Consider the problem of determining whether a Turing machine *M* on an input *w* ever attempts to move its head left when its head is on the left-most tape cell. This problem is formulated as a language:

$$L = \{< M, w >| M \text{ attempts to move its head left when its head is on the leftmost tape cell}\}$$

Assume that the language *L* is decidable and $\hat{M}$ be a TM that decides the language *L*. Construct a TM, *A* that decides the halting problem.

A = "on input $<M, w>$:

1. Construct a TM, A', from A. The TM A' moves w one tape cell to the right and marks the leftmost cell with #.
2. Run the TM A' on $<M, w>$.
3. If A' encounters # then A' moves to the right side and simulates M reaching the leftmost tape cell.
4. If M halts and accepts on w then A' simulates to move its head left when its head is on the leftmost tape cell."

Now, TM *A* runs $\hat{M}$ on the input $<A, w>$. If $\hat{M}$ accepts, *A* accepts. Otherwise, *A* rejects. It is assumed that $\hat{M}$ be a TM that decides the language *L*. If *M* halts and accepts on *w,* then only A' moves its head left when its head is on the left-most tape cell. If *A* decides the halting problem, then halting problem is decidable. Thus, the halting problem is undecidable. It is a contradiction.

Therefore, the language *L* is undecidable.