

No, A is not a regular language.

- Assume that the languages A is defined as follows:

$$A = \{ a^n b^n \mid n \geq 0 \} \text{ and } B = \{ b \}, \text{ over the input } \Sigma = \{ a, b \}.$$

- Specify the function  $f : \Sigma^* \rightarrow \Sigma^*$  in the following way:

$$f(w) = \begin{cases} b & \text{if } w \in A, \\ a & \text{if } w \notin A. \end{cases}$$

- Notice that if A is a context-free language, then it is Turing-decidable.
- Therefore, f is a computable function.
- Besides,  $w \in A$  if and only if  $f(w) = b$ , which is true if and only if  $f(w) \in B$ .

Hence it is proved that language A is not-regular, but language B is a regular language, because it is finite.