Assignment 3.1

D V K M Rishab
AI20MTECH14004

Download the python code, latex file and the pdf doc from

https://github.com/Rishab9991/EE5609/tree/master/Assignments/Assignment3.1

Solution:
Consider Fig. 1

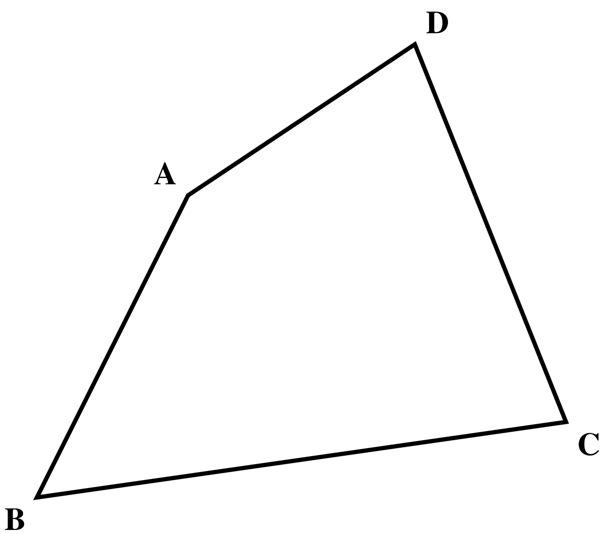

Fig. 1: Quadrilateral ABCD

AB is the smallest and CD is the largest side in the quadrilateral.

The sides AB, BC, CD and DA of the quadrilateral will be represented as direction vectors $\mathbf{m_{AB}}$, $\mathbf{m_{BC}}$, $\mathbf{m_{CD}}$ and $\mathbf{m_{DA}}$ which are obtained from Vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ which belong in the $\mathbb{R}^2$ space.

$$\mathbf{m_{AB}} = \mathbf{A} - \mathbf{B} \tag{1}$$
$$\mathbf{m_{BC}} = \mathbf{B} - \mathbf{C} \tag{2}$$
$$\mathbf{m_{CD}} = \mathbf{C} - \mathbf{D} \tag{3}$$
$$\mathbf{m_{AD}} = \mathbf{A} - \mathbf{D} \tag{4}$$

$$\cos\angle B = \frac{k}{(\|\mathbf{A} - \mathbf{B}\|)\,(\|\mathbf{B} - \mathbf{C}\|)} \tag{5}$$

$$\cos\angle D = \frac{p}{(\|\mathbf{C} - \mathbf{D}\|)\,(\|\mathbf{A} - \mathbf{D}\|)} \tag{6}$$

k,p are constants since the inner product of two vectors is always a constant. $\mathbf{A} - \mathbf{B}$, $\mathbf{B} - \mathbf{C}$, $\mathbf{C} - \mathbf{D}$ and $\mathbf{A} - \mathbf{D}$ are direction vectors (obtained using vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$) represent the sides of quadrilateral ABCD.
We know that,

$$\|\mathbf{A} - \mathbf{B}\| < \|\mathbf{B} - \mathbf{C}\| \tag{7}$$

$$\implies (\|\mathbf{A} - \mathbf{B}\|)\,(\|\mathbf{B} - \mathbf{C}\|) > \|\mathbf{A} - \mathbf{B}\|^2 \tag{8}$$

$$\implies \frac{1}{(\|\mathbf{A} - \mathbf{B}\|)\,(\|\mathbf{B} - \mathbf{C}\|)} > \frac{1}{\|\mathbf{A} - \mathbf{B}\|^2} \tag{9}$$

Similarly,

$$\frac{1}{(\|\mathbf{A} - \mathbf{D}\|)\,(\|\mathbf{C} - \mathbf{D}\|)} > \frac{1}{\|\mathbf{A} - \mathbf{D}\|^2} \tag{10}$$

$$i.e. \cos\angle B < \frac{k}{\|\mathbf{A} - \mathbf{B}\|^2} \tag{11}$$

$$\implies \cos\angle B = \frac{k}{\|\mathbf{A} - \mathbf{B}\|^2} \tag{12}$$

Since any constant less than k is also a constant.

Similarly for,

$$\cos\angle D = \frac{p}{\|\mathbf{A} - \mathbf{D}\|^2} \tag{13}$$

We know that,

$$||\mathbf{A} - \mathbf{B}|| < ||\mathbf{A} - \mathbf{D}|| \tag{14}$$

$$\frac{1}{||\mathbf{A} - \mathbf{B}||^2} < \frac{1}{||\mathbf{A} - \mathbf{D}||^2} \tag{15}$$