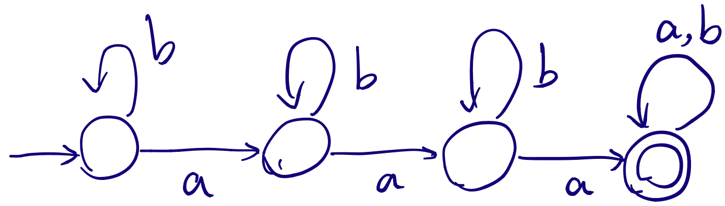
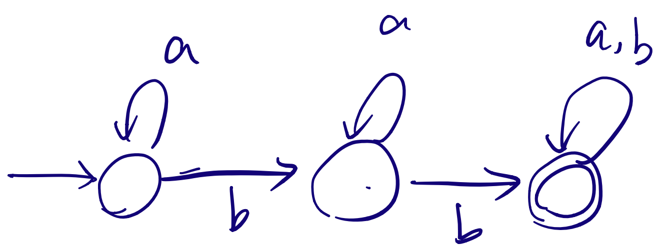
problem 1.4.

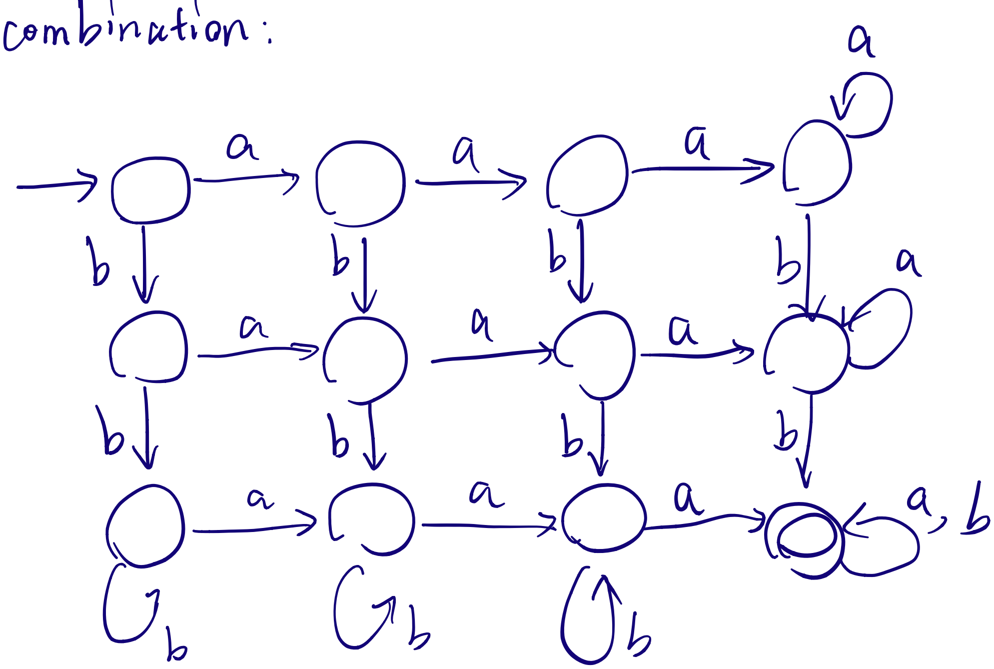
(a) one :



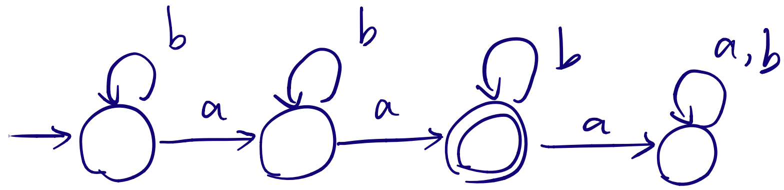
two :



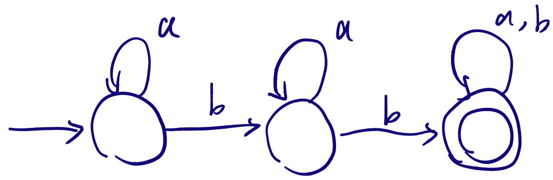
combination :



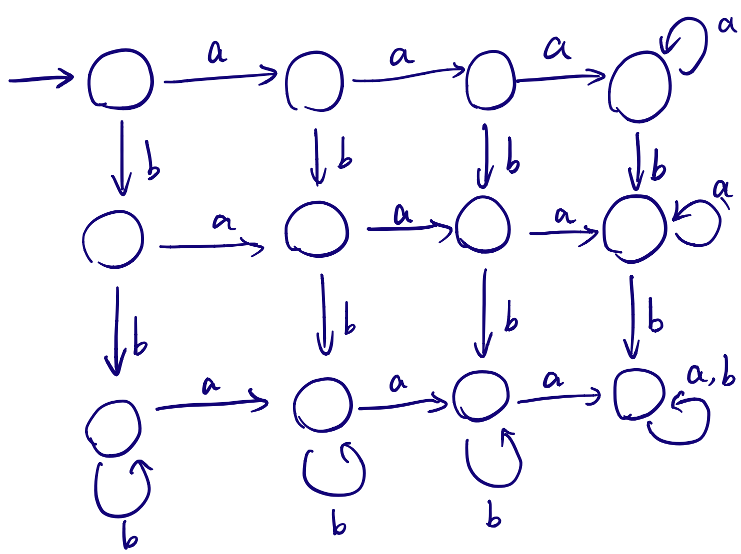
(b) one :



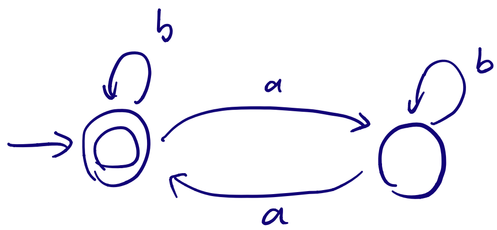
two :



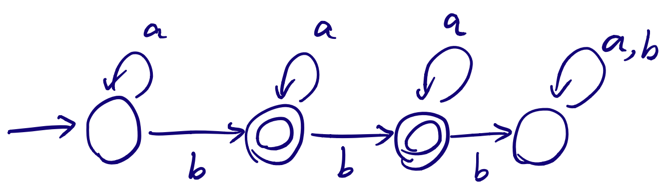
combination :



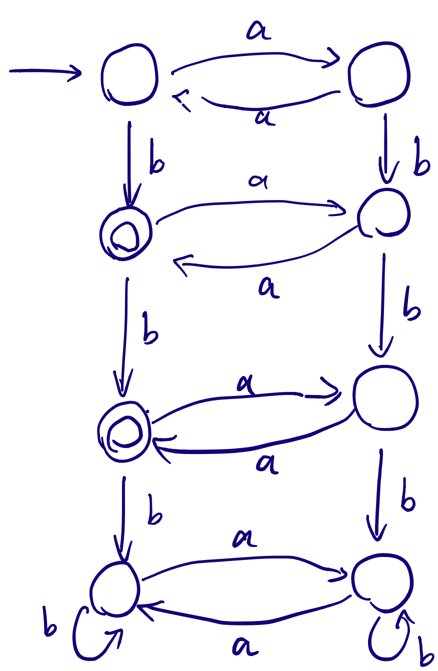
(c) one:



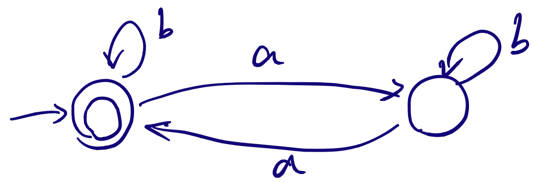
two:



Combination:

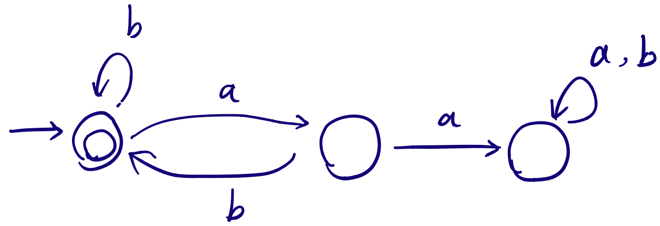


(d) one :

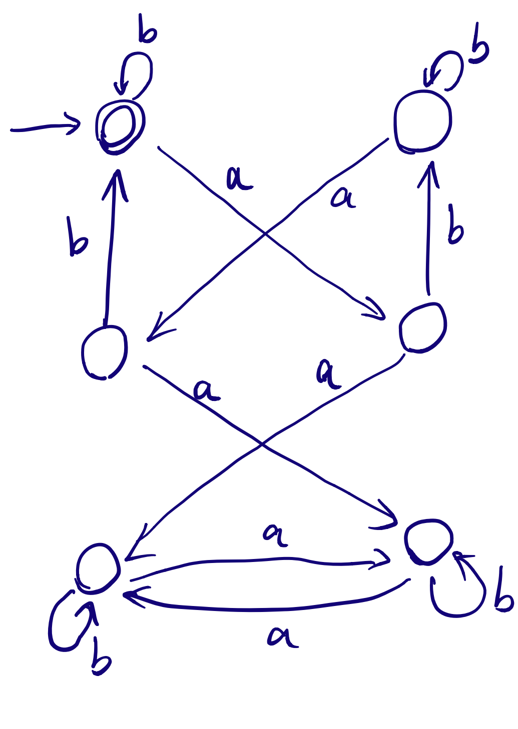


(2 , 1)

two :

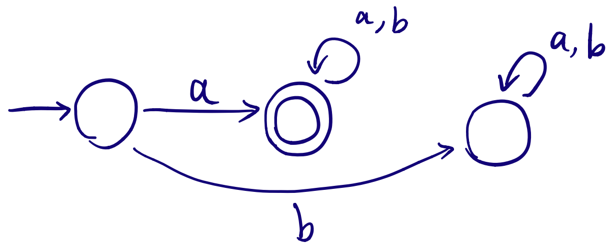


combination :

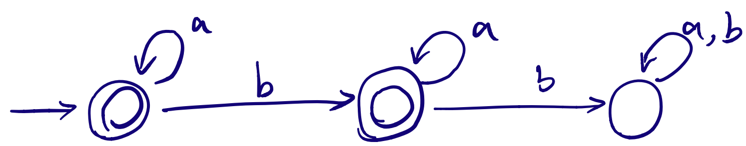


※ the bottom two nodes can be merged to one state

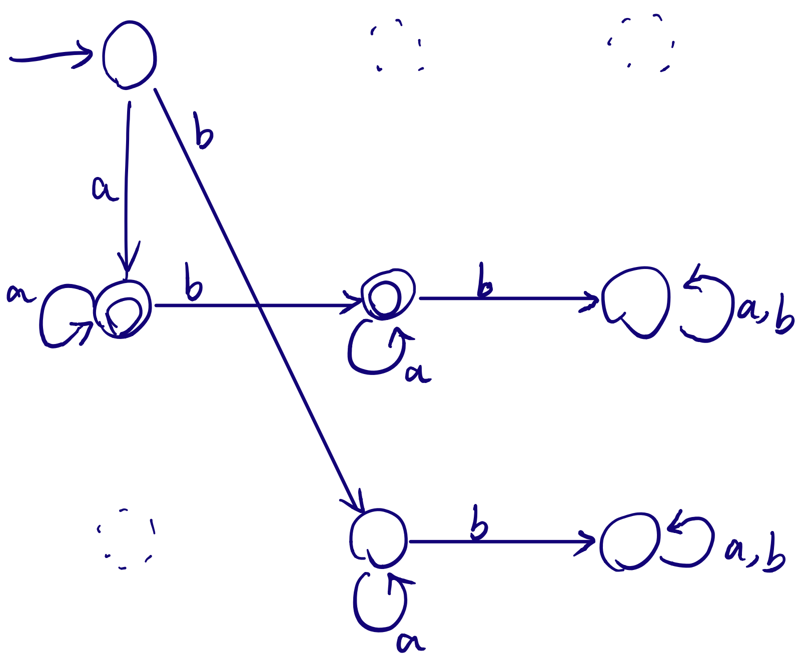
(e) one:



two:

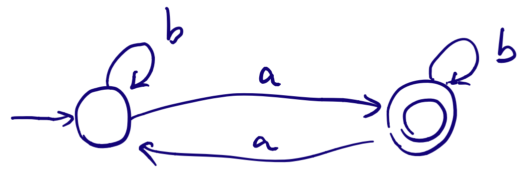


combination:

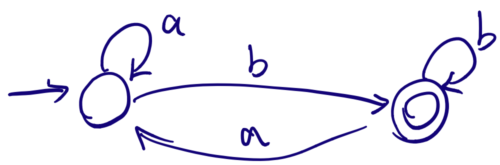


※ three nodes cannot be reached are removed.

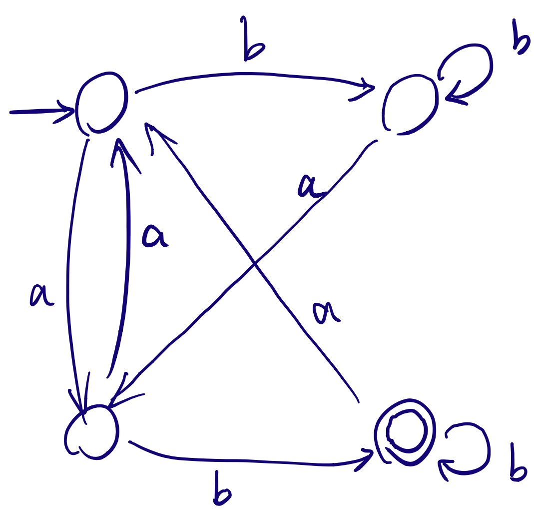
(f) one:



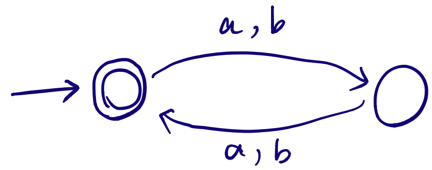
two:



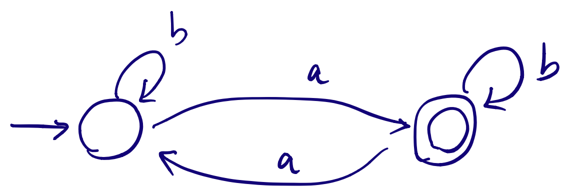
combination:



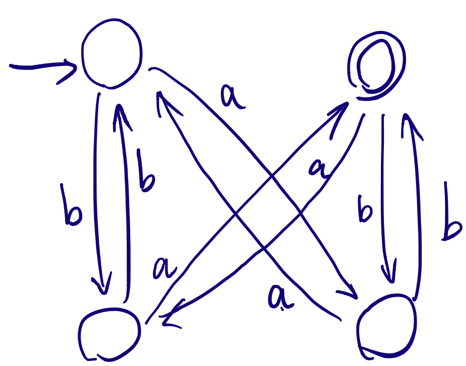
(g) one :



two :

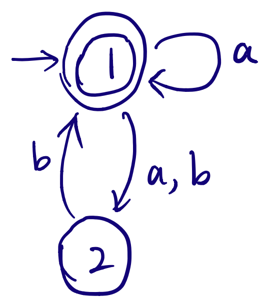


combination :



1.16

(a)



(1) Let $N = (Q, \Sigma, \delta, q_0, F)$ represent the original NFA.

so $Q = \{1, 2\}$, Let Q' be the power set $P(Q)$ of Q.

so $Q' = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

(2) For every $R \in Q'$, $a \in \Sigma$, the state transition function δ' can be calculate according to $\delta'(R, a) = \{q \in Q \mid q \in \delta(r, a)\}$, and we can calculate them as following:

$\because \delta(\emptyset, a) = \emptyset$, $\delta(\emptyset, b) = \emptyset$

$\delta(1, a) = \{1, 2\}$, $\delta(1, b) = \{2\}$

$\delta(2, a) = \emptyset$, $\delta(2, b) = \{1\}$.

$\therefore$ ① $\delta'(\emptyset, a) = \delta(\emptyset, a) = \emptyset$

② $\delta'(\emptyset, b) = \delta(\emptyset, b) = \emptyset$

③ $\delta'(\{1\}, a) = \delta(1, a) = \{1, 2\}$,

④ $\delta'(\{1\}, b) = \delta(1, b) = \{2\}$

⑤ $\delta'(\{2\}, a) = \delta(2, a) = \emptyset$

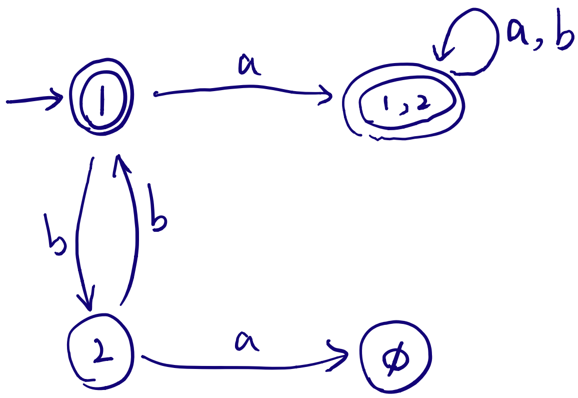
⑥ $\delta'(\{2\}, b) = \delta(2, b) = \{1\}$

⑦ $\delta'(\{1,2\}, a) = \delta(1, a) \cup \delta(2, a) = \{1,2\} \cup \emptyset = \{1,2\}$

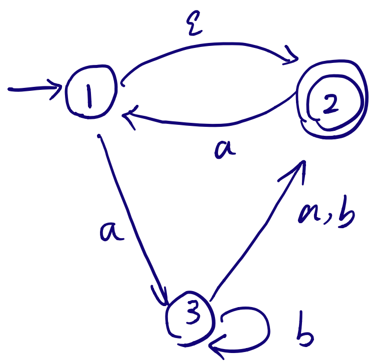
⑧ $\delta'(\{1,2\}, b) = \delta(1, b) \cup \delta(2, b) = \{1,2\} \cup \{1,2\} = \{1,2\}$

(3) Because $F = \{1\}$, $\therefore F' = \{\{1,2\}, \{1\}\}$.

Let $q_0' = \{q_0\}$ be the start state of DFN, we can get



(b)



(1) As stating above, $Q' = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\} \}$

(2) considering ε for $R \subseteq Q$, replace $\delta(R, a)$ by $E(\delta(R, a))$.

$E(R) = \{ q \mid q$ can be reached form R, by traveling along or more ε arrows$\}$

E can be calculated as following :

① $E(\emptyset) = \emptyset$

② $E(\{1\}) = \{1, 2\}$

③ $E(\{2\}) = \{2\}$

④ $E(\{3\}) = \{3\}$

⑤ $E(\{1,2\}) = \{1, 2\}$

⑥ $E(\{1, 3\}) = \{1, 2, 3\}$

⑦ $E(\{2, 3\}) = \{2, 3\}$

⑧ $E(\{1,2,3\}) = \{1,2,3\}$

$\therefore \quad \delta'(\emptyset, a) = E(\delta(\emptyset, a)) = E(\emptyset) = \emptyset$

$\delta'(\emptyset, b) = E(\delta(\emptyset, b)) = E(\emptyset) = \emptyset$

$\delta'(\{1\}, a) = E(\delta(1, a)) = E(\{3\}) = \{3\}$

$\delta'(\{1\}, b) = E(\delta(1, b)) = E(\emptyset) = \emptyset$

$\delta'(\{2\}, a) = E(\delta(2\ a)) = E(\{1\}) = \{1, 2\}$

$\delta'(\{2\}, b) = E(\delta(2, b)) = E(\emptyset) = \emptyset$

$\delta'(\{3\}, a) = E(\delta(3, a)) = E(\{2\}) = \{2\}$

$\delta'(\{3\}, b) = E(\delta\ 3, b)) = E(\{2,3\}) = \{2, 3\}$

$\delta'(\{1,2\}, a) = E(\delta(1,a)) \cup E(\delta(2,a)) = E(\{3\}) \cup E(\{1,2\}) = \{1,2,3\}$

$\delta'(\{1,2\}, b) = E(\delta(1,b)) \cup E(\delta(2,b)) = E(\emptyset) \cup E(\emptyset) = \emptyset$

$\delta'(\{1,3\}, a) = E(\delta(1,a)) \cup E(\delta(3,a)) = E(\{3\}) \cup E(\{2\}) = \{2,3\}$

$\delta'(\{1,3\}, b) = E(\delta(1,b)) \cup E(\delta(3,b)) = E(\emptyset) \cup E(\{2,3\}) = \{2,3\}$

$\delta'(\{2,3\}, a) = E(\delta(2,a)) \cup E(\delta(3,a)) = E(\{1\}) \cup E(\{2\}) = \{1,2\}$

$\delta'(\{2,3\}, b) = E(\delta(2,b)) \cup E(\delta(3,b)) = E(\emptyset) \cup E(\{2,3\}) = \{2,3\}$

$\delta(\{1,2,3\}, a) = E(\delta(1,a)) \cup E(\delta(2,a)) \cup E(\delta(3,a)) = E(\{3\}) \cup E(\{1\}) \cup E(\{2\}) = \{1,2,3\}$

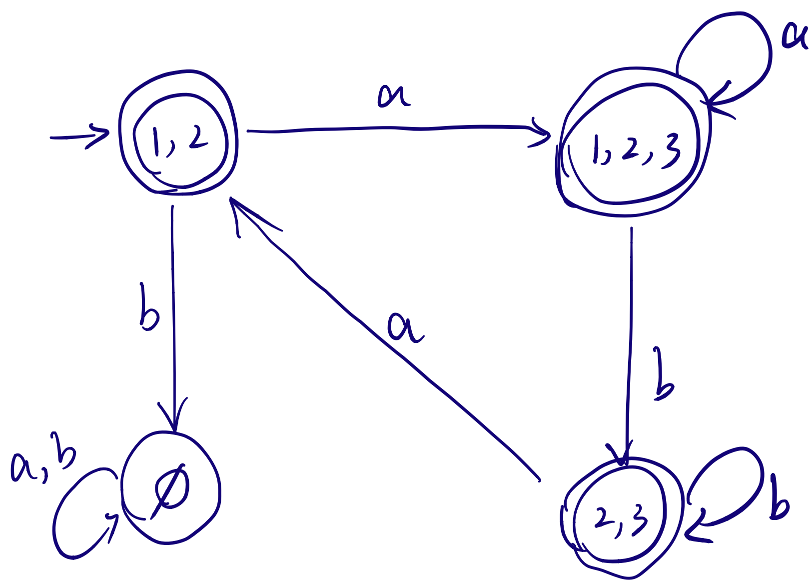
$\delta(\{1,2,3\}, b) = E(\delta(1,b)) \cup E(\delta(2,b)) \cup E(\delta(3,b)) = E(\emptyset) \cup E(\emptyset) \cup E(\{2,3\}) = \{2,3\}$

the DFA states are $\emptyset$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{2,3\}$, $\{1,2,3\}$

$\because q_0 = 1$, $\therefore q_0' = E(q_0) = E(\{1\}) = \{1,2\}$.

$\because F = \{2\}$, $\therefore F' = \{\{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}\}$.

$\therefore$ the state diagram for the DFA is as following:



state $\{2\}$ and $\{3\}$ cannot be reached from the start state $\{1,2\}$, therefore, they are removed from the state set Q' and do not appear in the DFA diagram