

Consider the following data:

From Exercise 2.16, it is clear that Context free language is closed under union operation, concatenation operation and star operation.

The definition of the regular expressions it can be inferred and known that regular expression consist of series of operations such as union, concatenation and star.

**Statement:** Every regular language is context free language

**Proof by using induction:**

Assume that  $E$  is a regular expression and there exists a CFG  $G$  such that  $L(E) = L(G)$ . The proof will be carried out by different number of operators in an expression  $E$ .

**Step-1:**

If  $Operation(E) = 0$  then  $E$  is either  $\phi, \epsilon$  or  $a$  in  $\Sigma$ .

**Step-2:**

- If  $E = \phi$ :  $G = (\{S\}, \{\}, P, S)$ . Here  $P = \{\}$ .
- If  $E = \epsilon$ :  $G = (\{S\}, \{\}, P, S)$ . Here  $P = \{S \rightarrow \epsilon\}$ .
- If  $E = a$ :  $G = (\{S\}, \{a\}, P, S)$ . Here  $P = \{S \rightarrow a\}$ .

For every regular expression consisting of  $\phi, \epsilon$  or  $a$  the CFG can be written. So, every regular expression is context free.

**Thus, every regular language is context free language.**

The context free languages are superset of regular languages. The rules for converting the regular languages consisting of union, concatenation and Kleene closure are as follows:

- If the regular expression  $E$  consists of  $E_1$  and  $E_2$  such that the concatenation  $E = E_1E_2$ , then the production can be easily expressed as  $S \rightarrow E_1E_2$ .
- If the regular expression  $E$  consists of  $E_1$  and  $E_2$  such that the union  $E = E_1 \mid E_2$ , then the production can be easily expressed as  $S \rightarrow E_1 \mid E_2$ .
- If the regular expression  $E$  consists of  $E_1$  such that the Kleene closure  $E = E_1^*$ , then the production can be easily expressed as  $E_1^* \rightarrow EE_1^* \mid \epsilon$ .