

In order to solve this problem, we need to know the definition of enumerator and some theorems

### Enumerator:-

An enumerator is a Turing machine that consist a work tape and the output tape. It outputs the strings by using the work tape without accepting any input.

Also we use the following theorem

### Theorem 1:

“A language is Turing – decidable if and only if some enumerator enumerates the strings of this language in lexicographic order”

Consider the language  $B = \{\langle M_1 \rangle, \langle M_2 \rangle, \dots\}$ .

$B$  is a Turing recognizable language.

$C$  is a language consisting of Turing machines descriptions.

Consider  $E$  be the enumerator for the Turing recognizable language  $B$ .

Construct an enumerator  $E_o$  which output the strings of  $C$  in lexicographic order.

From the above Theorem1,  $C$  is decidable.

Enumerator  $E_o$  simulates  $E$ .

When  $E$  gives the  $i^{\text{th}}$  TM  $\langle M_i \rangle$  as output, then enumerator  $E_o$  pads  $M_i$  by adding sufficiently many extra useless states to obtain a new TM  $M'_i$  where the length of  $\langle M'_i \rangle$  is greater than the length of  $\langle M'_{i-1} \rangle$ . Then  $E$  outputs  $\langle M'_i \rangle$ .

Thus simulation occurs in both directions.

Therefore,  $E_o$  and  $E$  are equivalent.