Incompressible strings:

Let w_i be a string. If w_i doesn't have any description shorter than itself then w_i is incompressible.

Now we have to show that set of incompressible strings contains no infinite subset that is Turing recognizable.

Let A be the set of incompressible strings and assume the contradiction A contains infinite subset and Turing recognizable.

We construct an enumeration function $f : N \rightarrow A$, A is recognized by machine M.

Enumeration: $f : N \rightarrow A$ such that $f(1) = w1, f(2) = w2, f(3) = w3...$ where first, second, and third enumerated strings are respectively $w1$, $w2$, $w3$, etc.

Since A reaches infinite there is a string $w_i \in A$.

Turing machine N which computes an incompressible string of length at least n.

$N =$" On input n (an integer in binary notation)

1. Call M to enumerate incompressible strings w_i.

2. If $|w_i| >= n$, output w_i and halt"

Now the length of $\langle N \rangle n$ is

$$\left| \langle N \rangle n \right| = \left| \langle N \rangle \right| + \log(n)$$

$= \log(n) + c$ where c is a constant

Now $|w_i| = n$

$n > \log n + c$ for large value of n.

$n > \left| \langle N \rangle n \right|$ $\left[\text{As} \left| \langle N \rangle \right| n = \log n + c \right]$

Therefore $|w_i| > n$

This contradicts the incompressibility of string w_i, therefore our assumption that M enumerates an infinite subset of incompressible strings w_i is wrong.

Hence set of incompressible strings contains no infinite subset is Turing – recognizable.