Question:

Say that a language is ***prefix-closed*** if all prefixes of every string in the language are also in the language. Let C be an infinite, prefix-closed, context-free language. Show that C contains an infinite regular subset.

Answer:

----SETP1----

**Given:** A language which is prefix closed. Here, it is required to prove that an infinite regular subset is contained in every context free language which is prefix-closed.

----SETP2----

**Proof:** Consider a language L which is context free and prefix closed. As the language L is context free and therefore, the principle of pumping lemma can be applied on it.

Let the length of the pumping lemma be P and the string that needs to be considered in the language is s.

The length of the pumping lemma P is lesser or smaller than length of the string. Now, it is possible to break the string into 'abcde' such that $ab^kcd^ke \in L$ for all $k \geq 0$ and $|bd| \geq 1$.

Now, it is already given that the language L is prefix closed and all the prefixes of the string s are also present in language L, therefore, $ab^k \in L$ for all $k \geq 0$.

Thus, it can be inferred that the language formed from the $ab^*$ is regular and it is a subset of L.

**Also, if $b \neq \in$ then it implies $ab^*$ is an infinite regular subset of Land thus it proves the required statement**.