Linear Algebra L-1

→ The Geometry of linear equations

We can view the geometry of linear equations through 2 perspectives:

1. The Row picture
2. The column Picture.

Consider two equations in 2D space

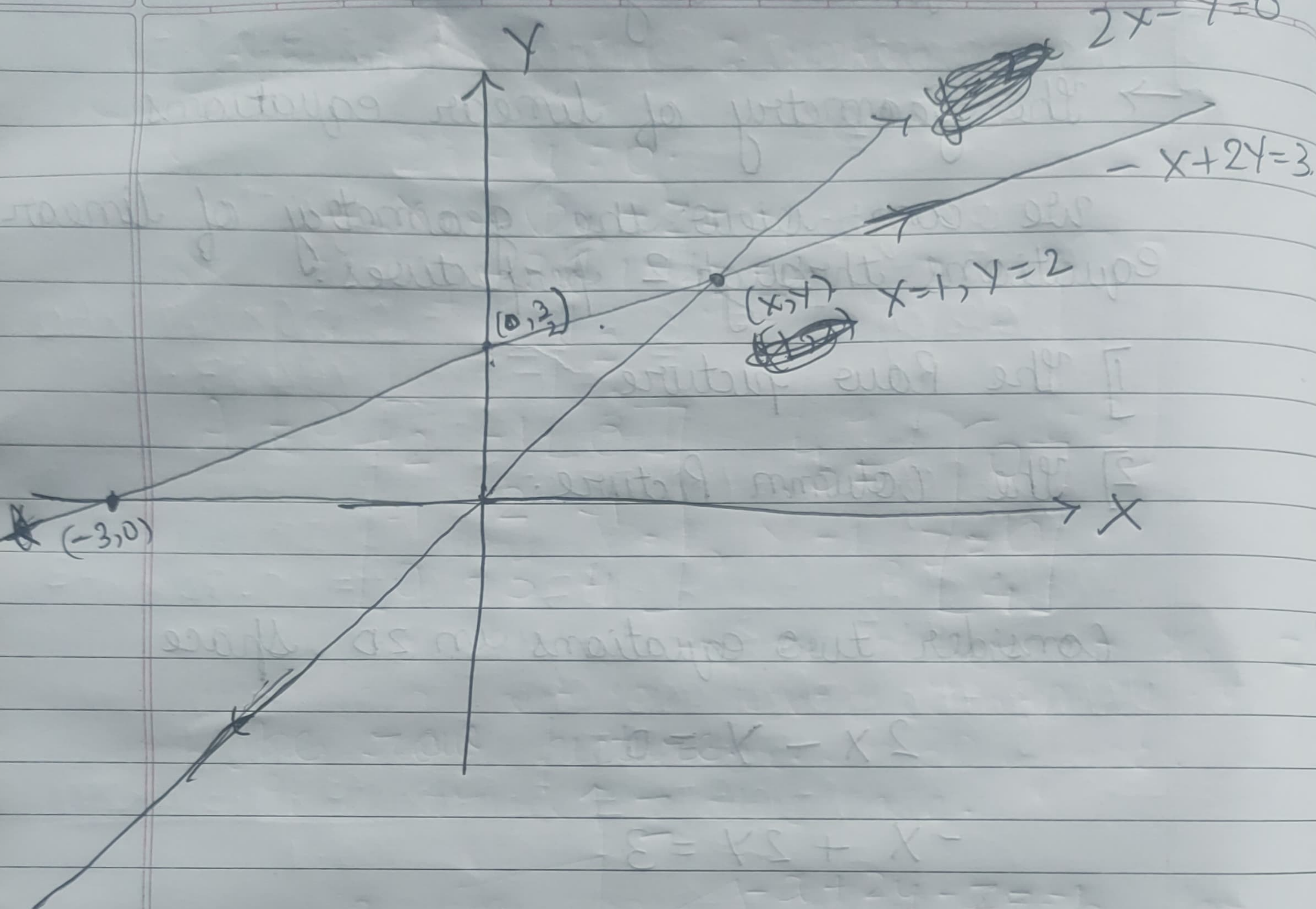
$$2x - y = 0$$

$$-x + 2y = 3$$

The matrix form is

$$\underbrace{\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}}_{A \text{ (co-efficient matrix)}} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 \\ 3 \end{bmatrix}}_{b \text{ output matrix.}}$$

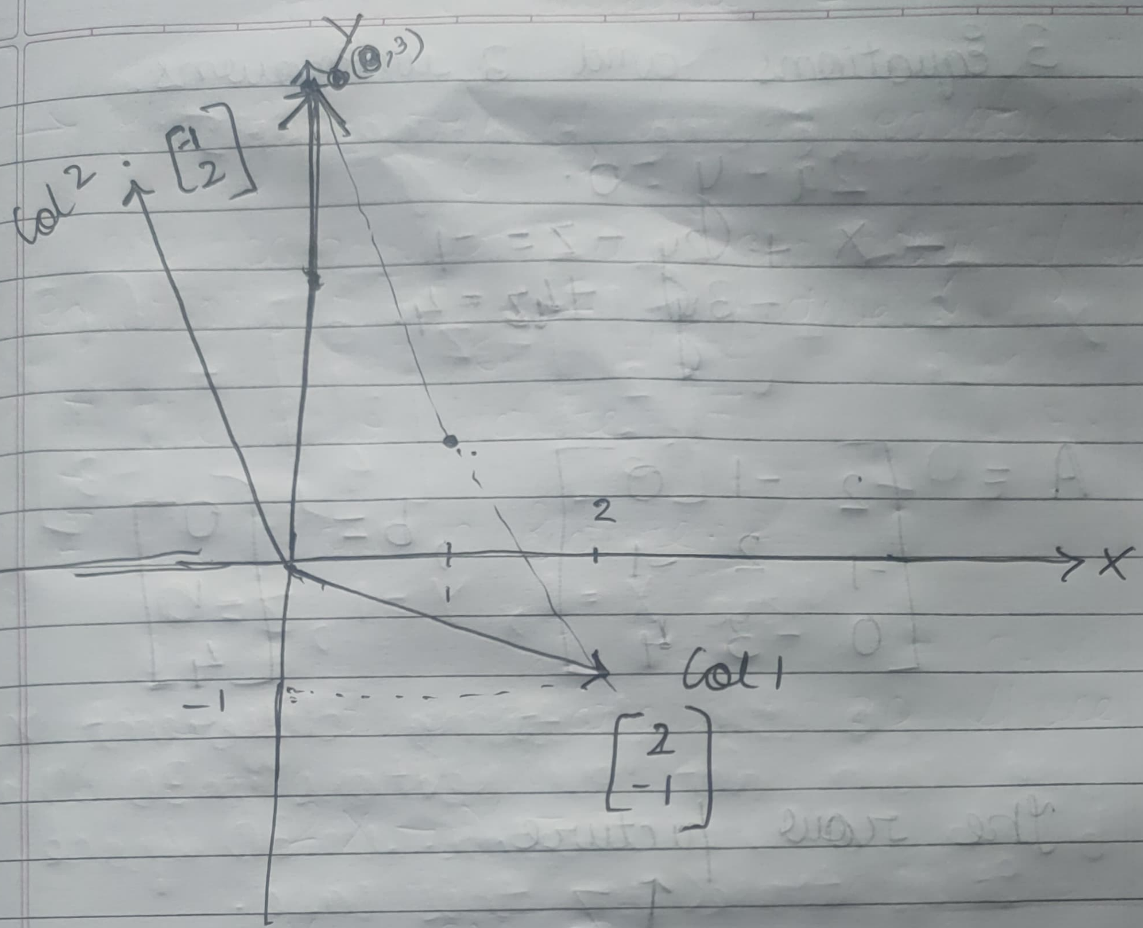
The row picture.



Column picture:

$$X\begin{bmatrix}2\\-1\end{bmatrix}+Y\begin{bmatrix}-1\\2\end{bmatrix}=\begin{bmatrix}0\\3\end{bmatrix}.$$

So we need to find the linear combination of columns which satisfies the R.H.S.



Its obvious $X = 1$, $Y=2$ here.

$$1\begin{bmatrix}2\\-1\end{bmatrix} + 2\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}0\\3\end{bmatrix}$$

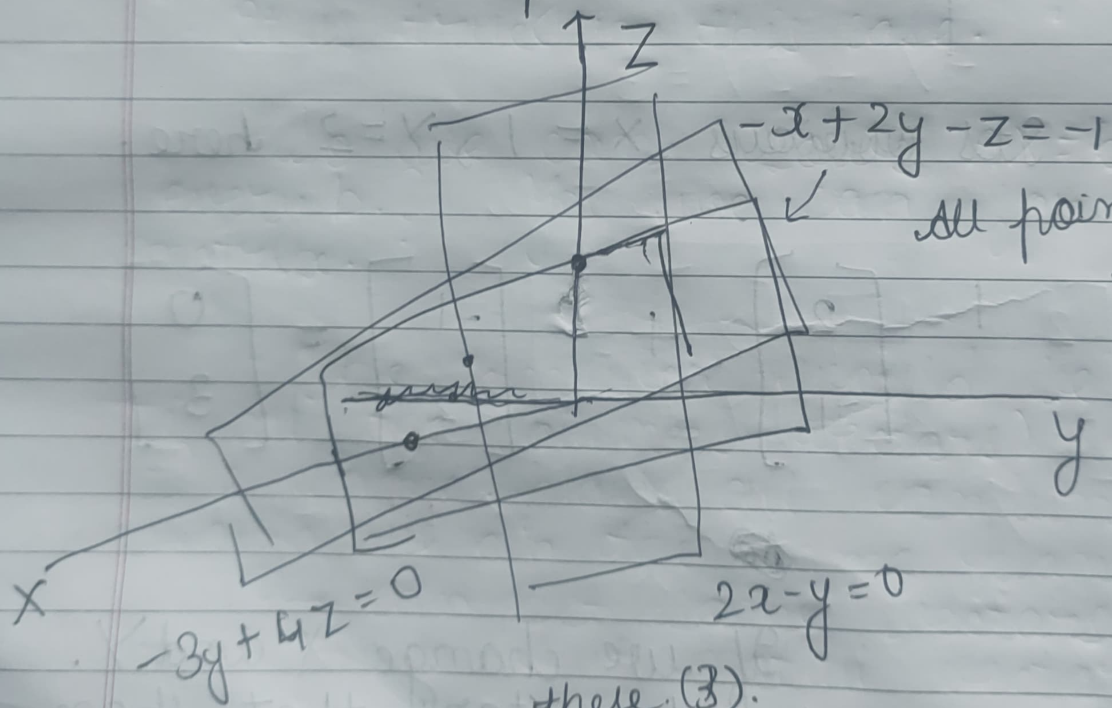
If we change X and Y. We can get all vectors that span the vector space.

3 Equations and 3 unknowns.

$$2x - y = 0$$
$$-x + 2y - z = -1$$
$$-3y + 4z = 4.$$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

The row picture.



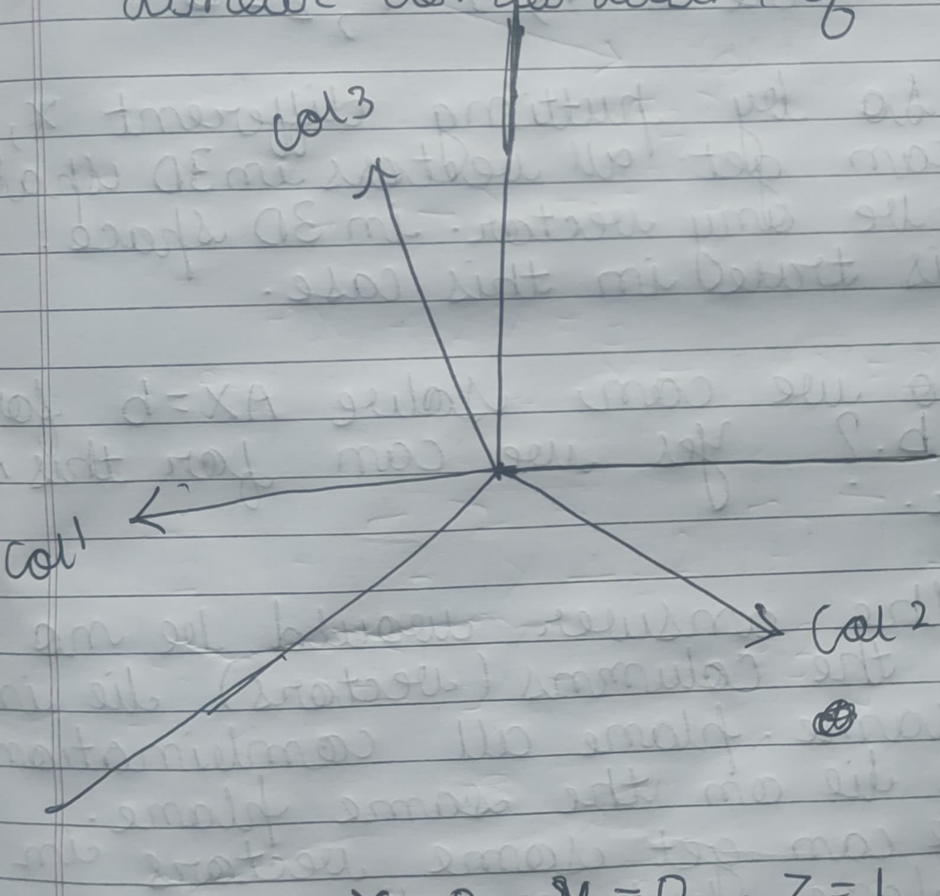
All these (3) planes meet at a point.

Row picture is very hard to visualize.

The column picture.

$$x\begin{bmatrix}2\\-1\\0\end{bmatrix} + y\begin{bmatrix}-1\\2\\-3\end{bmatrix} + z\begin{bmatrix}0\\-1\\4\end{bmatrix} = \begin{bmatrix}0\\-1\\4\end{bmatrix}$$

Linear combination of columns.



$$\begin{bmatrix}-1\\2\\-3\end{bmatrix}$$

$x=0, y=0, z=1$

PAGE No.
DATE / /

Suppose we change RHS

$$x \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} + z \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}$$

Here $X=1$, $Y=1$, $Z=0$

So by putting different x, y, z we can get all vectors in 3D space b can be any vector in 3D space. It is true in this case.

So we can solve $AX=b$ for every b.? Yes we can for this A matrix.

The answer would be no if all the columns (vectors) lie in the same plane all combinations would lie on the same plane. We can get some vectors in the plane but not all vectors in 3D space.

Suppose we have 9 equations and 9 unknowns (or any arbitrary number can be considered. So we have a 9D space. We can get every vector in 9D space.

But suppose if the 8th column and 9th column are similar they lie on the same 8D plane (Don't try to Imagine this XD!!). we will not get all vectors in 9D space. So we will get all vectors in 8D plane in 9D space.

Matrix multiplication column perspective

$$Ax = b.$$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

$$1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

Ax is a linear combination of columns of A.