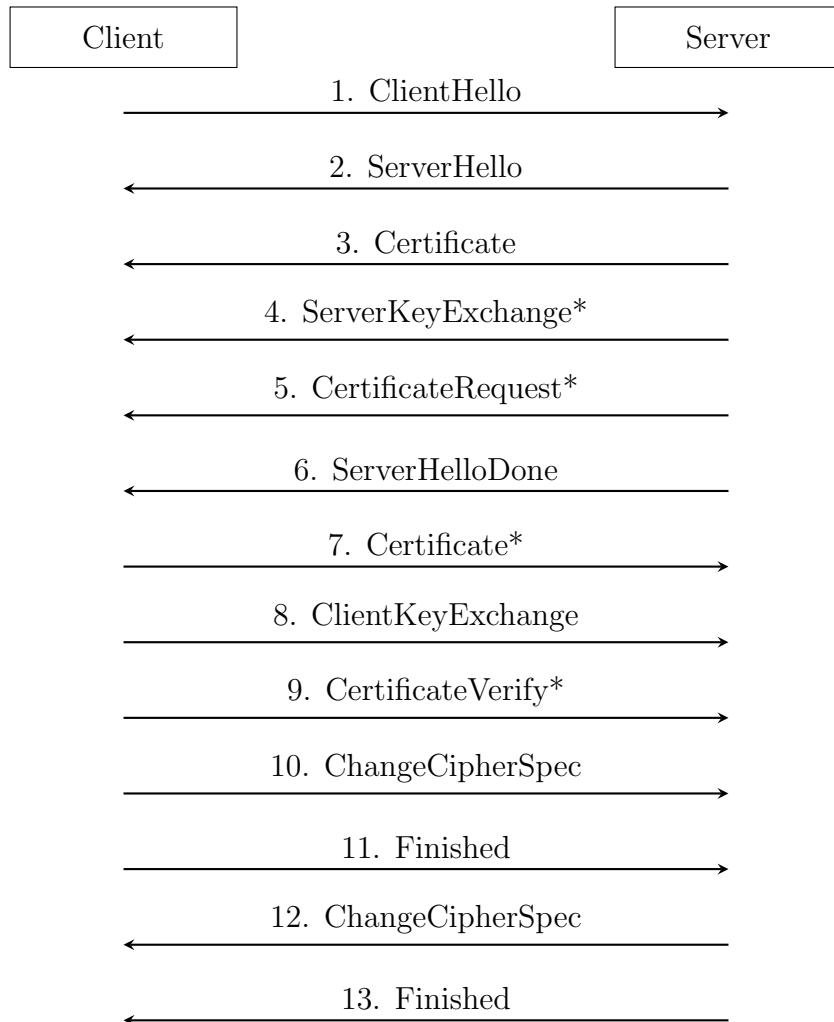


# Proposal for Multi-Party Authorization via TLS Transport Layer Security Protocol

November 15, 2025

## 1 TLS 1.2 Protocol Overview

Transport Layer Security (TLS) 1.2 establishes a secure session through authentication, cipher negotiation, and key establishment. The handshake protocol consists of 13 messages exchanged between client and server:



\* Optional or conditional messages

## 1.1 Message Descriptions

**1. ClientHello:** The client initiates the handshake with:

TLS version, 32-byte random, session ID, cipher suites list, compression methods, and extensions (SNI, ALPN, signature algorithms).

**2. ServerHello:** The server selects:

TLS 1.2, 32-byte server random, session ID, single cipher suite, compression method, and extension responses.

**3. Certificate:** X.509 certificate chain (server certificate + intermediate CAs). Client validates signature, validity, revocation status (CRL/OCSP), and hostname.

**4. ServerKeyExchange (Optional):** For DHE/ECDHE only. Contains DH parameters ( $p$ ,  $g$ ,  $g^a \bmod p$ ) or ECDH public key, signed with server's private key to prevent MITM.

**5. CertificateRequest (Optional):** If client authentication required. Specifies acceptable certificate types, trusted CAs, and signature algorithms.

**6. ServerHelloDone:** Indicates server completed its handshake phase.

**7. Certificate (Optional):** Client certificate chain if requested by server.

**8. ClientKeyExchange:** Key exchange material:

- **RSA:** 48-byte pre-master secret encrypted with server's public key
- **DHE/ECDHE:** Client's DH/ECDH public key value

**9. CertificateVerify (Optional):** If client sent certificate, signs hash of all handshake messages with client's private key.

## 1.2 Key Derivation

Both client and server compute:

master\_secret = PRF(pre-master\_secret, "master secret", ClientHello.random+ServerHello.random)

key\_block = PRF(master\_secret, "key expansion", ServerHello.random+ClientHello.random)

From the key block: client/server write MAC keys, encryption keys, and IVs (for block ciphers).

**10. ChangeCipherSpec (Client):** Activates encryption with newly negotiated keys.

**11. Finished (Client):** First encrypted message. Contains  $\text{verify\_data} = \text{PRF}(\text{master\_secret}, \text{"client finished"}, \text{Hash}(\text{handshake\_messages}))$ .

**12. ChangeCipherSpec (Server):** Server activates encryption.

**13. Finished (Server):** Encrypted message with  $\text{verify\_data} = \text{PRF}(\text{master\_secret}, \text{"server finished"}, \text{Hash}(\text{handshake\_messages}))$ . Handshake complete.

The standard TLS 1.2 key exchange uses RSA (no forward secrecy), DHE ( $g^{ab} \bmod p$ ), or ECDHE ( $d_s \cdot Q_c = d_c \cdot Q_s$ ) to establish a pre-master secret, from which the master secret is derived using PRF. Session resumption reuses cached master secrets to abbreviate the handshake.

## 2 Proposal: Multi-Party Authorization in TLS

Current TLS implementations vest complete control of the server's private key in a single entity. This creates a single point of failure and trust. We propose modifying TLS to require authorization from multiple parties before establishing a secure session.

### 2.1 Proposed Approaches

We present two approaches to enforce multi-party authorization in TLS by modifying the key derivation mechanism:

#### 2.1.1 Approach 1: Secret Sharing for Key Reconstruction

**Mechanism:** Use Shamir's Secret Sharing to split the server's private key among  $n$  parties.

##### Protocol Modification:

###### 1. Setup Phase:

- Server private key  $sk$  is split into  $n$  shares using  $(t, n)$ -threshold secret sharing
- Each party  $P_i$  receives share  $s_i$
- Public key remains in certificate

###### 2. Modified Handshake (RSA Key Exchange):

- Client sends ClientHello
- Server sends ServerHello, Certificate, ServerHelloDone
- Client sends ClientKeyExchange with encrypted pre-master secret:  $E_{pk}(PMS)$
- **NEW:** To decrypt, at least  $t$  parties must cooperate:
  - Each party  $P_i$  contributes their share  $s_i$
  - Reconstruct complete private key:  $sk = \text{Reconstruct}(s_1, s_2, \dots, s_t)$
  - Decrypt pre-master secret:  $PMS = D_{sk}(E_{pk}(PMS))$
  - **Critical:** Complete key  $sk$  exists temporarily during decryption
- Master secret derived from reconstructed PMS

### Implementation Details: Shamir's Secret Sharing Scheme:

To split the server's RSA private key  $sk$  into  $n$  shares with threshold  $t$ :

#### 1. Share Generation:

- Choose random polynomial  $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_{t-1}x^{t-1}$  where  $a_0 = sk \pmod{p}$  for large prime  $p$
- Generate shares:  $s_i = f(i) \pmod{p}$  for  $i = 1, 2, \dots, n$
- Distribute share  $s_i$  to party  $P_i$  over secure channel

#### 2. Key Reconstruction (during handshake):

- Collect  $t$  shares from cooperating parties:  $(i_1, s_{i_1}), (i_2, s_{i_2}), \dots, (i_t, s_{i_t})$
- Use Lagrange interpolation to recover  $sk = f(0)$ :

$$sk = \sum_{j=1}^t s_{i_j} \cdot \prod_{\substack{k=1 \\ k \neq j}}^t \frac{i_k}{i_k - i_j} \pmod{p}$$

- Decrypt PMS:  $PMS = sk^d \pmod{N}$  where  $d$  is RSA private exponent
- Securely erase reconstructed  $sk$  after use

#### 3. Security Properties:

- Any  $t-1$  or fewer shares reveal no information about  $sk$  (information-theoretic security)
- Shares must be refreshed periodically to prevent gradual compromise
- Communication between parties must be authenticated and encrypted

#### Example with $t = 3, n = 5$ :

Suppose server's private key  $sk = 12345$  (simplified), prime  $p = 15485863$ :

- Random polynomial:  $f(x) = 12345 + 7891x + 2468x^2 \pmod{p}$
- Generate shares:

$$\begin{aligned} s_1 &= f(1) = 12345 + 7891 + 2468 = 22704 \pmod{p} \\ s_2 &= f(2) = 12345 + 15782 + 9872 = 37999 \pmod{p} \\ s_3 &= f(3) = 12345 + 23673 + 22212 = 58230 \pmod{p} \\ s_4 &= f(4) = 12345 + 31564 + 39488 = 83397 \pmod{p} \\ s_5 &= f(5) = 12345 + 39455 + 61700 = 113500 \pmod{p} \end{aligned}$$

- Any 3 shares can reconstruct  $sk$ , but 2 or fewer cannot
- To decrypt during handshake: collect shares from any 3 parties, reconstruct  $sk$ , decrypt PMS

### 2.1.2 Approach 2: Threshold Decryption of Master Secret

**Mechanism:** Use threshold encryption (e.g., ElGamal, Paillier) where decryption inherently requires multiple parties without ever reconstructing the private key.

#### Protocol Modification:

##### 1. Setup Phase:

- Generate threshold public key  $pk$  through distributed key generation (DKG)
- Each party  $P_i$  holds private key share  $sk_i$
- $pk$  cannot decrypt without  $t$  parties cooperating
- Certificate contains threshold public key  $pk$

##### 2. Modified Handshake:

- Standard ClientHello, ServerHello, Certificate exchange
- Client sends ClientKeyExchange:  $C = E_{pk}(PMS)$
- **NEW:** Threshold decryption without key reconstruction:
  - Each party  $P_i$  computes decryption share:  $d_i = D_{sk_i}(C)$
  - Shares include zero-knowledge proofs of correctness
  - Combiner aggregates shares:  $PMS = \text{ThresholdDecrypt}(d_1, \dots, d_t, C)$
  - **Key property:** Individual  $sk_i$  never combined; PMS computed from shares
- Master secret derived:  $MS = \text{PRF}(PMS, \text{label}, \text{randoms})$
- Session keys derived from master secret

##### 3. Verification:

- Each party can publish proof that decryption share is correctly computed
- Client or auditor verifies proofs before accepting PMS
- Prevents malicious parties from contributing invalid shares

#### Advantages:

- Private key never exists in complete form (stronger security model)
- Individual compromise of  $< t$  parties reveals nothing
- Zero-knowledge proofs ensure correctness
- Can support proactive secret sharing (refresh shares periodically)
- Natural fit for distributed systems and multi-organizational trust

#### Disadvantages:

- Requires changes to TLS cipher suite specifications
- More complex DKG setup protocol
- Computational overhead for threshold operations