

A Survey on Quantum Image Processing*

CAI Yongquan, LU Xiaowei and JIANG Nan

(Faculty of Information Technology, Beijing University of Technology, Beijing 100124, China)

Abstract — Quantum image processing is the intersection of quantum computation and image processing. Due to it is a newly emerging thing, researchers are facing not only great opportunities but also many challenges to develop more efficient and practicable services. We provide a comprehensive survey on quantum image processing to gather the current mainstream and discuss the advances made in the area, including quantum image representations, processing algorithms, and image measurement. Moreover, some open challenges and future directions are pointed out to attract continued research efforts.

Key words — Quantum image processing, Quantum computation, Quantum information, Quantum image representation, Quantum image compression, Measurement.

I. Introduction

The study of quantum computation and quantum information has become increasingly mature^[1]. In addition to further advancing the mathematical and physical foundations of quantum computation and quantum information, a growing number of scientists and engineers are devoting their research efforts to identifying and developing cross-fertilizing initiatives of quantum information processing in various fields^[2]. Quantum image processing (QIP) is one of them and it is a discipline devoted to use quantum computers to process images^[3].

Classical digital image processing has been sophisticated after decades of development^[4,5]. Why do we study QIP? There are two reasons:

1) Many image processing algorithms are time consuming, such as image understanding, image classification, and so on. For example, Google TensorFlow can shorten the training process of machine learning for some models from weeks to hours. However, it still consumed 65 hours using 100 GPUs to do image classification^[6]. The physics properties of quantum computers make them more efficient. We need quantum computers to help us to solve such problems.

2) Quantum computers need images. Today, modern computers use images to increase the convenience of operations and to convey more information. A computer without images is unimaginable. Therefore, in the future, commercial quantum computers must inherit this attribute. They need images to enhance the interface friendliness.

Quantum image processing was first proposed by Russia researcher Vlasov in 1997^[7], but it did not attract much attention then. Until 2003, after Beach^[8] and Venegas-Andraca^[9,10] gave some quantum image processing algorithms, QIP began to be concerned. In 2005, Latorre presents a new quantum image representation method^[11]. From 2010, QIP becomes a hot topic and more and more prosperous. Fig.1 shows the number of papers that when we use “quantum image processing” as keywords to search SCI database. It obviously has an upward trend.

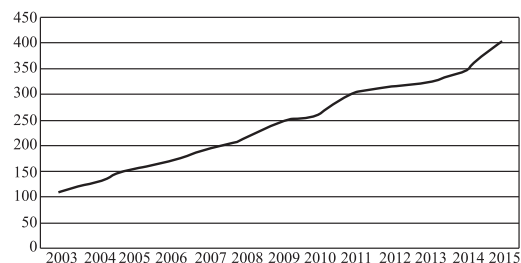


Fig. 1. The number of papers in each year

In general, QIP has three steps as Fig.2 shows:

1) Store the image into a quantum system, which is also known as quantum image preparation. This step is the classical-quantum interface. Now, there are multiple quantum image representation methods. Different representations have different preparation ways.

2) Process the quantum image. This is the main step of QIP, which means using a quantum computer to process quantum images. Many quantum algorithms have been proposed which can be divided into several categories:

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quantum geometric transformation, quantum color transformation, quantum image segmentation, quantum feature extraction, quantum image watermark and encryption, and so on.

3) Obtain the result by measurement. This step is the quantum-classical interface.

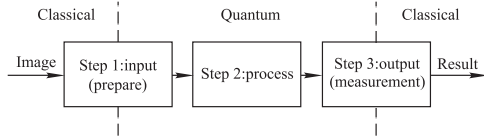


Fig. 2. The framework of QIP

In this paper, we provide a survey on quantum image processing. We first present the physics properties of quantum computation. Then, we summarize the structure of QIP, and introduce some QIP algorithms by category. Several open challenges and future directions are finally pointed out to recruit continued research efforts.

II. A Brief Introduction of Quantum Computation

Quantum image processing is based on quantum computation. In order to let readers understand QIP easily, we give a brief introduction of quantum computation.

1. Physics properties

Quantum has some physics properties that can't be explained with the macroscopic world, such as superposition, entanglement, collapses and *etc.*

If a quantum $|\psi\rangle$ is a superposition state, it stores a number of values $|\psi_1\rangle, |\psi_2\rangle, \dots, |\psi_m\rangle$ simultaneously:

$$|\psi\rangle = \sum_{i=1}^m \alpha_i |\psi_i\rangle \quad (1)$$

where, α_i are constants and $\sum_{i=1}^m |\alpha_i|^2 = 1$. However, we can not know the values of $\psi_1, \psi_2, \dots, \psi_m$ easily as we do in classical world.

In order to get the values, measurement is a necessary step. When measuring the superposition state $|\psi\rangle$, only one component state can be got and other states disappear. This is collapse. The probability of $|\psi_i\rangle$ being measured is $|\alpha_i|^2$.

In Eq.(1), $|\psi\rangle$ is a one-quantum state. If there are two or more quantum, some new phenomena will occur. For example, two two-quantum states $|\xi_1\rangle$ and $|\xi_2\rangle$ are shown below.

$$|\xi_1\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |01\rangle) = \frac{1}{\sqrt{2}} (|0\rangle \otimes (|0\rangle + |1\rangle))$$

$$|\xi_2\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

where, \otimes is tensor product. In $|\xi_1\rangle$, the first quantum is in state $|0\rangle$ and the second quantum is in state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.

They are separable. However, in $|\xi_2\rangle$, if the first quantum is in state $|0\rangle$, the second quantum also is in state $|0\rangle$; if the first quantum is in state $|1\rangle$, the second quantum also is in state $|1\rangle$. They entangle together.

2. The advantages of quantum computation

As early as 1982, the famous physicist Feynman has recognized the more powerful computing ability of quantum computers than classical computers^[12].

Quantum superposition significantly reduces both the space complexity and the time complexity. Considering the simplest case: each qubit is the superposition of only two values: 0 and 1. If we want to store all the 2^n signals from " $\underbrace{00 \dots 0}_n$ " to " $\underbrace{11 \dots 1}_n$ ", it needs $2^n \times n$ bit in a classical computer and in contrast, it needs only n qubit in a quantum computer. That is because in quantum computers, each qubit can store 0 and 1 simultaneously. Fig.3 gives an example.

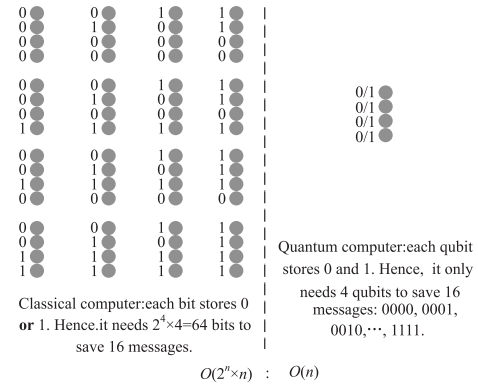


Fig. 3. Quantum superposition reduces the space complexity. An example: $n = 4$

If each number is added by 1, in the classical computer, the 2^n signals is processed one by one and the time complexity is $O(2^n)$. In the quantum computer, one operation is enough because all the signals stay at the same storage space. The time complexity is $O(1)$.

It is obviously that, no matter space complexity or time complexity is only $\frac{1}{2^n}$ as in classical computers. Hence, quantum computers can help us to solve problems that can not do in classical computers efficiently. This is the main reason to develop quantum image processing.

3. Quantum logical gate

Quantum logical gates are the basic operations in quantum computers. Table 1 gives some frequently-used gates in quantum image processing. Note that the description of the functions is not very strict, but might be easier to understand.

III. Quantum Image Representation and Preparation

1. The early representations

In 2003, the Qubit lattice model^[10] is derived from the classical image representation and see an image as a 2D matrix. If the image is Q and its size is $n_1 \times n_2$, $n_1 \times n_2$ qubits are logically arranged in a quantum matrix (*i.e.*, Qubit lattice), and each qubit store one pixel.

$$Q = \{|q\rangle_{i,j}\}, i \in \{1, 2, \dots, n_1\}, j \in \{1, 2, \dots, n_2\} \quad (2)$$

where,

$$|q\rangle_{i,j} = \cos \frac{\theta_{i,j}}{2} |0\rangle + e^{i\gamma} \sin \frac{\theta_{i,j}}{2} |1\rangle$$

and $\theta_{i,j}$ stores the color information.

Table 1. Frequently-used quantum gates

Gate name	Symbol	Function
Identity	$a \text{ --- } \boxed{I} \text{ --- } a$	Do nothing
Hadamard	$\text{---} \boxed{H} \text{---}$	Produce a superposition state that the probability of $ 0\rangle$ and $ 1\rangle$ are both $\frac{1}{2}$
NOT	$a \text{ --- } \oplus \text{ --- } \bar{a}$	Turn $ 0\rangle$ to $ 1\rangle$, and $ 1\rangle$ to $ 0\rangle$
CNOT	$\begin{array}{c} a \text{ --- } \bullet \text{ --- } a \\ \\ b \text{ --- } \oplus \text{ --- } a \oplus b \end{array}$	a is the control qubit and b is the target qubit. If a is in state $ 1\rangle$, b reverses
CCNOT	$\begin{array}{c} a \text{ --- } \circ \text{ --- } \\ \\ b \text{ --- } \oplus \text{ --- } \end{array}$	If a is in state $ 0\rangle$, b reverses
SWAP	$\begin{array}{c} a \text{ --- } \times \text{ --- } b \\ \\ b \text{ --- } \times \text{ --- } a \end{array}$	Exchange the value of a and b
Toffoli	$\begin{array}{c} a \text{ --- } \bullet \text{ --- } a \\ \\ b \text{ --- } \bullet \text{ --- } b \\ \\ c \text{ --- } \oplus \text{ --- } c \oplus ab \end{array}$	a and b are the control qubits and c is the target qubit. If a and b are in state $ 1\rangle$, c reverses

The Qubit lattice model is similar with the classical image representation to make it natural to transform from classical image to quantum image. However, it does not take advantage of the physics features of quantum system, such as superposition, entanglement and so on. It needs a large number of qubits to store an image.

In 2005, Real ket model^[11] casts an image into a real ket by dividing the image into four equal parts continually. A $2^n \times 2^n$ Real ket image is

$$|\psi_{2^n \times 2^n}\rangle = \sum_{i_1, \dots, i_n=1, \dots, 4} c_{i_n, \dots, i_1} |i_n, \dots, i_1\rangle \quad (3)$$

where, c is color information and i_n, \dots, i_1 are the location information. For example, a 4×4 image shown in Fig.4 can be represented as

$$\begin{aligned} |\psi_{2^2 \times 2^2}\rangle &= \sum_{i_1, i_2=1, \dots, 4} c_{i_2 i_1} |i_2 i_1\rangle \\ &= c_{11}|11\rangle + c_{12}|12\rangle + c_{13}|13\rangle + c_{14}|14\rangle \\ &\quad + c_{21}|21\rangle + c_{22}|22\rangle + c_{23}|23\rangle + c_{24}|24\rangle \\ &\quad + c_{31}|31\rangle + c_{32}|32\rangle + c_{33}|33\rangle + c_{34}|34\rangle \\ &\quad + c_{41}|41\rangle + c_{42}|42\rangle + c_{43}|43\rangle + c_{44}|44\rangle \end{aligned}$$

Real ket model takes advantage of superposition and only use n qubits to represent a $2^n \times 2^n$ image. (Note that

in Real ket, each qubit is the superposition of four values: 1, 2, 3, 4.)

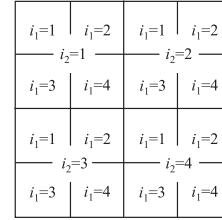


Fig. 4. Real ket representation

However, in Ref.[11], the authors do not give how to process a quantum image based on Real ket.

In 2010, Entangled image model is proposed, which is somewhat similar to Qubit lattice because they both use one qubit to store one pixel. However, it uses entanglement to denote the connection between the pixels. For example, Fig.5 gives a graph formed by 2 triangles with vertexes p, q, r and s, t, u . Its representation is

$$\begin{aligned} |I\rangle &= \otimes_{i=1, i \neq p, q, r, s, t, u}^n |0\rangle_i \otimes \frac{|000\rangle_{pqr} + |111\rangle_{pqr}}{\sqrt{2}} \\ &\quad \otimes \frac{|000\rangle_{stu} + |111\rangle_{stu}}{\sqrt{2}} \end{aligned}$$

where n is the number of pixels.

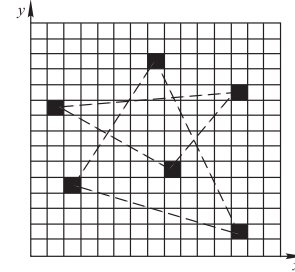


Fig. 5. Entangled image representation

2. The modern representations

There is a kind of quantum image representation that is similar to the pixel representation for images on traditional computers. It captures the essential information about the colors as well as the corresponding positions of every point in an image and integrates them into a quantum state^[13]. FRQI (Flexible representation for quantum images)^[14], MCQI (RGB Multi-channel representation for quantum images)^[15], NEQR (Novel enhanced quantum representation of digital images)^[16], INEQR (Improved NEQR)^[17], and GQIR (the Generalized quantum image representation)^[18,19] belong to this type.

FRQI was proposed in 2010^[14] which is the first representation model of this type. It utilizes the idea of using the angle θ to represent the color information of an image as in Ref.[10], and stores the two-dimensional position information (Y -axis and X -axis) in superposition states. FRQI fully exploits the physical properties of qubits and

reduces the number of qubits used to store a $2^n \times 2^n$ image from $2^n \times 2^n$ to $2n + 1$. Furthermore, FRQI resolves the real-time computation problem of image processing and provides a flexible method to process any part of a quantum image by using controlled quantum logic gates. However, it can not retrieve the pixels' value accurately through finite quantum measurements because it store the gray-scale information as the probability amplitude of a single qubit.

In 2013, two groups of researchers improve the color information of FRQI. MCQI uses 3 qubits to represent R, G, and B components of color images^[15]. NEQR uses the basis state of a qubit sequence with q -qubit to store the gray-scale value of every pixel^[16]. NEQR inherits the merits of FRQI and the color information of it can be accurately measured. Nevertheless, because NEQR does not improve the location information of FRQI, they only can represent square images in the size of $2^n \times 2^n$ which is not in accordance with the fact that most practical images are rectangular. In order to solve the problem, INEQR is proposed to represent quantum images having $2^{n_1} \times 2^{n_2}$ pixels by modifying the position information of NEQR^[17].

Furthermore, in 2015, Jiang *et al.* improve INEQR to GQIR to store arbitrary $H \times W$ quantum images^[18,19]. We introduce it in detail because a large part of quantum image algorithms are based on GQIR.

In GQIR, both the position information and the color information are captured into normalized quantum states: $|0\rangle$ and $|1\rangle$. It uses $h = \lceil \log_2 H \rceil$ qubits for Y -coordinate and $w = \lceil \log_2 W \rceil$ qubits for X -coordinate to represent a $H \times W$ image. However, this will generate a $(2^h - H)$ -row and $(2^w - W)$ -column redundancy. The redundancy is caused by the intrinsic property of the binary computation and is inevitable. For example, if we want to use binary codes to represent 5 symbols, the code word length is $\lceil \log_2 5 \rceil = 3$ and only 000, 001, 010, 011, 100 are useful. The remaining three code words 101, 110, 111 are redundancies which are inevitable and will not be processed.

In GQIR, a similar situation exists. Since Hadamard gates are used on the location qubits to let state $|0\rangle$ and state $|1\rangle$ appear with equal probability, $h + w$ location qubits will generate a $2^h \times 2^w$ blank image. We call it as a $2^h \times 2^w$ box. However, only $H \times W$ pixels of the box are useful, and other $2^h \times 2^w - H \times W$ "pixels" are redundancies. GQIR puts the effective $H \times W$ pixels, *i.e.*, the image to be represented, in the upper left corner of the box. All the redundant pixels are reserved as $|0\rangle$. Fig.6 is a diagram of GQIR box in which the white part is the image to be represented and the shaded area is the redundancy.

Hence, an GQIR image can be written as below.

$$|I\rangle = \frac{1}{\sqrt{2^{h+w}}} \left(\sum_{Y=0}^{H-1} \sum_{X=0}^{W-1} \otimes_{i=0}^{q-1} |C_{YX}^i\rangle |YX\rangle \right)$$

$$\begin{aligned} |YX\rangle &= |y_0 \dots y_{h-1}\rangle |x_0 \dots x_{w-1}\rangle, y_i, x_i \in \{0, 1\} \\ |C_{YX}\rangle &= |C_{YX}^0 C_{YX}^1 \dots C_{YX}^{q-1}\rangle, C_{YX}^i \in \{0, 1\} \end{aligned} \quad (4)$$

where

$$h = \begin{cases} \lceil \log_2 H \rceil, & H > 1 \\ 1, & H = 1 \end{cases} \quad (5)$$

$$w = \begin{cases} \lceil \log_2 W \rceil, & W > 1 \\ 1, & W = 1 \end{cases} \quad (6)$$

$|YX\rangle$ is the location information and $|C_{YX}\rangle$ is the color information. It needs $h + w + q$ qubits to represent a $H \times W$ image with gray range 2^q .

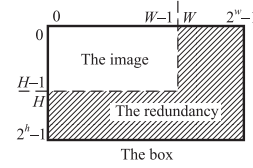


Fig. 6. The diagram of GQIR box

The box can be written as Eq.(7).

$$|B\rangle = |I\rangle + \frac{1}{\sqrt{2^{h+w}}} \left(\sum_{\substack{Y \in \{H, \dots, 2^h-1\} \\ X \in \{W, \dots, 2^w-1\}}} \otimes_{i=0}^{q-1} |0\rangle |YX\rangle \right) \quad (7)$$

Fig.7 shows a 1×3 gray-scale image and its representative expression in GQIR.

$$H = 1, W = 3 \Rightarrow h = 1, w = \lceil \log_2 3 \rceil = 2$$

The 1×3 image is put into a $2^1 \times 2^2 = 2 \times 4$ box. In fact, only three pixels ($Y = 0$ and $X = 00/01/10$) are effective, others are redundant but irremovable caused by the intrinsic property of binary expression.

	00	01	10	11
0	1	3	7	
1				

$$|I\rangle = \frac{1}{\sqrt{2}} (|001\rangle \otimes |000\rangle + |011\rangle \otimes |001\rangle + |111\rangle \otimes |010\rangle)$$

Fig. 7. A example image and its representative expression in GQIR

Note that GQIR can represent not only gray scale images but also color images because the color depth q is a variable. In most cases, when $q = 2$, it is a binary image; when $q = 8$, it is a gray scale image; and when $q = 24$, it is a color image. We define that $|R\rangle = |C_{YX}^0 \dots C_{YX}^7\rangle$, $|G\rangle = |C_{YX}^8 \dots C_{YX}^{15}\rangle$ and $|B\rangle = |C_{YX}^{16} \dots C_{YX}^{23}\rangle$ to represent Red (R), Green (G) and Blue (B) respectively.

The preparation is the process that transforms quantum computers from the initial state to the desired quantum image state^[14]. It has three steps.

Step 0 Prepare $h + w + q$ qubits and set all of them to $|0\rangle$. The initial state is:

$$|\Psi\rangle_0 = |0\rangle^{\otimes h+w+q} \quad (8)$$

Step 1 q identity gates and $h + w$ Hadamard gates are used to construct a blank $2^h \times 2^w$ box. The identity matrix and the Hadamard matrix are shown below.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (9)$$

In simple terms, the effect of identity gate is maintaining the qubit's original state unchanged and the effect of Hadamard gate is letting state $|0\rangle$ and state $|1\rangle$ appear with equal probability. After Step 1, the blank $2^h \times 2^w$ box is gained:

$$|\Psi\rangle_1 = \frac{1}{\sqrt{2^{h+w}}} \sum_{Y=0}^{2^h-1} \sum_{X=0}^{2^w-1} |0\rangle^{\otimes q} |YX\rangle \quad (10)$$

Step 2 Use $(h + w)$ -CNOT gates (CNOT gate with $h + w$ control qubits) to change the effective $H \times W$ pixels to the desired quantum image states:

$$|\Psi\rangle_2 = \frac{1}{\sqrt{2^{h+w}}} \left(\sum_{Y=0}^{H-1} \sum_{X=0}^{W-1} |C_{YX}\rangle |YX\rangle + \sum_{\substack{Y \in \{H, \dots, 2^h-1\} \text{ or} \\ X \in \{W, \dots, 2^w-1\}}} \otimes_{i=0}^{q-1} |0\rangle |YX\rangle \right) \quad (11)$$

Fig.8 shows the detailed quantum circuit for GQIR preparation for the example image shown in Fig.7. Only 3 effective pixels $|YX\rangle = |000\rangle, |001\rangle, |010\rangle$ are set to desired value. Others (the reserved pixels) remain unchanged, *i.e.*, are still in their initial state $|0\rangle$.

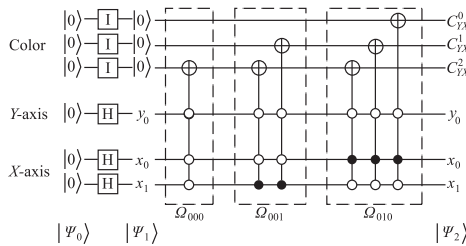


Fig. 8. The quantum circuit of the 1×3 image shown in Fig.7

Many other representations are proposed. The Caraiman's QIR approach (CQIR)^[20] is almost the same as NEQR that uses a qubit sequence to encode the color information in quantum images.

Most of the existing models can only store and process image sampled in Cartesian coordinates. However, the Quantum image representation for log-polar images (QUALPI)^[21] can store and process the log-polar images. The sampling resolutions of the log-radius and the angular orientations are assumed to be $2m$ and $2n$, respectively. The whole QUALPI quantum image is stored in a normalized and equiprobable quantum superposition, in which each basis state represents one pixel.

The Simple quantum representation of infrared images (SQR)^[22] is similar to Qubit lattice representation, in which radiation energy of objects is transformed into a quantum state $|I\rangle$.

QSMC & QSNCR^[23] uses two sets of quantum states to store an image, where QSMC represents M colors and QSNCR represents the coordinates of N pixels in an image.

Multi-dimensional image representation using a Normal arbitrary quantum superposition state (NAQSS)^[24] was proposed by Li *et al.*, where n qubits represent colors and coordinates of 2^n pixels and the remaining 1 qubit represents an image segmentation information.

IV. Quantum Image Processing Algorithm

1. Two simple examples

In this section, two simple examples are given to show how to process quantum images: one works on location information and the other works on color information.

In Ref.[25], Le *et al.* give a fast geometric transformation which can manipulate the geometric information of the GQIR images simply. Fig.9 presents an example, where Fig.9(a) is the circuit: when $y_2 = 1$, reverse all the bit of X -coordinate ($000 \leftrightarrow 111, 001 \leftrightarrow 110, 010 \leftrightarrow 101, 011 \leftrightarrow 100$). $y_2 = 1$ implies that only the lower half of the image is operated; $000 \leftrightarrow 111$ implies that Column 0 and Column 7 exchange position; $001 \leftrightarrow 110$ implies that Column 1 and Column 6 exchange position; and so on. Hence, the original image shown in Fig.9(b) is changed to Fig.9(c).

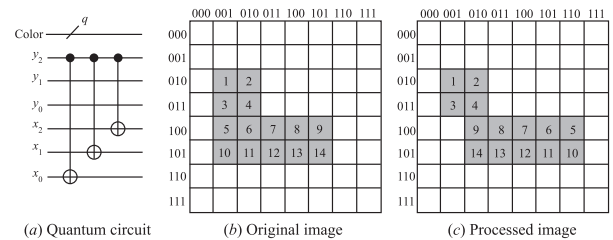


Fig. 9. The quantum geometric transformation circuit and its effect

Fig.10 is another example which is an excerpt from Ref.[16]. It processes a gray-scale image and inverts all the qubits in the color qubit sequence in NEQR and change the gray-scale value of every pixel in the image to its opposite value.

Although these two examples are not complicated, they demonstrate the basic idea of QIP well.

2. Quantum image compression

Quantum image compression is the procedure that reduces the quantum resources used to prepare quantum images^[14]. The main resource in quantum preparation is the number of quantum gates instead of the number of qubits, because as stated previously, the number of qubits

used in GQIR quantum images (*i.e.*, $h + w + q$) has been very fewer than the number of bits used in classical images (*i.e.*, HWq).

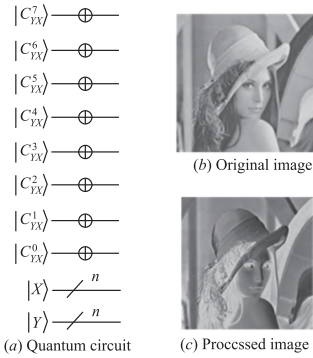


Fig. 10. The quantum color transformation circuit and its effect

In Refs.[14,16], the minimization of Boolean expressions is used to compress the image preparation (BEC). If x is a Boolean variable and the value of it is 1 then the lateral x is used in the minterm, otherwise the lateral \bar{x} is used. Then, for example, a Boolean expression

$$e = x_2\bar{x}_1x_0 + x_2x_1x_0$$

is minimized as

$$e = x_2x_0$$

That is to say, if two CNOT gates operate the same color qubit and only one of their control qubits is different, the two CNOT gates can be combined into one gate. Fig.11 gives an example.

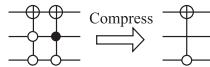


Fig. 11. BEC quantum image compression

However, the BEC method has some defects. In fact, BEC has two steps: 1) preprocess: determining which CNOT gates can be combined; and 2) input: using Hadamard gates and CNOT gates to input the image into quantum system. The preprocessing is time consuming because we have to look over all the CNOT gates before compression one pair by one pair. Its complexity is $O(q \cdot 2^{4n})$ which is even higher than the scheme without compression.

Hence, Ref.[26] gives a new compression scheme. The new one is based on JPEG which is the most widely used method for still image compression in classical computers. It inputs the quantized JPEG coefficients into qubits and then convert them into pixel values. Since the data amount of the JPEG coefficients are significantly less than that of the pixel values, the JPEG scheme can compress an image. Theoretical analysis and experimental results show that the compression ratio of JPEG scheme is about 90% which is obviously higher than that of BEC.

3. Quantum image geometric transformation

Fig.9 is an example about geometric transformation, but it is only a beginning. In Ref.[25], some classical-like transformations such as two-point swapping, flip, coordinate swapping, and orthogonal rotations can be performed on the images encoded in the FRQI representation using the basic quantum gates, *e.g.*, NOT, CNOT, and Toffoli gates.

Image translation, which maps the position of each picture element into a new position, is a basic image transformation. Quantum image translation (QIT) was first proposed in Ref.[27]. Two types of QIT based on NEQR representation were studied by giving the quantum translation circuits.

Jiang *et al.* propose quantum algorithms to realize the quantum image scaling based on GQIR for quantum images^[17,18]. It is necessary to use interpolation in image scaling because there is an increase or a decrease in the number of pixels. The interpolation method used in these papers is nearest neighbor which is simple and easy to realize. In Ref.[17], the scaling ratio is $2^{r_1} \times 2^{r_2}$ and in Ref.[18], the scaling ratio is extended to integers.

4. Quantum image color transformation

Fig.10 belongs to color transformation, but it is simple. Ref.[16] gives a series of color transformation, including inverse color, gray-range halving, *etc.*

Pseudocolor processing is a branch of image enhancement. It dyes gray-scale images to color images to make the images more beautiful or to highlight some parts on the images. Ref.[19] proposes a quantum image pseudocolor coding scheme based on the density-stratified method which changes the density value from gray to color parallel according to a colormap. The main advantages of the quantum version for pseudocolor processing over the classical approach are that it needs less memory and can speed up the computation.

5. Quantum image scrambling

Image scrambling, a technique widely used to transform an image into a disordered image, has been extensively studied. The popular traditional image scrambling algorithms, *i.e.*, Hilbert transform, Arnold transform, and Fibonacci transform, are extended to the quantum field^[28–30]. The Arnold and Fibonacci scrambling quantum circuits in Refs.[29,30] take advantage of quantum adder. The Hilbert scrambling quantum circuit in Ref.[28] is executed by a recursive generation algorithm.

Zhou *et al.* propose a quantum image gray-code and bit-plane scrambling scheme based on NEQR representations^[31], which scrambles not only position space but also bit-plane.

6. Quantum image segmentation

Image segmentation is the process of separating the foreground of one or more objects from the background

in a digital image.

In 2010, based on Entangled image model, Venegas-Andraca *et al.* stored the vertexes belong to one shape in one entangled state to represent the segmentation result^[32]. However, it only stored the result and did not say how to split.

In 2013, based on QSMC & QSNC model, Li *et al.* gave an image segmentation algorithm which is universal for gray-scale images and color images^[23]. It utilized Grover algorithm to find all the pixels whose value is bigger than a threshold f . If the searched-out colors are too few or too many, adjust f and repeat the above process until the image is split properly.

In 2015, Caraiman *et al.* gave a segmentation method similar to the scheme in Ref.[23]. The main difference is that the value of f is gained from histogram instead of by trying^[33,34].

7. Quantum image feature extraction

Feature extraction targets the detection and isolation of various desired portions or shapes of a digital image.

In 2014, Zhang *et al.* proposed a novel quantum image edge extraction algorithm (QSobel) based on FRQI representation and the classical edge extraction algorithm Sobel^[35]. Its time complexity for an n -length qubit sequence is no more than $O(n^2)$ which is a significant and exponential speedup compared with existing edge extraction algorithms.

In 2015, Zhang *et al.* gave a quantum feature extraction framework based on NEQR^[36]. They computed corner points using a gradient-based method. The time complexity is $O(n^2 + q^2)$.

8. Quantum image encryption

Image encryption offers a medium to translate the original image into a meaningless image (where visual acuity is lost) and become the information similar to the channel random noise. Zhou^[37,38], Wang^[39,40], Zhou^[41,42], Akhshani^[43], Yang *et al.*^[44,45] proposed their quantum image encryption algorithms respectively.

The procedures of these encryption algorithms are similar: using one or more geometric transformations or color transformations to make the image meaningless, and decryption is the reverse process of encryption. In the encrypted image, the correlation between the image pixels are destroyed and it looks like noise.

9. Quantum image watermarking

Quantum image watermarking is utilized to embed some watermark information into the carrier image and should not change the visual acuity of the carrier^[13]. Results in this area are relatively more.

In 2010, Ilyasu *et al.* pioneered what is today widely regarded as quantum image watermarking based on FRQI and restricted geometric transformations^[46]. However, it has some drawbacks: 1) the watermark is not actually

embedded in the carrier, and it is only a authentication scheme; and 2) it is a non-blind algorithm.

From then on, researchers presented a variety of quantum image watermarking algorithms such as Yan^[47], Zhang^[48,49], Song and Wang^[50–52], Jiang^[53,54], Heidari^[55], Miyake^[56], Wei *et al.*^[57] proposed their quantum image watermarking scheme respectively.

Their principles and effects are similar to classical watermarking schemes. The difference is that: quantum image watermarking is realized in quantum computers and classical image watermarking is realized in classical computers.

10. Other quantum image processing algorithms

In addition to the algorithms described above, there are other quantum image processing algorithms, such as morphology^[58,59], filtering^[60], denoising^[61], comparison^[62,63], searching^[64], *etc.* While these studies seem piecemeal, they are important to QIP.

IV. Quantum Image Measurement

The quantum image processing schemes introduced previously claim that their efficiency is theoretically higher than their corresponding classical schemes. However, most of them do not consider the problem of measurement. Measurement will lead to collapse. That is to say, executing the algorithm once, users can only measure the final state one time. Therefore, if users want to regain the results (the processed images), they must execute the algorithms many times and then measure the final state many times to get all the pixels' values. If the measurement process is taken into account, whether or not the algorithms are really efficient needs to be reconsidered.

In Refs.[65,66], Jiang *et al.* note this problem and try to solve it by giving a quantum image matching algorithm. Fig.12 is the comparison between ordinary quantum image processing algorithms and the scheme in Refs.[65,66]. The scheme modifies the probability of pixels to make the target pixel to be measured with higher probability, and the whole scheme is executed only once. Hence the complexity is really improved.

If define the two input images as A and B , the scheme is composed of 4 steps: 1) Find out the matched area, *i.e.*, the pixels that $I_A = I_B$. 2) Find out the upper left corner of the matched area. 3) Change the probability of subspace $|k_A\rangle$. 4) Measurement. Fig.13 is the circuit.

Theoretical analysis indicates that the network complexity of quantum image matching is $O(2^n)$. Compared with the complexity of the classical image matching: $O(2^{2n+2m})$, the quantum algorithm dropped the complexity obviously.

V. Open Challenges and Future Research Directions

Due to quantum image processing is in its infancy, it is a field full of exciting open problems for physicists, mathematicians, computer scientists, and engineers^[13].

1. More algorithms

Classic image processing (CIP) involves a rich content, such as intensity transformations, spatial filtering, filtering in the frequency domain, restoration, reconstruction, color processing, image compression, morphology, segmentation, object recognition and so on. Compared to CIP, quantum image processing algorithms are too little. It needs more researchers to do more work to promote the development of QIP and make it as a system.

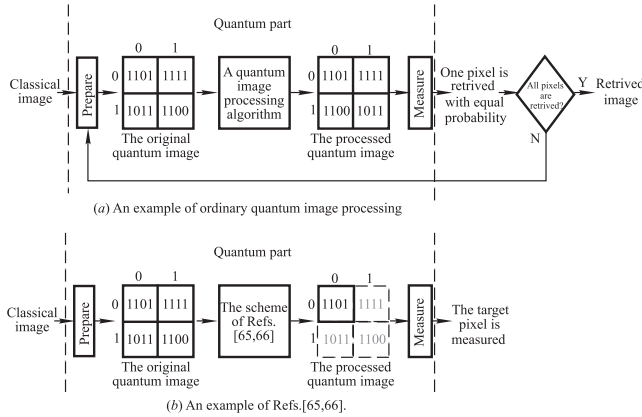


Fig. 12. Difference between ordinary quantum image processing algorithms and the scheme in Refs.[65,66]

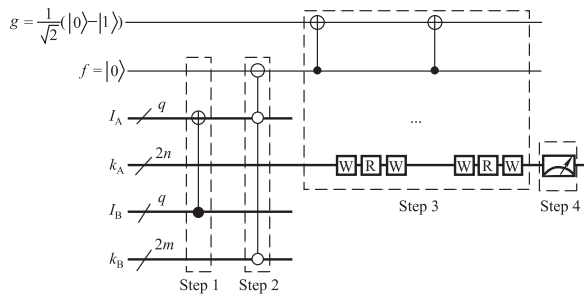


Fig. 13. Quantum image matching circuit

2. More accurate and comprehensive demonstration about the advantages of QIP

In general, when we analysis the advantages of a QIP scheme, we compare its network complexity with its classical counterpart. This is not compelling enough. Comparison with the best CIP algorithm is a better choice, where the best CIP algorithm indicates the algorithm that has the same effect with a QIP scheme instead of the same process.

3. Do physical experiments

At present, all QIP schemes live in theory. Researchers give the algorithms based on a certain quantum computing model and simulate them in classical computers. Network complexity is represented by $O(\cdot)$ in complexity theory, instead of the actual running time.

However, with that quantum computers ready to leap out of the lab^[1,67,68], doing physical experiments in really quantum computers will be possible. Moreover, researchers are eager to display and test their products in a real environment.

VI. Conclusions

Beyond the ongoing efforts toward physical realization of quantum computing hardware, research is focused on what can be done with these technologies when realization has been intensified^[69]. Quantum image processing is one of them, which is the intersection of quantum computation and image processing. Due to it is a newly emerging thing, researchers are facing great opportunities but also challenges to develop more efficient and practicable services.

In this paper, we have provided a comprehensive survey on quantum image processing to gather the current mainstream and discuss the advances made in the area, including quantum image representation, QIP algorithm, and quantum image measurement. Moreover, some open challenges and future directions are finally pointed out to attract continued research efforts.

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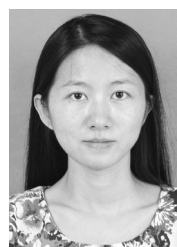
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CAI Yongquan was born in 1956. He is a professor and Ph.D. advisor of Beijing University of Technology. His main research interests are in the computer network and information security in which his accomplishments include extensive publications and software copyrights granted in China, and two technical books published in China. (Email: cyq@bjut.edu.cn)



LU Xiaowei was born in 1981. She received the B.E. degree in communication engineering and M.E. degree in communication and information system from Dalian Maritime University. She is a Ph.D. candidate of computer technology in Beijing University of Technology. Her research interests is in quantum image processing. (Email: 47949588@qq.com)



JIANG Nan (corresponding author) was born in 1977. She received the Ph.D. degree in cryptograph from Beijing University of Posts and Telecommunications, China, in 2006. From 2015 to 2016, she was a visiting scholar at the College of Science, Purdue University, USA. Now, she is an associate professor of computer science in Beijing University of Technology. Her research interests include quantum image processing and information hiding. (Email: jiangnan@bjut.edu.cn)