Question:

Let E = {$a^i b^j$ | i ≠ j and 2^i ≠ ^{j}}. Show that E is a context-free language.

Answer:

----SETP1----

Given language E is defined as follows:

$E = \{a^i b^j \mid \text{i} \neq \text{j} \text{ and } 2\text{i} \neq j\}$

In order to show that the language E as a context free language.

Consider the language E as the language of the following three languages:

$$E_1 = \left\{a^i b^j \mid j < i\right\}$$
$$E_2 = \left\{a^i b^j \mid i < j < 2i\right\}$$
$$E_3 = \left\{a^i b^j \mid j > 2i\right\}$$

Since context free languages are closed under the union operation. So for proving that the language E is context free user has to show that the all three languages which are written above are closed under union operation.

----SETP2----

For the language E_1 build the grammar as follows:

$$S \to aAB$$
$$A \to aA \mid \in$$
$$B \to aBb \mid \in$$

The non-terminal symbol B generates $a^i b^j$ for $j \geq 0$ and the non-terminal symbol A generates a^i for $i \geq 0$. The starting non terminal symbol S always includes an ***a*** so that user can conclude that any string which is generated by using above grammar has more $a's$ than $b's$.

Conversely, if the string w belongs to language E_1, then w can be written as $w = aa^{i-j-1}a^j b^j$ and assume that A generates a^{i-j-1} and B generates $a^j b^j$.

----SETP3----

For the language E_3 build the grammar as follows:

$$S \to ABb$$
$$A \to aAbb \mid \in$$
$$B \to Bb \mid \in$$

Now the non-terminal symbol A generates $a^i b^{2i}$ for $i \geq 0$ and the symbol B generates b^j for $j \geq 0$. Assume that this grammar derives the string w. Now suppose s is used for storing the total number of time user replaces A with $aAbb$ and t is used for storing the total number of times user replaces B with Bb. Then $w = a^s b^{2s+t+1}$ as $s,t \geq 0,\ 2s+t+1 > 2s$ (using *j>2i* assume *i=s* and *2s+t+1=j*) and w belongs to E_3.

Conversely, if the string w belongs to language E_3, then w can be written as $w = a^i b^{2i} b^{j-2i-1} b$ and assume that A derives $a^i b^{2i}$ and the non-terminal symbol B generate b^{j-2i-1}.

----SETP4----

For the language E_2 build the grammar as follows:

$S \to aAb$
$A \to aAb \mid aAbb \mid abb$

Assume this grammar generates the string w and assume that s is used for storing the total number of times the rule $A \to aAb$ is used and is used for storing the total number of times the rule $A \to aAbb$. But $S \to aAb$ and $A \to abb$ are used exactly once in the derivation of the string *w.* Then $w = a^{s+t+2} b^{s+2t+3}$ by assuming $i = s+t+2, j = s+2t+3$, here $s,t \geq 0$ and user has $i = s+t+2 < s+2t+3 = j$ and $2i = 2s+2t+4 > s+2t+3 = j$. Hence the string w belongs to E_2.

Conversely, if the string w belongs to language E_2 then w can be written as $w = a^i b^j$. Assume that $s = j-i-1$ and $t = 2i-j-1$. As $i < j < 2i$ and $s,t \geq 0$ this grammar generates the string w by using the rule $S \to aAb$ *s* times and $A \to aAbb$ t times.

Since E_1, E_2 and E_3 languages are closed under union operation.

Therefore, the language E is context free language.