Course Name: ___计算理论 A___   Lecturer: ___王轩___

Question	One	Two	Three	Four	Five	Six	Seven	Eight	Total
Mark									

Question One: For each of these questions below, select the correct answer. (3%*10=30%)

(1) Suppose L_1 and L_2 are regular languages. Then ___ is not regular language.
   a) $L_1 \cup L_2$ b) $L_1 \circ L_2$ c) L_1* d) neither of a,b,c

(2) Language $\{a^i b^i c^k | i=j$ or $j=k$ where $i,j,k \geq 0\}$ is ___.
   a) regular language   b) context-free language
   c) decidable language   d) Turing-recognizable

(3) ___ is not a decidable language.
   a) A_{DFA} b) E_{DFA} c) A_{LBA} d) E_{LBA}

(4) ___ is wrong for Turing machine.
   a) $aaq_i aa$ yields $aq_j aba$ b) $aaq_i aa$ yields $aabq_j a$
   c) $aaq_i aa$ yields $aq_j baa$ d) $aaq_i aa$ yields $aq_j aaa$

(5) which of the following is correct ___

   a) If $A \leq_m B$ and B is Turing-recognizable, then A is Turing-recognizable.

   b) If $A \leq_m B$ and A is not Turing-recognizable, then B is not Turing-recognizable.

   c) EQ_{TM} is neither Turing-recognizable nor co-Turing-recognizable.

   d) All of a,b,c

(6) Let M be an LBA with q states and g symbols in the tape alphabet. There are exactly ___ distinct configurations of M for a tape of length n.

   a) qng^n

   b) ngq^n

   c) qng^g

   d) ngq^g

(7) Let $t(n)$ be a function, where $t(n) \geq n$. Then every $t(n)$ time multi-tape Turing machine has an equivalent ___ time single-tape Turing machine.
   a) $O(t(n))$ b) $O(t^2(n))$ c) $O(t^3(n))$ d) $2^{O(t(n))}$

(8) ___ is not correct about CFG and PDA.
   a) There is no rule of the form A->bC or A->Bc in Chomsky normal form.
   b) A string w is derived ambiguously in context-free grammar G if it has two or more different rightmost derivations.
   c) A pushdown automata has an extra component called stack.
   d) If a pushdown automaton recognizes some language, then it is context free.

(9) ___ is not correct about P and NP.

   a) A language in P is decided by a polynomial time Turing machine
   b) A language in NP is decided by a polynomial time Turing machine
   c) If B is NP-complete and $B \in P$, then P=NP
   d) If B is NP-complete and $B \leq_p C$ for C in NP, then C is NP-complete

(10) ___ is wrong.
   a) a language in P is decided by a polynomial time Turing machine
   b) a language in NP is decided by a polynomial time Turing machine
   c) a language in NP is decided by a polynomial time nondeterministic Turing machine
   d) a language in NP is verified by a polynomial time Turing machine

Question Two:

Prove: $HALT_{TM} = \{<M,w>| $ M is a TM and M halts on input w$\}$ is undecidable. (10 %)

Question Three:
Convert the following nondeterministic finite automata (NFA) to deterministic finite automata (DFA) and regular expression (RE).(10 %)
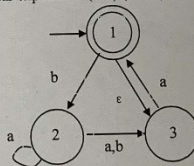
Question Four:

Let the TM be M=$(\{q_1,q_2,q_3\},\{0,1\},\{0,1,B\}, \delta ,q_1,B,\{q_2\})$, where δ consists of the rules:

$\delta (q_1,1)=(q_3,0,R)$
$\delta (q_3,0)=\{q_1,1,R\}$
$\delta (q_3,1)=\{q_2,0,R\}$
$\delta (q_3,B)=\{q_3,1,L\}$

1) The starting state will always be q_1, and q_2 will be the only accepting state.
2) The tape symbols are $X_1, X_2, ..., X_s$. X_1 will be the symbol 0, X_2 will be 1 and X_3 will be B, the blank.
3) We refer to direction L as D_1 and direction R as D_2.

Give the encoding sequence of TM M as what we talked about in class. (10%)

Question Five:
What is the different from P and NP class? What kind of problems is called NP-complete problems? You can answer this question in your own words. (10 %)

Question Six:
For the language $A = \{0^n 1^n | n \geq 0\}$, there is a TM M1 that decides it.

M1="On input string w:
1. Scan across the tape and *reject* if a 0 is found to the right of a 1.
2. Repeat the following as long as some 0s and some 1s remain on the tape:
3.   Scan across the tape, checking whether the total number of 0s and 1s remaining is even or odd. If it is odd, *reject*.
4.   Scan again across the tape, crossing off every other 0 starting with the first 0, and then crossing off every other 1 starting with the first 1.
5. If no 0s and no 1s remain on the tape, *accept*. Otherwise, *reject*."

Prove the correctness of the TM and analysis its time complexity. (10%)

Question Seven:
Prove PATH $\in$ P:

PATH = $\{<G,s,t>|$ G is directed graph that has a directed path from s to t$\}$. (10 %)

Question Eight:
Turing machine can decide the language C={ $a^i b^j c^k$ | i*j=k and i,j,k $\geq$ 1}. Please describe the process of recognization in your own words. (10%)

Student Name:

Student ID:

Major:

答题内容写在边线外视为无效

Course Name: 计算理论 B　　　　**Lecturer:** 王轩

Question	One	Two	Three	Four	Five	Six	Seven	Eight	Total
Mark									

Question One: For each of these questions below, select the correct answer. (3%*10=30%)

(1) Suppose L_1 and L_2 are regular languages. Then _____ is not regular language.
　　a) $L_1 \cup L_2$　　b) $L_1 \circ L_2$　　c) L_1*　　d) neither of a,b,c

(2) Language $\{a^i b^j c^k | i=j \text{ or } j=k \text{ where } i,j,k \geq 0\}$ is _____.
　　a) regular language　　　b) context-free language
　　c) decidable language　　d) Turing-recognizable

(3) _____ is wrong for Turing machine.
　　a) $aaq_i aa$ yields $aq_i aba$　　　　b) $aaq_i aa$ yields $aabq_i a$
　　c) $aaq_i aa$ yields $aq_i baa$　　　　d) $aaq_i aa$ yields $aq_i aaa$

(4) _____ is not a regular language.
　　a) $\{w | w \text{ begins with a 1 and ends with a 0}\}$
　　b) $\{w | \text{every odd position of } w \text{ is a 1}\}$
　　c) $\{w | w \text{ contains an even number of 0s, or contains exactly two 1s}\}$
　　d) $\{w | \text{the number of 0s and 1s in } w \text{ are equal}\}$

(5) _____ is not correct about variants of Turing machine.
　　a) Turing machines are robustness.
　　b) A language is Turing-recognizable if some multitape Turing machine recognizes it.
　　c) We use deterministic TM D to simulate nondeterministic TM N by a three-tape Turing machine. The three tapes are input tape, simulation tape and address tape
　　d) The single-tape Turing machine and the multitape Turing machine have the same function and efficiency

(6) _____ is not a decidable language.
　　a) A_{DFA}　　b) E_{DFA}　　c) A_{LBA}　　d) E_{LBA}

(7) Let $t(n)$ be a function, where $t(n) \geq n$. Then every $t(n)$ time multi-tape Turing machine has an equivalent _____ time single-tape Turing machine.
　　a) $O(t(n))$　　b) $O(t^2(n))$　　c) $O(t^3(n))$　　d) $2^{O(t(n))}$

(8) _____ is not correct about CFG and PDA.
　　a) There is no rule of the form A->bC or A->Bc in Chomsky normal form.
　　b) A string w is derived ambiguously in context-free grammar G if it has two or more different rightmost derivations.
　　c) A pushdown automata has an extra component called stack.
　　d) If a pushdown automaton recognizes some language, then it is context free.

(9) _____ is not P class.
　　a) PATH={<G,s,t>|G is a directed graph that has a directed path from s to t}.
　　b) SUBSET-SUM ={<S,t>| S={$x_1, ..., x_k$} and for some {$y_1,...,y_l$} $\subseteq$ {$x_1, ..., x_k$} , we have $\Sigma y_i = t$}.
　　c) RELPRIME = {<x,y>|x andy are relatively prime}.
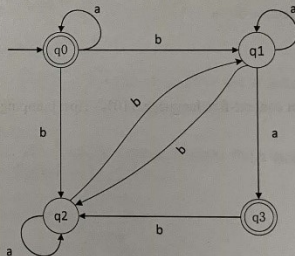　　d) Every context-free language.

(10) _____ is wrong.
　　a) a language in P is decided by a polynomial time Turing machine
　　b) a language in NP is decided by a polynomial time Turing machine
　　c) a language in NP is decided by a polynomial time nondeterministic Turing machine
　　d) a language in NP is verified by a polynomial time Turing machine

Question Two:
Prove that Language A= { $0^n 1^n 2^n$ | $n \geq 0$} is not regular. (10 %)

Question Three:
Convert the following nondeterministic finite automata (NFA) to deterministic finite automata (DFA) regular expression (RE).Show every step. (12 %)

Question Four:
Let the TM be M=({q_1,q_2,q_3},{0,1},{0,1,B}, δ ,q_1,B,{q_2}), where δ consists of the rules:
　δ (q_1,1)=(q_3,0,R)
　δ (q_3,0)={q_1,1,R}
　δ (q_3,1)={q_2,0,R}
　δ (q_3,B)={q_3,1,L}

1) The starting state will always be q_1, and q_2 will be the only accepting state.
2) The tape symbols are $X_1, X_2, ..., X_s$. X_1 will be the symbol 0, X_2 will be 1 and X_3 will be B, the blank.
3) We refer to direction L as D_1 and direction R as D_2.

Give the encoding sequence of TM M as what we talked about in class. (10%)

Question Five:

Prove PATH $\in$ P: (10 points)

PATH={<G,s,t>| G is directed graph that has a directed path from s to t}   (8%)

Question Six:

For the language A={0^n1^n|n≥0}, there is a TM M1 that decides it.

M1="On input string w:

1. Scan across the tape and *reject* if a 0 is found to the right of a 1.
2. Repeat the following as long as some 0s and some 1s remain on the tape:
3.   Scan across the tape, checking whether the total number of 0s and 1s remaining is even or odd. If it is odd, *reject*.
4.   Scan again across the tape, crossing off every other 0 starting with the first 0, and then crossing off every other 1 starting with the first 1.
5. If no 0s and no 1s remain on the tape, *accept*. Otherwise, *reject*."

Prove the correctness of the TM and analysis its time complexity. (10%).

Question Seven:

Turing machine can decide the language C={ $a^ib^jc^k$ | $i*j=k$ and i,j,k≥1}. Please describe the process of recognization in your own words. (10%)

Question Eight:

Prove that Language A={ $a^ib^jc^k$ |0≤ i≤ j≤ k } is not context-free language. (10%) Tips: pumping lemma