Question:

Let $A = \{wtw^{\mathcal{R}} \mid w, t \in \{0,1\}^* \text{ and } |w| = |t|\}$. Prove that A is not a CFL.

Answer:

----SETP1----

In the given function $A=\left\{wtw^R \middle| w,t\in\{0,1\}^* \text{ and } |w|=|t|\right\}$, every string $s=wtw^R\in A$ where $|w|=|t|=|w^R|$ and $|s|$ is a multiple of three. Let us assume that *A* is context free and reach to a contradiction.

- Let *p* be the pumping length for A that is guaranteed to exit by pumping lemma.

- Select string $s=0^{2p}0^p1^p0^{2p}\in A$ with $|s|>p$.

- Therefore, there exits $uvxyz$ such that

1) $uv^ixy^iz\in B$ for all $i\geq 0,$

2) $|uy|>0$,

3) $|vxy|\leq p$.

- Consider these cases for pumping lemma:

Case 1: $|vy|$ is not a multiple of 3. Then $s'=uv^2xy^2z\notin A$ since $|s'|$ is no longer a multiple of 3.

Case 2: vxy consist of only 0s from the prefix set of 0s and $|vy|=3r$ for some *r*. Then, $uv^2xy^2z=0^{3p+3r}1^p0^{2p}=0^{2p+r}0^{p+2r}1^{p-r}1^r0^{2p}\notin A$, since $w=0^{2p+r}$ and $w^R\neq 1^r0^{2p}$, the w^R of the string s.

Case 3: uxy consists of only 1s and $|vy|=3r$ for some *r.* Then, the string $uv^2xy^2z=0^{3p}1^{p+3r}0^{2p}=0^{2p+r}0^{p-r}1^{p+2r}1^r0^{2p}\notin A$, since $w=0^{2p+r}$ and $w^R\neq 1^r0^{2p}$, the w^R of the string s.

Case 4: vxy consist of only 0s from the suffix set of 0s and $|vy|=3r$ from some *r*. Then $uv^0xy^0z=0^{3p}1^p0^{2p-2r}\notin A,$ since $w=0^{2p-r}$ and $w^R\neq 1^{2r}0^{2p-3r}$, the w^R of the string s.

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Case 5: $uy=0^m1^n$ with $m,n>0$ and $m+n=3r$ from some *r.* Then, the string $uv^2xy^2z=0^{3p+m}1^{p+n}0^{2p}$ $=0^{2p+r}0^{p+m-r}1^{p+n-r}1^r0^{2p}\notin A,$ since $w=0^{2p+r}$ and $w^R\neq 1^r0^{2p}$, the w^R of the string s.

Case 6: $vy=1^m0^n$ with $m,n>0$ and $m+n=3r$ for some *r.*

- **Sub-case 6.1**: $n<r$. Then $uv^2xy^2z=0^{3p}1^{p+m}0^{2p+n}=0^{2p+r}0^{p-r}1^{p+m+n-r}1^{r-n}0^{2p+n}\notin A$, since $w=0^{2p+r}$ and $w^R\neq 1^{r-n}0^{2p+n}$, the w^R of the string s.

- **Sub-case 6.2**: $n>r$. Then $uv^0xy^0z=0^{3p}1^{p-m}0^{2p-n}=0^{2p-r}0^{p+r}1^{p+r-m-n}1^{n-r}0^{2p-n}\notin A$, since $w=0^{2p-r}$ and $w^R\neq 1^{n-r}0^{2p-n}$, the w^R of the string s.

- **Sub-case 6.3**: $n=r$. Then $uv^{p+2}xy^{p+2}z=0^{3p}1^{p+2r(p+2-1)}0^{2p+r(p+2-1)n}=0^{3p}1^{rp+r-p}1^{2r+rp+r}0^{2p+rp+r}\notin A$, since $w=0^{2p+r}$ and $w\neq 0^{3p}1^{rp+r-p}$ and $w^R\neq 0^{2p+rp+r}$, the w^R of the string s.

If $i < p+2,$ then take $r = 1$, $uv^i xy^i z = 0^{3p}1^{p+2(i-1)}0^{2p+(i-1)}$, $= 0^{2p+(i-1)}0^{p-(i-1)}1^{p+2(i-1)}0^{2p+(i-1)} \in A$. As there are not enough 1's to push or pump into *w*, $p+2$ is the first time that is guaranteed.

----SETP2----

Thus, in all the cases, the 1) of pumping lemma results in a contradiction. Therefore, the assumption that *A* is context free language, is false.