A PROJECT REPORT ON

VIBRATIONAL ANALYSIS OF CRACKED CATILEVER BEAM AND ITS VARIATION

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UNDER THE SUPERVISION OF

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LUCKNOW (U.P) INDIA

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CERTIFICATE

This is to certify that **Abhishek Singh (16502), AmanVerma (16504), Rishabh Kesarwani(16538) & Vijay Singh (16559)** have carried out the project work in this report entitled "**Vibrational Analysis of Cracked Cantilever Beam and Its Variation"** for the award of **Bachelor of Technology** in **Mechanical Engineering** at **Kamla Nehru Institute of Technology, Sultanpur** affiliated to **Dr. A. P. J. Abdul Kalam Technical University, Lucknow.**

This report is the record of the candidates' own work carried out by them under our supervision and guidance **Dr Amit Medhavi** .This project work is the part of their **Bachelor of Technology** in **Mechanical Engineering** curriculum.

Their performance was excellent and we wish them good luck for their future endeavors.

Dr. Amit Medhavi
(Project Guide)

(Prof.) S P Kutar
(In Charge Head of Department)

DECLARATION

We hereby declare that this submission is our own work and that, to the rest of our knowledge and belief, it contains no material previously or written by another person nor material which to a substantial extent has been accepted for the award of any other degree of the university or other institute of higher learning, except where due acknowledgement has been made in text.

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ABSTRACT

In these days of technology, we have new problem of cracked structures due to vibrations and other factors. In the field of analyzing the vibrating structure having crack, we did some progress but there is a lot future scope in this analysis. Due to vibration a lot of problem comes in the structure. Even some cracked structure going through vibration is big problem because its natural frequency is changed from the uncracked structure. We may get a lot of problems due to lack of analysis of this type of structure. Failure in the structures, reduced life cycle, excessive noise and severe financial loss due failure in structures are some losses which may occur due to crack in vibrating structures.

Most of the researchers have studied the effects of crack location, depth of crack on different vibrating properties like natural frequency, damping factor and resonance amplitude. It is very important to understand the dynamics of cracked cantilever beam because the change in natural frequency is generally used as that a crack is present in the beam. The change in natural frequency can show that how much damage is done. The results are compared with the previous researchers' results and verified also. ANSYS is used for obtaining the first three natural frequencies of different modes of un-crack beam and cracked beam.

In this project, we have taken the cantilever beam having crack. We have done vibrational analysis on Ansys of this beam in many conditions which is discussed in this project. Some of the results we obtained are that the natural frequencies of identical beam having top crack and bottom crack are almost same. When the depth of crack is increasing having fixed crack location, the natural frequencies of cracked beam decreases. We have found out that natural frequency found from the experiment of Shifferen and Routolo[1] and using Ansys are nearly same. Some application of this project are in mechanical systems like long shafts, turbine blades, aero plane blades, fan blades, in cranes etc.

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1. <u>Introduction</u>

Firstly we have studied about vibration in intact beam a lot but vibration in cracked beam is studied under this report. Since last two decades, problem of crack detection and crack effects in the structure has gained an important attention. A lot of numerical, analytical and theoretical studies on the crack has been done. Most of the researchers have studied the effects of crack location, depth of crack on different vibrating properties like natural frequency, damping factor and resonance amplitude. It is very important to understand the dynamics of cracked cantilever beam because the change in natural frequency is generally used as that a crack is present in the beam. The change in natural frequency can show that how much damage is done. The results are compared with the previous researchers' results and verified also. ANSYS is used for obtaining the first three natural frequencies of different modes of un-crack beam and cracked beam. Most of the structures and mechanical systems are failed due to the excessive vibrations in their working life. The vibrations having certain amplitude and certain natural frequency for which the material is designed is good for that beam. But excessive vibrations can have a drastic effects on beam performance or damage the structure. Since most human activities involves vibration in one form or other, so the study of vibration become important practically. To solve for beams euler bernoulli's beam is assumed.

The assumptions of classical theory of euler bernoulli's beam are the following-

- 1. Cross section plane perpendicular to the axis of beam remains plane after deformation.
- 2. The deformed cross section plane is still perpendicular to the axis after deformation.
- 3. Transverse Shear deformation is neglected where the transverse shear is determined by the equation of equilibrium.
- 4. Shear rotation effects are neglected.

1.1 Brief about vibration

With the discovery of musical instrument people started to notice about vibration. Many people applied some theories in vibration, sir Galileo contributed a lot in the field of vibration, he started the experiment of free vibration in simple pendulum. Then many people worked in this field and made contribution like Euler (Euler's beam theory), Newton (law of motion), Fourier (sine wave analysis) etc.

Any motion which repeats itself after certain period of time is called as vibration. The swing of pendulum is an example of vibration. Theory of vibration deals with the study of oscillatory motions of body and forces associated with them. Vibration can also be caused due to the external factors also like external unbalance force. A vibratory system consist of an elastic body like spring for storing potential energy, mass or inertia body for storing kinetic energy and damper for gradual loss of energy to take place. Like pendulum, suspension system, simply supported beam, cantilever beam, lateral vibrating string, vibration due to unbalance reciprocating or rotating force are example of vibrating system.

Vibration with certain amplitude and frequency may be reliable but excessive vibration causes the following

- Reduces the working life of component.
- Excessive noise and resonance.
- Loss in power transmission in case of gears.
- Failure in structure of machine components.
- Increase in bearing clearances.
- Severe effects on financial growth.

2.History and Literature

For vibrational analysis of cracked cantilever beam Shifrin and Rutolo proposed a new method of enumerating the natural frequencies in cracked cantilever beam with arbitrary number of transverse open cracks [1]. Compared to substitute methods continuous beam model was used so that less time to solve for natural frequencies of beam due to reduced dimension of matrix. Cracks are considered as massless rotational spring. Shen and Pierre (1986) proposed a finite element approach to predict the change in first few eigen frequencies, eigen modes due to presence of crack.[2] The experimental observations of effects of crack on first two mode of vibrating beams for both fixed-free and hinged-hinged boundary conditions are conducted by Owolabi.[3] The frequency response function (FRF) amplitude and changes in natural frequencies obtained from measurements of dynamic response of cracked beams as a function of crack depth and crack location are used for detection of crack. Zehang and kessissoglou obtained the natural frequencies and mode shapes of cracked beam using finite element method [4] (FEM). The total flexibility matrix is established by adding overall additional flexibility matrix to the flexibility matrix of corresponding intact beam. The results when compared with analytical results show more accuracy than when the local additional flexibility matrix was used in place of overall additional stiffness matrix. The local additional flexibility matrix is obtained from linear elastic fracture mechanics theory. Sadettin conducted number of experiments on edge cracked cantilever beam to see the effects of cracks on different vibration parameter [5]. Lee (2009) presented a simple method to recognize multiple cracks in a beam using finite element method [6] (FEM).

Rizos P. S., Aspragathos T., and Dimarogonas, A. D. (1989). Measurement of flexural vibrations of a cantilever beam with rectangular cross-section having a transverse surface crack extending uniformly along the width of the beam and analytical results are used to relate the measured vibration modes to the crack location and depth. From the measured amplitudes at two points of the structure vibrating at one of its natural modes, the respective vibration frequency and an analytical solution of the dynamic response, the crack location are found and depth is estimated.[7].Young-Shin., Myung-Jee., and Chung. (1999), In this paper, a simple and easy nondestructive evaluation procedure was presented for identifying a crack, the location and size of the crack, in a one-dimensional beam-type structure using the natural frequency data. F.E.M. model is adopted and the crack size was determined by F.E.M. Finally, the actual crack location can be identified by Gudmundson's equation using the determined crack size and the aforementioned natural frequencies. [8]

Kisa M., Brandon J., and Topcu M. (1998), In the present paper vibrational characteristics for a cracked Timoshenko beam were analyzed. Each substructure was modeled by Thimoshenko beam finite elements with 2 nodes and 3-degree of freedom (axial, transverse, rotation) at each node [9].Chondros T. G., Dimarogonas A. D., and Yao J. (2001), considered by a continuous cracked beam vibration theory for the prediction of changes in transverse vibration of a simply supported beam with a breathing crack. The equation of motion and the boundary conditions of the cracked beam considered as a one-dimensional continuum were used. [10]

Zheng D. Y., and Kessissoglou N. J. K. (2004), found the frequencies occurred naturally and mode pattern of a fractured beam by FEM. An overall additional Flexibility matrix, instead of the local additional Flexibility matrix is mixed-up to the present intact beam element to obtain the total flexibility matrix and also the stiffness matrix. In this paper a pattern function was constructed that will precisely answer the local flexibility positions of the location of the crack, which can tell more precisely vibration pattern[11] Kisa M., and Arif G. (2006), proposed a numerical model that combines the finite element and component mode synthesis methods for the modal analysis of beams with circular cross section and containing multiple non-propagating open cracks. Three numerical illustrations are explained to examine the effects and depth of crack on the frequency occurred naturally and beams with mode patterns. And examination of proposed model obtained modal data tells us regarding the data of the position and size of defect in the beams. [12]

3.Introduction to Crack

A crack in a structural member introduces local flexibility that would affect the vibration response of system. This property may be used to detect existence of a crack together its location and depth in the structure member. Under external load, crack changes the dynamic responses of crack.

3.1 Crack Classification

Based on geometry, cracks can be classified as following-

- **Transverse Crack:** These cracks are perpendicular to the beam axis. They are most common and most serious as they reduces the cross section as by weaken the beam.
- **Longitudinal Crack:** These cracks are parallel to the axis of beam. They are not most common but they pose danger to system when tensile load is applied at some angle.
- **Open Cracks:** These cracks always remain open. Practically they are called "notches". Most experiment work is done on these type of crack.
- **Breathing Cracks:** Cracks that open when affected part of the material is subjected to tensile stresses and close when the stress is reversed are known as "breathing cracks".
- **Slant Cracks:** These are cracks that are at an angle to beam axis, but are not very common.
- **Surface Cracks:** These are cracks that are open on surface. They can be detected by visual inspection.
- **Subsurface Cracks:** These are the cracks which do not open on surface. Special techniques such as ultrasonic, radiography are needed to detect them.

4.Introduction to Beam

Beam is generally a member subjected to transverse gravity or loading. There are many types of beams classified according to size, manner in which they are supported and their location in any structural system.

The beam may be classified as following-

- Simply Span: Both ends are supported and it is free to rotate.
- Continuous Beam: Beam is supported at more than two points.
- Cantilever Beam: Beam with one end fixed or supported and other end free.
- Fixed Beam: Beam is a type of Beam in which both the ends are fixed and they resist the rotation of beam.
- Overhanging Beam; Beam which is supported by more than two supports

There are other several types of beams such as reinforced beam, timber beam, steel beam, rectangular beam, t section beam, curved beam, tapered beam, straight beam which are of not our concern.

Our analysis will be concentrated only on the Cantilever type of beam. Cantilever beam has several uses such as bridges, towers, traffic light post, large warehouse, balconies of our house, bill board at the roadside of road and highway. If we talk about mechanical system then cantilever beam is used in the long shafts, turbine blades, aero plane blades, fan blades, in cranes etc.

5. Design and modelling of cantilever beam in Ansys Workbench

5.1 Dimensions and Parameter of the cantilever Beam

Design is the first basic step to be completed before we can start the analysis of the beam.

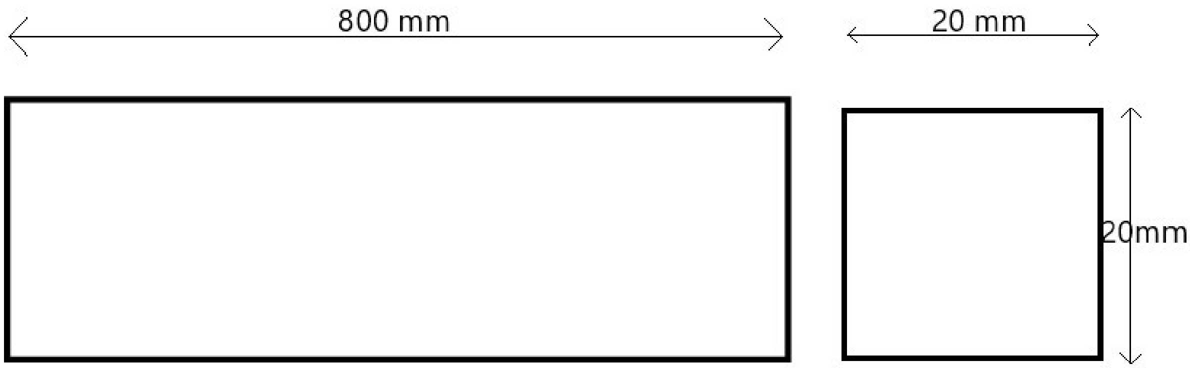

Fig 1: Basic 2D design of cantilever beam

Length-800mm

Modulus of Elasticity- 200 * 10^{11} N/m^2

Poisson's Ratio- .3

Cross sectional Area – 20 mm* 20mm

Density of Material – 7850 Kg / m^3

5.2 Steps Involved in making of Beam in Ansys Workbench

5.2.1 Making of the 3D model in Ansys

Step 1- Select static structural option opening the Ansys app and select Modal option

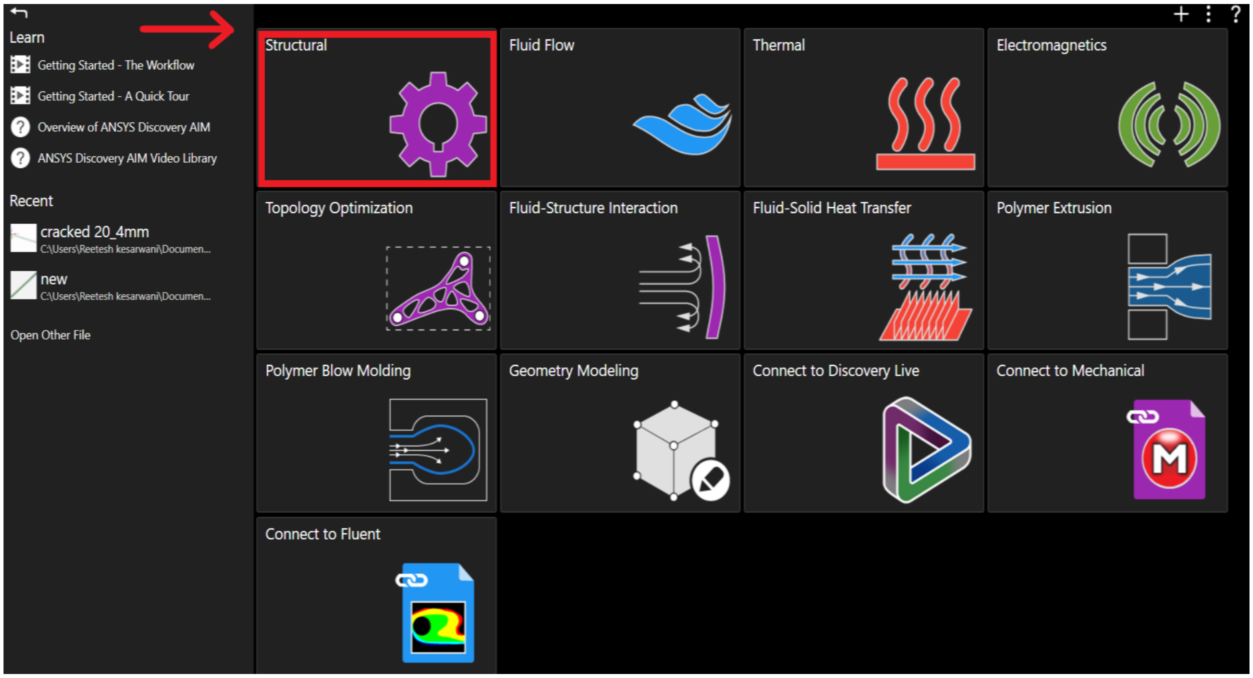

Fig 2: Selecting the Structural in Ansys 19.2

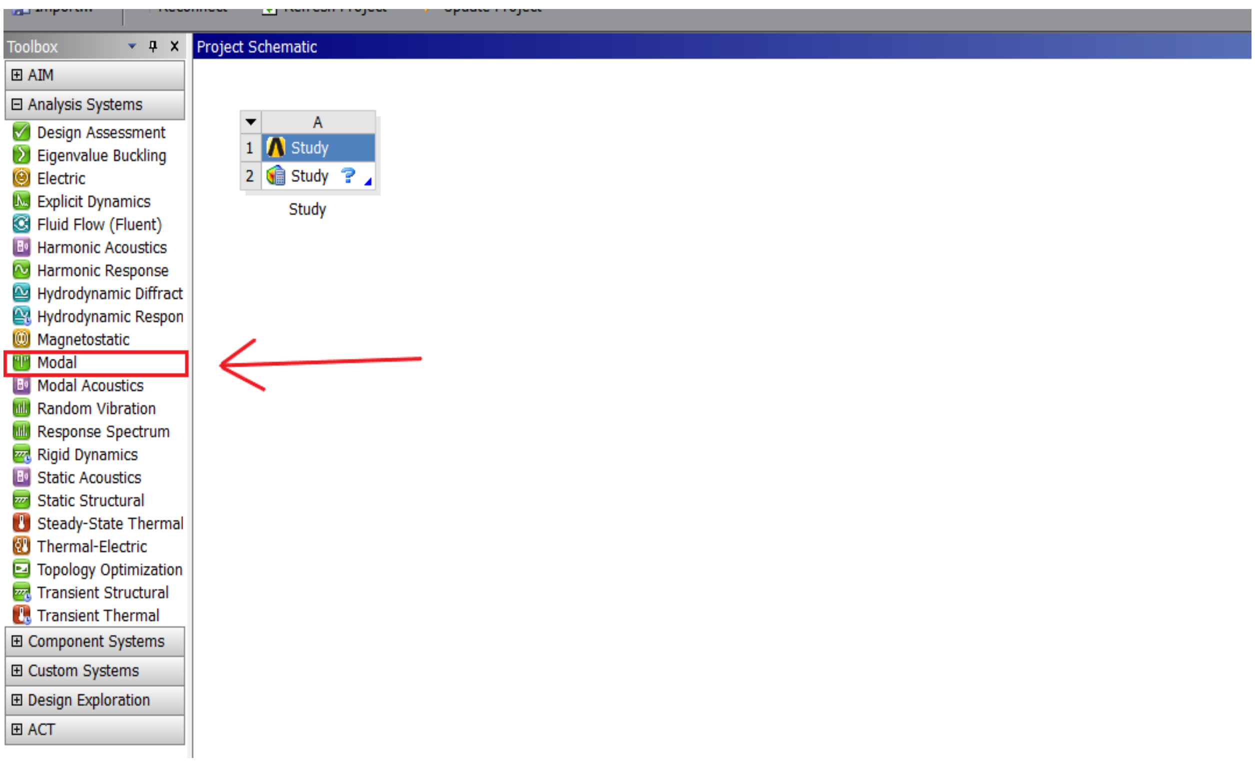

Fig 3: Selecting the Modal option after selecting the Structural option in Ansys

Step2- Fill the engineering data in the option present

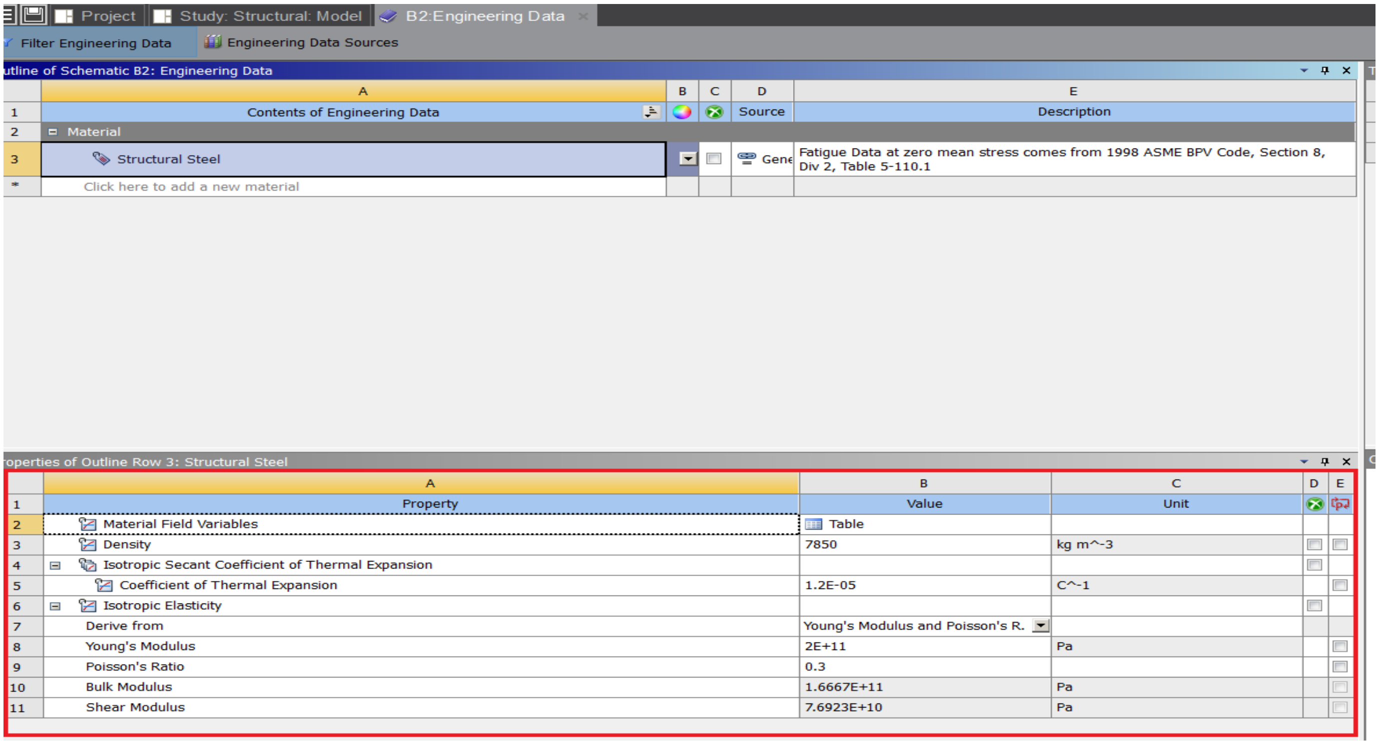

Fig 4: Filling the required engineering data for the material of beam

Step 3- Select the geometry option and then select the space claim option

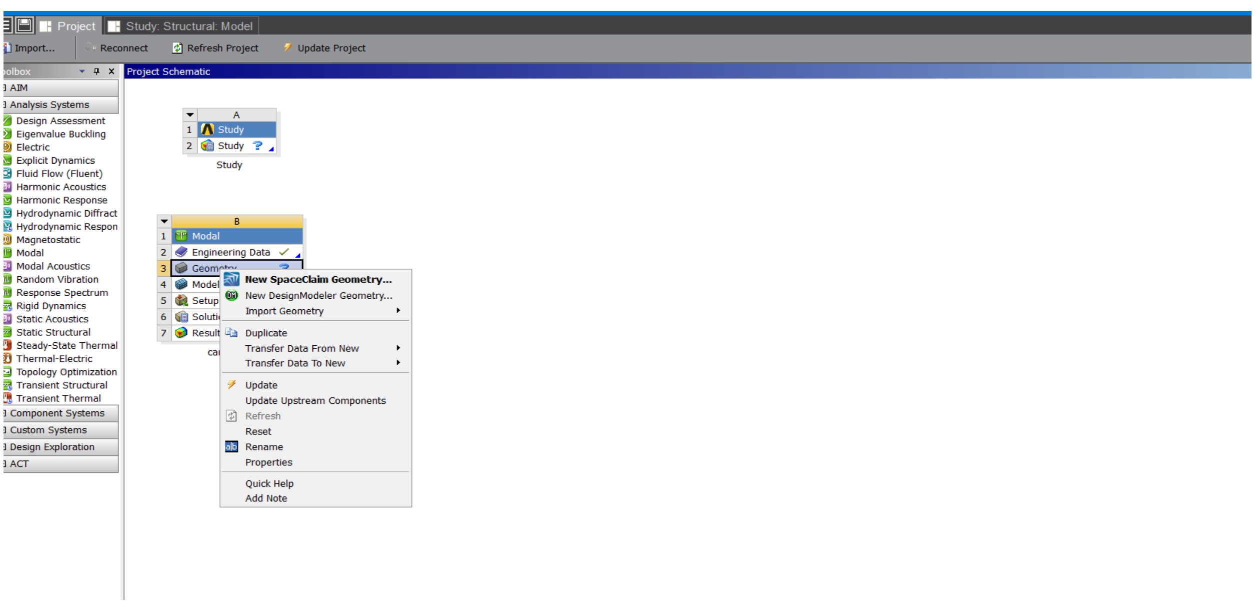

Fig 5: Selecting the SpaceClaim option to design the required beam

Step 4 –Make the 2D model in the space claim and then pull the model while to make its 3D model.

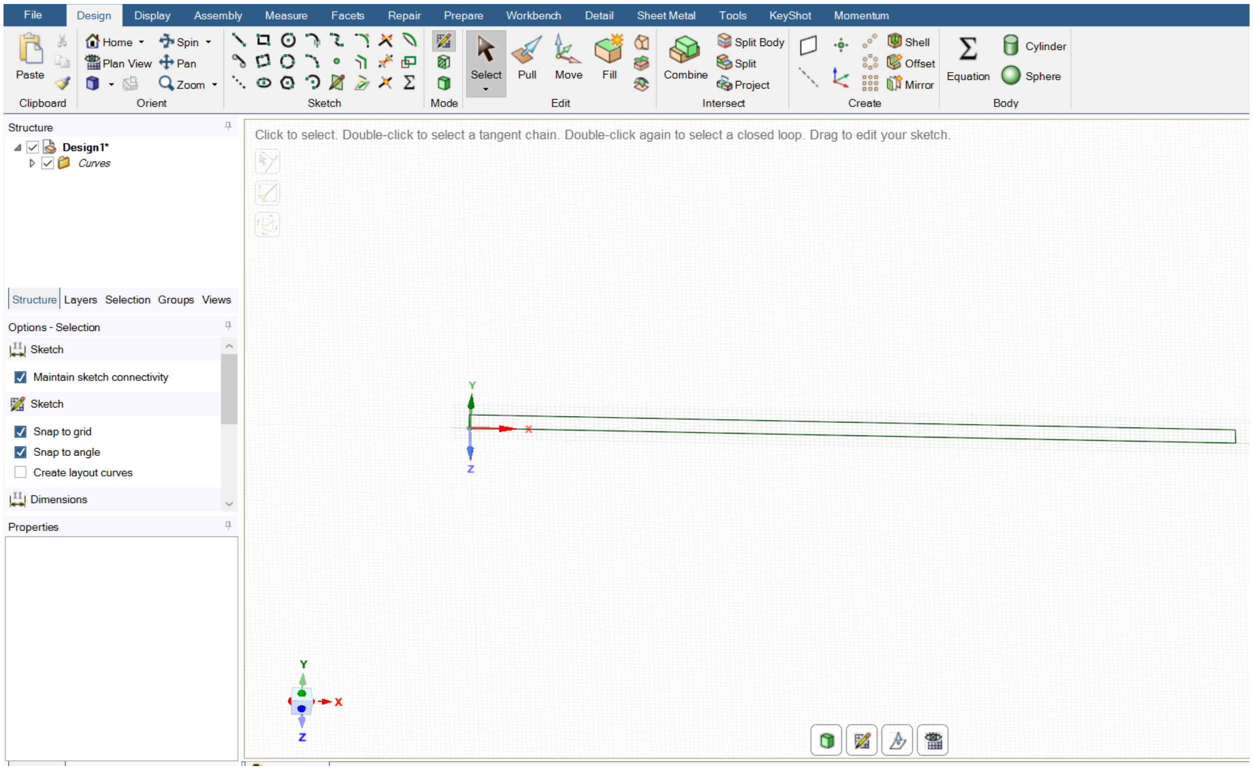

Fig 6: Making of the 2D design of the required beam

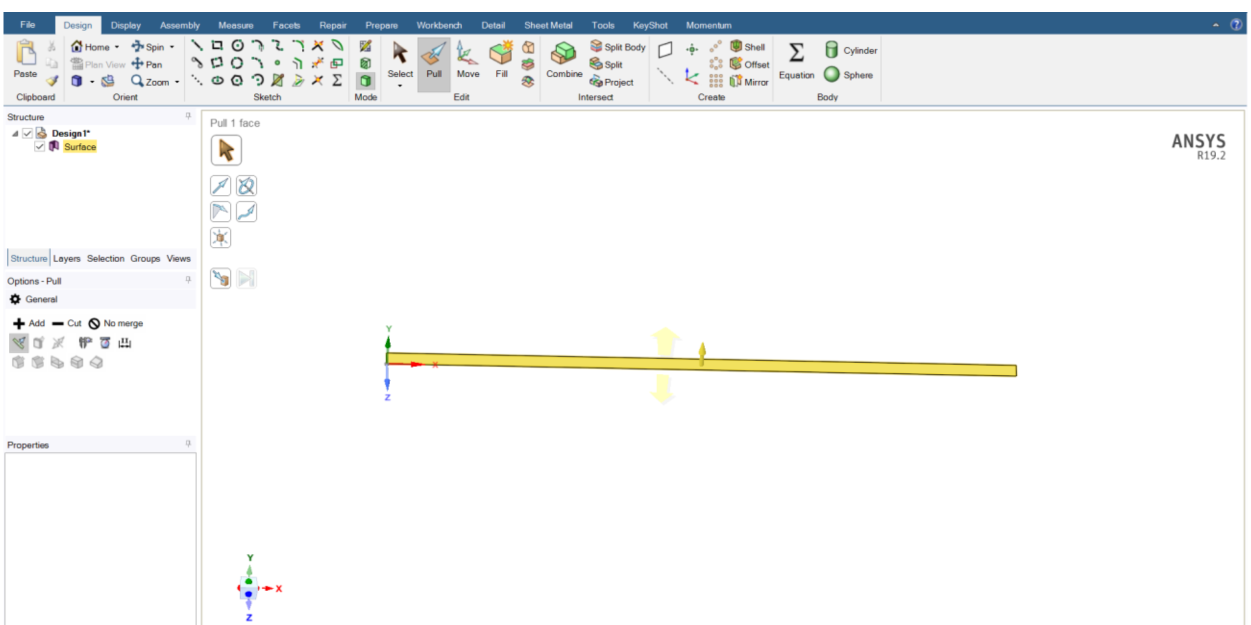

Fig 7: Getting ready to pull the 2D design to make 3D beam

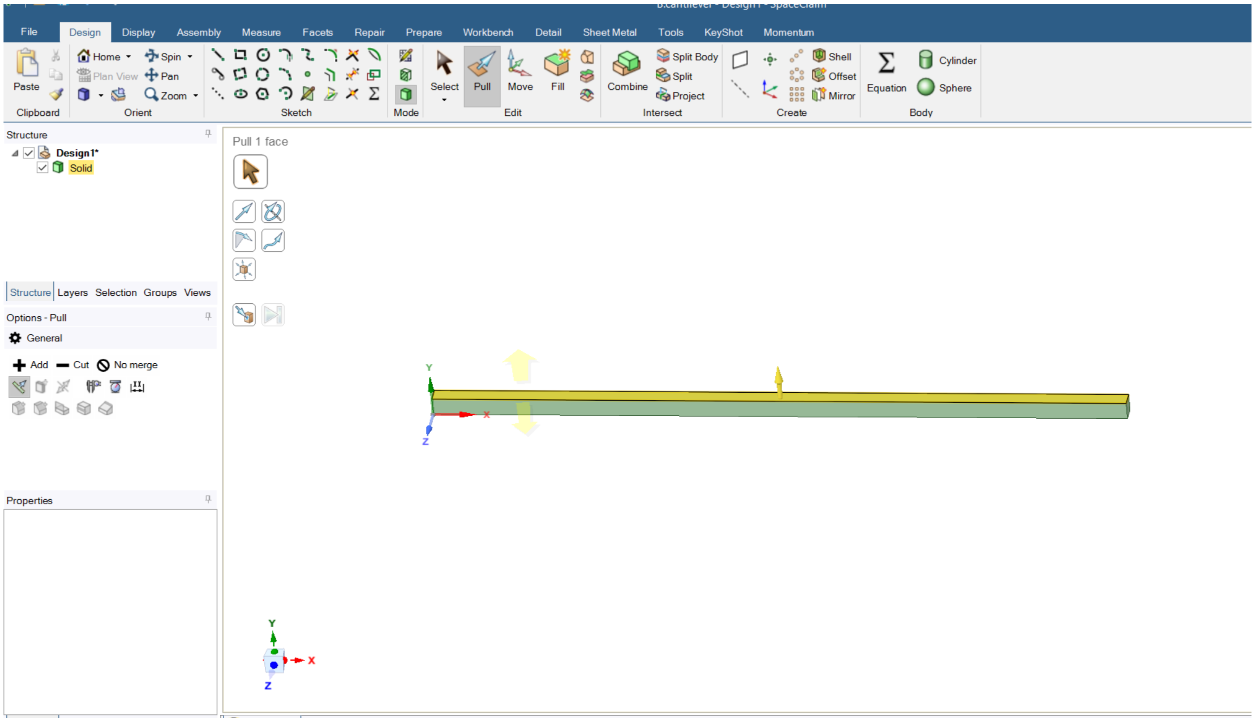

Fig 8: Finally the 3D beam is ready in the SpaceClaim

5.2.2 Meshing of the 3D Model in Ansys

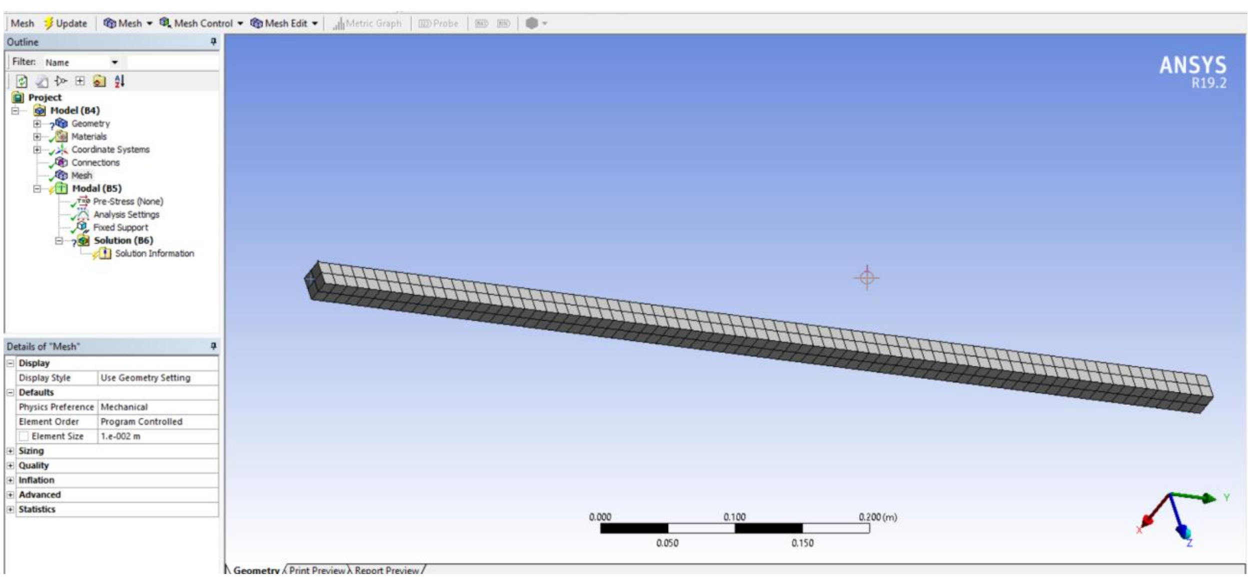

Fig 9: Meshing of the 3D beam

5.2.3 Fixing the one end of beam to make the beam as cantilever

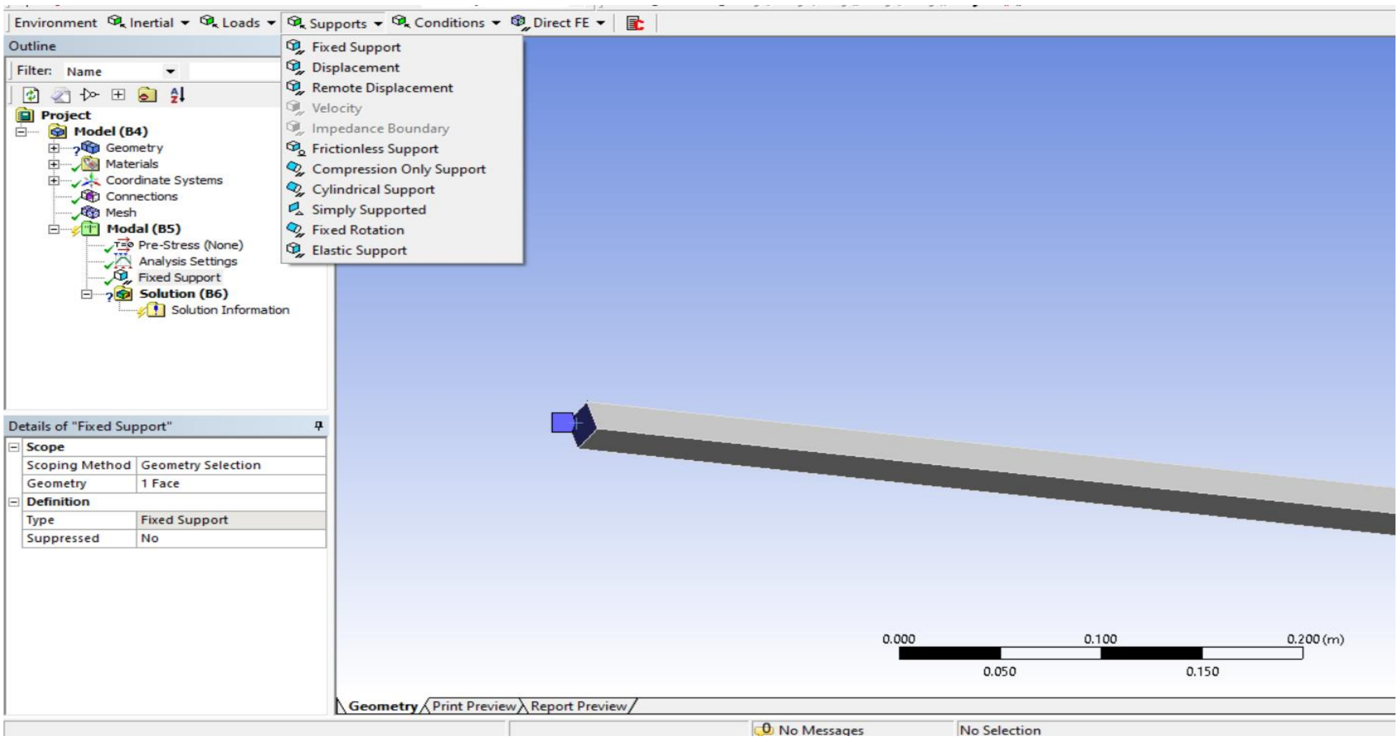

Fig 10: Fixing the one end of beam to make the given 3D beam Cantilever

5.2.4 Natural frequency of uncracked cantilever beam using the modal analysis present in the Ansys

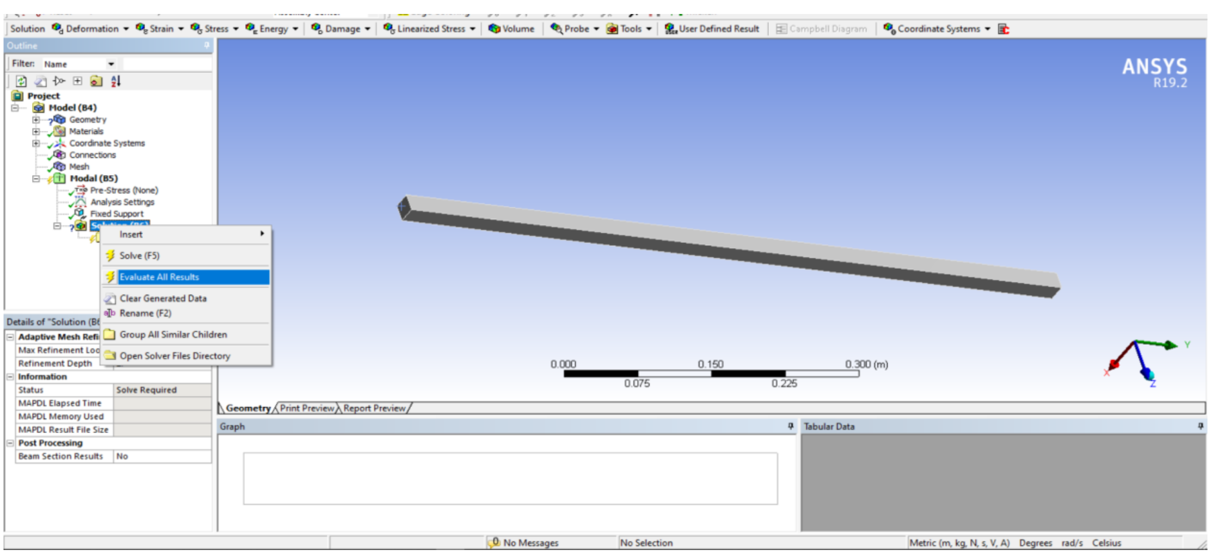

Fig 11: Select the Solve option to evaluate the natural frequency

5.3 Shifferen Beam dimensions & parameter of the cantilever beam

Length-800mm

Beam Material – Stainless Steel

Cross sectional area = 20mm * 20mm

Density- 7850 Kg / m^3

Modulus of Elasticity- 200 * 10^{11} N/m^2

Poisson's Ratio- .3

5.4 Natural frequency found by Shifferen using the experiment

Table 1- The data above mentioned have been taken from the research paper published by Shifferen and Routolo R [1]

Frequency Number	Value
1st Frequency (f1)	25.0954 Hz
2nd Frequency (f2)	163.3221 Hz
3rd Frequency (f3)	459.6011 Hz

5.5 Comparing with the Natural frequency found experimentally with the results found from Ansys

The natural frequency found using the Ansys software are as follows-

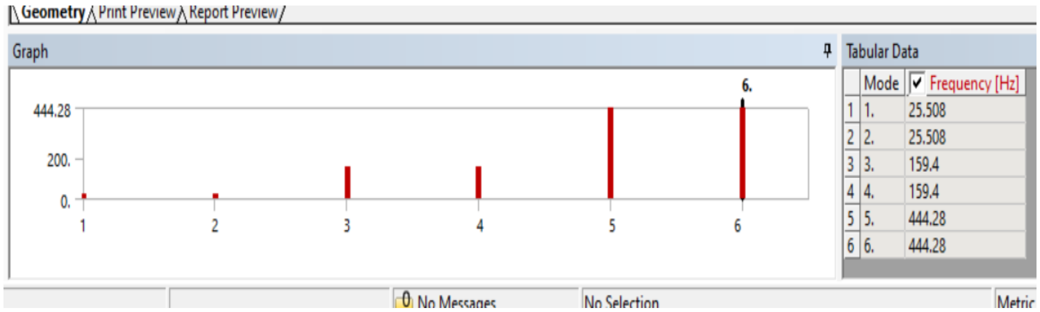

Fig 12: Natural frequency of beam using Ansys Software

Table 2- Natural frequency obtained from the Ansys software

Frequency Number	Value
1^{st} Frequency (f1)	25.508 Hz
2^{nd} Frequency (f2)	159.4 Hz
3^{rd} Frequency (f3)	444.28 Hz

We have found out that natural frequency found from the experiment and using Ansys are nearly same. These values have a very slight difference which may be due to the end conditions applied.

6. Results And Discussion

6.1 Natural frequency of top cracked cantilever when depth is constant and location varies

Length-700mm

Beam Material – Stainless Steel

Cross sectional area = 20mm * 20mm

Density- 7850 Kg / m^3

Modulus of Elasticity- 200 * 10^{11} N/m^2

Poisson's Ratio- .3

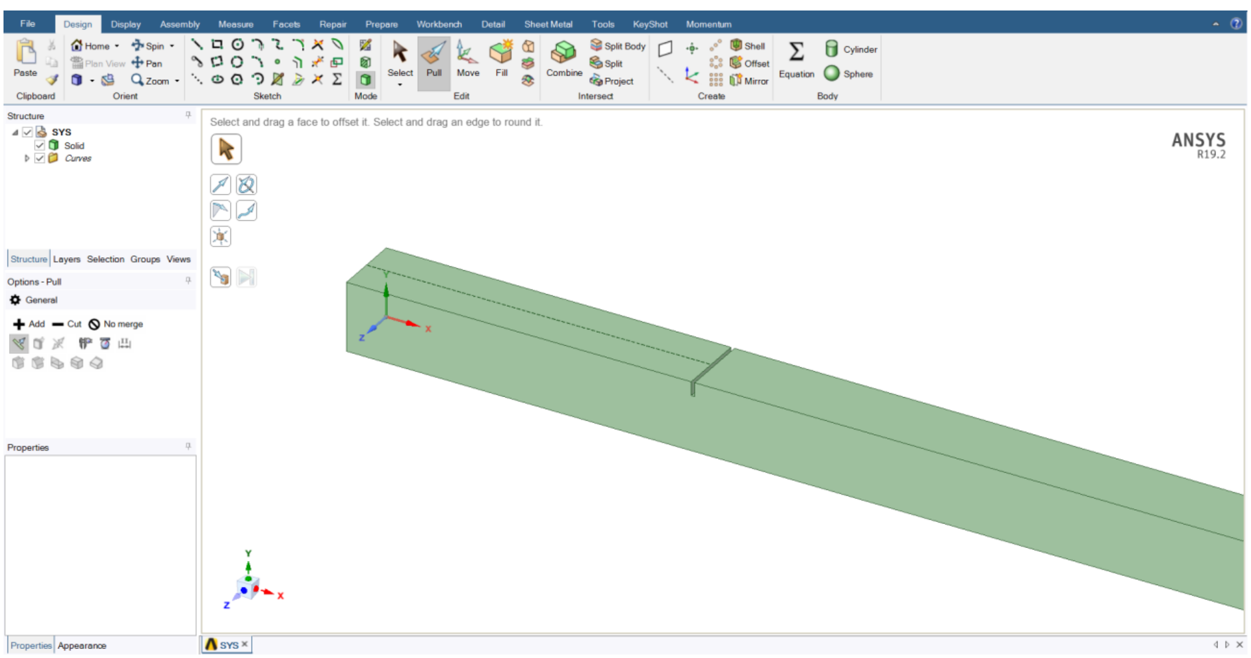

Fig 13: Beam having the top crack

Table 3- Natural frequency of top cracked cantilever beam

Node	4mm/100mm
1	32.83
2	207.54
3	578.98

6.2 Natural frequency of bottom cracked cantilever beam when depth is constant and location varies.

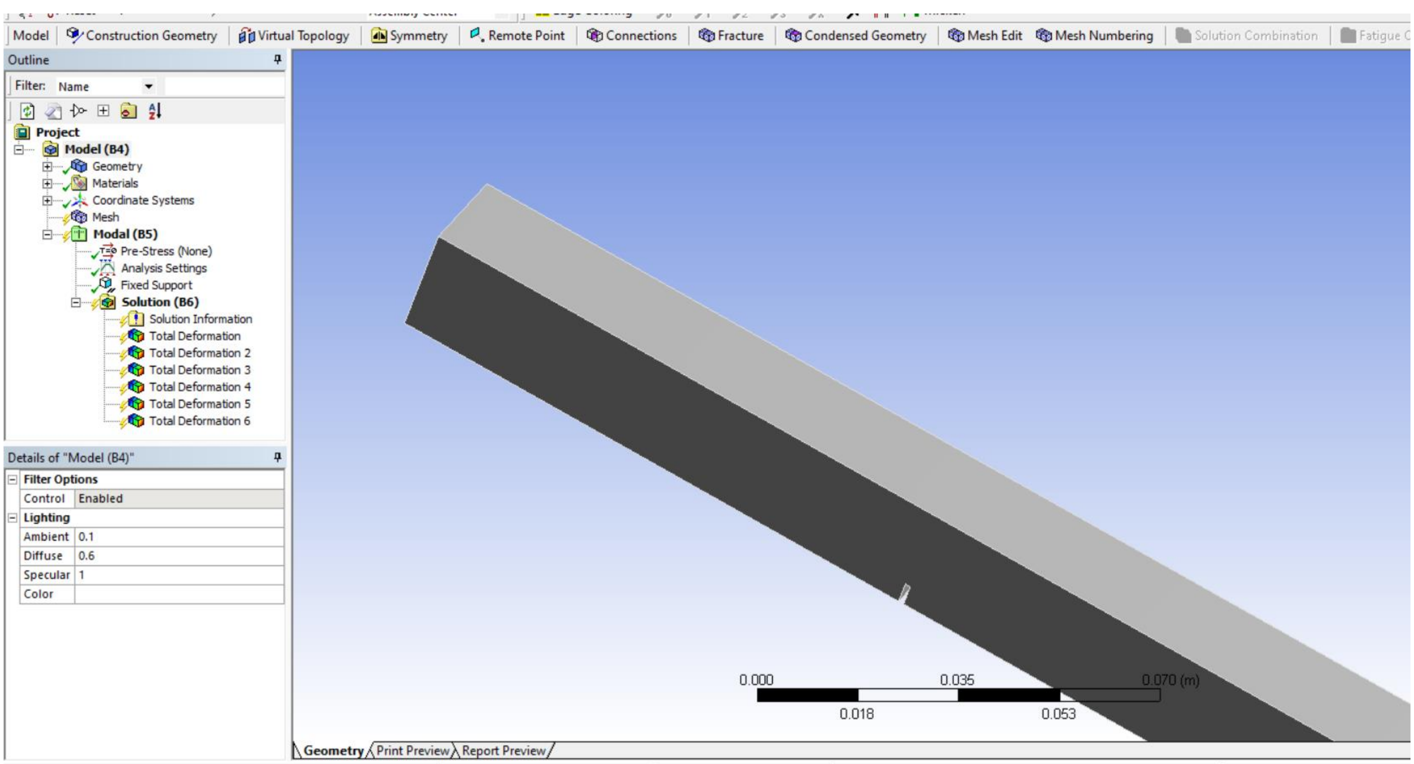

Fig 14: Beam having the bottom Crack

Table 4- Natural Frequency of Bottom cracked beam

Node	4mm/100mm
1	32.813
2	207.52
3	589.97

We have found that the frequency of the both top and bottom cracked beam at the same depth and same location the frequency values are same. We can draw this conclusion that frequency values of the cracked beam is the independent whether the beam has crack on the top side or the bottom side.

6.3 Natural frequency of top cracked when location kept constant beam when depth is varied

6.3.1 When the distance from fixed beam is 100mm and depth is varied

Table 5- Natural frequency of cracked beam of fixed location (100mm)

Node	4mm/100mm	8mm/100mm	12mm/100mm	16mm/100mm
1	25.146	21.283	21.283	14.299
2	158.86	157.26	153.91	147.76
3	444.27	444.01	444	443.32

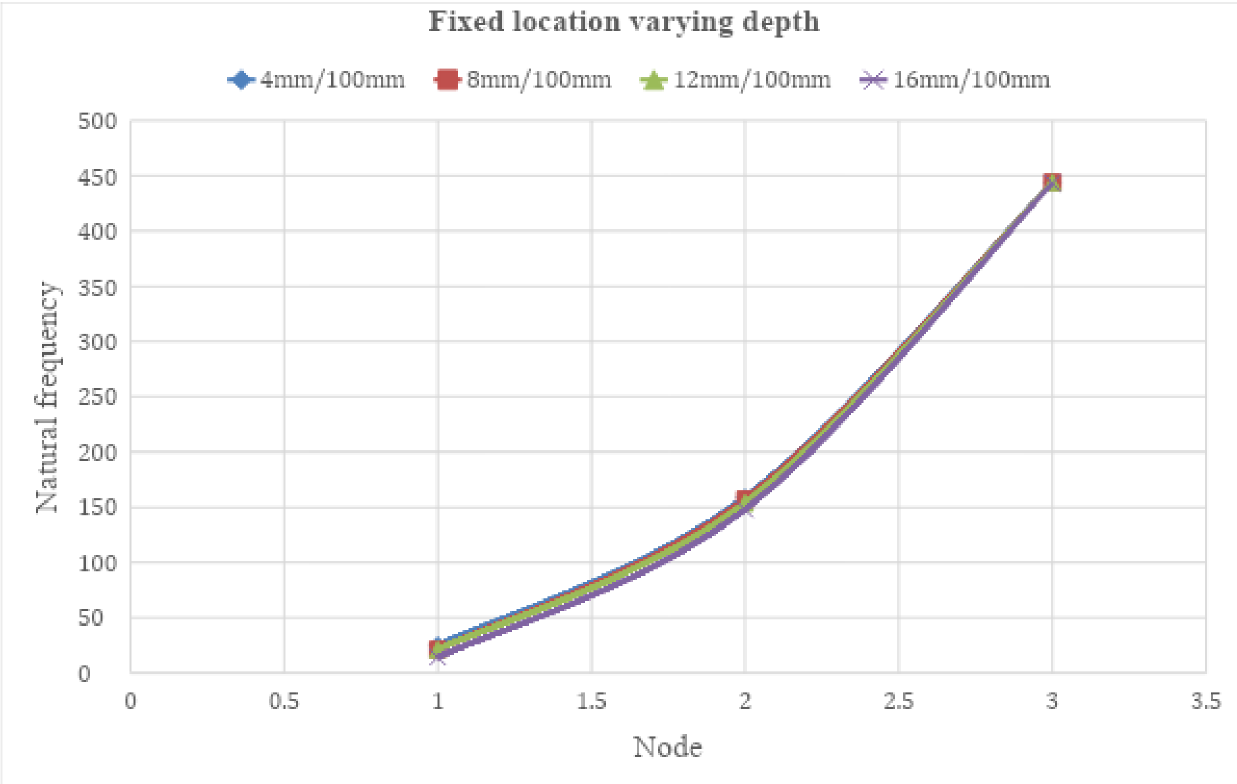

Fig 15: Chart for fixed location (100mm) and varying depth

6.3.2 When the distance from fixed axis is 150mm and depth is varied

Table 6- Natural frequency of cracked beam of fixed location (150mm)

Node	4mm/150mm	8mm/150mm	12mm/150mm	16mm/150mm
1	25.216	24.256	21.657	14.574
2	159.35	159.19	158.78	157.86
3	443.32	440.05	159.23	409.53

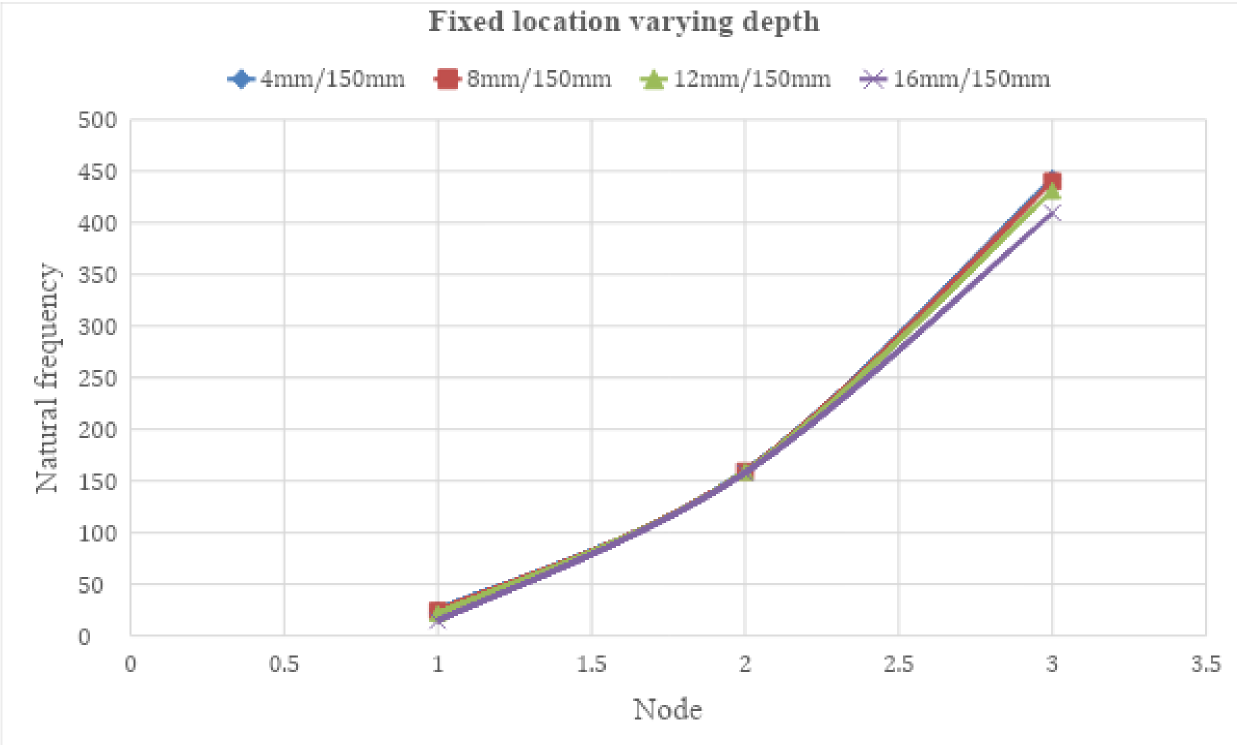

Fig 16: Chart for fixed location (150mm) and varying depth

6.3.3 When the distance from fixed axis is 200mm and depth is varied

Table 7- Natural frequency of cracked beam of fixed location (200mm)

Node	4mm/200mm	8mm/200mm	12mm/200mm	16mm/200mm
1	25.289	24.533	22.62	16.458
2	159.35	159.16	158.68	157.34
3	441.45	431.82	409.7	359.42

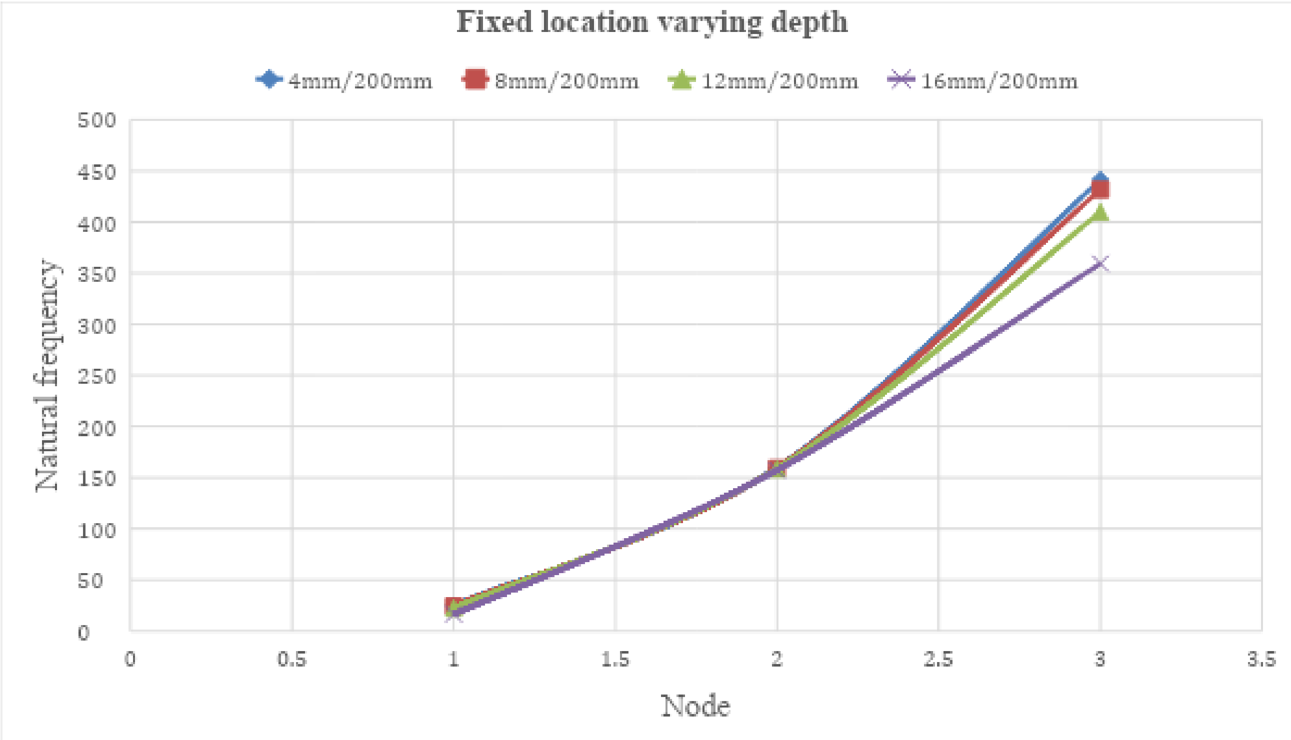

Fig 17: Chart for fixed location (200mm) and varying depth

6.3.4 When the distance from fixed axis is 300mm and depth is varied

Table 8- Natural frequency of cracked beam of fixed location (300mm)

Node	4mm/300mm	8mm/300mm	12mm/300mm	16mm/300mm
1	25.382	24.948	23.731	19.107
2	158.51	155.32	148.06	128.66
3	441.64	433.22	414.73	378.95

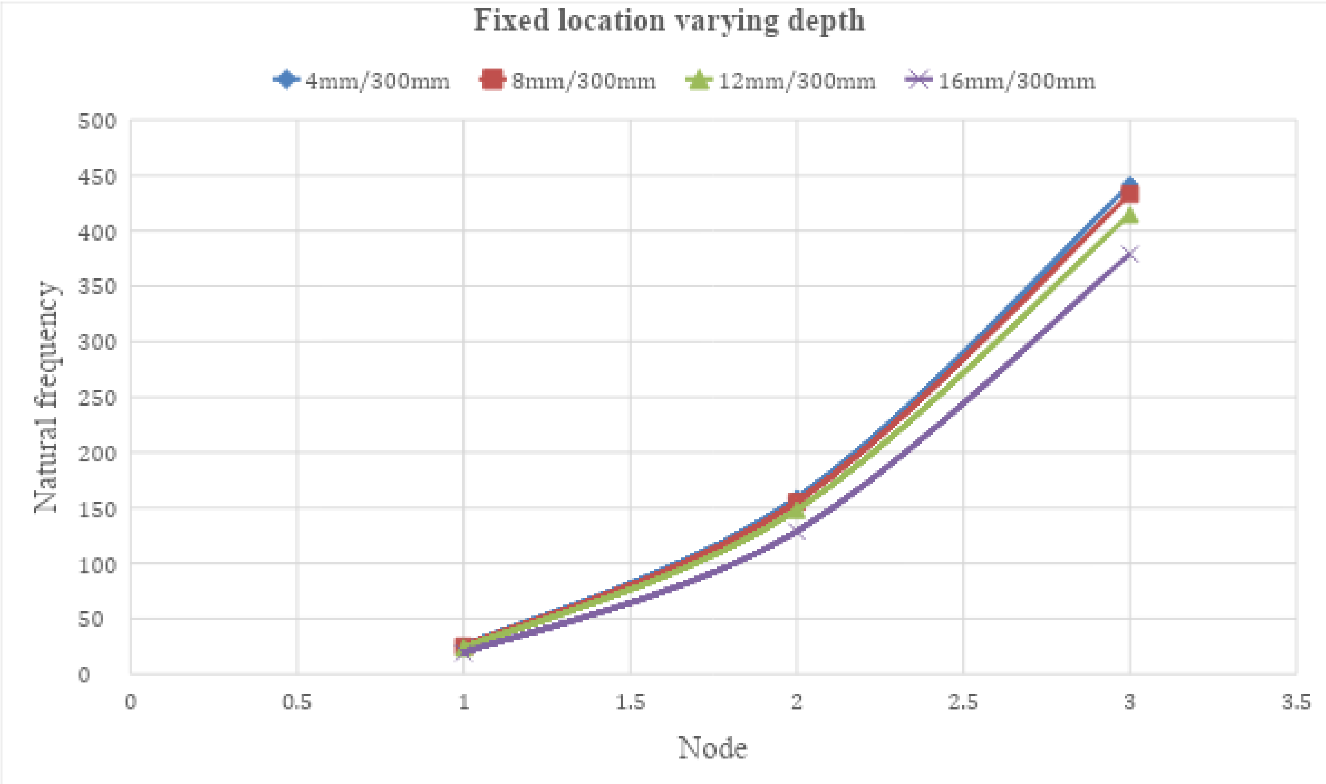

Fig 18: Chart for fixed location (300mm) and varying depth

6.3.5 When the distance from fixed axis is 400mm and depth is varied

Table 9- Natural frequency of cracked beam of fixed location (400mm)

Node	4mm/400mm	8mm/400mm	12mm/400mm	16mm/400mm
1	25.45	25.226	24.611	21.738
2	157.81	152.21	139.59	107.77
3	444.26	441.25	444.07	443.65

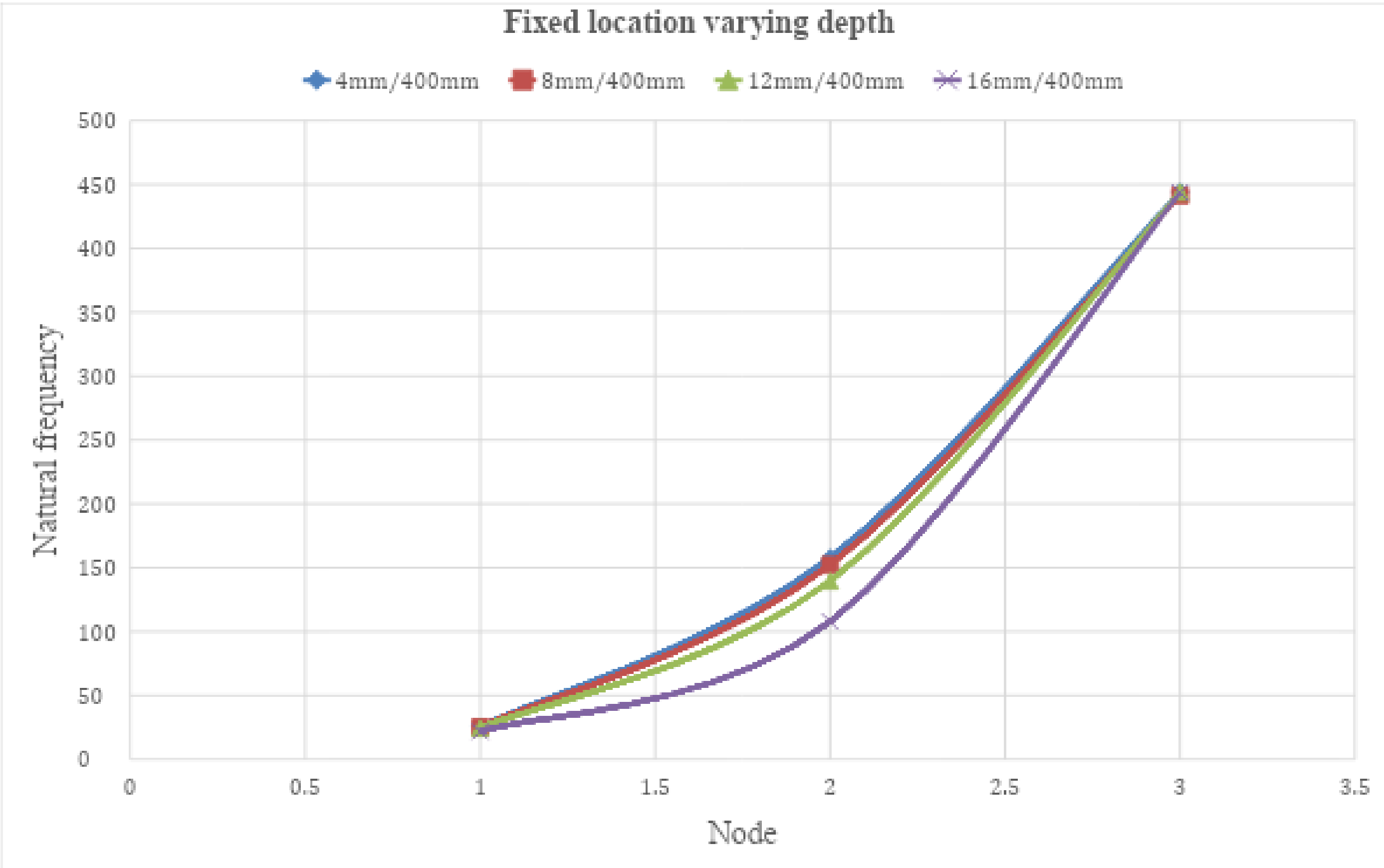

Fig 19: Chart for fixed location (400mm) and varying depth

It has been observed from the table and charts 8.1.1, 8.1.2, 8.1.3, 8.1.4 and 8.1.5 that at any fixed location with increase in depth of the crack, damping factor in the beam increases as stiffness of the beam decreases. This increase in damping factor results in decreasing the natural frequency of beam. Upto 20 % decrement in depth it has been observed that there is almost no change in natural frequency but when the depth is increased till 80 %, then we observe the very high decrement in the natural frequency.

6.4 Natural Frequency of top cracked beam when depth is constant and location is varied

6.4.1 When distance from the fixed axis is varying and depth is fixed (4mm)

Table 10- Natural frequency of cracked beam of fixed depth (4mm)

Node	4mm/100mm	4mm/200mm	4mm/300mm	4mm/400mm
1	25.146	25.289	25.382	25.45
2	158.86	159.35	158.51	157.81
3	444.27	441.45	441.64	444.26

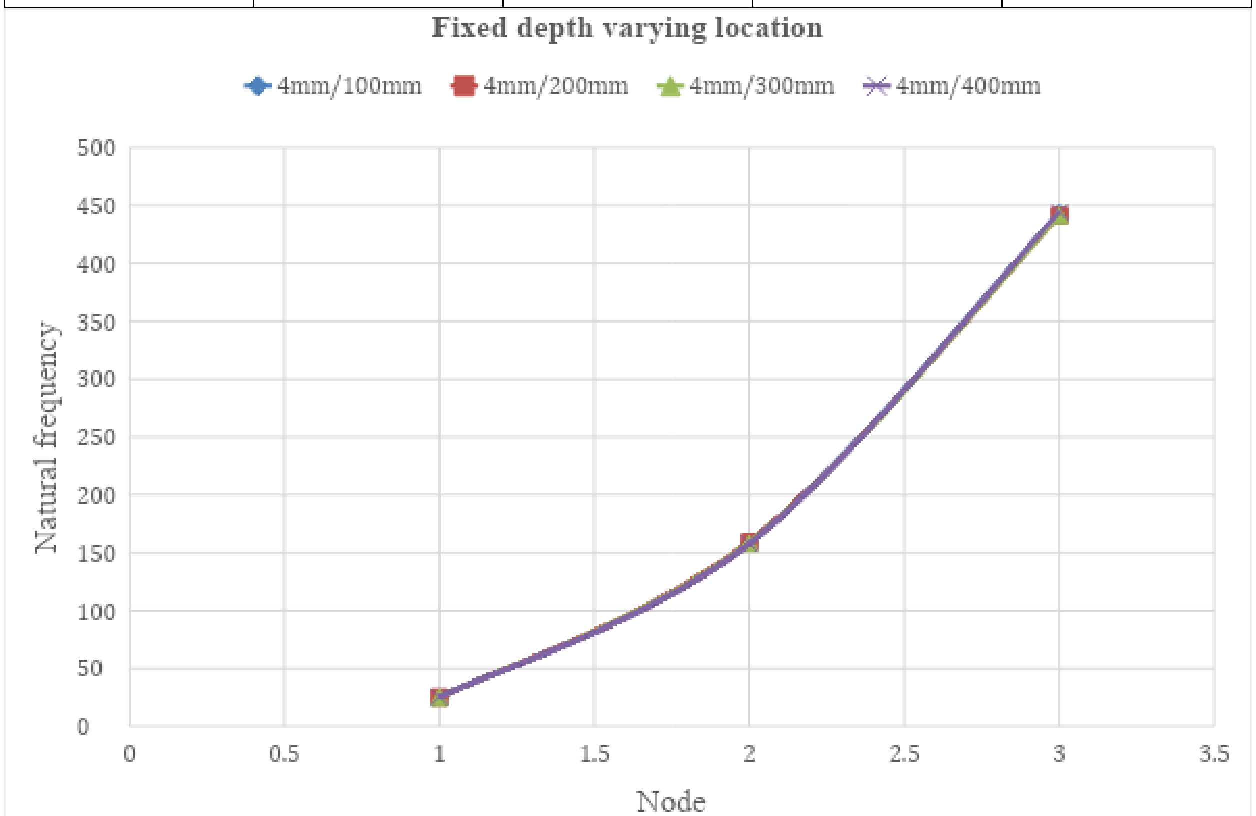

Fig 20: Chart for fixed depth (4mm) and varying location

6.4.2 When distance from the fixed axis is varying and depth is fixed (8mm)

Table 11- Natural frequency of cracked beam of fixed depth (8mm)

Node	8mm/100mm	8mm/200mm	8mm/300mm	8mm/400mm
1	24.948	25.226	24.948	25.45
2	155.52	152.21	155.52	152.81
3	433.23	441.21	433.23	441.21

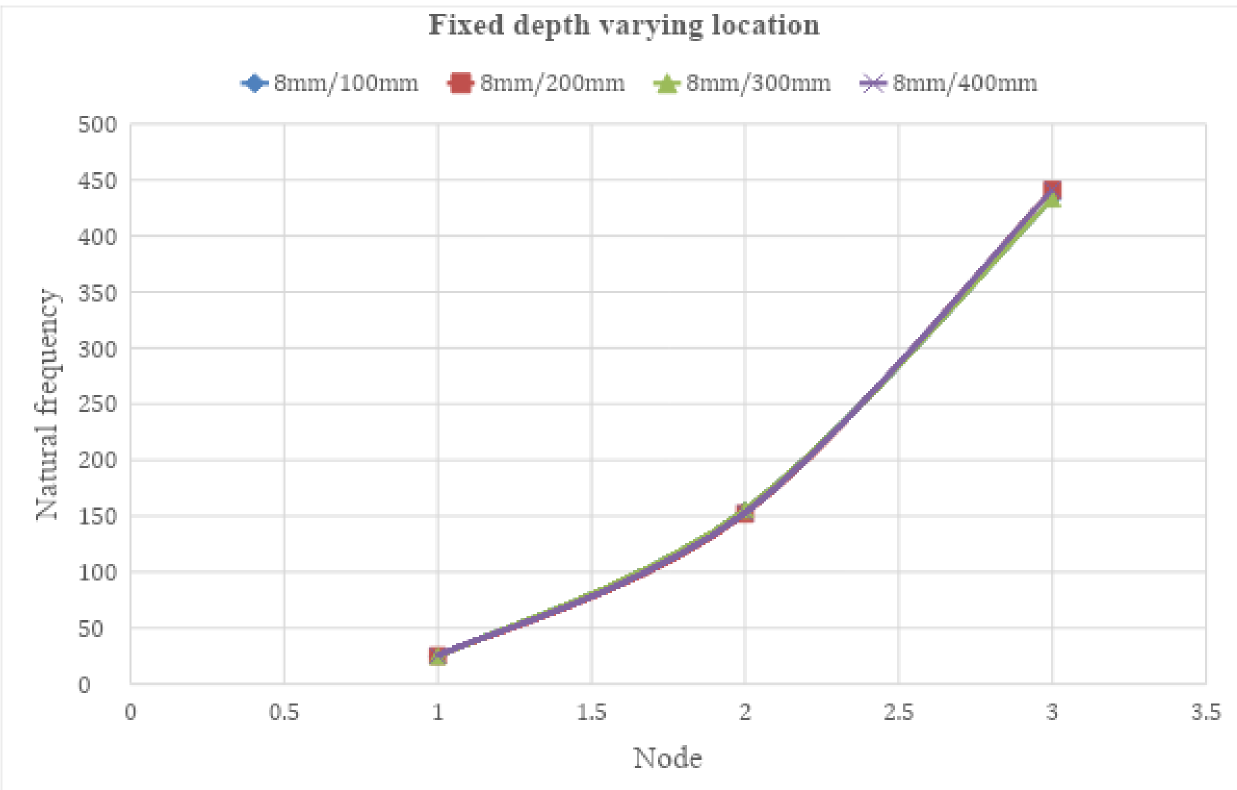

Fig 21: Chart for fixed depth (8mm) and varying location

6.4.3 When distance from the fixed axis is varying and depth is fixed (12mm)

Table 12- Natural frequency of cracked beam of fixed depth (12mm)

Node	12mm/100mm	12mm/200mm	12mm/300mm	12mm/400mm

1	21.283	22.62	23.731	24.611
2	153.91	158.68	148.06	139.59
3	444	409.7	414.73	444.07

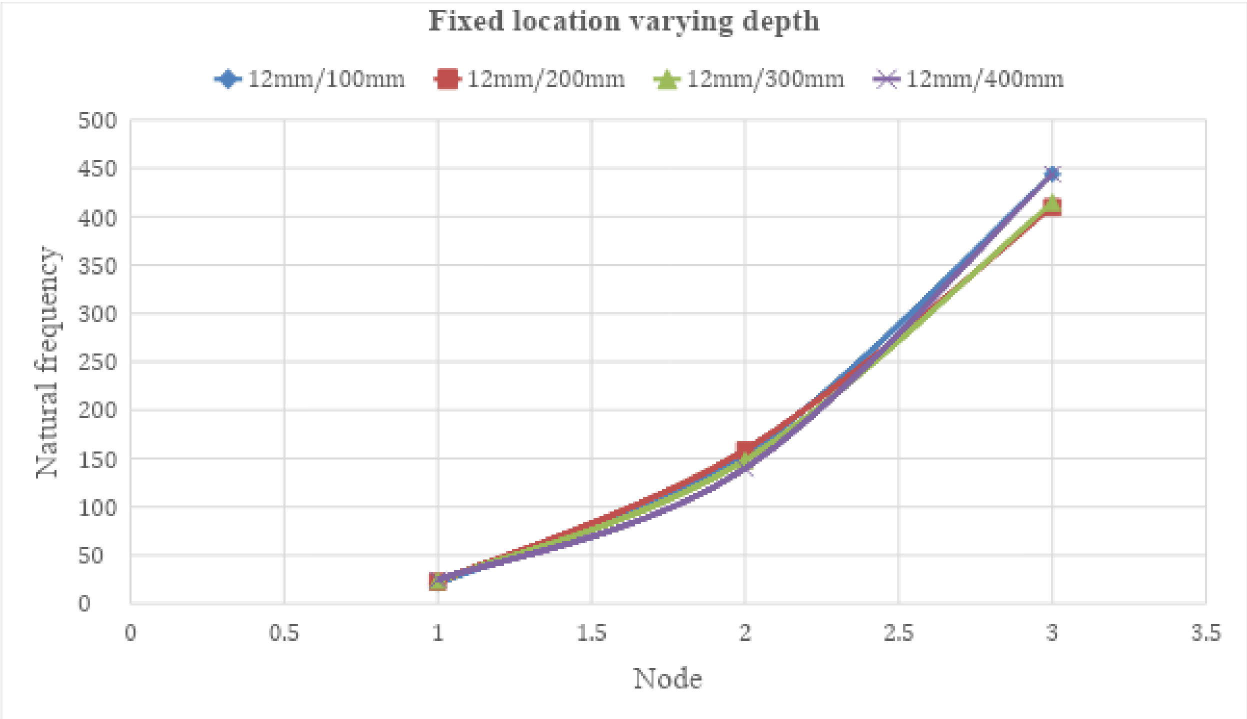

Fig 22: Chart for fixed depth (12mm) and varying location

6.4.4 When distance from the fixed axis is varying and depth is fixed (16 mm)

Table 13- Natural frequency of cracked beam of fixed depth (16mm)

Node	16mm/100mm	16mm/200mm	16mm/300mm	16mm/400mm
1	14.299	16.458	19.107	21.738
2	147.76	157.34	128.66	107.77

3	443.32	359.42	378.95	443.65

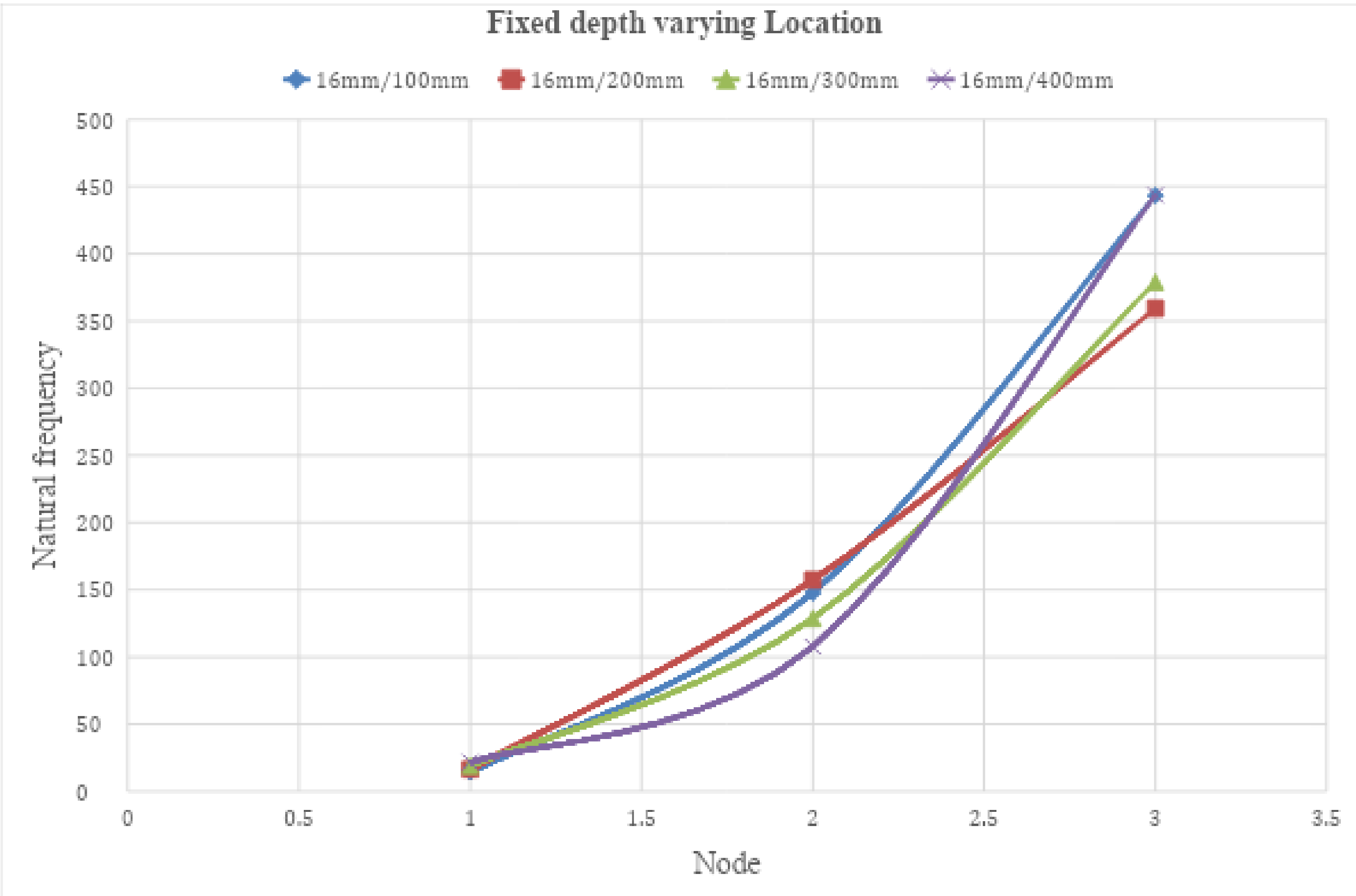

Fig 23: Chart for fixed depth (12mm) and varying location

When the crack depth was less than 20% of the total depth , here we observe that the natural frequency of the uncracked and the cracked beam at fixed depth and varying location on the beam have nearly no change. As the crack depth is more than 20 % then the natural frequency decreases less than as compared with the uncracked beam. This is mainly due to the decrease in the stiffness and increase in the damping factor.

7. Conclusion

In this project, we have taken the cantilever beam having crack. We have done vibrational analysis on Ansys of this beam in many conditions which is discussed in this project. Some of the results we obtained are that the natural frequencies of identical beam having top crack and bottom crack are almost same. When the depth of crack is increasing having fixed crack location, the natural frequencies of cracked beam decreases. We have found out that natural frequency found from the experiment of Shifferen and Routolo[1] and using Ansys are nearly same. Some application of this project are in mechanical systems like long shafts, turbine blades, aero plane blades, fan blades, in cranes etc.

- We have found this that natural frequency found from the Ansys using the FEM analysis is almost same as the natural frequency found by the Shiffrin and Routolo in 1999
- It has also been concluded that average natural frequency of the top and bottom side cracks are almost equal. Natural frequency is the independent of whether crack is on the top side or bottom side.
- When the crack location is fixed and its depth is increased then its natural frequency decreases
- When the crack depth is fixed and its distance from the fixed end is increased then its natural frequency increases

8.Future work

From above analysis we have got a rough idea how the natural frequency of the steel behaves with the crack propagation. Similarly we can do the analysis for different other kinds of material. With analysis we can get the variation of the frequency with the different parameters, we have pool of data of the different frequency value for the various conditions. We know the limits of the material and also the natural frequency if reached then will result in failure of the beam.

If we above got the data from the experiment then we can combine with machine learning to make a model and predict when the beam will fail. If we make this model and predict things then it would be great help. Before any mishap and accident we would be able to predict about the failure of the structure.

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