**Class *NP* :**

*NP* is a class of languages that are nondeterministic polynomial time on a non – deterministic single – tape Turing Machine.

From the definition 7.19 *NP* is the class of languages that have polynomial time verifies

Consider the given expression:

ISO $=\{\langle G,H\rangle \mid G \text{ and } H \text{ are isomorphic graphs}\}$

• If the nodes of *G* may be reordered so that it is identical to *H* then Graphs *G* and *H* are said to be isomorphic.

• Now it must be proved that $ISO \in NP$

• Let $G=(V_G,E_G)$ and $H=(V_H,E_H)$ be the two graphs

• Let $V_G=\{u_1,u_2,\ldots u_m\},V_H=\{v_1,v_2,\ldots v_n\}$ be the sets of vertices of *G* and *H*.

**Isomorphism:**

An isomorphism is defined by a mapping $f:V_G \to V_H$ with the property that it is a one – to –one correspondence. That means it is both one – to – one and onto.

• This one – to – one correspondence is possible only if $m=n$ and for all $u,v\in V_G$ we have $(v,v)\in E_G$ if and only if $(f(u),f(v))\in E_H$.

• Thus, the correspondence takes edges into edges and non – edges into non – edges.

• A mapping *f* can be represented. By a sequence $S=(S_1,S_2,\ldots S_m)$ of indices with the property that $f(u_i)=v_{s_i}$, that is $i^{th}$ point of *G* is mapped into the $S_i^{th}$ point of *H*.

• This sequence *S* can be taken as certificate.

Now *N* is the non – deterministic Turing machine $(NTM)$ that decides *ISO* in polynomial time.

$N=$ "On input $(\langle G,H\rangle,S)$:

Where *G* and *H* are graph as defined above *S* is the certificate.

1. Check whether *G* and *H* have same number of points.

2. If *G* and *H* have same number of points then checks that for each pair $i,j$

i. $E_U$ can be derived from the above mapping procedure,

$$\begin{aligned}&\Rightarrow (v_{S_i},v_{S_j})\in E_U \quad \ldots\ldots(1)\\&\Rightarrow (u_i,u_j)\in E_U \quad \ldots\ldots(2)\\&From\ (1)\ and\ (2)\\&f(u_i)=v_{s_i}=E_U\end{aligned}$$

ii. have $S_i\neq S_j$ and that $(u_i,u_j)\in E_U$ if and only if $(v_{S_i},v_{S_j})\in E_U$

iii. If the above condition satisfies, then "accept".

3. Otherwise "reject".

All these checking can be done in time $O(m^2)$, so in time polynomial in the description of $(G,H)$. **Therefore** $ISO\in NP$.