Question:

Let $D = \{xy | x, y \in \{0,1\}^* \text{ and } |x| = |y| \text{ but } x \neq y\}.$ Show that D is a context-free language.

Answer:

----SETP1----

By the definition of **Context free language,** for showing that the language D is a CFL i.e. context free language, generate a context free grammar CFG G.

Consider the following grammar G:

$S \to AB|BA$

$A \to 0|0A0|0A1|1A0|1A1$

$B \to 1|0B0|0B1|1B0|1B1$

The given grammar L (G) generates the language in the form $w_1xw_2v_1yv_2$, where $|w_1|=|w_2|=k, |v_1|=|v_2|=l, x \neq y$ for $\Sigma=(0,1)^*$.

----SETP2----

• By the definition, any language which is generated by a context-free grammar is termed as a context-free language.

• The grammar generated above is a Context Free Grammar. The language D can be generated using the above context free grammar G as follows:

• A string is in D iff it can be written as xy with $|x|=|y|$ s.t. for some i, the ith character of x and y are different from one another. The above grammar can be used to obtain the required string by generating the ith characters and filling up with the remaining characters.

• The generated language $w_1xw_2v_1yv_2$ can be subjected to nested induction over k and l with case distinction over pairs (x,y).

• Now, w2 and v1 can exchange symbols because both carry symbols that are independent of the rest of the string.

• Therefore, x and y in their respective half can have the same position, which implies L (G) = L because G doesn't impose any restrictions on its language.

Hence, the given language **D** is **context free language**.