

In order to show that context free language is closed under union operation considers two starts variable  $S_1$  and  $S_2$  for the two different languages  $L_1$  and  $L_2$ .

Grammar for union operation is as shown:

$$S \rightarrow S_1 \mid S_2$$

If both the language belongs to the context free language then union of both the language should belong to context free language.

By the above definition if user generates  $S_1$  and  $S_2$  string or both then in that case union of both the language is generated.

Hence,  $L_1 \cup L_2 \in CFL$ .

**This implies context free language is closed under union operation.**

In order to show that context free language is closed under concatenation operation considers two starts variable  $S_1$  and  $S_2$  for the two different languages  $L_1$  and  $L_2$ .

Grammar for union operation is as shown:

$$S \rightarrow S_1 S_2$$

If both the language belongs to the context free language then concatenation of both the language should belong to context free language.

$$\forall L_1, L_2 \in CFL$$

$$\{w_1 w_2 : w_1 \in L_1 \wedge w_2 \in L_2\} \in CFL$$

By the above definition if user generates  $S_1$  string for language  $L_1$  followed by  $S_2$  string of language  $L_2$ . Then it concatenation of both the language is generated.

Hence,  $\{w_1 w_2 : w_1 \in L_1 \wedge w_2 \in L_2\} \in CFL$

**This implies context free language is closed under concatenation operation.**

In order to show that context free language is closed under star operation.

Consider one start variable  $S_1$  for the languages  $L_1$

Grammar for union operation is as shown:

$$S \rightarrow S_1 S \mid \epsilon$$

If the language belongs to the context free language then star of the language should belong to context free language.

$$\forall L_1 \in CFL$$

By the above definition, if user generates zero or many string which is the definition of the star.

**This implies that context free language is closed under star operation.**