Assignments - IX

Definition 1 (Integer Partition) *Let n be a non-negative integer. A rank k partition of n is a k-tuple of positive integers $(\lambda_1, \lambda_2, \ldots, \lambda_k)$ satisfying*

$$n = \sum_{i=1}^{k} \lambda_i, \quad n \geq \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_k \geq 1. \tag{1}$$

The summands of the partition are known as parts. Hasse diagram is defined in terms of one partition is covered by another. Partition cover is formally defined as –

Definition 2 (Partition Cover) *If λ and λ' are two partitions of a positive integer n, then we say that λ is covered by λ', written as $\lambda \lessdot \lambda'$, if two summands of λ can be added to form λ'.*

Example 1 *Let two partitions of 7 be $\lambda = \langle 3, 2, 1, 1 \rangle$ and $\lambda' = \langle 5, 1, 1 \rangle$. λ is covered by λ' because two summands of λ, i.e., 2 and 3 is added to form λ'.*

Using the notion of the partition cover, Hasse diagram is defined as –

Definition 3 (Hasse Diagram) *Let $\mathcal{P}_n$ to be the set of all partitions of a positive integer n. We can organize $\mathcal{P}_n$ using a graphical representation called a Hasse diagram $\mathbb{G}_n$ in the following way. We create the graph $\mathbb{G}_n$ with vertices $V = \{\lambda : \lambda \in \mathcal{P}_n\}$ arranged in rows, according to rank. We then add edges*

$$E = \left\{ (\lambda, \lambda') : \lambda, \lambda' \in \mathcal{P}_n \text{ and either } \lambda \lessdot \lambda' \text{ or } \lambda' \lessdot \lambda \right\}$$

A node in a Hasse diagram $\mathbb{G}_n$ is represented as `Node` $= \langle$`nodeId`, `partition`$\rangle$.

- `nodeId`: It corresponds to the unique id assigned to each node. It is represented as `Node.nodeId`.
- `partition`: It represents the partition of an integer n corresponding to the node. It is denoted as `Node.partition`.

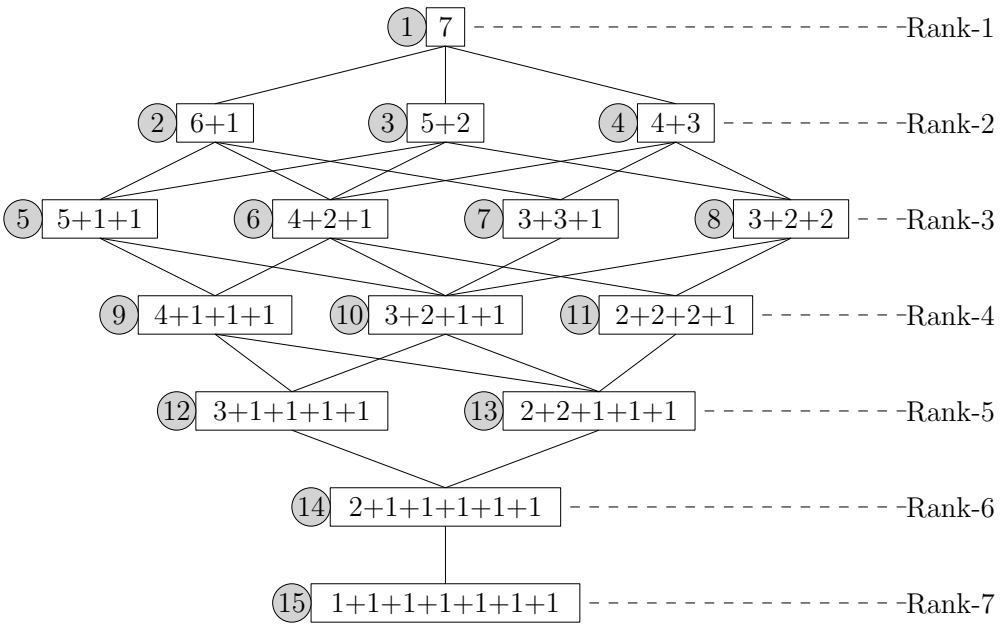

Figure 1: Hasse diagram for $\mathcal{P}_7$

The Hasse diagram $\mathbb{G}_7$ is shown in Figure 1. The `id` of each node is shown adjacent to it in the circle.

Write a program to generate the Hasse Diagram given a positive integer $n(1 \leq n \leq 50)$. Perform BFS and DFS on this Hasse Diagram.