Suppose that $MCP = \left\{ \begin{array}{l} \langle G,N\rangle \mid G \text{ is a graph } (V,E),\ N \text{ is labeling function } N:V \to \{X,0,1,2,...\} \\ \text{and there exists mine's placement on the set of nodes labeledso that every} \\ \text{numbers are consistence} \end{array} \right\}$

The language MCP is in NP, because in polynomial time one can easily test if a placement of mines is consistence. First users have to show $3SAT \le_P MCP$, to prove that it is also NP-complete.

Consider φ is a Boolean formula, now user want to convert it to G, which is an instance of the MCP problem.

Here, suppose $c_1, c_{2,...}$ are used to denote the clause appearing in φ. Now, suppose $x_1, x_{2,...}$

Denotes the variable used in φ. Here, a variable x_i 'appears' in φ if one of x_i or $\overline{x}_i$ appear in minimum single clause in φ.

For all the variables x_i which appears in φ:

1. Three nodes are created as x_i, x_i^f and x_i^t.

2. Edges are added (x_i, x_i^t) and (x_i, x_i^f), and

3. Now, set $N(x_i)=1,\ N(x_i^t)=X$ and $N(x_i^f)=X$

For all the clause c_i which appears in φ:

1. Three nodes are created as c_i, c_i^1 and c_i^2.

2. Edges are added from c_i to the nodes corresponding to the three literals in c_i

3. Now, set $N(c_i)=3,\ N(c_i^1)=X$ and $N(c_i^2)=X$

All instances of the 3SAT problem can be reduced to an instance of the Circuit-SAT problem in polynomial time in a trivial manner by changing the Boolean operators to a circuit of logic gates. The validity of this reduction needs to be proven.

• If there is an instance of the 3SAT problem it can be converted to an instance of the Circuit-SAT problem by simply mapping Boolean operators to logic gates and connections between the gates.

• Setting the corresponding logic inputs to the Boolean values that satisfy the 3SAT problem will satisfy this instance of the Circuit-SAT problem.

• Using an instance of the Circuit-SAT problem, an equivalent instance of the SAT problem is constructed by interchanging logic gates/wires with Boolean variables and operators. If the logical values that satisfy the instance of Circuit-SAT are changed into Boolean values, then the SAT instance will be satisfied.

The Circuit-SAT problem is reducible in polynomial time to an NP-complete problem, so it is also in NP-complete. Hence, the MCP problem is NP-complete.