Descriptive complexity of strings:-

If x be binary string, then the minimal description and descriptive complexity of x's are $d(x)$ and $\mathrm{K}(x)$ respectively. Turing machine M and small string w we get minimal description is $\langle M, w\rangle$. From several of such shorter strings we select lexicographically among them then we can get descriptive complexity of such strings $\mathrm{K}(x) = |d(x)|$.

Now we have to show how to compute $\mathrm{K}(x)$ with an oracle for A_{TM}.

- For the given string x, start testing all the strings 'S' up to the length $|x| + c$

Where $c =$ length of *TM* (Turing machine) that halts immediately upon starting.

- All the strings up to the length $|x| + c$ are potential description of x.
- If S is well formed as $\langle M, w\rangle$ from all binary strings in lexicographic order, then we simulate M with input w and see if it halts with x on the tape.
- Here we do not know whether M will halt on input w or not.
- An oracle for A_{TM} can determine this.

$$A_{TM} = \{\langle M, w\rangle \mid M \text{ is a } TM \text{ and } M \text{ accepts } w\}$$

- An oracle for A_{TM} will take $\langle M, w\rangle$ as input and determine whether M accepts w or not.
- If M doesn't halt we move on to the next string S, and so on.
- After that we will find lexicographically first string S among them.
- In this way shortest string will be determined and it is represented as minimal description $d(x)$.
- From $d(x)$, we find $\mathrm{K}(x)$ as

$$\mathrm{K}(x) = |d(x)|$$

- By this procedure, we will compute $\mathrm{K}(x)$ with an oracle for A_{TM}.