

P is a class of languages that are decidable in polynomial time on a deterministic single tape Turing machine.

Consider the language  $\text{CONNECTED} = \{\langle G \rangle \mid G \text{ is a connected graph}\}$ . Following is the algorithm for determining the decidability of the language CONNECTED:

Let  $M$  be the Turing machine that determines language CONNECTED.  $M$  can be described as follows:

$M = \text{"On input } \langle G \rangle \text{:}$

1. Select the first node of the graph  $G$  and mark it as selected.
2. Repeat the following step until no new nodes in the graph marked.
3. For each node in  $G$ , mark it if it is attached by an edge to node that is already marked.
4. Check all the nodes in  $G$  are marked. If they are marked accept; otherwise, reject."

The language is said to be in class P if it runs in polynomial time. The analysis for the algorithm is as follows:

- In the step 1 of the above algorithm, the first node will get selected and marked. It takes  $O(n)$  time to find and mark the first node.
- The step 2 runs for  $n+1$  times because it needs to repeat the step 3 until no new nodes in  $G$  marked.
- The step 3 takes  $O(n^2)$  time to check all the adjacent connected nodes which are marked.
- The step 2 and step 3 collectively takes  $O(n^3)$  time.
- The step 4 needs  $O(n)$  time to scan all the nodes in the graph.
- Thus, the algorithm takes  $O(n^3)$  time.

The algorithm runs in polynomial time. Therefore, the language CONNECTED is in class P.