Assignment 5

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Download the python code, latex file and the pdf doc from

https://github.com/Rishab9991/EE5609/tree/master/Assignments/Assignment5

Question

(QR decomposition of V from Assignment 4) Find the QR decomposition of,

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \tag{1}$$

Solution

A can be written as,

$$\mathbf{A} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \tag{2}$$

where **a** and **b** and are column vectors,

$$\mathbf{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \tag{3}$$

$$\mathbf{b} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{4}$$

The QR decomposition of a matrix **A** is given by,

$$\mathbf{A} = \mathbf{QR} \tag{5}$$

where **R** is a upper triangular matrix and **Q** such that,

$$\mathbf{Q^TQ} = \mathbf{I} \tag{6}$$

and

$$\mathbf{Q} = \begin{pmatrix} \mathbf{q_1} & \mathbf{q_2} \end{pmatrix} \; and \; \mathbf{R} = \begin{pmatrix} r_1 & r_2 \\ 0 & r_3 \end{pmatrix} \tag{7}$$

$\mathbf{q_1}$, $\mathbf{q_2}$ and the values in **R** are given by,

$$r_1 = \|\mathbf{a}\| \tag{8}$$

$$\mathbf{q_1} = \frac{\mathbf{a}}{r_1} \tag{9}$$

$$r_2 = \frac{\mathbf{q_1^T b}}{\|\mathbf{q_1}\|^2} \tag{10}$$

$$\mathbf{q_2} = \frac{\mathbf{b} - r_2\mathbf{q_1}}{\|\mathbf{b} - r_2\mathbf{q_1}\|} \tag{11}$$

$$r_3 = \mathbf{q_2^T b} \tag{12}$$

Using (3) and (4) we get,

$$r_1 = \sqrt{5} \tag{13}$$

$$\mathbf{q_1} = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} \end{pmatrix} \tag{14}$$

$$r_2 = \frac{-3}{\sqrt{5}} \tag{15}$$

$$\mathbf{q_2} = \sqrt{5}\begin{pmatrix} \frac{1}{5} \\ \frac{2}{5} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix} \tag{16}$$

$$r_3 = \frac{1}{\sqrt{5}} \tag{17}$$

Therefore,

$$\mathbf{QR} = \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix}\begin{pmatrix} \sqrt{5} & \frac{-3}{\sqrt{5}} \\ 0 & \frac{1}{\sqrt{5}} \end{pmatrix} \tag{18}$$

$$\implies \mathbf{QR} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \tag{19}$$

As (1) and (19) are equal, the QR decomposition holds.