Assignment 3

2.9 Let $G = \{V, \Sigma, R, S\}$, We know that $\Sigma = \{a, b, c\}$.R is following rules:

$$S \to U|V$$

$$U \to Uc|A$$

$$A \to aAb|\varepsilon$$

$$V \to aV|B$$

$$B \to bBc|\varepsilon$$

G is ambiguity. For abc, we have

$$S \Rightarrow U \Rightarrow Uc \Rightarrow Ac \Rightarrow aAbc \Rightarrow a\varepsilon bc \Rightarrow abc$$

and

$$S \Rightarrow V \Rightarrow aV \Rightarrow aB \Rightarrow abBc \Rightarrow ab\varepsilon c \Rightarrow abc$$

We have two different path.

Problem 2.14

1) We find there are two rules from S, $S \to TT$ and $S \to U$. From T we can get the language $L_1 = 0^i\#0^j, i, j \geq 0$, so from $S \to TT$ we get $L_2 = L_1L_1$. From $S \to U$ we have $L_3 = 0^p\#0^{2p}, p \geq 0$. So the language is $L(G) = L_2 \cup L_3$.
2) Let's prove L is not regular. Now we assume L is regular. Because of pumping lemma, if we let p be the pumping length given by the pumping lemma and $s = 0^p\#0^{2p} \in L$, we should have that s can be split in three pieces: $s = xyz$. Because $|xy| \leq p$, so y must all 0, that means $y = 0^k, k > 0$, so the string $xy^0z = 0^{p-k}\#0^{2p}$. It doesn't belong to L, so the language must be not regular.

Problem 4.3

To prove it, we construct the following TM:

M="on input $\langle A\rangle$ where $A = (Q, \Sigma, \delta, q, F)$ is a DFA,

1. Constract a new DFA $B = (Q, \Sigma, \delta, q, Q - F)$.
2. Run TM T in Theorem 4.4 on B to see if $L(B) = \emptyset$
3. If T accept, then accept
4. If T reject, then reject."

From the constraction we know that $L(B) = \emptyset \ iff \ L(B) = \Sigma^*$. So we use TM in Theorem 4.4 to find if $L(B) = \emptyset$.

Problem 5.1

To prove it, construct as follows: assume that D decides EQ_{CFG}

M="on input CFG G:

1. Construct CFG G_0 and $L(G_0) = \Sigma^*$.
2. Run D on input $\langle G, G_0 \rangle$.
3. If D accept, accept. Otherwise, reject

We find that D determines if $\mathrm{L(G) = L(G_0)}$, but $L(G_0) = \Sigma^*$. So it determines if $L(G) = \Sigma^*$. So it means D decides $\mathrm{ALL_{CFG}}$, but we know that $\mathrm{ALL_{CFG}}$ is undecidable, so $\mathrm{EQ_{CFG}}$ must undecidable.