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Heaps

Whenever there is a priority linked with anything, we have a data structure which is called Heap or Priority Queue.

Heap is a Complete Binary Tree.*

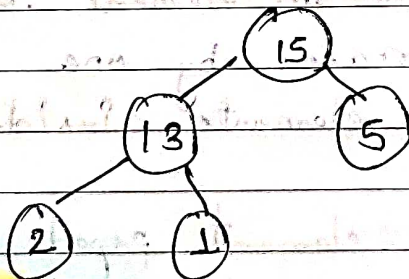
Types of Heap:-

Heap is like a queue but with certain priority
Two types of Heap are there.

- (1) Max Heap → Priority is given to the maximum element. It is sorted in descending order of elements.
ex. [2, 13, 5, 1, 15] → sample element.
Array implementation of Max heap.

Top → [15 | 13 | 5 | 2 | 1] → Bottom

Heap as a Complete Binary Tree



∴ Fill the left element first. then right.

deletion.

swap [arr[root], arr[last]
delete last

if (left < size && arr[i] < arr[left])

swap (arr[i], arr[left])

i = left; &

insertion in Max Heap

parent = $i/2$; index.

if (arr[parent] > arr[index]) return;

else swap (arr[parent], arr[index]);

index = parent;

else if (right < size && arr[i] < arr[right])

swap (arr[i], arr[right]); i = right; &

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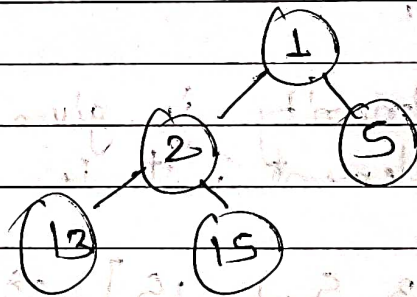
Min Heap - Priority is given to the smaller element. It is sorted in ascending order of elements.

ex \rightarrow [2, 13, 5, 1, 15]

Array implementation of Heap -

Top \rightarrow [1 | 2 | 5 | 13 | 15] \rightarrow Bottom.

Heap as a Complete Binary Tree



Operations in Heap.

1. Top \rightarrow It returns the topmost element of the heap.
if empty() then throw an error T.C \rightarrow O(1)
 2. Push \rightarrow It inserts an element into the heap. The size increases by one. T.C \rightarrow O(log n)
n = number of elements inside the heap already.
 3. Pop \rightarrow Topmost element popped from the heap.
Size decreases by one. If empty() throw an error T.C \rightarrow O(log n)
 4. Size \rightarrow Return size of the heap. T.C \rightarrow O(1)
if empty return 0.
- etc.

As, I already said Heaps are complete Binary Trees. Refer to Binary Tree section for more about BST.

* Conversion of Array into Heap.

Let us consider the given elements to be in a form of an array.

1. The first element is taken as the root.
2. Let the element at index i in the array be kept on the current root.
3. Then, the left child of that root will be the element at index $2*i + 1$.
4. Then, the right child of that root will be the element at index $2*i + 2$.
5. In this way only, the whole complete binary tree is created.
6. Index of array considered to be start from '0' if there is '1' indexed array, then

left $\rightarrow 2*i$, right $\rightarrow 2*i + 1$

* Declaration of Heap \rightarrow { C++ STL }

1. For Max Heap \rightarrow `priority_queue<int> max-pq;`
2. For Min Heap \rightarrow `priority_queue<int, vector<int>, greater<int>> min-pq;`

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Application → 'k' based question.

- ① Whenever we are asked to find the k^{th} smallest or largest element in the array. we can use heaps.
 1. One method is to sort the array & return $\text{arr}[k-1]$ element. T.C $\rightarrow O(n \log n)$.
 2. By using heap. we can insert the element in the heap one by one. As soon as the size of heap become greater than 'k' we pop the element. T.C $\rightarrow O(n \log k)$.
 3. For k^{th} smallest element we use max heap.
 4. For k^{th} largest we use min heap.

② Priority Based question → Some priority Linked in a question.

1. Returning min/max element at each query after performing some operations after each query.
2. Reserving the minimum element number query wise.

③ Huffman Coding → Heaps are usually used in Huffman coding while encoding & decoding the encrypted string message.