Rice's theorem: Let *P* be any nontrivial property of the language of a Turing machine.

In more formal terms, Let *P* be a Language consisting of Turing machine descriptions where *P* fulfills two conditions.

• First, *P* is nontrivial – It contains some, but not all, *TM* descriptions.

• Second, *P* is a property of the *TM's* language- whenever $L(M_1)=L(M_2)$, we have $\langle M_1\rangle \in P$ iff $\langle M_2\rangle \in P$. Here M_1 and M_2 are any TMs.

Then we have to prove that *P* is an undecidable Language.

Assume that *P* is a decidable Language satisfying the Properties of Rice theorem.

Let the Turing machine *T* decides the language *P*. According to the first condition of Rice theorem, *P* contains some, but not all, *TM* descriptions.

From above properties we assume $TM_1 \& TM_2$ be the Turing machines and $\langle TM_1\rangle \in P$ and $\langle TM_2\rangle \notin P$.

Now construct the Turing machine TM_3 using $TM_1 \& TM_2$.

TM_3"on input *w*:

1. By using recursion theorem, construct our own description for $\langle TM_3\rangle$

2. Run *T* on Turing machine $\langle TM_3\rangle$

3. If *T* accepts $\langle TM_3\rangle$, simulate $\langle TM_2\rangle$ on *w*.

If *T* rejects $\langle TM_3\rangle$, simulate $\langle TM_1\rangle$ on *w*.

• Suppose $\langle TM_3\rangle \in P$

Then *T* accepts $\langle TM_3\rangle$ and $L(TM_3)=L(TM_2)$

According to second condition of Rice's theorem $\langle TM_2\rangle \in P$ but our condition is $\langle TM_2\rangle \notin P$. It is a contradiction here.

• So, we will get similar contradiction if $\langle TM_3\rangle \notin P$.

As a result of that our assumption is wrong for *P* is a decidable language. Hence every property satisfying the conditions of Rice's theorem is undecidable.