Hybrid Realization of Quantum Mean Filters with Different Sized on E-NEQR Quantum Images by QFT Based Operations

Rısav Pokhrel, İbrahim Emre Kıvanççı
Classiq Technologies Inc, QWorld Association

Abstract- This paper presents a novel quantum-based method for noise reduction in quantum images using a combination of QFT-based mathematical functions and optimized classical partitioning techniques The method uses an image pixel values are mapped to real kets in Hilbert space using block structured addressing systems. The resulting state is then reformulated into a matrix-product-state representation, where quantum entanglement reveals classical correlations between different coarse-grained textures by using the Enanched novel enhanced quantum representation of digital images (E-NEQR) model to facilitate inversion classical image data into a grayscale quantum model. The study focuses on determining the optimal filtering actuator size to strike a balance between noise reduction and potential image blurring. This research contributes to theoretical insights and practical techniques for effectively reducing noise in quantum images, while preserving image quality

Keywords-Quantum Image Filtering, Quantum Circuit, Quantum Fourier Transform(QFT), Mean Filter, NEQR, ENEQR

I Introduction

In the landscape of quantum image representation and processing algorithms, this paper explores a broad range of methods with a specific focus on Mean filter integration. Quantum image representation deals with classical image representation using quantum properties.Two appear in this major area [6]:

1. **Encoding via angle parameter:** This probabilistic model includes FRQI (quantum representation of images using frequency spectrum information), QSMC (quantum spatial-spectral model), and related methods Despite its probabilistic nature , this method faces challenges in recovering accurate quantum images.

2. **Encoding Color Information into Qubit Sequences:** Notable examples in this category include NEQR (Novel Enhanced Quantum Representation) corresponding to grayscale images, cited as popular along with NCQI (Novel Quantum Representation of Color Digital Images) of three- channel images (RGB). The development of this model is exemplified in the E-NEQR extension. This paper focuses on exploring these quantum image interpretation techniques using E-NEQR[7] together with the Mean filter and investigating their effectiveness, limitations and potential applications.

This paper explores a novel approach to quantum mean filtering, taking advantage of QFT-based arithmetic[5] operation and the efficient ENEQR[7] quantum imaging model to achieve reduced circuit complexity and increased performance We depart from conventional methods that rely on quantum ripple full adders and multiplication, instead Using QFT based Arithmetic[5]. Using auxiliary qubits and an ENEQR[7] representation, we effectively reduce the circuit depth, resulting in an efficient and resource-efficient filtering scheme.

The study begins with a QFT-based analysis of transformation and divergence a comprehensive review, followed by an introduction to ENEQR. Next, we detail the implementation of the quantum mean filter[1] and carefully dissect its inner workings. To support the efficiency of our method, we combine the NEQR and ENEQR positions with depth-gate calculations, demonstrating the advantages of the latter. A thorough analysis of complexity confirms the computational advantages we have obtained through our proposed method. The paper concludes with a short but insightful conclusion, distilling the main takeaways and highlighting the promising possibilities of QFT-based arithmetic and ENEQR in quantum image processing

II Preliminaries

2.1 Quantum Fourier Transform

QFT basically transforms a state into a superposition of states with different phases on a computational basis, with phases associated with the frequency components of the initial state.QFT is designed to fine-tune the amplitudes of quantum states by redistributing their probabilities Mathematically,

QFT($|\Psi\rangle$) = $\frac{1}{\sqrt{N}} \sum_0^{N-1} e^{\frac{2\pi ijk}{N}} \ |\Psi\rangle$

2.2 Arithmetics used in this paper

As we work with qubits,which,like bits, can take the values of 0 or 1 we will represent numbers in binary form. While dealing with pixels, we only work with integers. So with n qubits, we can represent numbers from 0 to 2^n. Two numbers can be represented as a string of 1s and 0s, which we can represent as a multi-qubit state

$|m\rangle = |\overline{q_0 q_1 ... q_{n-1}}\rangle$

Where the formula to obtain the equivalent decimal number m will be

m= $\sum_{i=0}^{n-1} 2^{n-1-i} q_i$

Note that $|m\rangle$ refers to the basic state generated by the binary encoding of the number m

2.2.1 QFT based Addition

The procedure for applying this change is as follows

1. We apply QFT to the Sum Register and convert the state from Computational basis to Fourier basis.

2. We will apply controlled rotation to the jth qubit of the Sum Register, which leads to new phase of 2(m+k)$\frac{\pi}{2}\hat{\text{j}}$

3.We then return to the mathematical basis using Inverse QFT to obtain the combination (m+k).

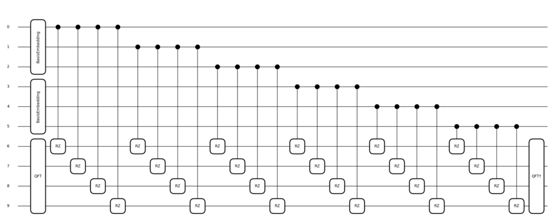

Figure 1. QFT based Addition source

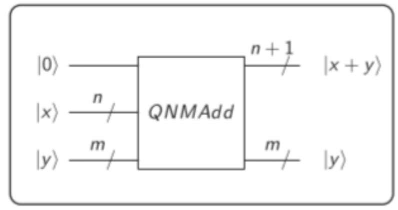

Figure 2. Block diagram of the QNMAddition [9]

2.2.2 Copy Module For copying the state we can use Quantum teleportation[8] OR also can use the series of CNOT gate.[9]

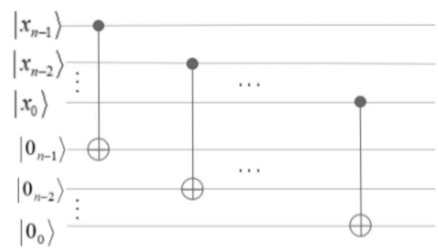

Figure 3. Circuit diagram of copy module

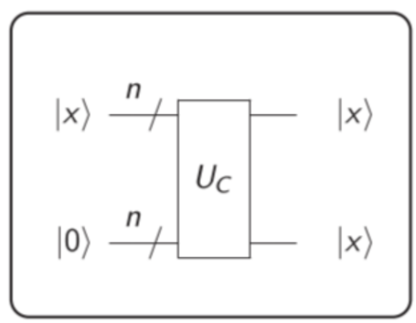

Figure 4. Figure 4. Data Unitary operator to copy the state of the qubits adapted from [9]

2.2.3 Cyclic shift module

The function of the cyclic shift module is to shift the entire image in a different direction, which is normally used when processing the image. To obtain grayscale information about neighboring pixels, we need to use these four modules: CS_{X+}, CS_{X-}, CS_{Y+} and CS_{Y-} [9]. The definitions of these four modules are shown in the equations below:

$CS_{X+}\ |I\rangle=\frac{1}{2^n}\sum_{Y=0}^{2^n-1}\sum_{X=0}^{2^n-1}\left|C_{(Y,X')}\right\rangle|Y\rangle\ |(X+1)mod2^n\rangle$

$CS_{X-}\ |I\rangle=\frac{1}{2^n}\sum_{Y=0}^{2^n-1}\sum_{X=0}^{2^n-1}\left|C_{(Y,X')}\right\rangle|Y\rangle\ |(X-1)mod2^n\rangle$

$CS_{Y+}\ |I\rangle=\frac{1}{2^n}\sum_{Y=0}^{2^n-1}\sum_{X=0}^{2^n-1}\left|C_{(Y',X)}\right\rangle\ |(Y+1)mod2^n\rangle|X\rangle$

$CS_{Y-}\ |I\rangle=\frac{1}{2^n}\sum_{Y=0}^{2^n-1}\sum_{X=0}^{2^n-1}\left|C_{(Y',X)}\right\rangle\ |(Y-1)mod2^n\rangle|X\rangle$

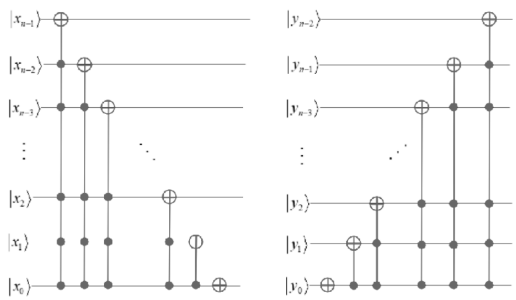

Figure 5. The quantum circuit of the CS_{X+} and CS_{Y-} cyclic shift transformations

The quantum circuit of the CS_{X+} and CS_{Y-} is shown above. Similarly, quantum circuits of CS_{Y+} and CS_{X-} can be formed by replacing the inputs $|X\rangle$ of CS_{X+} with $|Y\rangle$ and replacing input $|Y\rangle$ of CS_{Y-} with $|X\rangle$.

2.3 E-NEQR

ENEQR is a basis state representation that uses the CNOT gate to store the values of the pixels in a qubit sequence[7]. The basis states of qubit sequence are used to store the color and position information of image in the ENEQR model [7]. The ENEQR model represents only gray-scale images. Suppose the gray scale of image is 2^q. The binary sequence $C^0_{YX}C^1_{YX}...C^{q-2}_{YX}C^{q-1}_{YX}$ encodes the gray-scale value f(Y,X) of the corresponding pixel f(Y,X) as follows [7]:

$f(Y,X) = C^0_{YX}C^1_{YX}...C^{q-2}_{YX}C^{q-1}_{YX}$ where

$C^k_{YX} \in [0,1], f(Y,X) \in [0, 2^q - 1]$

The quantum state of a image for a $2^q * 2^q$ image can be written as follow [7]:

$|I\rangle=\frac{1}{2^n}\sum_{Y=0}^{2^n-1}\sum_{X=0}^{2^n-1}|f(Y,X)\rangle|YX\rangle$

$=\frac{1}{2^n}\sum_{Y=0}^{2^n-1}\sum_{X=0}^{2^n-1}\bigotimes_{i=1}^{q-1}\left|C^i_{YX}\right\rangle|YX\rangle$

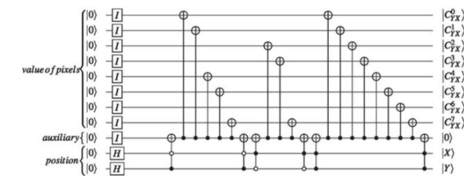

Figure 6. Eneqr Image Representation [7]

III Mean Filter

The mean filter, a fundamental image processing technique, helps reducing noise and in cost blurred images. It works by replacing the value of each pixel with the average of the neighboring pixels within a specified window or kernel. This process effectively reduces minor changes or irregularities in the image, resulting in a blurry, but consistent image. Its simplicity and effectiveness make it a widely used technique in image enhancement and noise reduction processing.The result value of the filtering operation is treated as the new value of the relevant pixel.

The main basis for this filtering is to use average values rather than pixel values in the original image. That is, we must choose a kernel that reflects the number of pixels in its neighborhood. [10]

g1(x,y)=$\frac{1}{M1}\sum_{f\in s1}$f(x,y)

Where s1 denotes the kernel, and M1 represents the total number of pixels in the kernel including the current pixel

Algorithm[1]: For filtering image with Different Sized:

This section presents an algorithm for implementation of a Quantum Mean Filter of different size kernel (s x s) based on the E-NEQR model.

The algorithm for applying sxs mask to an E-NEQR image($|I\rangle$) of size $2^n x 2^n$ with q-bit color depth is given in Algorithm 2 .

We used Built in function in Classiq platform for Addition operation. When doing division, because QFT division gives integer separation and can miss pixel value. So, we use classical division and for our study we use color depth q=8.

For the noise-free E-NEQR model however, the quantum state $|I_N\rangle$ of Color state $|0^r\rangle$ in the initial state of the r-qubit (where $r = log_2(2^q x s^2)$)) is ordered as follows.

$|I_N\rangle=\frac{1}{2^n}\sum_{y=0}^{2^n-1}\sum_{x=0}^{2^n-1}|0\rangle^{\bigotimes q}|yx\rangle$

So this Depending on the size of the kernel to be used, y and $x \leqslant (\frac{s}{2}-1)$ th and , y and $x \geqslant (2^n - \frac{s}{2})$) th are adjacent pixels with rows and columns missing . Therefore, the kernel operator is not used for those pixels, and the corresponding pixel value is efficiently taken from $|I\rangle$ to $|I_N\rangle$ by using the yx-controlled Uc function.

Algorithm 1 Algorithm for filtering image with different size.

Require: Inputs:$|I\rangle$, Outputs:$|I_N\rangle$
 y $\leftarrow roundup(\log_2(2^q x s^2))$
 Prepare the state $|I_N\rangle$ with color states $|0\rangle^{\bigotimes r}$for noiseless ENEQR Image
 for $y \leftarrow 2^{n-1}$ by 1 **do**
  for $x \leftarrow 2^{n-1}$ by 1 **do**
   if $((y \leqslant \frac{s}{2}-1)$ or $(y \geqslant 2^n - \frac{s}{2}) or (x \leqslant \frac{s}{2}-1)$ or $(y \geqslant 2^n - \frac{s}{2})$ **then**
    Assign the color value of the image $\left|I_{N(y,x)}\right\rangle$ by using the yx-controlled U_c operation
   else
    $calculatethevalue \left|V_{x,y)}\right\rangle$ for the corresponding pixel by evolving from the circuit $\left|QV_{yx)}\right\rangle$ to the $|I\rangle$
   end if
  end for
 end for
 Apply the Normalization Algorithm[3]

Algorithm[2]:Applying the filter weighted mask operator to the ENEQR Image

The process begins by adding the value of the pixel at (0,0) to $|Inyx\rangle$ using QFT based Addition, considering its neighboring pixels. The Cyclic Shift Transformation Operation is then applied to shift the next pixel in the image to the (0,0) position, and the OFT Based Adder operation is repeated. This cycle is repeated for all relevant pixels based on kernel size specified (s x s). At the same time, the CS_{X-} function is applied to $|I\rangle$ in the first image, shift the next pixel in each row to (0,0). This process is repeated for all pixels in the relevant row.

Algorithm 2 Applying the filter weighted mask operator QVxy[Reference][1]

for i $\leftarrow$ 1 to s by 1 **do**
 for j $\leftarrow$ 1 to s by 1 **do**
  $\left|I_{Nyx}\right\rangle \leftarrow \left|I_{Nyx}\right\rangle + \left|I_{(00)}\right\rangle$
  if j==s **then**
   for $k \leftarrow 1$ to $\frac{s}{2}$+1 $by 1$ **do**
    $C_{X+}|I\rangle$ shifting pixels right $\frac{s}{2}+1$ times by the x-axis to their original position
   end for
  else
   $C_{X-}|I\rangle$ The elements of $|I\rangle$ are shifted left by the x-axis
  end if
 end for
 if i==s **then**
  for $k \leftarrow 1$ to $\frac{s}{2}$+1 $by 1$ **do**
   $C_{Y+}|I\rangle$ All the pixels are shifted down $\frac{s}{2}+1$ times by the y-axis to their original position
  end for
 else
  $C_{Y-}|I\rangle$ All the pixels of the $|I\rangle$ are shifted up by the y-axis
 end if
end for

Algorithm[3]: Algorithm for normalisation of quantum circuit using classical division.

After this processing, new images are obtained, in which weighted pixels are added in their position. The classical division function is applied from pixel positions $\frac{s}{2}-1$ to $2^n - \frac{s}{2}$ in both rows and columns.

Input : Quantum register, s: Filter size operator
Output: Filtered mask Image

1) Measure the quantum register to obtain a classical outcome.
2) Filter out the values with a probability of $\frac{1}{(2^n * 2^n)} \pm 2\sigma$.
3) Convert the binary value obtained from measurement to decimal.
4) Normalize each element in the image array by dividing by s^2.

For $y \leftarrow \frac{s}{2}-1$ to $2^n - \frac{s}{2}$ by 1
 For $x \leftarrow \frac{s}{2}-1$ to $2^n - \frac{s}{2}$ by 1
  image[y][x]/= s^2

Convert the normalized array back to an image format.

The resulting image represents the filtered image, and shows the effect of quantum functions and classical division on the original image. This quantum image processing approach demonstrates the potential for efficient image manipulation and quantum filtering algorithms, and opens the way for improvements in quantum image processing techniques

IV Complexity analysis

The complexity analysis of the ENEQR quantum model $|I\rangle$ from a classical model involves several key steps and is roughly $O(n^2)$. The total circuit complexity of the quantum weighted addition complications arise from the subfunctions of the quantum weighted combination, including controlled Uc (copy module), cyclic shift conversion, and adder modules In particular, the use of s^2 Adder modules and s-1 Cyclic shift functions contribute to overall complexity.The Adder Module itself has a circuit complexity of $O(q^2)$, and the Cyclic shifts are at most of $O(n^2)$. Furthermore, the complexity of the Copy module is O(n). Considering the small kernel size, the total gate complexity can be expressed as $O(q^2+ns)$. Thus, the detailed circuit complexity of the quantum-weighted combination involving the weighted whole part would be $O(q^2+ns)$.

V Quantum Simulation Resources

This paper presents a novel approach to image cleaning by combining classical and quantum computing technologies. The entire project is simulated and programmed using Classiq, using Classiq-aer backend to ensure key sub-tasks like Controlled Uc (copy module), Cyclic shift transformations, Adder Module, E-NEQR etc. To validate the overall circuit, we leverage the capabilities of the IBM Quantum Vendor. Our study focuses on the use of the iconic "Lena" image as the primary noisy image, along with various other test images. The test includes different kernel sizes, mainly 2x2, 3x3, and 7x7 to evaluate the robustness and efficiency of the proposed quantum-inspired image denoising algorithm

VI Conclusion

In this paper, we present a novel approach to enhance imaging efficiency through quantum computing techniques, specifically targeting image filtering and noise reduction quantum computing these challenges with increasing image sizes and a corresponding increase in the computational requirements of various applications Emerging as a promising solution for remediation Our proposed approach uses QFT-based Arithmetic operation to reduce operations and provide a significant number of qubits.Quantum mean filtering in quantum images is greatly enhanced. We thoroughly investigate the effect of applying different size filtering operators on image quality It was developed. The effectiveness of our approach is demonstrated by clarifying the complexity of the circuit, illustrating the advantages offered by QFT-based mathematical operators. The Backbone for this paper is [1] where we modified Algorithm with use of E-NEQR instead of NEQR and also we use classical division for Division operation.

The proposed quantum models to represent images, and shows that they can be used for filters such as low-pass, high-pass, Butterworth etc. The simplicity and versatility of the method not only improves image quality but also opens the way to Quantum Digital Signal Processing. Specifically, it offers a highly efficient and quantum-based approach to image and signal processing.

VII Acknowledgments

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