Assignment 6

D V K M Rishab
AI20MTECH14004

Download the python code, latex file and the pdf doc from

QUESTION

(Q.No.32, UGC June 2017)
Find the Eigenvalues of the matrix,

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -2 & 5 \\ 2 & 5 & -3 \end{pmatrix} \quad (1)$$

Options:

1) -4, 3, -3
2) 4, 3, 1
3) 4, -4± $\sqrt{13}$
4) 4, -2± $\sqrt{7}$

SOLUTION

Using the characteristic equation of the matrix can find the Eigenvalues,

$$\left| \lambda \mathbf{I} - \mathbf{A} \right| = 0 \quad (2)$$

$$\implies \begin{vmatrix} \lambda - 1 & -1 & -2 \\ -1 & \lambda + 2 & -5 \\ -2 & -5 & \lambda + 3 \end{vmatrix} = 0 \quad (3)$$

The expression that is obtained after expanding the determinant and simplifying it is,

$$(\lambda - 1)(\lambda^2 + 5\lambda - 19) - (5\lambda + 31) = 0 \quad (4)$$

Further simplifying this we obtain the cubic equation,

$$\lambda^3 + 4\lambda^2 - 29\lambda - 12 = 0 \quad (5)$$

Solving this equation, the Eigenvalues obtained are,

$$\lambda_1 = -7.605, \ \lambda_2 = -0.394 \ and \ \lambda_3 = 4 \quad (6)$$

Therefore, the Eigenvalues of the given matrix are 4, -4 ± $\sqrt{13}$ (Option 3)