$\varphi = (1, \sqrt{2}x_1, \sqrt{2}x_2, \sqrt{2}x_1 x_2, x_1^2, x_2^2)^T$

$W = (w_1, w_2, w_3, w_4, w_5, w_6)$

$\min_{w,b} \ \frac{1}{2}||w||^2$

$y_j (w^T \varphi(x^j) + b) \geq 1 \quad j = 1, 2, 3, 4$

for $j = 1$ constraint is

$-1((w_1, w_2, w_3, w_4, w_5, w_6) \times (1, \sqrt{2}, \sqrt{2}, \sqrt{2}, 1, 1)^T + b) \geq 1$

$= -1((w_1 + \sqrt{2}w_2 + \sqrt{2}w_3 + \sqrt{2}w_4 + w_5 + w_6) + b) \geq 1$

for $j = 2$

$1((w_1, w_2, w_3, w_4, w_5, w_6) \times (1, \sqrt{2}, 0, 0, 1, 0)^T + b) \geq 1$

$= 1((w_1 + \sqrt{2}w_2 + w_5 + b) \geq 1$

for $j = 3$

$1((w_1, w_2, w_3, w_4, w_5, w_6)) \times (1, 0, \sqrt{2}, 0, 0, 1)^T + b) \geq 1$

$\Rightarrow 1((w_1 + \sqrt{2}w_3 + w_6) + b) \geq 1$

for $j = 4$

$-((w_1, w_2, w_3, w_5, w_6) \times (1, 0, 0, 0, 0, 0)^T + b) \geq 1$

$= -1(w_1 + b) \geq 1$

Using Lagrange we minimize

$\frac{1}{2}|w|^2 + \lambda_1(w_1 + \sqrt{2}w_2 + \sqrt{2}w_3 + \sqrt{2}w_4 + w_5 + w_6 + b + 1)$
$- \lambda_2(w_1 + \sqrt{2}w_2 + w_5 + b - 1) - \lambda_3(w_1 + \sqrt{2}W_3 + w_6 + b - 1)$
$+ \lambda_4(w_1 + b + 1)$

$$\Rightarrow X = \frac{1}{2}|w|^2 + \lambda_1 w_1 + \lambda_1\sqrt{2}w_2 + \lambda_1\sqrt{2}w_3 + \lambda_1\sqrt{2}w_4 + \lambda_1 w_5 + \lambda_1 w_6 + \lambda_1 b + \lambda_1 - \lambda_2 w_1 - \lambda_2\sqrt{2}w_2 - \lambda_2 w_5 - \lambda_2 b + \lambda_2 - \lambda_3 w_1 - \lambda_3\sqrt{2}w_3 - \lambda_3 w_6 - \lambda_3 b + \lambda_3 + \lambda_4 w_1 + \lambda_4 b + \lambda_4$$

$$\frac{dx}{db} = 0 = \lambda_1 - \lambda_2 - \lambda_3 + \lambda_4 = 0 \quad -\mathrm{I}$$

$$\frac{dx}{dw_1} \Rightarrow w_1 = \lambda_1 - \lambda_2 - \lambda_3 + \lambda_4 \quad -\mathrm{II}$$

$$\frac{dx}{dw_2} \Rightarrow w_2 = \sqrt{2}\lambda_1 - \sqrt{2}\lambda_2 \quad -\mathrm{III}$$

$$\frac{dx}{dw_3} \Rightarrow w_3 = \sqrt{2}\lambda_1 - \lambda_3\sqrt{2} \quad -\mathrm{IV}$$

$$\frac{dx}{dw_4} \Rightarrow w_4 = \sqrt{2}\lambda_1 \quad -\mathrm{V}$$

$$\frac{dx}{dw_5} \Rightarrow w_5 = \lambda_1 - \lambda_2 \quad -\mathrm{VI}$$

$$\frac{dx}{dw_6} \Rightarrow w_6 = \lambda_1 - \lambda_3 \quad -\mathrm{VII}$$

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$$\frac{dx}{d\lambda_1} = 0 = w_1 + \sqrt{2}\,w_2 + \sqrt{2}\,w_3 + \sqrt{2}\,w_4 + w_5 + w_6 + b + 1 \quad - \overline{\text{VIII}}$$

$$\frac{dx}{d\lambda_2} = 0 = -w_1 - \sqrt{2}\,w_2 - w_5 - b + 1 \quad - \overline{\text{IX}}$$

$$\frac{dx}{d\lambda_3} = 0 = -w_1 - \sqrt{2}\,w_2 - w_6 - b + 1 \quad - \overline{\text{X}}$$

$$\frac{dx}{d\lambda_4} = 0 = w_1 + b + 1 \quad - \overline{\text{XI}}$$

$\Rightarrow$ from I & II, $w_1 = 0$ — $\overline{\text{XII}}$

$b = -1$ [from $\overline{\text{XII}}$ & $\overline{\text{XI}}$] — $\overline{\text{XIII}}$

$w_2 = \sqrt{2}\,w_5$ — $\overline{\text{XIV}}$

$w_5 = 0$ — $\overline{\text{XIV}}$ & $\overline{\text{IX}}$ $\rightarrow$ $\overline{\text{XV}}$

$w_2 = 0$ — $\overline{\text{IX}}$ & $\overline{\text{XV}}$ $\rightarrow$ $\overline{\text{XVI}}$

$w_6 = 0$ — $\overline{\text{X}}$ & $\overline{\text{XV}}$ $\rightarrow$ $\overline{\text{XVII}}$

$\lambda_1 = \lambda_2 = \lambda_3$ using $\overline{\text{VI}}$, $\overline{\text{VII}}$, $\overline{\text{XV}}$, $\overline{\text{XVII}}$

$\therefore$ $w_3 = 0$

$\therefore$ $w_4 = 0$

$\therefore$ $\lambda_1 = 0$

$\therefore$ $\lambda_2 = 0$

$\therefore$ $\lambda_3 = 0$

$\therefore$ $\lambda_4 = 0$

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therefore values are :-

$b = -1$

$\omega_1 = 0$

$\omega_2 = 0$

$\omega_3 = 0$

$\omega_4 = 0$

$\omega_5 = 0$

$\omega_6 = 0$

$\lambda_1 = 0$

$\lambda_2 = 0$

$\lambda_3 = 0$

$\lambda_4 = 0$

$\therefore$ Our Equation of Hyperplane thus is

$$w^T \phi(x) + b = 0$$