Distributed Coordinated Tracking With Reduced Interaction via a Variable Structure Approach

ENPM667 Project 1 Report(Control of Mobile Robots)

Akash Ravindra(117422085) and Rishabh Singh (117511208)

Maryland Robotics Center

The University of Maryland, College Park

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ABSTRACT

The paper presented a situation of coordinated tracking with a leader and several followers. The conditions of both first order kinematics and second order dynamics are considered with varying as well as fixed velocity of the leader. The leader is defined as dynamic and a neighbour of atleast one follower to prove the algorithms proposed in the paper. The followers are allowed to have local interaction and partial measurement if they are neighbour of each other which can be considered as a minimum distance between two followers. The state of the leader is available to the followers who have the leader as their neighbour. In first order kinematics they proposed algorithms considering cases of distributed consensus tracking with both fixed and switching graph or network topologies and prove that the tracking with fixed network topology can be achieved in finite time and for switching network topology the tracking will be achieved but a fixed time cannot be defined for this because of the switching pattern of the network topology. The proposal is then extended to distributed swarm tracking with switching network topology but again with converging error but no bound of time. In second order dynamics, they again start with a case of distributed consensus tracking without any acceleration measurements with both switching and fixed network and by proposing a control parameter, algorithm and proving the derivative of the potential function to be negative definite they have proved that the algorithms at least guarantee global exponential tracking. Also, a mechanism is introduced for keeping adaptive connectivity maintenance by defining adjacency matrix according to three different scenarios based on distance between followers. Then a case is considered where leader's velocity is taken constant in both distributed consensus tracking and distributed swarm tracking. Again a mechanism is introduced using potential function derivative according to three different situations to avoid collision as well as maintaining connectivity.

Most of the simulations of this publication have been reproduced by us.

Keywords Position Tracking · Controllers · Swarm · Kinematics · Dynamics · Topology · Adjacency matrix · Lyapunov Function · Laplacian Matrix · Undirected Graphs · Variable Structures · MATLAB

1 INTRODUCTION

Connected vehicles could dramatically reduce the number of fatalities and serious injuries caused by accidents on our roads and highways. A connected vehicle is capable of connecting over wireless networks to nearby devices. Connected vehicles are an essential factor in the advance of IoT[1]. The use cases range from connected entertainment systems connected with the driver's mobile phone to Internet-connected vehicles with bi-directional communication with other vehicles, mobile devices, and city intersections. Connected vehicles connect to a network to enable bi-directional communications between vehicles (cars, trucks, buses, and trains) and other vehicles, mobile devices, and infrastructure for the purpose of triggering important communications and events. For example, in the case of city traffic and intersection safety, those communications can enable vehicles outfitted with connected vehicle technology to continuously communicate their locations and receive near real-time information that triggers an automated response.

Connected vehicles have significant advantages over new technologies now appearing in high-end vehicles, such as radar, lidar, cameras, and other sensors. For one thing, connected vehicle technologies and applications have a greater range than on-board vehicle equipment, which will allow you to receive alerts of hazardous situations much earlier, providing more time to react and prevent an accident. Also, connected vehicle technology doesn't depend on "line of sight" communications to be effective, unlike radar. So if a car ahead of you is braking hard on the other side of a hill due to an obstruction, you would receive notification even though you can't see and aren't aware of the dangerous situation developing. Connected vehicle technology is also less expensive to install than radar and camera equipment in vehicles.

While the number of people surviving crashes has increased thanks to airbags, anti-lock brakes, and other technology significantly, the US-DOT is shifting its focus from helping people survive crashes to preventing collisions from happening in the first place[2].

A distributed approach used in multi-vehicle cooperative control is consensus, which means that a group of vehicles reaches an agreement on a common value by interacting with their local (time-varying) neighbors. Consensus has been studied for systems with both first-order kinematics and second-order dynamics.Recent study of consensus and its applications in distributed multi-vehicle cooperative control can be found in [3] and [4]. Existing consensus algorithms were often studied either when there does not exist a leader or when the leader is static. Although consensus without a leader is useful in applications such as cooperative rendezvous of a group of vehicles, there are many applications that require a dynamic leader. Examples include formation flying, body guard, and coordinated tracking applications.
The author focus on solving a distributed coordinated tracking problem via a variable structure approach when there exists a dynamic virtual leader under the following three assumptions: 1) the virtual leader is neighbor of only subset of a group of followers;2) there exists only local interaction among all followers; and 3) the velocity measurements of the virtual leader and all followers in the case of first-order kinematics or the accelerations of the virtual leader and all followers in the case of second-order dynamics are not required. In the context of this paper, we use the term coordinated tracking to refer to both consensus tracking and swarm tracking.
We limited our review up until, distributed swarm tracking using dynamics.

2 BACKGROUND

The collection of leader and followers are organized into a Graph, which is a structure used to model pairwise relations between objects. A Graph is defined as an ordered pair of $G = (V, E)$ Where V represents the set of vertices and E represents the set of edges for a pair of vertices[5]. In this particular case the graph is an un-directed one, where the interactions between the pair is bidirectional.
An adjacency matrix is a square matrix used to represent a finite graph. The elements of the matrix indicate whether pairs of vertices are adjacent, that is, if they are connected by an edge. In the case of an undirected graph, the matrix will be square. This matrix encodes the information about the inter-vertex connections present in the graph.For a simple graph with vertex set $U = u_1, \ldots, u_n$, the adjacency matrix is symmetric $n \times n$ matrix A such that its element A_{ij} is one when there is an edge from vertex u_i to vertex u_j, and zero when there is no edge[6].
The Laplacian matrix, also called the Kirchhoff matrix, is a matrix representation of a graph. It can be used to calculate the number of spanning trees for a given graph. Defined for a simple graph G with n vertices as, $L_{n\times n}$ is defined as[7]: $L = D - A$ Where D is the degree matrix and A is the adjacency matrix of the graph. In this paper the authors have modelled the interaction between vehicles as a undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. The Edge is modelled as a link between vehicles to transfer information about position and velocity, hence $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ is a weighted adjacency matrix where is defined such that a_{ij} is a positive weight if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. As this is a simple graph, the matrix is symmetric, $a_{ij} = a_{ji} \forall i \neq j$ as both (j, i) and $(i, j) \in \mathcal{E}$ [8].
The authors have considered the Laplacian matrix to be defined as $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$ is derived from the adjacency matrix as $l_{ii} = \sum_{j=1,j\neq i}^{n} a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$. By definition, $\mathcal{L}$ is a symmetric positive semi definite matrix[8].

3 DISTRIBUTED COORDINATED TRACKING FOR FIRST-ORDER KINEMATICS

This section introduces the distributed coordinated tracking for the first order kinematics. This section consists of two subheadings and five proposed theorems along with remarks. We have tried to understand the mathematics in this section and have produced the missing steps in mathematical derivations. Other than the n vehicles that are followers there is one more vehicle considered as leader and marked as vehicle 0, with varying position r_0 and velocity $\dot{r_0}$. The maximum magnitude of the velocity of the leader has been defined as bounded in order to design the control input or algorithm in a way that the tracking can be achieved in finite time. Therefore, the assumption is:

$$|\dot{r_0}| \leq \gamma_l, \text{ where } \gamma_l \text{ is some positive constant.}$$

Now considering the situation of first order kinematics the positions of the followers is being controlled using the velocities. Therefore,

$$\dot{r_i} = u_i, i = 1, ..., n \tag{1}$$

where r_i is considered as position and v_i is the control input for the vehicle. The vehicles are assumed in a 1-D space for simplicity of presentation. However, all results hereafter are still valid for the m-dimensional case by introduction of the Kronecker product.[9].

3.1 Distributed Consensus Tracking Under Fixed and Switching Network Topologies

The control input for (1) has been designed in this subsection where the velocity measurements are not available and the followers will track the virtual leader defined as 0^{th} vehicle, also it is the neighbour of at-least one of the follower. The control input is proposed as follows by the author:

$$u_i = -\alpha \sum_{j=0}^{n} a_{ij}(r_i - r_j) - \beta sgn\left[\sum_{j=0}^{n} a_{ij}(r_i - r_j)\right] \tag{2}$$

where a_{ij}, *i, j = 1,...,n* is the (i,j)th entry of the adjacency matrix *A* which is same as defined before and is used for computing the difference in position of the neighbour when multiplied by $\tilde{r}$ from right as is seen in (5). Here, a_{i0}, *i* = 1,...,*n* is taken as a positive constant if the position of the virtual leader is available to follower *i* and is taken as 0 otherwise, α is a non-negative constant, β is a positive constant sgn(.) is the signum function.
The control input seems intuitive in a manner that when the position difference is positive, the negative β will tend to make the velocity as negative and the neighbour will tend to reduce the velocity, in the same manner, if the difference becomes negative the signum function will make the β value positive and hence the neighbour would try to increase its velocity to track the neighbour. So, the tracking control input seems well defined. Rest is being proved through the simulations.

Theorem 3.1: Suppose that the fixed undirected graph **G** is connected and at least one a_{i0} is nonzero (and hence positive). Using (2) for (1), if $\beta > \gamma_l$, then $r_i(t) \to r_0(t)$ in finite time. In particular, $r_i(t) = r_0(t)$ for any $t \geq \bar{t}$, , where

$$\bar{t} = \frac{\sqrt{\tilde{r}^T(0)M\tilde{r}}\sqrt{\lambda_{max}(M)}}{(\beta - \gamma_l)\lambda_{min}(M)} \tag{3}$$

where $\tilde{r}$ is the column stack vector of $\tilde{r}_i$, $i = 1, \ldots, n$ with $\tilde{r}_i = r_i - r_0$, $M = \mathcal{L} + diag(a_{10}, \ldots, a_{n0})$, where *L* is the Laplacian matrix which is formed by adding adjacency matrix M and a diagonal matrix having a_{i0} as its diagonal elements which are 0 or 1 as defined above. λ_min(.) and $lambda_max$(.) denote, the smallest and largest eigen value of the adjacency matrix M.

Proof: Now using the control input and some manipulations the proof is being derived by the author in the following lines. Noting that $\tilde{r}_i = r_i - r_0$, we can rewrite the closed loop system of (1) using (2)

$$\dot{\tilde{r}}_i = -\alpha \sum_{j=0}^{n} a_{ij}(\tilde{r}_i - \tilde{r}_j) - \beta sgn\left[\sum_{j=0}^{n} a_{ij}(\tilde{r}_i - \tilde{r}_j)\right] - \dot{r}_0 \tag{4}$$

The same thing can be written in the matrix form as follows. Following the rules defined as above for the laplacian matrix we will got to know when we do row wise multiplication between M and $\tilde{r}$ all the terms get cancelled out due to negative signs and 0 at all the places except diagonals, and we get the term $a_{ij}(\tilde{r}_i - \tilde{r}_j)$. Hence,

$$\dot{\tilde{r}}_i = -\alpha M\tilde{r} - \beta sgn(M\tilde{r}) - \mathbf{1}\dot{r}_0 \tag{5}$$

where $\tilde{r}$ and M are defined in (3), and sgn(.) is defined component wise and M is symmetric positive definite. Now the authors according to the rules for Lyapunov function considers the candidate as

$$V = (1/2)\tilde{r}^T M\tilde{r} \tag{6}$$

Now to check the feasibility of the Lyapunov candidate and that whether it is converging or diverging with time, we find the derivative as follows using the matrix calculus:

$$\dot{V} = \tilde{r}^T M\dot{\tilde{r}} \tag{7}$$

Now putting the value of $\dot{\tilde{r}}$ from (5),

$$\begin{aligned} \dot{V} &= \tilde{r}^T M[-\alpha M\tilde{r} - \beta sgn(M\tilde{r}) - 1\dot{r}_0] \\ &\leq -\alpha\tilde{r}^T M^2\tilde{r} - \beta\|M\tilde{r}^T\|_1 + |\dot{r}_0|\|M\tilde{r}\| \end{aligned} \tag{8}$$

This is from a fact that 1-norm of a vector $(x_1, x_2, \dots, x_n)$ is the sum $|x_1| + |x_2| + \cdots + |x_n|$, and considering that the sign of a vector is the vector of sign of its elements. And also if we multiply a matrix or vector to the signum function on it, it will always be positive in magnitude. Inequality sign comes from the properties for norms on symmetric matrices and vectors also known as Holder's inequality.$\|AB\| \leq |A||B|$

$$\leq -\alpha\tilde{r}^T M^2\tilde{r} - (\beta - \gamma_l)\|M\tilde{r}\|_1 \tag{9}$$

This inequality comes from the assumption the $|\dot{r}_0| \leq \gamma_l$. Now, we know that M^2 is symmetric positive definite, α is non-negative, and $\beta > \gamma_l$. Hence, because of the negative sign it is clear that $\dot{V}$ is negative definite and the potential function will be bounded and converge with time and hence, $\|\tilde{r}\| \to 0$ with $t \to \infty$. Now the same is being proved by integrating the above equation w.r.t time. After taking modulus on the right side of (6) and then using spectral radius property or Rayleigh-Ritz Inequality it has been written that $V \leq (1/2)\lambda_{max}(M)\|\tilde{r}\|_2^2$. The equation (9) has been re-written in the form below as,

$\dot{V} \leq -(\beta - \gamma_l)\|M\tilde{r}\|_2$
$= -(\beta - \gamma_l)\sqrt{\tilde{r}^T M^2 \tilde{r}}$
$\leq -(\beta - \gamma_l)\sqrt{\lambda_{min}^2(M)\|\tilde{r}\|_2^2}$

because of the negative sign the λ_{max} is replaced by λ_{min}, again due to Rayleigh-Ritz Inequality

$= -(\beta - \gamma_l)\lambda_{min}(M)\|\tilde{r}\|$
$\leq -(\beta - \gamma_l)\dfrac{\sqrt{2}\lambda_{min}(M)}{\sqrt{\lambda_{max}(M)}}\sqrt{V}$

This integration step was skipped by the author in the paper and hence we are writing it.

Let $V/\sqrt{V} = A$ where $A = -(\beta - \gamma_l)\dfrac{\sqrt{2}\lambda_{min}(M)}{\sqrt{\lambda_{max}(M)}}$ then, (10)

$\int \dfrac{\dot{V}}{V} \leq Adt$

$\left[2\sqrt{V}\right]_0^t = At$

$\left[2\sqrt{V(t)} - 2\sqrt{V(0)}\right] = At$

Substituting A back into the equation above we get

$2\sqrt{V(t)} \leq 2\sqrt{V(0)} - (\beta - \gamma_l)\dfrac{\sqrt{2}\lambda_{min}(M)}{\sqrt{\lambda_{max}(M)}}t$

By putting the value of $\bar{t}$ from (3) in above equation we found out that V(t) = 0 when $t \geq \bar{t}$. Hence, proved.

3.1.1 The case of switching network topology

In the previous case we just considered a single potential function since the topology was fixed. In this case, the switching network topology is considered and minimum communication/sensing radius is defined as a boundary where the potential function changes its value. So, a neighbour set $\bar{\mathcal{N}_i} \subseteq \{0, 1, \ldots, n\}$ of follower i is declared, which consists of *n* followers and a virtual leader. In this case, it is assumed that $j \in \bar{\mathcal{N}_i}(t), i = 1, \ldots, n, j = 0, \ldots, n$ if $|r_i - r_j| < R$ at time t and $j \notin \bar{\mathcal{N}_i}(t)$ otherwise, where R denotes the communication/sensing Radius of the vehicles. In this case also a similar algorithm for (1) is considered,

$$u_i = -\alpha \sum_{j \in \bar{\mathcal{N}_i}(t)}^{n} b_{ij}(r_i - r_j) - \beta sgn\left[\sum_{j \in \bar{\mathcal{N}_i}(t)}^{n} b_{ij}(r_i - r_j)\right] \tag{11}$$

where $b_{ij}, i = 1, dots, n\ j = 0, dots, n$, are positive constants, and α, β and sgn(.) are defined as before.

Theorem 3.2: Again the assumption is that the undirected graph G(t) is connected and the virtual leader is a neighbour of at-least one follower (i.e., $0 \in \bar{\mathcal{N}_i}$) for some i) at each time instant. Note: that the G is now a function of t because the edges are getting updated w.r.t time. Now the claim for (11) is made that, if $\beta > \gamma_l$, then then $r_i(t) \to r_0(t)$ as $t \to \infty$.

Proof: The new potential function is defined at each instant by checking the edge or the distant between the neighbour. So, $V_{ij} = (1/2)b_{ij}(r_i - r_j)^2$ when $|r_i - r_j| < R$ and $V_{ij} = (1/2)b_{ij}R^2$ when $|r_i - r_j| \geq R$. Also, a new potential is considered if the i_{th} vehicle has the leader as its neighbour as $V_{i0} = (1/2)b_{i0}(r_i - r_0)^2$ when $|r_i - r_0| \geq R$ and $V_{i0} = (1/2)b_{i0}R^2$ when $|r_i - r_0| \geq R$.

Accordingly, the potential function is divided into two parts i.e. the function that handles interaction between neighbours only and the interaction between the *i*th vehicle whose neighbour is the leader. So, we can see that V may not be smooth but is regular.

$$V = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}V_{ij} + \sum_{j=1}^{n} b_{i0}V_{i0} \tag{12}$$

The property of inclusions is defined for scenarios like static friction force, whose value is a subset of the maximum allowed friction or traction. Similarly here with the same analysis since $j \in \bar{\mathcal{N}}_i(t)$ where $\bar{\mathcal{N}}_i \subseteq \{0, 1, \ldots, n\}$ the value of j varies according to the subset. Hence, it has been said for $\dot{r}_i$ that,

$$\dot{r}_i \in^{a.e.} -K\left[\alpha \sum_{j\in\bar{\mathcal{N}}_i(t)}^{n} b_{ij}(r_i - r_j) - \beta sgn\left[\sum_{j\in\bar{\mathcal{N}}_i(t)}^{n} b_{ij}(r_i - r_j)\right]\right] \tag{13}$$

where a.e. stands for almost everywhere and K[.] is used to denote differential inclusion [10][11][12]
Now taking derivative of V defined as above and using the chain rule of partial derivatives, since V is a function of both r_i and r_j.

$$V^0 = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\left[\frac{\partial V_{ij}}{\partial r_i}\dot{r}_i + \frac{\partial V_{ij}}{\partial r_j}\dot{r}_j\right] + \sum_{j=1}^{n} b_i 0\left[\frac{\partial V_{i0}}{\partial r_i}\dot{r}_i + \frac{\partial V_{i0}}{\partial r_0}\dot{r}_0\right] \tag{14}$$

Now putting $V_{ij} = (1/2)b_{ij}(r_i - r_j)^2$ and $V_{i0} = (1/2)b_{i0}(r_i - r_0)^2$ and deriving it respectively with r_i, r_j and r_0 we get the equation below.

$$= \frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\bar{\mathcal{N}}_i(t), j\neq 0} b_{ij}\left[(r_i - r_j)\dot{r}_i - (r_j - r_i)\dot{r}_j\right] + \sum_{0\in\bar{\mathcal{N}}_i(t)}^{n} b_i 0\left[(r_i - r_0)\dot{r}_i - (r_0 - r_i)\dot{r}_0\right] \tag{15}$$

Note in the previous equation that $j \neq 0$ for the first term . But now replacing the value of $\dot{r}_i$ from (13) we get the equation below, where there are no such constraints.The first part of the equation below became square because of the multiplication between the same terms and when sgn(.) on the same term is multiplied to the term its norm or modulus can be taken since the value will always be positive. The restriction that $j \neq 0$ is removed because the term from the second part of the equation consisting of $b_{io}(r_i - r_o)^2$ after replacing the value of $\dot{r}_i$ gets added and now j=0 is also considered in the first part of the equation below.

$$= -\alpha\sum_{i=1}^{n}\left[\sum_{j\in\bar{\mathcal{N}}_i(t)} b_{ij}(r_i - r_j)\right]^2 - \beta\sum_{i=1}^{n}\left|\sum_{j\in\bar{\mathcal{N}}_i(t)} b_{ij}(r_i - r_j)\right| + \sum_{0\in\bar{\mathcal{N}}_i(t)} b_{i0}(r_0 - r_i)\dot{r}_0 \tag{16}$$

Now writing the above equation in the matrix form, we get the equation below.

$$= -\alpha\tilde{r}^T[\hat{M}(t)]^2\tilde{r} - \beta\|\hat{M}(t)\tilde{r}\|_1 + \dot{r}_0\sum_{i=1}^{n}\sum_{j\in\bar{\mathcal{N}}_i(t)} b_{ij}(r_i - r_j) \tag{17}$$

Now again using the Holder's inequality and the properties of norms, we get the equation and inequality below

$$\begin{aligned} &\leq -\alpha\tilde{r}^T[\hat{M}(t)]^2\tilde{r} - \beta\|\hat{M}(t)\tilde{r}\|_1 + \dot{r}_0\|\hat{M}(t)\tilde{r}\|_1 \\ &\leq -\alpha\tilde{r}^T[\hat{M}(t)]^2\tilde{r} - (\beta - \gamma_l)\|\hat{M}(t)\tilde{r}\|_1 \end{aligned} \tag{18}$$

The facts mentioned below are used to get the equation (18) where $\hat{M}(t)$ is also defined.

$\sum_{i=1}^{n}\sum_{j\in\bar{\mathcal{N}}_i(t), j\neq 0} b_{ij}(r_i - r_j) = 0$
$|\dot{r}_0| \leq \gamma_l$

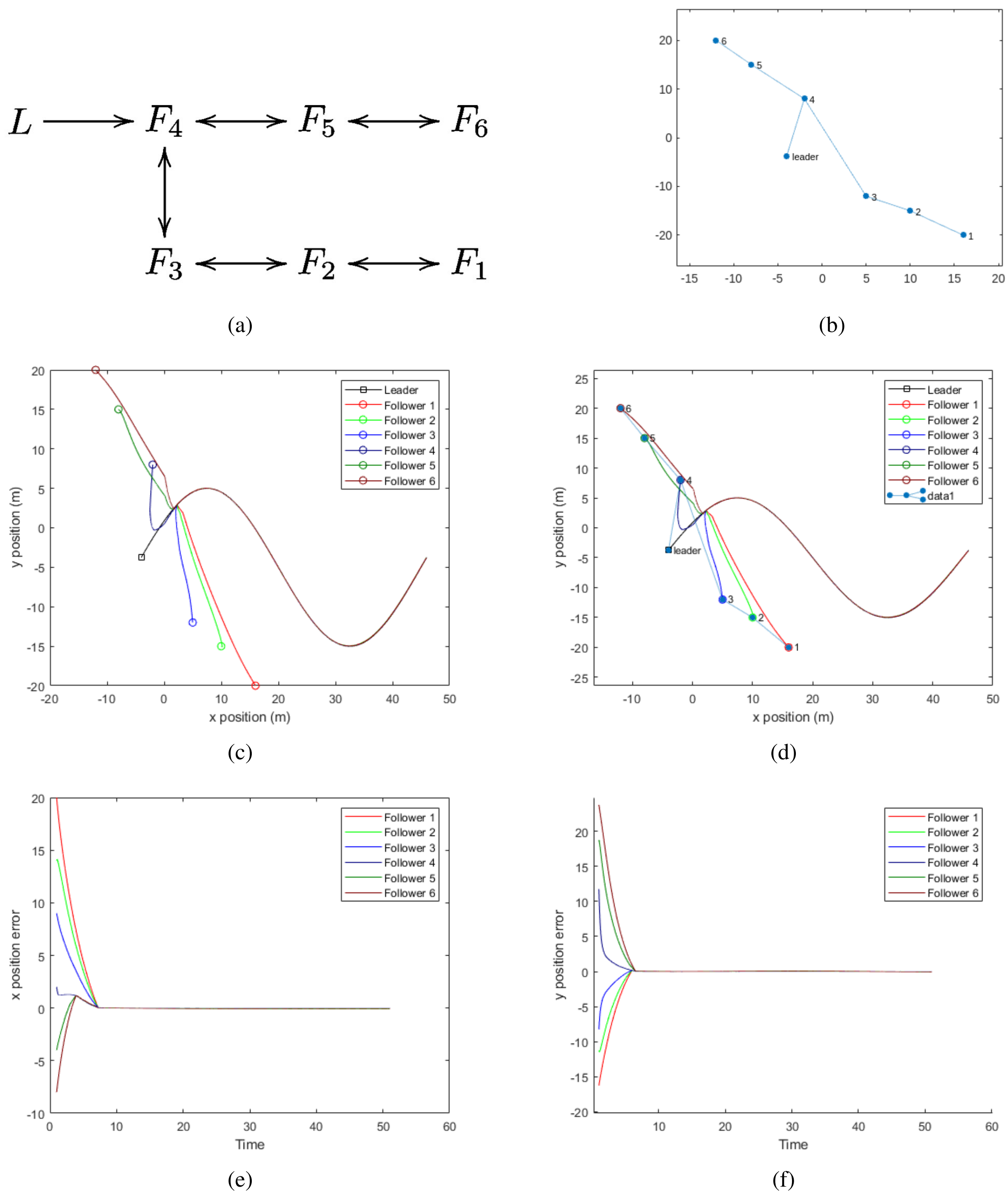


Figure 1: (a)Network topology for Distributed Consensus Tracking.(b) Undirected Graph representation for the above network topology.(c) Shows the path of the Leader and the followers as time progresses. (d) Undirected Graph representation for the above network topology. (e) Shows the error in X positions of each follower with respect to the virtual leader. (f) Shows the error in Y positions of each follower with respect to the virtual leader

$\hat{M}(t) = [\hat{m}_{ij}(t)] \in \mathbb{R}^{nn}$

$$\hat{m}_{ij}(t) = \begin{cases} -b_{ij}, & j \in \bar{\mathcal{N}}_i(t), j \neq i \\ 0, & j \notin \bar{\mathcal{N}}_i(t), j \neq i \\ \sum_{k \in \bar{\mathcal{N}}_i(t)} b_{ij}, & j = i \end{cases} \tag{19}$$

$\hat{M}(t)$ has been defined in a sense that it is symmetric positive definite at each time instant under the condition of the theorem. Also, $\beta \geq \gamma_l$. Hence equation (18) will always be negative, which makes $\dot{V}$ a negative definite. Hence, the equation for V will converge which gives the desired result as $||\tilde{r}(t)|| \to 0$ as $t \to \infty$.
Remark 3.3: The bound of time cannot be achieved just like theorem 3.1 because of the switching pattern of network topology. Which means a neighbour j may not belong into $\bar{\mathcal{N}}_i \subseteq \{0, 1, \ldots, n\}$ and hence may take a lot of time say infinite time to reduce the tracking error or to converge. Still it is considered as stable because of the convergence.

3.2 Distributed Swarm Tracking Under A Switching Network Topology

In this section the control input is defined to enhance swarm motion in the neighbours which means the neighbour will try to move in a pattern close to each other(cohesively) with the virtual leader and will also avoid inter vehicle collision with local interaction which has been defined using a new parameter d. Again, the measurements velocity is not available.
The new potential function has been defined by the author as follows:
Definition 3.4: The potential function V_{ij} is a differentiable, non-negative function of $||r_i - r_j$ satisfying the following conditions.
i) V_{ij} achieves its unique minimum when $\|r_i - r_j\|$ is equal to its desired value d_{ij}
ii) $V_{ij} \to \infty$ if $\|r_i - r_j\| \to 0$
iii) $(\partial V_{ij}/\partial(\|r_i - r_j\|)) = 0$ if $\|r_i - r_j\| \geq R$, where $R > max_{i,j} d_{ij}$ is a positive constant.
iv) $V_{ii} = c, i = 1, \ldots, n$, where c is a positive constant

Lemma 3.1: V_{ij} is defined as above. The following equality holds, which is proved below:

$$\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial V_{ij}}{\partial r_i}\dot{r}_i + \frac{\partial V_{ij}}{\partial r_j}\dot{r}_j\right) = \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial V_{ij}}{\partial r_i}\dot{r}_i \tag{20}$$

Proof: Note that

$$\begin{aligned} &\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial V_{ij}}{\partial r_i}\dot{r}_i + \frac{\partial V_{ij}}{\partial r_j}\dot{r}_j\right) \\ &\quad = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial V_{ij}}{\partial r_i}\dot{r}_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial V_{ij}}{\partial r_j}\dot{r}_j \end{aligned} \tag{21}$$

This is written using the condition in the theorem 3.4 that the potential function V_{ij} is a differentiable, non-negative function of $||r_i - r_j||$, i.e. $(\frac{\partial V_{ij}}{\partial r_i})$ = -$(\frac{\partial V_{ij}}{\partial r_j})$

$$
\begin{aligned}
&= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial V_{ij}}{\partial r_i}\dot{r}_i - \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{\partial V_{ji}}{\partial r_j}\dot{r}_i \rightarrow \text{The i and j have same values, hence are replaced} \\
&= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial V_{ij}}{\partial r_i}\dot{r}_i + \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{\partial V_{ij}}{\partial r_i}\dot{r}_i \rightarrow \text{Again using the condition in theorem 3.4} \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial V_{ij}}{\partial r_i}\dot{r}_i,
\end{aligned}
\tag{22}
$$

Hence, the lemma is proved to satisfy further results. Now using all definitions as defined before the distributed swarm tracking algorithm is defined for (1) as below:

$$
u_i = -\alpha \sum_{j\in\bar{\mathcal{N}}_i(t)}\frac{\partial V_{ij}}{\partial r_i} - \beta sgn\left(\sum_{j\in\bar{\mathcal{N}}_i(t)}\frac{\partial V_{ij}}{\partial r_i}\right) \tag{23}
$$

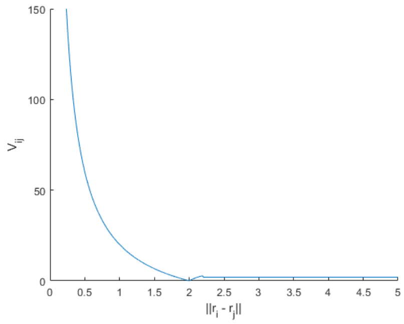
Theorem 3.5: Again it is assumed that the undirected graph *G*(t) is connected and the virtual leader is a neighbor of at least one follower at each time instant. It has been proposed that if we use (23) for (1) and if $\beta > \gamma_l$, the followers will stay close to the virtual leader and the inter-vehicle collision is avoided.

Proof: The Lyapunov function candidate is considered again as below which looks similar to the one proposed in the switching network topology for consensus tracking.

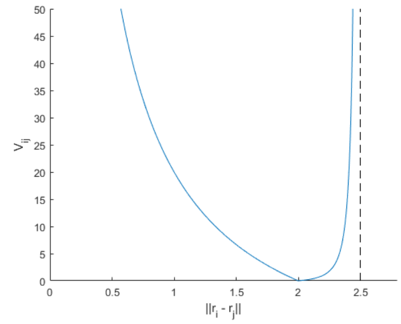
$$
V = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}V_{ij} + \sum_{i=1}^{n}V_{i0} \tag{24}
$$

Note that V is continuously differentiable with respect to r_i and r_j. Also note that the proof is very much similar to what we have explained in (20), (21) and (22). It follows that

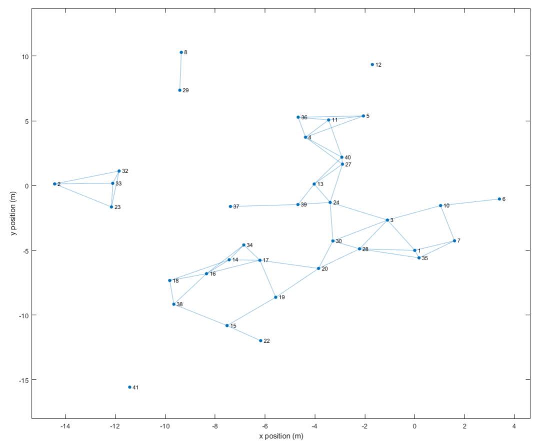
$$
\begin{aligned}
\dot{V} &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial V_{ij}}{\partial r_i}\dot{r}_i + \frac{\partial V_{ij}}{\partial r_j}\dot{r}_j\right) + \sum_{i=1}^{n}\left(\frac{\partial V_{i0}}{\partial r_i}\dot{r}_i + \frac{\partial V_{i0}}{\partial r_0}\dot{r}_0\right) \rightarrow \text{using chain rule of partial derivation} \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial V_{ij}}{\partial r_i}\dot{r}_i + \sum_{i=1}^{n}\left(\frac{\partial V_{i0}}{\partial r_i}\dot{r}_i + \frac{\partial V_{i0}}{\partial r_0}\dot{r}_0\right) \rightarrow \text{using the condition in lemma 3.1}
\end{aligned}
\tag{25}
$$



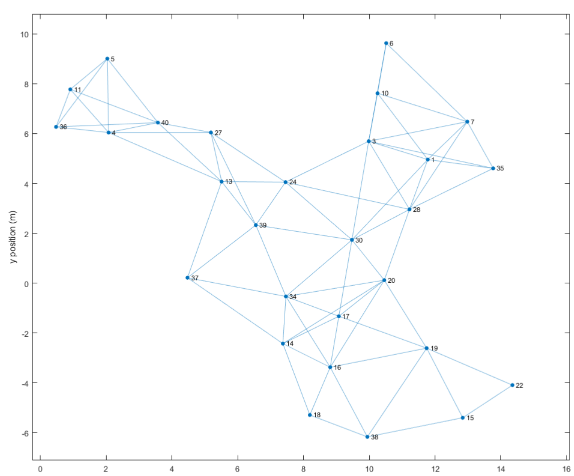
(a)



(b)



(c) Initial topology at $t = 0s$



(d) After $t = 11s$

Figure 2: (a)The characteristics of the first potential function as described in (53) (b) The characteristics of the first potential function as described in (53) (c) and (d) Swarm Tracking with varying topology

Substituting the above equation in place of $\dot{r}_i = u_i$. Also, the manipulations done in the calculations are same as we explained in (14), (15), (16), (17) and (18). Hence, the theorem is being proved.

$$\begin{aligned}
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial V_{ij}}{\partial r_i}\left[-\alpha\sum_{j=0}\frac{\partial V_{ij}}{\partial r_i} - \beta sgn\left(\sum_{j=0}\frac{\partial V_{ij}}{\partial r_i}\right)\right] \\
&+ \sum_{i=1}^{n}\frac{\partial V_{i0}}{\partial r_i}\left[-\alpha\sum_{j=0}\frac{\partial V_{ij}}{\partial r_i} - \beta sgn\left(\sum_{j=0}\frac{\partial V_{ij}}{\partial r_i}\right)\right] \\
&+ \sum_{i=1}^{n}\frac{\partial V_{i0}}{\partial r_0}\dot{r}_0 \\
&= -\alpha\sum_{i=1}^{n}\left(\sum_{j=0}^{n}\frac{\partial V_{ij}}{\partial r_i}\right)^2 - \beta\sum_{i=1}^{n}\sum_{j=0}^{n}\frac{\partial V_{ij}}{\partial r_i} + \sum_{i=1}^{n}\frac{\partial V_{i0}}{\partial r_0}\dot{r}_0 \\
&= -\alpha\sum_{i=1}^{n}\left(\sum_{j=0}^{n}\frac{\partial V_{ij}}{\partial r_i}\right)^2 - \beta\sum_{i=1}^{n}\sum_{j=0}^{n}\frac{\partial V_{ij}}{\partial r_i} + \sum_{i=1}^{n}\frac{\partial V_{i0}}{\partial r_0}\dot{r}_0 + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial V_{ij}}{\partial r_i}\dot{r}_0 \\
&\leq -\alpha\sum_{i=1}^{n}\left(\sum_{j=0}^{n}\frac{\partial V_{ij}}{\partial r_i}\right)^2 - \beta\sum_{i=1}^{n}\sum_{j=0}^{n}\frac{\partial V_{ij}}{\partial r_i} + |\dot{r}_0|\sum_{i=1}^{n}\left|\sum_{j=0}^{n}\frac{\partial V_{ij}}{\partial r_i}\right|
\end{aligned} \tag{26}$$

4 DISTRIBUTED COORDINATED TRACKING FOR SECOND-ORDER DYNAMICS

The section now introduces the virtual leader with measurements available for its position r_0 and velocity v_0 which are varying with time. Four different cases as mentioned in the introduction are considered.

4.1 Distributed Consensus Tracking With a Varying Virtual Leader's Velocity

Now the second order dynamics is given by

$$\dot{r_i} = v_i, \dot{v_i} = u_i, i = 1, \ldots, n \tag{27}$$

where $r_i \in R$ and $v_i \in R$ are, considered as the position and velocity of i^{th} follower and $u_i \in R$ is the control input. The maximum magnitude of the velocity of the leader is considered as $|\dot{v_0}| \leq \phi_l$ which is a positive constant. The control input u_i is designed for (27) such that all followers track the virtual leader with local interaction in the absence of acceleration measurements. The following tracking algorithm has been proposed by the author:

$$u_i = -\sum_{j=0}^{n} a_{ij}[(r_i - r_j) + \alpha(v_i - v_j)] - \beta sgn[\sum_{j=0}^{n} a_{ij}[\gamma(r_i - r_j) + (v_i - v_j)]] \tag{28}$$

All the parameters are same as defined before and α, β and γ are positive constants. The case of fixed network topology is considered first and for that a lemma is being defined as follows:

Lemma 4.1: Suppose that the fixed undirected graph *G* is connected and at least one follower has the virtual leader as its neighbour. Since now the state variables will be the stack of both velocities and position difference of followers hence a new 2X2 symmetric matrix is defined using the adjacency matrix as follows:

$$P = \begin{bmatrix}(1/2)M^2 & (\gamma/2)M \\ (\gamma/2)M & (1/2)M\end{bmatrix} \tag{29}$$

$$Q = \begin{bmatrix} \gamma M^2 & (\alpha\gamma/2)M^2 \\ (\alpha\gamma/2)M^2 & \alpha M^2 - \gamma M \end{bmatrix} \tag{30}$$

where γ and α are positive constants and adjacency matrix M is defined as the same as in the background i.e. $M = \mathcal{L} + diag(a_{10}, \ldots, a_{n0})$. It is being claimed that if:

$$0 < \gamma < min\left\{\sqrt{\lambda_{min}(M)}, \frac{4\alpha\lambda_{min}(M)}{4 + \alpha^2\lambda_{min}(M)}\right\} \tag{31}$$

then P and Q are symmetric positive definite.

Proof: Since M is a symmetric matrix it can be diagonalised as $M = \Gamma^{-1}\Lambda\Gamma$, where $\Gamma = diag[\lambda_1, \ldots, \lambda_n]$ where λ are the eigen values of M. Now we can write P in the form below:

$$P = \begin{bmatrix} \Gamma^{-1} & 0_{n\times n} \\ 0_{n\times n} & \Gamma^{-1} \end{bmatrix} \begin{bmatrix} \frac{1}{2}\Lambda^2 & \frac{\gamma}{2}\Lambda \\ \frac{\gamma}{2}\Lambda & \frac{1}{2}\Lambda \end{bmatrix} \begin{bmatrix} \Gamma & 0_{n\times n} \\ 0_{n\times n} & \Gamma \end{bmatrix} \tag{32}$$

where $\mathbf{0}_{n\times n}$ is the n x n zero matrix. Let μ be an eigen value of the F, where F is the middle matrix in P as defined below:

$$F = \begin{bmatrix} \frac{1}{2}\Lambda^2 & \frac{\gamma}{2}\Lambda \\ \frac{\gamma}{2}\Lambda & \frac{1}{2}\Lambda \end{bmatrix} \tag{33}$$

Now, Λ is a diagonal matrix, hence the characteristic equation for some λ_i can be written in a simplified manner as below:

$$\mu^2 - \frac{1}{2}(\lambda_i^2 + \lambda_i)\mu + \frac{1}{4}(\lambda_i^3 - \gamma^2\lambda_i^2) = 0 \tag{34}$$

The F matrix is symmetric and hence according to the properties of matrices the roots of equation (34) will be real. Using the quadratic formula it can be seen that all roots of equation (34) are positive if $\frac{1}{2}(\lambda_i^2 + \lambda_i) > 0$ and $\frac{1}{4}(\lambda_i^3 - \gamma^2\lambda_i^2) > 0$. We know that $\lambda_i > 0$ since M is a symmetric positive definite matrix, hence it is clear that $\frac{1}{2}(\lambda_i^2 + \lambda_i) > 0$. Now using the other condition $\frac{1}{4}(\lambda_i^3 - \gamma^2\lambda_i^2) > 0$ we get to know that this equation holds only when $\gamma^2 < \lambda_i$. Hence the roots of (34) will be positive using above two conditions. It can be seen that due to diagonalisation, P has the same eigen values as F, hence using the condition above it can be considered that if γ is less than the minimum value of λ_i, P will be positive definite. Now, since it has been already mentioned that γ is a positive constant the condition below holds for making P a positive definite:

$$0 < \gamma < \sqrt{\lambda_{min}(M)} \tag{35}$$

Similarly, the Q matrix can be written in the form of its diagonlisation using $M = \Gamma^{-1}\Lambda\Gamma$ as below:

$$Q = \begin{bmatrix} \Gamma^{-1} & 0_{n\times n} \\ 0_{n\times n} & \Gamma^{-1} \end{bmatrix} \begin{bmatrix} \gamma\Lambda^2 & \frac{\alpha\gamma}{2}\Lambda^2 \\ \frac{\alpha\gamma}{2}\Lambda^2 & \alpha\Lambda^2 - \gamma\Lambda \end{bmatrix} \begin{bmatrix} \Gamma & 0_{n\times n} \\ 0_{n\times n} & \Gamma \end{bmatrix} \tag{36}$$

Again let μ be the eigen value of H where H is defined as below:

$$H = \begin{bmatrix} \gamma\Lambda^2 & \frac{\alpha\gamma}{2}\Lambda^2 \\ \frac{\alpha\gamma}{2}\Lambda^2 & \alpha\Lambda^2 - \gamma\Lambda \end{bmatrix} \tag{37}$$

Now the new characteristic equation of μ can be written as follows:

$$\mu^2 - (\gamma\lambda_i^2 + \alpha\lambda_i + \gamma\lambda_i)\mu + \alpha\gamma\lambda_i^4 - \gamma^2\lambda_i^3 - \frac{\alpha^2\gamma^2\lambda_i^4}{4} = 0 \tag{38}$$

The H matrix is symmetric and hence according to the properties of matrices the roots of equation (34) will be real. Using the quadratic formula it can be seen that all roots of equation (34) are positive if $(\gamma\lambda_i^2 + \alpha\lambda_i + \gamma\lambda_i) > 0$ and $\alpha\gamma\lambda_i^4 - \gamma^2\lambda_i^3 - \frac{\alpha^2\gamma^2\lambda_i^4}{4} > 0$. We know that $_i > 0$, since M is a symmetric positive definite matrix, hence it is clear that$(\gamma\lambda_i^2 + \alpha\lambda_i + \gamma\lambda_i) > 0$. Now using the other condition $\alpha\gamma\lambda_i^4 - \gamma^2\lambda_i^3 - \frac{\alpha^2\gamma^2\lambda_i^4}{4} > 0$ we get to know that this equation holds only when $\gamma < \frac{4\alpha\lambda_i}{4+\alpha^2\lambda_i}$. Hence the roots of (38) will be positive using above two conditions. It can be seen that due to diagonalization, Q has the same eigen values as H, hence using the condition above it can be considered that if γ is less than the minimum value of i in the equation, Q will be positive definite always. Now, since it has been already mentioned that γ is a positive constant the condition below holds for making Q a positive definite. Hence, it proved that value of γ is same as mentioned in equation (34) for P and Q to be a symmetric positive definite matrix.
Theorem 4.1: Again the assumptions are same as before for the undirected graph *G* as connected and that the leader is the neighbour of at-least one follower at each time instant. The claim is made using (28) and (27), that if $\beta > \phi_l$ and that γ satisfies (31), then $r_i(t) \to r_0(t)$ and $v_i(t) \to v_0(t)$ globally exponentially as $t \to \infty$, which means:

$$\| \begin{bmatrix} \tilde{r}^T(t) & \tilde{v}^T(t) \end{bmatrix}^T \|_2 \leq k_1 r^{-k_2 t} \tag{39}$$

where $\tilde{r}$ and $\tilde{v}$ are the column stack vector of $\tilde{r}_i$ and $\tilde{v}_i$, where $\tilde{r}_i$ and $\tilde{v}_i$ are defined as before. P and Q are already defined in Lemma 4.1. and K_1 and K_2 are defined as below:

$$\begin{aligned} k_1 &= \sqrt{\begin{bmatrix} \tilde{r}^T(t) & \tilde{v}^T(t) \end{bmatrix} P \begin{bmatrix} \tilde{r}^T(t) & \tilde{v}^T(t) \end{bmatrix}^T / \lambda_{min}(P)} \\ k_2 &= \lambda_{min}(Q)/2\lambda_{max}(P) \end{aligned} \tag{40}$$

Proof: Using $\tilde{r}_= r_i - r_0$ and $\tilde{v}_i = v_i - v_0$ and the closed-loop equation for (27) can be written using (28) as follows:

$$\begin{aligned} \dot{\tilde{r}}_i &= \tilde{v}_i \\ \dot{\tilde{v}}_i &= -\sum_{j=0}^{n} a_{ij}[(r_i - r_j) + \alpha(v_i - v_j)] \\ &- \beta sgn\left[\sum_{j=0}^{n} a_{ij}[\gamma(r_i - r_j) + (v_i - v_j)]\right] - \dot{v}_0 \end{aligned} \tag{41}$$

The above can be written in matrix form as

$$\begin{aligned} &\dot{\tilde{r}} = \tilde{v} \\ &\dot{\tilde{v}}_i = -M\tilde{r} - \alpha M\tilde{v} - \beta sgn[M(\gamma\tilde{r} + \tilde{v})] - 1\dot{v}_0 \to \text{here the 1 denote a compatible identity matrix} \end{aligned} \tag{42}$$

where $\tilde{r}$ and $\tilde{v}$ are defined in (39) and $M = \mathcal{L} + diag(a_{10}, \ldots, a_{n0})$, the same as before.
The Lyapunov Function/candidate is considered as follows which is conceptually same as what the author defined in kinematics, the only difference is a new velocity stack is added because now we are defining it for second order dynamics.

$$\begin{aligned} &V = \begin{bmatrix} \tilde{r}^T(t) & \tilde{v}^T(t) \end{bmatrix} P \begin{bmatrix} \tilde{r}(t) \\ \tilde{v}(t) \end{bmatrix} \to \text{P is taken as 2x2 matrix from (29)} \\ &= \frac{1}{2}\tilde{r}^T M^2 \tilde{r} + \frac{1}{2}\tilde{v}^T M \tilde{v} + \gamma \tilde{r}^T M \tilde{v} \\ &\dot{V} = \tilde{r}^T M^2 \tilde{v} + \tilde{v}^T M \tilde{v} + \gamma \tilde{v}^T M \tilde{v} + \gamma \tilde{r}^T M \dot{\tilde{v}} \to \text{using matrix calculus and } \dot{\tilde{r}} \text{ is replaced by } \tilde{v} \text{ in the first term} \end{aligned} \tag{43}$$

Now using the value of Q from (30) we can re-write the equation (43) as below in terms of Q.

$$\begin{aligned} &= -\begin{bmatrix}\tilde{r}^T(t) & \tilde{v}^T(t)\end{bmatrix} Q \begin{bmatrix}\tilde{r}(t)\\ \tilde{v}(t)\end{bmatrix} - (\gamma\tilde{r}^T + \tilde{v}^T)M\beta sgn[M(\gamma\tilde{r}+\tilde{v}] + 1\dot{v}_0 \\ &\leq -\begin{bmatrix}\tilde{r}^T(t) & \tilde{v}^T(t)\end{bmatrix} Q \begin{bmatrix}\tilde{r}(t)\\ \tilde{v}(t)\end{bmatrix} - (\beta - \varphi_l)\|M(\gamma\tilde{r}+\tilde{v})\|_1 \end{aligned} \tag{44}$$

Again the vector in the above equation is multiplied by its sgn(.) and the maximum value of $\dot{v}_0$ is taken hence the inequality holds. We already defined that Q is symmetric positive definite when γ satisfies (31), β is already assumed to be greater than ϕ_l. Hence, $\dot{V}$ is negative definite. Hence, it becomes obvious from Lyapunov stability theory of non-smooth systems as mentioned in [13] that $\tilde{r}(t) \to 0_n$ and $\tilde{v}(t) \to 0_n$ as $t \to \infty$ where 0_n is a n X 1 zero vector. Also, it implies that $r_i(t) \to r_0(t)$ and $v_i(t) \to v_0(t)$ as $t \to \infty$.

Now to show that the distributed consensus tracking is achieved at least globally exponentially using the spectral radius property of norms of matrix/vector or Rayleigh-Ritz inequality we can re-write equation (43) as:

$$V \leq \lambda_{max}(P)\left\|\begin{bmatrix}\tilde{r}^T(t) & \tilde{v}^T(t)\end{bmatrix}^T\right\|_2^2 \rightarrow \text{after taking norm. } \tilde{r} \text{ X } \tilde{r}^T \text{ becomes square and hence second norm} \tag{45}$$

Using equation (44), since $\dot{V}$ is negative it will still hold the inequality sign if written as below:

$$\dot{V} \leq -\begin{bmatrix}\tilde{r}^T(t) & \tilde{v}^T(t)\end{bmatrix} Q \begin{bmatrix}\tilde{r}(t)\\ \tilde{v}(t)\end{bmatrix} \tag{46}$$

Again using the spectral radius property of norms of matrix/vector or Rayleigh-Ritz inequality we can re-write equation (43) as below due to its negative sign we are now again talking about minimum eigen value. After putting the value from (45) we get the equation below.

$$\leq -\lambda_{min}(Q)\left\|\begin{bmatrix}\tilde{r}^T(t) & \tilde{v}^T(t)\end{bmatrix}^T\right\|_2^2 \leq -\frac{\lambda_{min}(Q)}{\lambda_{max}(P)}V \tag{47}$$

On integrating (47) using the same step as we have shown in (10) from t=0 to t=t, we get,

$$\dot{V} \leq V(0)e^{-\lambda_{min}(Q)t/\lambda_{max}(P)} \tag{48}$$

Now for deriving the the inequality equation for exponential tracking from (39) we have included the missing steps.

We can write equation for V at t = 0 from (43) as

$$\begin{aligned} &V(0) = \begin{bmatrix}\tilde{r}^T(0) & \tilde{v}^T(0)\end{bmatrix} P \begin{bmatrix}\tilde{r}^T(0)\\ \tilde{v}^T(0)\end{bmatrix} \\ &\text{We know that, } V \geq \lambda_{min}(P)\left\|\begin{bmatrix}\tilde{r}^T(t) & \tilde{v}^T(t)\end{bmatrix}^T\right\|_2^2 \\ &\left\|\begin{bmatrix}\tilde{r}^T(t) & \tilde{v}^T(t)\end{bmatrix}^T\right\|_2^2 \leq \frac{V}{\lambda_{min}} \leq \frac{V(0)}{\lambda_{min}(P)}e^{(-\lambda_{min}(Q)/\lambda_{max}(P))t} \\ &\left\|\begin{bmatrix}\tilde{r}^T(t) & \tilde{v}^T(t)\end{bmatrix}^T\right\|_2^2 \leq \frac{\begin{bmatrix}\tilde{r}(0)^T & \tilde{v}(0)^T\end{bmatrix} P \begin{bmatrix}\tilde{r}(0)^T\\ \tilde{v}(0)^T\end{bmatrix}}{\lambda_{min}(P)}e^{(-\lambda_{min}(Q)/\lambda_{max}(P))t} \end{aligned} \tag{49}$$

Finally we get the equation below,

$$\left\|\begin{bmatrix}\tilde{r}^T(t) & \tilde{v}^T(t)\end{bmatrix}^T\right\|_2 \leq \left[\sqrt{\frac{\begin{bmatrix}\tilde{r}(0)^T & \tilde{v}(0)^T\end{bmatrix} P \begin{bmatrix}\tilde{r}(0)^T\\ \tilde{v}(0)^T\end{bmatrix}}{\lambda_{min}(P)}}\right] e^{(-\lambda_{min}(Q)/2\lambda_{max}(P))t} \tag{50}$$

which is equivalent in form to

$$\| \begin{bmatrix} \tilde{r}^T(t) & \tilde{v}^T(t) \end{bmatrix}^T \|_2 \leq k_1 r^{-k_2 t} \tag{51}$$

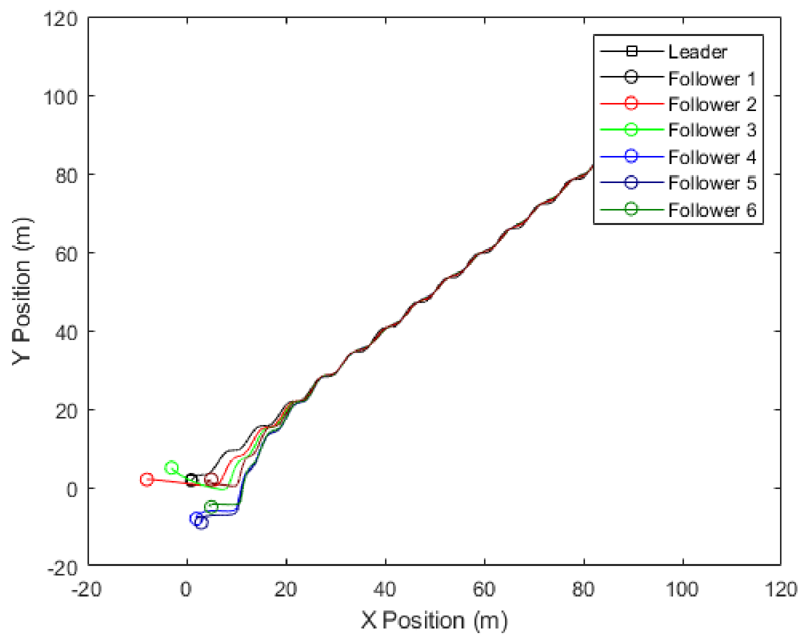
where,

$$\begin{aligned} k_1 &= \sqrt{\begin{bmatrix} \tilde{r}^T(0) & \tilde{v}^T(0) \end{bmatrix} P \begin{bmatrix} \tilde{r}^T(0) & \tilde{v}^T(0) \end{bmatrix}^T / \lambda_{min}(P)} \\ k_2 &= \lambda_{min}(Q)/2\lambda_{max}(P) \end{aligned} \tag{52}$$

Hence, the theorem 4.1 for exponential tracking is proved. we kept out scope of study til dynamics consensus tracking for fixed network topology and all the simulations and results are shown accordingly.

$$\begin{array}{ccccc} L & F_6 & \longleftrightarrow F_5 \longleftrightarrow & F_4 \\ & \searrow\ \updownarrow & & \updownarrow \\ & F_1 & \longleftrightarrow F_2 \longleftrightarrow & F_3 \end{array}$$

(a) Network topology of simulation 5.3



(b)

Figure 3: Coordinate tracking using dynamics

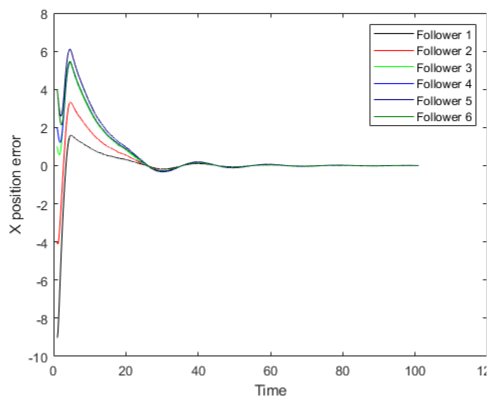
5 SIMULATION

In this section, we attempt to reproduce the simulation performed in the publication and apply the method on a new case to see its performance. All simulation were carried out in MATLAB using discrete timesteps. All the results found below both resemble and validate the (8) paper.

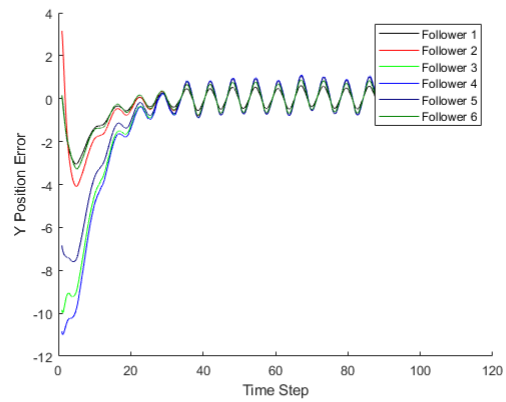
5.1 Distributed Consensus Tracking Under Fixed Network

We consider a group of six followers with a virtual leader in 2D space. Let the $r_i = [r_{ix}, r_{iy}]$ and its corresponding velocity $v_i = [v_{ix}, v_{iy}]$, where r_{ix} and r_{iy} denote, respectively, the x and y position of the follower i while the v_{ix} and v_{iy} denote, respectively, the x and y velocity of follower i. Let $a_{ij} = 1$ if vehicle

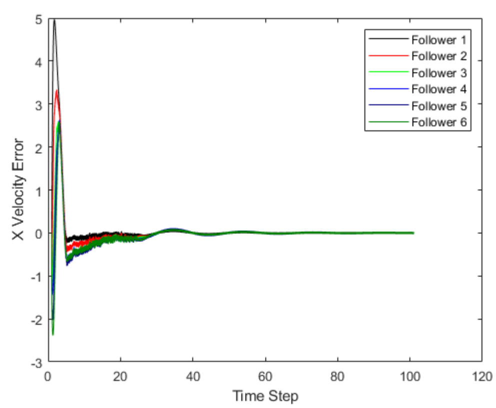
j is a neighbor of vehicle i, where $j = 0, 1, \ldots, 6$ and $i = 1, \ldots, 6$ and $a_{ij} = 0$. Neighbor vehicles are defined based on the network topology in Figure 1(a).
First we construct a 7×7 adjacency matrix based on the aforementioned topology. Using this we can create a undirected Graph structure on MATLAB as shown in figure 1(b). The leader is allotted a predefined trajectory $r_0(t) = [t - 5, -5 + 10sin(\pi t/25)]$. The values of the constants are taken as $\alpha = 1$ and $\beta = 1.5$, such that $\beta > \gamma_l$, where γ_l is the max velocity of the leader
The trajectory of the followers and the virtual leader are generated at every time step using (2). The path of the followers and the leader are shown in Figure 1(c) and its overlay with the graph is shown in Figure 1(d). The positional errors shown in Figure 1(e) and (f) shows that in finite time the error tends to zero and the followers reach and accurately track the leader (within a small margin).



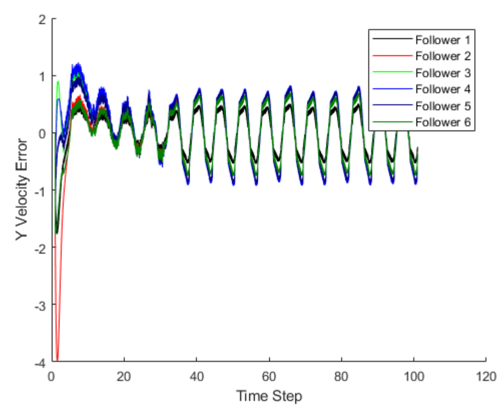
(a) X Position Errors



(b) Y Position Errors



(c) X Velocity Errors



(d) Y Velocity Errors

Figure 4

5.2 Distributed Swarm Tracking Under Varying Network

We consider for this simulation 40 followers and 1 virtual leader. The equation (23) is used to calculate the velocity of each of the followers. The partial derivative of the potential function chosen is as follows

$$\begin{aligned}\frac{\partial V_{ij}^1}{\partial r_i} &= \begin{cases} 0, & \|r_i - r_j\| > R \\ \frac{2\pi(r_i-r_j)\sin 2*\pi(\|r_i-r_j\|-d_{ij})}{\|r_i-r_j\|}, & d_{ij} < \|r_i - r_j\| \leq R \\ \frac{20(r_i-r_j)}{\|r_i-r_j\|}\frac{\|r_i-r_j\|-d_{ij}}{\|r_i-r_j\|}, & \|r_i - r_j\| \leq d_{ij} \end{cases} \\ \frac{\partial V_{ij}^2}{\partial r_i} &= \begin{cases} \frac{(r_i-r_j)}{\|r_i-r_j\|}\frac{\|r_i-r_j\|-d_{ij}}{(\|r_i-r_j\|-R)^2}, & d_{ij} < \|r_i - r_j\| \leq R \\ \frac{20(r_i-r_j)}{\|r_i-r_j\|}\frac{\|r_i-r_j\|-d_{ij}}{\|r_i-r_j\|}, & \|r_i - r_j\| \leq d_{ij} \end{cases}\end{aligned} \tag{53}$$

The characteristic of which can be seen in figure 2. For this simulation we only use the $\frac{\partial V_{ij}^1}{\partial r_i}$ potential function to drive the system. Intuitively, the function penalizes nodes that come closer than the defined constants d_{ij} while ignoring the ones that are outside the sensing radius R.

We chose $R = 4$, $d_{ij} = 2$, $\alpha = 1$ and $\beta = 3$. Figure 2(c)) depicts the initial positions and connections of all the nodes in the network. There exist some nodes that do not have any connections to the network and some that have formed their own closed chain. As we update the positions of the nodes after finding its velocity using (23), nodes that come within the R radius of any node gets added to the graph. Figure 2(d) clearly shows nodes that were not in the network at $t = 0$ and that came in close proximity have been added to the network, at the same time the nodes maintain a minimum distance of d_{ij} and hence avoiding collision.

5.3 Distributed Consensus Tracking Under Fixed Network Using Dynamics

While using only position to tracking the leader, the followers tend to miss the underlying type of motion that the leader is exhibiting. To account for this, the velocity of the leader is taken into consideration and the acceleration of each follower is calculated using (28).

Similar to 5.1, 6 follower and 1 virtual leader in a 2D space is taken for this study. The nodes follow the topology as depicted by Figure 3 (a), using which an adjacency matrix is created. The leader is allotted a predefined trajectory $r_0 = [t, t + sin(t)]$ and we set $\alpha = 1$, $\beta = 5$ and $\gamma = 1$. The trajectories of the followers and the virtual leader are shown in Figure 3(b). The tracking errors of the x and y positions are shown in Figure 4(a) and (b). The tracking errors of the x and y velocities are shown in Figure 4(c) and (d). It can be seen from Figure 4 that the tracking errors converges to zero. That is, all followers ultimately track the virtual leader as also shown in Figure 3(b)

6 Conclusion

Using techniques mentioned in (8) study, we were able to reproduce their results and extend them to other scenarios apart from what is mentioned in the paper. This indicates that the paper is not only accurate but also can be implemented with ease. The techniques detailed in this report can we extended for use in autonomous vehicles and swarm robotics, where autonomous tracking/following is one of its key principle.

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