

Consider,

$$\text{MODEXP} = \left\{ \langle a, b, c, p \rangle \mid a, b, c \text{ and } p \text{ are binary integers} \right. \\ \left. \text{such that } a^b \equiv c \pmod{p} \right\}$$

A polynomial time algorithm M for MODEXP is as follows:

$M =$ "On input $\langle a, b, c, p \rangle$, where a, b, c and p are binary integers.

- Calculate $x = a \bmod p$, initialize y to 1 and i to 0.
- For $b = b_n b_{n-1} \dots b_1 b_0$, do the following $n+1$ times:
 - if $b_i = 1$, then $y = y \cdot x \bmod p; x = x^2 \bmod p; i = i + 1$
- if $y \equiv c \pmod{p}$, accept. Otherwise, reject."

The algorithm runs in polynomial time. In the above algorithm, steps 1 and 4 will be executed once. The step 3 needs $O(n)$ time. Thus, M is a polynomial time algorithm for MODEXP .

Therefore, $\text{MODEXP} \in P$.