Zeta converter modeling analysis based on pulse waveform integral approach

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***Abstract*—**The circuit topology of a Zeta converter is very complicated，which made it difficult to use the conventional modeling approaches for its common modeling. In this paper, the pulse waveform integral approach is utilized to model and analyze the Zeta converter, then the unified circuit topology can be obtained, and in accordance with this unified circuit, the small-signal dynamic mathematical model has been established. Modeling results show that the mathematical model of control variables to the output voltage contains two zeros in the right half of the plane, and the system is a non-minimum phase system. This paper has analyzed the unified modeling theory by using the conventional modeling approaches for its common modeling in detail, and the unified modeling approach then has been given. The modeling and analysis process has a clear physical concept, and its course is simple, which contributes to an effective tool for unified modeling of high-order switching converters. Finally, the simulation results have been verified the validity of the modeling consequences which is based on the pulse waveform integral approach.

***Keywords*—**Modeling; pulse waveform integral approach ;Non-minimum phase system.

I. INTRODUCTION

A Zeta converter is a fourth-order DC-DC converter made up of two inductors and two capacitors and capable of operating in either step-up or step-down mode. It is widely used in the power factor correction, photovoltaic power generation system, portable electronic products, and many other fields. In order to achieve superior control performance, it is necessary to carry out the modeling analysis. There has been a widely active research on dc-dc converters. as a result, several modeling methods have been proposed[1-2]. In the middle of them, State-Space Averaging technique[3-4] is one of the Widely used methods. It provides a systematic way to model the converter and has gained widespread acceptance. But the State-Space Averaging is complex to establish the mathematical model of high order system such as Zeta converter. Pulse waveform integral method is an effective method for modeling high order systems. The Pulse waveform integral method was used to model the second-order converters such as buck, and boost converters[5-7].so far, the modeling of the fourth-order converters such as Cuk , Sepic and Zeta converters with Pulse waveform integral method has never been reported before in the literature. This paper, therefore, presents dynamic modeling and analysis of non-minimum phase on Zeta converter in Continuous Conduction Mode(CCM). Modeling results show control-to-output transfer function have two pairs of the complex poles on the left half plane and a pair of the complex zeros which locate on the right half plane.

II. ZETA CONVERTER

A Zeta converter is shown in Figure 1. It is comprised of the MOSFET switch(V_T), diode(V_D), two capacitors(C_1 and C_2), and two inductors(L_1 and L_2), and resistor. In CCM, the Zeta converter exhibits two circuit states. The first state is When the MOSFET switch turned on, During this time period, The diode V_D is close due to withstand reverse voltage. At this time the power input to the magnetizing filter inductor L_1, Energy transfer capacitors C_1 supply energy to the magnetizing filter inductor L_2 and the load. The second state is when the MOSFET switch turned off, As the inductor L_1 current cannot be mutated, So that the diode pass under the forward voltage. At this time, the total current of the inductor L_2 and the inductor L_1 is continued to flow through the diode, and the energy stored in the L_2 is supplied to the load, and the energy of the L_1 is added to the energy loss of the V_T when the C_1 is switched on.

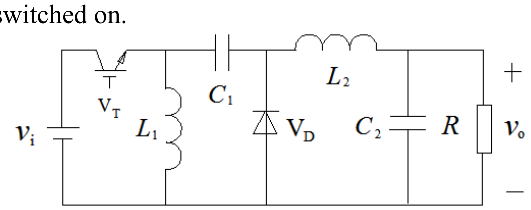

Fig.1 Zeta converter

III. MODELING OF ZETA CONVERTER BY PULSE WAVEFORM INTEGRAL APPROACH

Zeta converter is a nonlinear circuit, with the switch turn on and off, the circuit of the work mode is changed periodically, which makes the traditional linear circuit analysis method cannot be analyzed. The idea of integral approach of pulse waveform is based on the circuit topology and working condition of the converter, The introduction of discontinuous periodic pulse function, Using periodic impulse function, the

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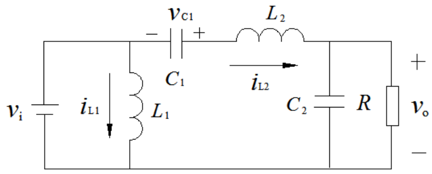
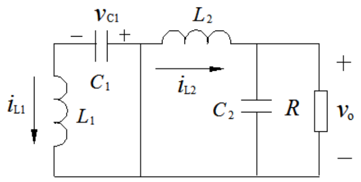
converter can be unified in one cycle of each sub circuit topology. A mathematical model of the dynamic small signal of the switching converter is established by this topology. The Zeta converter working in two sub areas corresponding to CCM circuit topology shown in figure 2.



(a) Switched on equivalent circuit



(b) Switched off equivalent circuit

Fig.2 The mode of CCM Zeta converter

Zeta converter is a periodic change due to the V_T and the V_D of the diode. So it can be unified into a circuit topology in one cycle by using the pulse function of figure 3.

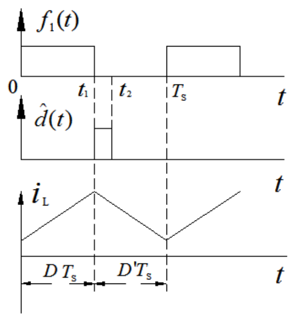


Fig.3 CCM Pulse waveform

where $f_1(t)=D$, $D+D'=1$, $\hat{d}(t)$ is for small signal perturbation components.

Uniform circuit flows through the power supply voltage V_i and the current pulse switch V_T equation $f_1(t)$ to a current source, The voltage at both ends of the diode V_D is changed into a voltage source by using the pulse equation $f_1(t)$. So that they can be time-varying networks into a circuit topology, as shown in Figure 4.

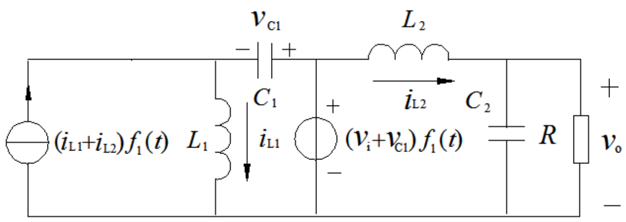


Fig.4 CCM Zeta converter circuit topology uniform

According to the unified circuit topology given in Figure 4, we can obtain the following equation of state

$$\begin{cases} \left(i_{L1}(t)+i_{L2}(t)\right)f_1(t)=i_{L1}(t)-C_1\dfrac{dv_{C1}(t)}{dt} \\ L_1\dfrac{di_{L1}(t)}{dt}+v_{C1}(t)=\left(v_i(t)+v_{C1}(t)\right)f_1(t) \\ i_{L2}(t)=C_2\dfrac{dv_o(t)}{dt}+\dfrac{1}{R}v_o(t) \\ \left(v_i(t)+v_{C1}(t)\right)f_1(t)=L_2\dfrac{di_{L2}(t)}{dt}+v_o(t) \end{cases}$$

(1)

In Figure 4, the $f_1(t)$ and the small signal disturbance components $\hat{d}(t)$ can be expressed by mathematical expressions (2)

$$\begin{cases} f_1(t)=\varepsilon(t-nT_s)-\varepsilon\left(t-(nT_s+t_1)\right) \\ \hat{d}(t)=\varepsilon\left(t-(nT_s+t_1)\right)-\varepsilon\left(t-(nT_s+t_1+t_2)\right) \end{cases}$$

(2)

Equation (1) is a nonlinear continuous-time equation. It can be linearized by small-signal perturbation. The equation (1) introduces the small signal perturbation quantity, the branch variables in Figure 4 are composed of two parts, the steady state component and the small signal dynamic component. In the formula "^" expresses the small signal component.

$$\begin{cases} \left(I_{L1}+\hat{i}_{L1}(t)+I_{L2}+\hat{i}_{L2}(t)\right)\left(f_1(t)+\hat{d}(t)\right)=I_{L1}+ \\ \qquad \hat{i}_{L1}(t)-C_1\dfrac{dV_{C1}}{dt}-C_1\dfrac{d\hat{v}_{C1}(t)}{dt} \\ L_1\dfrac{dI_{L1}}{dt}+L_1\dfrac{d\hat{i}_{L1}(t)}{dt}+V_{C1}+\hat{v}_{C1}(t)=(V_i+\hat{v}_i(t)+ \\ \qquad V_{C1}+\hat{v}_{C1}(t))\left(f_1(t)+\hat{d}(t)\right) \\ I_{L2}+\hat{i}_{L2}(t)=C_2\dfrac{dV_o}{dt}+C_2\dfrac{d\hat{v}_o(t)}{dt}+\dfrac{1}{R}V_o+\dfrac{1}{R}\hat{v}_o(t) \\ \left(V_i+\hat{v}_i(t)+V_{C1}+\hat{v}_{C1}(t)\right)\left(f_1(t)+\hat{d}(t)\right)=L_2\dfrac{dI_{L2}}{dt}+ \\ \qquad L_2\dfrac{d\hat{i}_{L2}(t)}{dt}+V_o+\hat{v}_o(t) \end{cases} \tag{3}$$

By analyzing the equation (3), the corresponding items on both sides of the equation must be equal. that is, the corresponding DC items are equal. we get:

$$\begin{cases} \left(I_{L1}+I_{L2}\right)f_1(t)=I_{L1}-C_1\dfrac{dV_{C1}}{dt} \\ L_1\dfrac{dI_{L1}}{dt}+V_{C1}=\left(V_i+V_{C1}\right)f_1(t) \\ I_{L2}=C_2\dfrac{dV_o}{dt}+\dfrac{1}{R}V_o \\ \left(V_i+V_{C1}\right)f_1(t)=L_2\dfrac{dI_{L2}}{dt}+V_o \end{cases} \tag{4}$$

Equation (3) in the first-order exchange of items and the exchange between the second-order items have the following relationship:

$$\begin{cases} \hat{i}_{\mathrm{L1}}(t)\hat{d}(t)+\hat{i}_{\mathrm{L2}}(t)\hat{d}(t) \ll \left| \hat{i}_{\mathrm{L1}}(t)-C_1\dfrac{\mathrm{d}\hat{v}_{\mathrm{C1}}(t)}{\mathrm{d}t} \right| \\ \hat{v}_{\mathrm{i}}(t)\hat{d}(t)+\hat{v}_{\mathrm{C1}}(t)\hat{d}(t) \ll L_1\dfrac{\mathrm{d}\hat{i}_{\mathrm{L1}}(t)}{\mathrm{d}t}+\hat{v}_{\mathrm{C1}}(t) \\ \hat{v}_{\mathrm{i}}(t)\hat{d}(t)+\hat{v}_{\mathrm{C1}}(t)\hat{d}(t) \ll L_2\dfrac{\mathrm{d}\hat{i}_{\mathrm{L2}}(t)}{\mathrm{d}t}+\hat{v}_{\mathrm{o}}(t) \end{cases} \tag{5}$$

Substituting the equation (4) into the equation (1) while ignoring the second-order exchange as equation (5), equation (3) can be simplified as in equation (6)

$$\begin{cases} (I_{\mathrm{L1}}+I_{\mathrm{L2}})\hat{d}(t)+\left(\hat{i}_{\mathrm{L1}}(t)+\hat{i}_{\mathrm{L2}}(t)\right)f_1(t)=\hat{i}_{\mathrm{L1}}(t)-C_1\dfrac{\mathrm{d}\hat{v}_{\mathrm{C1}}(t)}{\mathrm{d}t} \\ L_1\dfrac{\mathrm{d}\hat{i}_{\mathrm{L1}}(t)}{\mathrm{d}t}+\hat{v}_{\mathrm{C1}}(t)=(V_{\mathrm{i}}+V_{\mathrm{C1}})\hat{d}(t)+\left(\hat{v}_{\mathrm{i}}(t)+\hat{v}_{\mathrm{C1}}(t)\right)f_1(t) \\ \hat{i}_{\mathrm{L2}}(t)=C_2\dfrac{\mathrm{d}\hat{v}_{\mathrm{o}}(t)}{\mathrm{d}t}+\dfrac{1}{R}\hat{v}_{\mathrm{o}}(t) \\ (V_{\mathrm{i}}+V_{\mathrm{C1}})\hat{d}(t)+\left(\hat{v}_{\mathrm{i}}(t)+\hat{v}_{\mathrm{C1}}(t)\right)f_1(t)=L_2\dfrac{\mathrm{d}\hat{i}_{\mathrm{L2}}(t)}{\mathrm{d}t}+\hat{v}_{\mathrm{o}}(t) \end{cases}$$

(6)

Take the Laplace transform on the equation (6). The area enclosed by the pulse waveform shown in Figure 3 is actually equal to the integral of the small signal sampling value and the e^{-st} product at this period. The sum of all pulses is the Laplace transform of the sampling function

$$L[\hat{i}_{\mathrm{L1}}(t)]=\int_0^{T_s}\hat{i}_{\mathrm{L1}}(0)\mathrm{e}^{-st}\mathrm{d}t+\int_{T_s}^{2T_s}\hat{i}_{\mathrm{L1}}(T_s)\mathrm{e}^{-st}\mathrm{d}t+\cdots= \frac{1}{s}\sum_{\mathrm{n=0}}^{\infty}\hat{i}_{\mathrm{L1}}(nT_{\mathrm{s}})\mathrm{e}^{-snt}(1-\mathrm{e}^{-st})\approx\hat{i}_{\mathrm{L1}}(s)T_{\mathrm{s}}$$

(7)

$$L[\hat{d}(t)]=\int_0^{+\infty}\varepsilon\left(t-(nT_{\mathrm{s}}+t_1)\right)\mathrm{e}^{-st}\mathrm{d}t-\int_0^{+\infty}\varepsilon\left(t-(nT_{\mathrm{s}}+t_1+t_2)\right)\mathrm{e}^{-st}\mathrm{d}t= \frac{1}{s}\mathrm{e}^{-(nT_s+t_1)s}-\frac{1}{s}\mathrm{e}^{-(nT_s+t_1+t_2)s}= t_2-t_1=t_2-DT_{\mathrm{s}}=\hat{d}T_{\mathrm{s}}$$

(8)

In the same way

$$L[\hat{v}_{\mathrm{i}}(t)f_1(\mathrm{t})]\approx\hat{v}_{\mathrm{i}}(s)t_1=\hat{v}_{\mathrm{i}}(s)DT_{\mathrm{s}}$$

(9)

$$L[\hat{v}_{\mathrm{C1}}(t)f_1(\mathrm{t})]\approx\hat{v}_{\mathrm{C1}}(s)t_1=\hat{v}_{\mathrm{C1}}(s)DT_{\mathrm{s}}$$

(10)

$$L[\hat{i}_{\mathrm{L1}}(t)f_1(\mathrm{t})]\approx\hat{i}_{\mathrm{L1}}(s)t_1T_{\mathrm{s}}=\hat{i}_{\mathrm{L1}}(s)DT_{\mathrm{s}}$$

(11)

$$L[\hat{i}_{\mathrm{L2}}(t)f_1(\mathrm{t})]\approx\hat{i}_{\mathrm{L2}}(s)t_1T_{\mathrm{s}}=\hat{i}_{\mathrm{L2}}(s)DT_{\mathrm{s}}$$

(12)

$$L[\hat{v}_{\mathrm{o}}(t)]\approx\hat{v}_{\mathrm{o}}(s)T_{\mathrm{s}}$$

(13)

The above conclusions as equation (7), (8), (9), (10), (11), (12), (13) are brought to the equation (6) and take the Laplace transform and simplify as equation (14)

$$\begin{cases} (I_{\mathrm{L1}}+I_{\mathrm{L2}})\hat{d}(s)+\left(\hat{i}_{\mathrm{L1}}(s)+\hat{i}_{\mathrm{L2}}(s)\right)D=\hat{i}_{\mathrm{L1}}(s)-C_1s\hat{v}_{\mathrm{C1}}(s) \\ L_1s\hat{i}_{\mathrm{L1}}(s)+\hat{v}_{\mathrm{C1}}(s)=(V_{\mathrm{i}}+V_{\mathrm{C1}})\hat{d}(s)+\left(\hat{v}_{\mathrm{i}}(s)+\hat{v}_{\mathrm{C1}}(s)\right)D \\ \hat{i}_{\mathrm{L2}}(s)=C_2s\hat{v}_{\mathrm{o}}(s)+\hat{v}_{\mathrm{o}}(s)/R \\ (V_{\mathrm{i}}+V_{\mathrm{C1}})\hat{d}(s)+\left(\hat{v}_{\mathrm{i}}(s)+\hat{v}_{\mathrm{C1}}(s)\right)D=L_2s\hat{i}_{\mathrm{L2}}(s)+\hat{v}_{\mathrm{o}}(s) \end{cases}$$

(14)

Substituting $\hat{d}(s)$=0 into equation (14) may conclude the input-to-output voltage transfer function as

$$\left.\frac{\hat{v}_{\mathrm{o}}(s)}{\hat{v}_{\mathrm{i}}(s)}\right|_{\hat{d}(s)=0}=\frac{DL_1C_1s^2+DD'}{\alpha s^4+\beta s^3+\chi s^2+\gamma s+D'^2} \tag{15}$$

where $\alpha=L_1L_2C_1C_2$, $\beta=L_1L_2C_1/R$,

$\chi=L_1C_1+D^2L_1C_2+D'^2L_2C_2$, $\gamma=\left(L_1D^2+L_2D'^2\right)/R$,

According to the working principle of the Zeta converter, the steady state condition of the converter are $I_{\mathrm{L1}}=DV_{\mathrm{o}}/D'R$; $I_{\mathrm{L2}}=V_{\mathrm{o}}/R$; $V_{\mathrm{C1}}=V_{\mathrm{o}}$. By the relationship between the steady state conditions and the parameters of the converter and substituting $\hat{v}_{\mathrm{i}}(s)$=0 into equation (14) may conclude control-to-output transfer function as

$$\left.\frac{\hat{v}_{\mathrm{o}}(s)}{\hat{d}(s)}\right|_{\hat{v}_{\mathrm{i}}(s)=0}=\frac{\dfrac{L_1C_1V_{\mathrm{o}}}{D}s^2-\dfrac{L_1V_{\mathrm{o}}D}{RD'}s+\dfrac{D'V_{\mathrm{o}}}{D}}{\alpha s^4+\beta s^3+\chi s^2+\gamma s+D'^2}$$

(16)

By analysis of the equation (15) we can see that the mathematical model of zero and poles are located in the left half plane of the S plane, namely the input-output mathematical model is a minimum phase system; By analysis of the equation (16) we can see control-to-output transfer function has two zeros in the right half plane, indicating that the model is a non-minimum phase system.

IV. MODEL VERIFICATION ANALYSIS

The control-to-output transfer function includes two right half-plane zero, these right half-plane zero remarkable characteristic is when the converter duty factor changes suddenly in the situation, the output voltage can have the undershoot voltage. The undershoot voltage is that the output voltage in the duty cycle of the sudden increase (or decrease) of the start stage, will appear to rise after the first drop (or rise after the fall) of the changes. The undershoot voltage can prolong the time of the transition process, while the controller receives the feedback signal in the time of the negative feedback. Analysis equation (16) of zero point can be seen that the zero point is associated with the parameters of the Zeta converter, namely with L_1 inductor, capacitor C_1, duty ratio D, output voltage V_o, Namely the four parameters take different values, there might be two negative real root or two conjugate imaginary root, In order to verify the correctness of the modeling the use of power electronic simulation software PSIM6.0 specially for simulation. The simulation parameters are shown in table 1.

Tab. 1 CONVERTER PARAMETERS

Circuit Parameters	Values
V_i	12/V
D	0.7
R	5Ω
C_1 / C_2	100/100μF
L_1 / L_2	800/1000μH
T=1/f	20μs

Substituting Table 1 parameters in equation (16) can get pole-zero map as shown in figure 5.From figure 5, we can see that the system has four pole of the left half plane and two of the right half plane zero .

poles are: $P_{1,2} = -611 \pm 3276i$, $P_{3,4} = -389 \pm 789i$;

Zeros are: $Z_{1,2} = 1640 \pm 1029i$

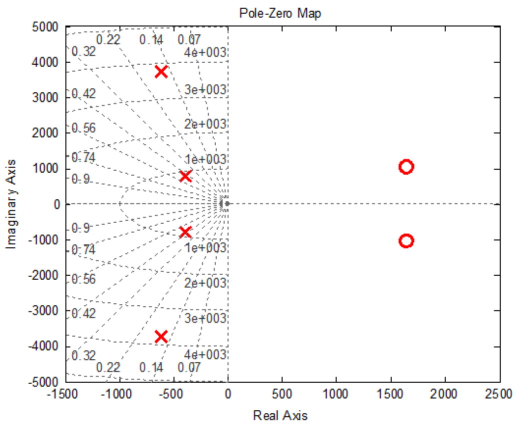


Fig.5 Pole-zero map of the control-to-output voltage transfer function

Right half plane zero system causes the undershoot voltage phenomenon, In order to verify the equation (16),take the duty ratio changed from 0.7 to 0.8, The following using PSIM software for simulation analysis as figure 6.

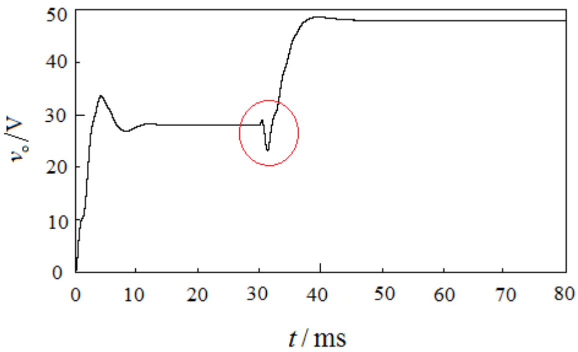


Fig.6 Duty ratio corresponding undershoot voltage simulation waveform

From figure 6, the simulation results it can be seen that when the duty ratio in 0.7 the output voltage of 28V, When the system is in 30ms duty cycle mutations from 0.7 to 0.8, the output voltage from 28V to 48V, When the duty factor has the sudden change, the output voltage appeared undershoot voltage phenomenon because of the right plane two conjugate complex root.

V. CONCLUSIONS

In this paper, modeling and non-minimum phase analysis of Zeta converter in Continuous Conduction Mode (CCM) has been presented. The Pulse waveform integral approach was applied to find small-signal linear dynamic model of the converter (equation(15) and (16)). The simulation results verify the correctness of the pulse waveform integral method based on the modeling results. show that the modeling of the high order Zeta converter based on pulse waveform integral approach is effective.

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