Question:

Show that if G is a CFG in Chomsky normal form, then for any string w ∑ L(G) of length n ≥ 1, exactly 2n − 1 steps are required for any derivation of w.

Answer:

----SETP1----

Given that *G* is a CFG in Chomsky Normal Form (CNF).

The length of the string $w \in L(G)$ is n ≥ 1 for the string *w.*

It is required to show that exactly 2*n*-1 steps are required for the derivation of string w.

It can be proved applying the **induction method** by on the string w of length *n*.

For $n=1$: Consider a string "**a**" of length 1 in Chomsky normal form, so the valid derivation for this will be $S \to a$, where $a \in \sum$ and S is starting symbol.

The number of steps can be obtained as follows:

$$\begin{aligned}2n-1 &= 2(1)-1\\ &= 2-1\\ &= 1\end{aligned}$$

Hence it is true that 2n-1 (for n=1) steps are required to derive a string **a**.

For $n=k$: Take a string of length $k \geq n$ in Chomsky normal form, so valid derivation for this will take $2k-1$ steps.

The number of steps can be obtained as follows:

$$\begin{aligned}2n-1 &= 2(k)-1\\ &= 2k-1\end{aligned}$$

Assume a string of length at most k≥1 terminal symbols and it has a string of length

n= $k+1$ is in Chomsky Normal Form

Since n>1, Consider a language as follows in CNF where derivation starts with start symbols S:

$$\begin{aligned}&S \to BC\\ &B \to^* x\\ &C \to^* y\end{aligned}$$

So length of the string starting with start symbol S is |*w*| = xy where |x| > 0 and | y | > 0.

Using the inductive hypothesis, for the above language in CNF the length of any derivation of string *w* must be

$$1+(2|x|-1)+(2|y|-1)=2|x|+2|y|+1-1-1=2(|x|+|y|)-1$$

Here n= $|x|+|y|$.

Since $B \to^* x$ has a length of |*x*| and $C \to^* y$ has a length of |*y*|.

Hence, it is proved that it requires **2n-1** steps required for the derivation of string

$w \in L(G)$ in Chomsky Normal Form(CNF).