

Consider the problem of determining whether a two-tape Turing machine ever writes a nonblank symbol on its second tape when it is run on input w .

The language that denotes the problem is,

$L = \{ \langle M, w \rangle \mid M \text{ is a two-tape Turing machine writes a nonblank symbol on second tape when it is run on input } w \}$

In order to check the decidability, show that Construct a TM A_{TM} reduces to L . Use proof by contradiction method to prove decidability of the language. Assume TM T decides L . Construct a TM B that uses T to decide A_{TM} .

$B =$ "On input $\langle M, w \rangle$:

1. Use M to construct the two-tape Turing machine S .

$S =$ "On input x :

1. Simulate M using the first tape on input x .

2. If M is accepted then write a nonblank symbol on the second tape."

2. Run T on $\langle S, w \rangle$ to check whether S on input w writes a nonblank symbol on second tape.

3. If T accepts, M accepts w then accept. Otherwise, reject."

Thus, T decides A_{TM} which is undecidable. Therefore, L is undecidable.