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Definition of NP- Complete:

A language B is NP- Complete if it satisfies two conditions.

1. B is in NP

2. Every A is NP is polynomial time reducible to B i.e. B is NP-hard.

Objective of the SOLITAIRE Game:

- SOLITAIRE Game requires a $m \times m$ board.

- On each n^2 positions of $m \times m$ board a blue stone or a red stone or nothing is placed.

- Now the game is to remove the stones so that each column contains only stones of single color and each row contains at least one stone.

- The people who achieve this objective will win the game.

Now we have to show that SOLITAIRE is NP-Complete. Before this, we have to show that SOLITAIRE is in NP.

SOLITAIRE is in NP:

SOLITAIRE $\in NP$ because it can be verified that a solution works in polynomial time.

Every Language in NP is polynomial time reducible to SOLITAIRE:

- We know that "$3SAT = \{\langle\phi\rangle \mid \phi \text{ is a satisfiable 3 cnf - formula}\}$", three variables.

And "3SAT is NP- complete"

So, if we show that 3SAT $\leq_p$ SOLITAIRE then SOLITAIRE is also NP- Complete.

- Given ϕ with m variables $V_1,...,V_m$ and k clauses $C_1,....,C_k$

- Now construct the following game g with $k \times m$ board.

Construction of $k \times m$ game of G:

Let us assume that ϕ has no clauses that contain both V_i *and* $\overline{V_i}$ because such clauses may

If the variable V_i is in clause C_i then put a blue stone in row C_i column V_i

If the variable $\overline{V_i}$ is in clause C_i then put a red stone in row C_i, column V_i then

$k \times m$ board can be changed to square board necessary without affecting solvability.

Now we need to show that ϕ is satisfiable if and only if G has a solution:

If ϕ is satisfiable then G has a solution(Forward direction):

- A Satisfying assignment is taken.

- If V_i is true, remove the rod stone from the corresponding column.

- If V_i is false, remove the blue stone from corresponding column.

- So, stones corresponding to true literals remains.

- Because every clause has a true literal, every row has a stone.

- Therefore G has a solute or.

If G has a solution then ϕ is satisfiable (*backward direction*):

- Take a game solution.

• If the red stone removed from a column, set the corresponding variable true.

• If the bluestone is removed from a column, set the corresponding variable false.

• Every row has a stone remaining, so every clause has a true literal.

• Therefore ϕ is satisfied

Thus, SOLITAIRE is NP-Complete.