NP – complete:

A language *B* is *NP – complete* if it satisfies two conditions

1. *B* is in *NP* and

2. Every *A* is *NP* is polynomial time reducible to *B*.

$\text{PATH} = \left\{ \langle G, s, t \rangle \mid G \text{ is a directed graph that has a directed path from s to t} \right\}$

1. **PATH is not *NP – complete*:**

◊ Let us assume that PATH would be *NP – complete*.

◊ From the definition of *NP – completeness*,

For all $L \in NP, L$ is polynomial time reducible to PATH.

◊ But this again implies that for all *L* in NP, L is in *P*. Thus $P = NP$ which we believe that it is not true.

Hence, PATH is not *NP – complete*.

2. Proving that PATH is not *NP – complete* implies that $NP \neq P$:

◊ Showing this by contraposition.

◊ Assume that $P = NP$ and then show that PATH is *NP – complete*.

◊ So assume $P = NP$.

◊ "If $P = NP$ then every language $A \in P$ is *NP – complete*".So, PATH is *NP – complete*.

Thus, if PATH is not *NP –complete*, then $NP \neq P$.