Assignment 2

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Download the python code, latex file and the pdf doc from

Problem 2.1

Solution:

$$A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \quad (1)$$

$$\implies A^T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \quad (2)$$

Need to prove that $A^T A = I$ (Identity Matrix)

For this we express each of the matrices as a complex number to find the product of those two matrices A^T and A.

We know that a complex number $\begin{pmatrix} a1 \\ a2 \end{pmatrix}$ can be represented as

$$\begin{pmatrix} a1 \\ a2 \end{pmatrix} = \begin{pmatrix} a1 & -a2 \\ a2 & a1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (3)$$

$$i.e \ A = \begin{pmatrix} \cos\alpha \\ -\sin\alpha \end{pmatrix} \quad (4)$$

$$A^T = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \quad (5)$$

$$\implies A^T A = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha \\ -\sin\alpha \end{pmatrix} \quad (6)$$

$$\implies A^T A = I \quad (7)$$

Using (3), the expressions in (6) and (13) were simplified to obtain I.

Problem 2.2

Solution:

$$A = \begin{pmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{pmatrix} \quad (8)$$

$$\implies A^T = \begin{pmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{pmatrix} \quad (9)$$

Need to prove that $A^T A = I$ (Identity Matrix)

For this we express each of the matrices as a complex number to find the product of those two matrices A^T and A.

Using (3), we get

$$i.e \ A = \begin{pmatrix} \sin\alpha \\ -\cos\alpha \end{pmatrix} \quad (10)$$

$$A^T = \begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix} \quad (11)$$

$$\implies A^T A = \begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix}\begin{pmatrix} \sin\alpha \\ -\cos\alpha \end{pmatrix} \quad (12)$$

$$\implies A^T A = I \quad (13)$$

Hence Proved for both **Problem 2.1** and **Problem 2.2**