Consider the language *A* is regular. Let $A_{\frac{1}{2}-}$ be the set of all first halves of strings in *A*.

$$A_{\frac{1}{2}-} = \{x \mid \text{ for some } y,\ |x| = |y| \text{ and } xy \in A\}$$

Since *A* is a regular language, the DFA *M* recognizes the language *A*.

$$M = (Q, \Sigma, \delta, q_0, F)$$

where, Q is the set of states

Σ is the input alphabet

δ is the transition function

q_0 is the start state

F is the final state

The language is said to be regular if there exists an FA for it. In this case, construct an NFA *N* that recognizes $A_{\frac{1}{2}-}$. Let *x* be the first part and choose *y* such that $|x| = |y|$. Here, $x \in A_{\frac{1}{2}-}$. To prove the language $A_{\frac{1}{2}-}$ is regular, run two DFAs at the same time one forward and the other backward. Run the DFA *M* on input *x* in forward direction and run the DFA *M* on input *y* in backward direction parallelly. The input string is accepted if both simulations reach the same state.

Construction of NFA N to recognize $A_{\frac{1}{2}-}$:

Let $N = (Q', \Sigma, \delta', q_0', F')$ where,

(i) $Q' = Q \times Q \cup \{q_0'\}$ set of states contains the following:

• Special start state q_0' and

• A cross product $q \times q \times q$ where

• The first part tracks the performance of *M* on *x*

• The second part does the same thing for *y*.

• The third part tests whether the guess on *M* is consistent or not.

(ii) $\Sigma =$ input alphabet

(iii) q_0' is the start state

(iv) $F' =$ set of final states $= \{\langle q_i, q_j, q_k \rangle \mid q_i, q_k, q_j \in Q\}$

(v) $\delta' =$ Rules of transition are as follows:

• There exists an ε move from the start state to the all the states in $\{(q_0, q_f) \mid q_f \in F\}$.

• Consider the states $q_i, q_j, q_k, q_l \in Q$. There exists a move from (q_i, q_j) to (q_k, q_l) on input symbol $a \in \Sigma$ if and only if $\delta(q_i, a) = q_k$ and $\delta(q_l, b) = q_j$.

The NFA *N* is constructed to recognize $A_{\frac{1}{2}-}$. Thus, $A_{\frac{1}{2}-}$ is regular.

Therefore, if *A* is regular then $A_{\frac{1}{2}-}$ is also regular.