Assignment 3.2

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Solution: Given,

The line equation,

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 1 \tag{1}$$

The circle equation,

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix} 8 & 6 \end{pmatrix}\mathbf{x} + 7 = 0 \tag{2}$$

The vector equation of a line can be expressed as,

$$\mathbf{x} = \mathbf{q} + \mu\mathbf{m} \tag{3}$$

The general equation of a second degree can be expressed as.

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{4}$$

Comparing (2) and (4) we get,

$$\mathbf{u} = \begin{pmatrix} -4 \\ -3 \end{pmatrix} \tag{5}$$

$$f = 7 \tag{6}$$

If **n** is the normal vector of a line, equation of the given line can be written as,

$$\mathbf{n}^T\mathbf{x} = c \tag{7}$$

Comparing (1) and (7) we get,

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{8}$$

The point of contact q, of a line with a normal vector **n** to the conic in (4) is given by,

$$\mathbf{q} = \mathbf{V}^{-1}(\kappa\mathbf{n} - \mathbf{u}) \tag{9}$$

$$\kappa = \pm\sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{\mathbf{n}^T\mathbf{V}^{-1}\mathbf{n}}} \tag{10}$$

For a circle,

$$\mathbf{V} = \mathbf{I} \tag{11}$$

where I is the Identity matrix.

Solving for κ using (10) we get,

$$\kappa = \pm 3 \tag{12}$$

$$i.e. \ \mathbf{q_1} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} \ for \ \kappa = 3 \tag{13}$$

and

$$\mathbf{q_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ for \ \kappa = -3 \tag{14}$$

To prove that the line touches the circle at **q** need to check that

$$\mathbf{m}^T(\mathbf{V}\mathbf{q} + \mathbf{u}) = 0 \tag{15}$$

We know that,

$$\mathbf{m}^T\mathbf{n} = 0 \tag{16}$$

$$\implies m = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{17}$$

Using (13), (14) and (17), we can say that (15) holds true for both $\mathbf{q_1}$ and $\mathbf{q_2}$ which means that both those points lie on the circle i.e. there will be a tangent passing through each of them which can be found out using (7)

$$i.e. \ \mathbf{n}^T\mathbf{q_1} = c_1 \tag{18}$$

$$\mathbf{n}^T\mathbf{q_2} = c_2 \tag{19}$$

Using (18) and (19) we get,

$$c_1 = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 7 \\ 6 \end{pmatrix} = 13 \qquad (20)$$

$$c_2 = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1 \qquad (21)$$

(19) is the given line (1)

Therefore, the line parallel to the given line is,

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 13 \qquad (22)$$

And the line perpendicular to the given line is the line passing through the normal vector **n** found out in (8).

The above is all represented in the form of a Fig.1.

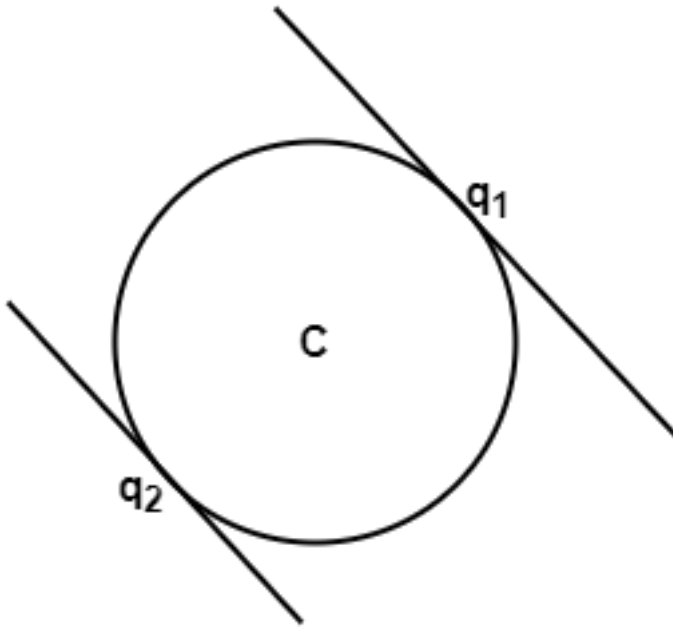**C** is the center of the circle. **q1** and **q2** are points of contact with the circle. (**Not drawn to scale**)

Fig. 1: Tangents and Normal on a Circle