Given: $S=\{\langle M\rangle \mid M \text{ is a DFA that accepts } w^R \text{ whenever it accepts } w\}$.

Here, M is a DFA that accepts w^R whenever it accepts w and M is recognizable and decidable on input w.

Note: - If a DFA accepts w^R whenever it accepts w, then L(M) = L(M^R), where M^R is the DFA that accepts the reverse of the strings accepted by M.

Proof that S is decidable is as follows:

Consider the following Turing Machine T = "On Input M, Where M is a DFA".

1) Construct DFA N which accepts the reverse of a string accepted by M.

2) Submit to the Decider for $EQ_{DFA.}$

3) If it accepts, accept.

4) If it rejects, reject.

T is a Decider since, steps 1, 3 and 4 will not create an infinite loop and step-2 calls a decider. Also, T accept M which is a DFA if L(M) = L(M^R).

Therefore, T decides S . Thus, S is decidable.

Construction:

DFA M^R can be constructed by first constructing NFA from M by reversing all transition in the following way:

• Change the initial state with new accepting state.

• After that, create a new initial start state with $\in$ transition to all earlier accepting state.

• Then construct a DFA from this NFA.

As, a DFA can accept only those particular languages that they are designed for, T is deciding the decidability of M. Decidability or Undecidability of a string depends upon recognition of its components.

It is eventually necessary that output of M is decidable by S. So, from the above proof it is clear that the language S is decidable.

Conclusion:

It is eventually necessary that output of M is decidable by S. So, from the above proof it is clear that the language S is decidable.