Consider a polynomial $f(x)=c_1x^n+c_2x^{n-1}+\cdots+c_nx+c_{n+1}$.

Since above polynomial has a root at

$$x=x_0$$

So $f(x_0)=0$ implies that

$$c_1x_0^{\ n}+c_2x_0^{\ n-1}+\cdots+c_nx_0+c_{n+1}=0$$

Rearrange the above equation as:

$$c_1x_0^n=-\left(c_2x_0^{n-1}+\ldots+c_nx_0+c_{n+1}\right)$$

Now, take modulus on both the sides of the equation as:

$$\left|c_1x_0^{\ n}\right|=\left|-\left(c_2x_0^{\ n-1}+\cdots+c_nx_0+c_{n+1}\right)\right|$$
$$\left|c_1\right|x_0^{\ n}=\left|\left(c_2x_0^{\ n-1}+\cdots+c_nx_0+c_{n+1}\right)\right|$$

Use sub-additive property $|a+b|\le|a|+|b|$ of modulus function,

$$\left|c_1x_0^{\ n}\right|\le\left|c_2x_0^{\ n-1}\right|+\ldots+\left|c_nx_0\right|+\left|c_{n+1}\right|$$

Also, as $c_{\max}$ is the largest absolute value of c_i then for each $i=1,2,\cdots,(n+1)$,

$$c_{\max}=\left|c_{n+1}\right|$$

So, from above equation,

$$\left|c_1x_0^{\ n}\right|\le c_{\max}\left(1+\left|x_0\right|+\ldots+\left|x_0^{\ n-1}\right|\right)$$

Substitute $n.x_0^{\ n-1}$ for $1+\left|x_0\right|+\ldots+\left|x_0^{\ n-1}\right|$ where, $x_0^{\ n-1}$ is the largest one if $x_0>1$:

$$\left|c_1x_0^{\ n}\right|\le\ c_{\max}\cdot n\left|x_0^{\ n-1}\right|$$
$$\left|\frac{x_0^{\ n}}{x_0^{\ n-1}}\right|\le\ n.\frac{c_{\max}}{\left|c_1\right|}$$
$$\left|x_0^{\ n-(n-1)}\right|\le\ n.\frac{c_{\max}}{\left|c_1\right|}$$
$$\left|x_0\right|\le\ n.\frac{\left|c_{\max}\right|}{\left|c_1\right|}$$

It also can be written as:

$$\left|x_0\right|\le n.\frac{c_{\max}}{\left|c_1\right|}$$

To make the term $n.\frac{c_{\max}}{\left|c_1\right|}$ strictly greater than $\left|x_0\right|$, we can re-write as:

$$\left|x_0\right|<(n+1)\frac{c_{\max}}{\left|c_1\right|}\quad(\text{since, } n<n+1\,\text{always holds})$$