

Assignment5 - Part1

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- $x = 0.84$
- $y = 0$

$P(\text{Observation} = \text{Red} \mid \text{State} = \text{Red})$	0.9
$P(\text{Observation} = \text{Green} \mid \text{State} = \text{Green})$	0.85

Belief State

- $b'(s') = (\propto * P(e \mid s') * \sum P(s' \mid s, a) * b(s)$

Where

- ❖ s = old state
- ❖ b = old belief state
- ❖ $b(s)$ = probability of s given belief state b
- ❖ a = action
- ❖ b' = new belief state
- ❖ $b'(s')$ = probability of s' given b'

• Step 1

Initially, the agent knows that it is in one of the red states i.e. S_1 , S_2 or S_5 . Therefore

Initial beliefs

0.3333	0.3333	0	0	0.3333
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1. Agent took the action Right and observed Red

$$\begin{aligned}
b'(s_1) &= \infty * P(\text{Red} | \text{Red}) * [P(s_1 | s_1, \text{Right}) * b(s_1) + P(s_1 | s_2, \text{Right}) * b(s_2) + P(s_1 | s_3, \text{Right}) * b(s_3) + P(s_1 | s_4, \text{Right}) * b(s_4) + P(s_1 | s_5, \text{Right}) * b(s_5)] \\
&= \infty * 0.9 * [(1 - x) * \frac{1}{3} + (1 - x) * \frac{1}{3} + 0 + 0 + 0] \\
&= \infty * 0.096
\end{aligned}$$

$$\begin{aligned}
b'(s_2) &= \infty * P(\text{Red} | \text{Red}) * [P(s_2 | s_1, \text{Right}) * b(s_1) + P(s_2 | s_2, \text{Right}) * b(s_2) + P(s_2 | s_3, \text{Right}) * b(s_3) + P(s_2 | s_4, \text{Right}) * b(s_4) + P(s_2 | s_5, \text{Right}) * b(s_5)] \\
&= \infty * 0.9 * [x * \frac{1}{3} + 0 + (1 - x) * 0 + 0 + 0] \\
&= \infty * 0.252
\end{aligned}$$

$$\begin{aligned}
b'(s_3) &= \infty * P(\text{Red} | \text{Green}) * [P(s_3 | s_1, \text{Right}) * b(s_1) + P(s_3 | s_2, \text{Right}) * b(s_2) + P(s_3 | s_3, \text{Right}) * b(s_3) + P(s_3 | s_4, \text{Right}) * b(s_4) + P(s_3 | s_5, \text{Right}) * b(s_5)] \\
&= \infty * 0.15 * [0 + x * \frac{1}{3} + 0 + (1 - x) * 0 + 0] \\
&= \infty * 0.042
\end{aligned}$$

$$\begin{aligned}
b'(s_4) &= \infty * P(\text{Red} | \text{Green}) * [P(s_4 | s_1, \text{Right}) * b(s_1) + P(s_4 | s_2, \text{Right}) * b(s_2) + P(s_4 | s_3, \text{Right}) * b(s_3) + P(s_4 | s_4, \text{Right}) * b(s_4) + P(s_4 | s_5, \text{Right}) * b(s_5)] \\
&= \infty * 0.15 * [0 + 0 + x * 0 + 0 + (1 - x) * \frac{1}{3}] \\
&= \infty * 0.008
\end{aligned}$$

$$\begin{aligned}
b'(s_5) &= \infty * P(\text{Red} | \text{Red}) * [P(s_5 | s_1, \text{Right}) * b(s_1) + P(s_5 | s_2, \text{Right}) * b(s_2) + P(s_5 | s_3, \text{Right}) * b(s_3) + P(s_5 | s_4, \text{Right}) * b(s_4) + P(s_5 | s_5, \text{Right}) * b(s_5)] \\
&= \infty * 0.9 * [0 + 0 + 0 + x * 0 + x * \frac{1}{3}] \\
&= \infty * 0.252
\end{aligned}$$

Now we will use :

$$\sum b(s) = 1$$

$$\infty * 0.096 + \infty * 0.252 + \infty * 0.042 + \infty * 0.008 + \infty * 0.252 = 1$$

$$\infty * 0.65 = 1$$

$$\infty = 1.538$$

Beliefs after Step 1

0.1477	0.3877	0.0646	0.0123	0.3877
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2. Agent took the action Left and observed Green

$$\mathbf{b'(s_1)} = \infty * P(\text{Green} | \text{Red}) * [P(s_1 | s_1, \text{Left}) * b(s_1) + P(s_1 | s_2, \text{Left}) * b(s_2) + P(s_1 | s_3, \text{Left}) * b(s_3) + P(s_1 | s_4, \text{Left}) * b(s_4) + P(s_1 | s_5, \text{Left}) * b(s_5)]$$

$$= \infty * 0.1 * [x * 0.1477 + x * 0.3877 + 0 + 0 + 0]$$

$$= \infty * 0.0449736$$

$$\mathbf{b'(s_2)} = \infty * P(\text{Green} | \text{Red}) * [P(s_2 | s_1, \text{Left}) * b(s_1) + P(s_2 | s_2, \text{Left}) * b(s_2) + P(s_2 | s_3, \text{Left}) * b(s_3) + P(s_2 | s_4, \text{Left}) * b(s_4) + P(s_2 | s_5, \text{Left}) * b(s_5)]$$

$$= \infty * 0.1 * [(1 - x) * 0.1477 + 0 + x * 0.0646 + 0 + 0]$$

$$= \infty * 0.0077896$$

$$\mathbf{b'(s_3)} = \infty * P(\text{Green} | \text{Green}) * [P(s_3 | s_1, \text{Left}) * b(s_1) + P(s_3 | s_2, \text{Left}) * b(s_2) + P(s_3 | s_3, \text{Left}) * b(s_3) + P(s_3 | s_4, \text{Left}) * b(s_4) + P(s_3 | s_5, \text{Left}) * b(s_5)]$$

$$= \infty * 0.85 * [0 + (1 - x) * 0.3877 + 0 + x * 0.0123 + 0]$$

$$= \infty * 0.0615094$$

$$\mathbf{b'(s_4)} = \infty * P(\text{Green} | \text{Green}) * [P(s_4 | s_1, \text{Left}) * b(s_1) + P(s_4 | s_2, \text{Left}) * b(s_2) + P(s_4 | s_3, \text{Left}) * b(s_3) + P(s_4 | s_4, \text{Left}) * b(s_4) + P(s_4 | s_5, \text{Left}) * b(s_5)]$$

$$= \infty * 0.85 * [0 + 0 + (1 - x) * 0.0646 + 0 + x * 0.3877]$$

$$= \infty * 0.2856$$

$$\mathbf{b'(s_5)} = \infty * P(\text{Green} | \text{Red}) * [P(s_5 | s_1, \text{Left}) * b(s_1) + P(s_5 | s_2, \text{Left}) * b(s_2) + P(s_5 | s_3, \text{Left}) * b(s_3) + P(s_5 | s_4, \text{Left}) * b(s_4) + P(s_5 | s_5, \text{Left}) * b(s_5)]$$

$$= \infty * 0.1 * [0 + 0 + 0 + (1 - x) * 0.0123 + (1 - x) * 0.3877]$$

$$= \infty * 0.0064$$

Now we will use :

$$\sum b(s) = 1$$

$$\infty * 0.0449736 + \infty * 0.0077896 + \infty * 0.0615094 + \infty * 0.2856 + \infty * 0.0064 = \infty * 0.406171 = 1$$

$$\mathbf{\infty = 2.461}$$

Beliefs after Step 2

0.1107	0.0192	0.1514	0.7030	0.0158
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3. Agent took the action Left and observed Green

$$\begin{aligned}
b'(s_1) &= \infty * P(\text{Green} | \text{Red}) * [P(s_1 | s_1, \text{Left}) * b(s_1) + P(s_1 | s_2, \text{Left}) * b(s_2) + P(s_1 | s_3, \text{Left}) * b(s_3) + P(s_1 | s_4, \text{Left}) * b(s_4) + P(s_1 | s_5, \text{Left}) * b(s_5)] \\
&= \infty * 0.1 * [x * 0.1107 + x * 0.0192 + 0 + 0 + 0] \\
&= \infty * 0.0109116
\end{aligned}$$

$$\begin{aligned}
b'(s_2) &= \infty * P(\text{Green} | \text{Red}) * [P(s_2 | s_1, \text{Left}) * b(s_1) + P(s_2 | s_2, \text{Left}) * b(s_2) + P(s_2 | s_3, \text{Left}) * b(s_3) + P(s_2 | s_4, \text{Left}) * b(s_4) + P(s_2 | s_5, \text{Left}) * b(s_5)] \\
&= \infty * 0.1 * [(1 - x) * 0.1107 + 0 + x * 0.1514 + 0 + 0] \\
&= \infty * 0.0144888
\end{aligned}$$

$$\begin{aligned}
b'(s_3) &= \infty * P(\text{Green} | \text{Green}) * [P(s_3 | s_1, \text{Left}) * b(s_1) + P(s_3 | s_2, \text{Left}) * b(s_2) + P(s_3 | s_3, \text{Left}) * b(s_3) + P(s_3 | s_4, \text{Left}) * b(s_4) + P(s_3 | s_5, \text{Left}) * b(s_5)] \\
&= \infty * 0.85 * [0 + (1 - x) * 0.0192 + 0 + x * 0.7030 + 0] \\
&= \infty * 0.5045532
\end{aligned}$$

$$\begin{aligned}
b'(s_4) &= \infty * P(\text{Green} | \text{Green}) * [P(s_4 | s_1, \text{Left}) * b(s_1) + P(s_4 | s_2, \text{Left}) * b(s_2) + P(s_4 | s_3, \text{Left}) * b(s_3) + P(s_4 | s_4, \text{Left}) * b(s_4) + P(s_4 | s_5, \text{Left}) * b(s_5)] \\
&= \infty * 0.85 * [0 + 0 + (1 - x) * 0.1514 + 0 + x * 0.0158] \\
&= \infty * 0.0318716
\end{aligned}$$

$$\begin{aligned}
b'(s_5) &= \infty * P(\text{Green} | \text{Red}) * [P(s_5 | s_1, \text{Left}) * b(s_1) + P(s_5 | s_2, \text{Left}) * b(s_2) + P(s_5 | s_3, \text{Left}) * b(s_3) + P(s_5 | s_4, \text{Left}) * b(s_4) + P(s_5 | s_5, \text{Left}) * b(s_5)] \\
&= \infty * 0.1 * [0 + 0 + 0 + (1 - x) * 0.7030 + (1 - x) * 0.0158] \\
&= \infty * 0.0115008
\end{aligned}$$

Now we will use :

$$\sum b(s) = 1$$

$$\alpha * 0.0109116 + \alpha * 0.0144888 + \alpha * 0.5045532 + \alpha * 0.0318716 + \alpha * 0.0115008 = \alpha * 0.573326 = 1$$

$$\alpha = 1.744$$

Beliefs after Step 3

0.0190	0.0253	0.8800	0.0556	0.0201
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