
Question:

Let $G = (V, \Sigma, R, \langle \text{STMT} \rangle)$ be the following grammar.

$$\begin{aligned}\langle \text{STMT} \rangle &\rightarrow \langle \text{ASSIGN} \rangle \mid \langle \text{IF-THEN} \rangle \mid \langle \text{IF-THEN-ELSE} \rangle \\ \langle \text{IF-THEN} \rangle &\rightarrow \text{if condition then } \langle \text{STMT} \rangle \\ \langle \text{IF-THEN-ELSE} \rangle &\rightarrow \text{if condition then } \langle \text{STMT} \rangle \text{ else } \langle \text{STMT} \rangle \\ \langle \text{ASSIGN} \rangle &\rightarrow \text{a:=1}\end{aligned}$$
$$\Sigma = \{\text{if, condition, then, else, a:=1}\}$$
$$V = \{\langle \text{STMT} \rangle, \langle \text{IF-THEN} \rangle, \langle \text{IF-THEN-ELSE} \rangle, \langle \text{ASSIGN} \rangle\}$$

G is a natural-looking grammar for a fragment of a programming language, but G is ambiguous.

- a. Show that G is ambiguous.
 - b. Give a new unambiguous grammar for the same language.
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Answer:

----SETP1----

Ambiguous and unambiguous grammar

a.

Consider $G = (V, \Sigma, R, \langle \text{STMT} \rangle)$

To show that G is ambiguous

It is required to find out two leftmost derivations:

$$\begin{aligned}\langle \text{Stmt} \rangle &\Rightarrow \langle \text{if-then} \rangle \\ &\Rightarrow \text{if condition then } \langle \text{Stmt} \rangle \\ &\Rightarrow \text{if condition then } \langle \text{if-then-else} \rangle \\ &\Rightarrow \text{if condition then if condition then } \langle \text{Stmt} \rangle \text{ else } \langle \text{Stmt} \rangle \\ &\Rightarrow \text{if condition then if condition then } \langle \text{Assign} \rangle \text{ else } \langle \text{Stmt} \rangle \\ &\Rightarrow \text{if condition then if condition then a:=1 else } \langle \text{Stmt} \rangle \\ &\Rightarrow \text{if condition then if condition then a:=1 else } \langle \text{Assign} \rangle \\ &\Rightarrow \text{if condition then if condition then a:=1 else a:=1}\end{aligned}$$

Now the second derivation will be:

$$\begin{aligned}\langle \text{Stmt} \rangle &\Rightarrow \langle \text{if-then-else} \rangle \\ &\Rightarrow \text{if condition then } \langle \text{Stmt} \rangle \text{ else } \langle \text{Stmt} \rangle \\ &\Rightarrow \text{if condition then } \langle \text{if-then} \rangle \text{ else } \langle \text{Stmt} \rangle \\ &\Rightarrow \text{if condition then if condition then } \langle \text{Stmt} \rangle \text{ else } \langle \text{Stmt} \rangle \\ &\Rightarrow \text{if condition then if condition then } \langle \text{Assign} \rangle \text{ else } \langle \text{Stmt} \rangle\end{aligned}$$

\Rightarrow if condition then if condition then a:=1 else $\langle \text{Stmt} \rangle$
 \Rightarrow if condition then if condition then a:=1 else $\langle \text{Assign} \rangle$
 \Rightarrow if condition then if condition then a:=1 else a:=1

----SETP2----

In both cases when it takes "if-then" or "if-then else" result is same. Hence both have same left derivation and it is ambiguous.

----SETP3----

For making unambiguous grammar, it is required to make correct interpretation of the above two. For that when "if-then-else" is derived, it should not allow then part for deriving "if-then". So introduce a new variable $\langle \text{Stmt1} \rangle$ the new grammar is:

$\langle \text{Stmt} \rangle \Rightarrow \langle \text{if-then} \rangle \mid \langle \text{Assign} \rangle \mid \langle \text{if-then-else} \rangle$
 $\langle \text{if-then-else} \rangle \Rightarrow \text{if condition then } \langle \text{Stmt1} \rangle \text{ else } \langle \text{Stmt} \rangle$
 $\langle \text{Stmt1} \rangle \Rightarrow \langle \text{if-then-else} \rangle \mid \langle \text{Assign} \rangle$
 $\langle \text{if-then} \rangle \Rightarrow \text{if condition then } \langle \text{Stmt} \rangle$
 $\langle \text{Assign} \rangle \Rightarrow \text{a:=1}$