Question:

Consider the language B = L(G), where G is the grammar given in Exercise 2.13. The pumping lemma for context free languages, Theorem 2.34, states the existence of a pumping length p for B. What is the minimum value of p that works in the pumping lemma? Justify your answer.

THEOREM **2.34**

Pumping lemma for context-free languages If A is a context-free language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five pieces s = uvxyz satisfying the conditions

1. for each $i \geq 0$, $uv^ixy^iz \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$.

When s is being divided into uvxyz, condition 2 says that either v or y is not the empty string. Otherwise the theorem would be trivially true. Condition 3 states that the pieces v, x, and y together have length at most p. This technical condition sometimes is useful in proving that certain languages are not context free.

Answer:

----SETP1----

Given:

The minimum value of the pumping length p for the language $B = L(G)$

----SETP2----

Finding minimum length of p :

The language $B = L(G)$ is defined by the

Grammar $G = (V, \Sigma, R, S)$ where $V = \{S, T, U\}$; $\Sigma = \{0, \#\}$ and the given set of rules R:

$$S \rightarrow TT \mid U$$
$$T \rightarrow 0T \mid T0 \mid \#$$
$$U \rightarrow 0U00 \mid \#$$

Theorem 2.34 states that the pumping lemma p, for a context-free language A, is the minimum length of any string s in A such that it may be split into five pieces $s = uvxyz$. The string s will also fulfill the following three conditions:

1. For all $i \geq 0$, the string uv^ixy^iz is a part of the context-free language A.

2. The strings which are pumped, which are v and y, cannot both be the empty string ε, that is $|vy| > 0$.

3. The combined length of the strings lying inside u and z must not be greater that the pumping length p. In other words $|vxy| \leq p$.

The string $s = uvxyz$ can be taken as $\#\#0$, with the substrings being as follows:

$$u = v = z = \varepsilon$$
$$x = \#\#$$
$$y = 0$$

Thus the pumping length p is $|vxy| = |\#\#0| = 3$

Since, $|vy| = |\varepsilon 0| = |0| = 1 > 0$, condition 2 of the theorem holds.

Condition 3 of Theorem 2.34 is also valid as $|vxy| = |\varepsilon\#\#0| = |\#\#0| = 3 \leq p = 3$.

To meet condition 1 it has to be proven that $uv^i xy^i z \in B$ for $i \geq 0$.

The string $uv^i xy^i z$, where $u = \varepsilon, v = \varepsilon, x = \#\#, y = 0, z = \varepsilon$, can be expressed as the regular expression $\#\#0^i$.

The derivation for the case when $i = 0$ is:

$$S \Rightarrow TT \Rightarrow \#T \Rightarrow \#\#$$

So the string $s = uv^0 xy^0 z$ lies in the language B.

When $i > 0$ the string $s = uv^i xy^i z$ will also lie in B as the derivation will be:

$$S \Rightarrow TT \Rightarrow \#T \Rightarrow \#T0 \stackrel{*}{\Rightarrow} \#T0^+ \Rightarrow \#\#0^+$$

Condition 1 has been proven true as $uv^i xy^i z \in B \forall i \geq 0$.

----SETP3----

Conclusion:

Thus the string satisfies all three conditions of Theorem 2.34 for a context-free language. Therefore, **the minimum value is 3** for the pumping lemma p.