Given language is

$$D = \left\{ w \in \Sigma_2^* \mid \text{ the top row of } w \text{ is the larger number } \text{ than is the bottom row} \right\}$$

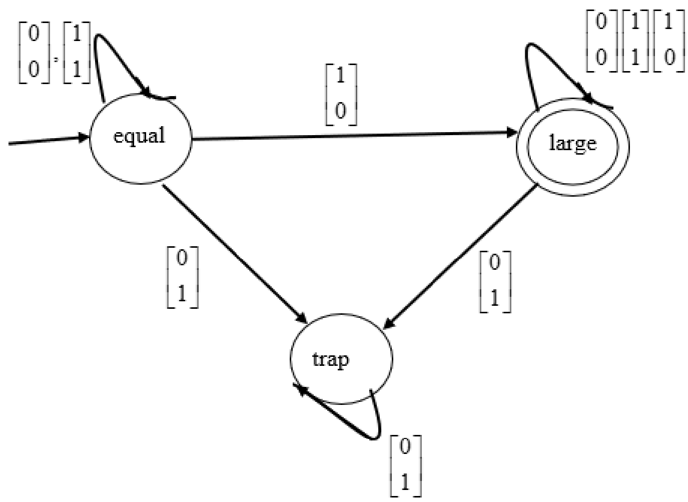
Over the alphabet $\Sigma_2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

Language for given expression L = $\left\{ \varepsilon, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \dots \right\}$

Here each row is binary number.

Let **M** be the DFA, over the input alphabet $\Sigma_2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$.

The state transition diagram of **M** is as follows:



We must prove that *D* is a regular language.

A language is said to be regular if it recognizes by a DFA.

Let take string form language D, $w = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$

Initial state of the above DFA is 'equal'

Parse string

$$\text{equal} \xrightarrow{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} \text{large} \xrightarrow{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} \text{large}$$

Here 'large' is final state , the string is accepted by the DFA.

Thus, language of given D is accepted by the given DFA.

we defined a DFA to recognize the language D.

Therefore, $\boxed{\text{D is a regular language.}}$