

Consider a problem of determining whether a Turing machine M on input w ever attempts to move its head left at any point during its computation on w . The language that describes the problem is,

$$L = \{ \langle M, w \rangle \mid M \text{ moves its head left on input } w \}$$

Construct a Turing machine A that decides the problem.

$A =$ "On input $\langle M, w \rangle$:

1. Run the machine for $|Q|+|w|+1$ steps.
2. If the Turing machine M 's head moved to the left then accept. Otherwise, reject."

Here, $|Q|$ represents the number of states and $|w|$ represents the length of the input string. The problem is said to be decidable, if the Turing machine M moves its head left on input w within the first $|Q|+|w|+1$ steps. The problem is decidable because, there exists a Turing machine for it.

Therefore, the language L is decidable.