Question:

Show that F = {$a^i b^j$ | i = k_j for some positive integer k} is not context free.

Answer:

----SETP1----

The language given in the question is as follows:

Language $F = \{a^i b^j \mid i = kj \text{ for some positive integer } k\}$.

A context free language (CFL) is generated by a context free grammar. **In order to prove that a language is not a CFL, Pumping Lemma is used.**

----SETP2----

Proof that Language L is not a Context Free Language:

Assume that *F* is CFL. Obtain a contradiction using pumping lemma to prove that the assumed statement is false.

• Let *p* be the pumping length for *F* that is guaranteed to exit by pumping lemma.

• Select string $s = a^p b^{2p} \in F$ where $k = 2$ and divide the string *s* into *uvxyz*.

• According to pumping lemma, *v* and *y* in string cannot be empty sets.

• Now, consider these two case, depending on whether substring *v* and *y* contain more than one type of alphabet symbol:

1. **Both *v* and *y* contain only one type of alphabet symbol:** In this case, both $v \, and \, y$ does not contain mixed a’s and b’s. Thus, the string uv^2xy^2z cannot contain equal number of a’s and b’s. Also, a pattern for a’s and b’s can be obtained that contains a relation between number of a’s and b’s. So, none of the conditions of lemma violates and thus it does not contradict.

For example:

Consider uv^ixy^iz such that $v = a, y = bb$ and $u = x = z = \phi$. Thus, the strings generated will be $s = abb, aabbbb, aaabbbbbb...$

All the strings *s* are a member of *F*. Hence, no contradiction is obtained.

2. **Either of *v* or *y* contains more than one type of alphabet symbols**: In this case, both $v \, and \, y$ contain mixed a’s and b’s. Thus, the string uv^2xy^2z will contain strings with some order of ab followed by some order of ab again. Thus, it produces a wrong order of strings thereby producing a contradiction

For example:

Consider uv^ixy^iz such that $v = ab \, and \, y = b$ and $u = x = z = \phi$. Thus the strings generated will be $s = abb, ababbb...$

The string s = $ababbb$ is not member of *F*. This violates our assumption and thus, a contradiction is obtained.

----SETP3----

The second case results in a contradiction. Hence, the assumption that *F* is context free language is false and therefore, F is not a context free language.