Given:

A function $f:A\to B$in which A and B are two sets. The function $f:A\to B$ is **one to one function** as it never maps two different elements of same set with one element of another set.

The function $f:A\to B$ is **onto function** as each and every element of the set B is hit by the element of set A.

As the function $f:A\to B$is one to one and onto at the same time then it means the set A and B has the same cardinality. If the cardinality of these two sets is same so these sets are of **same size or equinumerous**.

If the function $f:A\to B$is one to one and onto at the same time it means the function $f:A\to B$ is **correspondence function** also. **Correspondence function** is also known as **bijective function**.

If the function $f:A\to B$is one to one and onto or bijective function, then sets A and B are of **same size**.

Proof:

Equivalence relation: A relation is known as equivalence in nature if it is reflexive, transitive and symmetric.

Same size relation is equivalence relation if and only if it is symmetric, reflexive and transitive.

• **For reflexivity:** If the user checks the identity function on the set A then this identity function is a bijection from A to A.

Hence the **same size** relation is reflexive relation.

• **For symmetry: if the function** $f:A\to B$is a bijective function then it means the inverse function f^{-1} is also bijective function from the set B to set A.

Hence if $A\sim B$ then $B\sim A$, so **the same size** relation is symmetric relation also.

• **For transitivty: Assume that** $A\sim B$ and $B\sim C$. Then the function $f:A\to B$is bijective function from A to B and the function $g:B\to C$ from B to C.

Therefore the composition of two bijective functions $f$and $g$is also a bijective function from A to C.

Hence the **same size** relation is transitive relation as $A\sim C$.

Conclusion:

Hence the **same size** relation is reflexive, symmetric and transitive in nature so the **same size** relation is equivalence relation.