Pumping lemma:

For a regular language A with the pumping length P, if S is any string of A with minimum length of P, then the string S can be divided into 3 pieces x, y and z represented as S = xyz should satisfy the following conditions:

- for each
- |y| > 0, and
- $|xy| \le P$

a.

Consider the Language $L = \left\{0^n 1^m 0^n \mid m, n \ge 0\right\}$.

Assume that *L* is regular language and a string $S = 0^P 10^P$. Divide the string into three pieces *x*, *y* and *z*. So, $S = 0^P 10^P = xyz$ where, *P* is the pumping length.

Assume that $x = 0^{P-K}$, $y = 0^K$ and $z = 10^P$ (where K > 0)

Now
$$xy^0z = 0^{P-K} (0^K)^0 1 0^P$$

$$=0^{P-K}10^{P} \notin L \quad \left[\because y^{0} = \in \right]$$

The String xy^0z does not belong to L because P-K < P.

So, the assumption that L is regular is a contradiction. Thus, by using pumping lemma it is proved that L is not regular.

h

Consider the Language $L = \{0^m 1^n \mid m \neq n\}$.

Assume that L is regular language and string $S = 0^P 1^{P+P!}$. Divide the string into three pieces x, y and z. So, $S = 0^P 1^{P+P!} = xyz \in L$ and $|S| \ge P$ where, P is the pumping length.

Note: P! is divisible by all integers from 1 to P, where $P!=P\times (P-1)\times (P-2)\times ...\times 1$.

Assume that $x = 0^a$, $y = 0^b$ and $z = 0^c 1^{P+P!}$, where $b \ge 1$ and a + b + c = P.

Now take string $s' = xy^{i+1}z$, where $i = \frac{P!}{h}$.

Then $y^i = 0^{P!}$ so $y^{i+1} = 0^{b+P!}$, and

So
$$xyz = 0^{a+b+c+P!}1^{P+P!}$$
.

That gives $xyz = 0^{P+P!}1^{P+P!} \notin L$, a contridiction.

Here, m = P + P!, n = P + P! and m = n. This is a contradiction to the assumption because m = n. Thus, by using pumping lemma it is proved L is not regular.

c.

Consider the Language $L = \{w \mid w \in \{0,1\}^* \text{ is not a palindrome}\}$.

Assume that L is regular language.

The compliment of the language L is $\overline{L} = \{w \mid w \in \{0,1\}^* \text{ is palindrome}\}$ is also regular.

Assume a string $S = 0^P 1 \ 0^P$. Divide the string into three pieces x, y and z. So, $S = 0^P 1 \ 0^P = xyz \in L$ where, P is the pumping length.

Assume that $x = 0^{P-K}$, $y = 0^K$ and $z = 10^P$ where K > 0

Now
$$xy^0z = 0^{P-K} (0^K)^0 10^P$$

$$=0^{P-K}10^P \notin L \quad \left[\because y^0 = \in \right]$$

The String xy^0z is not same from forward and backward direction because P-K < P.

So, the string xy^0z does not belong to \overline{L} . So, the assumption is a contradiction.

Thus, by using pumping lemma it is proved L is not regular.

d.

Consider the Language
$$\ L = \left\{ wtw \, | \, w,t \in \left\{0,1\right\}^+ \right\}$$
 .

Assume that *L* is regular language.

Assume a string $S = 0^P 1 \ 0^P$. Divide the string into three pieces x, y and z. So, $S = 0^P 1 \ 0^P = xyz \in L$ where, P is the pumping length.

Assume that
$$x = 0^{P-K}$$
, $y = 0^K$ and $z = 10^P$ where $K > 0$

Now
$$xy^0z = 0^{P-K} (0^K)^0 10^P$$

$$=0^{P-K}10^P \notin L \quad \left[\because y^0 = \in \right]$$

The String $\chi V^0 Z$ does not belong to L because P-K < P and not of the form wtw as in L.

So, the assumption is a contradiction. Thus, by using pumping lemma it is proved L is not regular.