

Let  $A$  and  $B$  are two disjoint languages. Consider that language  $C$ , that separates  $A$  and  $B$  if  $A \subseteq C$  and  $B \subseteq \bar{C}$ . It can be proved as follows:

- It is quite easy to understand this statement. Read the statement carefully things are pretty obvious all you need to understand is  $A \subseteq C$  and  $B \subseteq \bar{C}$ .
- Here,  $A$  is a set of  $C$  and  $B$  is a set of Complement of  $C$ . So, it is quite obvious now  $A$  and  $B$  both are not related to each other anyhow.
- Here, no use of  $C$  because even if we use  $C$  it won't be able to prove decidability of  $A$  on the basis of  $B$  and Turing reducibility of  $A$  on the basis of  $B$ . If it comes to languages those are fully different and belong to different sets then no separators are required. So, here it is worthless to use  $C$  as separator.

Now, consider a Turing machine  $T$  that will work as a decider for the language  $C$  that separates  $A$  and  $B$ . **Consider both languages as Regular Expressions that will be decided by  $M_1$  and  $M_2$ .**

1.  $S = \langle M, w \rangle$  Where  $M$  is a Turing machine.

2. Now, run  $\langle M_1, W \rangle$  and  $\langle M_2, W \rangle$

3. If  $M_1$  accepts then  $M$  rejects and if  $M_2$  accepts  $M$  rejects.

• **Remember  $M$  will always halt in each situation. Where  $C$  decides A or B. Now it is pretty easy to understand the situation.**

• Therefore, it can be said that only the first statement of question is enough to prove the concept " **No decidable languages can be used to separate two disjoint Turing recognizable languages**".