Consider the following language over the alphabet $\Sigma = \{0,1,+,=\}$.

$ADD = \{x = y + z \mid x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}$.

The language is said to be regular if it is satisfied by the pumping lemma, otherwise the language is not regular.

Pumping Lemma:

If *A* is any regular language then there is a number p (the pumping length) where *S* is any string that belongs to *A* of length at least *P*, then *S* may be divided into three pieces, $S = uvw$, satisfying the following conditions.

1. For each $i \geq 0, uv^i w \in A$

2. $|v| > 0$, and

3. $|uv| \leq p$

Assume that ADD is a regular language.

Let p be the pumping length given by the pumping lemma. The strings of the language *ADD* are of the form $x = y + z$. Consider a string $1^{p+1} = 10^p + 1^p \in ADD$.

Let $111 = 100 + 11$ be the string that belongs to *ADD*. The pumping length of the string is 2. To satisfy the conditions of the pumping lemma, divide the considered string into three parts. Here, *u* is equal to 1, *v* is equal to 1, *w* is equal to 1=100+11 (the remaining part of the string).

$$
\begin{aligned}
S &= 111 = 100 + 11 \\
&= \underset{u}{\underline{1}}\ \underset{v}{\underline{1}}\ \underset{w}{\underline{1 = 100 + 11}}
\end{aligned}
$$

Pump the middle part such that $uv^i w\ (i \geq 0)$. For *i*=2, the *v* becomes 11.

$$
\begin{aligned}
S &= (1)\ (1)^i\ (1 = 100 + 11) \\
&= \underset{u}{\underline{1}}\ \underset{v}{\underline{11}}\ \underset{w}{\underline{1 = 100 + 11}} \qquad [when\ i{=}2]
\end{aligned}
$$

The string after pumping is $1111 = 100 + 11$.

The string $1111 = 100 + 11 \notin ADD$ because, addition of two binary numbers 100, 11 is not equal to the binary number 1111. It is a contradiction. So, the pumping lemma is violated.

Therefore, ADD is not a regular language.