Backward Propagation(BP) in Convolutional Neural Network(CNN)

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https://github.com/RipperLom/CNN_BackwardPropagation_PythonCoding

summary

Introduction - BP

Figura: Hinton

$dW = \frac{\partial L}{\partial W}$, $db = \frac{\partial L}{\partial b}$ (L: Lost function)

Introduction - CNN

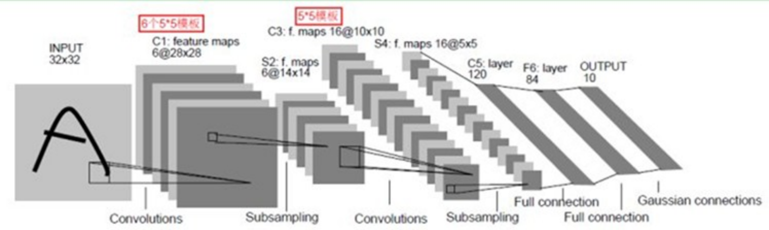

Figura: LeNet5

$$tf.nn.conv2d(input, filter, strides, padding)$$

Model - Structure

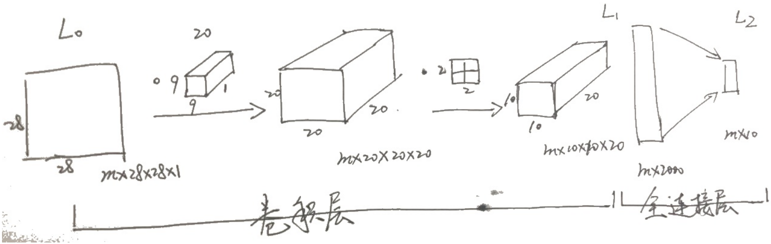

Figura: Structure

$W_{1(9,9,1,20)}$ $W_{2(2000,10)}$

$b_{1(1,20)}$ $b_{2(1,10)}$

Forward Propagation - Layer1

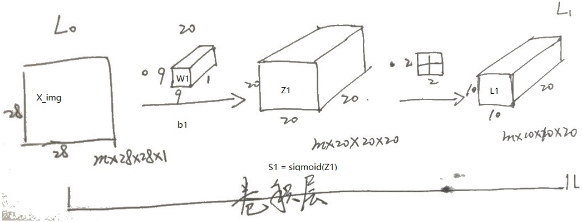

Figura: Layer1

$$
\begin{aligned}
Z_{1(m,20,20,20)} &= cov2d(X\_img_{(m,28,28,1)}, W_{1(9,9,1,20)}) + b_{1(1,20)} \\
S_{1(m,20,20,20)} &= sigmoid(Z_{1(m,20,20,20)}) \\
L_{1(m,10,10,20)} &= pool(S_{1(m,20,20,20)})
\end{aligned}
$$

Forward Propagation - Layer2

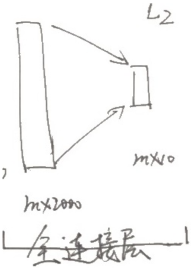

Figura: Layer2

$$L_1\_flat_{(m,2000)} = reshape(L_{1(m,10,10,20)})$$
$$Z_{2(m,10)} = L_1\_flat_{(m,2000)} .* W_{2(2000,10)} + b_{2(1,10)}$$
$$S_{2(m,10)} = sigmoid(Z_{2(m,10)})$$

Preparation - Kron

$$A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix}.$$

Figura: Kron

$$N_{(2k,2d)} = (kron(M_{(k,d)}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}))_{(2k,2d)}$$

Preparation - $\frac{\partial L}{\partial W}$

$$A_{(3,3)} \circlearrowright W_{(2,2)} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \circlearrowright \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix} =$$

$$\begin{pmatrix} a_{11}w_{11}+a_{12}w_{12}+a_{21}w_{21}+a_{22}w_{22} & a_{12}w_{11}+a_{13}w_{12}+a_{22}w_{21}+a_{23}w_{22} \\ a_{21}w_{11}+a_{22}w_{12}+a_{31}w_{21}+a_{32}w_{22} & a_{22}w_{11}+a_{23}w_{12}+a_{32}w_{21}+a_{33}w_{22} \end{pmatrix} =$$

$$\begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} = Z_{(2,2)}$$

$$\left(\frac{\partial L}{\partial Z}\right)_{(2,2)} = \begin{pmatrix} \left(\frac{\partial L}{\partial Z}\right)_{11} & \left(\frac{\partial L}{\partial Z}\right)_{12} \\ \left(\frac{\partial L}{\partial Z}\right)_{21} & \left(\frac{\partial L}{\partial Z}\right)_{22} \end{pmatrix} = \begin{pmatrix} \left(\frac{\partial L}{\partial Z_{11}}\right) & \left(\frac{\partial L}{\partial Z_{12}}\right) \\ \left(\frac{\partial L}{\partial Z_{21}}\right) & \left(\frac{\partial L}{\partial Z_{22}}\right) \end{pmatrix}$$

$$\left(\frac{\partial L}{\partial W}\right)_{(2,2)} = \begin{pmatrix} \left(\frac{\partial L}{\partial W_{11}}\right) & \left(\frac{\partial L}{\partial W_{12}}\right) \\ \left(\frac{\partial L}{\partial W_{21}}\right) & \left(\frac{\partial L}{\partial W_{22}}\right) \end{pmatrix} = \begin{pmatrix} \left(\frac{\partial L}{\partial Z}\right)\left(\frac{\partial Z}{\partial W_{11}}\right) & \left(\frac{\partial L}{\partial Z}\right)\left(\frac{\partial Z}{\partial W_{12}}\right) \\ \left(\frac{\partial L}{\partial Z}\right)\left(\frac{\partial Z}{\partial W_{21}}\right) & \left(\frac{\partial L}{\partial Z}\right)\left(\frac{\partial Z}{\partial W_{22}}\right) \end{pmatrix}$$

Preparation - $\frac{\partial L}{\partial W} = \frac{\partial L}{\partial Z}\frac{\partial Z}{\partial W}$

$$(\frac{\partial L}{\partial Z})(\frac{\partial Z}{\partial W_{11}}) = a_{11}(\frac{\partial L}{\partial Z_{11}}) + a_{12}(\frac{\partial L}{\partial Z_{12}}) + a_{21}(\frac{\partial L}{\partial Z_{21}}) + a_{22}(\frac{\partial L}{\partial Z_{22}})$$
$$(\frac{\partial L}{\partial Z})(\frac{\partial Z}{\partial W_{12}}) = a_{12}(\frac{\partial L}{\partial Z_{11}}) + a_{13}(\frac{\partial L}{\partial Z_{12}}) + a_{22}(\frac{\partial L}{\partial Z_{21}}) + a_{23}(\frac{\partial L}{\partial Z_{22}})$$
$$(\frac{\partial L}{\partial Z})(\frac{\partial Z}{\partial W_{21}}) = a_{21}(\frac{\partial L}{\partial Z_{11}}) + a_{22}(\frac{\partial L}{\partial Z_{12}}) + a_{31}(\frac{\partial L}{\partial Z_{21}}) + a_{32}(\frac{\partial L}{\partial Z_{22}})$$
$$(\frac{\partial L}{\partial Z})(\frac{\partial Z}{\partial W_{22}}) = a_{22}(\frac{\partial L}{\partial Z_{11}}) + a_{23}(\frac{\partial L}{\partial Z_{12}}) + a_{32}(\frac{\partial L}{\partial Z_{21}}) + a_{33}(\frac{\partial L}{\partial Z_{22}})$$

$$A_{(3,3)} \circlearrowleft (\frac{\partial L}{\partial Z})_{(2,2)} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \circlearrowleft \begin{pmatrix} (\frac{\partial L}{\partial Z_{11}}) & (\frac{\partial L}{\partial Z_{12}}) \\ (\frac{\partial L}{\partial Z_{21}}) & (\frac{\partial L}{\partial Z_{22}}) \end{pmatrix} =$$
$$\begin{pmatrix} (\frac{\partial L}{\partial Z})(\frac{\partial Z}{\partial W_{11}}) & (\frac{\partial L}{\partial Z})(\frac{\partial Z}{\partial W_{12}}) \\ (\frac{\partial L}{\partial Z})(\frac{\partial Z}{\partial W_{21}}) & (\frac{\partial L}{\partial Z})(\frac{\partial Z}{\partial W_{22}}) \end{pmatrix} = \begin{pmatrix} (\frac{\partial L}{\partial W_{11}}) & (\frac{\partial L}{\partial W_{12}}) \\ (\frac{\partial L}{\partial W_{21}}) & (\frac{\partial L}{\partial W_{22}}) \end{pmatrix} = (\frac{\partial L}{\partial W})_{(2,2)}$$

Backward Propagation - Layer2 - $\frac{\partial L}{\partial Z_2}_{(m,10)}$

$$L = -(Y * ln(S_2) + (1 - Y) * ln(1 - S_2))$$

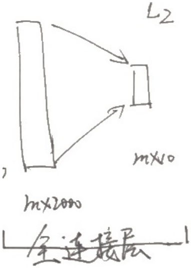

Figura: Layer2

$$\frac{\partial L}{\partial Z_2}_{(m,10)} = \frac{\partial L}{\partial S_2}\frac{\partial S_2}{\partial Z_2} = (-\frac{Y}{S_2} + \frac{1-Y}{1-S_2}) * [S_2 * (1 - S_2)] = (S_2 - Y)_{(m,10)}$$

Backward Propagation - Layer2 - $\frac{\partial L}{\partial W_2}_{(2000,10)}$

$$\frac{\partial L}{\partial Z_2}_{(m,10)} = \frac{\partial L}{\partial S_2}\frac{\partial S_2}{\partial Z_2} = \left(-\frac{Y}{S_2} + \frac{1-Y}{1-S_2}\right) * [S_2 * (1 - S_2)] = (S_2 - Y)_{(m,10)}$$

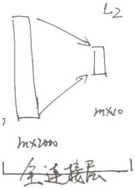

Figura: Layer2

$$\frac{\partial L}{\partial W_2}_{(2000,10)} = \frac{\partial L}{\partial Z_2}\frac{\partial Z_2}{\partial W_2} = (S_2 - Y)_{(m,10)} \Diamond (L_1\_flat)^T_{(2000,m)} = (1/m)(L_1\_flat)^T_{(2000,m)}(S_2 - Y)_{(m,10)}$$

Backward Propagation - Layer2 - $\frac{\partial L}{\partial b_2}_{(1,10)}$

$$\frac{\partial L}{\partial Z_2}_{(m,10)} = \frac{\partial L}{\partial S_2}\frac{\partial S_2}{\partial Z_2} = \left(-\frac{Y}{S_2}+\frac{1-Y}{1-S_2}\right)*[S_2*(1-S_2)] = (S_2-Y)_{(m,10)}$$

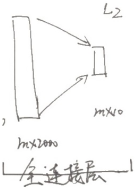

Figura: Layer2

$$\frac{\partial L}{\partial b_2}_{(1,10)} = \frac{\partial L}{\partial Z_2}\frac{\partial Z_2}{\partial b_2} = (S_2 - Y)_{(m,10)} \Diamond (\vec{1})^T{}_{(1,m)} = (1/m)\sum_{k=1}^{m}(s_2-y)_{(1,10)}$$

Backward Propagation - Layer2 - $\frac{\partial L}{\partial L_1\_flat}_{(m,2000)}$

$$\frac{\partial L}{\partial Z_2}_{(m,10)} = \frac{\partial L}{\partial S_2}\frac{\partial S_2}{\partial Z_2} = \left(-\frac{Y}{S_2}+\frac{1-Y}{1-S_2}\right)*[S_2*(1-S_2)] = (S_2 - Y)_{(m,10)}$$

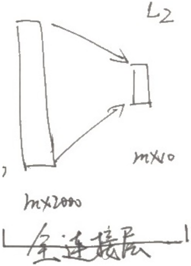

Figura: Layer2

$$\frac{\partial L}{\partial L_1\_flat}_{(m,2000)} = \frac{\partial L}{\partial Z_2}\frac{\partial Z_2}{\partial L_1\_flat} = (S_2 - Y)_{(m,10)}\,{W_2}^T{}_{(10,2000)}$$

Backward Propagation - Layer1 - $\frac{\partial L}{\partial L_1}_{(m,10,10,20)}$

$$\frac{\partial L}{\partial L_1\_flat}_{(m,2000)} = \frac{\partial L}{\partial Z_2}\frac{\partial Z_2}{\partial L_1\_flat} = (S_2 - Y)_{(m,10)} {W_2}^T{}_{(10,2000)}$$

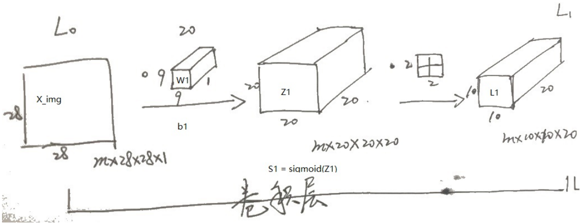

Figura: Layer2

$$\frac{\partial L}{\partial L_1}_{(m,10,10,20)} = reshape(\frac{\partial L}{\partial L_1\_flat}_{(m,2000)})$$

Backward Propagation - Layer1 - $\frac{\partial L}{\partial S_1}{}_{(m,20,20,20)}$

$$\frac{\partial L}{\partial L_1}{}_{(m,10,10,20)} = reshape(\frac{\partial L}{\partial L_{1\_flat}}{}_{(m,2000)})$$

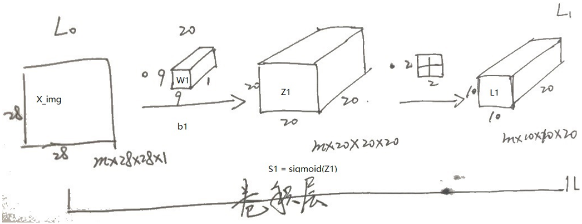

Figura: Layer2

$$\frac{\partial L}{\partial S_1}{}_{(m,20,20,20)} = \frac{1}{2*2} kron(\frac{\partial L}{\partial L_1}{}_{(m,10,10,20)}, np.ones([2,2]))$$

Backward Propagation - Layer1 - $\frac{\partial L}{\partial Z_1}{(m,20,20,20)}$

$$\frac{\partial L}{\partial S_1}{(m,20,20,20)} = \frac{1}{2*2} kron(\frac{\partial L}{\partial L_1}{(m,10,10,20)}, np.ones([2,2]))$$

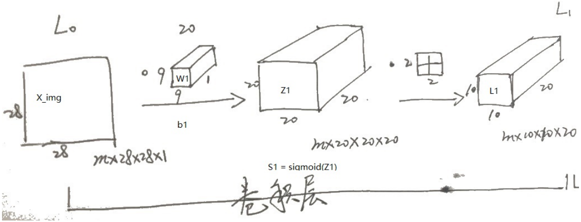

Figura: Layer2

$$\frac{\partial L}{\partial Z_1}{(m,20,20,20)} = \frac{\partial L}{\partial S_1}\frac{\partial S_1}{\partial Z_1} =$$
$$\frac{\partial L}{\partial S_1}{(m,20,20,20)} * [(S_1)_{(m,20,20,20)} * (1 - S1)_{(m,20,20,20)}]$$

Backward Propagation - Layer1 - $\frac{\partial L}{\partial b_1}{}_{(1,20)}$

$$\frac{\partial L}{\partial Z_1}{}_{(m,20,20,20)} \longrightarrow \frac{\partial L}{\partial Z_1}{}_{(20*20*m,20)}$$

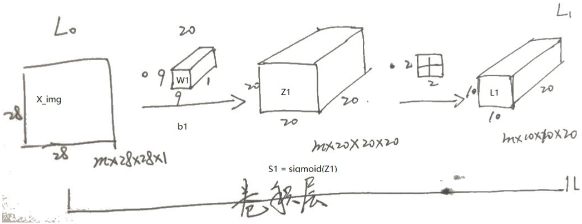

Figura: Layer2

$$\frac{\partial L}{\partial b_1}{}_{(1,20)} = \frac{\partial L}{\partial Z_1}\frac{\partial Z_1}{\partial b_1} = \frac{1}{20*20*m}\Big(\sum_{k=1}^{20*20*m} \frac{\partial L}{\partial Z_1}{}_{(20*20*m,20)}\Big)_{(1,20)}$$

Backward Propagation - Layer1 - $\frac{\partial L}{\partial W_1}_{(9,9,1,20)}$

$$\frac{\partial L}{\partial Z_1}_{(m,20,20,20)} \longrightarrow \frac{\partial L}{\partial Z_1}_{(20,20,m,20)}$$

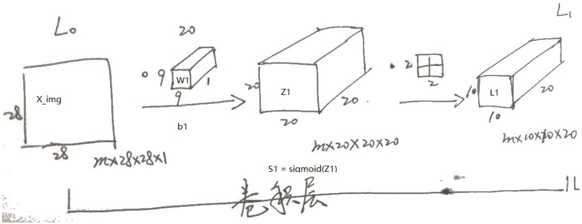

Figura: Layer2

$$\frac{\partial L}{\partial W_1}_{(9,9,1,20)} = \frac{\partial L}{\partial Z_1}\frac{\partial Z_1}{\partial W_1} =$$
$$\frac{1}{m}\sum_{k=1}^{m} cov2d(X\_img_{(1,28,28,1)}, \frac{\partial L}{\partial Z_1}_{(20,20,1,20)})_{(1,9,9,20)}$$

Reference

- https://www.cnblogs.com/tornadomeet/p/3468450.html
- Paper: Jake Bouvrie — Notes on Convolutional Neural Networks

— http://cogprints.org/5869/1/cnn_tutorial.pdf

- GitHub: https://github.com/amaas/stanford_dl_ex