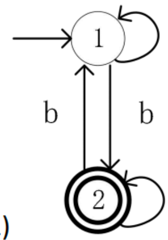
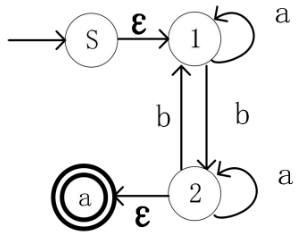
1 Answer:

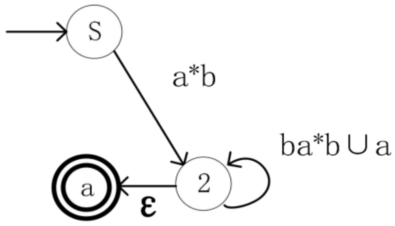
(a)



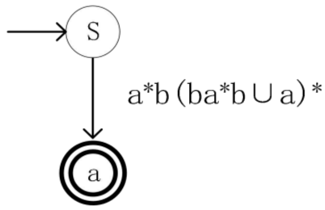
(1)



(2)



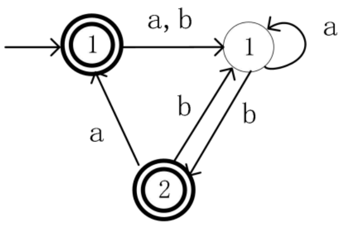
(3)



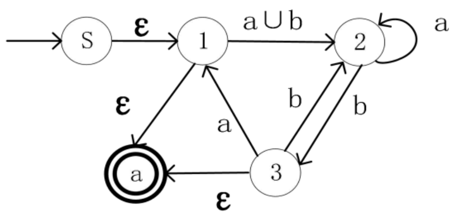
(4)

Hence, the regular expression is a*b(ba*b ∪ a)* .

(b)



(1)



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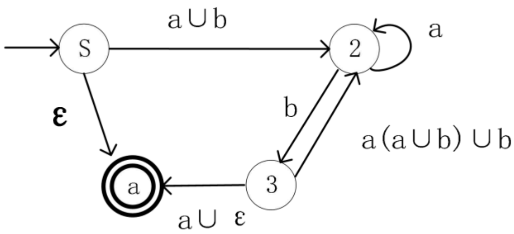

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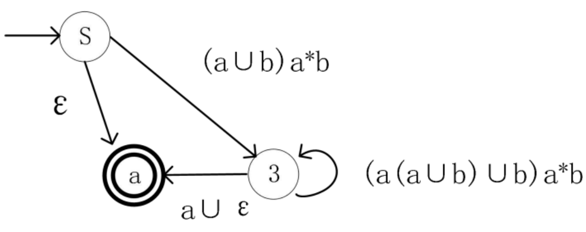

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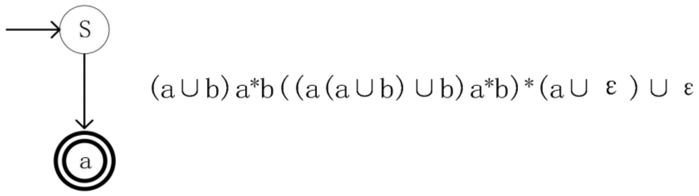
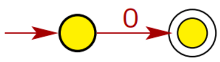


(5)

Hence, the regular expression is $(a \cup b)a^*b((a(a \cup b) \cup b)a^*b)^*(a \cup \varepsilon) \cup \varepsilon$

2 Answer:

0



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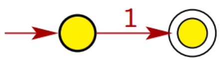

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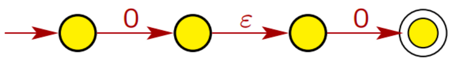

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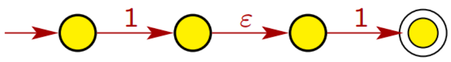

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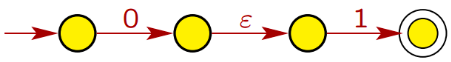
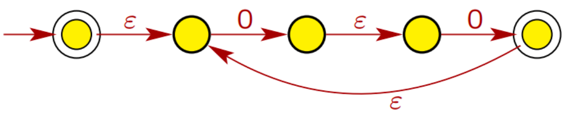
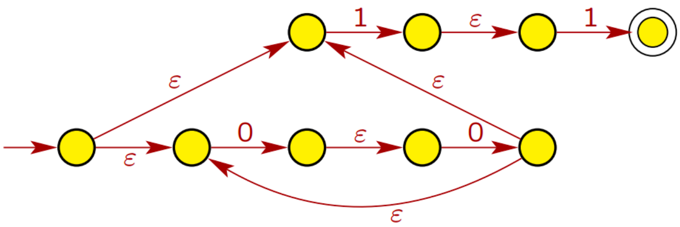
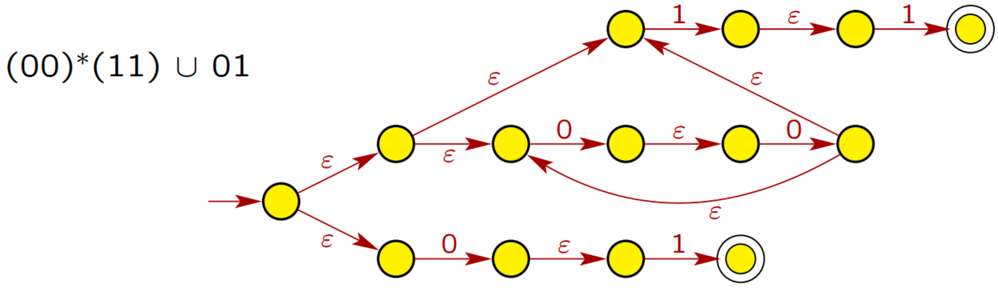


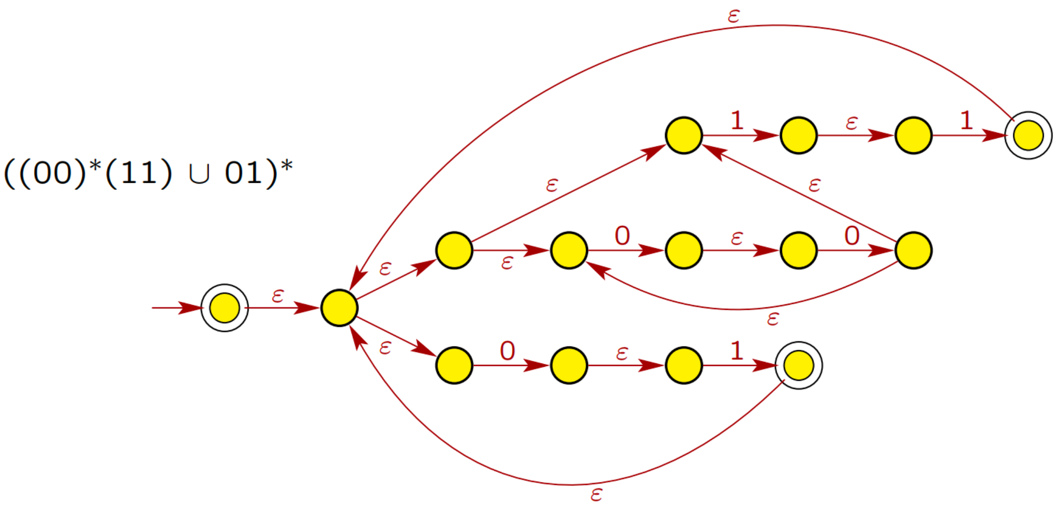
(00)*



(00)*(11)







3 Answer:

Answer: Suppose that A_2 is a regular language. Let p be the "pumping length" of the Pumping Lemma. Consider the string $s = a^p b a^p$. Note that $s \in A_2$ since $s = s^{\mathcal{R}}$, and $|s| = 2p + 1 \geq p$, so the Pumping Lemma will hold. Thus, we can split the string s into 3 parts $s = xyz$ satisfying the conditions

i. $xy^i z \in A_2$ for each $i \geq 0$,

ii. $|y| > 0$,

iii. $|xy| \leq p$.

Since the first p symbols of s are all a's, the third condition implies that x and y consist only of a's. So z will be the rest of the first set of a's, followed by ba^p. The second condition states that $|y| > 0$, so y has at least one a. More precisely, we can then say that

$$\begin{aligned} x &= a^j \text{ for some } j \geq 0, \\ y &= a^k \text{ for some } k \geq 1, \\ z &= a^m b a^p \text{ for some } m \geq 0. \end{aligned}$$

Since $a^p b a^p = s = xyz = a^j a^k a^m b a^p = a^{j+k+m} b a^p$, we must have that $j + k + m = p$. The first condition implies that $xy^2 z \in A_2$, but

$$\begin{aligned} xy^2 z &= a^j a^k a^k a^m b a^p \\ &= a^{p+k} b a^p \end{aligned}$$

since $j+k+m=p$. Hence, $xy^2z \notin A_2$ because $(a^{p+k}ba^p)^{\mathcal{R}} = a^pba^{p+k} \neq a^{p+k}ba^p$ since $k \geq 1$, and we get a contradiction. Therefore, A_2 is a nonregular language.

4 Answer:

For each $n \geq 1$, we can construct a DFA with the n states ($q_0, q_1, \ldots, q_{n-1}$) to recognize a^n, the start state q_0 is also accept state.

The transition function is $\delta(q_j,a)=q_{j+1}$, for j=0,1,…,n-2, and $\delta(q_{n-1},a)=q_0$.

Then add an ϵ transition from q_{n-1} to q_0, this NFA can recognize a^k, k is a multiple of n, for each n.

5 Answer:

a.

If $x \in B$, then there is some positive integer k such that $x = 1^ky$ where y is over {0,1} and has at least k 1s.

So $x = 1^k(0^*1)^k\{0,1\}^* \Rightarrow x = 11^{k-1}(0^*1)^k\{0,1\}^* \Rightarrow x = 1z$ where z over {0,1} and has at least 2k − 1 1s $\Rightarrow$ $x = 1z'$ where z' has at least one 1 (because $k \geq 1$, so $2k-1 \geq 1$) $\Rightarrow x \in \{1y \mid y \in \{0,1\}^*$ and y contains at least one 1}.

Let $A = \{1y \mid y \in \{0,1\}^*$ and y contains at least one 1}. As proven above, if $x \in B \Leftrightarrow x \in A$.

And A can described by the regular expression $10^*1\{0 \cup 1\}^*$. Hence, B is regular.

b.

Assume that C is regular. Let p be the pumping length given by the Pumping Lemma. Choose s to be the string $1^p0^p1^p$. Because s is a member of C and s is longer than p, the pumping lemma guarantees that s can be split into three pieces, s = xyz, satisfying the three conditions of the pumping lemma.

Since the first p symbols of s are all 1's, the third condition ($|xy| \leq p$) implies that x and y consist only of 1's. So z will be the rest of the first set of 1's, followed by 0^p1^p.

The second condition states that $|y| > 0$, so y has at least one 1. So we can say that $x = 1^j$ for some $j \geq 0$, $y = 1^k$ for some $k \geq 1$, $z = 1^{p-j-k}0^p1^p$.

The first condition implies that $xz \in C$, but $xz = 1^j\, 1^{p-j-k}0^p1^p = 1^{p-k}0^p1^p \notin C$, and we get a contradiction. Therefore, C is not regular.