• Consider if M accepts the string $s = s_1 s_2 s_3 \ldots s_l$ of the length l then there must be some states in sequence $\left\{ \begin{array}{l} \left(q_{j_0}, q_{j_1}, q_{j_2} \ldots\ldots, q_{j_l}\right) : q_{j_0} = q_0 \\ \text{and } q_{j_i} = \delta\left(q_{j_{i-1}}, s_i\right) \end{array} \right\}$ and $\left[q_{j_l}\right] \in F$. That shows M' accept s depends on states sequence $\left\{ \begin{array}{l} \left[q_{j_0}\right], \left[q_{j_1}\right], \left[q_{j_2}\right] \ldots\ldots \left[q_{j_l}\right] : \left[q_{j_0}\right] = q_0 \\ \text{and } q_{j_i} = \delta'\left(q_{j_{i-1}}, s_i\right) \end{array} \right\}$

and $\left[q_{j_l}\right] \in F'$.

Hence, $L(M)$ is subset of $L(M')$.

• Now if some other string $u = u_1 u_2 u_3 \ldots u_l$ of l length accepted with M', consider $\left\{\left[q_{j_0}\right], \left[q_{j_1}\right], \left[q_{j_2}\right] \ldots\ldots \left[q_{j_l}\right] : \left[q_{j_0}\right] = q_0 \text{ and } q_{j_i} = \delta'\left(q_{j_{i-1}}, s_i\right)\right\}$ and $\left[q_{j_l}\right] \in F'$. By induction when u is input to M, now the corresponding sequence state which are visited by M, say

$p_0, p_1, \ldots, p_l$ such that $p_0 = q_0$ and $r_i \in \left|q_{j_i}\right| \forall i$.

As $\left[q_{j_l}\right] \in F'$ and $\left[q_{j_l}\right] \in F$, hence $p_l \in F$ therefore $L(M') \subseteq L(M)$

• Consider $\delta(q_0, s)$ is state of M after reading s after start from q_0. Two different states p and q in graph which is undirected is connected by edge iff there exist s and u strings. Such that $\delta(q_0, s) = q$ and $\delta(q_0, u) = p$.

Based on given case $|q|$ will store states q' so that s and u with $\delta(q_0, s) = q$ and $\delta(q_0, u) = q'$ such that s and u indistinguishable. For $q' \in [q]$ hence $q' = [q]$

• As statement given by theorem Myhill Nerode Deterministic Finite Automata recognize $L(M)$ must be $|Q'|$ number of states.

Hence, M' also have $|Q'|$ number of states and $L(M)$ will be equal to $L(M')$ and M' will minimal.

• Consider $|Q| = n_1$. In given algorithm Step 1 will take $O\left(n_1^2 |\Sigma| + n_1^3\right)$, by Brute force algorithm. Also 3rd 5th and 6th Step will take $O\left(n_1^2\right)$ time and each repetition will take $O\left(n_1^2 \Sigma\right)$ time.

• 10th Step will complete in $O\left(n_1^3\right)$. In 8th step it will check that either $|q| = |r|$ or not that takes $O\left(n_1^2\right)$

• When construct final Deterministic Finite Automata M', will additional take additional $O\left(n_1^2 \Sigma\right)$ time.