

**Proving relation is transitive**

Transitive relations are those in which one element is related to the second element and second element is related to the third element.

In this situation, the first element should be related to the third element. If the condition is fulfilled, then it can be said that the relation is transitive.

Now, for proving transitivity of  $\leq_m$  two relations are required.

Consider the first relation for showing the transitivity:

$$A \leq_m B$$

Consider the second relation for showing the transitivity:

$$B \leq_m C$$

Now, computational function for first relation is as shown:

$$x \in A \Leftrightarrow f(x) \in B$$

Here,  $x$  is considered as input string on  $A$  and output of  $A$  is  $f(x)$

Now, computational function for second relation is as shown:

$$y \in B \Leftrightarrow g(y) \in C$$

Here,  $y$  is considered as input string on  $B$  and output of  $B$  is  $g(y)$

Now, consider a composition function that can be considered as the mapping function between  $f$  and  $g$ .

$$h(x) = g(f(x))$$

Now, find the mapping between these two functions, it may be required to create a Turing Machine that computes  $h$  as follows:

- Simulate a Turing Machine on input  $x$  and call the output  $y$ , this will simulate a Turing Machine for  $g$  on  $y$ .
- Now, the output of this input can be viewed as considered function that is,  $h(x) = g(f(x))$ .

In this way:

$$x \in A \Leftrightarrow h(x) \in C$$

Thereby,  $h(x)$  is computable function.

Here, input  $x$  on  $A$  is directly dependent on  $h(x)$ .

Hence, if  $A \leq_m B$  and  $B \leq_m C$  then  $A \leq_m C$ .

Therefore,  $\leq_m$  is Transitive Relation.