

A language B is said to be NP-complete if the following conditions are satisfied:

1. $B \in NP$

2. Every language L can be polynomial-time reduced to B.

Let $P = NP$ and let $A \in P$ such that $A \neq \emptyset$ and $A \neq \Sigma^*$ so, it is required to prove that for every $A \in NP$ and $L \in NP$, $L \leq_p A$.

Let there exist an arbitrary language L from $NP=P$. Hence, the language L has polynomial decider X_L so, the polynomial reduction f from L to A will be as follows:

When the input string is x:

1. Run X_L on the string x.
2. If the decider X_L accepts the string then output x_{in}
3. If the decider X_L rejects the string then output x_{output}

Thus, there exists a poly-time reduction from L to A, so, A is NP-complete.