Chapter 4.

Question 1.

The procedure FIND-MAX pseudo code is as following.

```
FIND_MAX(T)

    max ← -INF

    for i ← 0 to m-1 do

        if T[i] ≠ NIL and max < T[i] then

            max = T[i]

    return max
```

It is obvious that the worst case running time is $O(m)$.

Question 2.

$h(5) = 5\%9 = 5$

$h(28) = 28\%9 = 1$

$h(19) = 19\%9 = 1$

$h(15) = 15\%9 = 6$

$h(20) = 20\%9 = 2$

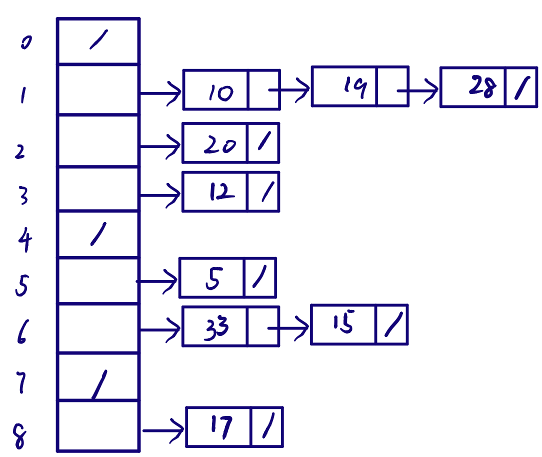
$h(33) = 33\%9 = 6$

$h(12) = 12\%9 = 3$

$h(17) = 17\%9 = 8$

$h(10) = 10\%9 = 1$

the table T is :



Question . 3

$m = 1000$, $A = (\sqrt{5} - 1)/2$, $h(k) = \lfloor m(k \cdot A \bmod 1) \rfloor$

$$\therefore \; h(61) = \left\lfloor 1000 \times \left(61 \times \frac{\sqrt{5}-1}{2} - \left\lfloor 61 \times \frac{\sqrt{5}-1}{2} \right\rfloor \right) \right\rfloor = 700$$

$$h(62) = \left\lfloor 1000 \times \left(62 \times \frac{\sqrt{5}-1}{2} - \left\lfloor 62 \times \frac{\sqrt{5}-1}{2} \right\rfloor \right) \right\rfloor = 318$$

$$h(63) = \left\lfloor 1000 \times \left(63 \times \frac{\sqrt{5}-1}{2} - \left\lfloor 63 \times \frac{\sqrt{5}-1}{2} \right\rfloor \right) \right\rfloor = 936$$

$$h(64) = \left\lfloor 1000 \times \left(64 \times \frac{\sqrt{5}-1}{2} - \left\lfloor 64 \times \frac{\sqrt{5}-1}{2} \right\rfloor \right) \right\rfloor = 554$$

$$h(65) = \left\lfloor 1000 \times \left(65 \times \frac{\sqrt{5}-1}{2} - \left\lfloor 65 \times \frac{\sqrt{5}-1}{2} \right\rfloor \right) \right\rfloor = 172$$

Question 4 .

$h'(k) = k \bmod m$, $m = 11$, key $= \{10, 22, 31, 4, 15, 28, 17, 88, 59\}$

(1) linear probing. $h(k,i) = (h'(k) + i) \bmod m$

① $h(10,0) = 10 \% 11 = 10$

② $h(22,0) = 22 \% 11 = 0$

③ $h(31,0) = 31 \% 11 = 9$

④ $h(4,0) = 4 \% 11 = 4$

⑤ $h(15,0) = 15 \% 11 = 4$,
collision
$h(15,1) = 16 \% 11 = 5$

⑥ $h(28,0) = 28 \% 11 = 6$

⑦ $h(17,0) = 17 \% 11 = 6$
collision
$h(17,1) = 18 \% 11 = 7$

⑧ $h(88,0) = 88 \% 11 = 0$
collision
$h(88,1) = 89 \% 11 = 1$

⑨ $h(59,0) = 59 \% 11 = 4$
collision
$h(59,1) = 60 \% 11 = 5$
collision
$h(59,2) = 61 \% 11 = 6$
collision
$h(59,3) = 62 \% 11 = 7$
collision
$h(59,4) = 63 \% 11 = 8$

0	22
1	88
2	
3	
4	4
5	15
6	28
7	17
8	59
9	31
10	10

(2) quadratic probing, $c_1=1$, $c_2=3$, $h(k,i) = (h'(k)+i+3i^2) \bmod m$.

① $h(10,0) = 10\%11 = 10$

② $h(22,0) = 0\%11 = 0$

③ $h(31,0) = 9\%11 = 9$

④ $h(4,0) = 4\%11 = 4$

⑤ $h(15,0) = 4\%11 = 4$, collision

$h(15,1) = (4+1+3)\%11 = 8$,

⑥ $h(28,0) = 6\%11 = 6$

⑦ $h(17,0) = 6\%11 = 6$

collision

$h(17,1) = (6+1+3)\%11 = 10$

collision

$h(17,2) = (6+2+12)\%11 = 9$

collision

$h(17,3) = (6+3+27)\%11 = 3$

⑧ $h(88,0) = 0\%11 = 0$

collision

$h(88,1) = (0+1+3)\%11 = 4$

collision

$h(88,2) = (0+2+12)\%11 = 3$

collision

$h(88,3) = (0+3+27)\%11 = 8$

collision

$h(88,4) = (0+4+48)\%11 = 8$

collision

$h(88,5) = (0+5+75)\%11 = 3$

collision

$h(88,6) = (0+6+108)\%11 = 4$

collision

$h(88,7) = (0+7+147)\%11 = 0$

collision

$h(88,8) = (0+8+192)\%11 = 2$

$h(59,0) = 4\%11 = 4$

collision

$h(59,1) = (4+1+3)\%11 = 8$

collision

$h(59,2) = (4+2+12)\%11 = 7$

0	22
1	
2	88
3	17
4	4
5	
6	28
7	59
8	15
9	31
10	10

(3) double hashing. $h_1(k) = k \bmod m$, $h_2(k) = 1 + k \bmod (m-1)$

$h(k,i) = [h_1(k) + i \cdot h_2(k)] \bmod m$.

① $h(10,0) = 10 \% 11 = 10$

② $h(22,0) = 0 \% 11 = 0$

③ $h(31,0) = 9 \% 11 = 9$

④ $h(4,0) = 4 \% 11 = 4$

⑤ $h(15,0) = 4 \% 11 = 4$

collision

$h(15,1) = (4 + 1 \times 6) \% 11 = 10$

collision

$h(15,2) = (4 + 2 \times 6) \% 11 = 5$

⑥ $h(28,0) = 6 \% 11 = 6$

⑦ $h(17,0) = 6 \% 11 = 6$

collision

$h(17,1) = (6 + 1 \times 8) \% 11 = 3$

⑧ $h(88,0) = 0 \% 11 = 0$

collision

$h(88,1) = (0 + 1 \times 9) \% 11 = 9$

collision

$h(88,2) = (0 + 2 \times 9) \% 11 = 7$

⑨ $h(59,0) = 4 \% 11 = 4$

collision

$h(59,1) = (4 + 1 \times 10) \% 11 = 3$

collision

$h(59,2) = (4 + 2 \times 10) \% 11 = 2$

0	22
1	
2	59
3	17
4	4
5	15
6	28
7	88
8	
9	31
10	10

Question 5.

Firstly, according to Group Theory, when $h_2(k)$ and the table size m are relatively prime, namely, $gcd(h_2(k), m) = 1$ and then, a mod-m addition cyclic group could be generated by some generator $h_2(k)$,

According to the properties of the group, it contains all integers from 0 to $m-1$, in other word, using $(h_1(k) + ih_2(k)) \% m$ can iterate trough all the table when $d = gcd(h_2(k), m) = 1$.

However, if $d = gcd(h_2(k), m) > 1$, it will cause whe collision happens, the probing stride increase to d. But d is a factor of m, it will make some positions cannot be visit. For example, in this case, the position sequence could be $\{0, d, 2d, \cdots, m-d\}$, so, only $(\frac{1}{d})$th positions in table can be probin; and it will cause possibly no position for elements even if the table is not be completely filled yet.