Turing – recognizable Language:-

Language is Turing – recognizable language if some Turing machine recognizes it.

(a) **:**

Let L_1 and L_2 be the Turing – recognizable languages that are recognized by the machines M_1 and M_2

We want to show that

• There is a Turing machine M' such that $L(M') = L_1 \cup L_2$.

The description of M' is as follow:

$M' =$ "on input *w*:

1. Run M_1 on *w*. if M_1 accepts then ***accept***

2. Else run M_2 on *w*. if M_2 accept then ***accept***

3. Else ***reject."***

If any of M_1 and M_2 accept *w*, then M' will accept *w*.

Since the accepting TM will come to its accepting state after a finite number of steps.

If both M_1 and M_2 reject and either of them does so by looping then M' will loop.

Thus $L(M') = L_1 \cup L_2$ and Turing recognizable languages are closed under union.

(b) **Concatenation:**

Let L_1 and L_2 be the Turing – recognizable languages run by the machines M_1 and M_2

We want to show that

• There is a Turing machine M' such that $L(M') = L_1 o L_2$.

The description of M' is as follow:

$M' =$ "on input *w*:

1. Non – deterministically cut input *w* into w_1 and w_2.

2. Run M_1 on w_1. If it halts and rejects, ***reject***.

3. Else Run M_2 on w_2. If M_2 rejects then ***reject***

4. Else ***accept***.

Note the difference between the Turing machines for recognizable and decidable languages, here we need to take care of the fact that the machines M_1 and M_2 need not halt. Thus $L(M') = L_1 o L_2$ and Turing – recognizable languages are closed under concatenation.

(c) **Star:**

Let L_1 be the Turing – recognizable language that are recognized by the machine M_1.

We want to show that

• there is a Turing machine M' such that $L(M') = L_1^*$

The description of M' is as follow:

$M' =$ "On input *w*:

1. Cut *w* into parts $w_1, w_2, \ldots w_n$

2. Run M_1 on w_i for $i = 1, \ldots n$

3. If M_1 accepts all of them, ***accept***

4. if M_1 halts and rejects for any *i*, ***reject***"

If there is a way to cut *w* into strings $w_1 w_2 \ldots w_n$ such that each w_i ? L_1, then there is a computation path in M' that accepts *w* in a finite number of steps. Thus $L(M') = L^*$and Turing – recognizable languages are closed under star.

(d) **Intersection:**

Let L_1 and L_2 be the Turing – recognizable languages that are recognized by the machines M_1 and M_2

We want to show that

• There is a Turing machine M' such that *L*(M')= $L_1 \cap L_2$.

The description of M' is as follows:

$M' =$"on input *w*:

1. Run M_1 on *w*. if it halts and rejects, ***reject***. If it accepts, go to step 2.

2. Run M_2 on *w*. if it halts and rejects, ***reject***.

If it accepts, ***accept***.

M' accepts a string *w* if both M_1 and M_2 accepts, thus *w* belongs to $L_1 \cap L_2$.

Thus $L(M') = L_1 \cap L_2$and Turing – recognizable languages are closed under intersection.