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Design and implementation of an output feedback controller for the Cuk converter

Satyajit Chincholkar, Chok-You Chan
School of Electrical and Electronic Engineering,
Nanyang Technological University, Singapore

Abstract— **The Cuk converter is a non-isolated DC-DC converter having fourth-order and non-linear dynamic characteristics. In this paper, an output feedback controller is proposed for the regulation of the negative output Cuk converter. The main advantage of the proposed controller is only the output voltage state variable is used for feedback purposes. This eliminates the use of the current sensor. Various issues concerning the controller design like the detailed stability and feasibility analysis are also discussed to get some insight into the controlled system. Experimental results showing the feasibility of the proposed controller under the load and reference input variations are also provided to validate the theoretical design.**

Keywords— Cuk converter, Output feedback Control

I. INTRODUCTION

The Cuk converter is a dc-dc converter which is essentially a cascade connection of the conventional boost converter and the buck converter having a capacitor to couple the energy [1]. Its advantages mainly include the use of fewer number of switches, smooth input as well as output current and magnetic component integrability, etc. [2].

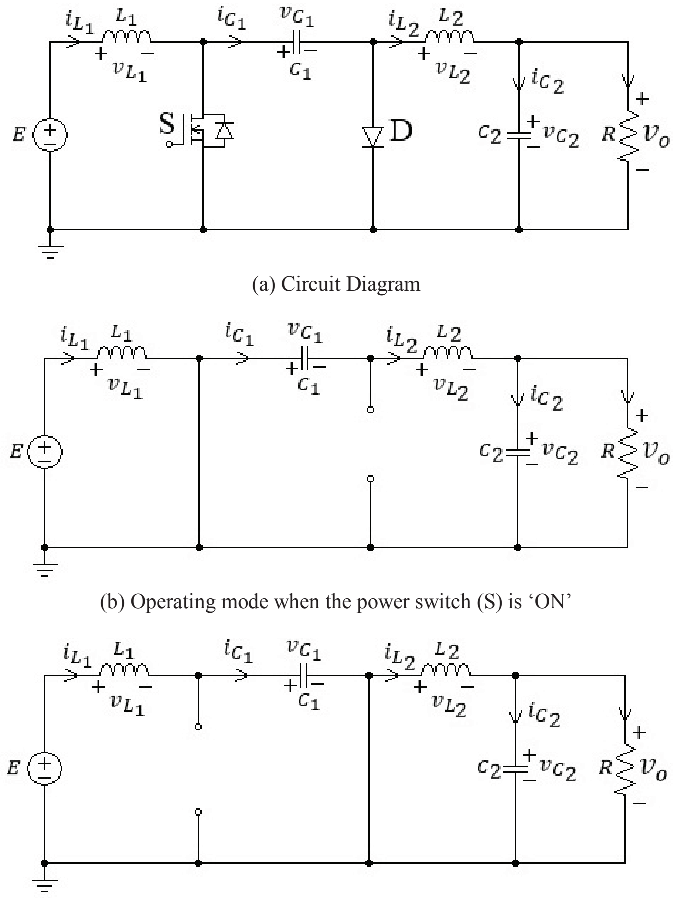
The voltage-mode control and the current-mode control are two widely used methodologies for regulating the output voltage in dc-dc converters. However, for boost-derived topologies like the Cuk converter, non-minimum phase zeroes which may be present in the control to output transfer function of the dc-dc converter, makes it difficult to design a controller using a single voltage-loop. To solve this problem, the current-mode control has been already applied to many boost-derived topologies in the past to regulate their output voltages [3], [4]. It offer several advantages like ease of design, improved current accuracy and increased settling time. However, since its design is based on the small-signal model of the converter, the controller gives satisfactorily performance only in the vicinity of the steady-state operating point. As such some non-linear means of controlling are usually required to achieve the tight regulation of these converters over a wide range of operating conditions. Sliding-mode (SM) control is a one of the widely used non-linear control methodology for dc-dc converters [2], [5]-[7]. It offers several advantages like its ability to handle the large deviations introduced about the steady state equilibrium point as well as guaranteed stability and robustness against load and line variations [8]. Pulse-width-modulation (PWM) and hysteresis-based modulation (HM) are two widely used modulation techniques for implementing sliding-mode controller in dc-dc converters. Even though the hysteresis-based modulation minimizes the risk of saturation when operating at high values of duty cycle [9], this approach uses the variable switching frequency which may increase power losses like inductor and switching losses and may also lead to electromagnetic-interference (EMI) generation [10]. Even though the sliding-mode controller operating at a fixed switching frequency is proposed in the past, it may require more number of current-sensors for its implementation which can increase the cost and complexity of the implementation [2], [11]. Considering this, the challenge is to design some kind of non-linear controller using the least number of state-variables and at the same time without compromising the transient response specifications. Recently, the output feedback control strategy in [12] and [13], is found to be useful for the regulation of the output voltage of higher order converters. However, it should be noted that the control law derived for the non-inverting converters like the fifth-order dc-dc boost converter and the 2-stage cascade boost converter cannot be readily applied to the inverting Cuk converter.

In this paper, the output feedback controller is designed for the regulation of the output voltage of the inverting Cuk converter. It should be noted that the control law used in this paper is motivated by the control law implemented in [14] for the simple boost converter and that of the higher-order converters in [12] and [13]. However, the structure is entirely different from the both. It differs from that of [14], in the sense that now both proportional and integral actions are included to improve the performance of the controller. The main advantage of the proposed controller is that it requires only one state variable i.e. output voltage for feedback purposes.

The paper is organized as follows. The modelling and the steady state analysis of the Cuk converter are given in Section II. The structure of the proposed controller as well as its stability analysis and feasibility analysis are given in Section III, followed by the experimental results in Section IV. Section V presents the conclusion.

II. MODELLING OF THE CUK CONVERTER

Fig. 1(a) shows the circuit diagram of the inverting, non-isolated Cuk converter. It has two operating modes, namely, when the power switch (S) is ‘ON’ (fig. 1(b)), and when the power switch is ‘OFF’ (fig.1(c)). Unlike most other types of dc-dc converter, it uses a capacitor instead of an inductor as its main energy-storage component.



(a) Circuit Diagram

(b) Operating mode when the power switch (S) is 'ON'

(c) Operating mode when the power switch (S) is 'OFF'

Fig.1. Non-isolated Cuk converter and its operating modes

The capacitor C_1 is alternately connected to the input and the output and is used to transfer the energy via the commutation of the transistor-switch (S) and the diode (D). The average state-space model of the inverting Cuk converter operating in continuous conduction mode of operation is given as follows [7]:

$$\frac{di_{L_1}}{dt} = -\frac{1-d}{L_1}v_{c1} + \frac{E}{L_1} \tag{1}$$

$$\frac{di_{L_2}}{dt} = -\frac{d}{L_2}v_{c1} - \frac{1}{L_2}v_{c2} \tag{2}$$

$$\frac{dv_{C_1}}{dt} = \frac{1-d}{C_1}i_{L1} + \frac{d}{C_1}i_{L2} \tag{3}$$

$$\frac{dv_{C_2}}{dt} = \frac{1}{C_2}i_{L2} - \frac{1}{RC_2}v_{c2} \tag{4}$$

where i_{L_1}, i_{L_2}, v_{C_1}, and $v_{C_2} = v_o$ represent the state variables of the averaged model and E is the input voltage. The scalar d denotes the duty ratio, with $0 \le d \le 1$. From (1) – (4), the following equilibrium values are obtained:

$$I_{L_1} = \frac{V_{C_2}^2}{RE}, I_{L_2} = \frac{V_{C_2}}{R},\ V_{C_1} = E - V_{C_2},\ D = 1 - \frac{E}{E - V_{C_2}} \tag{5}$$

where D represents the steady-state value of the duty ratio and I_{L_1}, I_{L_2}, V_{C_1} and V_{C_2} represent the steady state values of the state variables i_{L_1}, i_{L_2}, v_{C_1}, and v_{C_2} respectively. Setting V_{C_2} at the desired voltage value V_d (with $E < |V_d| < \infty$) gives the following desired constant values:

$$I_{L_1} = \frac{V_d^2}{RE},\quad I_{L_2} = \frac{V_d}{R},\quad V_{C_1} = E - V_d,\quad D = 1 - \frac{E}{E - V_d} \tag{6}$$

The problem at hand is to find a suitable output feedback controller to regulate the output voltage of the Cuk converter.

III. Proposed Controller

In this section, an output feedback controller suitable for the regulation of the inverting Cuk converter is proposed. The control law uses only the output voltage state variable feedback and as such eliminates the use of the current sensor.

A. Control law

The proposed output feedback control law can be expressed as below:

$$d = \frac{-(x_d - K_p e_4 - K_i \int e_4 d\tau)}{E - x_d} = 1 - \frac{E - K_p e_4 - K_i \int e_4 d\tau}{E - x_d} \tag{7}$$

$$\frac{dx_d}{dt} = \frac{1}{C_2}\{-(K_1 + K_2)x_d + K_2 v_0 + K_1 V_d\} \tag{8}$$

where K_1, K_2, K_p, and K_i are the adjustable values of the controller gains specified by the user and $e_4 = v_o - V_d$ is the output voltage error. It should be noted that the form of (7) and (8) is originally derived for the conventional boost converter in [14] and is now used here. However in contrast to [14], additional proportional and integral actions are added, like what was done in [12] and [13], for the improved performance of the controller. Also the expression (7) is obtained from the expression of d, i.e., $d = -x_d/(E - x_d) = 1 - E/(E - x_d)$, where x_d is the solution of (8).

B. Stability analysis

Define the errors as follows:

$$e_1 = i_{L_1} - I_{L_1}, e_2 = i_{L_2} - I_{L_2},\ e_3 = v_{C_1} - V_{C_1},$$

$$e_4 = v_o - V_d \tag{9}$$

Substituting (7) – (8) into (1) – (4) and using (9) yields the following set of equations:

$$\dot{e}_1 = -\frac{1}{L_1}\left(\frac{E - K_p e_4 - \sigma}{E - x_d}\right)\left(e_3 + V_{C_1}\right) + \frac{E}{L_1} \tag{10}$$

$$\dot{e}_2 = -\frac{1}{L_2}\left(\frac{-x_d + K_p e_4 + \sigma}{E - x_d}\right)\left(e_3 + V_{C_1}\right) - \frac{1}{L_2}(e_4 + V_d) \tag{11}$$

$$\dot{e}_3 = \frac{1}{C_1}\left(\frac{E - K_p e_4 - \sigma}{E - x_d}\right)\left(e_1 + I_{L_1}\right) + \frac{1}{C_1}\left(\frac{-x_d + K_p e_4 + \sigma}{E - x_d}\right)\left(e_2 + I_{L_2}\right) \tag{12}$$

$$\dot{e}_4 = \frac{1}{C_2}\left(e_2 + I_{L_2}\right) - \frac{1}{RC_2}\left(e_4 + V_d\right) \tag{13}$$

$$\dot{x_d} = \frac{K_2}{C_2}(e_4 + V_d) - \frac{(K_1+K_2)}{C_2}x_d + \frac{K_1}{C_2}V_d \tag{14}$$

$$\dot{\sigma} = K_i e_4 \tag{15}$$

The unique equilibrium point of (10) – (15) is given by

$$(e_{1\infty}, e_{2\infty}, e_{3\infty}, e_{4\infty}, x_{d\infty}, \sigma_\infty) = (0,\ 0,\ 0,\ 0, V_d, 0) \tag{16}$$

Linearizing (10) – (15) about equilibrium point (16) gives the following system:

$$\dot{\beta} = M\beta \tag{17}$$

where $\beta = [\beta_1\ \beta_2\ \beta_3\ \beta_4\ \beta_5\ \beta_6]^T$, and

$$\beta_1 = e_1 - e_{1\infty},\ \beta_2 = e_2 - e_{2\infty},\ \beta_3 = e_3 - e_{3\infty},$$

$$\beta_4 = e_4 - e_{4\infty},\ \beta_5 = x_d - x_{d\infty},\quad \beta_6 = \sigma - \sigma_\infty$$

$$M = \begin{bmatrix} 0 & 0 & \frac{E}{L_1(V_d - E)} & \frac{K_p}{L_1} & \frac{E}{L_1(V_d - E)} & \frac{1}{L_1} \\ 0 & 0 & -\frac{V_d}{L_2(V_d - E)} & \frac{-(1+K_p)}{L_2} & -\frac{E}{L_2(V_d - E)} & -\frac{1}{L_2} \\ \frac{E}{C_1(E - V_d)} & \frac{V_d}{C_1(V_d - E)} & 0 & \frac{K_p V_d}{C_1 RE} & \frac{V_d}{RC_1(V_d - E)} & \frac{V_d}{C_1 RE} \\ 0 & \frac{1}{C_2} & 0 & -\frac{1}{RC_2} & 0 & 0 \\ 0 & 0 & 0 & \frac{K_2}{C_2} & -\frac{(K_1+K_2)}{C_2} & 0 \\ 0 & 0 & 0 & K_i & 0 & 0 \end{bmatrix}$$

The stability analysis can now be performed by finding the eigenvalues of matrix M, i.e., the roots of $|sI - M| = 0$, where s is a complex variable. The system will be stable if and only if all eigenvalues lie in the open left-half complex plane. The root locus method can be used to analyze system stability as shown below.

For the purpose of illustration, consider the Cuk converter with following circuit parameter values:

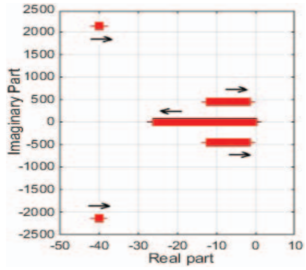
$$E = 5\,V,\ \ V_d = -10\,V,\ \ L_1 = 0.68\,mH,\ \ L_2 = 0.68\,mH,$$

$$C_1 = 470\,\mu F,\ C_2 = 470\,\mu F,\ R = 1\,K\Omega$$

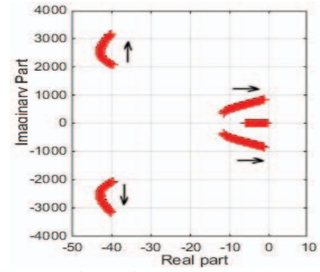
Fig. 2(a) shows the root locus plot for $K_1 = 1$, $K_2 = 1$, $K_P = 0.1$ and $0 < K_i < 7$ and Fig. 2(b) shows the root locus plot for $K_1 = 1$, $K_2 = 1$, $K_i = 1$ and $0 < K_p < 2$. The arrows show how the poles are moving from $K_i = 0$ and $K_p = 0$, respectively. The system is stable for all values of K_i and K_p in this range.

C. Feasibility of the proposed controller

Next, in order to demonstrate the feasibility of the proposed controller, the expressions for $\dot{d}$ and x_d can be obtained from (7).



(a) Root locus for $K_P = 0.1$, $0 < K_i < 7$



(b) Root locus for $K_i = 1,\ 0 < K_p < 2$

Fig. 2. Root locus plot for varying controller gains

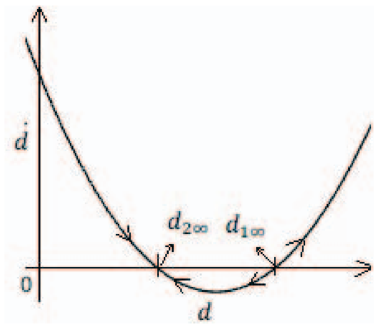


Fig. 3. “Remaining dynamics” of the controlled Cuk converter

$$\dot{d} = \frac{\dot{x}_d - K_p\dot{e}_4 - \dot{\sigma}}{(x_d - E)} - \frac{\dot{x}_d(x_d - K_p e_4 - \sigma)}{(x_d - E)^2} \tag{18}$$

$$x_d = \frac{-Ed + K_p e_4 + \sigma}{1-d} \tag{19}$$

Using (4), (8) – (9), (15) and (18) – (19) yields

$$\dot{d} = \frac{\left(\frac{-(K_1+K_2)}{C_2}\left\{\frac{-Ed+K_p e_4+\sigma}{(1-d)}\right\} + \frac{K_2}{C_2}v_{c2} + \frac{K_1}{C_2}V_d\right) - K_p\left\{\frac{1}{C_2}i_{L_2} - \frac{1}{RC_2}v_0\right\} - K_i(v_{c2} - V_d)}{\frac{-Ed+K_p e_4+\sigma}{1-d} - E} - \frac{d\left(\frac{-(K_1+K_2)}{C_2}\left\{\frac{-Ed+K_p e_4+\sigma}{(1-d)}\right\} + \frac{K_2}{C_2}v_{c2} + \frac{K_1}{C_2}V_d\right)}{\frac{-Ed+K_p e_4+\sigma}{1-d} - E} \tag{20}$$

Now, by equating i_{L_2}, v_{c2} and σ to coincide with their desired values, namely, $i_{L_2} = V_d/R$, $v_{c2} = V_{C2} = V_d$, and $\sigma = 0$, the ‘remaining dynamics’ can be easily obtained as:

$$\dot{d} = \frac{(K_1+K_2)(E - V_d)}{C_2 E}(d-1)\left(d - \left(-\frac{V_d}{E - V_d}\right)\right) \tag{21}$$

The equilibrium points of (21) are given by

$$d_{1\infty} = 1,\ d_{2\infty} = -\frac{V_d}{E-V_d} \tag{22}$$

The phase diagram of (21) is shown in Fig. 3. It is evident that that $d_{2\infty} = -V_d/(E - V_d)$ is a locally stable equilibrium point whereas $d_{1\infty} = 1$ is unstable. Also, $d_{1\infty} = 1$ corresponds to $V_d = \infty$. Thus, the proposed controller is stable for $0 \leq d < 1$.

IV. EXPERIMENTAL RESULTS

Next, in order to verify the effectiveness of the proposed controller, a prototype of output feedback controlled Cuk converter was built in the laboratory. The controller (7) and (8) was built using simple analog components and the AD633 chip. The following circuit parameters were used for implementation:

$$E = 2.5V,\ \ V_d = -10V,\ \ L_1 = 0.68\,mH,\ \ L_2 = 0.68\,mH,$$

$$C_1 = 470\,\mu F,\ C_2 = 470\mu F,\ R = 1\,k\Omega$$

Also, for the sake of the implementation, a scaling factor β is used and the modified control law is given as:

$$u = 1 - \frac{E_s - K_p(v_{os} - V_{ds}) - K_i\int(v_{0s} - V_{ds})d\tau}{-z_d + E_s} \tag{23}$$

$$\frac{dz_d}{dt} = \frac{1}{C_{2s}}\{-(K_1 + K_2)z_d + K_2 v_{0s} + K_1 V_{ds}\} \tag{24}$$

where $v_{os} = \beta v_o$, $V_{ds} = \beta V_d$, and $E_s = \beta E$. The scaling factor β was 0.1, $C_{2s} = 395 \times 10^{-6}$ and z_d and u are the scaled values of x_d and d respectively. The division function was implemented using the AD633 chip and the switching frequency used was 10 kHz. The PWM signal was generated using the LM311 comparator and a $1\,V_{p-p}$ triangular waveform. It should be noted that all the waveforms of the output voltage were inverted for the sake of easy visualization. The experimental diagram is shown in fig. 4 [15].

Fig. 5(a) shows the transient output response and corresponding control signal waveform when a step change in the reference voltage from $V_d = 0\,V$ to $V_d = -10\,V$ was applied. Here $K_1 = K_2 = 1, K_p = 0.55$ and $K_i = 13$ were used. It can be observed that the smooth output response having an overshoot of ~2V and the settling time of ~0.5s was obtained. Fig. 5(b) shows the transient output response and the corresponding inductor current waveform when a similar step change in the reference voltage from $V_d = 0\,V$ to $V_d = -10\,V$ was applied. Fig. 6(a) shows the output response and the control signal when the reference voltage was changed from $V_d = -6\,V$ to $V_d = -11\,V$ and then back to $V_d = -6\,V$. Fig. 6(b) shows the output response and its corresponding control signal when the load was changed from $R = 1\,k\Omega$ to $R = 320\,\Omega$ (~70% reduction) and then back to $R = 1\,k\Omega$. All these results show that the proposed output feedback controller is suitable for the regulation of the inverted Cuk converter.

V. CONCLUSION

In this paper, an output feedback controller for the inverted, non-isolated Cuk converter was proposed. The stability analysis and the feasibility issues of the controller were also addressed. The proposed controller can be implemented using only the output voltage feedback and using simple analog components along with the AD633 chip. Experimental results demonstrate the ability of the proposed controller to track the changes in the load and the reference input.

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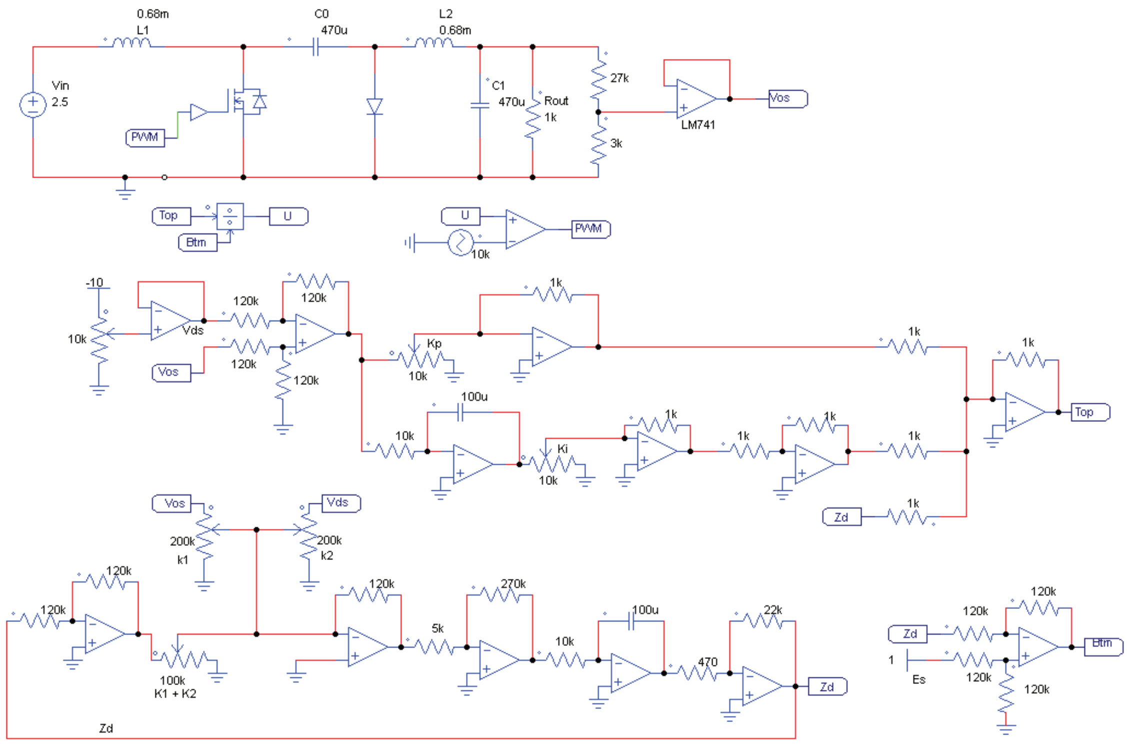
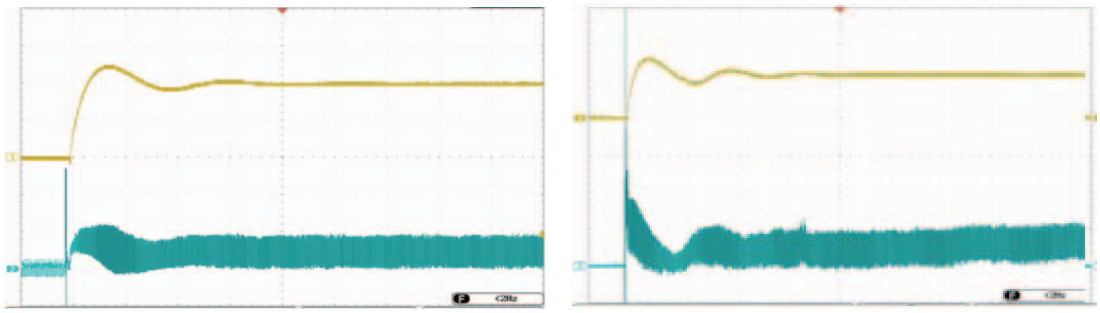


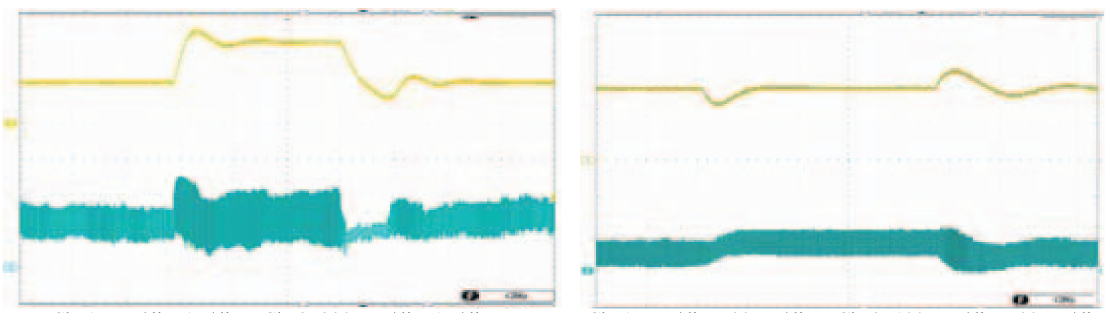
Fig. 4. Experimental diagram of output feedback controller for the Cuk converter



Ch. 1: 5 V/div, 500 ms/div Ch. 2: 100 mV/div, 500 ms/div
(a)

Ch. 1: 5 V/div, 500 ms/div Ch. 2: 200 mV/div, 500 ms/div
(b)

Fig. 5. System response for the Cuk converter
(a). Transient Output response and control signal waveform for step change in reference voltage ($V_d = 0V$ to $V_d = -10V$)
(b). Transient Output response and inductor current waveform for step change in reference voltage ($V_d = 0V$ to $V_d = -10V$)



Ch. 1: 5 V/div, 1 s/div Ch. 2: 200 mV/div, 1 s/div
(a)

Ch. 1: 5 V/div, 500 ms/div Ch. 2: 100 mV/div, 500 ms/div
(b)

Fig. 6. System response for the Cuk converter
(a). Output response and control signal waveform for step change in reference voltage ($V_d = -6V$ to $V_d = -11V$)
(b). Output response and control signal waveform for change in load ($R = 1\ k\Omega$ to $R = 320\ \Omega$)