MODELING BIOLOGICAL SYSTEMS

INSTRUCTIONS TP-TD4

Nota bene :
These two exercises and those of the next Practical Tutorial (TD-TP5) will all be part of a **single report due on the 4th of December**. Please refer to the TD-TP4 instructions for the general rules of report presentation.

The following two exercises can be achieved by hand. **However, a numerical evolution of the population size in the report is required and will count for the final grade. This numerical analysis has to be done for each of the 4 models to check if your theoretical analysis is correct**. The choice of the parameters you implement will be discussed.

Exercise 1 :

Consider the following equation, describing a continuous-time logistic population growth model:

$$N'(t) = rN(t)\big(1 - \frac{N(t)}{K}\big)$$

A. Demonstrate that the following function is a solution of the logistic equation:

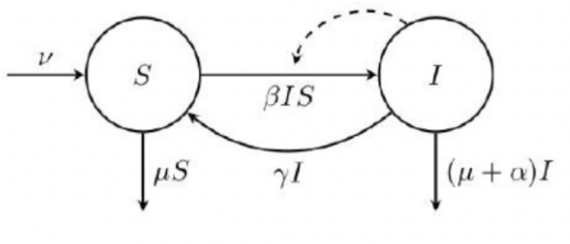
$$N(t) = N_0 \frac{Ke^{rt}}{K + N_0(e^{rt} - 1)}$$

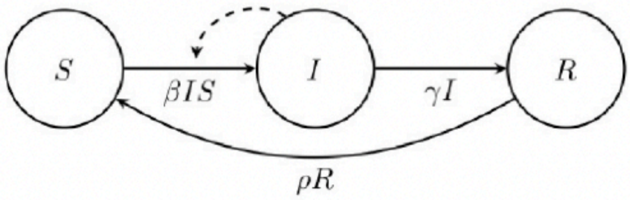
Facultative : can you demonstrate that this is the unique solution for $N(t = 0) = N_0$?

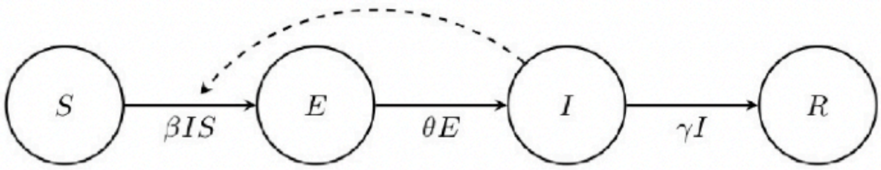
B. Check that this solution follows the equilibria and stability discussed in the lecture (*i.e.* prove that this function behaves like shown on the plots during the lecture).

Exercice 2 :

Consider the flow diagrams for the following epidemiological models:







For each of the three models:

A. Explain what the model is, what is happening and what the parameters might represent.

B. Write down the corresponding ODE (Ordinary Differential Equation) system.

C. Determine the equilibria of the system.

D. Guess how the system will behave (provide and use the value R_0).

>> Do not forget the graphics of the evolution of each population! <<