

Useless states in automata are the **states whose removal from the automata does not make any difference or put any impact on the language accepted by the automaton.**

Given: A pushdown automaton.

Here, it is required to test whether the machine has useless states or not and whether the problem of testing is decidable or not.

- Let P be the set of all the strings accepted by a pushdown automaton.
- Let the language $L = \{x \in P \mid x \text{ contains a useless state}\}$.
- To show that language L is decidable, construct a Turing machine which accepts strings in language L .
- In reference to this book, consider that the question of whether a PDA has an empty language is decidable.
- It can reduce the question of whether a given state n is useless to this question by making n the only accept state and then determine whether the resulting push down automata has an empty language.
- If it does then, n is a useless state.

Hence, our Turing machine successfully decides whether there is any useless state, by performing this test for each and every state in order.