2018 IEEE 13th International Conference on Industrial and Information Systems (ICIIS)

Study of Closed loop Cuk converter controlled by Loop Shaping Method

ZEESHAN RAYEEN
Dept. of Electrical Engg.
NIT Uttarakhand
Srinagar, India
zeeshan.rayeen.eee17@nituk.ac.in

SOURAV BOSE
Dept. of Electrical Engg.
NIT Uttarakhand
Srinagar, India
souravbose@nituk.ac.in

PRAKASH DWIVEDI
Dept. of Electrical Engg.
NIT Uttarakhand
Srinagar, India
prakashdwivedi@nituk.ac.in

***Abstract*—This paper presents the detailed analysis of the Cuk converter in closed loop operation. The average modeling is done and the small signal analysis is carried out for linearising the system. The Cuk converter is inherently a Non-Minimal Phase (NMP) system hence, the controller design for it is a challenging task. The paper presents the Graphical Loop shaping method for controller design of the Cuk converter and the sensitivity is analyzed. The whole system is validated using the MATLAB/SIMULINK tool.**

***Index Terms*—Cuk converter, Small signal analysis, Non minimal phase (NMP) system, Loop shaping controller design technique.**

I. Introduction

The application of DC-DC converters in the power electronic society has made it indispensable. The concept of switching converter was investigated in 1960. After that, R.D. Middlebrook and Cuk made significant contributions to design and modeling of DC-DC converter [1]- [3] leading to the development of the Cuk converter [4]. The Cuk converter is basically a dual of buck-boost converter or it can be said a boost converter cascaded with a buck converter. The advantages of the basic converters (buck, boost and buck-boost) are incorporated to design the Cuk converter which has numerous advantages like less ripple content, capacitive energy transfer, full transformer utilization, better steady state performance, continuous input as well as output current hence, eliminates the use of filters on both sides therefore it reduces the cost of the system. The output voltage can be less than or greater than the input voltage [5]- [7]. These converters find its application in the recent developments for harnessing renewable power [8], power factor correction [5], [7], electric vehicles [9], [10], switched mode power supplies [10] etc. owing to the aforementioned properties. This has motivated a lot of experts to make their contributions in upgradation of topologies, analysis as well as its controlling. A small review of the history of cuk converter is presented here. The very first contributor for the development of the Cuk model was B.Cuk [4]. The [10]- [13], show the design approaches and its analysis. The [6], [8] show the various topologies which are developed from Cuk converter topology.

Till now, many topologies and the designing approaches of DC-DC converter are already present. Recently, the main concern is to efficiently control the converter. For controlling the converter, many linear and non-linear controlling techniques are presented in literature like PID controller, Fuzzy logic controller, Sliding mode controller [14], Feedback linearization [15], Loopshaping controller design [16], [17], H_∞ controller design [18] etc. Out of which PID controller was mostly used in industries for the control process because of its simplicity, flexibility in use and good steady state performance. For the satisfactory control operation, tuning of PID controller is required. There are some prominent names who worked in tuning method of PID control like Zeigler and Nichols, Parker (introduced optimal tuning method), Rivera et al(introduced internal model control), Deshpande (introduced application of optimization technique) etc. Instead of following advantages mentioned above, the PID controller has some cons,

- large value of time delay
- poor performance in integrating process
- poor controlling performance in transient response

These limitations have increased the scope of other controllers, now fuzzy logic controller and sliding mode controller attract more for controlling because of their better transient as well as steady state performance, large range of controllability and practically implementable. In this paper, the Graphical loop shaping technique for controller design of Cuk converter is presented. The cuk converter is discussed here by deriving its averaged state space model and small signal analysis is done. Further, controller designing using loop shaping method has been implemented.

In this paper, Section II describes the mathematical modeling of Cuk converter and its modes of operation, the loop shaping technique for compensator design is detailed in Section III. The simulation results are presented and discussed in Section IV, and at last section V concludes this paper.

Part No: CFP1858A-ART; ISBN No:978-1-5386-8492-4/18/$31.00 ©2018 IEEE

II. Cuk Converter Circuit Model

A. Cuk converter

Cuk converter is a type of DC-DC converter. The Fig. 1 shows the circuit diagram of non isolating type Cuk converter with component non idealities. It comprises of two inductors (L_1 L_2), two capacitors (C_1 C_2), a switch and a diode.

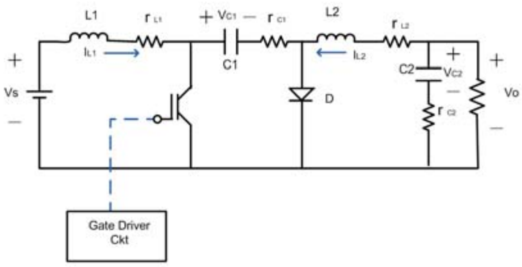

Fig. 1. Circuit diagram of Cuk converter.

In Cuk converter, The capacitor C_1 is used to transfer energy. The two inductors L_1 and L_2 are used to convert the input voltage and output voltage source into current sources respectively. For a small interval, inductor can be considered as a current source as it maintains constant current. This conversion is necessary to prevent the high energy loss (flow of excessive current) when voltage source is directly connected.

B. Mathematical modelling of Cuk converter

Mode-1: Switch in Closed. The switch is turned ON for DT period. where D is duty cycle and T is time period. When the switch is closed, the current on the inductors increases linearly. The capacitor provides the energy to load and hence, the voltage across it gets reduced. The diode gets reverse biased because of capacitor C_1 voltage polarity.

On applying KVL and KCL in the circuit and rearranging them, we get the state-space matrices.

The state space model of mode-1 operation is:

$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{V}_{C1} \\ \dot{V}_{C2} \end{bmatrix} = \begin{bmatrix} -\frac{r_{L1}}{L_1} & 0 & 0 & 0 \\ 0 & -\frac{1}{L_2}(r_{C1}+r_{L2}+\frac{Rr_{C2}}{R+r_{C2}}) & \frac{1}{L_2} & (-\frac{R}{R+r_{C2}})\frac{1}{L_2} \\ 0 & -\frac{1}{C_1} & 0 & 0 \\ 0 & (\frac{R}{R+r_{C2}})\frac{1}{C_2} & 0 & -\frac{1}{(R+r_{C2})C_2} \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_{C1} \\ V_{C2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} V_s \tag{1}$$

$$V_o = \begin{bmatrix} 0 & \frac{Rr_{C2}}{R+r_{C2}} & 0 & \frac{R}{R+r_{C2}}1 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_{C1} \\ V_{C2} \end{bmatrix} \tag{2}$$

Mode-2: Switch in Open.
The switch is open for the period (1-D)T. In this mode, the switch is turned OFF and the diode gets turned ON, the capacitor C_1 gets charged by source and its voltage increases linearly.

On applying KVL and KCL in the circuit and rearranging them, we get the state-space matrices. The state space model of mode-2 operation is:

$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{V}_{C1} \\ \dot{V}_{C2} \end{bmatrix} = \begin{bmatrix} -\frac{(r_{L1}+r_{C1})}{L_1} & 0 & -\frac{1}{L1} & 0 \\ 0 & -\frac{1}{L_2}(r_{L2}+\frac{R}{R+r_C2}) & 0 & -\frac{R}{(R+r_{C2})} \\ \frac{1}{C1} & 0 & 0 & 0 \\ 0 & \frac{R}{(R+r_{C2})C_2} & 0 & -\frac{1}{C_2(R+r_{C2})} \end{bmatrix} \tag{3}$$

$$V_o = \begin{bmatrix} 0 & \frac{Rr_{C2}}{R+r_{C2}} & 0 & \frac{R}{R+r_{C2}}1 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_{C1} \\ V_{C2} \end{bmatrix} \tag{4}$$

C. Circuit Averaging

1) Average Large-Signal Model: The Eq. (1-2) and Eq. (3-4) shows the state space model of the cuk converter during switch turn ON and turn OFF period respectively. The converter alters between two switching states at a high frequency. To represent the converter through a single equivalent dynamic representation which is valid for both the ON and OFF periods, the averaged model is used. The Average Large-signal model for a system is:

$$x = \dot{x}_{avg}T_S = \dot{x}_{DT_S}DT_S + \dot{x}_{(1-D)T_S}(1-D)T_S \tag{5}$$

where, $\dot{x}_{avg}$ is an averaged dynamic variable, $\dot{x}_{DT_S}DT_S$ and $\dot{x}_{(1-D)T_S}(1-D)T_S$ dynamic variables during switch turn ON and turn OFF time.
The averaged model state space matrices is:

$$A = A_1D + A_2(1-D) \tag{6}$$

$$B = B_1D + B_2(1-D) \tag{7}$$

$$C = C_1D + C_2(1-D) \tag{8}$$

where, A_1 and A_2 are the plant transition matrices during switch turn OFF and turn ON periods respectively. Similarly, B_1, B_2 are input matrices and C_1, C_2 are the output matrices. Eq. (9) and (10) shows average large signal state space model,

$$\dot{x} = Ax + Bu \tag{9}$$

$$V_o = Cx \tag{10}$$

2) Small Signal Model: Eq. (9) and (10) is average large signal model of converter which is linear but time variant. Hence it is required to model the system in the neighborhood of the operating point. Such a model is referred to as the small signal model which is analysed now. The instantaneous values can be represented as:

$$d = D + \hat{d},\ x = X + \hat{x},\ v_S = V_S + \hat{v_S}$$

where, d and v_S is the input and x is system state variable. The capital letter shows the steady state value, the small letter with cap shows the dynamic value and the small letter shows the instantaneous value. Here input voltage v_S is considered as a constant value, hence $\hat{v_S}$ gets zero.

On adding the deviations in Eq. (9) and Eq. (10) and simplifying them we will get,

$$\dot{\hat{x}} = (A_1D + A_2(1-D))\hat{x} + ((A_1 - A_2)X + (B_1 - B_2)V_S)\hat{d} \tag{11}$$

$$\hat{y} = (C_1D + C_2(1-D))\hat{x} \tag{12}$$

Laplace transform of Eq. (11) and Eq. (12) and arranging them we get the output voltage as:

$$\hat{V_o}(s) = C(sI-A)^{-1}((A_1-A_2)X+(B_1-B_2)V_S)\hat{d}(s) \tag{13}$$

$$\frac{\hat{V_o}(s)}{\hat{d}(s)} = C(sI-A)^{-1}((A_1-A_2)X+(B_1-B_2)V_S) \tag{14}$$

Eq. (14) is transfer function of the converter. On subsituting the design specification values of converter shown in Table I in Eq. (14), will yeild as:

$$G(s) = \frac{1465s^3 + 1.887e8s^2 - 2.017e11s + 2.611e14}{s^4 + 1521s^3 + 8.416e6s^2 + 3.633e9s + 3.335e12} \tag{15}$$

TABLE I
DESIGN SPECIFICATIONS OF CUK CONVERTER

Parameters Description	Symbols	Design Value
Input voltage	V_s	12 V
Output voltage	V_o	-15 V
Input current	I_s	1.6667 A
Inductor L_1 current	i_{L1}	2.2469 A
Inductor L_2 current	i_{L2}	1.4970 A
Output current	I_o	1.67 A
Duty cycle	D	0.6
Switching frequency	f_s	50 KHz
Inductor L_1 ripple current	$\triangle I_{l1}$	0.224 A
Inductor L_2 ripple current	$\triangle I_{l2}$	0.149 A
Voltage ripple	$\triangle V_o$	0.5 V
Inductor	L_1	324 μH
Inductor	L_2	28.91 μH
Capacitor	C_1	90 μF
Capacitor	C_2	77 μF
Load Resistance	R	9 Ω
L_1 parastic Resistance	r_{L1}	0.01 Ω
L_2 parastic Resistance	r_{L2}	0.01 Ω
C_1 parastic Resistance	r_{C1}	0.15 Ω
C_2 parastic Resistance	r_{C2}	0.1 Ω

III. CONTROLLER DESIGN

In this paper, Graphical Loop Shaping technique is used for this purpose. For applying the loop shaping technique, the plant transfer function should be known. Fig. 2 shows the schematic diagram of conventional feedback control, where G(s) is a plant transfer function, K(s) is controller transfer function and L(s) is closed loop gain. The closed loop transfer function of this system is:

$$\frac{C(s)}{R(s)} = \frac{G(s)K(s)}{1 + G(s)K(s)} \tag{16}$$

The closed loop gain of Fig. 2 is:

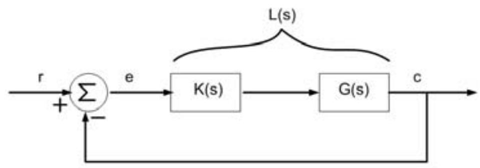

Fig. 2. Typical block diagram of the closed loop system.

$$L(s) = G(s)K(s) \tag{17}$$

A. Loop Shaping Technique

Loop Shaping method is mainly the designing of a frequency response of L(s) satisfying the control system and then with the given transfer function G(s), calculate the controller transfer function K(s), and finally design the required closed loop gain transfer function L(s).

Keeping in mind the stability considerations of the plant transfer function, the author has used the Loop shaping technique in such a manner, that it should reshape the plot. The need for reshaping the plot comes from the non-minimal phase behaviour of the plant transfer function as it contains a zero on the right hand side which is causing the instability. The zero is at ω = 1171.32 rad/sec. Now to reshape the plot, a new plot L(s) is drawn consisting of a -20dB line drawn for the first five decades and then the new plot will follow the original plot till the point where the original point has intersected the 0 dB line. The aim is to modify the plot in such a way that the gain crossover frequency should be less than the phase cross over frequency for stable system. For this, the new plot should cut the zero dB line before ω = 1171.32 rad/sec, and through this the controller transfer function K(s) has been deduced given by the Eq. (18). The transfer function of the desired plot L(s) is computed using the relation L(s)= G(s)*K(s) which is given by:

The controller transfer function K(s) is:

$$k(s) = 1.273\frac{(\frac{s^2}{4.441e5} + \frac{418.6s}{4.441e5} + 1)(\frac{s^2}{7.51e6} + \frac{1102s}{7.51e6} + 1)}{s(\frac{s^2}{1.372e6} + \frac{1071s}{1.372e6} + 1)(\frac{s}{1e4} + 1)} \tag{18}$$

and L(s) is

$$L(s) = \frac{7.69(s + 1.29e5)}{s(s + 1e4)} \tag{19}$$

IV. Control aspects and Experimental Result

This section deals with some of the controlling aspects of the plant and their results are simulated using the MATLAB/SIMULINK tool.

A. Time response and Frequency response analysis of the system

1) Time response analysis: The Fig. 3 shows the step response of the Cuk converter. The step response shows undershoot at the beginning, this characteristic is generally shown by the non-minimal phase system.

The Step response of the closed loop transfer function is given

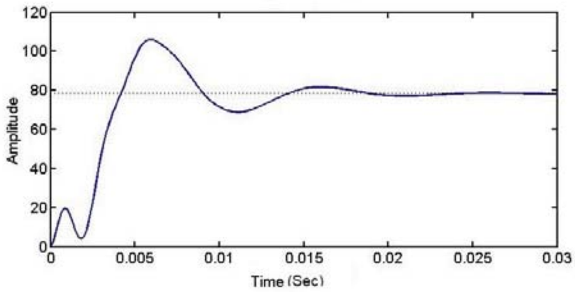

Fig. 3. Step Response of the plant.

in Fig. 4, shows the rise time is $T_r = 0.0177$ sec, peak value is 0.9982, peak time is $T_s = 0.0533$ sec The response settles at $T_s = 0.0338$ sec with the minimum and maximum magnitude of 0.9010 and 0.9982 respectively.

By changing the controller gain, the response of a system can

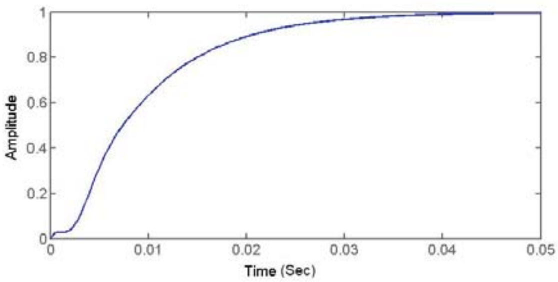

Fig. 4. Step response of the closed loop system.

be made better, the Fig. 5 and Table II shows the step response of the closed loop system at different value of controller gain K_C.

2) Frequency response analysis : The Bode plot of the plant is shown in the Fig. 6. Gain margin and Phase margin of a plant is -37.7 dB and 16.9 degrees respectively, this shows the closed loop of the plant is unstable.

The Fig. 7 shows the Bode plot of closed loop gain of system.

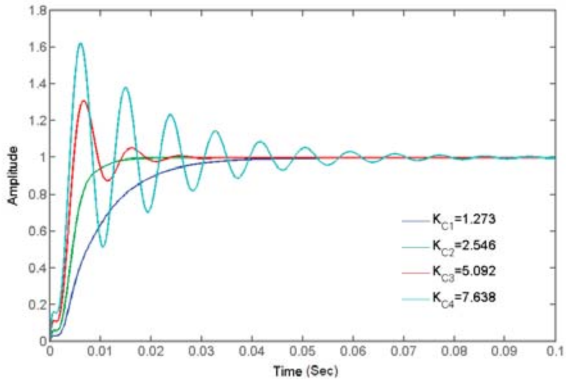

Fig. 5. Effect of variation of K_C on time response.

TABLE II
STEP RESPONSE ANALYSIS AT DIFFERENT VALUE OF K_C.

Controller gain	Rise time	settling time	Peak
K_{C1} =1.273	0.0177	0.0338	0.9982
K_{C2}=2.546	0.0059	0.0135	0.9999
K_{C3}=5.092	0.0039	0.0214	1.3105
K_{C4}=7.638	0.0035	0.0683	1.6224

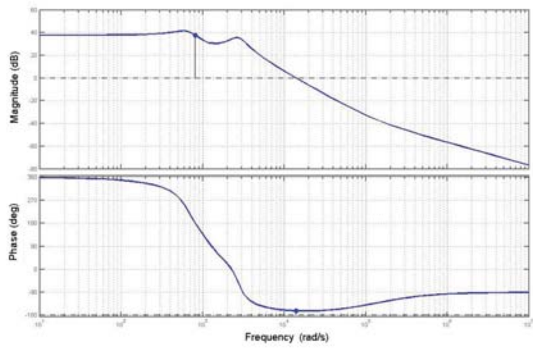

Fig. 6. Bode plot of the plant.

The Gain margin and Phase margin is improved as 17.3 dB and 80.5 degree respectively, this shows the closed loop of the plant becomes stable.

From the Table III, the variation in the frequency response of the system is analyzed by varying the controller gain which is shown in Fig. 8.

TABLE III
STEP RESPONSE ANALYSIS AT DIFFERENT VALUE OF K_C.

Controller gain	Gain margin(dB)	Phase margin
K_{C1} =1.273	17.3	80.5
K_{C2}=2.546	11.3	70.7
K_{C3}=5.092	5.26	49
K_{C4}=7.638	1.74	22.2

3) Sensitivity analysis for robustness: For robust control, the two crucial parameters are sensitivity function (S) and

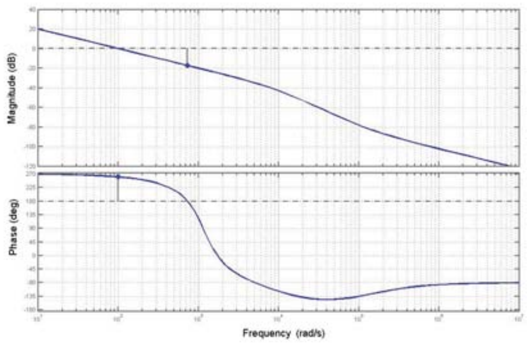

Fig. 7. Bode plot of G(s)K(s).

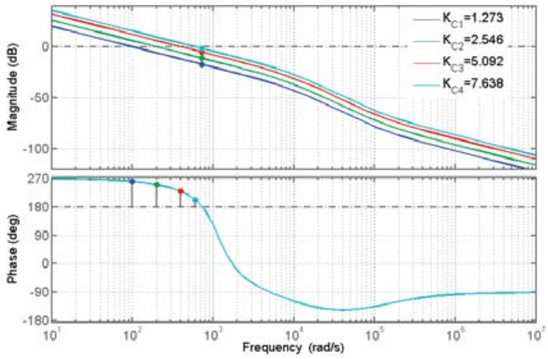

Fig. 8. Effect of variation of Kc on frequency response.

complimentary sensitivity function (T) such that $S + T = 1$ for any frequency. Sensitivity is the amount of change in output due to undesired change in any system variable. The mathematical expression is as follows:

$$S_{\infty} = \frac{1}{1+L} \quad and \quad T_{\infty} = \frac{L}{1+L} \tag{20}$$

where L is the closed loop gain.

The sensitivity function "S" gives the closed loop transfer function between the output and the load perturbations while "T" will give the closed loop transfer function between the output and reference and noise signal. For better rejection of the noise signal the sensitivity should be closed to zero. Hence, the complimentary sensitivity should be close to 1 [19].

The Fig. 9 shows the frequency response of sensitivity function for different values of controller gain K_C. To ensure robustness, the peak of the sensitivity function should be less than 2 (6 dB) [20] and from the figure obtained, the peak value comes out to be 1.45 dB which validates the desirable condition to achieve robust control.

The effect of varying the controller gain is also discussed here. As seen from the Fig. 9, the gain has been varied from 1.273 to 7.638 and the corresponding peak value comes out to be 1.45 dB to 15.5 dB. However, beyond the gain of 7.638, the practical limits is crossed which is undesirable. Hence, the

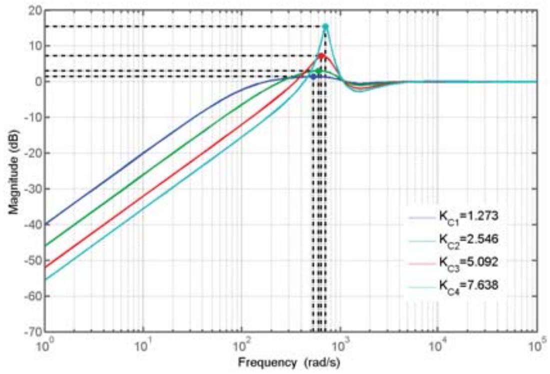

Fig. 9. Sensitivity plot for different gain.

gain can only be varied up to 7.638. The above condition is based on the ideal peak value of the sensitivity function which should be 1 dB. Therefore, the gain value can be reduced to achieve the robust behaviour however, choosing very low value of gain will make the system slow and a high value of gain will speed up the system.

B. Experimental Result

1) Case-1 When input voltage is constant: The Fig. 10 shows the waveform of the output voltage. It is set to reference of -15 V. This waveform is obtained by applying a constant 12 V input. It is shown in the figure that the controller settles the output voltage to -15 V.

Fig. 11 (a) and Fig. 11 (b) show the waveform of the current

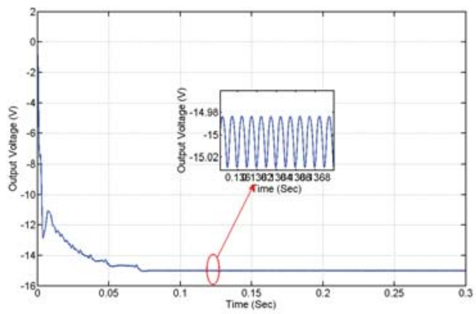

Fig. 10. Output voltage at constant voltage source.

flow in inductor L_1 and L_2 respectively.

2) Case-2 When the input voltage changes abruptly: The input voltage is varied abruptly between 12V to 18V and the corresponding output voltage is shown in the Fig. 12.The factors considered in transient analysis are the settling time and the peak transient voltage. The settling time of output voltage is less than t=0.1 sec which is within the limits of the settling time required (0.2-0.5 sec) for good response.

Note:- In case-2 and case-3 the absolute value of output is taken.

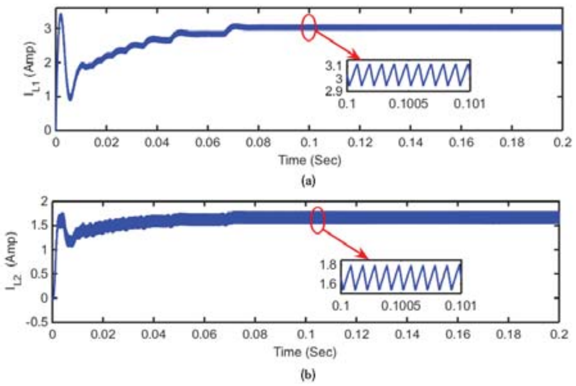

Fig. 11. (a) Current in inductor L_1; (b) Current in inductor L_2.

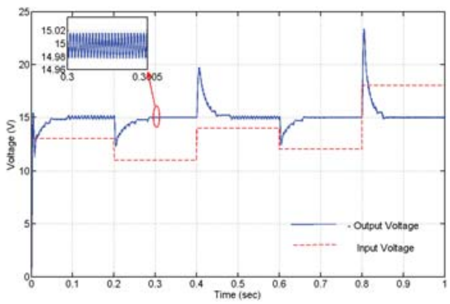

Fig. 12. Output voltage with abrupt change in input.

3) Case-3 When the input voltage is changed smoothly.: Now the input voltage is varied between 12V to 25V smoothly. The input and output waveform is shown in the Fig. 13.

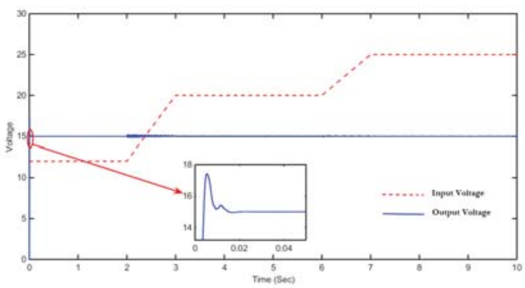

Fig. 13. Output voltage with smooth variation in input voltage.

C. Conclusion

The author has computed the state space model of Cuk converter from its mathematical equations and the averaged model is developed on which the small signal analysis is carried out. The obtained transfer function of the plant has a complex zero on the right side of s-plane, which is a non minimal phase system. The gain margin and phase margin of the plant is -37.7 dB and 16.9 degree respectively. This system is then modelled using the controller by reshaping the plant plot into a desired plot such that the overall system becomes stable. The loop shaping technique is applied for controller design, the improved gain margin and phase margin is 173 dB and 80.5 degree, respectively. As shown in the simulations results, the controller works well in all three cases.

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