(a)

The language given is as follows:

$B = \{1^k y \mid y \in \{0,1\}^*\}$ and y contains at least k 1s, for $k \geq 1\}$

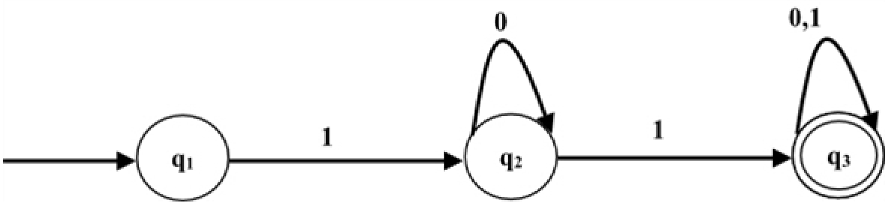
From the definition 1.16: A language is said to be Regular language if some finite automaton recognizes it.

String in language B must start with a 1 and contains at least one other 1, if k=1. So, if k is positive that, any string that start with a 1 and contains at least one other 1 matches in the y. B is defined by regular expression $10^*1(1 \cup 0)^*$ and therefore B is regular.

To show *B* is regular, the **definition 1.16** can be used.

Let *M* be the DFA recognizes the language *B*.

The state diagram of *M* is as follows:



Since there is a DFA recognizing the language *B*.

Hence it is proved that *B* is regular language.

(b)

The language given is as follows:

$C = \{1^k y \mid y \in \{0,1\}^* \text{ and } y \text{ contains at most } k \text{ 1s, for } k \geq 1\}$

The proof for to prove *C* is not regular language is as follows:

Assume that *C* is a regular language.

Assume *P* as the pumping length.

Consider a string $S = 1^P 0 1^P \in C$

Using the pumping lemma, *S* can be written as

$S = 1^P 0 1^P = uvw$ such that $|uv| \leq P, |y| > 0$ and $uv^i w \in C \ \ \forall i \geq 0$

When $i = 0,\ uw = 1^{P-t} 0\ 1^P$ for some *k* where the number of 1's in *y* is less than or equal to *k*. $1^k y = 1^{P-t} 0 1^P$.

From this, *y* must contain the substring 01^P.

So $k \leq P - t$ with the number of 1s in $y \geq P$.

So, the number of 1's in *y* is always greater than *k* since $t > 0$.

Therefore, it is proved that uw does not belong to language *C*.

Since the above statement results a contradiction.

Hence, it is proved that the given language *C* is not a regular language.