

Un-decidable subset of  $\{1\}^*$  exists. Existence of un-decidable subset can be found as follows:

Consider there is an  $i^{\text{th}}$  position in any string generated from  $\{1\}^*$ . It is quite obvious that there are uncountable number of strings can be made using  $\{1\}^*$ .

Let  $S$  be the subset of  $\{1\}^*$ . The  $i^{\text{th}}$  position of the generated string would be 1 if the generated string is in  $S$ . The number of subsets of  $\{1\}^*$  is equal to the infinite number of binary strings which is uncountable. The uncountable set is not Turing recognizable because there is more number of subsets than the number of Turing machines.

Therefore, as it is proved above it is not recognizable so it will be un-decidable.