

获得的答案

Consider the problem statement provided in the textbook.

Let $T = \{ \langle M \rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w \}$.

- It is already known that $L = \{ \langle w, M \rangle : w \text{ is accepted by } M \}$ is undecidable.
- Assume that T is decidable, then there must exist a TM by which T can be decided. Let's say P is the Turing Machine that decides T .

For any input $\langle w, M \rangle$, M' can be constructed as follows:

If $w = w^R$, simulate M on w . The Σ is the alphabet set of M and let $a, b \notin \Sigma$.

Let $\Sigma \cup \{a, b\}$ be the alphabet set of M' . Then for input ab , M' will reject all the other strings except ab .

Now, simulate M on w .

- If M accepts w , M' rejects.
- If M rejects w , M' accepts.

Claim: P accepts M' iff M accepts w .

Proof: If P accepts M' . Since, M' rejects all the other strings which include ba also, then M' rejects ab which implies M accepts w .

If w is accepted by M , then M' rejects ab . Since, M' rejects all the other strings, M' is accepted by P .

Now, construct a TM, Q for L . Construct M' on input $\langle w, M \rangle$ and run P on it. Q accepts iff P accepts.

This contradicts the fact that L is undecidable.

Therefore, T is undecidable. Hence Proved.