

获得的答案

Let B be any language over the alphabet Σ .

To Prove: $B = B^+$ iff $BB \subseteq B$

The requirement is to prove both the directions of iff.

One direction:

Assume: $B = B^+$ (1)

To show: $BB \subseteq B$

Since, for every language $BB \subseteq B^+$ (2)

By substituting (1) in (2), it can be obtained that $BB \subseteq B$

Hence, it has been proved that: $BB \subseteq B$ iff $B = B^+$

Other direction:

Assume: $BB \subseteq B$

To prove: $B = B^+$

It is known that for every language $BB \subseteq B^+$ (3).

Let w be a string of elements, such that $w \in B^+$ then $w \in B$.

- If $w \in B^+$ then the string w can be split into elements $x_1, x_2, x_3, \dots, x_k$ such that $w = x_1, x_2, \dots, x_k$ for $x_i \in B$ and $k \geq 1$. Since, $x_1, x_2 \in B$ and it is assumed that $BB \in B$.
- Similarly, it holds for the element x_3 , such that $x_3 \in B$ and $BB \subseteq B$. Thus, $x_1, x_2, x_3 \in B$.
- If this procedure is continued, then $x_1, x_2, x_3, \dots, x_k \in B$ that is $w \in B$.
- It means that, if all the elements of the string (x_1, x_2, \dots, x_k) belong to B , then the string $w \in B$.

Thus, $B^+ \subseteq B$ (4)

From, (3) and (4) it can be concluded that $B = B^+$.

Hence, it has been proved for both the directions that, for any language B , $B = B^+$ iff $BB \subseteq B$.