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The three methods for integration, namely Euler's explicit method, Euler's implicit method and Crank Nicholson method.

Explicit :

$$y_{n+1} = y_n + hf(x_n, y_n), \quad n = 0, \dots, N - 1;$$

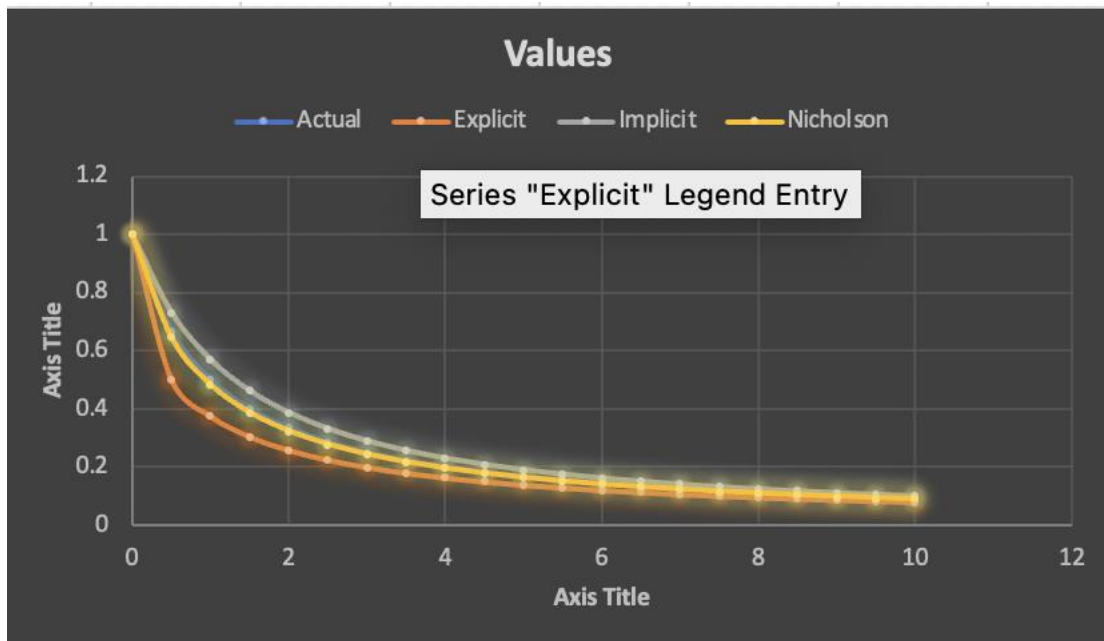
Implicit :

$$y_{n+1} = y_n + hf(x_{n+1}, y_{n+1}), \quad n = 0, \dots, N - 1;$$

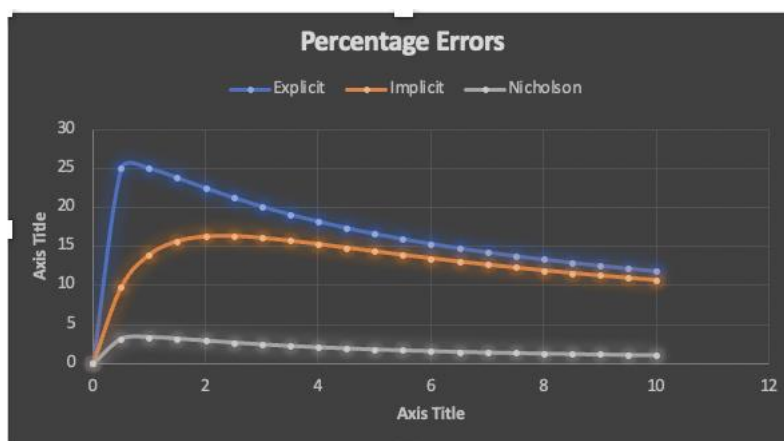
Crank Nicholson :

$$y_{n+1} = y_n + \frac{1}{2}h[f(x_n, y_n) + f(x_{n+1}, y_{n+1})], \quad n = 0, \dots, N - 1;$$

On analysing the graph between values of 'Y' and ' τ ', it is observable that Euler's explicit method is the least accurate while Nicholson's method is the most accurate one being the average of Euler's explicit and Euler's implicit method.



On plotting the graph for percentage errors, I found that the initial error for Euler's explicit method is about 25% highest among all three methods. Although, as the τ increases, the error for the explicit method goes down and the graph tends to converge with the implicit one.



The Crank Nicholson method is the most accurate one among all due to the modification of each 'Y' value. This is because it is the average value of implicit and explicit methods.

I have also mentioned the actual values of the integral by integrating it manually at every τ for $h=0.5$, to get a basic idea about the error witnessed.

It can also be concluded that as we increase the number of divisions, or as we decrease the value of 'h', the accuracy of the result increases. Also, the nature of graph Y v/s τ for every 'h' is kind of hyperbolic decreasing. Thus, the concentration decreases hyperbolically along with time.