

As per the theorem 3.16, Every DTM (Deterministic Turing machine) has an equivalent NDTM (Non-deterministic Turing machine). The corollary 3.19, states that a language is decidable iff some NDTM (Non-deterministic Turing machine) decides it.

In order to modify the proof of theorem 3.16 to obtain corollary 3.19, prove the following:

(i) If a Language L is decidable then the language L can be decided by NDTM (non-deterministic Turing machine).

Let L be a language. The Language L is decidable if it is decided by a deterministic Turing machine. Any DTM (Deterministic Turing machine) is automatically a NDTM (Non-deterministic Turing machine).

Thus, language L is decidable if it is decided by NDTM (Non-deterministic Turing machine) TM also.

(ii) If some NDTM (Non-deterministic Turing machine) decides a language then it is decidable.

Let L be an any language and it is decided by NDTM (Non-deterministic Turing machine).

Construct a deterministic TM D that decides L and prove that D simulates N .

Construction of D and Simulation of D with N :

The simulating deterministic TM ' D ' has 3 tapes

1. Input tape
2. Simulating tape
3. Address tape

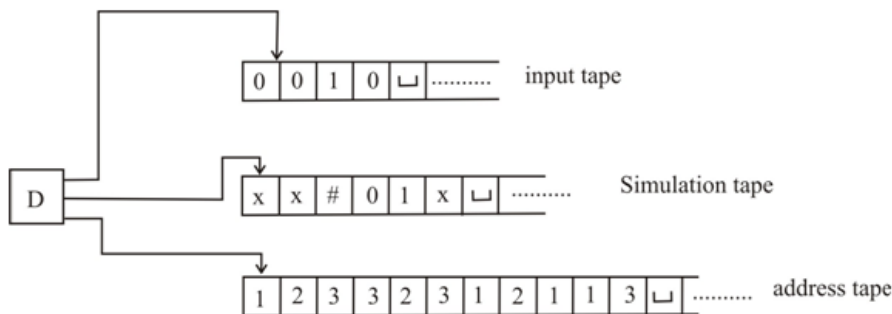
By the theorem

"Every multi tape TM has an equivalent single tape TM"

D 's arrangement is equivalent to having a single tape TM.

The machine D uses its three tapes in the following way:

- Tape 1 always contains the input string and is never altered.
- Tape 2 maintains a copy of N 's tape on some branch of its non-deterministic computation
- Tape 3 keeps track of D 's location in N 's non-deterministic computation tree.



The description of D in detail as follows:

Stage 1: Initially tape 1 contains the input w , and tapes 2 and 3 are empty.

Stage 2: Copy tape 1 to tape 2.

Stage 3: Use tape 2 to simulate N with input w on one branch of its non-deterministic computation. Before each step of N check the next symbol on tape 3 to determine which choice to make among those allowed by N 's transition function. If no more symbols left on tape 3 or if this non-deterministic choice is invalid, abort this branch by going to stage 4. Also go to stage 4 if a rejecting configuration is encountered. If an accepting configuration is encountered, then accept the input.

Stage 4: Replace the string on tape 3 with the lexicographically next string. Simulate the next branch of N 's computation by going to stage 2.

Stage 5: Reject if all branches of non-determinism of N are exhausted.

Now it can be said that D is a decider of L .

- If N accepts its input, then D will find an accepting branch and accept.
- If N rejects its input, then all its branches halt and reject.

Thus, the language L is decidable.

From (i) and (ii),

The language L is decidable if and only if some non-deterministic Turing machine decides it.