Consider the following information:

• String *x* is a prefix of string *y* if a string *z* exists such that *xz=y*.

• String *x* is a proper prefix of *y* if *xz=y* and $x \neq y$.

• The language *A* is regular language. Assume $M=(Q,\Sigma,\delta,q_0,F)$ be a DFA recognizing *A*.

a.

$NOPREFIX(A)=\{w \in A \mid no\ proper\ prefix\ of\ w\ is\ a\ member\ of\ A\}$

1. Let $M=(Q,\Sigma,\delta,q_0,F)$ be a DFA recognizing *A*.

2. Initially, find all words that have a proper prefix in *A*. The language *L* is represented as $L=\{w \in \Sigma^* : x \in A\ and\ z \in \Sigma^*\ such\ that\ xz=y\}$.

3. Now, construct the NFA $M^1=(Q^1,\Sigma,\delta^1,q_0{}^1,F^1)$ for all its components such that:

• $Q^1 = Q \cup \{q_f\}$ and $q_f \notin Q$

• For $q \in Q^1$ and $a \in \Sigma$ define $\delta^1(q,a)=\begin{Bmatrix}\delta(r,a) & if\ r \notin F \\ \phi & if\ r \in F\end{Bmatrix}$

• $q_0{}^1 = q_0$

• $F^1 = q_f$

Proof:

• If *w* is a string in Language *L*, there is a string *y* in *A*. Here, *x* is a proper prefix of *y* such that *xz=y* and *x* is non-empty.

• If *w* is taken as input of M^1, the computation on *x* ends at an accepting state in *M* and some computation on *z* ends at state q_f.

• So *w* is accepted by M^1, which means that there is a computation that ends at q_f.

• From the construction of M^1, the computation arrives at one of the accepting states in *M* before it reaches q_f.

• If we conclude that String *x* is a proper prefix of *y*, *M* on input *x* ends in one of its accepting states. So, *w* is a member of *L*, and *x* is in A.

• As, *NOPREFIX*(*A*) is defined as $A \cap \overline{L}$ and class of regular languages are closed under intersection and complement, *NOPREFIX*(*A*) is also regular.

b.

$NOEXTEND(A)=\{w \in A \mid w\ is\ not\ proper\ prefix\ of\ any\ string\ in\ A\}$

• Let $M=(Q,\Sigma,\delta,q_0,F)$ be a DFA recognizing *A*.

• Assume that the DFA for language M accepts that only the strings reaching the final state but not those strings that are added to reach a final state again.

• So, the strings exactly ending in final states are accepted.

• For a state $q \in F$, check whether there is a path from $q \in Q$ to any state in *F* (or a cycle involving *q*) using Depth First Search.

• Let $F^1 \subseteq F$ be the set of all the states from which there is no such path.

• Now, changing the set of final states *F* to F^1 gives a DFA for *NOEXTEND* (*A*).

• Thus, *NOEXTEND(A)* is also regular.