Question:

Read the definitions of NOPREFIX(A) and NOEXTEND(A) in Problem 1.40.

a. Show that the class of CFLs is not closed under NOPREFIX.

b. Show that the class of CFLs is not closed under NOEXTEND.

Problem 1.40

Recall that string x is a ***prefix*** of string y if a string z exists where xz = y, and that x is a ***proper prefix*** of y if in addition x ≠ y. In each of the following parts, we define an operation on a language A. Show that the class of regular languages is closed under that operation.

A**a.** NOPREFIX(A) = {w ? A| no proper prefix of w is a member of A}.

b. NOEXTEND(A) = {w ? A|w is not the proper prefix of any string in A}.

Answer:

----SETP1----

a)

Consider the $NOPREFIX$ operation. For a language A, the $NOPREFIX$ operation is defined as:

$NOPREFIX(A) = \{w \in A \mid \text{no proper prefix of } w \text{ is a member of } A\}$

• Now consider the language P, defined as $P = P_1 \cup P_2$ where $P_1 = \{x^a y^b z \mid a \neq b, a, b \geq 1\}$ and $P_2 = \{x^a y^b z^b \mid a, b \geq 1\}$.

• If a string $x^a y^b z$ of P_1 is considered, then the proper prefix of it is the string that consists only x and y and all the string in P_1 and P_2 consists minimum one z. Therefore, all strings in P_1 is in $NOPREFIX(P)$.

----SETP2----

Now, if a string $x^a y^b z^b$ in P_2 is considered. It is not in $NOPREFIX(P)$, if and only if there is proper prefix of it that is in P.

• As no proper prefix exists in P_2, the proper prefix will have to come from P_1 and hence the $a \neq b$. Thus, the string in P_2 which are in $NOPREFIX(P)$ are $\{x^a y^a z^a \mid a \geq 1\}$. Therefore, $NOPREFIX(P) = P_1 \cup \{x^a y^a z^a \mid a \geq 1\}$

• P is a context free language since P_1 and P_2 are both context-free and context-free languages are closed under union. However, $NOPREFIX(P)$ is not context-free.

• In other way, context-free behavior is shown by $NOPREFIX(P) \cap P(x^* y^* zzz^*) = \{x^a y^a z^a \mid a \geq 2\}$ **that is a contradiction**. Therefore, **a context-free language** P **exists in such a way that** $NOPREFIX(P)$ **is not context-free.**

Hence, from the above discussion, it can be said that **context-free languages are not closed under** $NOPREFIX$ **operation.**

----SETP3----

b)

Consider the $NOEXTEND$ operation. For a language A, the $NOEXTEND$ operation is defined as:

$NOEXTEND(P) = \{w \in A \mid w \text{ is not a proper prefix of any string in } A\}$

Now consider the language $P = P_1 \cup P_2$ where $P_1 = \{x^a y^b z^c \mid a \neq b, a,b,c \geq 1\}$ and $P_2 = \{x^a y^b z^b \mid a,b \geq 1\}$.

• Consider the string $x^a y^b z^c \in P_1$, the given string is not in $NOEXTEND(P)$ since $x^a y^b z^{c+1}$, which is an extension of the string is in P.

• Now, the string $x^a y^b z^b$ is considered. Now any extension of this string in P should belong to P_1. Hence this string will not exist in $NOEXTEND(P)$, if and only if an extension of it belongs to P_1 if $a \neq b$. Therefore**, the string of the form $x^a y^a z^a$ belongs to $NOEXTEND(P)$.**

• Hence, $NOEXTEND(A) = \{x^a y^a z^a \mid a \geq 1\}$. As it is known **that P is context-free but $NOEXTEND(P)$ is not context-free**.

Hence from the above explanation, it can be said that "**the context-free language are not closed under $NOEXTEND$ operation".**