Question:

Let B be the language of all palindromes over {0,1} containing equal numbers of 0s and 1s. Show that B is not context free.

Answer:

----SETP1----

Let B be the language of all palindromes over $\{0, 1\}$ containing the equal numbers of 0's and 1's. To prove B is not a context free language by taking a contradiction.
Assume that B is a context free language.

Since B is a context free language, then by pumping lemma, there is a number p (the pumping length) where, if s is any string in B of length at least p, then s may be divided into five pieces $s = uvxyz$ satisfying the conditions

1. for each $i \geq 0, uv^i xy^i z \in A,$
2. $|vy| > 0$, and
3. $|vxy| \leq p.$

----SETP2----

Case 1:

Now select a string $s = 0^n 1^{2n} 0^n$.

Clearly s is a member of B of length at least p.

Assume the value of n=2 for the string *s*.

s = $0^2 1^4 0^2$

s = 00111100

The string s can be divided into *uvxyz* as follows:

$$\frac{00}{u}\ \frac{11}{v}\ \frac{1}{x}\ \frac{10}{y}\ \frac{0}{z}$$

Now apply the first condition of the pumping lemma.

for each $i \geq 0, uv^i xy^i z \in A$

For i = 2:

$$\frac{00}{u}\left(\frac{11}{v}\right)^2 \frac{1}{x}\left(\frac{10}{y}\right)^2 \frac{0}{z}$$

$$\frac{00}{u}\left(\frac{1111}{v}\right) \frac{1}{x}\left(\frac{1010}{y}\right) \frac{0}{z}$$

Assume the obtained string 00 1111 1 1010 0 as s'.

The obtained string s' is not a palindrome after applying the first condition of pumping lemma and $s' \notin$ B. In the pumped string, the number of 0’s and 1’s is not equal.

So, the language B is not following the condition1 of the pumping lemma.

----SETP3----

Case 2:

The same string is selected as $s = 0^n 1^{2n} 0^n$

Clearly s is a member of B of length at least p.

Assume the value of n=2 for the string s.

$s=0^2 1^4 0^2$

s=00111100

The string s can be divided into *uvxyz* as follows:

$$\frac{0}{u}\ \frac{01}{v}\ \frac{11}{x}\ \frac{1}{y}\ \frac{00}{z}$$

Now apply the first condition of the pumping lemma.

$$\text{for each } i \geq 0, uv^i xy^i z \in A$$

For i=2:

$$\frac{0}{u}\left(\frac{01}{v}\right)^2 \frac{11}{x}\left(\frac{1}{y}\right)^2 \frac{00}{z}$$

$$\frac{0}{u}\left(\frac{0101}{v}\right) \frac{11}{x}\left(\frac{11}{y}\right) \frac{00}{z}$$

Assume the obtained string 0 0101 11 11 00 as s'.

The obtained string s' is not a palindrome after applying the first condition of pumping lemma and $s' \notin \mathrm{B}$. In the pumped string, the number of 0’s and 1’s is not equal.

Hence, the assumption B is a context free language is wrong.

Therefore, by the two cases it can be proved that B is not a context free language.