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a) Let C be a context free language and R be a regular language.

Now we have to prove that the language $C \cap R$ is a context free.

In order to prove, let us consider

P be the PDA (Push Down Automata) recognizes C, and

D be the DFA (Deterministic Finite Automata) recognizes R.

Now we construct a PDA that recognizes $C \cap R$ with the set of states $Q \times Q'$

where

Q be the set of states of P and

Q' is the set of states of D

Here P' will do what P does and keep track of the states of D.

The PDA that recognizes $C \cap R$ accepts the string w if and only if it stops a state $q \in F_P \times F_D$

where

F_P is the states of accepts of P and

F_D is the states of accepts of D.

So $C \cap R$ is recognized by P'.

Therefore, $C \cap R$ is context free.

b) Given the language is

$$A = \left\{ w \mid w \in \{a,b,c\}^* \text{ and contains equal number of } a\text{'s, } b\text{'s and } c\text{'s} \right\}$$

Now we have to prove A is not a CFL (Context Free Language) .

Let R be the regular language $a^* b^* c^*$.

If A were a CFL (Context Free Language) then $A \cap R$ would be a CFL (using the result proved above in part (a) of this problem).

Hence, inorder to prove that A is not a CFL it is enough to prove that $A \cap R$ is not a CFL.

We have $A \cap R = \left\{ a^n b^n c^n \mid n \geq 0 \right\}$.

We will prove $A \cap R$ is not a CFL by taking a contradiction.

Assume that $A \cap R$ is a CFL.

Using the pumping lemma, which states thet every context-free language has a special calue called *pumping length* such that all longer strings in the language can be "pumped", let p be the pumping length for $A \cap R$.

Consider a string $s = a^p b^p c^p$.

Clearly s is a member of $A \cap R$ and of length at least p .

Now we prove that one condition of pumping lemma violated by proving s cannot be pumped.

If we divide s into $wxyz$, condition 2 stipulates that either v or y is non-empty.

Now consider one of the two cases, depending on wheather substring v and y contains more than one type of alphabet symbol.

1. If both v and y contain only one type of symbol, v doesn't contain both a's and b's or both b's and c's, and the same holds for y . Here the string $uv^2 xy^2 z$ cannot contain equal number of a's, b's and c's. Therefore it cannot be a member of $A \cap R$ which violates the first condition of the pumping lemma and thus is a contradiction to our hypothsis.

2. If either v or y contain more than one type of symbol uv^2xy^2z may contain equal number of the three alphabet symbols but not in the correct order. Hence it cannot be a member of $A \cap R$ and thus is a contradiction to our hypothsis.

One of the above two case must occur. However, both the cases raised contradiction. This is because of our assumption $A \cap R$ is a CFL.

Hence our assumption is false and $A \cap R$ is not a CFL

Therefore, A is not a CFL.

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Therefore, A is not a CFL.