

The Language $D_k = \Sigma^* a (\Sigma \cup \varepsilon)^{k-1}$ for each $k \geq 1$, over the alphabet $\Sigma = \{a, b\}$. Here, D_k is the language consisting of all strings that have at least one a among the last k symbols. The DFA (deterministic finite automata) that recognizes the language D_k .

The construction of M where $M = (Q_k, \Sigma, \delta_k, q_0, F)$ is as follows:

- $Q_k = \{q_0, q_1, q_2, \dots, q_k\}$ = set of states
- $\Sigma = \{a, b\}$ = set of alphabets
- $q_0 = \{q_0\}$ = start state.
- $F = \{q_1, q_2, \dots, q_k\}$ = set of final states
- δ_k = Transition function is given as follows

$$\delta_k(q, l) = \begin{cases} q_1 & i = 0 \wedge l = a \\ q_0 & i = 0 \wedge l = b \\ q_1 & i \neq 0 \wedge l = a \\ q_{(i+1) \bmod k} & i \neq 0 \wedge l = b \end{cases}$$

In other words,

- M is in state q_0 if it has not seen and a within the past k letters then it will reject.
- It is in state q_i if it saw a from i letters ago.

The complete DFA is as follows:

