

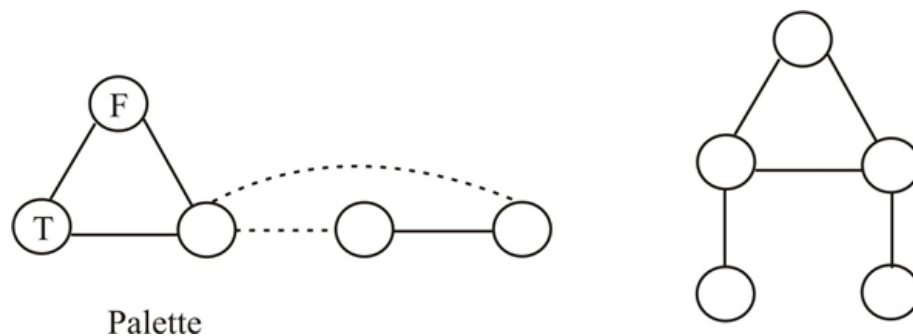
NP – complete:

A language B is NP – complete if it satisfies following two conditions:

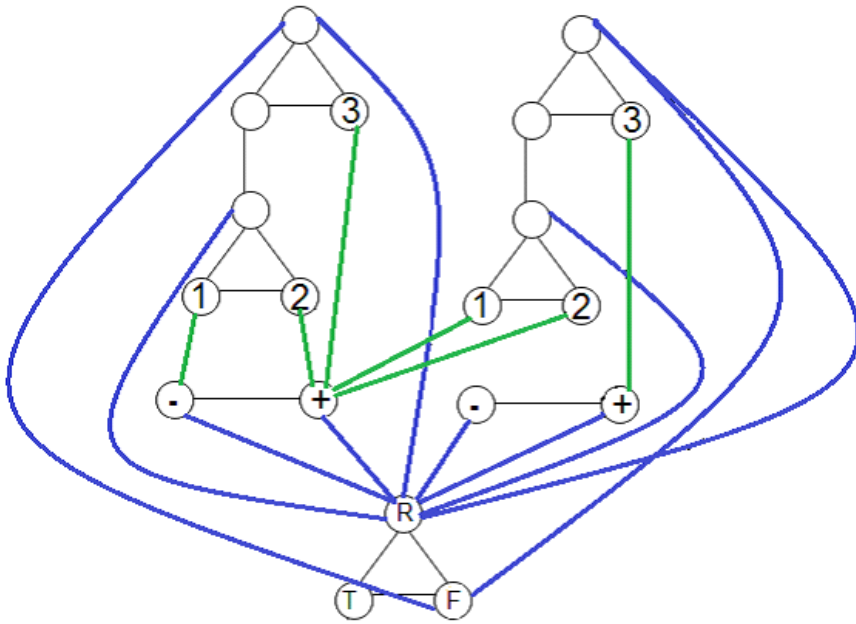
1. B is in NP
 2. Every A in NP is polynomial time reducible to B .
- 1.3 COLOR is in NP because a coloring can be verified in polynomial time.

2. $3SAT \leq_p 3COLOR$:

- “ $3SAT = \{\langle \phi \rangle \mid \phi \text{ is a satisfiable 3cnf – formula}\}$ ” and “3cnf –formula is the one in which all the clauses have three literals”
- Let $\phi = c_1 \wedge c_2 \wedge \dots \wedge c_l$ be a 3cnf formula over the variable x_1, \dots, x_n .
- To build a graph G with $2n + 6l + 3$ nodes, containing a variable gadget for each variable x_i , one clause gadget for each clause and one palette gadget as follows.
- Label the nodes of the palette gadget T, F and R .
- Label the node since each variable gadget + and – and cannot reach to the R node in the palette gadget.
- For each clause, create a gadget.
- Given three sub graphs.



- Connect the F and R nodes to the top of the clause gadget in the palette.
- Also, connect the top of its bottom triangle to the R node.
- For every clause c_j , connect the i^{th} ($1 \leq i \leq 3$) bottom node of its clause gadget to the literal node that appears in its i^{th} location.
- An example is shown below.



- To show that the construction is correct, we first demonstrate that if ϕ is satisfiable, the graph is 3- colored.
- The three colors are called T, F and R.
- Color the palette with its labels.
- For each variable, color the + node T and – node F if the variable is true in a satisfying assignment: otherwise reverse the colors.
- Because each clause has one True literal in the assignment, we can color the nodes of that clause so that the node connected to the F node in the palette is not colored F.
- Hence we have proper 3-coloring.
- Similarly, if we are given a 3-coloring, we can obtain a satisfying assignment by taking the colors assigned to the + nodes of each variable.
- Observe that neither node of the variable gadget can be colored R, because all variable nodes are connected to the R node in the palette.
- Furthermore, if both bottom nodes of the clause gadget are colored F, the top node must be colored T, and hence, each clause must contain a true literal.

Hence, $3SAT \leq_p 3COLOR$

Therefore, from (1) and (2) 3 COLOR is NP- complete.