Proof of the decidability of the language:

• Express the language as $L = \langle R,S \rangle$ | R is a Deterministic Finite Automata(DFA) and S is a regular expression with $L(R) = L(S)$)}.

• Recollect the Theorem 4.5 states a Turing machine *T* that decides the language EQ_{DFA} = {$\langle P,Q \rangle$ | P and Q are Deterministic Finite Automata's(DFA) $L(P) = L(Q)$ }.

• Assume that *T* is the Turing Machine which decides language *L*.

• It can be defined as follows:

• T = "On input $L = \langle R,S \rangle$, where *R* is a Deterministic Finite Automata(DFA) and *S* is a regular expression:

• Convert R into a Deterministic Finite Automata(DFA) D_R using the algorithm in the proof of Kleene's Theorem.

• Operate a Turing machine *TM* as a decider *F* using Theorem 4.5 on input $\langle R, D_S \rangle$.

• If *F* accepts, accept the language *L*.

• If *F* rejects, reject the language *L*.