

## 获得的答案

Compute descriptive complexity of strings  $K(x)$  with an oracle for  $A_{TM}$ .

- For the given string  $x$ , start testing all the strings 'S' up to the length  $|x| + c$

Where  $c$  = length of  $TM$  (Turing machine) that halts immediately upon starting.

- All the strings up to the length  $|x| + c$  are potential description of  $x$ .
- If  $S$  is well formed as  $\langle M, w \rangle$  from all binary strings in lexicographic order, then we simulate  $M$  with input  $w$  and see if it halts with  $x$  on the tape.
- Here we do not know whether  $M$  will halt on input  $w$  or not.
- An oracle for  $A_{TM}$  can determine this.

$$A_{TM} = \{ \langle M, w \rangle \mid M \text{ is a TM and } M \text{ accepts } w \}$$

- An oracle for  $A_{TM}$  will take  $\langle M, w \rangle$  as input and determine whether  $M$  accepts  $w$  or not.
- If  $M$  doesn't halt we move on to the next string  $S$ , and so on.
- After that we will find lexicographically first string  $S$  among them.
- In this way shortest string will be determined and it is represented as minimal description  $d(x)$ .
- From  $d(x)$ , we find  $K(x)$  as

$$K(x) = |d(x)|$$

- By this procedure, we will calculate  $K(x)$  with an oracle for  $A_{TM}$ .

From the above procedure,  $K(x)$  is the descriptive complexity of strings and computed with an oracle  $A_{TM}$ .

Now, we have to know the definition of incompressible strings.

Incompressible strings:

Let  $x$  be a string. If  $x$  doesn't have any description shorter than itself then  $x$  is incompressible.

Let  $f$  be the function that is computable with an oracle for  $A_{TM}$ , for each  $n$ , calculate  $f(n)$  i.e., incompressible length of string  $n$ .

Create an oracle  $TM$  (Turing machine) that does the following :

1. For a given number  $n$ , begin
2. Enumerate all strings  $x$  of length  $n$ .
3. For each  $x$ , calculate  $K(x)$  by using oracle for  $A_{TM}$ .
4. As soon as we find  $x = f(n)$  with  $K(x) \geq |x|$  output  $x$ .

[ $K(x) \geq |x|$  means getting a string which is greater than its description.]

5. Halt.