Consider the data

- $\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$
- A string of symbols in Σ_3 gives 3 rows of $0s$ and $1s$.
- Each row to be a binary number
- $B = \{ w \in \Sigma_3^*$ the bottom row of w is the same of the top two row $\}$ is the language over Σ_3.

Already know that "regular languages are closed under reversal".
Then, if prove that B^R is regular, then automatically B is regular and vice – versa.
So, first have to prove that B^R is regular.
A language is said to be regular if some automaton recognizes it.

Let M be the automaton that recognizes B^R.

- M has 2 states.

(i) c_0, which denotes that the string that we have read so for leads to a carry 0.

(ii) c_1, that stands for carry 1.

Now $M = (Q, \Sigma, \delta, q_0, F)$

Where $Q = \{c_0, c_1\}$
= set of states

$\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \ldots \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

= set of alphabets

$q_0 = c_0$
= start state

$F = \{c_0\}$
= set of final states.

δ is given as:

- $\delta(c_0, a) = c_0$ if $a = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$
- $\delta(c_0, a) = c_1$ if $a = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$
- $\delta(c_1, a) = c_1$ if $a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$
- $\delta(c_1, a) = c_0$ if $a = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

All other arrows go to trap state. Then, the defined a automation M to recognize B^R
Therefore B^R is a regular language. As B^R is regular, B is also a regular language.