

Assignment 1

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Assignment 2

Problem 2.1

Solution: Given,

$$A = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \quad A^T = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \quad (1)$$

We have to prove that $A^T A = I$ (Identity matrix)

LHS:

$$A^T A = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \quad (2)$$

$$= \begin{pmatrix} \cos^2 \alpha + \sin^2 \alpha & \cos \alpha \sin \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \cos \alpha \sin \alpha & \sin^2 \alpha + \cos^2 \alpha \end{pmatrix} \quad (3)$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (4)$$

$$= I \quad (5)$$

RHS:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (6)$$

Therefore, **LHS = RHS** i.e. $A^T A = I$

Hence Proved.

Problem 2.2

Solution: Given,

$$A = \begin{pmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{pmatrix} \quad A^T = \begin{pmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{pmatrix} \quad (7)$$

We have to prove that $A^T A = I$ (Identity matrix)

LHS:

$$A^T A = \begin{pmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{pmatrix} \begin{pmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{pmatrix} \quad (8)$$

$$= \begin{pmatrix} \sin^2 \alpha + \cos^2 \alpha & \sin \alpha \cos \alpha - \cos \alpha \sin \alpha \\ \cos \alpha \sin \alpha - \sin \alpha \cos \alpha & \cos^2 \alpha + \sin^2 \alpha \end{pmatrix} \quad (9)$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (10)$$

$$= I \quad (11)$$

RHS:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (12)$$

Therefore, **LHS = RHS** i.e. $A^T A = I$

Hence Proved.