

**Countability:** A set is countable if a set is either finite or has the same cardinality as the set of positive integers.

Given  $T = \{\langle i, j, k \rangle \mid i, j, k \in \mathbb{N}\}$

- The goal is to prove that  $T$  is a countable set.
- First, let's define a set  $P = \{\langle i, j, k \rangle \in T \mid i+j+k = s\}$  for each triple  $\langle i, j, k \rangle$  where  $i, j, k \in \mathbb{N}$ , let  $i+j+k$  be the sum  $s$  of the triplet.
- Now, for each number  $s \in \mathbb{N}$ ,
- There are finitely many triples that has sum equal to  $s$ .
- Enumerating the triples with sum zero, then triples with sum equal to 1, then sum equal to 2 and so on.
- The previous step will follow all the triples in  $T$ .
- Hence, set  $P$  is finite for every  $s \in \mathbb{N}$ .

Now, since  $P$  is finite and according to the given definition, it is countable too, therefore the set  $P' = \{\langle i', j', k' \rangle \in T \mid i'+j'+k' = s'\}$  is also countable.

Therefore, any set  $P_i$  where  $i \in \mathbb{N}$ , the union  $T = \bigcup_{i \in \mathbb{N}} P_i$  is also countable since, a countable union of a number of finite sets is countable.

**Hence, it is proved that  $T$  is countable.**