

Class P : P is a class of languages that are decidable in polynomial time on a deterministic single tape Turing machine. That is $P = \bigcup_k \text{TIME}(n^k)$.

Now, prove that P is closed under star operation. Consider a language $A \in P$. The following procedure decides A^* :

$M =$ "On input assume $y = y_1y_2\dots y_n \in \Sigma$.

- 1) if $y = \varepsilon$, output ACCEPT and halt.
- 2) Initialize the table $T[i, j] = 0$ for $i \leq j$.
- 3) For $i = 1$ to n
 - a) Run M on y_i , if $y_i \in L$ then set $T[i, i] = 1$.
- 4) For $k = 2$ to n
 - For $i = 1$ to $n - k + 1$
 - a) Assume $j = i + k - 1$
 - b) Run M on $y_i\dots y_j$, if $y_i\dots y_j \in L$ then set $T[i, j] = 1$.
 - c) For $l = i$ to $j - 1$
 - Set $T[i, j] = 1$, if $T[i, l] = 1$ and $T[l, j] = 1$
- 5) Output ACCEPT if $T[1, n] = 1$; otherwise output REJECT.

The above algorithm is a polynomial time algorithm. Therefore, P is closed under star operation.