

# Lecture 2

After the introduction of word2vec, this lecture is about the part of **Optimization**

## 1. Gradient Descent

- so as said, there is a cost function  $J(\theta)$  to be minimized, by calculating the gradient of  $J(\theta)$  and moving  $\theta$  in small steps to the direction of the gradient
- $\theta^{new} = \theta^{old} - \alpha \nabla_{\theta} J(\theta)$
- $\alpha$  is the learning rate/step size, which is a hyperparameter
- $\nabla$  is the gradient

## 2. Stochastic Gradient Descent (SGD)

- obviously, the  $J(\theta)$  in the gradient calculation is too massive, so for a single update, it would take a lot of time
- so this **SGD** method, repeatedly takes small random window size with **less center words**, and update the  $\theta$  parameters in the same way
- does a better job, and also in a quicker time**

## 3. Negative Sampling (skip Gram model)

- In **Softmax** the **denominator for the norm** consists of the huge sum of all the words in the vocabulary, which is very costly to compute

### Regression (side quest)

**Linear regression:** is training a model in a form of  $y = wx + b$ , where w is the weight, and b is the bias, and learns w and b to predict y as close as possible to the real value of y

**Logistic regression:** here the trained model predicts the probability of y, which is in the form of  $y = \frac{1}{1+e^{-wx}}$  using a sigmoid.

- so in the Skip Gram model, it uses logistic regression. And is trained in a way such that it uses a **true pair versus a noise pair**, ( pair consists of a center word and a context word, and noise refers to a random context word for the same center word). so it gives the probability if it is true pair then  $P = 1$ , else  $P = 0$ , it is a **classifier**
- $J_t(\theta) = \log(\sigma(u_o^T v_c)) + \sum_{i=1}^k E_{j \text{ in } P(w)} [\log(\sigma(-u_j^T v_c))]$
- $\sigma$  as the sigmoid/logistic function, so the goal is to minimize the cost, here which means maximizing  $\sigma$  in which when the input dot product is large to infinity, output of  $\sigma$  is 1, and  $\log 1 = 0$ , similarly for noise pairs, the dot product should have negative infinity, and putting it into  $\sigma(-\infty)$  also gives out 0, so  $0 + 0 = 0$ , (maximized). cuz everything else is negative.
- and negating works due to the sigmoid function is symmetric around 0.5
- And again, we are taking the **negative log likelihood**, because before 0 was max, still maximizing, but we want to minimize the cost function.
- $J_{neg\_sample}(\theta) = -\log(\sigma(u_o^T v_c)) - \sum_{i=1}^k E_{j \text{ in } P(w)} [\log(\sigma(-u_j^T v_c))]$
- maximize real pair, minimize random pair

## 4. Co-Occurrence Matrix $\neq$ Word2vec

- it is a symmetric matrix, depending on the window size, it places the number of occurrences of a word in the context window of another center word. like row for context, column for center word.
- Reducing dimension on Co-occurrence Matrix:** By decomposing the matrix through **SVD**, getting n singular values,
- Stores most information in a smaller number of dimensions.
- Problems:** for processing the values:
  - As running raw counts on SVD, it wouldn't work well
- Fix:**
  - make the maximum limit of 100
  - we can log the frequencies
  - also remove the frequently occurring function words like "the/he/has"
  - so you get more useful word vectors

## Comparisons, introducing GloVe

LHS uses Linear algebra

RHS uses gradient neural models

- LSA, HAL (Lund & Burgess),
- COALS, Hellinger-PCA (Rohde et al, Lebret & Collobert)

- Fast training
- Efficient usage of statistics
- Primarily used to capture word similarity
- Disproportionate importance given to large counts

- Skip-gram/CBOW (Mikolov et al)
- NNLM, HLBL, RNN (Bengio et al; Collobert & Weston; Huang et al; Mnih & Hinton)

- Scales with corpus size
- Inefficient usage of statistics
- Generate improved performance on other tasks
- Can capture complex patterns beyond word similarity

- skip gram lacks statistics , because it only looks at one word at a time, unlike the co-occurrence matrix which looks at the whole window

## 5. GloVe

Meaning Components: words co-occurring with each other

- level of co-occurrence represented with the **ratio of probabilities**.
- for example **ice to solid** rather than gas

	$x = \text{solid}$	$x = \text{gas}$	$x = \text{water}$	$x = \text{random}$
$P(x \text{ice})$	large	small	large	small
$P(x \text{steam})$	small	large	large	small
$\frac{P(x \text{ice})}{P(x \text{steam})}$	large	small	$\sim 1$	$\sim 1$

- so here, you can see that ice and steam is **both similar** to water, so the ratio is  $\sim 1$ , nice!! however for both random words to ice and steam with **no correlation**, the ratio is also  $\sim 1$
- So for plotting we would normally try to find one that is related but the other one is not.

Capturing the ratio:

- using a log bilinear model
- The GloVe model takes both the co-occurring and neural model's properties. (count based & dot product learning)

◦

Q: How can we capture ratios of co-occurrence probabilities as linear meaning components in a word vector space?

A: Log-bilinear model:  $w_i \cdot w_j = \log P(i|j)$

with vector differences

$$w_x \cdot (w_a - w_b) = \log \frac{P(x|a)}{P(x|b)}$$

**Loss:**  $J = \sum_{i,j=1}^V f(X_{ij}) (w_i^T \tilde{w}_j + b_i + \tilde{b}_j - \log X_{ij})^2$



- Fast training
- Scalable to huge corpora

- the bilinear model is just the defining the dot product to be the log of the ratio of the probabilities
- For the **loss function**  $X_{ij}$  is form the co-occurrence matrix, how many times the context j appeared in the window of center word i
- so then, the model needs to adjust the weight and biases so for every pair  $(i, j)$ , you want the model's prediction  $w_i^T \tilde{w}_j + b_i + \tilde{b}_j$  to be close to the "target" value  $\log X_{ij}$ .
- $f(X_{ij})$  is for scaling the words that are more common

• **Intrinsic way of evaluation:** (局部子任务做起)

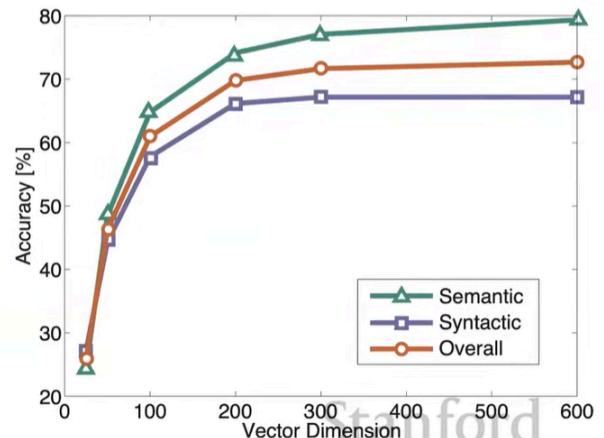
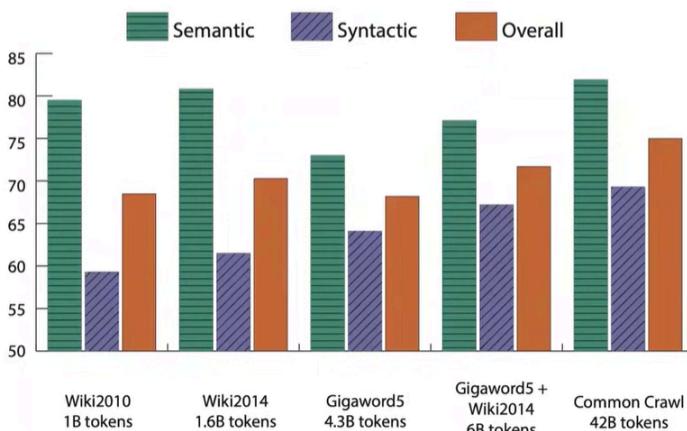
- evaluate directly on a specific subtask
- helps to understand the system
- fast
- not clear if there is correlation established between the real task

• **Extrinsic way of evaluation:** (全局的任务)

- evaluate on a specific task (whole)
- long time to evaluate
- Unclear if the subsystem has problems

## 6. Results

- More data helps
- Wikipedia is better than news text!
- Dimensionality
- Good dimension is ~300



## Word vector distances and their correlation with human judgments

Model	Size	WS353	MC	RG	SCWS	RW
SVD	6B	35.3	35.1	42.5	38.3	25.6
SVD-S	6B	56.5	71.5	71.0	53.6	34.7
SVD-L	6B	65.7	<u>72.7</u>	75.1	56.5	37.0
CBOW <sup>†</sup>	6B	57.2	65.6	68.2	57.0	32.5
SG <sup>†</sup>	6B	62.8	65.2	69.7	<u>58.1</u>	37.2
GloVe	6B	<u>65.8</u>	<u>72.7</u>	<u>77.8</u>	53.9	<u>38.1</u>
SVD-L	42B	74.0	76.4	74.1	58.3	39.9
GloVe	42B	<b>75.9</b>	<b>83.6</b>	<b>82.9</b>	<b>59.6</b>	<b>47.8</b>
CBOW*	100B	68.4	79.6	75.4	59.4	45.5

### On extrinsic task - In a real NLP task

Extrinsic evaluation of word vectors: All subsequent NLP tasks in this class. More examples soon.

One example where good word vectors should help directly: **named entity recognition**: identifying references to a person, organization or location

Model	Dev	Test	ACE	MUC7
Discrete	91.0	85.4	77.4	73.4
SVD	90.8	85.7	77.3	73.7
SVD-S	91.0	85.5	77.6	74.3
SVD-L	90.5	84.8	73.6	71.5
HPCA	92.6	<b>88.7</b>	81.7	80.7
HSMN	90.5	85.7	78.7	74.7
CW	92.2	87.4	81.7	80.2
CBOW	93.1	88.2	82.2	81.1
GloVe	<b>93.2</b>	88.3	<b>82.9</b>	<b>82.2</b>

Stanford

- **Extra:** words with multi meaning are made into multiple word vectors.
- and when computing they are made into 1 single vector through linear combination of the multi meaning vectors.
- however they could be separated again due to the high dimensionality of the vectors and how sparse the vectors are.