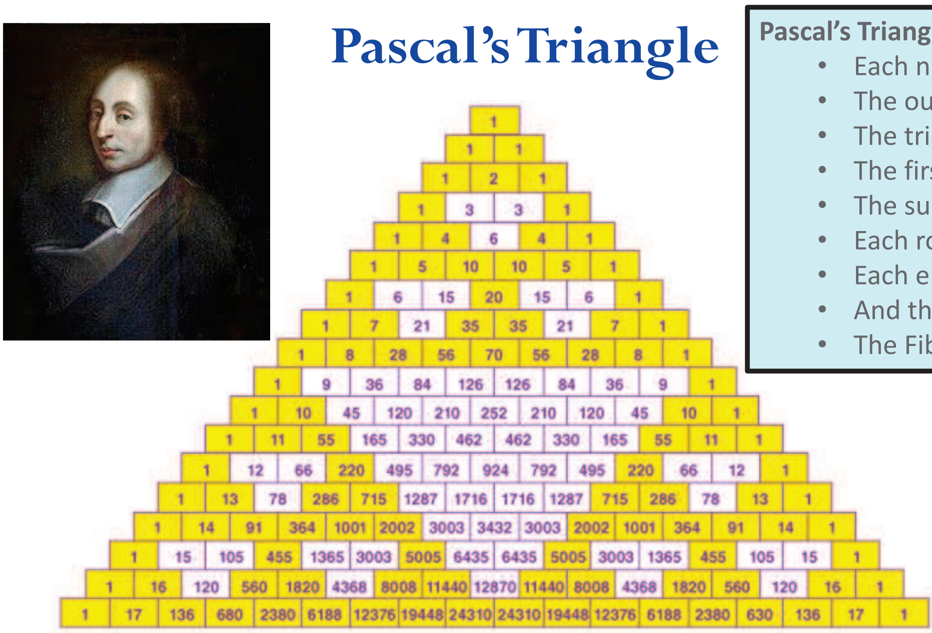
Lagakriyā: Piṅgala ‘s Varṇa Meru





French mathematician Blaise Pascal (1655 CE)

Pascal’s Triangle



Pascal’s Triangle Properties

- Each number is the sum of the two numbers above it.
- The outside numbers are all 1.
- The triangle is symmetric.
- The first diagonal shows the counting numbers.
- The sums of the rows give the powers of 2.
- Each row gives the digits of the powers of 11.
- Each entry is an appropriate “choose number.”
- And those are the “binomial coefficients.”
- The Fibonacci numbers are there along diagonals.

Varāhamihira's Bṛhatsaṃhitā (550 CE)

Varāhamihira's Bṛhatsaṃhitā (550 CE), mentions 1820 different combinations that can be obtained by choosing **4 perfumes from a set of 16 basic perfumes.** ($16_{C_4} = 1820$).
Varāhamihira also discusses the construction of a Meru – tabular form for arriving at this binomial coefficient.

Magic Squares in India

12	3	6	13
14	5	4	11
7	16	9	2
1	10	15	8

(a) Magic square

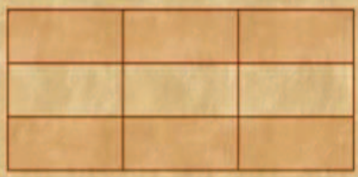
10	3	13	8
5	16	2	11
4	9	7	14
15	6	12	1

(b) Pan-diagonal magic square

Magic and Pan-diagonal Magic Square; Magic Sum = 34

Indian mathematicians specialized in the construction of a special class of magic squares called **sarvatobhadra** or **pan-diagonal magic squares**.
Bhadra-gaṇita is the name for the study of magic squares in the Indian mathematical tradition.

Magic Squares in India



Garga (100 BCE)

An Old Indian Tradition:

- ✓ Kakṣapuṭa of Nāgārjuna (100 CE)
- ✓ Varāramihira (587 CE)
- ✓ Jaina inscription (11th century CE)
- ✓ Gwalior fort (1480 CE)
- ✓ Srinivasa Ramanujan

Nāgārjuna's Scheme for Generating a 4 × 4 Pan-diagonal Magic Square with an Illustration for n = 50

Construction of a 4 × 4 Pan-diagonal Magic Square

Formula to construct generic magic square

"अर्क इन्दुनिधा नारी तेन लग्न विनासनम्"
(arka indunidhā nārī tena lagna vināsanam).

0	1	0	8
0	9	0	2
6	0	3	0
4	0	7	0

(a)

n-3	1	n-6	8
n-7	9	n-4	2
6	n-8	3	n-1
4	n-2	7	n-9

(b)

47	1	44	8
43	9	46	2
6	42	3	49
4	48	7	41

(c)

Nārāyaṇa classifies the magic squares into three categories:

a. if n = 4m, then it is called ***Samagarbha*** (double even squares)

b. if n = 4m + 2, then it is ***Viṣamagarbha***, and (semi-double even squares)

c. if n = 2m + 1, then it is ***Viṣama*** (odd squares), where m = 1, 2, 3, …

	3		
5	16	2	
	9		
			1

(a)

10	3	13	
5	16	2	
4	9	7	
			1

(b)

10	3	13	8
5	16	2	
4	9	7	14
15		12	1

(c)

10	3	13	8
5	16	2	11
4	9	7	14
15	6	12	1

(d)

384 pan-diagonal matric can be constructed

Construction of a 4 x 4 Pan-diagonal Magic Square

Properties are useful for constructing a 4 × 4 pan-diagonal magic square:

Property-1

- In a pan-diagonal 4 × 4 magic, the entries of any 2 × 2 sub-square formed by consecutive rows and columns add up to the magic

Property-2

- In a pan-diagonal 4 × 4 magic square the sum of an entry with another which is two squares away from it along a diagonal is always half the magic sum

Property-3

- In a 4 × 4 magic square with entries, 1, 2, …, 16, each element has the same set of neighbors in each of the 384 pan-diagonal versions. In particular, the element 16 has as neighbours 2, 3, 5, 9.

Using these properties, we can construct a 4 × 4 pan-diagonal matrix using numbers 1, 2, 3, …, 16 as follows:
Step 1: First we place 1 in any of the cells and place 16 two cells diagonally away from 1.

Step 2: Using property 3, we generate the first few elements of the 4 × 4 pan-diagonal magic square by placing 2, 3, 5, and 9 in any order as neighbours of 16 (Figure (a)).

Step 3: The magic sum for a 4 × 4 pan-diagonal magic square for numbers 1 to 16 is 34. We use the above properties to fill all the remaining cells in the magic square.

For example,

a. Using property 1 we fill in some of the adjacent cells as shown in Figure (b).
b. Using property 2 to fill in some more cells (highlighted cells in Figure (c)).
c. Finally using property 1, we fill the balance cells (highlighted cells in Figure (d)).

	3		
5	16	2	
	9		
			1

(a)

10	3	13	
5	16	2	
4	9	7	
			1

(b)

10	3	13	8
5	16	2	
4	9	7	14
15		12	1

(c)

10	3	13	8
5	16	2	11
4	9	7	14
15	6	12	1

(d)

Construction of a 4 x 4 pan-diagonal magic square (n = 34)

Property 3:
In a 4 x 4 magic square with entries, 1, 2, ... 16, each element has the same set of neighbours in each of the 384 pan-diagonal versions.
In particular, the element 16 has as neighbours 2, 3, 5, 9.

	3		
5	16	2	
	9		
			.

Property 2:
In a pan-diagonal 4 x 4 magic square the sum of an entry with another which is two squares away from it along a diagonal is always half the magic sum.

	3		
5	16	2	
	9		
			1

Property 1:
In a pan-diagonal 4 x 4 magic, the entries of any 2 x 2 sub-square formed by consecutive rows and columns add up to the magic

10	3	13	
5	16	2	
4	9	7	
			1

Property 2:
In a pan-diagonal 4 x 4 magic square the sum of an entry with another which is two squares away from it along a diagonal is always half the magic sum.

10	3	13	8
5	16	2	
4	9	7	14
15		12	1

Property 1:
In a pan-diagonal 4 x 4 magic, the entries of any 2 x 2 sub-square formed by consecutive rows and columns add up to the magic

10	3	13	8
5	16	2	11
4	9	7	14
15	6	12	1

આભાર شکریہ ধন্যবাদ ਤੁਹਾਡਾ ਧੰਨਵਾਦ நன்றி ధన్యవాదాలు
ಧನ್ಯವಾದ നന്ദി धन्यवाद આભાર شکریہ ধন্যবাদ ਤੁਹਾਡਾ ਧੰਨਵਾਦ
நன்றி ధన్యవాదాలు ಧನ್ಯವಾದ നന്ദിઆભાર شکریہ ধন্যবাদ ਤੁਹਾਡਾ
ਧੰਨਵਾਦ நன்றிధన్యవాదాలు ধন্যবাদ ಧನ್ಯವಾದനന്ദിઆભાર شکریہ

धन्यवाद

ধন্যবাদ ਤੁਹਾਡਾ ਧੰਨਵਾਦ நன்றிధన్యవాదాలు ಧನ್ಯವಾದനന്ദി
આભાર شکریہ ধন্যবাদ ਤੁਹਾਡਾ ਧੰਨਵਾਦ धन्यवाद

Thank you

நன்றிధన్యవాదాలు धन्यवाद ಧನ್ಯವಾದനന്ദി આભાર شکریہ
ধন্যবাদ ਤੁਹਾਡਾ ਧੰਨਵਾਦ நன்றிధన్యవాదాలు ಧನ್ಯವಾದനന്ദി
धन्यवाद ಧನ್ಯವಾದനന്ദി આભાર شکریہ ধন্যবাদ ਤੁਹਾਡਾ ਧੰਨਵਾਦ
நன்றிధన్యవాదాలు ಧನ್ಯವಾದനന്ദി આભાર ধন্যবাদ ਤੁਹਾਡਾ
ਧੰਨਵਾਦ நன்றிధన్యవాదాలు ধন্যবাদ ਤੁਹਾਡਾ ਧੰਨਵਾਦ
நன்றிధన్యవాదాలు