

Consider the language  $C_k = \Sigma^* a \Sigma^{k-1}$  for each  $k \geq 1$ , over the alphabet  $\Sigma = \{a, b\}$ .  $C_k$  be the language consisting of all strings that contain an 'a' exactly  $k$  places from the right-hand end.

Now it is required to prove that, no DFA (Deterministic finite automation) can recognize  $C_k$  with fewer than  $2^k$  states.

If a DFA enters two different states after reading two different strings  $lz$  and  $mz$  with same suffix  $z$ , then the DFA enters two different states after reading the strings  $l$  and  $m$ . Take two different strings of length  $k$  such that  $l = l_1 \dots l_i$  and  $m = m_1 \dots m_i$ . Let  $i$  be some position such that  $l_i \neq m_i$ . One of the strings contains  $a$  in its  $i^{\text{th}}$  position and the other string contains  $b$  in its  $i^{\text{th}}$  position.

Consider the suffix string  $z = b^{i-1}$ . In this case, either the string  $lz$  or  $mz$  has the  $k^{\text{th}}$  bit from the end as  $a$ . The total number of strings of length  $k$  over the input alphabet  $\{a, b\}$  is  $2^k$ . Thus, the DFA needs  $2^k$  states in order to distinguish  $2^k$  strings.

Therefore, the DFA that recognizes the language  $C_k$  has at least  $2^k$  states.