

The formula  $(x \vee y) \wedge (x \vee \bar{y}) \wedge (\bar{x} \vee y) \wedge (\bar{x} \vee \bar{y})$  is not satisfiable.

**Explanation:**

- A Boolean formula is satisfiable if some assignment of 0s and 1s to the variables makes the formula evaluate to 1.
- We know that

$$\begin{array}{lll} 0 \wedge 0 = 0 & 0 \vee 0 = 0 & \bar{0} = 1 \\ 0 \wedge 1 = 0 & 0 \vee 1 = 1 & \bar{1} = 0 \\ 1 \wedge 0 = 0 & 1 \vee 0 = 1 & \\ 1 \wedge 1 = 1 & 1 \vee 1 = 1 & \end{array}$$

Specified Boolean formula is

$$(x \vee y) \wedge (x \vee \bar{y}) \wedge (\bar{x} \vee y) \wedge (\bar{x} \vee \bar{y})$$

Here consider  $x$  and  $y$  are variables.

**Case 1:**

Assign  $x = 0$  and  $y = 1$

$$\text{Then } (0 \vee 1) \wedge (0 \vee \bar{1}) \wedge (\bar{0} \vee 1) \wedge (\bar{0} \vee \bar{1})$$

$$= 1 \wedge (0 \vee 0) \wedge (1 \vee 1) \wedge (1 \vee 0)$$

$$= 1 \wedge 0 \wedge 1 \wedge 1$$

$$= 0 \wedge 1$$

$$= 0$$

**Case 2:**

Assign  $x = 1$  and  $y = 0$

$$\text{Then } (1 \vee 0) \wedge (1 \vee \bar{0}) \wedge (\bar{1} \vee 0) \wedge (\bar{1} \vee \bar{0})$$

$$= 1 \wedge (1 \vee 1) \wedge (0 \vee 0) \wedge (0 \vee 1)$$

$$= 1 \wedge 1 \wedge 0 \wedge 1$$

$$= 1 \wedge 0$$

$$= 0$$

From case 1 and case 2 of the Boolean values for  $x$  and  $y$ , the formula always evaluated to 0, but we know that if some assignment of 0s and 1s to the variables makes the formula evaluate to 1 then a Boolean formula is satisfiable.

So, the formula  $(x \vee y) \wedge (x \vee \bar{y}) \wedge (\bar{x} \vee y) \wedge (\bar{x} \vee \bar{y})$  is not satisfiable.