1. Solve the following linear program using SIMPLEX:

Maximize $18x_1 + 12.5x_2$

Subject to $x_1 + x_2 \le 20$

$x_1 \le 12$

$x_2 \le 16$

$x_1, x_2 \ge 0$

Sol: Now the linear program is in standard form, and we convert it into slack form as follows.

Maximize $18x_1 + 12.5x_2$

Subject to $\begin{cases} x_3 = 20 - x_1 - x_2 \\ x_4 = 12 - x_1 \\ x_5 = 16 - x_2 \\ x_1, x_2, x_3, x_4, x_5 \ge 0 \end{cases}$

(1) Add the value of x_1

Obviously, the tightest constraint is the equation $x_4 = 12 - x_1$. We rewrite it as $x_1 = 12 - x_4$, then substitute it into other equations and target function. The result is as follows.

Maximize $18(12 - x_4) + 12.5x_2$

Subject to $\begin{cases} x_1 = 12 - x_4 \\ x_3 = 8 + x_4 - x_2 \\ x_5 = 16 - x_2 \\ x_1, x_2, x_3, x_4, x_5 \ge 0 \end{cases}$

(2) Add the value of x_2

The tightest constraint is the equation $x_3 = 8 + x_4 - x_2$. We rewrite it as $x_2 = 8 + x_4 - x_3$, then substitute it into other equations and target function. The result is as follows.

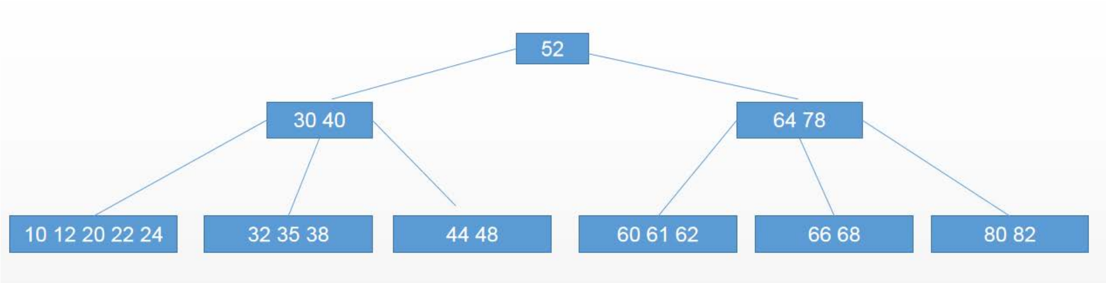
Maximize $316 - 5.5x_4 - 12.5x_3$

Subject to $\begin{cases} x_1 = 12 - x_4 \\ x_2 = 8 + x_4 - x_3 \\ x_5 = 8 - x_4 + x_3 \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{cases}$

Now all coefficients in the target function are negative. Let all nonbasic variables be 0, then we get the optimal solution $(x_1, x_2, x_3, x_4, x_5) = (12,8,0,0,8)$ as well as the optimal solution of original linear program $(x_1, x_2) = (12,8)$.

2.

B tree ,minimum degree t=3



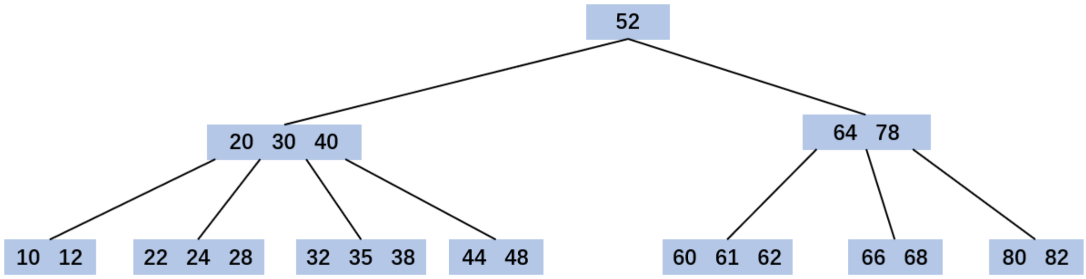
Draw the figure to show

(A).Insert 28

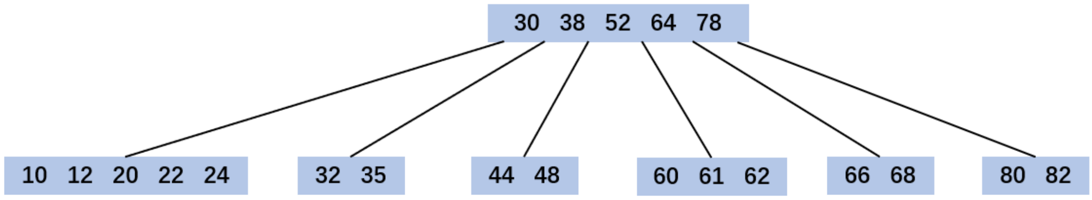
(B) Delete 40

Sol:

(A)



(B)



3.The interval-tree builds the red-black tree according to the preceding segment of the interval, with each node of the red-black tree appended with an x.max, which is the maximum value of the endpoints of all intervals of the x-rooted subtree.

(A) The interval-tree has a new operation, INTERVAL-SEARCH(T, i), which is used to find the node in the tree that overlaps the interval i. If no node in the tree overlaps with interval i, T.nil is returned. Write the pseudocode for this operation.

(B) With interval set {[0, 3], [5,8], [6, 10], [8, 9], [15, 23], [16, 21], [17,19], [19, 20], [25, 30], [26, 26]}, please build an interval tree (write simple process in drawing.)

Sol:

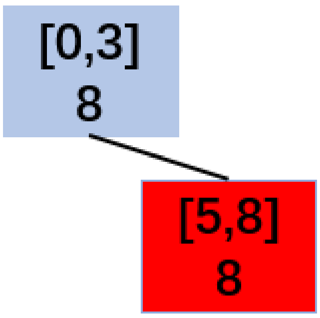
(A)

INTERVAL-SEARCH(T, i)
$x \leftarrow T.root$
while $x \neq T.nil$ and i does not overlap $x.int$ do
if $x.left \neq T.nil$ and $x.left.max \geq i.low$ then
$x \leftarrow x.left$
else $x \leftarrow x.right$
return x

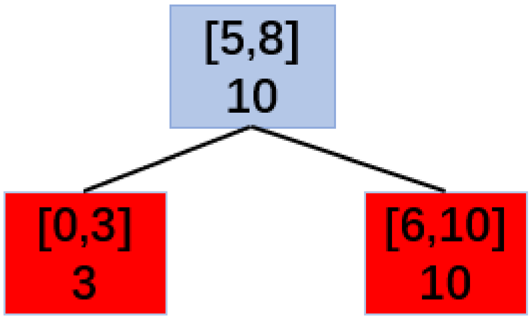
(B)

-insert [0,3]

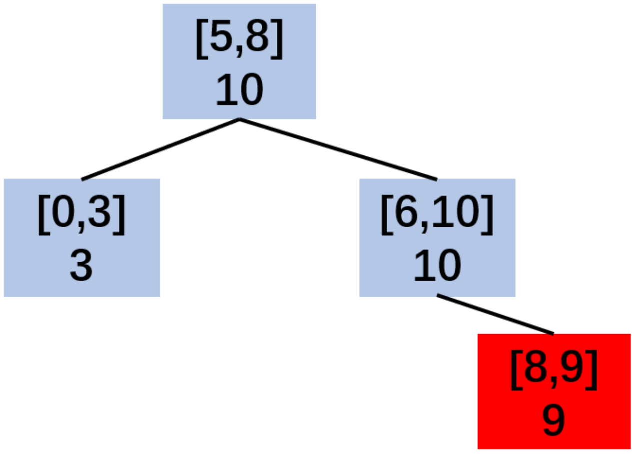
-insert [5,8]



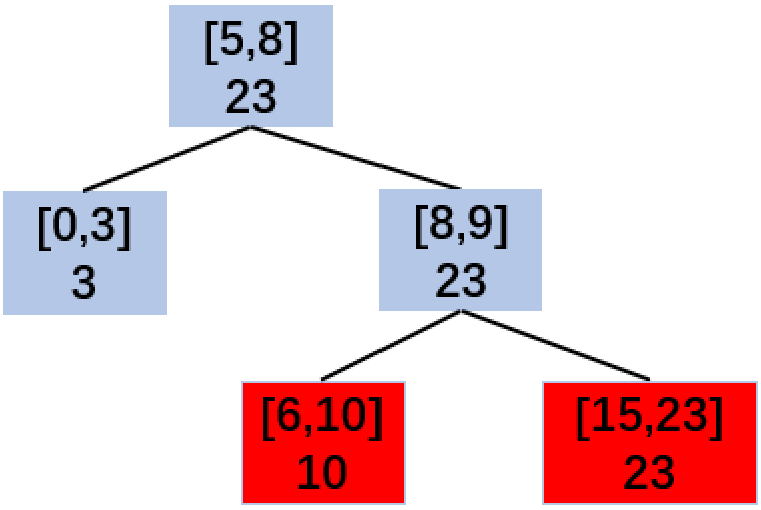
-insert [6,10]



-insert [8,9]



-insert [15,23]

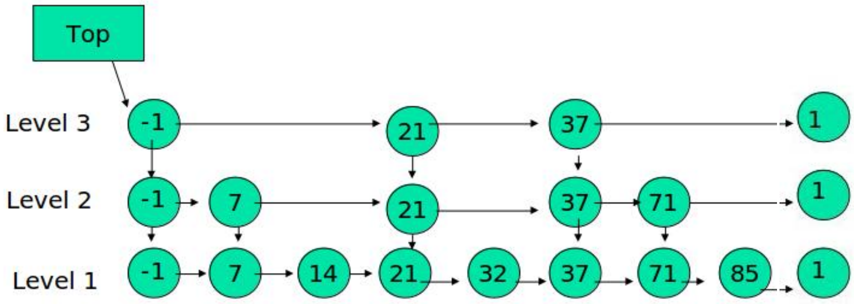


4. Skip List（-1 represents flag of begin, 1 represent flag of end）

(A) Delete (x) is an algorithm for delete element x in a skip list.

Write its pseudocode.

(B) There is a skip list as shown in the following figure. Insert element 119 in the skip list at level 4. (write simple process in drawing.)



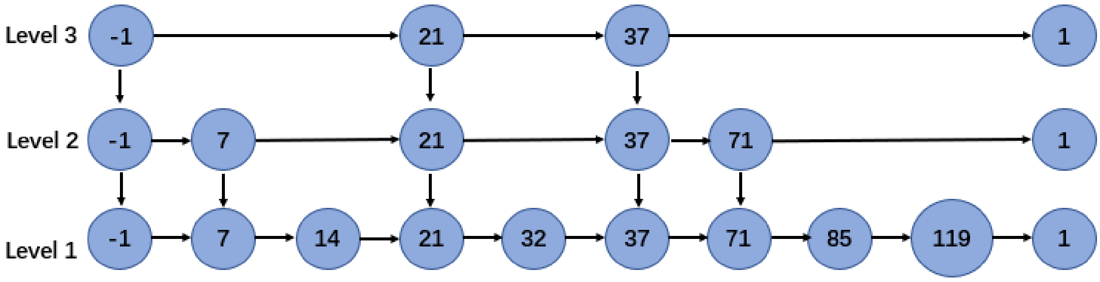
Sol:

(A)

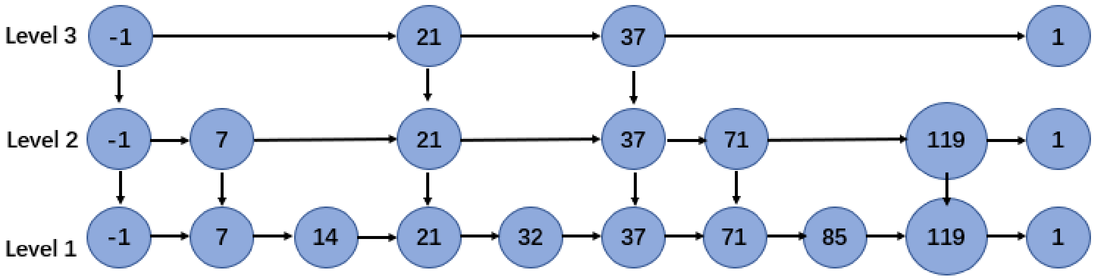
Delete(x)
$p \leftarrow \text{Top}$
while $p \neq \text{NULL}$ do
while $p.\text{next.value} < x$ do
$p \leftarrow p.\text{next}$
if $p.\text{next.value} = x$ then
$p.\text{next} \leftarrow p.\text{next.next}$
$p \leftarrow p.\text{down}$

(B)

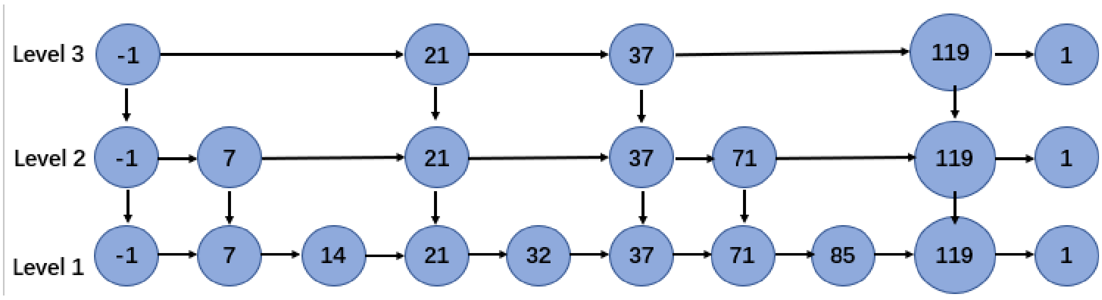
Level 1:



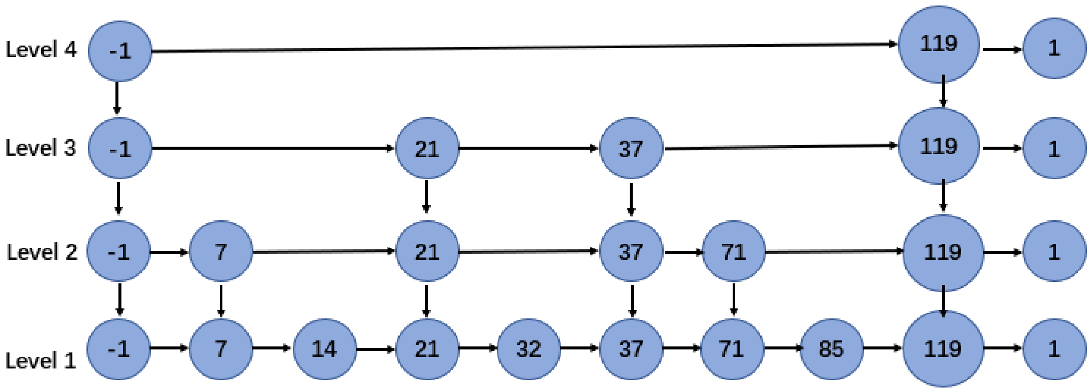
Level 2:



Level 3:



Level 4:



5. Problem X: Does the bool sequence $\{x_1, x_2, \ldots, x_n\}$ have at least one

value x_i is false.

Problem Y: Integer sequence $\{y_1, y_2, \ldots, y_n\}$. Is the minimum value y_i negative.

What is the Construct function T to reduce problem X to problem Y.

Sol:

The Construct function T is as follows.

$$y_i = \begin{cases} 0, & if\ x_i = true \\ -1, & if\ x_i = false \end{cases}$$

Obviously, $\{x_1, x_2, \ldots, x_n\}$ having at least one false is equivalent to that the minimum value y_i is negative.