All NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if every possible state that M could be in after reading input x is a state from F.

There are two steps to prove that all – NFAs recognize the class of regular languages.

Step 1: Every regular language is recognized by some all – NFA

Step 2: Every all – NFA recognizes a regular language.

Proof for the Step 1 as follows:

Clearly every DFA can be viewed as an all - NFA.

- Let *L* be the any regular language.
- Let $M = (Q, \Sigma, \delta, q_0, F)$ be the DFA that recognizes L.
- Clearly for each input string x,

There is exactly one possible state $q \in Q$ that M could be in after reading x.

- Hence, if M is viewed as an all-NFA, then it accept x if and only if $q \in F$, which happens if and only if $x \in L$.
- Therefore, when M is viewed as an all NFA, it also recognizes the language L.

Thus, every regular language is recognized by some all – NFA.

Proof for the Step 2 as follows:

- Let $N = (Q, \Sigma, \delta, q_0, F)$ be an all NFA.
- Let A be the language recognized by N.

Now, prove that A is regular.

• Constructing a DFA $M = (Q', \Sigma, \delta', q'_0, F)$ that recognizes A.

$$M = (Q', \Sigma, \delta', q'_0, F)$$

- Where Q' = P(Q), where P(Q) is the set of subset of Q
- For $R \in Q'$ and $a \in \Sigma$

$$\delta'(R,a) = \bigcup_{r \in R} E(\delta(r,a))$$

• For any subset $S \subset Q, E(S)$ is the set of all sates $q \in Q$ that can reached from S by travelling along \in arrows including the members of S themselves.

$$q_0' = E(\lbrace q_0 \rbrace)$$

$$F' = \{ R \in Q' \mid R \subset F \}$$

Now, consider an input string $x \in \Sigma^*$.

- Let R be the set of states that the all NFA N could be in after reading x.
- Then x is fed to the DFA M defined above, M will end at state R.
- By the definition of all-NFA , have $x \in A \Leftrightarrow R \subset F$.
- By the definition of M, M accepts x if and only if $R \in F'$, which is equivalent to $R \subset F$.

Hence M recognize A.

Therefore, every all- NFA recognizes a regular language.

From step 1 and step 2 all- NFAs recognizes the class of regular language.