Numbers Ex.1

1. By checking the options, only (D) satisfies.
   Answer option is (D).

2. Let natural number is x and its reciprocal is $\frac{1}{x}$

As we know for positive numbers AM≥GM

$$\frac{x+\frac{1}{x}}{2} \geq \sqrt{x \times \frac{1}{x}}$$

⇨ $x+\frac{1}{x} \geq 2$

Therefore minimum value is 2.

3. (3!)!
   3! = 1×2×3 = 6
   6! = 1×2×3×4×5×6 = 720

4. Given number is 56314.09287
   Digit at thousands place = 6
   Digit at thousandths place = 2
   Therefore, sum is 2+6 = 8
   Answer option is (B).

5. 165-9 = 156 = n×k
   495-27 = 468 = n×m
   n is common factor of 156 & 468 and greater than 27.
   HCF (156,468) = 156
   Maximum value of n = 156
   Here, n is any factor of 156 and n>27
   Answer options are (A),(D) & (F).

6. Greatest prime factor of 56 is 7.
   $56=2^3 \times 7^1$
   Greatest prime factor of 4! i.e. 24 is 3.
   $24=2^3 \times 3^1$
   7+3 = 10
   Out of the given options 10 is a factor of 30.
   Answer option is (D).

7. Given
   $a^2=16$
   ⇨ $a = \pm 4$
   Therefore, b = 4 and c = -4
   Hence, c-b = -4-4 = -8
   Answer option is (A)

8. $f(x)=-2x^3+y^2-16$

x is an integer and 16 is also an integer.

Whether f(x) is an integer or not depends on y^2.

If y^2 is an integer then f(x) is also an integer.

Since $-1<y<1$ then for y = 0, only y^2 is an integer.

Answer option is (B).

9. If unit digit of y^2 is 5 then unit digit of y is also 5.

   In the question, unit digit of $(x-1)^2$ is 5.

   Therefore, Unit digit of (x-1) = 5 and unit digit of x = 5+1 = 6

   Square of any number with unit digit 6 is 6.

   Hence, Unit digit of x^2 is 6.

   Answer is 6.

10. Given: Sum of 3 different prime numbers is even.

    Let, 3 prime numbers are a, b & c such that a is the smallest.

    a+b+c = even

    Case I: Even+even+even

    This case is not possible because 2 is the only even prime number.

    Case II: Even+odd+odd

    This is possible when one of the prime number is 2 and 2 is the smallest prime.

    Answer option is (A).

11. Given, $x^2+y^2+z^2=\text{even}$

    If x is even then its square is also even.

    $$x^2+y^2+z^2=\text{even}$$

    Case I: Even + even + even

    This is possible when x,y & z all 3 are even.

    Case II: Even + odd + odd

    This is possible when any one of them is even and remaining 2 are odd.

    In option (C) one number is even and 2 numbers are odd.

    Answer option is (C).

12. Let, integers are a & b.

    Given, a+b = odd

    Sum of 2 integers is odd means one of them is odd and other is even.

    Odd×even = even

    Therefore, their product is always even.

    Answer option is (B).

13. Since question is mark one or more correct therefore check options.

    (A) $\underset{\text{odd}}{n^2}+\underset{\text{odd}}{1}=\text{Even}$

    (B) $\underset{\text{odd}}{n^3}-\underset{\text{even}}{2}=\text{odd}$

(C) $\frac{n+1}{2}$

If n = 3 $\frac{3+1}{2}$ =2 i.e. even	If n = 5 $\frac{5+1}{2}$ = 3 i.e. odd

(D) $\frac{n-1}{2}$

If n = 3 $\frac{3-1}{2}$ =1 i.e. odd	If n = 5 $\frac{5-1}{2}$ = 2 i.e. even

(E) $\underset{\text{odd}}{n^2}+\underset{\text{even}}{4n}=\text{odd}$

Answer options are (B) & (E).

14. Given x & y are prime numbers.

As we know 2 is the even prime number. It is important that one of the prime numbers 2 there or not since question is related to odd-even.

(A)x+y

If x =2 & y = 3 x+y = 2+3 = 5 i.e. odd	If x=3 & y=5 x+y = 3+5 = 8 i.e. even

(B) x-y

If x =3 & y = 2 x-y = 3-2 = 1 i.e. odd	If x=5 & y=3 x-y = 5-3 = 2 i.e. even

(C) xy

If x =3 & y = 2 xy = 3×2 = 6 i.e. even	If x=5 & y=3 x×y = 5×3 = 15 i.e. odd

(D) x+y-1

If x =2 & y = 3 x+y-1 = 2+3-1 = 4 i.e. even	If x=3 & y=5 x+y-1 = 3+5-1 = 7 i.e. odd

None of the options is definitely odd i.e. depends on the values of x & y.

Therefore, answer is none of these.

Answer option is (E).

15. n is non-zero integer

$n^6+n^4+n^2$

Since, all the powers of n are even and n≠0 therefore it is always positive irrespective of value of n.

Note: Its odd –even depends on the value of n.

Answer option is (C).

16. $x = n^2-3n-10$

If n = 0 i.e. even then x = -10 i.e. even	If n = 1 i.e. odd then x = 1-3-10 = -12 i.e. even

x is even for odd as well as even values of n.

Therefore, x is always even.

Answer option is (B).

17. Let x is even then x and (x+2) are 2 consecutive even numbers.

$$(x+2)^2-x^2 = 4x+4$$
$$= 4(\underset{\text{even}}{x}+\underset{\text{odd}}{1}) = 4\times\text{odd}$$

(4×odd) is always divisible by 4.

Answer option is (B).

18. Let x is even then x and (x+2) are 2 consecutive even numbers.

$$(x+2)^2-x^2 = 4x+4$$
$$= 4(\underset{\text{even}}{x}+\underset{\text{odd}}{1}) = 4\times\text{odd}$$

(4×odd) can never be divisible by 8.

Answer option is (E).

19. 11,13,31,17,71,37,73,79,97

20. Given, a is even and b is odd.

(A) $\underset{\text{Even}}{a}+\underbrace{2\times b}_{\text{Even}}=\text{Even}$

(B) $\underbrace{2\times a}_{\text{Even}} + \underbrace{b}_{\text{odd}} = \text{Odd}$

(C) a×b = Even × Odd = Even

(D) $a^{b} = (\text{Even})^{\text{Odd}} = \text{Even}$

(E) $\left(\underset{\text{Even}}{a} + \underset{\text{odd}}{b}\right)^2 = \left(\text{Odd}\right)^2 = \text{Odd}$

(F) $\underset{\text{Even}}{a^2} - \underset{\text{Odd}}{b^2} = \text{Odd}$

21. 2n is a multiple of 7, implies n is a multiple of 7 i.e. 7,14,21,28.. ... and so on.
When n is multiplied by any natural number it is still divisible by 7.
Therefore, options (A), (B) & (D)
Options (C) & (E) depends on the value on n but in question it is asked must be therefore these 2 options are not included in the answers.

22. All the even numbers are multiple of 2.
Out of 2 consecutive even numbers one is definitely multiple of 4.

r × s
= (Multiple of 2) × (Multiple of 4)
= Definitely multiple of 8

Answer is option (A).

23. As we know, $\frac{\text{Odd}}{\text{Even}} \neq \text{Integer}$

Out of given options, option D is of the form $\frac{\text{Odd}}{\text{Even}}$

Therefore, answer is option (D)
24. A number is divisible by 9 if sum of its digits is multiple of 9.
Given number is 56a46,
Sum of digits = 5+6+a+4+6 = 21+a
27 is the multiple of 9 just more than 21.
Therefore, 21+a = 27
a = 6
Answer is 6.

25. Sum or difference of any 2 multiples of 5 is again multiple of 5.
Therefore all the 3 options satisfy the condition.
Answer options are (A),(B) & (C).

26. $3^8 - 1 = \left(3^4\right)^2 - \left(1^4\right)^2$

=> $(3^4+1)(3^4-1)$

Therefore options (B) & (D) which can be expressed in the given form.
Answer is (B) & (D).
27. 6n is a multiple of 11, implies n is a multiple of 11 i.e. 11,22,33,44.. ...and so on.
When n is multiplied by any natural number it is still divisible by 11.
Therefore, options (A) &(B)
Options (C),(D) & (E) depends on the value on n but in question it is asked must be therefore only 2 options i.e. (A) & (B) are included in the answer.

28. When 8545 is divided by 15 remainder is 10.
8545 – 10 = 8535
8535 is multiple of 15.
Answer is option (C)

29. 9999 is the largest 4 digit number.
When 9999 is divided by 75 remainder is 24.
9999-24 = 9975
Therefore 9975 is divisible by 75.

Alternate solution

75 = 25×3
A number is divisible by 75 if it is divisible by 25 & 3 both.
Any number ending with 25,50,75 or 00 is multiple of 25.
Largest number divisible by 25 is 9975 and it is also divisible by 3.
Hence, answer is 9975.

30. From 71 to 102, 72 & 102 are smallest and largest multiple of 3.
72 = 24×3 and 102 = 34 × 3
Total multiples of 3 are 11 i.e. 24 to 34.
Answer is 11.
31. Given x is an odd number
Let x = 1 and check the options.
Only option (D) gives odd value.
Hence, answer option is (D).
32. Given x is an odd number
Let, x = 1 and check the options.
Only option (A) gives odd value.
Hence, answer option is (A).
33. Remove
34.
In given question, it is mentioned that which **can be** true and therefore if any option is true for even a single value one need to consider that option for answer.

(A) (a+b) is prime
Consider, a=3 & b=2
Then, a+b = 3+2 = 5 i.e. prime number.
(B) ab is odd
Consider, a =5 & b = 3
Then, ab = 5×3 = 15 i.e. odd
(C) a(a-b) is odd
Consider, a=3 & b=2
Then, a(a-b) = 3(3-2) = 3 i.e. odd
(D) a-b is prime
Consider, a =5 & b = 3
a-b = 5-3 = 2 i.e. prime number
(E) a^b is even if a is even when a is even prime i.e. 2 .
But a>b i.e. b is the smaller prime and therefore a cannot be 2.
Hence, not possible.
Answer options are (A),(B),(C) & (D).

35.42xy60
72 = 8×9
If a number is divisible by 8 & 9 both then number is divisible by 72.
For divisibility of we need to check last 3 digits i.e. y60

When y = 1,3,5,7 or 9 then it is divisible by 8.
For divisibility of 9 we need to check sum of the digits.
For each value of y there is one value of x and therefore total 5 pairs.
Answer option is (D)
36. r is odd & s is even
(A) r is prime: All the odd numbers are not prime and therefore, it is not always true.
(B) r^2 is even: Square of an odd number is always odd.
This option is always false.
(C)
37. 1000 = 142×7+6
Therefore, 142 multiples of 7 are till 1000.
Note: Consider only integer.
Hence, answer is option (C).

38. REmove
39. Given number is 58215x237
Using the rule of 11: (7+2+5+2+5)-(3+x+1+8) = 21-(12+x) = 9-x
For x = 9, (9-x) becomes 0.
Hence, answer option is (A).

40.
Given number is 2506x8
A number is divisible by 8 if number formed by last 3 digits is a multiple of 8.
Last 3 digits are 6x8
6x8 = 600+x8
600 is divisible by 8 and x8 is divisible 8 when x is 0,4 or 8.
Thus, x can take 3 values.
Answer option is (D).
41. Remove
42. 8537x54 is divisible by 3.
A number is divisible by 3 if sum of digits is divisible by 3.
8+5+3+7+x+5+4 = 32+x
x is a digit from 0 to 9.
For x =1,4,7 i.e. for 3 values (32+x) is divisible by 3.
x can take 3 values.
Answer option is (D).

43. 12 = 3×4, where 3 & 4 are co-prime
If a number is divisible by 3 & 4 both then number is divisible by 12.
Rule of 4: A number is divisible by 4 if number formed by last 2 digits is divisible by 4.
Last 2 digits of the given number are x4.
For x = 0, 4, 8 i.e. for 3 values x4 is divisible by 4.
Rule of 3: sum of the digits should be multiple of 3.
5+1+0+6+2+x+4 = x+18
For x = 0,3,6 & 9 i.e. for 4 values it is divisible by 3.
There is only one value is common in divisibility of 4 & 3 which is 0.
Hence, answer option is (B)
44. Remove
45. Given number is 5821x59x243
Using the rule of 11: (3+2+9+x+2+5) – (4+x+5+1+8) = 3
Therefore, Remainder is 3.

Hence, answer option is (A).

46. 99 = 9×11, where 9 & 11 are co-prime
If a number is divisible by 9 and 11 both then number is divisible by 99.
Given number is 4432364xy
For divisibility of 9: sum of digits = 4+4+3+2+3+6+4+x+y = 26+x+y
Out of given options only 10 satisfies the condition.
26+10 = 36
Hence, answer option is (C).
Note: Here, no need to check the rule of 11 because only one option satisfies for 9.
If more than 1 option satisfy then we need to check for 11 also.

47. Given P is any prime number.
If P =2, then Both the quantities are equal.
If P =3 then quantity A is greater than quantity B.
Therefore, answer option is (D).

48. Given prime number P is odd
Any odd prime number is always greater than 2.
Therefore, answer option is (A).
49. a, b & c are consecutive odd numbers such that a>b>c

Quantity A: a-b = 2
Quantity B: c-b = -2
Quantity A> Quantity B
Hence, answer option is (A).
50. $5x^2=25$

$x = \pm\sqrt{5}$

When x is +ve then x>2
When x is –ve then x<2
Answer option is (D)

51. Maximum value of $\frac{y}{x} = \frac{y_{max}}{x_{min}} = \frac{44}{14} = 3.\overline{142857}$

Quantity A > Quantity B
Answer option is (A).

52. Quantity A

$\frac{5}{6} - \frac{6}{5} < 0$

Quantity B : $\frac{6}{5} - \frac{5}{6} > 0$

Quantity A < Quantity B
Hence, answer option is (B)

53. $4\times10^a > 54321$

Quantity A: Minimum value of a = 5
Quantity A = Quantity B
Hence, answer option is (C).

54. $\frac{40}{3} = 13\frac{1}{3}$

$\frac{13}{3} = 4\frac{1}{3}$

$\frac{4}{3} = 1\frac{1}{3}$

Highest power of 3 = Summation of quotient = 13+4+1 = 18

12 = 3×4

$8^{12} \times 9^{20} = (2^3)^{12} \times (3^2)^{20} = 4^{18} \times 3^{40} = (4^{18} \times 3^{18}) \times 3^{22}$

Therefore, highest power of 12 is 18.

Thus, both the quantities are equal.
Answer option is (C).

55. $72=2^3 \times 3^2$

A number is perfect cube when powers of all its prime factors are multiple of 3.

$N^3 = 2^3 \times 3^2 \times 3^1 = 6^3$

Minimum value of N = 6.
Quantity A < Quantity B
Hence, answer option is (B).