Assignment 2

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Download the python code, latex file and the pdf doc from

https://github.com/Rishab9991/EE5609/tree/master/Assignments/Assignment2

Problem 2.1

Solution:

$$A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ \text{-}\sin\alpha & \cos\alpha \end{pmatrix} \tag{1}$$

$$\implies A^T = \begin{pmatrix} \cos\alpha & \text{-}\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \tag{2}$$

Need to prove that A^TA = I (Identity Matrix)

For this we express each of the matrices as a complex exponential to find the product of A^T and A using it.

$$i.e\ A = \begin{pmatrix} \frac{(e^{i\alpha}+e^{-i\alpha})}{2} & \frac{(e^{i\alpha}-e^{-i\alpha})}{2i} \\ \text{-}\frac{(e^{i\alpha}-e^{-i\alpha})}{2i} & \frac{(e^{i\alpha}+e^{-i\alpha})}{2} \end{pmatrix} \tag{3}$$

$$A^T = \begin{pmatrix} \frac{(e^{i\alpha}+e^{-i\alpha})}{2} & \text{-}\frac{(e^{i\alpha}-e^{-i\alpha})}{2i} \\ \frac{(e^{i\alpha}-e^{-i\alpha})}{2i} & \frac{(e^{i\alpha}+e^{-i\alpha})}{2} \end{pmatrix} \tag{4}$$

$$\implies A^T A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I \tag{5}$$

$$Since,\ \frac{(e^{i\alpha}+e^{-i\alpha})^2}{4} - \frac{(e^{i\alpha}-e^{-i\alpha})^2}{4} = 1 \tag{6}$$

$$\frac{(e^{i\alpha})^2-(e^{-i\alpha})^2}{4i} - \frac{(e^{i\alpha})^2-(e^{-i\alpha})^2}{4i} = 0 \tag{7}$$

Equations (6) and (7) are the expressions that are obtained in the resultant A^TA matrix. In (6) the i^2 in the denominator of the second term will lead to the subtraction between the two terms in the expression.

Therefore, A^TA = I (Identity matrix)

Hence Proved.

Problem 2.2

Solution:

$$A = \begin{pmatrix} \sin\alpha & \cos\alpha \\ \text{-}\cos\alpha & \sin\alpha \end{pmatrix} \tag{8}$$

$$\implies A^T = \begin{pmatrix} \sin\alpha & \text{-}\cos\alpha \\ \cos\alpha & \sin\alpha \end{pmatrix} \tag{9}$$

Need to prove that A^TA = I (Identity Matrix)

For this we express each of the matrices as a complex exponential to find the product of A^T and A using it.

$$i.e\ A = \begin{pmatrix} \frac{(e^{i\alpha}-e^{-i\alpha})}{2i} & \frac{(e^{i\alpha}+e^{-i\alpha})}{2} \\ \text{-}\frac{(e^{i\alpha}+e^{-i\alpha})}{2} & \frac{(e^{i\alpha}-e^{-i\alpha})}{2i} \end{pmatrix} \tag{10}$$

$$A^T = \begin{pmatrix} \frac{(e^{i\alpha}-e^{-i\alpha})}{2i} & \text{-}\frac{(e^{i\alpha}+e^{-i\alpha})}{2} \\ \frac{(e^{i\alpha}+e^{-i\alpha})}{2} & \frac{(e^{i\alpha}-e^{-i\alpha})}{2i} \end{pmatrix} \tag{11}$$

$$\implies A^T A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I \tag{12}$$

Similarly using equations (6) and (7) we get 1 and 0. The same terms appear in the resultant A^TA matrix of this problem as in **Problem 2.1**

Therefore, A^TA = I (Identity matrix)

Hence Proved.