

Consider the two languages A and B . The language *shuffle* on A and B is as follows: $\{w \mid w = a_1b_1...a_kb_k, \text{ where } a_1...a_k \in A \text{ and } b_1...b_k \in B, \text{ each } a_i, b_i \in \Sigma^*\}$.

Assume, $DFA_A = (Q_A, \Sigma, \delta_A, S_A, F_A)$ and $DFA_B = (Q_B, \Sigma, \delta_B, S_B, F_B)$ be two DFAs that recognize A and B respectively. $DFA_{shuffle} = (Q, \Sigma, \delta, S, F)$ recognizes the language perfect shuffle on A and B . For each character read, $DFA_{shuffle}$ may move from running DFA_A to running DFA_B . The NFA is more flexible when compared to the DFA . In this case, $NFA_{shuffle} = (Q, \Sigma, \delta, S, F)$ has to be constructed to allow more flexibility.

The $NFA_{shuffle}$ keeps track the current states of DFA_A and DFA_B . For each character read, $NFA_{shuffle}$ makes moves in the corresponding DFA (either DFA_A or DFA_B). After the whole string is read, if both DFA_A and DFA_B reaches to the final state, then the input string is accepted by $NFA_{shuffle}$.

The $NFA_{shuffle}$ can be defined as follows:

- $Q = (Q_A \times Q_B) \cup \{q_0\}$: The set of all possible states of DFA_A and DFA_B which should match with $NFA_{shuffle}$. Here, q_0 denotes the initial state.
- $q = q_0$
- $F = (F_A \times F_B) \cup \{q_0\}$: F_A and F_B are the final states for DFA_A and DFA_B respectively. The $NFA_{shuffle}$ accepts the string if both DFA_A and DFA_B are in accept states or $NFA_{shuffle}$ accepts the empty string.
- δ is as follows:
 - $\delta(q_0, \epsilon) = (q_A, q_B)$: At the start state q_0 , the current state of DFA_A is q_A and the current state of DFA_B is q_B without reading anything.
 - $(\delta_A(m, a), n) \in \delta((m, n), a)$: Change the current state of A to $\delta_A(m, a)$ when the character a is read. Here, the current state of D_A is m and the current state of D_B is n .
 - $(m, \delta_B(n, a)) \in \delta((m, n), a)$: Change the current state of B to $\delta_B(n, a)$ when the character a is read. Here, the current state of D_A is m and the current state of D_B is n .

The language L is said to be regular if there exist an FA that recognizes the language L . Here, the $NFA_{shuffle}$ is defined for the language *shuffle*.

Therefore, the class of regular languages is closed under shuffle.