Perceptron Learning Algorithm

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Mathematical Background

The perceptron is used whenever the data we have is linearly separable i.e., when the data forms clusters that can be separated by a hyperplane. Our goal is to find this hyperplane.

Let the data points in such a data be represented as, $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N$ be inputs with the labels $y_1, y_2, \cdots, y_N$ respectively where $\mathbf{x}_i \in \mathcal{X}$ *is* $\mathbf{d} - dimensional$ and $\mathcal{Y} = \{-1, 1\}$.

These labels are as given by a target function $f : \mathcal{X} \to \mathcal{Y}$ such that $f(\mathbf{x}) = y$
Let $h : \mathcal{X} \to \mathcal{Y}$ be the hypothesis that approximates the target f. Let $\mathbf{w}$ be the weight vector such that if Let $\mathbf{x} = (x_1, x_2, \cdots, x_d)$ and $\mathbf{w}$=$(w_1, w_2, \cdots, w_d)$,

$$
\begin{aligned}
h(\mathbf{x}) &= sgn(\sum_{i=1}^{d} w_i x_i - threshold) \\
&\quad By\ setting\ w_0 = -threshold \\
&= sgn(\sum_{i=1}^{d} w_i x_i + w_0(1)) \\
&\quad By\ setting\ x_0 = 1 \\
&= sgn(\sum_{i=0}^{d} w_i x_i) \\
h(\mathbf{x}) &= sgn(\mathbf{w}^T \mathbf{x})
\end{aligned}
$$

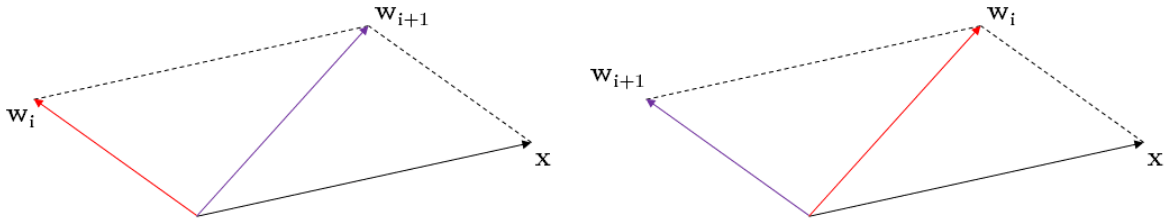
Selection and updating the weights

To start with, we can set the $\mathbf{w}_0$ vector randomly. Let $\mathbf{w}_i$ be the weight in i^{th} iteration.
***Definition*:** A point $\mathbf{x}$ is said to be misclassified if $y = f(\mathbf{x}) \neq h(\mathbf{x})$.

***Perceptron Learning Algorithm*:**(in i^{th} iteration)

1. Choose a misclassified point, say $\mathtt{x}_r$.
2. Update the weight vector as $\mathtt{w}_{i+1} \leftarrow \mathtt{w}_i + y_r \mathtt{x}_r$.
3. Continue till there are no misclassified points.

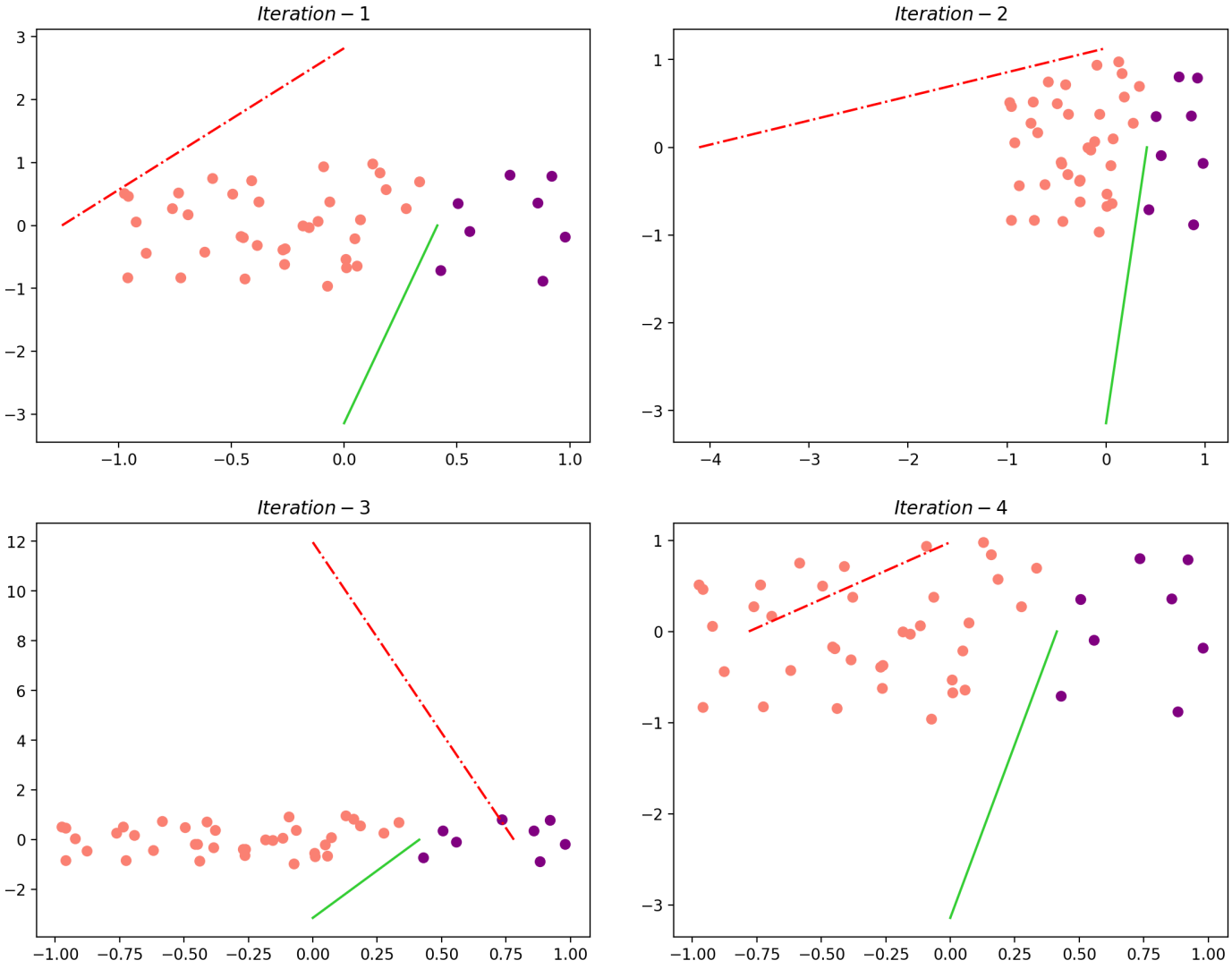
Why does this work?

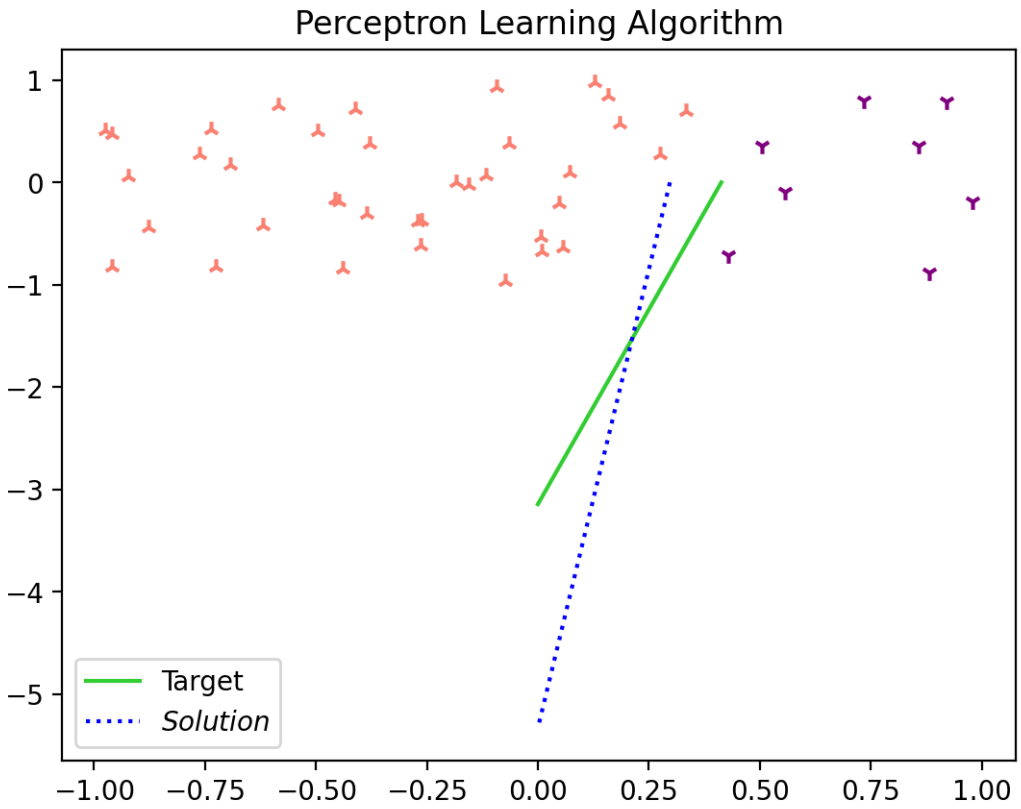


$y = +1$ $y = -1$

Updating the points at misclassified points

Visualization of the results in 2-d





Final Solution as obtained by the PLA

Some observations:

The algorithm converges to the target much better to the target function as the number of data points increases as the available 'whitespace' decreases with increasing points.