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Part A:
Applying Trapezoidal, Simpson's 1/3 rule, Simpson's 3/8 rule, from previous assignment I am using codes for the same by changing the function to :

$$\int_0^1 x^{0.1}(1.2 - x)(1 - e^{20(x-1)})dx$$

The true error and value is (For Trapezoidal rule) :

```
Value is : 0.335764
Error is : 0.266534
Value is : 0.371322
Error is : 0.230976
Value is : 0.404866
Error is : 0.197432
Value is : 0.436315
Error is : 0.165983
Value is : 0.465597
Error is : 0.136701
Value is : 0.492646
Error is : 0.109652
Value is : 0.517386
Error is : 0.0849116
Value is : 0.53972
Error is : 0.0625785
Value is : 0.559478
Error is : 0.0428198
Value is : 0.576333
Error is : 0.0259651
Value is : 0.589588
Error is : 0.0127097
Value is : 0.597781
Error is : 0.00451724
Value is : 0.597986
Error is : 0.00431243
Value is : 0.585041
Error is : 0.0172575
Value is : 0.552466
Error is : 0.0498324
Value is : 0.503266
Error is : 0.0990321
Value is : 0.502036
Error is : 0.100262
Value is : 0.878854
Error is : -0.276556
Value is : 2.95391
Error is : -2.35162
Value is : 11.4734
Error is : -10.8711
Value is : 42.5057
Error is : -41.9034
```

The true error is coming negative after some value hence it will never always decrease.

For Simpsons 1/3 rule is :

```
Value is : 0.381047
Error is : 0.221251
Value is : 0.403593
Error is : 0.198705
Value is : 0.444782
Error is : 0.157516
Value is : 0.464466
Error is : 0.137832
Value is : 0.499883
Error is : 0.102415
Value is : 0.516516
Error is : 0.085782
Value is : 0.545746
Error is : 0.056552
Value is : 0.559033
Error is : 0.0432645
Value is : 0.581069
Error is : 0.0212289
Value is : 0.589992
Error is : 0.0123055
Value is : 0.600623
Error is : 0.00167541
Value is : 0.6008
Error is : 0.00149826
Value is : 0.583536
Error is : 0.0187621
Value is : 0.561747
Error is : 0.0405507
Value is : 0.49687
Error is : 0.105428
Value is : 0.495789
Error is : 0.106509
Value is : 0.998236
Error is : -0.395938
Value is : 2.3837
Error is : -1.7814
Value is : 13.6969
Error is : -13.0946
Value is : 34.4317
Error is : -33.8294
```

The true error is coming negative after some value hence it will never always decrease.

For Simpson's 3/8 is :

```
Error is :0.159415
Value is :0.442883
Error is :0.137384
Value is :0.464914
Error is :0.107365
Value is :0.494933
Error is :0.0797295
Value is :0.522568
Error is :0.0629254
Value is :0.539373
Error is :0.0410098
Value is :0.561288
Error is :0.0221932
Value is :0.580105
Error is :0.0122148
Value is :0.590083
Error is :0.00318406
Value is :0.599114
Error is :0.00301472
Value is :0.599283
Error is :0.0127242
Value is :0.589574
Error is :0.0491888
Value is :0.553109
Error is :0.104306
Value is :0.497992
Error is :0.105339
Value is :0.496959
Error is :-0.318596
Value is :0.920894
Error is :-2.65039
Value is :3.25269
Error is :-9.04481
Value is :9.6471
Error is :-43.8364
Value is :44.4387
```

The true error is coming negative after some value hence it will never always decrease.

Simpson's ⅓ Rule for Integration

We can get a quick approximation for definite integrals when we divide a small interval [a, b] into two parts. Therefore, after dividing the interval, we get;

x_0= a, x_1= a + b, x_2 = b

Hence, we can write the approximation as;

$\int_a^b f(x)\,dx \approx S_2 = h/3[f(x_0) + 4f(x_1) + f(x_2)]$

$S_2 = h/3\ [f(a) + 4\ f((a+b)/2) + f(b)]$

Where h = (b – a)/2

This is the Simpson's ⅓ rule for integration.

Simpson's 3/8 Rule

Another method of numerical integration is called "Simpson's 3/8 rule". It is completely based on the cubic interpolation rather than the quadratic interpolation. Simpson's 3/8 or three-eight rule is given by:

$$\int_a^b f(x)\,dx = 3h/8\ [(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \dots + y_{n-1}) + 2(y_3 + y_6 + y_9 + \dots + y_{n-3})]$$

This rule is more accurate than the standard method, as it uses one more functional value. For 3/8 rule, the composite Simpson's 3/8 rule also exists which is similar to the generalized form. The 3/8 rule is known as Simpson's second rule of integration.

Derivation of Trapezoidal Rule Formula

We can calculate the value of a definite integral by using trapezoids to divide the area under the curve for the given function.

Trapezoidal Rule Statement: Let f(x) be a continuous function on the interval (a, b). Now divide the intervals (a, b) into n equal sub-intervals with each of width,

Δx = (b - a)/n, such that $a = x_0 < x_1 < x_2 < x_3 < \dots < x_n = b$

Then the Trapezoidal Rule formula for area approximating the definite integral $\int_a^b f(x)dx$ is given by:

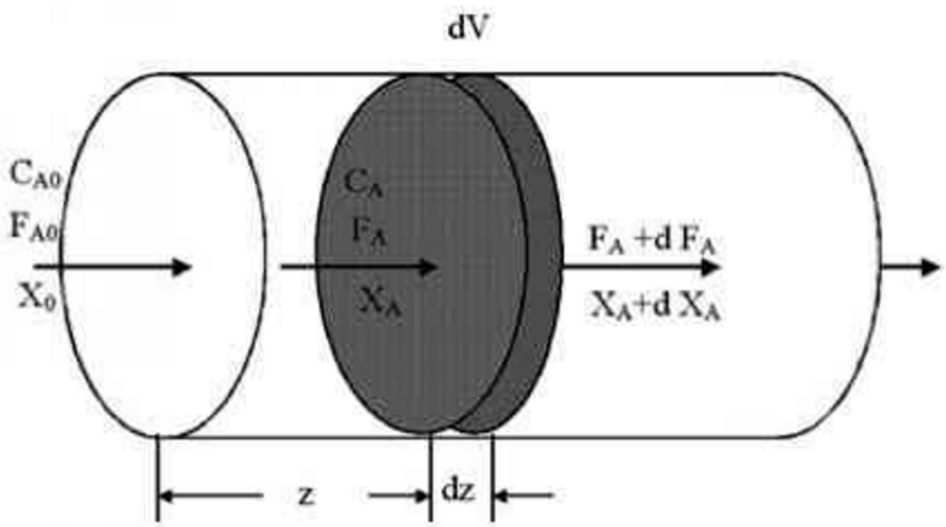
$$\int_a^b f(x)\,dx \approx T_n = \triangle x/2\ [f(x_0) + 2f(x_1) + 2f(x_2) + \dots 2f(x_{n-1}) + f(x_n)]$$

where, $x_i = a + i\triangle x$

If $n \to \infty$, R.H.S of the expression approaches the definite integral $\int_a^b f(x)dx$

Part B :

```
volume required :  4321.11
number of subdomains for simpson_1/3:  7
Volume required is :  2434.96
number of subdomains for trapezoidal:  680
Volume required is :  3609.17
number of subdomains for simpson_3/8:  11
rishabh@Rishabhs-MacBook-Air PartB %
```



In the plug flow reactor

$$V = \frac{F_{A0}}{kC_{A0}^{n}} \int_{0}^{X_{A\_EXIT}} \frac{dx_A}{(1-x_A)^n}$$