

## 获得的答案

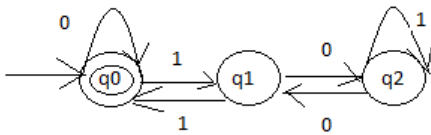
Given language is

$$C_n = \{x \mid x \text{ is a binary number that is a multiple of } n\}$$

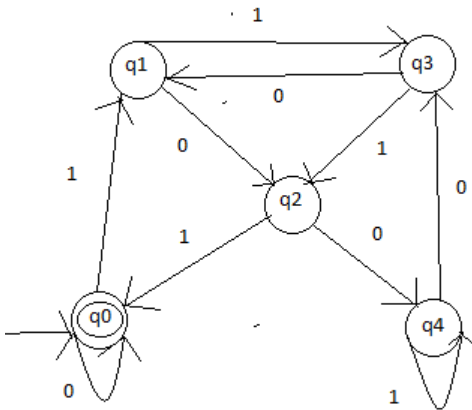
- A language is regular if it is recognized by a DFA.
- So construct a DFA to keep track of the remainder of the input when divided by  $n$ .

Construction of DFA is as follows:

- Assume the value of  $n$  as 3.
- To determine whether a binary number is a multiple of 3, it is necessary to find its remainder modulo 3.
- If it ends up with remainder zero then accept.
- Otherwise reject.
- Every time read a digit, the preceding string is shifted left one position thereby doubling its value  $x$ .
- If the current digit is 0, then the new value is  $2x \pmod{3}$ .
- If the current digit is 1, then the new value is  $2x + 1 \pmod{3}$ .
- The DFA for the above example is as follows:



- Similarly for  $n=5$  the DFA is as follows:



The other case can be proved similarly.

For example:

$$C_6 = C_3 0$$

$$C_{12} = C_6 00$$

$$C_{15} = C_3 \cap C_5$$

....

Thus  $M = (\{q_0, q_1, \dots, q_n\}, \{0, 1\}, \delta, q_0, \{q_0\})$

Since  $M$  recognizes language  $C_n$ . Hence, it is proved that  $C_n$  is regular.