L be any language and x and y are the strings

<u>Distinguishable by L :</u>

Strings x and y are distinguishable by L if $\exists$ some z such that exactly one of the strings xz and yz belongs to L.

<u>Indistinguishable by L :</u>

Strings x and y are indistinguishable by L if for every string z , $xz \in L$ whenever $yz \in L$

If x and y are indistinguishable by L then we write $x \equiv_L y$

Now we have to show that $\equiv_L$ is an equivalence relation.

• To show that $\equiv_L$ is an equivalence relation, we have to show that $\equiv_L$ is

(i) Reflexive

(ii) Symmetric

(iii) Transitive

• According to the given data, $x \equiv_L y$ means "for every string *z*, *xz* is in L whenever *yz* is in L". That means, "for every string z, xz is in L iff yz is in L"

(i) <u>Reflexivity:</u> $x \equiv_L x$ is true

For all strings z, xz is in L iff xz is in L

Therefore $x \equiv_L x$ is true.

Hence $\equiv_L$ is reflexive.

(ii) <u>Symmetry:</u> $x \equiv_L y$ implies $y \equiv_L x$

If $x \equiv_L y$ is true then "for all *z*, *xz* is in L iff *yz* is in L"

Which is equivalent to "for all *z*, *yz* is in L iff *xz* is in L"

Therefore $y \equiv_L x$ is also true.

Hence $\equiv_L$ is symmetric.

(iii) <u>Transitivity:</u> If $a \equiv_L b$ and $b \equiv_L c$ then $a \equiv_L c$

This means that

"for all z, az is in L iff bz is in L and

For all z, bz is in L iff cz is in L".

Therefore, "for all z, az is in L iff cz is in L".

That is, $a \equiv_L c$ is true

Hence $\equiv_L$ is transitive.

From (i), (ii) and (iii)

$\equiv_L$ is equivalence relation.