

### NP –Complete:

A language  $B$  is NP-complete if it satisfies 2 conditions

1.  $B$  is in NP
2. Every  $A$  in NP is polynomial time reducible to  $B$ .

#### 1. SET – SPLITTING is in NP :

SET – SPLITTING is in NP because we can verify in polynomial time that no subset  $C_i$  is monochromatic.

#### 2. $3 SAT \leq_p SET - SPLITTING$ :

To prove that the problem is NP complete, we give a polynomial time reduction from 3SAT to SET-SPLITTING.

Given an instance of 3SAT  $\phi$ , set  $S = \{x_1, \bar{x}_1, \dots, x_n, \bar{x}_n, y\}$ , where  $x_i$ 's are the variables and  $y$  is a special color variable.

#### The splitting is done as follows:

For every clause  $C_i$  in  $\phi$ , Let  $C_i$  be a subset of  $S$  containing the elements corresponding to the literals in  $C_i$  and the special elements  $y \in S$ . Then  $C = C_1, \dots, C_k$

If  $\phi$  is satisfiable, consider a satisfying assignment.

If we color all the true literals red, all the false ones are blue, and  $y$  blue, then every subset  $C_i$  of  $S$  has at least one red element (because it is satisfiable and it also contain one blue element  $y$ ).

In addition, for a given splitting  $\langle S, C \rangle$ , we can able to set the literals that are colored differently from  $y$  to true.

In the same way, we can able to set the literals that have the same color as  $y$  to false.

This concludes that satisfying assignment for  $\phi$ .

**Thus, SET – SPLITTING is NP-Complete.**