

**Given:**

$M$  is a DFA, accepts a string which contains equal number of 0s and 1s.

**Proof:**

For proving  $BAL_{DFA}$  is decidable use the concept of context free language which is as follows:

Now, consider a string  $w$  that contains equal no of 0s and 1s that is accepted by DFA  $M$ . If a regular language is Context Free Language then it is decidable and it is decidable then it would be recognizable as well.

Now use the approach in the same way as uses for CFL.

String containing equal no of 0s and 1s can be generated by a particular grammar pattern or can be said by using a Context Free Grammar then it'd be a Context Free Language.

Now, it is already known that each and every context free language is decidable and it can be proved as follows:

Here  $M$  is a DFA that accepts string  $w$  that contains equal no of 0s and 1s consider  $A$  be a CFG that generates  $w$ .

**Construction:**

Consider a Turing machine that decides  $w$ , and built a copy of  $w$  in Turing machine  $A$ . This process works as follows:

$T$  = Runs on input  $w$

- Run Turing Machine  $S$  on input  $\langle M, w \rangle$

Here, consider another Turing machine that generates output by considering input from  $M$  by running it on input  $w$

- If machine accepts, accept; if rejects, reject

**Conclusion:**

The above Turing machine decides  $w$  running on  $M$ , this way  $BAL_{DFA}$  is decidable.