The pumping lemma is used as a negative test to prove that the given language is non-regular. The language that violates the any of the three conditions of the pumping lemma is classified as non-regular.

Since the pumping lemma starts by assuming that the given language is regular, the belongingness of the string, which is used as a counter example, is tested only for the given language and not for all the languages that accepts the string.

The proof given in the example 1.73 (refer to the textbook example 1.73) is for a different language. The counter examples given to prove the language as non-regular does not hold true for the language given in the question as shown below:

Let A be the language $\{0^*1^*\}$ given in the question.

Let B be the language $\{0^n1^n \mid n>=0\}$ given in the example 1.73.

As per the example 1.73, s is 0^p1^p and hence as per the pumping lemma, s should be split into three pieces as s = xyz, where for any i>=0, the string xy^iz is in B. The cases are as follows:

• The string y contains all 0s: The string xyyz will result in more number of 0s than the number of 1s which does not belong to the language B but the string belongs to language A and hence, the pumping lemma is not violated.

• The same reason holds true for the case when the string y contains all 1s. The string xyyz will results in more number of 1s which is again accept by the given language A.

Since the above conditions are satisfied, the given language cannot be classified as non-regular by pumping lemma.

Therefore, the error in the given proof is that a string which is used as a counter example to prove a certain language as non-regular does not signifies that all the languages that accept that string are considered as non-regular.

The above problems arise when the pumping lemma is used on a language which is regular since violating a condition of the pumping lemma indicates that the language is non-regular but the vice versa is not always true.