

## 获得的答案

**The class NP is closed under union and concatenation****NP – class:**

NP is a class of languages that are decidable in nondeterministic polynomial time on a non – deterministic Turing machine.

**Union:**

- Let  $A$  and  $B$  be languages are decided by NP – machines  $T_A$  and  $T_B$ .
- Now we want to show that, there is a non – deterministic poly time decider  $T_{A \cup B}$  that decides union of  $A$  and  $B$ .
- The construction of  $T_{A \cup B}$  is as follows:

$T_{A \cup B}$  = " On input  $S$ :

1. Run  $T_A$  on  $S$ . If  $T_A$  accepts  $S$ , then accept.
2. Else run  $T_B$  on  $S$ . If  $T_B$  accepts  $S$ , then accept.
3. Else reject"

As the new TM  $T_{A \cup B}$  calls  $T_A$  and  $T_B$  each once, it runs on  $O(T_A + T_B)$ ,

as both are NP is  $T_{A \cup B}$ .

**Concatenation:**

- Let  $A$  and  $B$  be languages are decided by NP – machines  $T_A$  and  $T_B$ .
- Now we want to show that, there is a non – deterministic poly time decider  $T_{A \circ B}$  that decides concatenation of  $A$  and  $B$ .
- The construction of  $T_{A \circ B}$  is as follows:

$T_{A \circ B}$  = "On input  $S$ :

1. Split  $S$  into  $S_1, S_2$  such that  $S = S_1 S_2$ .
2. Run the NP machine  $T_A$  on  $S_1$ . If  $T_A$  is rejected, then reject.
3. Else run  $T_B$  on  $S_2$ . If  $T_B$  is rejected, then reject.
4. Else accept.

The time taken by step 1 is  $O(n)$  in a two tape Turing Machine. Thus,  $T$  is a poly-time non-deterministic decider for  $A \circ B$ .