Question:

Let ∑ = {1, 2, 3, 4} and C = {w ? ∑*| in w, the number of 1s equals the number of 2s, and the number of 3s equals the number of 4s}. Show that C is not context free.

Answer:

----SETP1----

Context Free Language

Consider the language:

$$C = \left\{ \begin{array}{l} w \in \{0,1,2,3,4\}^* \mid \text{in w the number of 1s and the number of 2s} \\ \text{are equal,the number of 3s and the number of 4s are equal.} \end{array} \right\}$$

On the contrary consider C is context free. So, C has a pumping length p.

Take $s = 1^p 3^p 2^p 4^p \in C \text{ with } |s| > p$.

Therefore, there exist $uvxyz$ such that

(a) $uv^i xy^i z \in C \text{ for all } i \geq 0$ ……(1)

(b) $vy > 0$ ……(2)

(c) $vxy \leq p$ ……(3)

Now it has to prove all the cases by contradiction, no matter what the value of $uvxyz$.

Case 1: If vxy contains a 1. Then $uv^2xy^2z \notin C$, since it cannot be same number of 1s and 2s. Hence due to equation (3), vxy cannot contain any 2s.

Case 2: If vxy contains a 2. Then $uv^2xy^2z \notin C$, since it cannot be same number of 1s and 2s. Hence due to equation (3), vxy cannot contain any 1s.

Case 3: If vxy contains a 3. Then $uv^2xy^2z \notin C$, since it cannot be same number of 3s and 4s. Hence due to equation (3), vxy cannot contain any 4s.

Case 4: If vxy contains a 4. Then $uv^2xy^2z \notin C$, since it cannot be same number of 3s and 4s. Hence due to equation (3), vxy cannot contain any 3s.

Hence from equation 2, it contradicts equation 1 in all the cases which shows *C* is not context free language.