

TRUE (or) FALSE

Big – O Notation:

Let f and g be functions $f, g : N \rightarrow R^+$ say that $f(n) = O(g(n))$ if positive integers c and n_0 exist such that for every integer $n \geq n_0$

$$f(n) \leq c(g(n))$$

When $f(n) = O(g(n))$ we say that $g(n)$ is an upper bound for $f(n)$.

(a)

True.

The statement $2n = O(n)$ is valid, because from the definition of Big-O notation it is clear that $f(n) = c(g(n))$.

(b)

False.

The statement $n^2 = O(n)$ is not valid, because $n^2 = n \cdot n$ which will grow faster than n .

That contradicts Big – O notation. Thus, $n^2 = O(n)$ is False.

(c)

False.

The statement $n^2 = O(n \log^2 n)$ is not valid, because factor n grows faster than the factor $\log^2 n$. That means $f(n) > g(n)$, which contracts the Big –O notation.

Hence $n^2 = O(n \log^2 n)$ is false.

(d)

True.

The statement $n \log n = O(n^2)$ is valid, because the factor $\log n$ grows slower than the factor n . That means $f(n) < g(n)$. From Big-O notation $n \log n = O(n^2)$ is true.

(e)

True.

The statement $3^n = 2^{O(n)}$ is valid, because $3^n = 2^{n \log 3}$ and $n \log 3 = O(n)$.

From Big-O notation $3^n = 2^{O(n)}$ is true.

(f)

True.

The statement $2^{2^n} = O(2^{2^n})$ is valid, because from Big- O notation $f(n) = O(f(n))$ for any function $f(n)$. Hence $2^{2^n} = O(2^{2^n})$ is true.