Consider theorem 6.12 given in the textbook where it is proved that Th(◊, +) is decidable where + refers to the relation. As decidability has been proved for Th(◊,+), reduce Th(◊,+) to Th(◊,<).

The decidability of Th(◊, <) can be proved by converting sentence S_1 of language Th(◊,<) into S_2 sentence preserving all the truth or falsity related to models.

Now, replace the occurrences of S_1 in S_2 where $\wedge$ and $\vee$ with formula:

$\exists k[(i+k=j)\wedge(k+k\neq k)]$

k refers to new variable which will be different for every case.

S_2 is equivalent to S_1 because the value of i is less than the value of j. So, the value of j can be obtained by adding non-zero value in i. "i is less than j" that means 'j' can be obtained by adding non zero value to 'i'.

To prove the decidability of Th(◊,+) S_2 is supposed to be put in prenex-normal form.

For bringing existential qualifiers to front of the sentence, quantifiers are supposed to pass through Boolean operations that are being appeared in the sentence.

When the quantifiers are brought, the operations ^ and V will not be changed. When a null is brought it changes to and vice-versa. So, $\neg\exists k\varphi$ becomes $\forall k\neg\varphi$ and $\neg\forall k\varphi$ becomes $\exists k\neg\varphi$.

So, Th(◊,<) is decidable.