
Question:

Show that A is Turing-recognizable iff $A \leq_m A_{TM}$.

Answer:

----SETP1----

Turing Machine:

- This is a type of mathematical model of computation that specify the conceptual machine.
- The Turing machine that operate a strings symbols on the tape as per given set of rules.
- The Turing machine can be manufactured in order to imitate the algorithms.

----SETP2----

Turing Recognizable:

- Any language will be recognizable or not if and only if it depends on Turing machine which will stop and receive only strings in that language and for strings not in the language.
- So, the Turing machine then refuse or does not stop at all.

----SETP3----

Theorem of Turing Recognizable:

- As per given in the theorem 5.28 from the book.
- If $A \leq_m A_{TM}$ in this A_{TM} is Turing Recognizable then A is Turing Recognizable.
- A is Turing recognizable that can be prove below which is as follows:

----SETP4----

Consider the following details which is as follows:

A is Turing Recognizable iff $A \leq_m A_{TM}$.

- If $A \leq_m A_{TM}$ that means then A is Turing recognizable since A_{TM} is Turing recognizable.
- Suppose that A is Turing Recognizable then there will exists a Turing machine T that identify A.
- That means T gets the inputs which is w and receive if $w \in A$ that means w belongs to A.
- Apart from that T does not receive.
- In order to exhibit the $A \leq_m A_{TM}$.
- Describe a Turing machine that does the following works which is as follows:

- Take input w and write the $\langle T, w \rangle$ on the tape and stop.
- Then check the $\langle T, w \rangle$ is in A_{TM} .
- The above condition can be checked if and only if w is in A .
- So, get the mapping reduction of A to A_{TM} .
- So, the reduction proves that A is Turing Recognizable.

Hence, it is proved that A is Turing Recognizable.