Given: A language C, and this language is Turing recognizable if and only if there exist y such that $\langle x,y\rangle \in D$.

Proof:

Assume that a language C. This language is recognized by a Turing machine M. When the input x is passed to the Turing machine this machine simulates machine M_d. The decider for the language D on the input string $\langle x,y_i\rangle$, here y_1, y_2, y_3........ string in lexicographic order.

Assume that the language C is recognizable. Suppose $L(M)=C$ which means that Turing machine M recognize the language C. If $L(M)=C$ then for every string x which belongs to the language C, an accepting computation of M on the input string x is present.

Suppose the language D on input $\langle x,y\rangle$ verifies whether y encodes an accepting computation of the input string x on machine M.

Example of such encoding is $c_0\#c_1\#c_2......\#c_n$

Here in the above encoding c_i's configurations of the Turing machine M on the input string x. c_0 Is the initial configuration of the input string x on machine M. The configuration c_n is the accepting configuration for input x. After each c_i, c_{i+1} come.

So it is clearly seen that D is decider.

Construction:

Here user supposed to construct Turing machine that will decide the decidability of C. Now follow these terms:

• For proving decidability of C one needs a Turing Machine so consider a Turing Machine T.

• Construct Turing Machine in a way so that each possible string of Y can be searched or found.

• Test the string whether it is according to the predefined rules $C=\{x \mid \exists y \langle x,y\rangle \in D\}$ in the question.

• If $\langle x,y\rangle \in D$ then T accepts otherwise rejects.

Conclusion:

Here D is Recognizable by C and C is recognizable by Turing machine T and C is also decided by Turing Machine T so $\langle x,y\rangle \in C$ as well this way C is Turing recognizable.