Consider the language:

$$F = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$$

The language F is the union of three disjoint languages.

$$F_0 = \{b^j c^k \mid j, k \geq 0\}, F_1\{ab^j c^j \mid j \geq 0\}, F_2 = \{a^i b^j c^k \mid i \geq j, k \geq 0\}$$

Clearly F_0 and F_2 are regular languages.

The class of regular languages is closed under union and complement.

Thus $\overline{F_0 \cup F_2}$ is also regular.

We have $F_1 = F - (F_0 \cup F_2)$

$$= F \cap \overline{F_0 \cup F_2}$$

Since the class of regular languages is closed under intersection if *F* is regular, then so is F_1.

a.

Use pumping lemma to show that F_1 is not regular and hence neither is F.

Assume that F_1 is regular language.

Let *P* be the pumping length given by pumping lemma.

Consider a string $S = ab^P c^P \in F_1$

Clearly $|S| > P$.

By using the pumping lemma, *S* can be divided into three pieces.

i.e., $S = ab^P c^P = uvw$ such that $|uv| \leq P, |v| > 0$ and $uv^i w \in F_1 \ \forall i \geq 0$.

Take $u = a \quad v = b^P \quad w = c^P$

$$uv^0 w = a(b^P)^0 c^P \quad \therefore (i = 0)$$
$$= ac^P \notin F_1$$

The string *w* consist of '*c*' s. In this case, string $a(b^P)^0 c^P$ has more *c*s than '*b*'s and so is not a member of F_1, violating the condition of pumping lemma.

This is contradiction. The previous assumption that F_1 is regular is wrong. Thus F_1 is not regular.

Therefore, *F* is also not a regular language.

b.

Let pumping length $P = 2$

- Show that every string $S \in F$ of length at least *P* can be divided into three pieces $S = uvw$ such that, $|uv| \leq P, |v| > 0$ and $uv^l w \in F \ \forall l \geq 0$.
- Consider a string $a^i b^j c^k \in F$ of length at least 2.
- Choose *x* to be the empty string.
- If $i \neq 2$, then choose *y* to be the first symbol in $a^i b^j c^k$.
- If $i = 2$, then choose $y = aa$

Clearly these chosen x, y satisfies the three conditions of pumping lemma.

c.

part (a) and part(b) do not contradict the pumping lemma because the pumping lemma just says if a language is regular then there is a pumping length for that language. But, if the language satisfies the pumping lemma, the language may not be regular.