

User contains an NDTM M_L , for any given NP language, in such a way that $\forall x \in L$, M_L accepts x on minimum single branch in maximum $p_L(|x|)$ steps. Here, $p_L(\)$ denotes a fixed polynomial depending on the machine.

- It is also known that any $x \notin L$ will not be accepted by M_L . Then, user can use a polynomial time to create $y = \langle M_L, x, \#^{p_L(|x|)} \rangle$ for the given x .
- Consider the previous argument, according to this argument, $x \in L$ if and only if $y \in U$. Therefore, **U is NP-hard**.

To show that U is also in NP, users have to create an NDTM M_U , which given an input $y = \langle M_L, x, \#^t \rangle$, simulates M on x for t steps.

- Every branches of M are guessed by M_U non-deterministically and accepts u if and only if u is accepted by M .
- Since the minimum length of input is t and user simulate M for maximum t steps, the running time is polynomial in the length of the input.
- Now it can be easily seen that the language U is exactly accepted by M_U , therefore, $U \in NP$.

Hence, **U is NP-complete**.