Given that

A is any language and

$DROP\_OUT(A) = \{xz \mid xyz \in A \text{ where } x,z \in \Sigma^*, y \in \Sigma\}$

We have to prove that class of regular languages closed under $\mathrm{DROP\_OUT}$ operation.

i.e. if *A* is a regular language then $DROP\_OUT(A)$ is also regular.

We have to take that *A* is regular and we have to prove that $DROP\_OUT(A)$ is regular.

Since *A* is a regular language, it must be recognized by a DFA.

Let $M = (Q, \Sigma, \delta, q_0, F)$ be the DFA recognizes *A*.

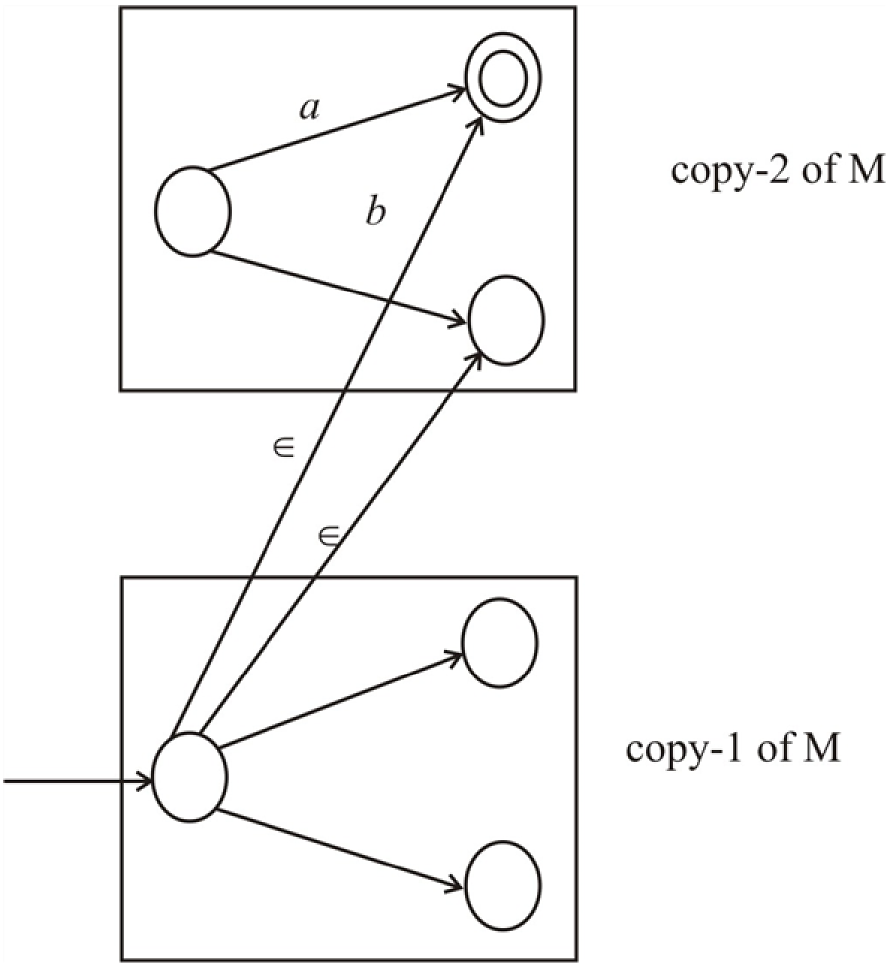
Now we will construct an NFA $N = (Q', \Sigma \cup \{\in\}, \delta', q_0', F')$ that we recognize $DROP\_OUT(A)$.

There are two copies of Machine *M*.

• Copy 1: Copy 1 corresponds to the state of having 'not yet skipped a symbol'

• Copy 2: Copy 2 corresponds to the state of having "already skipped a symbol".

(i) Proof by picture:-



$\mathrm{N} = (\mathrm{Q}', \Sigma \cup \{\in\}, \delta', q_0', F')$

• $Q' = \{(q,b) \mid q \in Q, b \in \{0,1\}\}$ = set of states

• q_0' = start state

$= (q_0, 0)$

• F' = set of final states

$= \{(q,1) \mid q \in F\}$

- δ' is gives as follows:

$\rightarrow$ $\delta'((q,b),a) = \{(\delta(q,a),b)\} \forall q \in Q, b \in \{0,1\}, a \in \Sigma$

This means that both the copy1 and copy2 of the machine *M* do exactly as the original machine does on every symbol a of the alphabet

$\rightarrow$ $\delta'((q,0),\in) = \{(\hat{q},1) \mid \exists a \in \Sigma, \delta(q,a) = \hat{q}\}$

Also at every stage, the machine has the option to skip a character. The only accepting sates are in copy 2. This means, the machine cannot accept a string without skipping a character.

(ii) Formal proof:

$\rightarrow$ The formal proof is given by induction on the length the string.

$\rightarrow$ An appropriate inductive hypothesis is to assume that, for any string *w* of length *k*,

- The machine *M* stays in the copy -1 iff it has not yet skipped a symbol.

i.e. $\delta'^*((q_0,0),w) = (q_1,0)$ iff $\delta^*(q_0,w) = q_1$

- The machine *M* jumps to the copy-2 iff there is some symbol a that is skipped.

i.e. $\delta'^*((q_0,0),w) = (q_1,1)$ iff $\delta^*(q_0,w_1 a w_2) = q_1$.

So in both (i) and (ii) we constructed an NFA N that recognizes the language $DROP\_OUT(A)$.

Thus $DROP\_OUT(A)$ is regular.

Hence class of regular languages closed under $\mathrm{DROP\_OUT}$ operation.