Step 1: HAMPATH $\in$ NP

CERTIFICATE: A sequence of vertices y.

VERIFICATION: $a(<G,u,v),<y>)$

• Check whether y has n vertices. If not, return 0.

• Check whether $y=(y_1,y_2\cdots\cdots,y_n)$ 0; Repeated vertices; if so, return 0.

• Check whether $(y_i,y_{i+1})\in$ E for $i=1,\cdots,n-1$ and whether $\{y_n,y_1\}\in E$. If some of the tests fail, then return 0.

• Check whether $y_1=u$ and $y_n=V$. If not, return 0; otherwise return 1.

A runs in Polynomial Time since

1. runs in O(n) steps

2. runs in O(n) steps

3. runs in O(n) steps

4. runs in O(n) steps

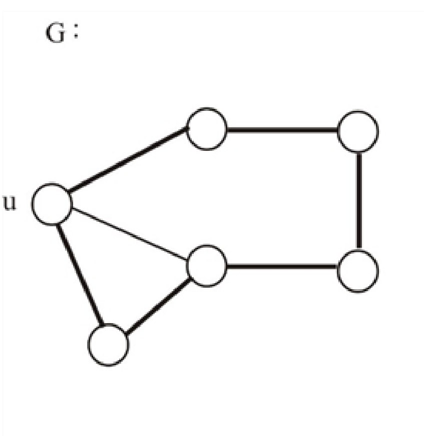
So, A runs in O(n) steps

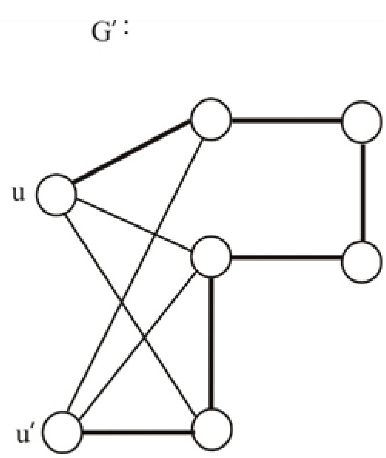
It is easy to see A is a correct verification algorithm for HAMPATH.

Step 2: HAMCYCLE $\leq$ p HAMPATH

Idea for the reduction. Given G, pick an arbitrary vertex u and create a new Vertex u′ connected it to all the neighbors of u, in the new graph G′. A Hamiltonian in G corresponds to a Hamiltonian path from u to u′ in G′.

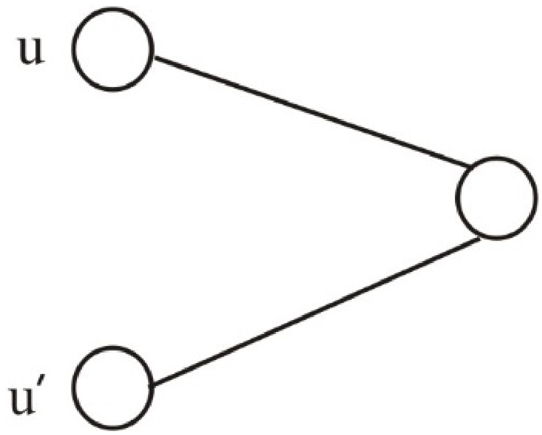
Example:





The only case for which this reduction fails is $G:u\,0-0$ since G has no Hamiltonian cycles but has a Hamiltonian path between u and u′

G′ :



Therefore, we treat the case $(V)=2$ separately in the algorithm.

Step 3: Reduction Algorithm

Algorithm $F(<G>)$

Let $G=(V,E)$. If $|V|=2$, then return $<G'=(V'=\{u,v\},E'=\phi)<G'=(V'=\{u,v\},E'=\phi),\mu,V>$

Select a vertex u in G $E'\leftarrow E$.

For each v in V do

If $\{u,v\}\in E$ then $E'\leftarrow E'\cup\{u'v\}$

Return $<G'=(V',E'),u,u'>$;

Step 4:

• The reduction works that is G has a Hamiltonian cycle if and only if G′ has a Hamiltonian path from u to u′ $(\Rightarrow)$ let $C=(V_1=u,V_2,\cdots\cdots V_n)$ be a Hamiltonian cycle in G (we can assume $V_i=u$ with loss of generate) it is easy to see that $(V_1=u,V_2,V_3------V_n,V_{n+1}=u')$ is a Hamiltonian path between u and u' in G′ since $(V_1,V_2--------V_n)$ are distinct vertices in G so $(V_1=u,V_2,------V_n,V_{n+1}=u')$ are distinct vertices in G′: more over if $\{V_i,V_{i+1}\}\in E$ $i=1,2\cdots\cdots,n$ and $\{V_n,u\}\in E$ then $\{V_i,V_{i+1}\}\in E'$, $i=1,2,\cdots\cdots n$ and $\{V_n,u'\}\in E'$ $(\Leftarrow)$

• Let $P=(u=V_1,V_2,V_3-----,V_n,V_{n+1}=u')$ be a Hamiltonian path in G′ then $(V_1,V_2,\cdots\cdots V_n,V_{n+1}=u')$ are distinct vertices in G′. So $(V_1,V_2,------V_n)$ are distinct in G. more over, since $\{V_i,V_{i+1}\}\in E'$, $i=1,2,------n$ and $\{u',V,\}\in E'$ then we conclude $\{V_i,V_{i+1}\}\in E$, $i=1,2,\cdots\cdots n$ and $\{u,v_1\}\in E'$.Moreover, since $n>2$, then $\{V_1,V_2\}\neq\{V_n,V_i\}$ So, $(V_1,V_2,-------,V_n)$ is a Hamiltonian cycle in G.

Step 5: F runs in Polynomial time

Copying G into G′, takes time $O(n^2)$ where $n=|V|$. Creating u′ and its incidence edges takes time in $O(n)$. Therefore, F runs in $O(n^2)$.