

Theory of  $N$  :-

If  $N$  is a model, theory of  $N$ , written  $Th(N)$ , be the collection of true sentences in the language of that model.

• Given sentence is  $\phi_1 = \exists x \forall y [x + y = y]$  and the given theory of model is  $Th(N, +)$ .

The statement  $\phi_1$  is true in model  $(N, +)$ . Because for  $N = \{0, 1, 2, 3, \dots\}$  and for  $x = 0$  the statement  $0 + y = y$  is true.

So the statement  $\exists x \forall y [x + y = y]$  is a member of  $Th(N, +)$

• Given statement is  $\phi_2 = \exists x \forall y [x + y = x]$

This statement  $\phi_2$  is false in the model  $(N, +)$

Because for any  $x$  the statement  $x + y = x$  is false, for all  $y$  (except if  $y = 0$ ).

So the statement  $\phi_2$  is false in the model  $(N, +)$

Hence the statement  $\exists x \forall y [x + y = x]$  is not a member of  $Th(N, +)$