

Assume that $PAL_{DFA} = \{\langle M \rangle \mid M \text{ is a DFA that accepts some palindrome}\}$. If a Turing machine can be presented for the given DFA that runs finitely and halts, then the PAL_{DFA} is decidable.

Construct a decider D for PAL_{DFA} and a Turing machine K that can decides E_{CFG} :

$D =$ "On input $\langle M \rangle$,

1. A PDA P is constructed as: $L(P) = \{w \mid w \text{ is a palindrome}\}$
2. A PDA P' is constructed so that $L(P') = L(P) \cap L(M)$
3. Now P' is converted into an equivalent CFG G .
4. Check if $L(G)$ is empty using Theorem 4.8 over Turing Machine K .
5. If $L(G)$ is empty then reject, otherwise, accept.

For Turing machine K :

- Both steps 1 and 2 can be done in finite steps.
- Step 3 also takes finite steps to convert P' into its equivalent CFG.
- In step 4, the decider K checks whether the language $L(G)$ is empty or not. It can also be done in a finite step.

Since D takes finite steps for any input, it means that it is a decider. Hence, PAL_{DFA} is decidable.