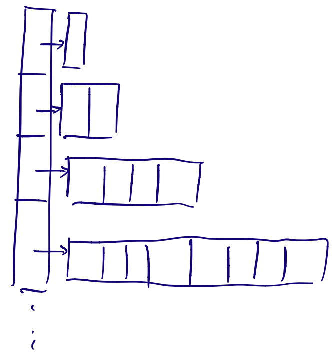
1. Solution

(a) the data structure of sorted array is like the diagram on the right. the top is corresponding to the lowest bit of n represted by binary code.



If some bit of n is zero, it means the sorted array at corresponding row is empty.

SEARCH: Since all arrays are sorted, so we just need to do binary search on each of them. The worst case occurs when do binary search on all arrays, and it takes $T(n) = \sum_{i=1}^{\lceil \lg n \rceil} c \cdot \lg(i) = \Theta(\lg^2 n)$

(b) INSERT: Since all arrays are sorted, when a new value comes, we just need to merge some rows. These rows are determined by the size n, spercificly, continous 1's bits of n from low position.

For example, let $n = 11 = (1011)_2$, the continuous 1's bit is (1011),

so we need to merge the first. second rows and the new element, the amound of them is $1+2+1=4$, we need to transform them to third row, then $n = (1100)$

The insertion process is as following:

1. store the new element in a new array A^* ($|A| = 1$)
2. find $A_{\lg|A^*|}$ which has the same length with A^*
3. if $A_{\lg|A^*|}$ is full, then expand A^* to double size and merge $A_{\lg|A^*|}$ and original A^* to expanded A^* --- $O(|A^*|)$
4. if $A_{\lg|A^*|}$ is empty, let $A_{\lg|A^*|}$ pointer re link to A^* -- $O(1)$
5. if $A_{\lg|A^*|}$ does not exist, create $A_{\lg|A^*|}$ pointer and let it link to A^* --- $O(1)$

We can notice step 3 cost main time, since it is a repeating operation

The worst case occurs when merge all elements and move them to a new row, and we consider the cost of searching for a "1" row. So, $T(n) = \sum |A^*| = \sum_{i=0}^{\lceil \lg n \rceil + 1} c \cdot 2^i = O(n)$

Then analysis the amortized time. For $i > j$, when A_i becomes full, A_j is set to be full and empty $\frac{2^i/2^j}{2} = 2^{i-j-1}$ times, Let the dynamic binary arrays be initialized to 0, and when inserting n elements, the total cost of A_i is the time of merging timing the times of reset, i.e.

$T(A_i) = |A_i| \cdot (2^{\lceil \lg n \rceil - i - 1} + 2^{\lceil \lg n \rceil - i - 2} + \cdots + 2^i + 1) = 2^i \cdot (\sum_{j=1}^{\lceil \lg n \rceil - i - 1} 2^j + 1) = 2^i (2^{\lceil \lg n \rceil - i} - 1) = \Theta(n)$

Thus, $T(n)$ is the sum of $T(A_i)$, i.e. $T(n) = \sum_{i=0}^{\lceil \lg n \rceil - 1} T(A_i) = \lg n \cdot \Theta(n) = \Theta(n \lg n)$

2. Intuitively, we can use dynamic programming to solve the problem.

Let $dp[i][j]$ be the least edit distance between strings $A[1:i]$ and $B[1:j]$, (assume the lengths of them are m and n).

Initialize $dp[][]$:

$dp[i][0] = i$, $dp[0][j] = j$, for $i = 0, 1, \ldots, m$, $j = 0, 1, \ldots, n$.

Thus, the transition function is:

$$dp[i][j] = \min(\, d[i-1][j] + 1,\; d[i][j-1] + 1,\; d[i-1][j-1] + A[i] == B[j]\ ?\ 0 : 1\,)$$

for $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

3. (a)

if $n = 0$, the optimal choise is to give none.

if $n > 0$, we consider the following 4 cases:

1. if $1 \le n < 5$, then the solution only contains 1, it mush greedy choise
2. if $5 \le n < 10$, then the solution contains 1 and 5, for any n pennies, we can us a nikle to replace 5 pennies, then n reduce to $n-5+1 = n-4$.
3. if $10 \le n < 25$, then the solution contains 1, 5, and 10, we can replace

every 2 nikles by a dime for any n to reduce. like $n \to n-2+1$

4. if $n \geq 25$, then - solution contains 1, 5, 10 and 25, we can replce every 3 dimes and by a quartor and a nikle. if it contains at most 2 dimes, then it can be sum up to 25 by a nikle, so we can replace at least 3 cents by one. $n \to n-3+1$

$n \to n-2-1+1$, $m \geq 1$

Thus, there is always an optimal solution that includes the greedy choies.

(b) Assume a non-greedy solution is optimal. we can define the solution as:

① let the sum of all coins be n

② let a_i represent the amount of c^i, $i = 0, 1, \cdots, j-1$, where j is the maximum power satisfying $c^j \leq n$, so, we can get $\sum_{i=0}^{j-1} a_i c^i = n$. Since $c^j \leq n$, so $\sum_{i=0}^{j-1} a_i c^i \geq c^j$. (1)

③ It is obvious that for any a_i, $a_i < c$, otherwise we can use c^{i+1} to replace c c^i's.

To sum up, $a_i < c$, $\therefore a_i \le c-1$, $i = 0, 1, \dots, \hat{j}-1$,

Thus, $\sum_{i=0}^{\hat{j}-1} a_i c^i \le \sum_{i=0}^{\hat{j}-1} (c-1) c^i$

$$= (c-1) \underbrace{\sum_{i=0}^{\hat{j}-1} c^i}_{\text{this is a geometric series}}$$

$$= (c-1) \frac{1-c^{\hat{j}}}{1-c} = c^{\hat{j}} - 1 < c^{\hat{j}} \quad (2)$$

(1) and (2) are contradictory, so our assumption is wrong.

Hence, a non-greedy solution is not optimal. So, only the greedy solution is optimal.

(c). we can use coins of denomination only containing penny, dime and quarter, i.e. 1, 10, 25,

and let $n = 30$.

if we use greedy solution, we get $25+1+1+1+1+1$ which contains 6 coins in total.

but if we only use dimes, we get $10+10+10$ which contains only 3 coins in total.

(d) This problem is like "complete knapsack problem"

Let coins are $c_1, c_2, \dots, c_k$, and total value is n.

using dynamic programming :

$$dp[0] = 0$$

$$dp[i] = \min_{1 \le j \le k} (dp[i - c_j]) + 1 \quad , \quad i = 1, 2, \dots, n$$

4. let the production be x_1, x_2, x_3, x_4 in 4 quarters.

$$\min \ 15x_1 + 14x_2 + 15.3x_3 + 14.8x_4 + 0.2(x_1 - 20) + 0.2(x_1 + x_2 - 20) + 0.2(x_1 + x_2 + x_3 - 70)$$

s.t. $x_1 + x_2 \geq 40$

$x_1 + x_2 + x_3 \geq 70$

$x_1 + x_2 + x_3 + x_4 = 80$

$20 \le x_1 \le 30$

$0 \le x_2 \le 40$

$0 \le x_3 \le 20$

$0 \le x_4 \le 10$

transite to standard form :

$$\max \ 26 - 15.6x_1 - 14.4x_2 - 15.5x_3 - 14.8x_4$$

s.t. $-x_1 - x_2 \le -40$

$$-x_1 - x_2 - x_3 \le -70$$

$$-x_1 - x_2 - x_3 - x_4 \le -80$$

$$x_1 + x_2 + x_3 + x_4 \le 80$$

$$-x_1 \le -20$$

$$x_1 \le 30$$

$$x_2 \le 40$$

$$x_3 \le 20$$

$$x_4 \le 10.$$

$$x_1, x_2, x_3, x_4 \ge 0.$$

$$\max \quad 26 - 15.6x_1 - 14.4x_2 - 15.5x_3 - 14.8x_4$$

$$x_5 = -40 + x_1 + x_2$$

$$x_6 = -70 + x_1 + x_2 + x_3$$

$$x_7 = -80 + x_1 + x_2 + x_3 + x_4$$

$$x_8 = 80 - x_1 - x_2 - x_3 - x_4$$

$$x_9 = -20 + x_1$$

$$x_{10} = 30 - x_1$$

$$x_{11} = 40 - x_2$$

$$x_{12} = 20 - x_3$$

$$x_{13} = 10 - x_4$$

$$x_1, x_2, \ldots, x_{13} \ge 0$$

use auxilary linear programming:

$$
\begin{aligned}
\max \quad & -x_0. \\
\text{s.t.} \quad & -x_1 - x_2 - x_0 \le -40 \\
& -x_1 - x_2 - x_3 - x_0 \le -70 \\
& -x_1 - x_2 - x_3 - x_4 - x_0 \le -80 \\
& x_1 + x_2 + x_3 + x_4 - x_0 \le 80 \\
& -x_1 - x_0 \le -20 \\
& x_1 - x_0 \le 30 \\
& x_2 - x_0 \le 40 \\
& x_3 - x_0 \le 20 \\
& x_4 - x_0 \le 10 \\
& x_0, x_1, x_2, x_3, x_4 \ge 0.
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
\max \quad & -x_0 \\
\text{s.t.} \quad & x_5 = -40 + x_1 + x_2 + x_0 \\
& x_6 = -70 + x_1 + x_2 + x_3 + x_0 \\
& x_7 = -80 + x_1 + x_2 + x_3 + x_4 + x_0 \\
& x_8 = 80 - x_1 - x_2 - x_3 - x_4 + x_0 \\
& x_9 = -20 + x_1 + x_0 \\
& x_{10} = 30 - x_1 + x_0 \\
& x_{11} = 40 - x_2 + x_0 \\
& x_{12} = 20 - x_3 + x_0 \\
& x_{13} = 10 - x_4 + x_0 \\
& x_1, x_2, \dots, x_{13} \ge 0
\end{aligned}
$$

$$-50 + 20 - x_0 + x_9 + x_2 + x_4 + x_6 - x_7 = -30 - x_0 + x_2 + x_4 + x_6 - x_7 + x_9$$

$$30 - \cancel{x_0} - 80 + x_1 + x_2 + x_4 + \cancel{x_0} - x_7 + x_6 = -50 + x_1 + x_2 + x_4 + x_6 - x_7$$

$$-40 + \cancel{x_1} + 70 - \cancel{x_1} - x_3 - x_0 + x_6 = 30 - x_0 - x_3 + x_6$$

$$-40 + \underline{x_1 + x_2} - x_5$$

$$
\begin{aligned}
\max \quad & -x_0 \\
\text{s.t.} \quad & x_5 = -40 + x_1 + x_2 + x_0 \Rightarrow x_0 = 40 - x_1 - x_2 + x_5 \\
& x_6 = -70 + x_1 + x_2 + x_3 + x_0 \Rightarrow x_2 = 70 - x_1 - x_3 - x_0 + x_6 \\
& x_7 = -80 + x_1 + x_2 + x_3 + x_4 + x_0 \Rightarrow x_3 = 80 - x_1 - x_2 - x_4 - x_0 + x_7 \\
& x_8 = 80 - x_1 - x_2 - x_3 - x_4 + x_0 \\
& x_9 = -20 + x_1 + x_0 \Rightarrow x_1 = 20 - x_0 + x_9 \\
& x_{10} = 30 - x_1 + x_0 \\
& x_{11} = 40 - x_2 + x_0 \\
& x_{12} = 20 - x_3 + x_0 \\
& x_{13} = 10 - x_4 + x_0 \\
& x_1, x_2, \dots, x_{13} \ge 0
\end{aligned}
$$

$$\max \quad -x_0$$

$$\text{s.t.} \quad x_5 = -40 + x_1 + x_2 + x_0,$$

$$x_6 = -70 + x_1 + x_2 + x_3 + x_0$$

$$x_7 = -80 + x_1 + x_2 + x_3 + x_4 + x_0$$

$$x_8 = 80 - x_1 - x_2 - x_3 - x_4 + x_0$$

$$x_9 = -20 + x_1 + x_0$$

$$x_{10} = 30 - x_1 + x_0$$

$$x_{11} = 40 - x_2 + x_0$$

$$x_{12} = 20 - x_3 + x_0$$

$$x_{13} = 10 - x_4 + x_0$$

$$x_1, x_2, \cdots, x_{13} \geq 0$$

$\Rightarrow$

$$\max \quad -40 + x_1 + x_2 - x_5$$

$$\text{s.t.} \quad x_0 = 40 - x_1 - x_2 + x_5$$

$$x_6 = -30 + x_3 + x_5$$

$$x_7 = -40 + x_3 + x_4 + x_5$$

$$x_8 = 120 - 2x_1 - 2x_2 - x_3 - x_4 + x_5$$

$$x_9 = 20 - x_2 + x_5$$

$$x_{10} = 70 - 2x_1 - x_2 + x_5$$

$$x_{11} = 80 - x_1 - 2x_2 + x_5$$

$$x_{12} = 60 - x_1 - x_2 - x_3 + x_5$$

$$x_{13} = 50 - x_1 - x_2 - x_4 + x_5$$

$$x_1, x_2, \cdots, x_{13} \geq 0$$

$$x_5 = 30 - x_3 + x_6$$

$$\max \quad -40 + x_1 + x_2 - x_5 \Rightarrow z = -70 + x_1 + x_2 + x_3 - x_6$$

$$\text{s.t.} \quad x_0 = 40 - x_1 - x_2 + x_5 \Rightarrow x_0 = 40 - x_1 - x_2 + 30 - x_3 + x_6 = 70 - x_1 - x_2 - x_3 + x_6$$

$$x_6 = -30 + x_3 + x_5 \Rightarrow x_5 = 30 - x_3 + x_6$$

$$x_7 = -40 + x_3 + x_4 + x_5 \Rightarrow x_7 = -40 + x_3 + x_4 + 30 - x_3 + x_6 = -10 + x_4 + x_6$$

$$x_8 = 120 - 2x_1 - 2x_2 - x_3 - x_4 + x_5 \Rightarrow x_8 = 120 - 2x_1 - 2x_2 - x_3 - x_4 + 30 - x_3 + x_6 = 150 - 2x_1 - 2x_2 - 2x_3 - x_4 + x_6$$

$$x_9 = 20 - x_2 + x_5 \Rightarrow x_9 = 20 - x_2 + 30 - x_3 + x_6 = 50 - x_2 - x_3 + x_6$$

$$x_{10} = 70 - 2x_1 - x_2 + x_5 \Rightarrow x_{10} = 70 - 2x_1 - x_2 + 30 - x_3 + x_6 = 100 - 2x_1 - x_2 - x_3 + x_6$$

$$x_{11} = 80 - x_1 - 2x_2 + x_5 \quad x_{11} = 80 - x_1 - 2x_2 + x_5 = 80 - x_1 - 2x_2 + 30 - x_3 + x_6 = 110 - x_1 - 2x_2 - x_3 + x_6$$

$$x_{12} = 60 - x_1 - x_2 - x_3 + x_5 \quad x_{12} = 60 - x_1 - x_2 - x_3 + 30 - x_3 + x_6 = 90 - x_1 - x_2 - 2x_3 + x_6$$

$$x_{13} = 50 - x_1 - x_2 - x_4 + x_5 \quad x_{13} = 50 - x_1 - x_2 - x_4 + 30 - x_3 + x_6 = 80 - x_1 - x_2 - x_3 - x_4 + x_6$$

$$x_1, x_2, \cdots, x_{13} \geq 0$$

$x_6 = 10 - x_4 + x_7$

$$\max\ z = -70 + x_1 + x_2 + x_3 - x_6 \Rightarrow z = -70 + x_1 + x_2 + x_3 - 10 + x_4 - x_7 = -80 + x_1 + x_2 + x_3 + x_4 - x_7$$

s.t.

$$x_0 = 70 - x_1 - x_2 - x_3 + x_6 \Rightarrow x_0 = 80 - x_1 - x_2 - x_3 - x_4 + x_7$$

$$x_5 = 30 - x_3 + x_6 \Rightarrow x_5 = 30 - x_3 + 10 - x_4 + x_7 = 40 - x_3 - x_4 + x_7$$

$$x_7 = \underbrace{-10 + x_4 + x_6} \Rightarrow x_6 = 10 - x_4 + x_7$$

$$x_8 = 150 - 2x_1 - 2x_2 - 2x_3 - x_4 + x_6 \quad x_8 = 150 - 2x_1 - 2x_2 - 2x_3 - x_4 + 10 - x_4 + x_7 = 160 - 2x_1 - 2x_2 - 2x_3 - 2x_4 + x_7$$

$$x_9 = 50 - x_2 - x_3 + x_6 \quad x_9 = 50 - x_2 - x_3 + 10 - x_4 + x_7 = 60 - x_2 - x_3 - x_4 + x_7$$

$$x_{10} = 100 - 2x_1 - x_2 - x_3 + x_6 \quad x_{10} = 100 - 2x_1 - x_2 - x_3 + 10 - x_4 + x_7 = 110 - 2x_1 - x_2 - x_3 - x_4 + x_7$$

$$x_{11} = 110 - x_1 - 2x_2 - x_3 + x_6 \quad x_{11} = 110 - x_1 - 2x_2 - x_3 + 10 - x_4 + x_7 = 120 - x_1 - 2x_2 - x_3 - x_4 + x_7$$

$$x_{12} = 90 - x_1 - x_2 - 2x_3 + x_6 \quad x_{12} = 90 - x_1 - x_2 - 2x_3 + 10 - x_4 + x_7 = 100 - x_1 - x_2 - 2x_3 - x_4 + x_7$$

$$x_{13} = 80 - x_1 - x_2 - x_3 - x_4 + x_6 \quad x_{13} = 80 - x_1 - x_2 - x_3 - x_4 + 10 - x_4 + x_7 = 90 - x_1 - x_2 - x_3 - 2x_4 + x_7$$

$\max\ z = -80 + x_1 + x_2 + x_3 + x_4 - x_7$	use x_1 rotate.	
s.t. $x_0 = 80 - x_1 - x_2 - x_3 - x_4 + x_7$,	$x_1 = 80$	
$x_5 = 40 - x_3 - x_4 + x_7$	.	
$x_6 = 10 - x_4 + x_7$	.	
$x_8 = 160 - 2x_1 - 2x_2 - 2x_3 - 2x_4 + x_7$	$x_1 = 80$	
$x_9 = 60 - x_2 - x_3 - x_4 + x_7$	.	$2x_1 = 110 - x_2 - x_3 - x_4 + x_7 - x_{10}$
$x_{10} = 110 - 2x_1 - x_2 - x_3 - x_4 + x_7$	$x_1 = 55$ ✓	$x_1 = 55 - \frac{1}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 + \frac{1}{2}x_7 - \frac{1}{2}x_{10}$
$x_{11} = 120 - x_1 - 2x_2 - x_3 - x_4 + x_7$	$x_1 = 120$	
$x_{12} = 100 - x_1 - x_2 - 2x_3 - x_4 + x_7$	$x_1 = 100$	
$x_{13} = 90 - x_1 - x_2 - x_3 - 2x_4 + x_7$	$x_1 = 90$	

$$\max\ z = -25 + \frac{1}{2}x_2 + \frac{1}{2}x_3 + \frac{1}{2}x_4 - \frac{1}{2}x_7 - \frac{1}{2}x_{10}$$

$$\text{s.t.}\quad x_0 = 25 - \frac{1}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 + \frac{1}{2}x_7 + \frac{1}{2}x_{10} \qquad x_2 = 50$$

$$x_5 = 40 - x_3 - x_4 + x_7$$

$$x_6 = 10 - x_4 + x_7$$

$$x_8 = 50 - 3x_2 - 3x_3 - 3x_4 + 2x_7 - x_{10} \qquad x_2 = \frac{50}{3} \checkmark$$

$$x_9 = 60 - x_2 - x_3 - x_4 + x_7 \qquad x_2 = 60$$

$$x_1 = 55 - \frac{1}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 + \frac{1}{2}x_7 - \frac{1}{2}x_{10} \qquad x_2 = 110$$

$$x_{11} = 65 - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 + \frac{1}{2}x_7 + \frac{1}{2}x_{10} \qquad x_2 = \frac{130}{3}$$

$$x_{12} = 45 - \frac{1}{2}x_2 - \frac{3}{2}x_3 - \frac{1}{2}x_4 + \frac{1}{2}x_7 + \frac{1}{2}x_{10} \qquad x_2 = 90$$

$$x_{13} = 35 - \frac{1}{2}x_2 - \frac{1}{2}x_3 - \frac{3}{2}x_4 + \frac{1}{2}x_7 + \frac{1}{2}x_{10} \qquad x_2 = 70$$

$$\therefore\quad x_2 = \frac{50}{3} - x_3 - x_4 + \frac{2}{3}x_7 - \frac{1}{3}x_{10} - \frac{1}{3}x_8$$

$$z = -25 + \frac{1}{2}\left(\frac{50}{3} - x_3 - x_4 + \frac{2}{3}x_7 - \frac{1}{3}x_{10} - \frac{1}{3}x_8\right) + \frac{1}{2}x_3 + \frac{1}{2}x_4 - \frac{1}{2}x_7 - \frac{1}{2}x_{10}$$

$$= -\frac{25}{3} + \frac{1}{3}x_7 - \frac{1}{6}x_{10} - \frac{1}{6}x_8 - \frac{1}{2}x_7 - \frac{1}{2}x_{10}$$

$$= -\frac{25}{3} - \frac{1}{6}x_7 - \frac{1}{6}x_{10} - \frac{1}{6}x_8$$

$$\max\ z = -\frac{25}{3} - \frac{1}{6}x_7 - \frac{1}{6}x_{10} - \frac{1}{6}x_8$$

$$\text{s.t.}\quad x_0 = \frac{25}{3} + \frac{1}{6}x_7 + \frac{1}{6}x_{10} + \frac{1}{6}x_8$$

$$x_5 = 40 - x_3 - x_4 + x_7$$

$$x_6 = 10 - x_4 + x_7$$

$$x_2 = \frac{50}{3} - x_3 - x_4 + \frac{2}{3}x_7 - \frac{1}{3}x_{10} - \frac{1}{3}x_8$$

$$x_9 = \frac{130}{3} + \frac{1}{3}x_7 + \frac{1}{3}x_{10} + \frac{1}{3}x_8$$

$$x_1 = \frac{140}{3} + \frac{1}{6}x_7 - \frac{1}{3}x_{10} + \frac{1}{6}x_8$$

$$x_{11} = 40 + x_3 + x_4 - \frac{1}{2}x_7 + x_{10} + \frac{1}{2}x_8$$

$$x_{12} = \frac{110}{3} - x_3 + \frac{1}{6}x_7 + \frac{2}{3}x_{10} + \frac{1}{6}x_8$$

$$x_{13} = \frac{80}{3} - x_4 + \frac{1}{6}x_7 + \frac{2}{3}x_{10} + \frac{1}{6}x_8$$

The optimal solution is $\left(\frac{25}{3}, \frac{140}{3}, \frac{50}{3}, 0, 0, 10, 0, 0, \frac{130}{3}, 0, 40, \frac{110}{3}, \frac{80}{3}\right)$

we can get when $x_0 = \frac{25}{3}$, z get max $-\frac{25}{3}$, let we consider the original problem:

$$\max \quad z = 26 - 15.6x_1 - 14.4x_2 - 15.5x_3 - 14.8x_4$$

s.t.

$$x_5 = -40 + x_1 + x_2$$

$$x_6 = -70 + x_1 + x_2 + x_3$$

$$x_7 = -80 + x_1 + x_2 + x_3 + x_4$$

$$x_8 = 80 - x_1 - x_2 - x_3 - x_4$$

$$x_9 = -20 + x_1$$

$$x_{10} = 30 - x_1$$

$$x_{11} = 40 - x_2$$

$$x_{12} = 20 - x_3$$

$$x_{13} = 10 - x_4$$

$x_1, x_2, \cdots, x_{13} \geq 0$

max

$$Z = 26 - 15.6\left(\frac{140}{3} + \frac{1}{6}x_7 - \frac{1}{3}x_{10} + \frac{1}{6}x_8\right) - 14.4\left(\frac{50}{3} - x_3 - x_4 + \frac{2}{3}x_7 - \frac{1}{3}x_{10} - \frac{1}{3}x_8\right)$$
$$- 15.5x_3 - 14.8x_4$$

$$= -260.7 - 1.1x_3 + 0.4x_4 - 12.2x_7 + 10x_{10} + 2.2x_8$$

s.t.

$$x_0 = \frac{25}{3} + \frac{1}{6}x_7 + \frac{1}{6}x_{10} + \frac{1}{6}x_8 \quad , \; x_{10} = 50$$

$$x_5 = 40 - x_3 - x_4 + x_7 \qquad \times$$

$$\times$$

$$x_6 = 10 - x_4 + x_7$$

$$x_2 = \frac{50}{3} - x_3 - x_4 + \frac{2}{3}x_7 - \frac{1}{3}x_{10} - \frac{1}{3}x_8 \qquad x_{10} = 50. \checkmark$$

$$x_9 = \frac{130}{3} + \frac{1}{3}x_7 + \frac{1}{3}x_{10} + \frac{1}{3}x_8 \qquad x_{10} = 130$$

$$x_1 = \frac{140}{3} + \frac{1}{6}x_7 - \frac{1}{3}x_{10} + \frac{1}{6}x_8 \qquad x_{10} = 140$$

$$x_{11} = 40 + x_3 + x_4 - \frac{1}{2}x_7 + x_{10} + \frac{1}{2}x_8 \qquad x_{10} = -40$$

$$x_{12} = \frac{110}{3} - x_3 + \frac{1}{6}x_7 + \frac{2}{3}x_{10} + \frac{1}{6}x_8 \qquad x_{10} = -55$$

$$x_{13} = \frac{80}{3} - x_4 + \frac{1}{6}x_7 + \frac{2}{3}x_{10} + \frac{1}{6}x_8 \qquad x_{10} = -40.$$

$$\Downarrow$$

$Z = 1165 + \cdots + 2.2x_8 + 10(\cdots - x_8)$, other variables are all less than zero

so, the solution is 1165

5. $x = (0, 1, 2, 3)$

$x^0 = (0, 2)$

$x^1 = (1, 3)$

$w = 1$

$w_n = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$

recurrence

$y_k^0 = fft(x[0]) = [2, -2]$

$y_k^1 = fft(x[1]) = [4, -2]$

recurrence

$y_0 = 2 + 1 \times 4 = 6$

$y_2 = 2 - 1 \times 4 = -2$

$w = w \times w_n = i$

$y_1 = -2 + (-2)i = -2 - 2i$

$y_3 = -2 - (-2)i = -2 + 2i$

$w = w \times w_n = -1$

$y = [6, -2-2i, -2, -2+2i]$

Hence, the solution is $[6, -2-2i, -2, -2+2i]$.

$x = (0, 2)$

$x^0 = [0]$

$x^1 = [2]$

$w = 1$

$w_n = \cos\pi + i\sin\pi = -1$

$y_k^0 = fft(x[0]) = [0]$ (no change)

$y_k^1 = fft(x[1]) = [2]$ (no change)

$y_0 = 0 + 1 \times 2 = 2$

$y_1 = 0 - 1 \times 2 = -2$

$x = (0, 2)$

$x^0 = [1]$

$x^1 = [3]$

$w = 1$

$w_n = \cos\pi + i\sin\pi = -1$

$y_k^0 = fft(x[0]) = [1]$ (no change)

$y_k^1 = fft(x[1]) = [3]$ (no change)

$y_0 = 1 + 1 \times 3 = 4$

$y_1 = 1 - 1 \times 3 = -2$

6. proof

① prove HAMPATH $\in$ NP:

proof: let HAMPATH be represented by a sequence of vertices y, and let $G\langle V, E\rangle$ be the graph, u, v be the begin and end vertex. Then we nee to verify whet er ~~hecing~~ 'd' $\langle\langle G, u, v\rangle, y\rangle$ in polynomial time:

check: ① whether y has n vertices ------ $O(n)$

② whether $y_i \in y$ are different --- $O(n)$

③ whether $(y_i, y_{i+1}) \in E$ ---- $O(n)$

④ whether $y_1 = u$ ---- $O(1)$

⑤ whether $y_n = v$ ----- $O(1)$

So, the deciding time cost is $O(n)$, so HAMPATH $\in$ NP

② reduce HAMCYCLE $\leq_p$ HAMPATH

choose a vertex w arbitrarily, add a new vetex w' and then add edge (w', t), t is one-order neibours of w. Thus, if there is a HAMCYCLE start from w and end at w, there will be a HAMPATH start from w and end at w'.

Thus, HAMCYCLE $\leq_p$ HAMPATH

Hence, HAMPATH is NPC