Decidability of the language

Given: In this a language $A_{\varepsilon CFG}$ is given.

Proof: For showing that the language $A_{\varepsilon CFG}$ is decidable, build a Turing machine T for deciding the language $A_{\varepsilon CFG}$. For all Context free grammars G

• If the grammar G derives ε then $T(\langle G \rangle)$ accepts

• If the grammar G does not derive $\in$ then $T(\langle G \rangle)$ rejects.

Constructions:

For proofing the decidability of $A_{\varepsilon CFG}$ firstly convert the context free grammar G into an equivalent G' in CNF. If $S \to \varepsilon$ is the rule in the CFG G′ then it means that G' derives ε.

If the CFG G' derives ε then G also derives it as $L(G) = L(G')$. As G' is in CNF so only possible $\in$-rule in G' is $S \to \varepsilon$. If G' contains $S \to \varepsilon$ in production rules then $\varepsilon \in L(G')$. If G' does not contains the rule $S \to \varepsilon$ then $\varepsilon \notin L(G')$.

Turing machine T = on input $\langle G \rangle$ where G is a context free grammar

• Convert the grammar G in CFG G'.

• If G' contains the production rule $S \to \varepsilon$ then accept it.

• Otherwise reject it.

Conclusion:

From the above construction it is clear that $\langle G \rangle \in A_{\varepsilon CFG}$ iff $\langle G, \varepsilon \rangle$ is also belongs to the A_{CFG}. So the above construction is correct. Hence the language $A_{\in CFG}$ is decidable.