

Definition of $A \leq_r B$:

Language A is Turing reducible to Language B , written as $A \leq_r B$, if A is decidable relative to B . That is the oracle for language B decides Language A .

Given that

- $A \leq_r B$

That is, Let the oracle M_1^B for Language B decides the Language A . and

- $B \leq_r C$

That is, Let the oracle M_2^C for Language C decides the Language B .

We have to prove that

- $A \leq_r C$

That is, there exists an oracle M_3^C for Language C which decides the Language A .

That means, machine M_3 have to simulate machine M_1 .

We will explain this simulation in detail as follows

- Let M_1 queries an oracle about some String x .
- M_3 does not have an oracle for B . M_3
- So M_3 does not perform the test whether $x \in B$ or not directly.
- Thus M_3 first simulates M_2 on input x and get the result.
- Then M_3 provides that answers to M_1 .
- But the queries of machine M_2 are directly answered by M_3 . Because M_3 and M_2 use same oracle C .

In this way the oracle for Language C decides Language A . That is $A \leq_r C$.

Thus If $A \leq_r B$ and $B \leq_r C$ then $A \leq_r C$.