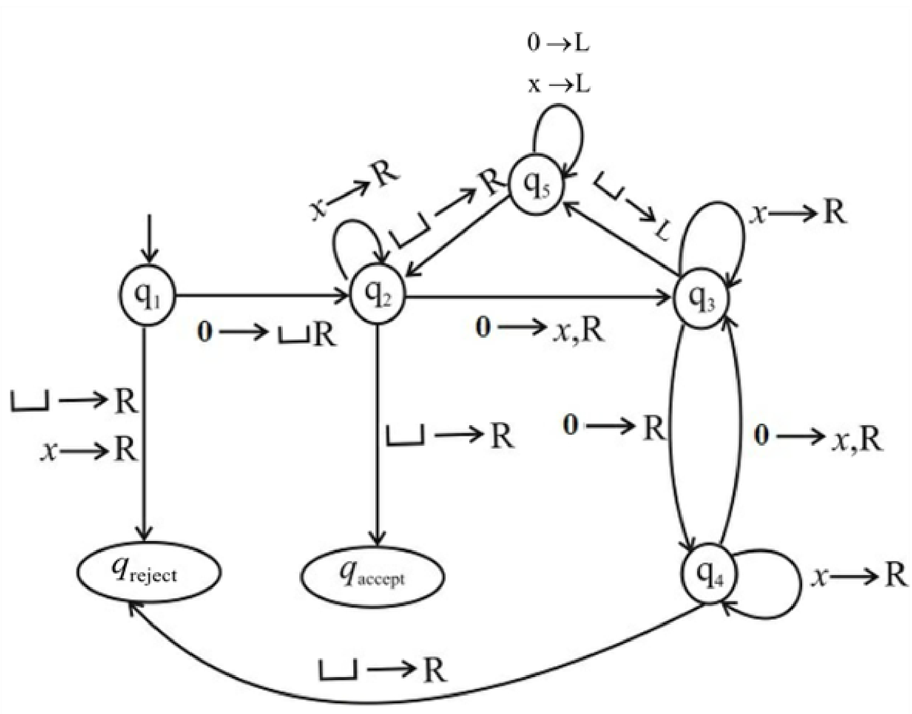
Consider the language $A = \left\{0^{2^n} \mid n \geq 0\right\}$, consisting of all strings of 0s whose length is a power of 2.

Turing machine M_2 decides a language A.

The state diagram for M_2 is as follows.



In this state diagram the label $0 \to x, R$, appears on the transition from q_4 to q_3. This label signifies that, the state q_4 with head reading 0, the machine goes to state q_3, writes *x*, and moves the head to the right. In the similar manner, other transitions also occur.

a. 0

Run the machine M_2 on the input 0. The starting configuration is $q_1 0$. The sequence of configurations that the machine enters when started on the input string is as follows:

$q_1 0$ (At q_1, $0 \to ⊔$,R goes to the q_2)
$⊔ q_2 ⊔$ (At q_2, $⊔ \to$ R goes to the accept state)
$⊔⊔ q_{accept}$

The state q_1 on 0, the machine goes to state q_2, writes ⊔ and moves the head to the right. The state q_2 on ⊔, the machine goes to state q_{accept}, then halts.

As M_2 enters q_{accept} state the input 0 is accepted.

b. 00

Run the machine M_2 on the input 00. The starting configuration is $q_1 00$. The sequence of configurations that the machine enters when started on the input string is as follows:

$q_1 00$ (At q_1, $0 \to \sqcup$,R goes to the q_2)
$\sqcup q_2 0$ (At q_2, $0 \to$ x,R goes to the q_3)
$\sqcup x q_3 \sqcup$ (At q_3, $\sqcup \to$ L goes to the q_5)
$\sqcup q_5 x \sqcup$ (At q_5, x $\to$ L goes to the q_5)
$q_5 \sqcup x \sqcup$ (At q_5, $\sqcup \to$ R goes to the q_2)
$\sqcup q_2 x \sqcup$ (At q_2, x $\to$ R goes to the accept state)
$\sqcup x q_2 \sqcup$ (At q_2, $\sqcup \to$ R goes to the accept state)

$\sqcup x \sqcup q_{accept}$

The state q_1 on 0, the machine goes to state q_2, writes $\sqcup$ and moves the head to the right. The state q_2 on 0, the machine goes to state q_3, writes x and moves the head to the right. The state q_3 on $\sqcup$, the machine goes to state q_5, moves the head to the left. The state q_5 on x, the machine goes to state q_5, moves the head to the left. The state q_5 on $\sqcup$, the machine goes to state q_2, moves the head to the right. The state q_2 on x, the machine goes to state q_2 itself, moves the head to the right. The state q_2 on $\sqcup$, the machine goes to state q_{accept}, then halts.

Finally, M_2 enters q_{accept} state. Thus, the input 00 is accepted.

c. 000

Run the machine M_2 on the input 000. The starting configuration is $q_1 000$.The sequence of configurations that the machine enters when started on the input string is as follows:

$q_1 000$ (At q_1, $0 \to \sqcup$,R goes to the q_2)
$\sqcup q_2 00$ (At q_2, $0 \to$ x,R goes to the q_3)
$\sqcup x q_3 0$ (At q_3, $0 \to$ R goes to the q_4)
$\sqcup x 0 q_4 \sqcup$ (At q_4, $\sqcup \to$ R goes to the q_{reject})
$\sqcup x 0 \sqcup q_{reject}$

The state q_1 on 0, the machine goes to state q_2, writes $\sqcup$ and moves the head to the right. The state q_2 on 0, the machine goes to state q_3, writes x and moves the head to the right. The state q_3 on 0, the machine goes to state q_4, moves the head to the right. The state q_4 on $\sqcup$, the machine goes to state q_{reject}, moves the head to the right.

Finally, M_2 enters q_{reject} state. Thus, input 000 is rejected.

d. 000000

Run the machine M_2 on the input 000000. The starting configuration is $q_1 000000$.The sequence of configurations that the machine enters when started on the input string is as follows:

$q_1000000$	(At q_1,0 $\rightarrow \sqcup$,R goes to the q_2)
$\sqcup q_200000$	(At q_2,0$\rightarrow$ x,R goes to the q_3)
$\sqcup x q_30000$	(At q_3,0 $\rightarrow$R goes to the q_4)
$\sqcup x0q_4000$	(At q_4,0 $\rightarrow$x,R goes to the q_3)
$\sqcup x0xq_300$	(At q_3,0 $\rightarrow$R goes to the q_4)
$\sqcup x0x0q_40$	(At q_4,0 $\rightarrow$x,R goes to the q_3)
$\sqcup x0x0xq_3\sqcup$	(At q_3,$\sqcup$ $\rightarrow$L goes to the q_5)
$\sqcup x0x0q_5x\sqcup$	(At q_5,x $\rightarrow$L goes to the q_5)
$\sqcup x0xq_50x\sqcup$	(At q_5,0 $\rightarrow$L goes to the q_5)
$\sqcup x0q_5x0x\sqcup$	(At q_5,x $\rightarrow$L goes to the q_5)
$\sqcup xq_50x0x\sqcup$	(At q_5,0 $\rightarrow$L goes to the q_5)
$\sqcup q_5x0x0x\sqcup$	(At q_5,x $\rightarrow$L goes to the q_5)
$q_5\sqcup x0x0x\sqcup$	(At q_5,$\sqcup$ $\rightarrow$R goes to the q_2)
$\sqcup q_2x0x0x\sqcup$	(At q_2,x $\rightarrow$R goes to the q_2)
$\sqcup xq_20x0x\sqcup$	(At q_2,0 $\rightarrow$x,R goes to the q_3)
$\sqcup xxq_3x0x\sqcup$	(At q_3,x $\rightarrow$R goes to the q_3)
$\sqcup xxxq_30x\sqcup$	(At q_3,0 $\rightarrow$R goes to the q_4)
$\sqcup xxx0q_4x\sqcup$	(At q_4,x $\rightarrow$R goes to the q_4)
$\sqcup xxx0xq_4\sqcup$	(At q_4,$\sqcup$$\rightarrow$R goes to the q_{reject})
$\sqcup xxx0x\sqcup q_{reject}$	

Finally, M_2 enters q_{reject} state. Hence, the input 000000 is rejected.