Let *L* be an infinite Turing recognizable language.

We know that

"A language is Turing recognizable if and only if some enumerator enumerates it"

So, let M be the enumerator that enumerates L.

Consider a language $L' = \{w_1, w_2, w_3 ....\}$ where

$\rightarrow w_1$ is the first string enumerated by *M*

$\rightarrow$ For every $i > 1$,

w_i is the first string enumerated by *M* and that is lexicographically larger than w_{i-1}.

(i) **First we prove L' is infinite and is subset of L:**

Let us assume that L' is finite

Then w_i is the last and lexicographically largest element in L' and all strings enumerated by M must be lexicographically less than w_i.

Since there are only a finite number of strings that are less than w_i, L would then be finite.

This is a contradiction because *L* is infinite.

Therefore, our assumption that L' is finite is wrong.

Hence L' is infinite.

All the strings in L' were at some point enumerated by M.

So clearly L' is subset of *L*.

(ii) **Next we have to prove L' is decidable**:

Now we will show that L' is decidable for the given enumerator *M*, we can construct an enumerator M' in lexicographic order.

M' does the following

• Let *w* be the last string that M' emitted, and initialize *w* to a dummy value that comes lexicographically before all strings.

• Simulate *M* until it emits a string *t*

• If: $t > w$ lexicographically

Then set $w = t$ and Let M' emits *t*.

• Else: Ignore *t*.

• Resume simulating *M*, and go to step 2

Thus M' is constructed and that enumerated L' in lexicographic order.

We know that,

"A language is decidable if and only if some enumerator enumerates the language in lexicographic order"

So by this theorem L' is decidable.

Therefore from (i) and (ii) L' is an infinite decidable subset of *L*.