

Let  $UNARY-SSUM$  be the subset sum problem in which all numbers are represented in unary.

The complexity of a function can be measured from the input size and number of steps required for that input.

- If the input is in unary, then the size of the input is equal to the value. When the input value is  $x$ , then number of squares needed to represent the input is  $x$ .
- If the input is in binary, then  $\log x$  squares needed to represent the input value  $x$ . Thus, if the input size in binary is  $N$ , then the input size in unary is  $2^N$ .
- It can be concluded that the size of the input is increasing exponentially.
- The reduction proof requires more than polynomial time to reduce  $3SAT$  to  $UNARY-SSUM$ .

NP-completeness proof for  $SUBSET-SUM$  fails to show  $UNARY-SSUM$  is NP-complete because, the reduction is not polynomial time. The input size in unary is  $2^N$  which takes exponential time.

Therefore, the NP-completeness proof for  $SUBSET-SUM$  fails to show  $UNARY-SSUM$  is NP-complete.

To prove the language is in P, need to show that the polynomial time algorithm decides it. The standard algorithm for  $SUBSET-SUM$  runs in the time bounded by a polynomial in the input size  $N$  and the target integer  $S$ . The polynomial time algorithm for  $UNARY-SSUM$  is as follows:

- Compute the sum of numbers in the input.
- Check if the sum of the numbers is at least the target integer  $S$ .
- If the sum of the numbers is at least the target integer  $S$ , then run the standard dynamic programming algorithm of  $SUBSET-SUM$ .
- Otherwise, reject.

There exists a polynomial time algorithm to decide  $UNARY-SSUM$ . Therefore,  $UNARY-SSUM \in P$ .