

Consider the following sentence that is provided in the problem 6.10 in the textbook,

$$\begin{aligned}\phi_{eq} = & \forall x [R_1(x, x)] \\ & \wedge \forall x, y [R_1(x, y) \leftrightarrow R_1(y, x)] \\ & \wedge \forall x, y, z [(R_1(x, y) \wedge R_1(y, z)) \rightarrow R_1(x, z)]\end{aligned}$$

The above statement explains about the conditions of the equivalence relation. A model (A, R_1) , where A is any universe and R_1 is the equivalence relation over the elements of A . The line 1 describes that for all x , x is equal to itself. The line 2 describes that for all x and y , if and only if x is equal to y then y is equal to x . The line 3 describes that for all x , y , and z , if x is equal to y and y is equal to z then x is equal to z .

Consider the following sentence,

$$\begin{aligned}\phi_{lt} = & \phi_{eq} \\ & \wedge \forall x, y [R_1(x, y) \rightarrow \neg R_2(x, y)] \\ & \wedge \forall x, y [\neg R_1(x, y) \rightarrow (R_2(x, y) \oplus R_2(y, x))] \\ & \wedge \forall x, y, z [(R_2(x, y) \wedge R_2(y, z)) \rightarrow R_2(x, z)] \\ & \wedge \forall x \exists y [R_2(x, y)]\end{aligned}$$

The above statement explains about the conditions of the equivalence relation and less than relation. A model (A, R_1, R_2) , where A is any universe, R_1 is the equivalence relation over the elements of A and R_2 is the less than relation over A .

- The line 1 describes the conditions of the equivalence relation.
- The line 2 describes that for all x and y , if x is equal to y then it is tends to complement of less than relation. This means if x is equal to y then it can be said that x is not less than y .
- The line 3 describes that for all x and y , if x is not equal to y then either x less than y or y less than x is true.
- The line 4 describes that for all x , y and z , if x is less than y and y is less than z then x is less than z .
- The line 5 describes that for all x there exist y , x is less than y .