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**Undecidability of the Turing machine problem:**

• The given problem is defined as the following language:

$USELESS_{TM} = \{\langle T, q\rangle \mid q \text{ is a useless state in TM T}\}.$

• Show that $USELESS_{TM}$ is undecidable by reducing $E_{TM}$ *to* $USELESS_{TM}$ ,where $E_{TM} = \{\langle T1\rangle \mid \text{T1 is a TM and } L(T1) = \varnothing\}$.

• Using the Theorem 5.2. it is already proved that $E_{TM}$ is undecidable

• Suppose that $USELESS_{TM}$ is decidable and that *TM R* decides it.

• Note that for any Turing machine M with accept state $q_{accept}$, $q_{accept}$ is useless if and only if $L(T1) = \varnothing$.

• Accordingly, since *TM* R solves $USELESS_{TM}$, *R* can be used to check if $q_{accept}$ is a useless state to decide $E_{TM}$ .

Specifically, below is a *TM S* that decides $E_{TM}$ by using the decider R for $USELESS_{TM}$ as a subroutine:

S = "On input $\langle T\rangle$ , where *M* is a *TM*:

1. Run TM R on input $\langle T, q_{accept}\rangle$ , where $q_{accept}$ is the accept state of *T*.

2. If *R* accepts, accept. If *R* rejects, reject."

However, since it is known $E_{TM}$ is undecidable and there cannot be a *TM* that decides $USELESS_{TM}$.

Hence it is proved, that the given problem is undecidable.