

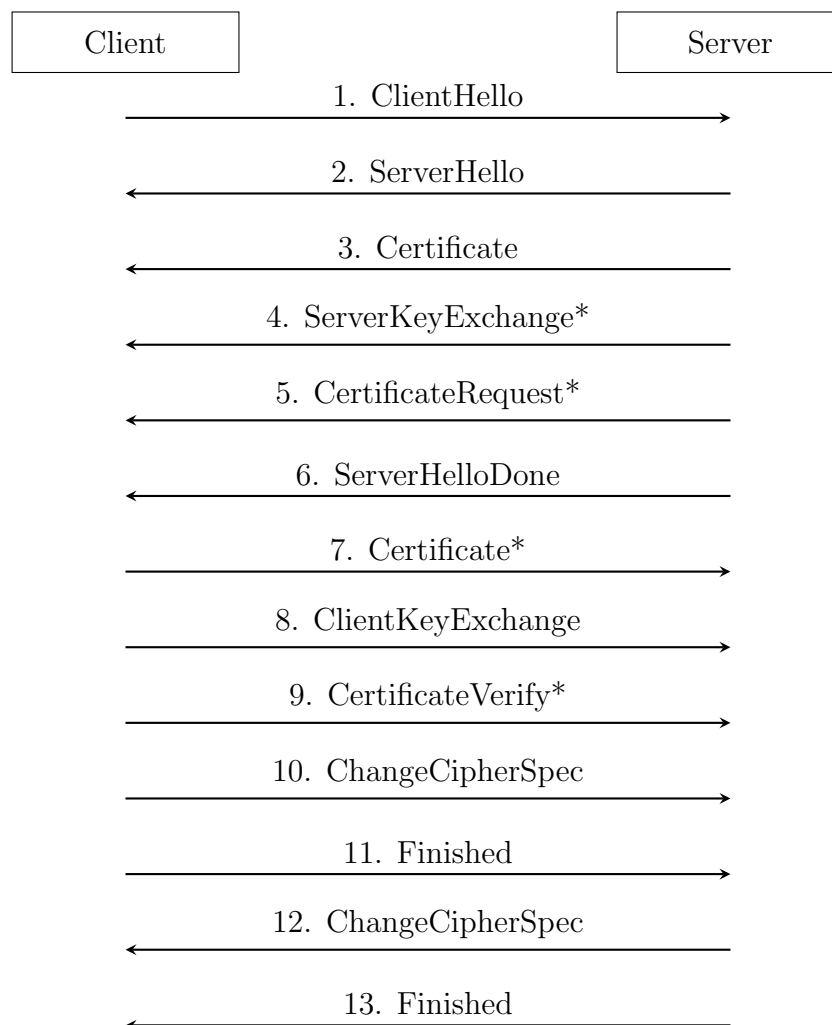
Proposal for Multi-Party Authorization via TLS

Transport Layer Security Protocol

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1 TLS 1.2 Protocol Overview

Transport Layer Security (TLS) 1.2 establishes a secure session through authentication, cipher negotiation, and key establishment. The handshake protocol consists of 13 messages exchanged between client and server:



* Optional or conditional messages

1.1 Message Descriptions

1. ClientHello: The client initiates the handshake with:

TLS version, 32-byte random, session ID, cipher suites list, compression methods, and extensions (SNI, ALPN, signature algorithms).

2. ServerHello: The server selects:

TLS 1.2, 32-byte server random, session ID, single cipher suite, compression method, and extension responses.

3. Certificate: X.509 certificate chain (server certificate + intermediate CAs). Client validates signature, validity, revocation status (CRL/OCSP), and hostname.

4. ServerKeyExchange (Optional): For DHE/ECDHE only. Contains DH parameters (p , g , $g^a \bmod p$) or ECDH public key, signed with server's private key to prevent MITM.

5. CertificateRequest (Optional): If client authentication required. Specifies acceptable certificate types, trusted CAs, and signature algorithms.

6. ServerHelloDone: Indicates server completed its handshake phase.

7. Certificate (Optional): Client certificate chain if requested by server.

8. ClientKeyExchange: Key exchange material:

- **RSA:** 48-byte pre-master secret encrypted with server's public key
- **DHE/ECDHE:** Client's DH/ECDH public key value

9. CertificateVerify (Optional): If client sent certificate, signs hash of all handshake messages with client's private key.

1.2 Key Derivation

Both client and server compute:

$\text{master_secret} = \text{PRF}(\text{pre-master_secret}, \text{"master secret"}, \text{ClientHello.random} + \text{ServerHello.random})$

$\text{key_block} = \text{PRF}(\text{master_secret}, \text{"key expansion"}, \text{ServerHello.random} + \text{ClientHello.random})$

From the key block: client/server write MAC keys, encryption keys, and IVs (for block ciphers).

10. ChangeCipherSpec (Client): Activates encryption with newly negotiated keys.

11. Finished (Client): First encrypted message. Contains $\text{verify_data} = \text{PRF}(\text{master_secret}, \text{"client finished"}, \text{Hash}(\text{handshake_messages}))$.

12. ChangeCipherSpec (Server): Server activates encryption.

13. Finished (Server): Encrypted message with $\text{verify_data} = \text{PRF}(\text{master_secret}, \text{"server finished"}, \text{Hash}(\text{handshake_messages}))$. Handshake complete.

The standard TLS 1.2 key exchange uses RSA (no forward secrecy), DHE ($g^{ab} \bmod p$), or ECDHE ($d_s \cdot Q_c = d_c \cdot Q_s$) to establish a pre-master secret, from which the master secret is derived using PRF. Session resumption reuses cached master secrets to abbreviate the handshake.

2 Proposal: Multi-Party Authorization in TLS

Current TLS implementations vest complete control of the server's private key in a single entity. This creates a single point of failure and trust. We propose modifying TLS to require authorization from multiple parties before establishing a secure session.

2.1 Proposed Approaches

We present two approaches to enforce multi-party authorization in TLS by modifying the key derivation mechanism:

2.1.1 Approach 1: Secret Sharing for Key Reconstruction

Mechanism: Use Shamir's Secret Sharing to split the server's private key among n parties.

Protocol Modification:

1. Setup Phase:

- Server private key sk is split into n shares using (t, n) -threshold secret sharing
- Each party P_i receives share s_i
- Public key remains in certificate

2. Modified Handshake (RSA Key Exchange):

- Client sends ClientHello
- Server sends ServerHello, Certificate, ServerHelloDone
- Client sends ClientKeyExchange with encrypted pre-master secret: $E_{pk}(PMS)$
- **NEW:** To decrypt, at least t parties must cooperate:
 - Each party P_i contributes their share s_i
 - Reconstruct complete private key: $sk = \text{Reconstruct}(s_1, s_2, \dots, s_t)$
 - Decrypt pre-master secret: $PMS = D_{sk}(E_{pk}(PMS))$
 - **Critical:** Complete key sk exists temporarily during decryption
- Master secret derived from reconstructed PMS

Implementation Details: Shamir's Secret Sharing Scheme:

To split the server's RSA private key sk into n shares with threshold t :

1. Share Generation:

- Choose random polynomial $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_{t-1}x^{t-1}$ where $a_0 = sk \mod p$ for large prime p
- Generate shares: $s_i = f(i) \mod p$ for $i = 1, 2, \dots, n$
- Distribute share s_i to party P_i over secure channel

2. Key Reconstruction (during handshake):

- Collect t shares from cooperating parties: $(i_1, s_{i_1}), (i_2, s_{i_2}), \dots, (i_t, s_{i_t})$
- Use Lagrange interpolation to recover $sk = f(0)$:

$$sk = \sum_{j=1}^t s_{i_j} \cdot \prod_{\substack{k=1 \\ k \neq j}}^t \frac{i_k}{i_k - i_j} \mod p$$

- Decrypt PMS: $PMS = sk^d \mod N$ where d is RSA private exponent
- Securely erase reconstructed sk after use

3. Security Properties:

- Any $t-1$ or fewer shares reveal no information about sk (information-theoretic security)
- Shares must be refreshed periodically to prevent gradual compromise
- Communication between parties must be authenticated and encrypted

Example with $t = 3, n = 5$:

Suppose server's private key $sk = 12345$ (simplified), prime $p = 15485863$:

- Random polynomial: $f(x) = 12345 + 7891x + 2468x^2 \mod p$
- Generate shares:

$$\begin{aligned} s_1 &= f(1) = 12345 + 7891 + 2468 = 22704 \mod p \\ s_2 &= f(2) = 12345 + 15782 + 9872 = 37999 \mod p \\ s_3 &= f(3) = 12345 + 23673 + 22212 = 58230 \mod p \\ s_4 &= f(4) = 12345 + 31564 + 39488 = 83397 \mod p \\ s_5 &= f(5) = 12345 + 39455 + 61700 = 113500 \mod p \end{aligned}$$

- Any 3 shares can reconstruct sk , but 2 or fewer cannot
- To decrypt during handshake: collect shares from any 3 parties, reconstruct sk , decrypt PMS

2.1.2 Approach 2: Threshold Decryption of Master Secret

Mechanism: Use threshold encryption (e.g., ElGamal, Paillier) where decryption inherently requires multiple parties without ever reconstructing the private key.

Protocol Modification:

1. Setup Phase:

- Generate threshold public key pk through distributed key generation (DKG)
- Each party P_i holds private key share sk_i
- pk cannot decrypt without t parties cooperating
- Certificate contains threshold public key pk

2. Modified Handshake:

- Standard ClientHello, ServerHello, Certificate exchange
- Client sends ClientKeyExchange: $C = E_{pk}(PMS)$
- **NEW:** Threshold decryption without key reconstruction:
 - Each party P_i computes decryption share: $d_i = D_{sk_i}(C)$
 - Shares include zero-knowledge proofs of correctness
 - Combiner aggregates shares: $PMS = \text{ThresholdDecrypt}(d_1, \dots, d_t, C)$
 - **Key property:** Individual sk_i never combined; PMS computed from shares
- Master secret derived: $MS = \text{PRF}(PMS, \text{label}, \text{randoms})$
- Session keys derived from master secret

3. Verification:

- Each party can publish proof that decryption share is correctly computed
- Client or auditor verifies proofs before accepting PMS
- Prevents malicious parties from contributing invalid shares

Advantages:

- Private key never exists in complete form (stronger security model)
- Individual compromise of $< t$ parties reveals nothing
- Zero-knowledge proofs ensure correctness
- Can support proactive secret sharing (refresh shares periodically)
- Natural fit for distributed systems and multi-organizational trust

Disadvantages:

- Requires changes to TLS cipher suite specifications
- More complex DKG setup protocol
- Computational overhead for threshold operations