Question:

Let G be a CFG in Chomsky normal form that contains b variables. Show that if G generates some string with a derivation having at least 2^b steps, L(G) is infinite.

Answer:

----SETP1----

CFG:

• A finite set of grammar rules is known as **CFG (context free grammar)**. It consisting of that is quadruple (*N*, *T*, *P*, *S*).

• Where, set of non-terminal symbol represented by *N*.

• **T** is group of terminal $N \cap T = \text{NULL}$.

• **P** is group of rule, $P : N \to (N \cup T)*$.

• **S** is start symbol.

----SETP2----

CNF:

• In **CNF (Chomsky normal form)** we have a restriction on the length of RHS, which is a CFG (context free grammar) is in Chomsky normal form if the productions are in the following terms:

$U \to u$
$U \to VW$

• Where *U*, *V* and *W* are non-terminals and *u* is a terminal.

----SETP3----

• At most two terminal can generate in every deviation.

• In any parse string with use of *G*.

• An internal node can have at most two children.

• Parse tree with height k has at most o $2^k - 1$ internal node.

• If several string generated by *G* with a derivation taking at least 2^b steps.

• At least 2^b inner node takes parse tree of that string.

• At least consuming $b+1$ height in Parse tree.

• It exists a path from root to leaf containing $b+1$ variable.

• In this one variable occurring twice.

Hence, user can use the technique in proof of pumping lemma to construct infinity many string

which are all in L(G).