Question:

Let
$Y = \{w \mid w = t_1\#t_2\# \cdots \#t_k \text{ for } k \geq 0, \text{ each } t_i \in 1^*, \text{ and } t_i \neq t_j \text{ whenever } i \neq j\}$.
Here ∑ = {1, #}. Prove that Y is not context free.

Answer:

----SETP1----

CFG:

- A fixed set of grammar rules is known as **CFG (context free grammar)**. It consisting of that is augment (N, T, P, S).
- Where, **N** is set of non-terminal symbol.
- **T** is set of terminal $N \cap T = NULL$.
- **P** is set of rule, $P: N \rightarrow (N \cup T)^*$.
- **S** is start symbol.

----SETP2----

Consider the following details:

The language is $Y = \{w \mid w = t_1\#t_2\#....\#t_k \text{ where } k \geq 0, t_i \in 1^* \text{ and } t_i \neq t_j \text{ when } i \neq j\}$ with the terminals being $\Sigma = \{1, \#\}$.

----SETP3----

Proof:

Theorem 2.34: Any string s in A, the pumping lemma p is the minimum length such that It could make part under five ends $s = uvxyz$. The string s also satisfies the following conditions for a context-free language A:

1. The string what's to come for uv^ixy^iz only those context-free dialect A, the point when every $i \geq 0$.

$uv^ixy^iz \in A$

2. Those strings that need aid pumped, v furthermore y, can't both make the void string ε.

$|vy| > 0$

3. The joined together period of the strings lying inside what's to come for u Also z must not a chance to be more stupendous that those pumping length p.

$|vxy| \leq p$

----SETP4----

This problem is solved by the proof of contradiction.

- The language Y is supposed to be a context-free language.
- Theorem 2.34 is shown not to hold for the language.
- This makes the assumption, which is that Y is a CFL, invalid.
- Assume language Y is a context-free language.
- Now it can be seen that either x or y cannot have any $\#$’s.
- This is as when user pump the string then user will get strings of the form $s = t_1\#t_2\#....\#t_k$ where $t_i = t_j$ when $i \neq j$.

• Such strings do not lie in A. Consequently, to get $v, y = 1^*$.

----SETP5----

Construction:

Consider the string $s = uv^i xy^i z$ with $v = 1^*, y = 1^*$. The two cases possible for the substring vxy are:

• The substring contains $\#$: as the $\#$ symbol cannot lie in either v or y, it must lie in x.

• When the string $s = uv^i xy^i z$ is pumped with $x = 1^* \# 1^*$, the case $t_i = t_j$ when $i \neq j$ can occur.

• Thus, pumping this string does not necessarily produce strings that lie in Y.

• It does not contain $\#$: the substring xyz will be just be a sequence of 1 s. As was argued for the previous case, on applying the condition 1 of theorem 2.34 to pump the string will result in strings wherein $t_i = t_j$ in cases when $i \neq j$. As has been seen these strings are not part of language Y.

----SETP6----

Conclusion:

The language Y does not satisfy the pumping lemma. Consequently, it is not a CFL.