获得的答案

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Given sentence is

$$\phi_{eq} = \forall x\left[R_1(x,x)\right]$$
$$\wedge \forall x,y\left[R_1(x,y) \leftrightarrow R_1(y,x)\right]$$
$$\wedge \forall x,y,z\left[\left(R_1(x,y)\right) \wedge R_1(y,z) \rightarrow R_1(x,z)\right]$$

ϕ_{eq} gives three conditions of equivalence relations i.e., Reflexive relation, Symmetric relation and Transitive relations.

Let R_{11} and R_{12} are two equivalence relations on some set, definition of R_1 by

$$\forall x,y\left[R_1(x,y) \equiv R_{11}(x,y) \wedge R_{12}(x,y)\right]$$

Where *x, y* are elements from set.

Reflexive relation: $\forall x\left[R_1(x,x)\right]$

$\equiv$ {definition of R_1}
$R_{11}(x,x) \wedge R_{12}(x,x)$

$\equiv$ {R_{11}, being an equivalence relation, is reflexive}
similarly R_{12}

$\equiv \{true\} \wedge \{true\}$
$\equiv true$

Symmetric relation: $\forall x,y\left[R_1(x,y) \leftrightarrow R_1(y,x)\right]$

$\equiv$ {definition of R_1}
$R_{11}(x,y) \wedge R_{12}(x,y)$

$\equiv$ {R_{11}, being an equivalence relation, is symmetric}
similarly R_{12}

$\equiv$ {definition of R_1}
$R_1(y,x)$

Transitive relation: $\forall x,y,z\left[\left(R_1(x,y)\right) \wedge R_1(y,z) \rightarrow R_1(x,z)\right]$

$\equiv$ {definition of R_1}
$\left(R_{11}(x,y) \wedge R_{12}(x,y)\right) \wedge \left(R_{11}(y,z) \wedge R_{12}(y,z)\right)$

$\equiv$ {rearranging the conjucts}
$\left(R_{11}(x,y) \wedge R_{11}(y,z)\right) \wedge \left(R_{12}(x,y) \wedge R_{12}(y,z)\right)$

$\equiv$ {R_{11}, being an equivalence relation, is transitive}
similarly R_{12}

$\equiv$ {definition of R_1}
$R_1(x,z)$

A model (U, R_1), where *U* is universe and R_1 is equivalence relation over *U*, is a model of ϕ_{eq}.