Given that

$Infinite_{PDA} = \{ \langle M \rangle \mid \text{M is a PDA and L(M) is an infinite language} \}$

The decidability of *Infinite* $_{PDA}$ is as follows:

• To decide the $Infinite_{PDA}$ convert the given PDA *M* into a CFG *G*.

• Now convert into an equivalent grammar G' in Chomsky Normal Form (CNF).

• Generate a graph A1: Each Variable is a vertex, and for a rule or production

• $T \rightarrow UV$ add edges (T, U) from T to U. and (T, V) from T to V.

• Apply DFS or BFS from the start state to see if A1 has a directed cycle. If it does, accept. Otherwise reject.

• If $\langle M \rangle \in \text{INFINITE}_{PDA}$, then it has a string of length greater than pumping length of L (M).

• Following the proof of pumping lemma, it means there is a Variable V which can derive sVt for some strings s, t. (Since it is in CNF it must be non-empty).

• This implies that the graph A1 has a cycle involving the variable V.

• Assume there is a cycle in A1. Then it is clear that for some variable V, a derivation $V \rightarrow *sVt$ must exist and s or t must be a non-empty, since G' is in CNF.

• Since there is a scope for finding a cycle from S. there is rule $S \rightarrow *aVb.$

• Thus, $S \rightarrow *au^iVv^ib.$ for all i>=0 and so L(M) is infinite.

Hence, basing on the above discussion it is proved that $Infinite_{PDA}$ is decidable.