Remember that  $EQ_{CFG}$  is co-Turing-recognizable language if and only if its complement  $\overline{EQ_{CFG}}$  is a Turing-recognizable language.

Now, 
$$\overline{EQ_{CFG}} = A \cup B$$
, where

 $A = \{ w \mid w \text{ does not have the form } \langle G1, G2 \rangle \text{ for some CFGs } G1 \text{ and } G1 \},$ 

B = 
$$\{ \langle G1, G2 \rangle \mid G1 \text{ and } G2 \text{ are CFGs and } L(G1) \neq L(G2) \}.$$

- A contains strings that defy the syntax for encoding  $\langle G1,G2 \rangle$ ) is simple to accept.
- The set B is realized in the following way:
- Transform the G1, G2 to Chomsky Normal Form (CNF).
- Then begin numbering strings in  $\sum$ \* lexicographically, where  $\sum$  is the group of terminals for G1, G2.
- For every string w numbered, check if it is produced by G1 and by G2.
- If the 2 Context Free Grammars or both of them cannot produce w, then TM goes on to generate the next string in the lexicographic order.
- Else, precisely one of the CFGs produces the string, and the TM accepts.
- Therefore, B is a Turing recognizable language.
- It is already proved that Turing-recognizable languages are closed under union, so EQCFG is Turing-recognizable language.
- Let the list of strings that are listed in lexicographic order are as follows:

s1, s2, s3 . . . over the input 
$$\Sigma^*$$
 .

Now EQ<sub>CFG</sub> is realized by the following TM M:

M = "On input  $\langle G1, G2 \rangle$ , where G1, G2 are Context Free Grammars:

- 1. Examine if G1, G2 are valid Context Free Grammars. If atleast 1 does not, accept.
- 2. Transform G1, G2 into corresponding Context Free Grammars  $\ensuremath{G^{\prime}_{1}}$  ,  $\ensuremath{G^{\prime}_{2}}$  .both into CNF.
- 3. Replicate the below step 4 for  $j = 1, 2, 3 \dots$
- 4. Examine if  $G_1$  and  $G_2$  produce  $s_j$  if precisely one of them does, accept."

Hence it is proved that EQ CFG is co-Turing-recognizable language.