Seventh Edition

ELEMENTS OF MECHANICAL ENGINEERING

Er. R.K. RAJPUT

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MECHANICAL ENGINEERING

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As per latest syllabus of Punjab Technical University, Jalandhar

S.I. UNITS

By
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PART B

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PREFACE TO THE SEVENTH EDITION

I am pleased to present the **"Seventh Edition"** of this book titled **"Elements of Mechanical Engineering"**. The warm reception which the previous editions and reprints have enjoyed has been a matter of great satisfaction to me.

As per new syllabus of PTU, Jalandhar, three new chapters namely, **"Engineering Materials" "Centre of Gravity and Centroid"** and **"Moment of Inertia"** have been added to make the book complete in every respect.

Any suggestions for the improvement of this book will be thankfully received and incorporated in the next edition.

Er. R.K. Rajput
(Author)

PREFACE TO THE FIRST EDITION

This book has been written specifically to meet the exhaustive requirements of the subject **"Elements of Mechanical Engineering"** of *B.E. examination of* **Punjab Technical University, Jalandhar.**

The book includes comprehensive treatment of the subject matter under wide range of topics mentioned in the syllabus, with a large number of solved examples to support the text. Besides this, *Highlights, Objective Type Questions, Theoretical Questions* and *Unsolved Examples* have been added at the end of each chapter to make the book a comprehensive unit in all respects.

Any suggestions for the improvement of this book will be thankfully acknowledged and incorporated in the next edition.

Er. R.K. Rajput
(Author)

SYLLABUS (*Latest*)

Punjab Technical University, Jalandhar

B. Tech. 1st and 2nd Semester

BTME 101: ELEMENTS OF MECHANICAL ENGINEERING

PART-A

1.   BASIC CONCEPTS OF THERMODYNAMICS   **(08 hours)**

Definition of thermodynamics: Need to study thermodynamics; Application areas of thermodynamics; Difference between Microscopic (or, Statistical) thermodynamics and Macroscopic (or, Classical) thermodynamics; Brief concept of continuum. **Thermodynamic System:** Definition, types (Open, Closed and Isolated) and their examples; **Thermodynamic System Boundary:** Definition, types and their examples; **Surroundings**; Control (fixed) mass and Control Volume concept and their examples; Thermodynamic State; **Thermodynamic Property:** Definition, types citing their examples; condition for any quantity to be a property; State postulate; Thermodynamic equilibrium (which includes Thermal, Mechanical and Chemical equilibrium etc.); Thermodynamic path; **Thermodynamic process:** Definition, **concept of reversible process**, quasi-static (or, quasi-equilibrium) process, irreversible process, conditions for reversibility and how these are met with, non-flow processes and flow processes, method of representation of reversible and irreversible process on property diagrams; Cyclic process; **Thermodynamic Cycle:** Definition and its concept; Energy and its forms (microscopic and macroscopic); Physical insight to internal energy; Energy transfer across system boundary *i.e.,* transient energies (heat and work); Difference between heat and work; Sign conventions for heat and work interactions; heat and work as path functions; Equality of Temperature and Zeroth law of Thermodynamics.

2.   FIRST LAW OF THERMODYNAMICS AND ITS APPLICATIONS   **(12 hours)**

Definition, essence and corollaries or consequences of first law of Thermodynamics; Expressions for First law of Thermodynamics for a control mass undergoing a Cycle and for a process (*i.e.*, a change in state of a control mass); Concept of Enthalpy and total energy and differentiation between the two – a thermodynamic property; Compressible and incompressible substances, Specific heats, Difference between Internal Energy and Enthalpy of compressible and incompressible substances; Representation of first law of thermodynamics as rate equation; Analysis of non-flow/flow process for a control mass undergoing constant volume, constant pressure, constant temperature, adiabatic and polytropic processes; Free Expansion Process and its examples, its representation on Property diagram; Review of concepts of control volume; Expressions of first law of thermodynamics for a control volume (*i.e.*, open system); Steady State Steady Flow

process and its examples; First law analysis of Steady State Flow process *e.g.*, isochoric, isobaric, isothermal, isentropic and polytropic process; Throttling process and its applications; Flow energy or inertial energy of flowing fluids or, Energy transport by mass; Application of Steady State Flow Energy Equation to various engineering devices.

3. SECOND LAW OF THERMODYNAMICS (16 hours)

Limitations of first law of thermodynamics; and how second law is fully able to explain away and thus overcome those shortcomings of first law; Thermal Reservoirs, source and sink (Low temperature and high temperatures); **Heat Engine, Heat pump and Refrigerator:** Definitions, working, efficiency/performance and their real life examples. Justification as to why the actual efficiency of Heat Pump and Refrigerator shall also be ≤ 100% though on the face of it seems to be more than 100%; Various statements of Second Law of Thermodynamics and their equivalence; Philosophy of Carnot cycle and its consequences *viz.*, how each of the individual four processes constituting the cycle contribute in optimizing the output and efficiency of the cycle; **Carnot Engine, Carnot Refrigerator and Carnot Heat Pump:** Definitions, working, efficiency/ performance and Limitations of the cycle; Carnot theorem for heat engines, refrigerators and heat pumps; Derivation of Carnot efficiency/COP (which seems to be more than 100%); Thermodynamic Temperature Scale; Clausius theorem and Inequality; Philosophy and concept of entropy; Entropy changes during various processes; Temperature–Entropy Chart and representation of various processes on it; Principle of Increase of Entropy; Applications of Entropy Principle; Quality of Energy *viz.*, high and low grade energies; Degradation of Energy; Third Law of Thermodynamics.

PART-B

4. GAS POWER CYCLES (12 hours)

Introduction; Concept and philosophy of Air Standard Cycle along with associated assumptions and advantages; Air Standard Efficiency; Nomenclature of reciprocating piston-cylinder arrangement with basic definitions such as swept volume, clearance volume, compression ratio, mean effective pressure etc., Otto cycle (or constant volume heat addition cycle). Diesel cycle (or constant pressure heat addition cycle) and Dual cycle (Mixed or Composite or Limited Pressure cycle) with their representation on P-V and T-S Charts, their Air-standard (thermal) Efficiencies; Comparison of Otto, Diesel and Dual cycle under some defined similar parametric conditions; Introduction to heat engines; Merits of I.C. Engines and their important applications, Classification and constructional features of I.C. Engines; Working of two stroke and four stroke Petrol and Diesel engines and their comparison.

5. ENGINEERING MATERIALS (05 hours)

Materials and Civilization, Materials and Engineering, Classification of Engineering Materials, Mechanical Properties of Materials: elasticity, plasticity, strength, ductility, brittleness, malleability, toughness, resilience, hardness, machinability, formability, weldability. Properties, Composition, and Industrial Applications of materials: Metals

(ferrous-cast iron, tool steels, stainless steels and non- ferrous-Aluminium, brass, bronze), Polymers (natural and synthetic, thermoplastic and thermosetting), Ceramics (glass, optical fiber glass, cements), Composites (fiber reinforced, metal matrix), Smart materials (piezoelectric, shape memory, thermochromic, photochromic, magnetorheological), Conductors, Semi-conductors and Insulators, Organic and Inorganic materials. Selection of materials for engineering applications.

6. CENTROID, CENTRE OF GRAVITY AND MOMENT OF INERTIA   **(08 hours)**

Difference between centre of gravity and centroid. Determination of position of centroid of plane geometric figures of I, U, H, L, T, C, Circular and Triangular Sections. Centroid of Composite Areas. Determination of position of Centre of Gravity (CG) of regular solids *viz*. Right Circular Cone, Solid Hemisphere, thin Hollow Hemisphere. Area moment of inertia and mass moment of inertia, Polar moment of inertia, Parallel axes Theorem (or transfer formula), Perpendicular axes Theorem, Radius of gyration, determination of area Moment of Inertia of I, U, H, L, T, C, Circular and Triangular Sections along various axes. Mass moment of Inertia of Circular Ring, Disc, Cylinder, Sphere and Cone about their axis of symmetry and other axes.

PART A

1

Basic Concepts of Thermodynamics

1.1. DEFINITION OF THERMODYNAMICS

Thermodynamics may be defined as follows :

Thermodynamics is an axiomatic science which deals with the relations among heat, work and properties of system which are in equilibrium. It describes state and changes in state of physical systems.

Or

Thermodynamics is *the science of the regularities governing processes of energy conversion.*

Or

Thermodynamics is *the science that deals with the interaction between energy and material systems.*

Thermodynamics, basically entails *four laws* or axioms known as Zeroth, First, Second and Third law of thermodynamics.

— the *First law* throws light on *concept of internal energy*.

— the *Zeroth law* deals with *thermal equilibrium* and establishes a *concept of temperature*.

— the *Second law* indicates the limit of *converting heat into work* and introduces the *principle of increase of entropy*.

— third law defines the *absolute zero of entropy*.

These laws are based on experimental observations and have no *mathematical proof*. Like all physical laws, these laws are based on *logical reasoning*.

1.2. THERMODYNAMIC SYSTEMS

1.2.1. System, Boundary and Surroundings

System. A system is *a finite quantity of matter or a prescribed region of space* (Refer Fig. 1.1)

3

Boundary. The *actual or hypothetical envelope enclosing the system* is the boundary of the system. The boundary may be fixed or it may move, as and when a system containing a gas is compressed or expanded. The boundary may be *real* or *imaginary*. It is not difficult to envisage a real boundary but an example of imaginary boundary would be one drawn around a system consisting of the fresh mixture about to enter the cylinder of an I.C. engine together with the remanants of the last cylinder charge after the exhaust process (Refer Fig. 1.2).
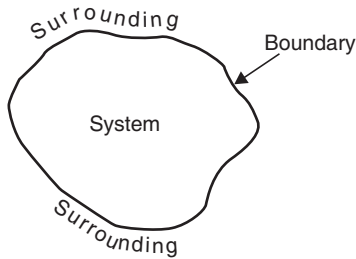
Fig. 1.1. The system.
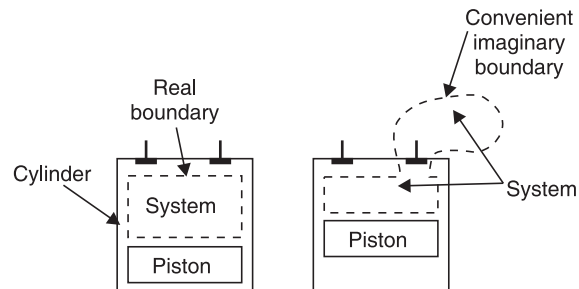
Fig. 1.2. The real and imaginary boundaries.

1.2.2. Closed System

Refer Fig. 1.3. If the boundary of the system is impervious to the flow of matter, it is called a *closed system*. An example of this system is mass of gas or vapour contained in an engine cylinder, the boundary of which is drawn by the cylinder walls, the cylinder head and piston crown. Here the *boundary is continuous and no matter may enter or leave*.
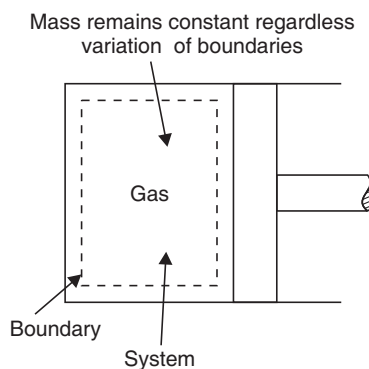
Fig. 1.3. Closed system.
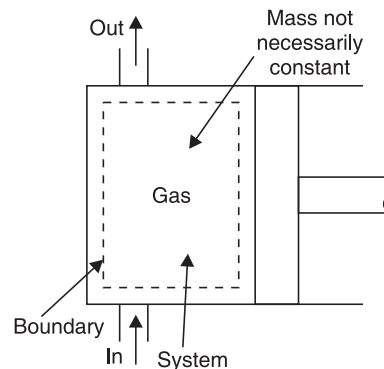
Fig. 1.4. Open system.

1.2.3. Open System

Refer Fig. 1.4. An open system is one in which *matter flows into or out of the system*. Most of the engineering systems are open.

1.2.4. Isolated System

An isolated system is that system *which exchanges neither energy nor matter with any other system or with environment*.

1.2.5. Adiabatic System

An adiabatic system is one *which is thermally insulated from its surroundings*. It can, however, *exchange work with its surroundings*. If it does not, it becomes an isolated system.

Phase. A phase is a quantity of matter which is homogeneous throughout in chemical composition and physical structure.

1.2.6. Homogeneous System

A system which consists of a single phase is termed as *homogeneous system*. Examples : Mixture of air and water vapour, water plus nitric acid and octane plus heptane.

1.2.7. Heterogeneous System

A system which consists of two or more phases is called a *heterogeneous system*. Examples : Water plus steam, ice plus water and water plus oil.

1.3. MACROSCOPIC AND MICROSCOPIC POINTS OF VIEW

Thermodynamic studies are undertaken by the following two different approaches.

1. Macroscopic approach—(*Macro* mean *big* or *total*)

2. Microscopic approach—(*Micro* means *small*)

These approaches are discussed (in a comparative way) below :

S. No.	Macroscopic approach	Microscopic approach
1.	In this approach a certain quantity of matter is considered *without* taking into account the events occurring at molecular level. In other words this approach to thermodynamics is concerned with *gross or overall behaviour*. This is known as *classical thermodynamics*.	The approach considers that the system is made up of a very large number of discrete particles known as *molecules*. These molecules have different velocities and energies. The values of these energies are constantly changing with time. This approach to thermodynamics which is concerned directly with the *structure of the matter* is known as *statistical thermodynamics*.
2.	The analysis of macroscopic system requires simple mathematical formulae.	The behaviour of the system is found by using statistical methods as the number of molecules is very large. So advanced statistical and mathematical methods are needed to explain the changes in the system.
3.	The values of the properties of the system are their average values. For example, consider a sample of a gas in a closed container. The *pressure* of the gas is the average value of the pressure exerted by millions of individual molecules. Similarly the *temperature* of this gas is the average value of translational kinetic energies of millions of individual molecules. These properties like *pressure* and *temperature* can be measured very easily. *The changes in properties can be felt by our senses.*	The properties like *velocity, momentum*, impulse, kinetic energy, force of impact etc. which describe the molecule *cannot be easily measured by instruments. Our senses cannot feel them*.
4.	In order to describe a system only a few properties are needed.	Large number of variables are needed to describe a system. So the approach is complicated.

Note. Although the macroscopic approach seems to be different from macroscopic one, there exists a relation between them. Hence when both the methods are applied to a particular system, they give the same result.

1.4. PURE SUBSTANCE

A pure substance is one that has a homogeneous and invariable chemical composition even though there is a change of phase. In other words, it is a system which is (*a*) homogeneous in composition, (*b*) homogeneous in chemical aggregation. Examples : Liquid, water, mixture of liquid water and steam, mixture of ice and water. The mixture of liquid air and gaseous air is not a pure substance.

1.5. THERMODYNAMIC EQUILIBRIUM

A system is in *thermodynamic equilibrium* if the temperature and pressure at all points are same ; there should be no velocity gradient ; the chemical equilibrium is also necessary. Systems under temperature and pressure equilibrium but not under chemical equilibrium are sometimes said to be in metastable equilibrium conditions. *It is only under thermodynamic equilibrium conditions that the properties of a system can be fixed.*

Thus for attaining a state of *thermodynamic equilibrium* the following three types of equilibrium states must be achieved :

1. **Thermal equilibrium.** The temperature of the system does not change with time and has same value at all points of the system.

2. **Mechanical equilibrium.** There are no unbalanced forces within the system or between the surroundings. The pressure in the system is same at all points and does not change with respect to time.

3. **Chemical equilibrium.** No chemical reaction takes place in the system and the chemical composition which is same throughout the system does not vary with time.

1.6. PROPERTIES OF SYSTEMS

A property of a system is a characteristic of the system which depends upon its state, but not upon how the state is reached. There are two sorts of property :

1. **Intensive properties.** These properties *do not depend on the mass of the system. Examples* : Temperature and pressure.

2. **Extensive properties.** These properties *depend on the mass of the system. Example* : Volume. Extensive properties are often divided by mass associated with them to obtain the intensive properties. For example, if the volume of a system of mass m is V, then the specific volume of

matter within the system is $\dfrac{V}{m} = v$ which is an intensive property.

1.7. STATE

State is the condition of the system at an instant of time as described or measured by its properties. Or each unique condition of a system is called a state.

It follows from the definition of state that each property has a single value at each state. Stated differently, all properties are *state* or *point functions*. Therefore, all properties are identical for identical states.

On the basis of the above discussion, we can determine if a given variable is *property* or not by applying the following *tests* :

— *A variable is a property, if and only if, it has a single value at each equilibrium state.*

— A variable is a property, if and only if, the change in its value between any two prescribed equilibrium states is single-valued.

Therefore, *any variable whose change is fixed by the end states is a property*.

1.8. PROCESS

A process occurs when the system undergoes a change in a state or an energy transfer at a steady state. A process may be *non-flow* in which a fixed mass within the defined boundary is undergoing a change of state. Example : a substance which is being heated in a closed cylinder undergoes a non-flow process (Fig. 1.3). *Closed systems undergo non-flow processes.* A process may be a flow process in which mass is entering and leaving through the boundary of an open system. In a steady flow process (Fig. 1.4) mass is crossing the boundary from surroundings at entry, and an equal mass is crossing the boundary at the exit so that the total mass of the system remains constant. In an open system it is necessary to take account of the work delivered from the surroundings to the system at entry to cause the mass to enter, and also of the work delivered from the system at surroundings to cause the mass to leave, as well as any heat or work crossing the boundary of the system.

Quasi-static process. Quasi means 'almost'. A quasi-static process is also called a *reversible process. This process is a succession of equilibrium states and infinite slowness is its characteristic feature.*

1.9. CYCLE

Any process or series of processes whose end states are identical is termed a *cycle*. The processes through which the system has passed can be shown on a state diagram, but a complete section of the path requires in addition a statement of the heat and work crossing the boundary of the system. Fig. 1.5 shows such a cycle in which a system commencing at condition '1' changes in pressure and volume through a path 123 and returns to its initial condition '1'.

1.10. POINT FUNCTION

When two properties locate a *point* on the graph (co-ordinate axes) then those properties are called as *point function*.

Examples. Pressure, temperature, volume etc.

$$\int_1^2 dV = V_2 - V_1 \text{ (an } exact\ differential\text{)}.$$

Fig. 1.5. Cycle of operations.

1.11. PATH FUNCTION

There are certain quantities which cannot be located on a graph by a *point* but are given by the *area* or so, on that graph. In that case, the area on the graph, pertaining to the particular process, is a function of the path of the process. Such quantities are called *path functions*.

Examples. Heat, work etc.

Heat and work are *inexact differentials*. Their change cannot be written as difference between their end states.

Thus $\int_1^2 \delta Q \neq Q_2 - Q_1$ and is shown as $_1Q_2$ or Q_{1-2}

Similarly $\int_1^2 \delta W \neq W_2 - W_1$ and is shown as $_1W_2$ or W_{1-2}

Note. The operator δ is used to denote inexact differentials and operator d is used to denote exact differentials.

1.12. TEMPERATURE

- *The temperature is a thermal state of a body which distinguishes a hot body from a cold body*. The temperature of a body is *proportional to the stored molecular energy i.e.* the average molecular kinetic energy of the molecules in a system. (A particular molecule does not have a temperature, it has energy. The gas as a system has temperature).

- Instruments for measuring ordinary temperatures are known as thermometers and those for measuring high temperatures are known as *pyrometers*.

- It has been found that a gas will not occupy any volume at a certain temperature. This temperature is known as *absolute zero temperature*. The temperatures measured with absolute zero as basis are called *absolute temperatures*. Absolute temperature is stated in degrees centigrade. The point of absolute temperature is found to occur at 273.15°C below the freezing point of water.

Then : Absolute temperature = thermometer reading in °C + 273.15.

Absolute temperature is degree centigrade is known as degrees kelvin, denoted by K (SI unit).

1.13. ZEROTH LAW OF THERMODYNAMICS

- **'Zeroth law of thermodynamics'** *states that if two systems are each equal in temperature to a third, they are equal in temperature to each other*.

Example. Refer Fig. 1.6. System '1' may consist of a mass of gas enclosed in a rigid vessel fitted with a pressure gauge. If there is no change of pressure when this system is brought into contact with system '2' a block of iron, then the two systems are equal in temperature (assuming that the systems 1 and 2 do not react each other chemically or electrically). Experiment reveals that if system '1' is brought into contact with a third system '3' again with no change of properties then systems '2' and '3' will show no change in their properties when brought into contact provided they do not react with each other chemically or electrically. Therefore, '2' and '3' must be in equilibrium.

- This law was enunciated by R.H. Fowler in the year 1931. However, since the first and second laws already existed at that time, it was designated as *zeroth law* so that it *precedes* the first and second laws *to form a logical sequence*.
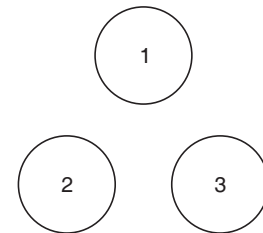
Fig. 1.6. Zeroth law of thermodynamics.

1.14. THE THERMOMETER AND THERMOMETRIC PROPERTY

1.14.1. Introduction

The zeroth law of thermodynamics provides the basis for the measurement of temperature. It enables us to compare temperatures of two bodies '1' and '2' with the help of a third body '3' and say that the temperature of '1' is the same as the temperature of '2' without actually bringing '1' and '2' in thermal contact. In practice, body '3' in the zeroth law is called the **thermometer**. It is brought into thermal equilibrium with a set of standard temperature of a body '2', and is thus calibrated. Later, when any other body '1' is brought in thermal communication with the thermometer, we say that the body '1' has attained equality of temperature with the thermometer, and hence with body '2'. This way, the body '1' has the temperature of body '2' given for example by, say the height of mercury column in the thermometer '3'.

- The height of mercury column in a thermometer, therefore, becomes a **thermometric property**.

There are other methods of temperature measurement which utilize various other properties of materials, those are functions of temperature, as thermometric properties.

Six different kinds of thermometers, and the names of the corresponding thermometric properties employed are given below :

Thermometer	Thermometric property
1. Constant volumes gas	Pressure (p)
2. Constant pressure gas	Volume (V)
3. Alcohol or mercury-in-glass	Length (L)
4. Electric resistance	Resistance (R)
5. Thermocouple	Electromotive force (E)
6. Radiation (pyrometer)	Intensity of radiation (I or J)

1.14.2. Measurement of Temperature

Temperature can be depicted as a *thermal state which depends upon the internal or molecular energy of the body*.

1.14.2.1. Temperature Measuring Instruments

These instruments may be classified in two broad categories :

1. **Non-electrical methods :**

 (*i*) By using change in volume of a liquid when its temperature is changed.

(*ii*) By using change in pressure of a gas when its temperature is changed.

(*iii*) By using changes in the vapour pressure when the temperature is changed.

2. Electrical methods :

 (*i*) By thermocouples.

(*ii*) By change in resistance of material with change in temperature.

(*iii*) By comparing the colours of filament and the object whose temperature is to be found out.

(*iv*) By ascertaining the energy received by radiation.

 The **thermometers** may also be classified as follows :

 1. Expansion thermometers

 (*i*) Liquid-in-glass thermometers

(*ii*) Bimetallic thermometers.

 2. Pressure thermometers

 (*i*) Vapour pressure thermometers

(*ii*) Liquid-filled thermometers

(*iii*) Gas-filled thermometers.

 3. Thermocouple thermometers

 4. Resistance thermometers

 5. Radiation pyrometers

 6. Optical pyrometers.

 1. Expansion Thermometers. The expansion thermometers make use of the differential expansion of two different substances. Thus in liquid-in-glass thermometers, it is the difference in expansion of liquid

Fig. 1.7. Mercury-in-glass thermometer.

and the containing glass. And in bimetallic thermometers, the indication is due to the difference in expansion of the two solids. These thermometers are discussed below :

(*i*) **Liquid-in-glass thermometers.** This is a very familiar type of thermometer. The mercury or other liquid fills the glass bulb and extends into the bore of the glass stem. Mercury is the most suitable liquid and is used from – 38.9°C (melting point) to about 600°C. The thermometers employed in the laboratory have the scale engraved directly on the glass stem. A usual type of mercury-in-glass thermometer is shown in Fig. 1.7. An expansion bulb is usually provided at the top of the stem to allow room for expansion of mercury, in case the thermometer is subjected to temperature above its range. The upper limit for mercury-in-glass thermometers is about 600°C. As the upper limit is far above the boiling point of mercury, some inert gas *i.e.* nitrogen is introduced above the mercury to prevent boiling.

Pentane, ethyl alcohol and toluene are the other liquids which can be used for liquid-in-glass thermometers. Since these liquids are normally colourless a dye is added to facilitate reading. These liquids have a low freezing point as shown below and are suitable for low temperature thermometer.

Liquid	Boiling point	Freezing point
Pentane	36°C	– 130°C
Ethyl alcohol	78°C	– 100°C
Toluene	110°C	– 92°C

(*ii*) **Bimetallic thermometers.** In a bimetallic thermometer differential expansion of bimetallic strips is used to indicate the temperature. It has the advantage over the liquid-in-glass thermometer, that it is *less fragile* and is *easier to read*. In this type of thermometer two flat strips of different metals are placed side by side and are welded together. Many different metals can be used for this purpose. Generally one is a *low expanding metal* and the other is *high expanding metal*. The bimetal strip is coiled in the form of a spiral or helix. *Due to rise in temperature, the curvature of the strip changes. The differential expansion of a strip causes the pointer to move on the dial of the thermometer*.

2. Pressure thermometers. In pressure thermometers liquids, gases and vapour can all be used. The principle on which they work is quite simple. The fluid is confined in a closed system. In this case the *pressure is a function of the temperature*, so that when the fluid is heated, the pressure will rise. And the temperature can be indicated by Bourdon type pressure gauge. In general, the thermometer consists of a bulb which contains bulk of the fluid. The bulb is placed in the region whose temperature is required. A capillary tube connects the bulb to a Bourdon tube, which is graduated with a temperature scale.

Pressure thermometers are discussed below :

(*i*) **Vapour pressure thermometers.** A schematic diagram of a vapour pressure thermometer is shown in Fig. 1.8. When the bulb containing the fluid is installed in the region whose temperature is required, some of the fluid vapourizes, and increases the vapour pressure. This change of pressure is indicated on the Bourdon tube. The relation between temperature and vapour pressure of a volatile liquid is of the *exponential form*. Therefore, the scale of a vapour pressure thermometer will not be linear.

(*ii*) **Liquid-filled thermometers.** A liquid-filled thermometer is shown in Fig. 1.9. In this case, the *expansion of the liquid causes the pointer to move in the dial*. Therefore liquids having high co-efficient of expansion should be used. In practice many liquids *e.g.* mercury, alcohol, toluene and glycerine have been successfully used. The operating pressure varies from about 3 to 100 kgf/cm^2. These type of thermometers could be used for a temperature upto 650°C in which mercury could be used as the liquid.
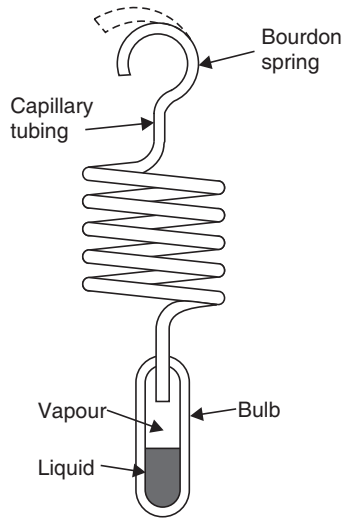
Fig. 1.8. Vapour pressure thermometer.
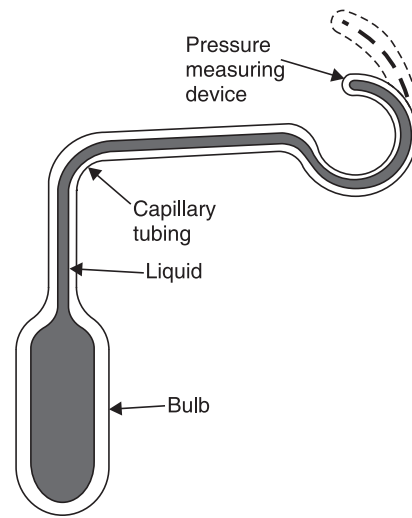
Fig. 1.9. Liquid-filled thermometer.

In actual design, the internal diameter of the capillary tube and Bourdon tube is, made much smaller than that of the bulb. This is because the capillary tube is subjected to a temperature which is quite different from that of the bulb. Therefore, to minimise the effect of variation in temperature to which the capillary tube is subjected, the *volume of the bulb is made as large as possible as compared with the volume of the capillary*. However, large volume of bulb tends to increase time lag, therefore, a *compensating device is usually built into the recording or indicating mechanism, which compensates the variations in temperature of the capillary and Bourdon tubes*.

(*iii*) **Gas-filled thermometers.** The temperature range for gas thermometer is practically the same as that of liquid filled thermometer. The gases used in the gas thermometers are *nitrogen* and *helium*. Both these gases are chemically inert, have good values for their co-efficient of expansion and have low specific heats. The construction of this type of thermometer is more or less the same as mercury-thermometer in which Bourdon spring is used. The errors are also compensated likewise. The only difference in this case is that bulb is made much larger than used in liquid-filled thermometers. For good performance the volume of the bulb should be made at least 8 times than that of the rest of the system.

These thermometers are generally used for pressures below 35 kgf/cm^2.

3. Thermocouple thermometers. For higher range of temperature *i.e.* above 650°C, filled thermometers are unsuitable. For higher range of temperature, thermocouples and pyrometers are used.
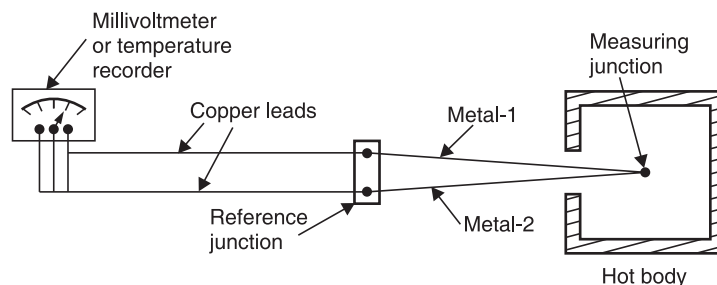
Fig. 1.10. Thermocouple.

In its smallest form a thermocouple consists of *two dissimilar metals or alloys which develop e.m.f. when the reference and measuring functions are at different temperatures*. The reference junction or cold junction is usually maintained at some constant temperature, such as 0°C. Fig. 1.10, shows a simple circuit of a thermocouple and the temperature measuring device. In many industrial installations the instruments are equipped with automatic compensating devices for temperature changes of the reference junction, thus eliminating the necessity of maintaining this junction at constant temperature.

Table 1.1 gives the composition, useful temperatures range and temperature versus e.m.f. relationship for some commercial thermocouples.

Table 1.1. Composition, Useful Temperature Range and e.m.f. produced for Some Thermocouples

| S.No. | Thermocouple | Composition | Temperature (°C) | | Thermoelectric power | | Remarks |
			Useful range	Max.	°C	Millivolt	
1.	Platinum vs Platinum-rhodium	Pure platinum vs Pt + 10 or 13% Rh	400 to 1450	1700	0 500 1000 1500	0.0 4.219 9.569 15.498	Used for high temperature measurements
2.	Chromel vs alumel	90% Ni + 10% Cr vs 95% Ni +5% (Al + Sn)Ma	– 200 to 1200	1450	– 200 0 300 600 900 1200	– 5.75 0.0 12.21 24.90 37.36 48.85	High resistance to oxidation
3.	Iron vs constan-tan	Pure iron vs 45–60% Cu + 55–40% Ni	– 200 to 750	1000	– 200 0 300 600 900	– 8.27 0.0 16.59 33.27 52.29	—
4.	Copper vs constantan	Pure copper vs Cu-Ni constan-tan	– 200 to 350	600	– 200 0 200 400	– 5.539 0.0 9.285 20.865	Not suitable in air due to exces-sive oxidation

4. Resistance thermometers. The electrical resistance of the metals increases with the temperature. This fact is made use of in resistance thermometers which are purely electrical in nature. *A resistance thermometer is used for precision measurements below 150°C*.

A simple resistance thermometer consists of a *resistance element* or *bulb, electrical loads* and *a resistance measuring or recording instrument*. The resistance element (temperature sensitive element) is usually supplied by the manufacturers with its protecting tube and is ready for electrical connections. The resistance of the metal used as resistance element should be reproducible at any given temperature. *The resistance is reproducible if the composition or physical properties of the metal do not change with temperature. For this purpose platinum is preferred*. A *platinum resistance thermometer* can measure temperatures to within ± 0.01°C. However, because of high cost of platinum, nickel and copper are used as resistance elements

for industrial purposes for low temperatures. The fine resistance wire is wound in a spiral form on a mica frame. The delicate coil is then enclosed in a porcelain or quartz tube. The change of resistance of this unit can be measured by instruments such as Wheatstone bridge, potentiometer or galvanometer.

Advantages :

The resistance thermometers possess the following **advantages** over other devices :

1. A resistance thermometer is very accurate for low ranges below 150°C.
2. It requires no reference junction like thermocouples and as such is more effective at room temperature.
3. The distance between the resistance element and the recording element can be made much larger than is possible with pressure thermometers.
4. It resists corrosion and is physically stable.

Disadvantages :

1. The resistance thermometers cost more.
2. They suffer from time lag.

5. Radiation pyrometers. A device which measures the total intensity of radiation emitted from a body is called **radiation pyrometer**.

The elements of a total radiation pyrometer are illustrated in Fig. 1.11. It collects the radiation from an object (hot body) whose temperature is required. A mirror is used to focus this radiation on a thermocouple. This energy which is concentrated on the thermocouple raises its temperature, and in turn generates an e.m.f. This e.m.f. is then measured either by the galvanometer or potentiometer method. Thus rise of temperature is a function of the amount of radiation emitted from the object.
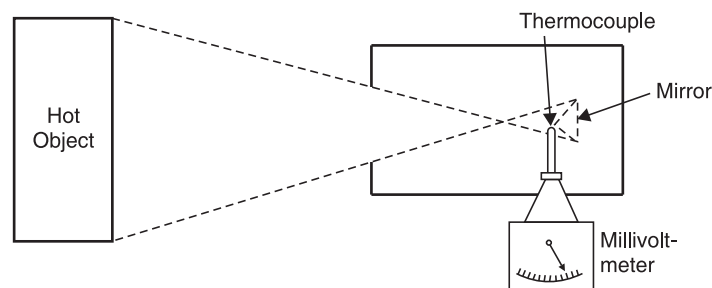
Fig. 1.11. A schematic diagram of radiation pyrometer.

Advantages of the pyrometers :

1. The temperature of moving objects can be measured.
2. A higher temperature measurement is possible than that possible by thermocouples etc.
3. The average temperatures of the extended surface can be measured.
4. The temperature of the objects which are not easily accessible can be measured.

6. Optical pyrometers. An optical pyrometer works on the principle *that matters glow above 480°C and the colour of visible radiation is proportional to the temperature of the glowing matter*. The amount of light radiated from the glowing matter (solid or liquid) is measured and employed to determine the temperature.

Fig. 1.12 shows a *disappearing filament pyrometer*.
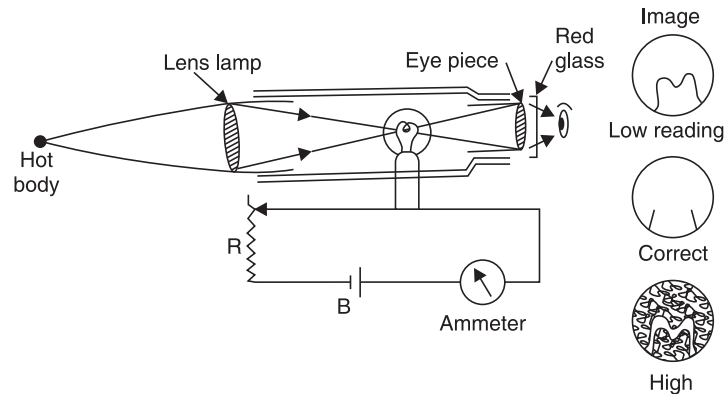
Fig. 1.12. Optical pyrometer.

Operation :

- The optical pyrometer is sighted at the hot body and focused.
- In the beginning filament will appear dark as compared to the background which is bright (being hot).
- By varying the resistance (R) in the filament circuit more and more current is fed into it, till filament becomes equally bright as the background and hence disappears.
- The current flowing in the filament at this stage is measured with the help of an ammeter which is calibrated directly in terms of temperature.
- If the filament current is further increased, the filament appears brighter as compared to the background which then looks dark.
- An optical pyrometer can measure temperature ranging from 700 to 4000°C.

Table 1.2 gives the summary of temperature range of different instruments on next page.

1.14.3. The International Practical Temperature Scale

For the calibration of thermometric instruments the Seventh General Conference on Weight and Measures held in 1927 formulated a convenient scale known as the **International Practical Temperature Scale**. It was revised at Thirteenth General Conference in 1968. It consists of *reproducible reference temperatures or primary fixed points* defined by a number of pure substances with assigned values of temperatures determined with precision on ideal or perfect gas temperature scale as given in Table 1.3.

Table 1.2. Summary of Temperature Range of Different Instruments

Type of Instru-ment	Liquid in Glass Thermo-meter	Bimetal-lic Ther-mometer	Pressure Thermometers			Thermocouples				Resistance Thermometers			Radia-tion Pyro-meter	Optical Pyro-meters
	Mercury		Vapour Pres-sure	Liquid filled	Gas filled	Iron Con-stantan	Copper Con-stantan	Chro-mel Alumel	Platinum Platinum-Rhodium	Copper	Nickel	Plati-num	—	—
Low tempe-rature limit	– 39°C	– 75°C	– 250°C	– 87°C	– 269°C	– 196°C	– 250°C	– 184°C	0°C	– 140°C	– 184°C	– 184°C	about room temp.	760°C
High tempe-rature limit	600°C	540°C	316°C	650°C	540°C	760°C	350°C	1260°C	1540°C	120°C	316°C	760°C	as high as desired	as high as desired
Remarks	Low price, Fairly ac-curate, Easily broken	Easier to read than Liquid-in-glass thermo-meters	Normal-ly scale non-linear	To be compen-sated for varia-tion in am-bient tempe-ratures	Re-quires large bulb, Linear scale	Widely used in Industry	Con-stantan is 60% Cu, 40% Ni	Chromel is 20% Cr, 80% Ni Alumel is 2% Al, 98% Ni	Expen-sive	Very sensitive. Suitable for narrow range Instruments.			—	—

**Table 1.3. Fixed Points of the International
Practical Temperature Scale of 1968**

Equilibrium state	Assigned value of temperature	
	T, K	t, °C
1. Triple point of hydrogen	13.81	– 259.34
2. Boiling point of hydrogen at 33.306 kPa	17.042	– 266.108
3. Normal boiling point of hydrogen	20.28	– 252.87
4. Normal boiling point of neon	27.102	– 246.048
5. Triple point of oxygen	54.361	– 218.789
6. Normal boiling point of oxygen	90.188	– 182.962
7. Triple point of water	273.16	0.01
8. Normal boiling point of water	373.15	100.00
9. Normal freezing point of antimony (antimony point)	630.74	357.59
10. Normal freezing point of zinc (zinc point)	692.73	419.58
11. Normal freezing point of silver (silver point)	1235.08	961.93
12. Normal freezing point of gold (gold point)	1337.58	1064.43

It is stated here that :

— The *triple point* represents an equilibrium state between solid, liquid and vapour phases of a substance.

— Normal *boiling point* is the temperature at which the substance boils at standard atmospheric pressure of 760 mm Hg.

— *Normal freezing point* is the solidification or the melting point temperature of the substance at standard atmospheric pressure.

Based on the available method of measurement, the whole temperature scale may be divided into *four ranges*. The equations for interpolation for each range are as follows :

1. From – 259.34°C (triple point of hydrogen) to 0°C :

A *platinum resistance thermometer of a standard design* is used and a polynomial of the following form is fitted between the resistance of the wire R_t and temperature t

$$R_t = R_0 (1 + At + Bt^2 + Ct^3) \qquad \qquad ...(1.1)$$

where, R_0 = resistance at the ice point.

2. From 0°C to 630.74°C (Antimony point) :

• It is also based on *platinum resistance thermometer.*

• The diameter of the platinum wire must lie between 0.05 and 0.2 mm.

3. From 630.74°C to 1064.43°C (Gold point) :

• It is based on *standard platinum versus platinum-rhodium thermocouple.*

• Following equation between e.m.f. E and temperature t is employed :

$$E = a + bt + ct^2 \qquad \qquad ...(1.2)$$

4. Above 1064.43°C :

• It is *based on the intensity of radiation J_T at temperature T emitted by a black body at a wavelength λ in the visible spectrum and by comparing this to the intensity of radiation J_{Au} at the same wavelength emitted by a black body at the gold point.*

- The temperature is calculated from Planck's equation for black body radiation

$$\frac{J_T}{J_{Au}} = \frac{\exp.\left(\dfrac{C_2}{\lambda T_{Au}}\right) - 1}{\exp.\left(\dfrac{C_2}{\lambda T}\right) - 1} \qquad \text{...(1.3)}$$

where, C_2 = 0.01438 in °C, and λ = wavelength in metres.

Following points are worth noting for gas thermometers :

- *The gas thermometers are never used for the measurement of temperatures.* However, *they are ideal when used for calibration for establishing the ideal gas temperature scale, and for establishing a standard because of precision, reproducible results, and their reading being independent of the thermometric substance used.*
- The gas thermometers can be used only for temperatures upto which gases do not liquify.

Method in use before 1954 :

- Celsius and Fahrenheit scales are the two commonly used scales for the measurement of temperature. Symbols C and F are respectively used to denote the readings on these two scales. Until 1954 the temperature scales were based on two fixed points : (*i*) the *steam point* (boiling point of water at standard atmospheric pressure), and (*ii*) the *ice point* (freezing point of water).

The fixed points for these temperature scales are :

Temperature	Celsius scale	Fahrenheit scale
Steam point	100	212
Ice point	0	32
Interval	100	180

- The relation between a particular value C on celcius scale and F on Fahrenheit scale is found to be as mentioned below :

$$\frac{C}{100} = \frac{F - 32}{180} \quad \text{or} \quad \frac{C}{5} = \frac{F - 32}{9} \qquad \text{...(1.4)}$$

Further the relation between a temperature difference Δt_F on Fahrenheit scale and Δt_C on celsius scale is

$$\Delta t_F = \frac{180}{100}\ \Delta t_C = \frac{9}{5}\ \Delta t_C = 1.8\ \Delta t_C \qquad \text{...(1.5)}$$

- The use of two fixed points was found unsatisfactory and later *abandoned* because of the following reasons :

  (*i*) It is difficult to achieve equilibrium between pure ice and air-saturated water (since when ice melts, it surrounds itself only with pure water and prevents intimate contact with air-saturated water).

  (*ii*) There is extreme sensitiveness of the steam point to the change in pressure.

Method in use after 1954 :

It was suggested by kelvin that a single fixed point only was necessary to establish a temperature. He pointed out that *tripe point of water* (the state at which ice, liquid water and water vapour coexist in equilibrium), could be used as the single point. The tenth CGPM, in 1954, adopted this fixed point, and value was set at 0.01°C or 273.16 K in the Kelvin scale thus established. Correspondingly, the ice point of 0°C on the celsius scale becomes equal to 273.15 K on the kelvin scale. Celsius and kelvin scales are distinguished by using distinct symbols t and T, the relation between these two is then given by :

$$T(K) = t(°C) + 273.15 \qquad \text{...(1.6)}$$

1.14.4. Ideal Gas

From experimental observations it has been established that an ideal gas to a good approximation behaves according to the simple equation

$$pV = mRT \qquad \qquad \text{...(1.7)}$$

where p, V and T are the pressure, volume and temperature of gas having mass m and R is a constant for the gas known as its *gas constant*.

Eqn. (1.7) can be written as $\qquad pv = RT \qquad \qquad \text{...(1.8)} \qquad \left(where\ v = \dfrac{V}{m} \right)$

In reality there is no gas which can be qualified as an ideal or perfect gas. However *all gases tend to ideal or perfect gas behaviour at all temperatures as their pressure approaches* **zero pressure**.

For two states of the gas, (1.7) can be written as,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad \text{or} \quad \frac{T_2}{T_1} = \frac{p_2}{p_1} \times \frac{V_2}{V_1} \qquad \qquad \text{...(1.9)}$$

With the help of this eqn. (1.9), the temperatures can be measured or compared.

1.15. PRESSURE

1.15.1. Definition of Pressure

Pressure is defined as a *force per unit area*. Pressures are exerted by gases, vapours and liquids. The instruments that we generally use, however, record pressure as the difference between two pressures. Thus, it is the *difference between the pressure exerted by a fluid of interest and the ambient atmospheric pressure*. Such devices indicate the pressure either above or below that of the atmosphere. When it is *above the atmospheric pressure*, it is termed *gauge pressure* and is *positive*. When it is *below* atmospheric, it is *negative* and is known as *vacuum*. Vacuum readings are given in millimetres of mercury or millimetres of water below the atmosphere.

It is necessary to establish an absolute pressure scale which is independent of the changes in atmospheric pressure. A pressure of absolute zero can exist only in complete vacuum. *Any pressure measured above the absolute zero of pressure* is termed an '*absolute pressure*'.

A schematic diagram showing the *gauge pressure, vacuum pressure* and the *absolute pressure* is given in Fig. 1.13.
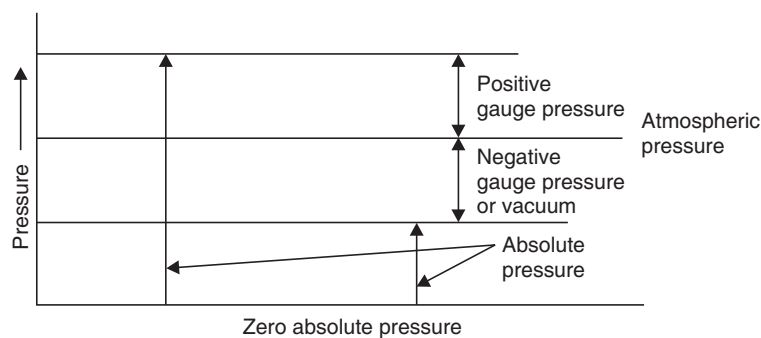
Fig. 1.13. Schematic diagram showing gauge, vacuum and absolute pressures.

Mathematically :

(*i*) Absolute pressure = Atmospheric pressure + Gauge pressure

$$p_{\text{abs}} = p_{atm.} + p_{\text{gauge}}.$$

(*ii*) Vacuum pressure = Atmospheric pressure – Absolute pressure.

Vacuum is defined as the *absence of pressure*. A *perfect vacuum* is obtained when *absolute pressure is zero*, at this instant *molecular momentum is zero*.

Atmospheric pressure is measured with the help of barometer.

1.15.2. Unit for Pressure

The fundamental SI unit of pressure is N/m^2 (sometimes called *pascal*, *Pa*) or bar. 1 bar = 10^5 N/m^2 = 10^5 Pa. Standard atmospheric pressure = 1.01325 bar = 0.76 m Hg.

Low pressures are often expressed in terms of mm of water or mm of mercury. This is an abbreviated way of saying that the pressure is such that which will support a liquid column of stated height.

1.15.3. Types of Pressure Measurement Devices

The pressure may be measured by means of indicating gauges or recorders. These instruments may be mechanical, electro-mechanical, electrical or electronic in operation.

1. **Mechanical instruments.** These instruments may be classified into following two groups :

— The *first group* includes those instruments in which the *pressure* measurement is made by *balancing an unknown force with a known force*.

— The *second group* includes those employing *quantitative deformation of an elastic member for pressure measurement*.

2. **Electro-mechanical instruments.** These instruments usually *employ a mechanical means for detecting the pressure and electrical means for indicating or recording the detected pressure*.

3. **Electronic instruments.** Electronic pressure measuring instruments normally depend on some physical change that can be detected and indicated or recorded electronically.

1.15.4. Mechanical-type Instruments

The mechanical-type instruments are classified as follows :

1. **Manometer gauges**

(*i*) U-tube manometer   (*ii*) Cistern manometer

(*iii*) Micro-manometer etc.

2. **Pressure gauges**

(*i*) Bourdon tube pressure gauge   (*ii*) Diaphragm gauge

(*iii*) Vacuum gauge.

1.15.4.1. Liquid Manometers

Low pressures are generally determined by *manometers* which employ liquid columns. It is difficult and costly to construct manometers to measure high pressures, as otherwise the liquid column will become unwieldy and temperature corrections will also be difficult. Their use is, therefore, *restricted to low pressures only*, and for such purposes they are *quite accurate*.

The liquids commonly employed for manometers are mercury and water. *Mercury is used for high and water for low pressures*. For this purpose a liquid is suitable if it has a low viscosity, so that it can adjust itself quickly, and also a low co-efficient of thermal expansion, so that density changes with temperature are minimum.

1. U-tube manometer. A U-tube manometer is in the form of U-tube and is made of glass. When no pressure is applied, the height of the liquid in the two legs is the same. The pressure is then applied to one leg, whilst the other is open to the atmosphere. Under this pressure the liquid will *sink* in this leg and will *rise* in the other. As the other leg is open to air, therefore, the pressure on this side is known, and is barometric. Now the pressure applied to the first leg can be calculated. This is explained with reference to Fig. 1.14. This consists of a water manometer.

Considering equilibrium condition, we have

$$p_{atm} + w_a h_a = p_i + w_i h_i$$
$$p_i = p_{atm} + w_a h_a - w_i h_i$$

where $p_{atm.}$ = atmospheric pressure,

   p_i = pressure over water surface in the container,

   h_a = height of liquid in U-tube manometer,

   h_i = difference between water surface and lower surface of the liquid in manometer,

   w_a = specific weight of liquid, and

   w_i = specific weight of water.

Fig. 1.14. Principle of U-tube manometer.
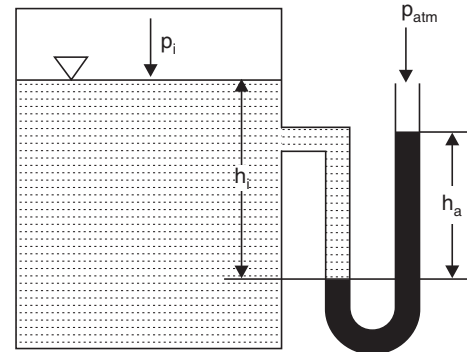
The U-tube manometer shown in Fig. 1.15 is of the simplest form. However, readings have to be taken *at two different places*. Moreover, the *deflection of the two columns may not be the same*. To avoid this difficulty cistern or well type manometer is used.
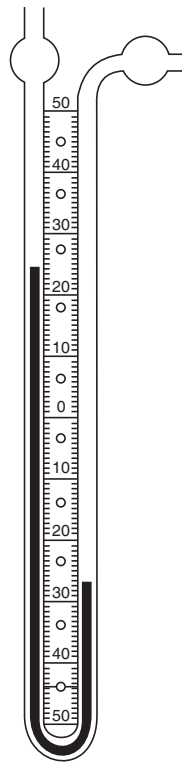
Fig. 1.15. U-tube manometer.
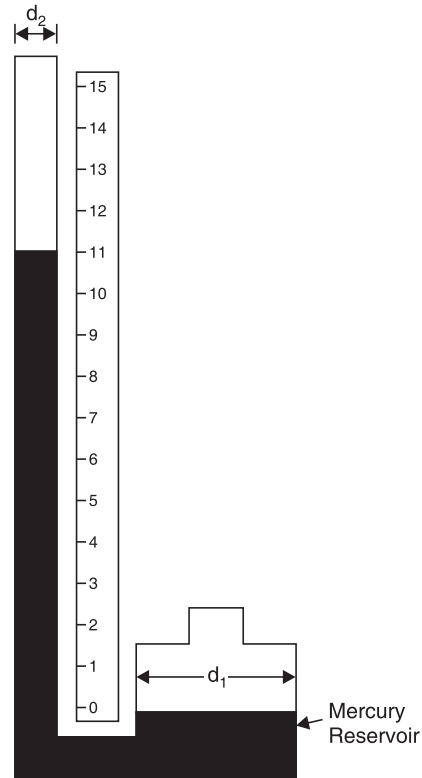
Fig. 1.16. Cistern manometer.

2. Cistern manometer. Fig. 1.16 shows a cistern manometer. The mercury reservoir A is made large enough so that change of level in the reservoir is negligible. This form of manometer is generally used for measuring pressures above atmospheric. In this case, *only one reading of the level in the column is required*. However, a zero setting is necessary.

3. Micro-manometer. The U-tube manometer (discussed above) is not very suitable for measuring very low pressures. Therefore, for such purposes, precision types are required. They are called *multiplying* or *micro-manometers*, because they multiply the movement of the level of the liquid. By far the most widely used type of multiplying manometer is the *inclined manometer*. If the tube is inclined as shown in Fig. 1.17 the sensitiveness of the U-tube manometer is increased. The inclined tube *causes a larger displacement of the liquid along the tube for a given pressure difference*.
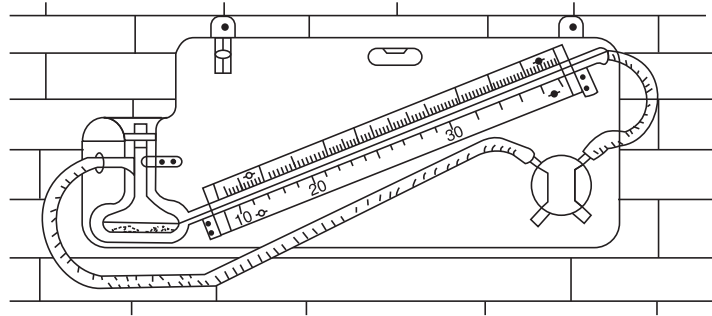
Fig. 1.17. Inclined manometer.

The principle of the inclined manometer is explained in Fig. 1.18. If pressure $p_1 = p_2$, then the level of liquid is shown by *LM*. However, when p_1 is slightly greater than p_2, the level is the reservoir sinks by h_2, whilst level in the tube rises by a greater distance h_1 as shown in the diagram. If h is the vertical distance between the two surfaces due to difference of pressure, then
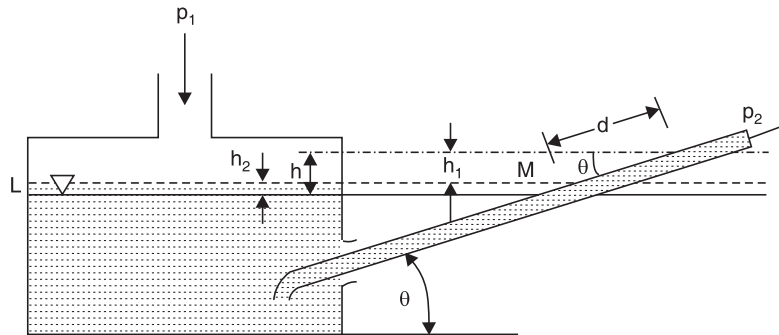
Fig. 1.18. Principle of inclined manometer.

$$h = h_1 + h_2$$

also
$$h_1 = d \sin \theta$$

and
$$h_2 \times A = d \times a \quad \text{or} \quad h_2 = d \times \frac{a}{A}$$

where A = area of cross-section of the reservoir, and

a = area of cross-section of the inclined tube.

Also the pressure difference Δp *i.e.* $(p_1 - p_2)$

$$= hw = \left(d \times \sin \theta + d \times \frac{a}{A} \right) w = dw \left(\sin \theta + \frac{a}{A} \right)$$

where w is the specific weight of the liquid.

The sensitiveness of the instrument can be varied by *changing the slope of the inclined tube*.

The position of the inclined tube is so arranged that $\left(\sin \theta + \dfrac{a}{A} \right)$ is round figure.

The multiplication factor of the gauge is $\dfrac{d}{h} = \dfrac{1}{\sin \theta + \dfrac{a}{A}}$

Thus the multiplication factor depends on θ and $\dfrac{a}{A}$. The *smaller the values of* θ *and* $\dfrac{a}{A}$, *the greater the multiplication factor*.

Advantages and disadvantages of Manometers :

Advantages. The manometer claims the following *advantages* :

1. Relatively inexpensive.
2. Easy to fabricate.
3. Requires very little maintenance.
4. Good accuracy and sensitivity.
5. Their sensitivity can be changed by changing manometric fluids.
6. Particularly suitable to low pressures and low differential pressures.

Disadvantages. The *disadvantages* of manometers are as follows :

1. Unsuitable for recording.
2. Generally large, bulky and fragile.
3. Their calibration is affected by changes in gravitational force and density of fluids and their calibration changes with altitude and temperature.
4. Surface tension of manometric fluid creates a capillary effect and possible hysteresis.
5. A particular manometer can be used only for measurement of a particular fluid/fluids.
6. Meniscus height has to be determined by accurate means to ensure improved accuracy.

Elastic pressure elements :

Elastic pressure elements or mechanical type of transducers are used for measurement of very high pressures upto about 700 MN/m^2. There are three main types of pressure elements.

    (*a*) Bourdon tube   (*b*) Diaphragms   (*c*) Bellows.

The action of these mechanical transducers depends upon the *displacement caused by the pressure*. The displacement produced may actuate a pointer whose deflection may be direct measure of the pressure applied or the displacement is measured with the help of a secondary transducer which is electrical in nature. The output of the secondary transducer which is electrical in format is a measure of the pressure applied.

    (*a*) **Bourdon tubes/elements.** Fig. 1.19 shows Bourdon tube configurations.

Advantages. The Bourdon tube element has the following *advantages* :

1. Simple in construction and cheap.
2. Available in several different ranges.
3. Capability to measure gauge, absolute and differential pressures.
4. The sensitivities of Bourdon tube may be changed by changes in their dimensions.
5. Excellent sensitivity.
6. Simple and straight forward calibration with deal weight tester.
7. Easily adapted to strain, capacitance, magnetic and other electrical transducers.

Disadvantages :
1. Susceptibility to shock and vibration.
2. Inherent hysteresis and slow response to pressure changes.
3. Unsuitable for low pressure applications.
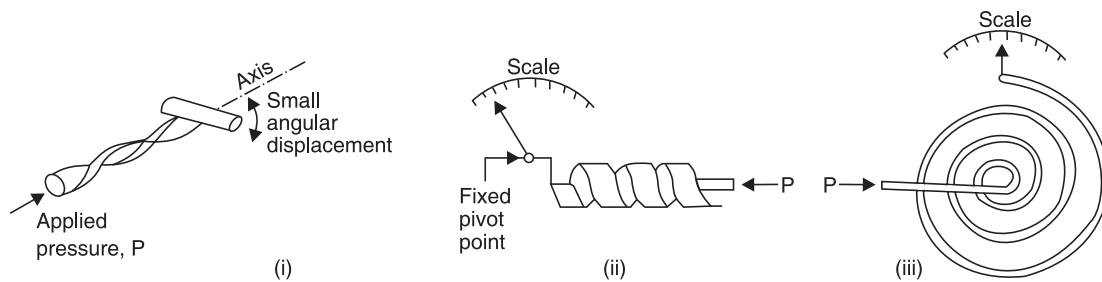
Fig. 1.19. (*i*) Twisted tube, (*ii*) Helical, (*iii*) Spiral.

(*b*) **Diaphragm elements.** Fig. 1.20 shows basic diaphragm types.
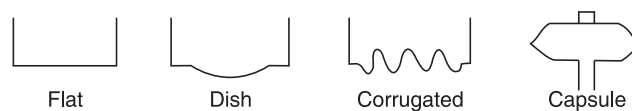
Fig. 1.20. Basic diaphragm types.

Advantages :
1. Small in size and moderately priced.
2. Wide linear range.
3. Can withstand high over pressures and under pressures.
4. Small hysteresis.
5. Can be used for measurement of absolute and differential pressures as also vacuum.

Disadvantages :
1. Need protection against shocks and vibrations.
2. Cannot be used to measure high pressures.
3. Difficult to repair.

(*c*) **Bellow gauges/elements.** Fig. 1.21 shows some bellow gauges.
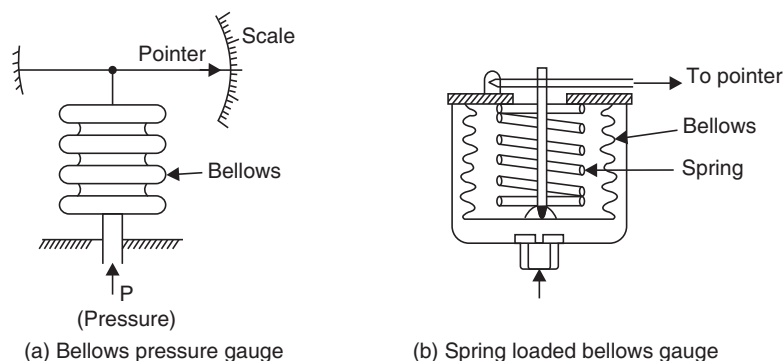
Fig. 1.21. Bellow gauges.

Advantages :
1. Simple and rugged construction.
2. Useful for measurement of low and medium pressures.
3. Moderate cost.
4. Can be used for measurement of absolute, gauge and differential pressures.

Disadvantages :
1. Need spring for accurate characterisation.
2. Greater hysteresis and zero drift problems.
3. Unsuitable for transient measurements due to longer relative motion and mass.
4. Requires compensation for ambient temperature changes.

1.15.5. Important Types of Pressure Gauges

The manometers and U-types (discussed earlier) are suitable for comparatively *low pressures*. For high pressures they become unneccessarily larger even when they are filled with heavy liquids. Therefore for measuring medium and high pressures, we make use of elastic pressure gauges. They employ different forms of elastic systems such as tubes, diaphragms or bellows etc. to measure the pressure. The elastic deformation of these elements is used to show the effect of pressure. Since these elements are deformed within the elastic limit only, therefore these gauges are sometimes called *elastic gauges*. Sometimes they are also called *secondary instruments*, which implies that they must be calibrated by comparison with primary instruments such as manometers etc.

Some of the important types of these gauges are enumerated and discussed below :

1. Bourdon tube pressure gauge
2. Diaphragm gauge
3. Vacuum gauge.

1. Bourdon tube type pressure gauge. A Bourdon type tube pressure gauge is used for measuring *high as well as low pressures*. A simple form of this gauge is shown in Fig. 1.22. In this case the pressure element consists of a metal tube of approximately *elliptical cross-section*. This tube is bent in the form of a segment of a circle and responds to pressure changes. When one end of the tube which is attached to the gauge case, is connected to the source of pressure, the internal
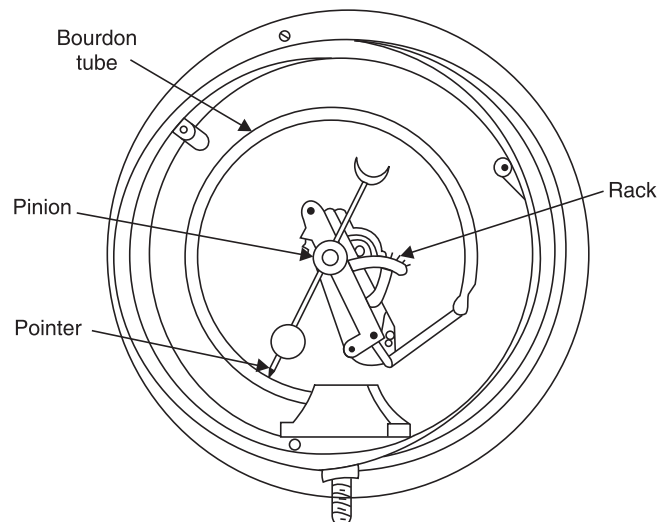
Fig. 1.22. Burdon tube pressure gauge.

pressure causes the tube to expand, whereby circumferential stress *i.e.*, hoop tension is set up. The free end of the tube moves and is in turn connected by suitable levers to a rack, which engages with a small pinion mounted on the same spindle as the pointer. Thus the pressure applied to the tube causes the rack and pinion to move. The pressure is indicated by the pointer over a dial which can be graduated in a suitable scale.

The Bourdon tubes are generally made of *bronze or nickel steel*. The former is generally used for low pressures and the latter for high pressures.

Depending upon the purpose for which they are required Bourdon tube gauges are made in different forms, some of them are :

(*i*) **Compound Bourdon tube** used for measuring pressures both above and below atmospheric.

(*ii*) **Double Bourdon tube** used where vibrations are encountered.

2. Diaphragm gauge. This type of gauge employs a metallic disc or diaphragm instead of a bent tube. This disc or diaphragm is used for *actuating* the indicating device.

Refer Fig. 1.23 when pressure is applied on the lower side of the diaphragm, it is deflected upward. This movement of the diaphragm is transmitted to a rack and pinion. The latter is attached to the spindle of needle moving on a graduated dial. The dial can again be graduated in a suitable scale.

3. Vacuum gauge. Bourdon gauges discussed earlier can be used to measure vacuum instead of pressure. Slight changes in the design are required for this purpose. Thus, in this case, the tube be *bent inward instead of outward* as in pressure gauges. *Vacuum gauges* are graduated in millimetres of mercury below atmospheric pressure. In such cases, therefore, absolute pressure in millimetres of mercury is the difference between barometer reading and vacuum gauge reading.

Fig. 1.23. Principle of diaphragm gauge.

Vacuum gauges are used to measure the vacuum in the *condensers* etc. If there is leakage, the vacuum will drop.

The pressure gauge installation require the following considerations :

(*i*) Flexibile copper tubing and compression fittings are recommended for most installations.

(*ii*) The installation of a gauge cock and tee in the line close to the gauge is recommended because it permits the gauge to be removed for testing or replacement without having to shut down the system.

(*iii*) Pulsating pressures in the gauge line are not required.

(*iv*) The gauge and its connecting line is filled with an inert liquid and as such liquid seals are provided. Trapped air at any point of gauge lines may cause serious errors in pressure reading.

1.16. SPECIFIC VOLUME

The *specific volume* of a system is the volume occupied by the unit mass of the system. The symbol used is v and units are ; for example, m^3/kg. The symbol V will be used for volume. (Note that specific volume is *reciprocal of density*).

Example 1.1. *Convert the following readings of pressure to kPa assuming that barometer reads 760 mm of Hg.*

   (i) 80 cm of Hg   *(ii) 30 cm Hg vacuum*

(iii) 1.35 m H$_2$O gauge   *(iv) 4.2 bar.*

Solution. Assuming density of Hg, ρ_{Hg} = 13.596 × 1000 kg/m^3

Pressure of 760 mm of Hg will be

$$= \rho \times g \times h = 13.596 \times 1000 \times 9.806 \times \frac{760}{1000}$$

$$= 101325 \text{ Pa} = 101.325 \text{ kPa}.$$

(*i*) **Pressure of 80 cm of Hg**

$$= \frac{800}{760} \times 101.325 = \textbf{106.65 kPa.} \quad \textbf{(Ans.)}$$

(*ii*) **30 cm Hg vacuum**

$$= 76 - 30 = 46 \text{ cm of Hg absolute.}$$

Pressure due to 46 cm of Hg

$$= \frac{460}{760} \times 101.325 = \textbf{61.328 kPa.} \quad \textbf{(Ans.)}$$

(*iii*) **Pressure due to 1.35 m H$_2$O gauge**

$$= 1000 \times 9.806 \times 1.35 = 13238 \text{ Pa} = \textbf{13.238 kPa.} \quad \textbf{(Ans.)}$$

(*iv*) **4.2 bar**

$$= 4.2 \times 10^2 \text{ kPa} = \textbf{420 kPa.} \quad \textbf{(Ans.)}$$

Note. Pressure of 1 atmosphere = 760 mm of Hg   or   = 101325 N/m^2.

The above values are standard. To get this value we have to use ρ_{Hg} = 13596 kg/m^3 and g = 9.806 m/s^2. When we use ρ_{Hg} = 13600 kg/m^3 and g = 9.81 m/s^2, we get $p_{atm.}$ = 101396 N/m^2 which is slightly different from 101325 N/m^2. It is recommended that for pressure of 1 atm. the value 101325 N/m^2 should be used.

Example 1.2. *On a piston of 10 cm diameter a force of 1000 N is uniformly applied. Find the pressure on the piston.*

Solution. Diameter of the piston, d = 10 cm (= 0.1 m)

Force applied on the piston, F = 1000 N

∴   Pressure on the piston, $p = \dfrac{\text{force}}{\text{area}} = \dfrac{F}{A} = \dfrac{1000}{\pi/4 \times (0.1)^2}$

$$= 127307 \text{ N/m}^2 = \textbf{127.307 kN/m}^2. \quad \textbf{(Ans.)}$$

Example 1.3. *A tube contains an oil of specific gravity 0.9 to a depth of 120 cm. Find the gauge pressure at this depth (in kN/m^2).*

Solution. Specific gravity of oil = 0.9

Depth of oil in the tube, h = 120 cm = (1.2 m)

We know that $p = wh$

$$= \rho.g.h, \rho \text{ being the mass density}$$

$$= (0.9 \rho_w) \times g \times h, \rho_w \text{ being mass density of water}$$

$$\left[\text{Specific gravity} = \frac{\rho}{\rho_w} \right]$$

$$= 0.9 \times 1000 \times 9.81 \times 1.2 \text{ N/m}^2$$

$$= 10594.8 \text{ N/m}^2 = \textbf{10.595 kN/m}^2. \quad \textbf{(Ans.)}$$

Example 1.4. *A vacuum recorded in the condenser of a steam power plant is 740 mm of Hg. Find the absolute pressure in the condenser in Pa. The barometric reading is 760 mm of Hg.*

Solution. Vacuum recorded in the condenser   = 740 mm of Hg

Barometric reading   = 760 mm of Hg

We know that,

Actual pressure in the condenser

= Barometric reading – vacuum in the condenser

= 760 – 740 = 20 mm of Hg

= 20 × 133.4 N/m^2 ($\because$ 1 mm of Hg = 133.4 N/m^2)

= 2668 N/m^2 = **2668 Pa.   (Ans.)**

Example 1.5. *A vessel of cylindrical shape is 50 cm in diameter and 75 cm high. It contains 4 kg of a gas. The pressure measured with manometer indicates 620 mm of Hg above atmosphere when barometer reads 760 mm of Hg. Determine :*

(i) The absolute pressure of the gas in the vessel in bar.

(ii) Specific volume and density of the gas.

Solution. Diameter of the vessel, d = 50 cm (= 0.5 m)

Height of the vessel, h = 75 cm (= 0.75 m)

Mass of gas in the vessel, m = 4 kg

Manometer reading   = 620 mm of Hg above atmosphere

Barometer reading   = 760 mm of Hg

Now, volume of the vessel = $\dfrac{\pi}{4}\, d^2 \times h = \dfrac{\pi}{4} \times (0.5)^2 \times (0.75) = 0.147$ m^3.

(*i*) **Total pressure in the vessel**

= 760 + 620 = 1380 mm of Hg

= 1380 × 133.4 N/m^2 [$\because$ 1 mm of Hg = 133.4 N/m^2]

= 1.841 × 10^5 N/m^2 = **1.841 bar.   (Ans.)**   [$\because$ 1 bar = 10^5 N/m^2]

(*ii*) **Specific volume**   = $\dfrac{0.147}{4}$ = **0.03675 m^3/kg.   (Ans.)**

Density   = $\dfrac{4}{0.147}$ = **27.21 kg/m^3.   (Ans.)**

Example 1.6. *In a pipe line the pressure of gas is measured with a mercury manometer having one limb open to the atmosphere (Fig. 1.24). If the difference in the height of mercury in the two limbs is 550 mm, calculate the gas pressure.*

Given : Barometric reading = 761 mm of Hg

Acceleration due to gravity = 9.79 m/s^2

Density of mercury = 13640 kg/m^3.

Solution. At the plane *LM*, we have

$$p = p_0 + \rho g h$$

Now $p_0 = \rho g h_0$

where h_0 = barometric height ; ρ = density of mercury ; p_0 = atmospheric pressure

Therefore, $p = \rho g h_0 + \rho g h = \rho g\,(h_0 + h)$

$$= 13640 \times 9.79 \left(\frac{761}{1000} + \frac{550}{1000} \right) = 13640 \times 9.79\,(0.761 + 0.55)$$

$$= 175.065 \times 10^3 \text{ N/m}^2 = 175.065 \text{ kPa} = \textbf{1.75 bar.   (Ans.)}$$
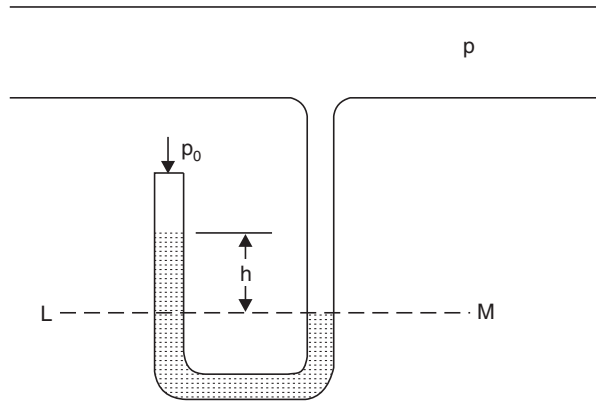
Fig. 1.24

Example 1.7. *A U-tube mercury manometer with one arm open to atmosphere is used to measure pressure in a steam pipe. The level of mercury in open arm is 97.5 mm greater than that in the arm connected to the pipe. Some of steam in the pipe condenses in the manometer arm connected to the pipe. The height of this column is 34 mm. The atmospheric pressure is 760 mm of Hg. Find the absolute pressure of steam.*   **(Poona University, Nov. 1995)**

Solution. Equating the pressure in mm of Hg on both arms above the line *XX* (Fig. 1.25), we get

$$p_{\text{abs.}} + p_{\text{water}} = p_{\text{Hg}} + p_{\text{atm.}}$$

Now, $p_{\text{water}} = \dfrac{34}{13.6} = 2.5$ mm of Hg.

∴ $p_{\text{abs}} + 2.5 = 97.5 + 760$

or $p_{\text{abs}} = 97.5 + 760 - 2.5$

$$= 855 \text{ mm of Hg.}$$

$$= 855 \times p_{\text{Hg}} \times g \times 10^{-5} \text{ bar}$$

$$= \frac{855}{1000} (m) \times (13.6 \times 1000) \text{ (kg/m}^3)$$

$$\times 9.81 \times 10^{-5}$$

$$= \textbf{1.14070 bar.}   \textbf{(Ans.)}$$

Fig. 1.25

Example 1.8. *A U-tube manometer is connected to a gas pipe. The level of the liquid in the manometer arm open to the atmosphere is 170 mm lower than the level of the liquid in the arm connected to the gas pipe. The liquid in the manometer has specific gravity of 0.8. Find the absolute pressure of the gas if the manometer reads 760 mm of Hg.*   **(Poona University, Dec. 1996)**

Solution. Equating pressure on both arms above the line *XX* (Fig. 1.26), we get

$$p_{\text{gas}} + p_{\text{liquid}} = p_{\text{atm.}} \qquad \qquad ...(i)$$

Now, $p_{\text{liquid}} = \rho.g.h$

$$= (0.8 \times 1000) \times 9.81 \times \frac{170}{1000}$$
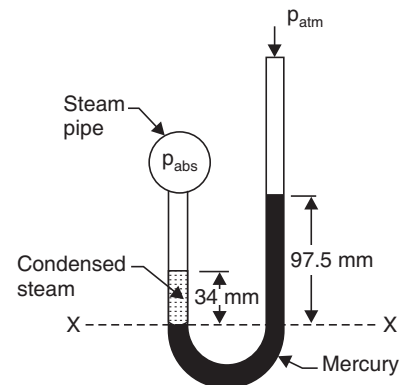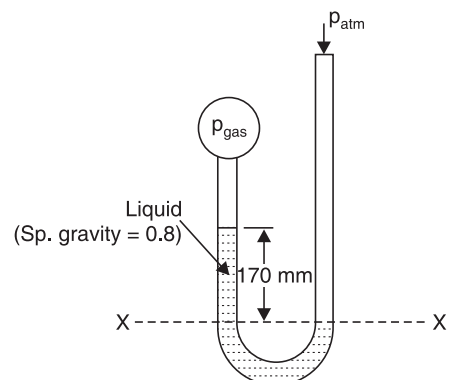
Fig. 1.26

$$= 1334.16 \text{ N/m}^2$$
$$= 0.0133416 \text{ bar}$$
$$p_{\text{atm.}} = 760 \text{ mm of Hg} = 1.01325 \text{ bar}$$

Substituting these value is eqn. (i) above, we have

$$p_{\text{gas}} + 0.0133416 = 1.01325$$

∴ $p_{\text{gas}} = $ **0.9999 bar.   (Ans.)**

Example 1.9. *Estimate the mass of a piston that can be supported by a gas entrapped under the piston in a 200 mm diameter vertical cylinder when a manometer indicates a difference of 117 mm of Hg column for the gas pressure.*   **(Poona University, May 1994)**

Solution. Refer Fig. 1.27.

Let m = mass of the piston, kg

p = pressure of the gas

= 117 mm of Hg column

Dia. of vertical cylinder, d = 200 mm

Now, downward force   = $m.g$ ...(i)

and   upward force   = $p \times \pi/4 \, d^2$ ...(ii)

Fig. 1.27

Equating eqns. (i) and (ii), we get

$$m.g = p \times \pi/4 \, d^2$$

$$m \times 9.81 = \left(13.6 \times 1000 \times 9.81 \times \frac{117}{1000}\right) \times \frac{\pi}{4} \times \left(\frac{200}{1000}\right)^2 \qquad (\because \quad p = \rho g h)$$

∴   **m = 49.989 kg.   (Ans.)**
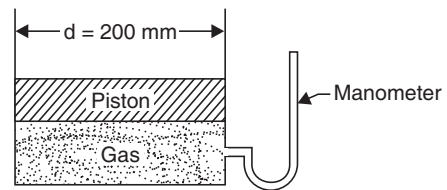
1.17. REVERSIBLE AND IRREVERSIBLE PROCESSES

Reversible process. *A reversible process (also sometimes known as quasi-static process) is one which can be stopped at any stage and reversed so that the system and surroundings are exactly restored to their initial states.*

This process has the following *characteristics* :

1. It must pass through the same states on the reversed path as were initially visited on the forward path.

2. This process when undone will leave no history of events in the surroundings.

3. It must pass through a continuous series of equilibrium states.
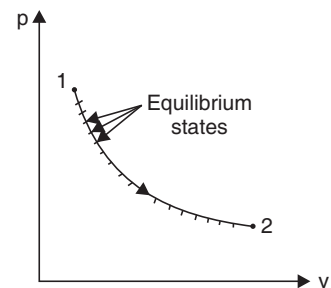
Fig. 1.28. Reversible process.

No real process is truely reversible but some processes may approach reversibility, to close approximation.

Examples. Some examples of nearly reversible processes are :

 (i) Frictionless relative motion.

(ii) Expansion and compression of spring.

(iii) Frictionless adiabatic expansion or compression of fluid.

(iv) Polytropic expansion or compression of fluid.

 (v) Isothermal expansion or compression.

(vi) Electrolysis.

Irreversible process. An *irreversible process* is one *in which heat is transferred through a finite temperature*.

Examples.

(*i*) Relative motion with friction   (*ii*) Combustion

(*iii*) Diffusion   (*iv*) Free expansion

(*v*) Throttling   (*vi*) Electricity flow through a resistance

(*vii*) Heat transfer   (*viii*) Plastic deformation.

An irreversible process is usually represented by a dotted (or discontinuous) line joining the end states to indicate that the intermediate states are indeterminate (Fig. 1.29).

Irreversibilities are of *two types* :

1. **External irreversibilities.** These are associated with *dissipating effects outside the working fluid*.

Example. *Mechanical friction occurring during a process due to some external source.*

2. **Internal irreversibilities.** These are associated with *dissipating effects within the working fluid*.

Example. *Unrestricted expansion of gas, viscosity and inertia of the gas*.
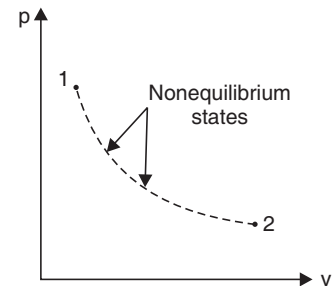
Fig. 1.29. Irreversible process.

1.18. ENERGY, WORK AND HEAT

1.18.1. Energy

Energy is a general term embracing *energy in transition* and *stored energy*. The stored energy of a substance may be in the forms of *mechanical energy* and *internal energy* (other forms of stored energy may be chemical energy and electrical energy). Part of the stored energy may take the form of either potential energy (which is the gravitational energy due to height above a chosen datum line) or kinetic energy due to velocity. The balance part of the energy is known as *internal energy*. In a *non-flow process* usually there is no change of potential or kinetic energy and hence change of mechanical energy will not enter the calculations. In a *flow process*, however, there may be changes in both potential and kinetic energy and these must be taken into account while considering the changes of stored energy. *Heat and work* are the forms of energy in transition. These are the only forms in which energy can cross the boundaries of a system. Neither heat nor work can exist as stored energy.

1.18.2. Work and Heat

Work

Work is said to be done when a *force moves through a distance*. If a part of the boundary of a system undergoes a displacement under the action of a pressure, the work done *W* is the product of the force (pressure × area), and the distance it moves in the direction of the force. Fig. 1.30 (*a*) illustrates this with the conventional piston and cylinder arrangement, the heavy line defining the boundary of the system. Fig. 1.30 (*b*) illustrates another way in which work might be applied to a system. A force is exerted by the paddle as it changes the momentum of the fluid, and since this force moves during rotation of the paddle room work is done.

Work is a transient quantity which only appears at the boundary while a change of state is taking place within a system. Work is 'something' which appears at the boundary when a system changes its state due to the movement of a part of the boundary under the action of a force.
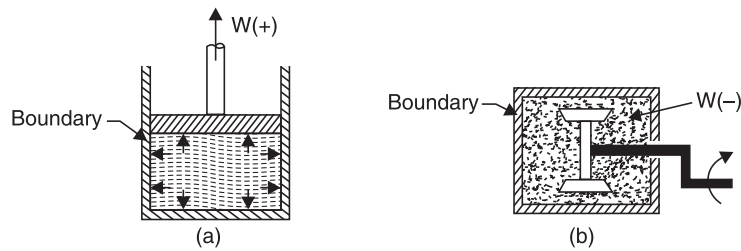
Fig. 1.30

Sign convention :

- If the work is done *by* the system *on* the surroundings, *e.g.* when a fluid expands pushing a piston outwards, the work is said to be *positive*.

  i.e.,   *Work output of the system = + W*

- If the work is done *on* the system *by* the surroundings, *e.g.*, when a force is applied to a rotating handle, or to a piston to compress a fluid, the work is said to be *negative*.

  i.e.,   *Work input to system = – W*

Heat

Heat (denoted by the symbol *Q*), may be, defined in an analogous way to work as follows :

"*Heat is 'something' which appears at the boundary when a system changes its state due to a difference in temperature between the system and its surroundings*".

Heat, like work, is a transient quantity which only appears at the boundary while a change is taking place within the system.

It is apparent that neither δW or δQ are exact differentials and therefore any integration of the elemental quantities of work or heat which appear during a change from state 1 to state 2 must be written as

$$\int_1^2 \delta W = W_{1-2} \text{ or } {}_1W_2 \text{ (or } W\text{), and}$$

$$\int_1^2 \delta Q = Q_{1-2} \text{ or } {}_1Q_2 \text{ (or } Q\text{)}$$

Sign convention :

If the heat flows *into* a system *from* the surroundings, the quantity is said to be *positive* and, conversely, if heat flows *from* the system to the surroundings it is said to be *negative*.

In other words :

Heat received by the system = + Q

Heat rejected or given up by the system = – Q.

Comparison of Work and Heat

Similarities :

 (*i*) Both are *path functions and inexact differentials*.

 (*ii*) Both are boundary phenomenon *i.e.* both are recognized at the boundaries of the system as they cross them.

 (*iii*) Both are associated with a process, not a state. Unlike properties, work or heat has no meaning at a state.

 (*iv*) Systems possess energy, but not work or heat.

Dissimilarities :

(*i*) In heat transfer temperature difference is required.

(*ii*) In a stable system there cannot be work transfer, however, there is no restriction for the transfer of heat.

(*iii*) The sole effect external to the system could be reduced to rise of a weight but in the case of a heat transfer other effects are also observed.

1.19. REVERSIBLE WORK

Let us consider an ideal frictionless fluid contained in a cylinder above a piston as shown in Fig. 1.31. *Assume that the pressure and temperature of the fluid are uniform and that there is no friction between the piston and the cylinder walls.*

Let A = cross-sectional area of the piston,

p = pressure of the fluid at any instant,

$(p - dp)A$ = restraining *force exerted* by the surroundings on the piston, and

dl = the distance moved by the piston under the action of the *force exerted*.

Then work done by the fluid on the piston is given by force times the distance moved,

i.e.,   Work done by the fluid

$$= (pA) \times dl = pdv$$

(where dV = a small increase in volume)

Or considering unit mass

Work done = pdv (where v = specific volume)

This is only *true* when (*a*) the *process is frictionless and (b) the difference in pressure between the fluid and its surroundings during the process is infinitely small.* Hence when a reversible process takes place between state 1 and state 2, we have

Work done by the unit mass of fluid

$$= \int_1^2 pdv \qquad \qquad ...(1.10)$$

When a fluid undergoes a reversible process a series of state points can be joined up to form a line on a diagram of properties. The work done by the fluid during any reversible process is therefore given by the area under the line of process plotted on a *p-v* diagram (Fig. 1.32).
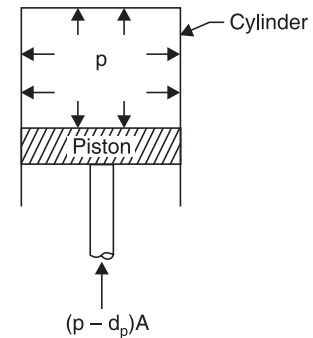
Fig. 1.31
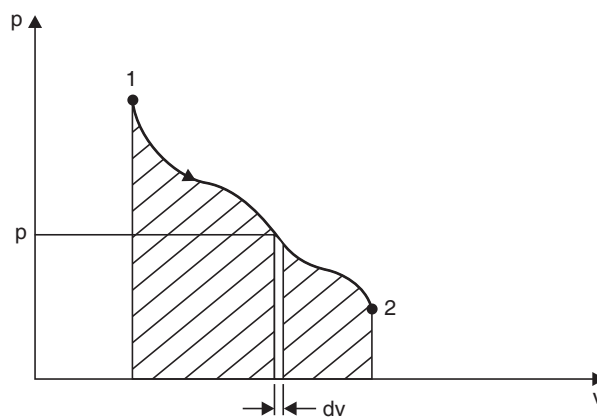
Fig. 1.32

i.e.,   Work done = shaded area on Fig. 1.32

$$= \int_1^2 pdv.$$

When p can be expressed in terms of v then the integral, $\int_1^2 pdv$, can be evaluated.

Example 1.10. *An artificial satellite revolves round the earth with a relative velocity of 800 m/s. If acceleration due to gravity is 9 m/s² and gravitational force is 3600 N, calculate its kinetic energy.*

Solution. Relatively velocity of satellite, $v = 800$ m/s

Acceleration due to gravity, $g = 9$ m/s²

Gravitational force, $m.g = 3600$ N

∴   Mass, $m = \dfrac{3600}{g} = \dfrac{3600}{9} = 400$ kg.

Kinetic energy $= \dfrac{1}{2} mv^2 = \dfrac{1}{2} \times 400 \times (800)^2$ J $= 128 \times 10^6$ J or **128 MJ.   (Ans.)**

Example 1.11. *The specific heat capacity of the system during a certain process is given by*
$$c_n = (0.4 + 0.004\ T)\ kJ/kg°C.$$
If the mass of the gas is 6 kg and its temperature changes from 25°C to 125°C find :
 (*i*) *Heat transferred* ;   (*ii*) *Mean specific heat of the gas.*

Solution. Mass of the gas, $m = 6$ kg

Change in temperature of the gas = 25°C to 125°C

(*i*) **Heat transferred, Q :**

We know that heat transferred is given by,

$$Q = \int m\ c_n\ dT = 6 \int_{25}^{125} (0.4 + 0.004\ T)\ dT$$

$$= 6 \left[0.4T + 0.004 \left(\frac{T^2}{2} \right) \right]_{25}^{125}$$

$$= 6[0.4\ (125 - 25) + 0.002\ (125^2 - 25^2)]$$

$$= 6(40 + 30) = \textbf{420 kJ.   (Ans.)}$$

(*ii*) **Mean specific heat of the gas, c$_n$:**

$$Q = m.c_n.dT$$

i.e., $420 = 6 \times c_n \times (125 - 25)$

∴ $c_n = \dfrac{420}{6 \times 100} = \textbf{0.7 kJ/kg°C.   (Ans.)}$

Example 1.12. *A temperature scale of certain thermometer is given by the relation*
$$t = a\ ln\ p + b$$
where a and b are constants and p is the thermometric property of the fluid in the thermometer. If at the ice point and steam point the thermometric properties are found to be 1.5 and 7.5 respectively what will be the temperature corresponding to the thermometric property of 3.5 on Celsius scale.

(Poona University)

Solution. $t = a \ln p + b$ ...(Given)

On Celsius scale :

$$\text{Ice point} = 0°C, \text{ and}$$
$$\text{Steam point} = 100°C$$

∴   From given conditions, we have

$$0 = a \ln 1.5 + b \qquad \qquad ...(i)$$

and $100 = a \ln 7.5 + b$ ...(ii)

i.e. $0 = a \times 0.4054 + b$ ...(iii)

and $100 = a \times 2.015 + b$ ...(iv)

Subtracting (iii) from (iv), we get

$$100 = 1.61a$$

or $a = 62.112$

Substituting this value in eqn. (iii), we get

$$b = -0.4054 \times 62.112 = -25.18$$

∴   When $p = 3.5$ the value of temperature is given by

$$t = 62.112 \ln (3.5) - 25.18 = \textbf{52.63°C.} \quad \textbf{(Ans.)}$$

Example 1.13. *A thermocouple with test junction at t°C on gas thermometer scale and reference junction at ice point gives the e.m.f. as*

$$e = 0.20\, t - 5 \times 10^{-4} t^2 \text{ mV.}$$

The millivoltmeter is calibrated at ice and steam points. What will be the reading on this thermometer where the gas thermometer reads 70°C.

Solution. $e = 0.20\, t - 5 \times 10^{-4} t^2$ mV   (Given)

At ice point :   When $t = 0°C$, $e = 0$

At steam point :   When $t = 100°C$,

$$e = 0.20 \times 100 - 5 \times 10^{-4} \times (100)^2 = 15 \text{ mV}$$

Now,   when $t = 70°C$

$$e = 0.20 \times 70 - 5 \times 10^{-4} \times (70)^2 = 11.55 \text{ mV}$$

∴   When the gas thermometer reads 70°C the thermocouple will read

$$t = \frac{100 \times 11.55}{15} = \textbf{77°C.} \quad \textbf{(Ans.)}$$

☞ **Example 1.14.** *Comment whether the following quantities can be called as properties or not :*

(i) $\displaystyle\int pdV,$ (ii) $\displaystyle\int Vdp,$ *and*   (iii) $\displaystyle\int pdV + \int Vdp.$

Solution. (i) $\displaystyle\int pdV$:

p is a function of V and integral can only be evaluated if relation between p and V is known. It is thus an *inexact differential* and hence **not a property.**

(ii) $\displaystyle\int Vdp$:

It is **not a property** for the *same reason* as mentioned in (i).

(iii) $\displaystyle\int pdV + \int Vdp$:

$$\int pdV + \int Vdp = \int (pdV + Vdp) = \int d(pV) = pV.$$

Thus the integral can be evaluated without knowing the relation between p and V. It is an *exact differential* and hence **it is a property.**

Example 1.15. *Gas from a cylinder of compressed helium is used to inflate an inelastic flexible balloon, originally folded completely flat, to a volume 0.6 m³. If the barometer reads 760 mm Hg, what is the amount of work done upon the atmosphere by the balloon ? Sketch the system before and after the process.*

Solution. Refer Fig. 1.33. The firm line B_1 shows the boundary of the system before the process, and dotted line B_2 shows the boundary after the process.

Valve

Final volume of balloon = 0.6 m³

Balloon initially flat

B_1

B_2

Helium cylinder

p = 760 mm Hg = 101.325 k Pa
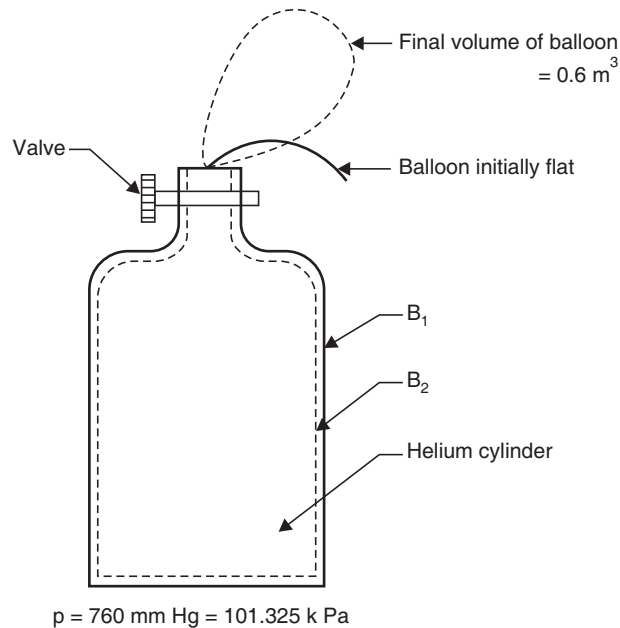
Fig. 1.33

The displacement work,

$$W_d = \int_{\text{cylinder}} p\ dV + \int_{\text{balloon}} p\ dV = 0 + \int_{\text{balloon}} p\ dV$$

$$= 101.325 \times 0.6 \qquad\qquad [\because \quad dV = 0.6 \text{ m}^3]$$

$$= \textbf{60.795 kJ.} \quad \textbf{(Ans.)}$$

This is a positive work, because the *work is done by the system*. Work done by the atmosphere is – 60.795 kJ. Since the wall of the cylinder is rigid there is no *pdV*-work involved in it.

It is assumed that the pressure in the balloon is atmospheric at all times, since the balloon fabric is light, inelastic and unstressed. If the balloon were elastic and stressed during the filling process, the work done by the gas would be *greater* than 60.795 kJ by an amount equal to the work done in stretching the balloon, although the displacement work done by atmosphere is still –60.795 kJ. However, if the system includes both the gas and the balloon, the displacement work should be 60.795 kJ, as estimated above.

Example 1.16. *Determine the work done by the air which enters into an evacuated vessel from atmosphere when the valve is opened. The atmospheric pressure is 1.013 bar and 1.5 m³ of air at atmospheric condition enters into the vessel.*

Solution. Fig. 1.34 shows the initial and final conditions of the system.
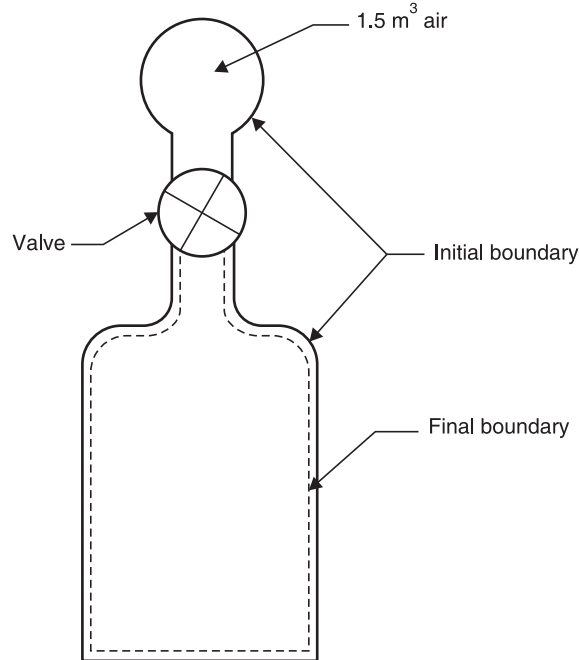
Fig. 1.34

No work is done by the boundary in contact with the vessel as the boundary does not move. Work is done by the external boundary at constant pressure.

$$\therefore \qquad W = \int_{V_1}^{V_2} p\, dV = \int_{1.5}^{0} p\, dV \qquad\qquad [\because \quad V_1 = 1.5 \text{ m}^3 \text{ and } V_2 = 0]$$

$$= p(0 - 1.5) = 1.013 \times 10^5 \times (-1.5)$$

$$= -1.5195 \times 10^5 \text{ J} = -\textbf{151.95 kJ.} \quad \textbf{(Ans.)}$$

Since the free air boundary is contracting, the work done by the system is negative, and the surroundings do positive work upon the system.

☞**Example 1.17.** *A piston and cylinder machine containing a fluid system has a stirring device as shown in Fig. 1.35. The piston is frictionless, and it is held down against the fluid due to atmospheric pressure of 101.3 kPa. The stirring device is turned 9500 revolutions with an average torque against the fluid of 1.25 mN. Meanwhile the piston of 0.65 m diameter moves out 0.6 m. Find the net work transfer for the system.*

Solution. Refer Fig. 1.35.

Work done by *the stirring device* upon the system,

$$W_1 = 2\pi NT$$

where T = torque = 1.25 mN

N = number of revolutions = 9500

$$W_1 = 2\pi \times 9500 \times 1.25 = 74622 \text{ Nm} = 74.622 \text{ kJ}$$

This is *negative work* for the system.
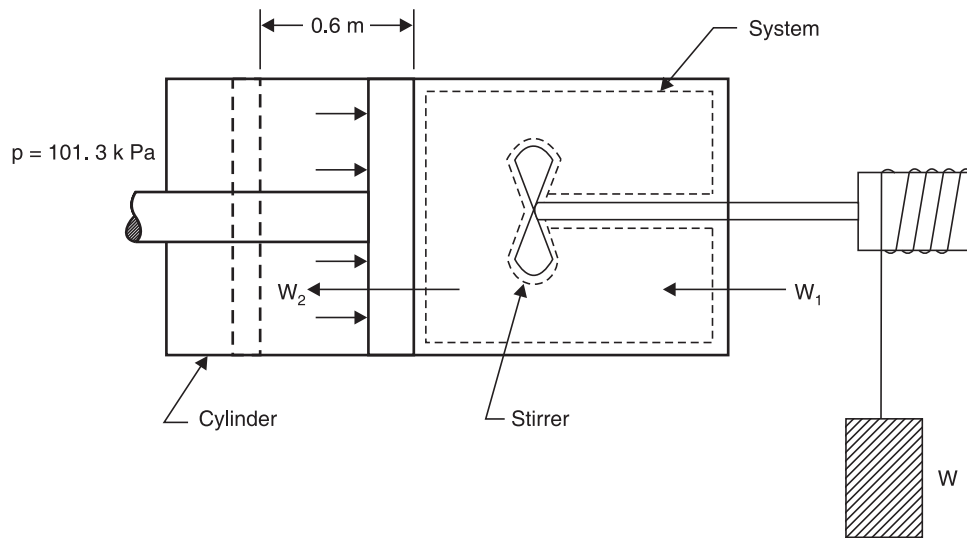
Fig. 1.35

Work done *by the system* upon the surroundings

$$W_2 = (pA).\, L$$

where p = pressure = 101.3 kPa

A = area of the piston = $\pi/4 \times (0.65)^2$ = 0.3318 m^2

L = distance moved by the piston = 0.6 m

$$W_2 = 101.3 \times 0.3318 \times 0.6 = 20.167 \text{ kJ}$$

This is a *positive work for the system*.

Hence, the net work transfer for the system

$$W_{net} = W_1 + W_2 = -74.622 + 20.167 = \textbf{– 54.455 kJ.}\quad \textbf{(Ans.)}$$

Example 1.18. *A diesel engine piston which has an area of 45 cm² moves 5 cm during part of suction stroke. 300 cm³ of fresh air is drawn in from the atmosphere. The pressure in the cylinder during suction stroke is 0.9 × 10⁵ N/m² and the atmospheric pressure is 1.013 × 10⁵ N/m². The difference between the suction and atmospheric pressure is accounted for flow resistance in the suction pipe and inlet valve. Find the net work done during the process.*

Solution. Area of diesel engine piston

$$= 45 \text{ cm}^2 = 45 \times 10^{-4} \text{ m}^2$$

Amount of fresh air drawn in from atmosphere

$$= 300 \text{ cm}^3 = 300 \times 10^{-6} \text{ m}^3$$

The pressure inside the cylinder during suction stroke

$$= 0.9 \times 10^5 \text{ N/m}^2$$

Atmospheric pressure = 1.013×10^5 N/m^2

Initial and final conditions of the system are shown in Fig. 1.36.

Net work done = work done by free air boundary + work done on the piston

The work done by the free air = – ve because *boundary contracts*

The work done by the cylinder on the piston = +ve because the *boundary expands*
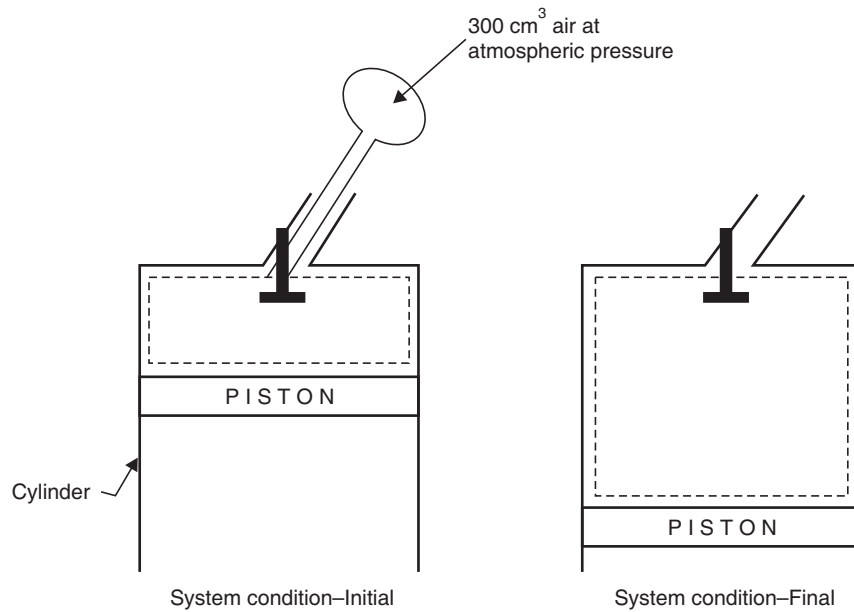
Fig. 1.36

$\therefore$ **Net work done** $= \int pdV + \int pdV$

piston   free air
            boundary

$$= \left[0.9 \times 10^5 \times 45 \times 10^{-4} \times \frac{5}{100} - 1.013 \times 10^5 \times 300 \times 10^{-6} \right]$$

$= [20.25 - 30.39] = -\textbf{10.14 Nm or J.}$ **(Ans.)**

Example 1.19. *The properties of a closed system change following the relation between pressure and volume as pV = 3.0 where p is in bar V is in m³. Calculate the work done when the pressure increases from 1.5 bar to 7.5 bar.*

Solution. Initial pressure, $p_1 = 1.5$ bar
Final pressure, $p_2 = 7.5$ bar
Relation between p and V, $pV = 3.0$
Work done, $W :$
The work done during the process is given by

$$W = \int_{V_1}^{V_2} pdV$$

$$V_1 = \frac{3.0}{p_1} = \frac{3.0}{1.5} = 2 \text{ m}^3$$

$$V_2 = \frac{3.0}{p_2} = \frac{3.0}{7.5} = 0.4 \text{ m}^3$$

$\therefore$ $W = 10^5 \int_{2}^{0.4} \frac{3.0}{V} dV$ Nm $[\because \quad 1 \text{ bar} = 10^5 \text{ N/m}^2]$

$$= 10^5 \times 3.0 \left[\log_e V\right]_2^{0.4} = 10^5 \times 3.0(\log_e 0.4 - \log_e 2)$$

$$= -3 \times 10^5 \log_e (2/0.4) = -3 \times 10^5 \times \log_e 5 = -3 \times 10^5 \times 1.61 \text{ Nm}$$

$$= -4.83 \times 10^5 \text{ Nm} = -4.83 \times 10^5 \text{ J} = -\textbf{483 kJ. (Ans.)}$$

Example 1.20. *To a closed system 150 kJ of work is supplied. If the initial volume is 0.6 m³ and pressure of the system changes as p = 8 − 4V, where p is in bar and V is in m³, determine the final volume and pressure of the system.*

Solution. Amount of work supplied to a closed system = 150 kJ

$$\text{Initial volume} = 0.6 \text{ m}^3$$

$$\text{Pressure-volume relationship, } p = 8 - 4V$$

The work done during the process is given by

$$W = \int_{V_1}^{V_2} pdV$$

$$= 10^5 \int_{0.6}^{V_2} (8 - 4V)dV = 10^5 \left[8V - 4 \times \frac{V^2}{2}\right]_{0.6}^{V_2}$$

$$= 10^5[8(V_2 - 0.6) - 2(V_2{}^2 - 0.6^2)]$$
$$= 10^5[8V_2 - 4.8 - 2V_2{}^2 + 0.72]$$
$$= 10^5[8V_2 - 2V_2{}^2 - 4.08] \text{ Nm or J}$$

But this work is equal to − 150 × 10³ J as this work is supplied to the system.

$$\therefore \qquad -150 \times 10^3 = 10^5[8V_2 - 2V_2{}^2 - 4.08]$$

or $\qquad 2V_2{}^2 - 8V_2 + 2.58 = 0$

$$V_2 = \frac{8 \pm \sqrt{64 - 4 \times 2 \times 2.58}}{4} = \frac{8 \pm 6.585}{4} = 0.354 \text{ m}^3$$

Positive sign is incompatible with the present problem, therefore it is not considered.

$\therefore$ **Final volume,** $V_2 = \textbf{0.354 m}^3$. **(Ans.)**

and **final pressure,** $p_2 = 8 - 4V = 8 - 4 \times 0.354$

$$= 6.584 \text{ bar} = \textbf{6.584} \times \textbf{10}^5 \textbf{ N/m}^2 \textbf{ or Pa. (Ans.)}$$

Example 1.21. *A fluid at a pressure of 3 bar, and with specific volume of 0.18 m³/kg, contained in a cylinder behind a piston expands reversibly to a pressure of 0.6 bar according to a law, $p = \dfrac{C}{v^2}$ where C is a constant. Calculate the work done by the fluid on the piston.* **(PTU)**

Solution. Refer Fig. 1.37.

$$p_1 = 3 \text{ bar} = 3 \times 10^5 \text{ N/m}^2$$
$$v_1 = 0.18 \text{ m}^3/\text{kg}$$

Work done = shaded area = $\int_1^2 p \, dv$

i.e., Work done, $W = \displaystyle\int_1^2 \frac{C}{v^2} \cdot dv = C\int_1^2 \frac{dv}{v^2} = C\left|\frac{v^{-2+1}}{-2+1}\right|_{v_1}^{v_2}$

$$= C\left[-v^{-1}\right]_{v_1}^{v_2} = C\left[-\frac{1}{v}\right]_{v_1}^{v_2} = C\left[\frac{1}{v_1} - \frac{1}{v_2}\right] \qquad \qquad ...(i)$$
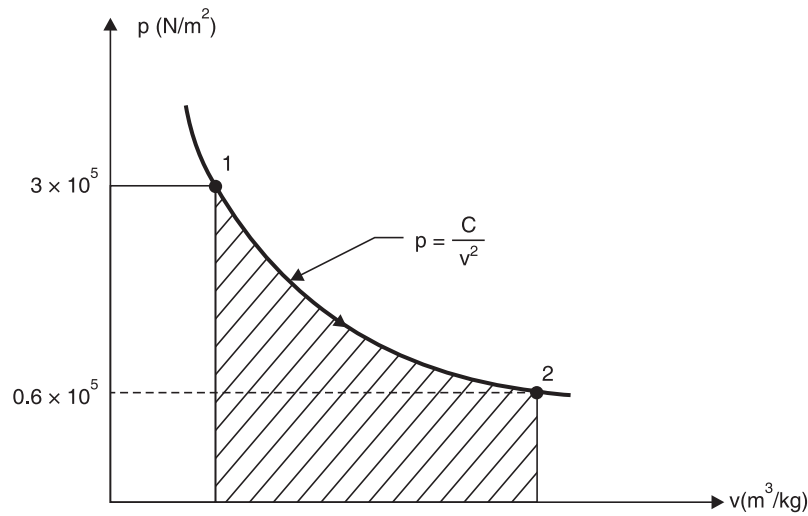
Fig. 1.37

Also $\qquad\qquad C = pv^2 = p_1v_1{}^2 = 3 \times 0.18^2 = 0.0972$ bar $(m^3/kg)^2$

and $\qquad\qquad v_2 = \sqrt{\dfrac{C}{p_2}} = \sqrt{\dfrac{0.0972}{0.6}} = 0.402 \text{ m}^3/\text{kg}$

Substituting the values of C, v_1 and v_2 in equation (i), we get

Work done, $\qquad W = 0.0972 \times 10^5 \left[\dfrac{1}{0.18} - \dfrac{1}{0.402}\right]$ Nm/kg

$\qquad\qquad\qquad\qquad = \textbf{29840 Nm/kg.}\quad \textbf{(Ans.)}$

☞**Example 1.22.** *A cylinder contains 1 kg of a certain fluid at an initial pressure of 20 bar. The fluid is allowed to expand reversibly behind a piston according to a law pV^2 = constant until the volume is doubled. The fluid is then cooled reversibly at constant pressure until the piston regains its original position ; heat is then supplied reversibly with the piston firmly locked in position until the pressure rises to the original value of 20 bar. Calculate the net work done by the fluid, for an initial volume of 0.05 m^3.*

Solution. Refer Fig. 1.38.

Mass of fluid, $\qquad m = 1$ kg

$\qquad\qquad p_1 = 20$ bar $= 20 \times 10^5$ N/m^2

$\qquad\qquad V_1 = 0.05$ m^3

Considering the process 1-2

$\qquad\qquad p_1V_1{}^2 = p_2V_2{}^2$

∴ $\qquad\qquad p_2 = p_1 \left(\dfrac{V_1}{V_2}\right)^2 = 20 \left(\dfrac{V_1}{2V_1}\right)^2 \qquad\qquad [\because\;\; V_2 = 2V_1 \text{ (given)}]$

$\qquad\qquad\qquad = \dfrac{20}{4} = 5$ bar

Work done by the fluid from 1 to 2 = Area 12 ML1 $= \displaystyle\int_1^2 p\ dV$
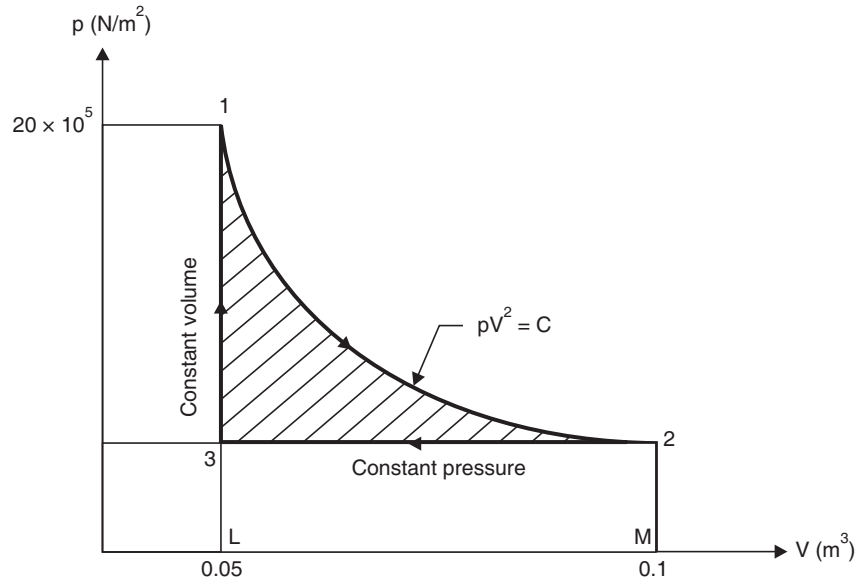
Fig. 1.38

i.e., $W_{1-2} = \int_{v_1}^{v_2} \dfrac{C}{V^2}\, dV$, where $C = p_1 V_1^2 = 20 \times 0.05^2$ bar m^6

$\therefore$ $W_{1-2} = 10^5 \times 20 \times 0.0025 \left[-\dfrac{1}{V} \right]_{0.05}^{0.1}$

$= 10^5 \times 20 \times 0.0025 \left(\dfrac{1}{0.05} - \dfrac{1}{0.1} \right) = 50000$ Nm

Work done on fluid from 2 to 3

$= \text{Area } 32 \text{ ML } 3 = p_2 (V_2 - V_3) = 10^5 \times 5 \times (0.1 - 0.05) = 25000$ Nm

Work done during the process 3-1

$= 0$, because piston is locked in position (*i.e., volume remains constant*)

$\therefore$ **Net work done by the fluid**

$= \text{Enclosed area } 1231 = 50000 - 25000$

$= \textbf{25000 Nm.}$ **(Ans.)**

| **HIGHLIGHTS** |

1. *Thermodynamics* is an axiomatic science which deals with the relations among heat, work and properties of systems which are in equilibrium. It basically entails four laws or axioms known as *Zeroth, First, Second* and *Third* law of thermodynamics.
2. A *system* is a finite quantity of matter or a prescribed region of space.
   A system may be a *closed, open* or *isolated* system.
3. A *phase* is a quantity of matter which is homogeneous throughout in chemical composition and physical structure.
4. A *homogeneous system* is one which consists of a *single phase*.

5. A *heterogeneous system* is one which consists of *two or more phases*.

6. A *pure substance* is one that has a homogeneous and invariable chemical composition even though there is a change of phase.

7. A system is in *thermodynamic equilibrium* if temperature and pressure at all points are same ; there should be no *velocity gradient*.

8. A *property of a system* is a characteristic of the system which depends upon its state, but not upon how the state is reached.
Intensive properties do not depend on the mass of the system.
Extensive properties depend on the mass of the system.

9. *State* is the condition of the system at an instant of time as described or measured by its properties. Or each unique condition of a system is called a state.

10. A *process* occurs when the system undergoes a change in state or an energy transfer takes place at a steady state.

11. Any process or series of processes whose end states are identical is termed a *cycle*.

12. The *pressure* of a system is the force exerted by the system on unit area of boundaries. Vacuum is defined as the absence of pressure.

13. A *reversible process* is one which can be stopped at any stage and reversed so that the system and surroundings are exactly restored to their initial states.
An *irreversible process* is one in which heat is transferred through a finite temperature.

14. Zeroth law of termodynamics states that if two systems are each equal in temperature to a third, they are equal in temperature to each other.

15. Infinite slowness is the characteristic feature of a quasi-static process. A quasi-static process is a succession of equilibrium states. It is also called a reversible process.

OBJECTIVE TYPE QUESTIONS

Choose the correct answer :

1. A definite area or space where some thermodynamic process takes place is known as
(*a*) thermodynamic system   (*b*) thermodynamic cycle
(*c*) thermodynamic process   (*d*) thermodynamic law.

2. An open system is one in which
(*a*) heat and work cross the boundary of the system, but the mass of the working substance does not
(*b*) mass of working substance crosses the boundary of the system but the heat and work do not
(*c*) both the heat and work as well as mass of the working substances cross the boundary of the system
(*d*) neither the heat and work nor the mass of the working substances cross the boundary of the system.

3. An isolated system
(*a*) is a specified region where transfer of energy and/or mass take place
(*b*) is a region of constant mass and only energy is allowed to cross the boundaries
(*c*) cannot transfer either energy or mass to or from the surroundings
(*d*) is one in which mass within the system is not necessarily constant
(*e*) none of the above.

4. In an extensive property of a thermodynamic system
(*a*) extensive heat is transferred   (*b*) extensive work is done
(*c*) extensive energy is utilised   (*d*) all of the above
(*e*) none of the above.

5. Which of the following is an intensive property of a thermodynamic system ?
(*a*) Volume   (*b*) Temperature
(*c*) Mass   (*d*) Energy.

6. Which of the following is the extensive property of a thermodynamic system ?
   (a) Pressure   (b) Volume
   (c) Temperature   (d) Density.

7. When two bodies are in thermal equilibrium with a third body they are also in thermal equilibrium with each other. This statement is called
   (a) Zeroth law of thermodyamics   (b) First law of thermodynamics
   (c) Second law of thermodynamics   (d) Kelvin Planck's law.

8. The temperature at which the volume of a gas becomes zero is called
   (a) absolute scale of temperature   (b) absolute zero temperature
   (c) absolute temperature   (d) none of the above.

9. The value of one bar (in SI units) is equal to
   (a) $100 \ N/m^2$ (b) $1000 \ N/m^2$
   (c) $1 \times 10^4 \ N/m^2$ (d) $1 \times 10^5 \ N/m^2$
   (e) $1 \times 10^6 \ N/m^2$.

10. The absolute zero pressure will be
    (a) when molecular momentum of the system becomes zero
    (b) at sea level   (c) at the temperature of $-$ 273 K
    (d) under vacuum conditions   (e) at the centre of the earth.

11. Absolute zero temperature is taken as
    (a) $-$ 273°C   (b) 273°C
    (c) 237°C   (d) $-$ 373°C.

12. Which of the following is correct ?
    (a) Absolute pressure = gauge pressure + atmospheric pressure
    (b) Gauge pressure = absolute pressure + atmospheric pressure
    (c) Atmospheric pressure = absolute pressure + gauge pressure
    (d) Absolute pressure = gauge pressure – atmospheric pressure.

13. The unit of energy in SI units is
    (a) Joule (J)   (b) Joule metre (Jm)
    (c) Watt (W)   (d) Joule/metre (J/m).

14. One watt is equal to
    (a) 1 Nm/s   (b) 1 N/min
    (c) 10 N/s   (d) 100 Nm/s
    (e) 100 Nm/m.

15. One joule (J) is equal to
    (a) 1 Nm   (b) kNm
    (c) 10 Nm/s   (d) 10 kNm/s.

16. The amount of heat required to raise the temperature of 1 kg of water through 1°C is called
    (a) specific heat at constant volume   (b) specific heat at constant pressure
    (c) kilo calorie   (d) none of the above.

17. The heating and expanding of a gas is called
    (a) thermodynamic system   (b) thermodynamic cycle
    (c) thermodynamic process   (d) thermodynamic law.

18. A series of operations, which take place in a certain order and restore the initial condition is known as
    (a) reversible cycle   (b) irreversible cycle
    (c) thermodynamic cycle   (d) none of the above.

19. The condition for the reversibility of a cycle is
    (a) the pressure and temperature of the working substance must not differ, appreciably, from those of the surroundings at any stage in the process
    (b) all the processes, taking place in the cycle of operation, must be extremely slow
    (c) the working parts of the engine must be friction free
    (d) there should be no loss of energy during the cycle of operation.
    (e) all of the above   (f) none of the above.

20. In an irreversible process, there is a
    (*a*) loss of heat   (*b*) no loss of heat
    (*c*) gain of heat   (*d*) no gain of heat.
21. The main cause of the irreversibility is
    (*a*) mechanical and fluid friction   (*b*) unrestricted expansion
    (*c*) heat transfer with a finite temperature difference
    (*d*) all of the above   (*e*) none of the above.
22. According to kinetic theory of heat
    (*a*) temperature should rise during boiling   (*b*) temperature should fall during freezing
    (*c*) at low temperature all bodies are in solid state
    (*d*) at absolute zero there is absolutely no vibration of molecules
    (*e*) none of the above.
23. A system comprising a single phase is called a
    (*a*) closed system   (*b*) open system
    (*c*) isolated system   (*d*) homogeneous system
    (*e*) heterogeneous system.

Answers

1. (*a*)	**2.** (*c*)	**3.** (*c*)	**4.** (*e*)	**5.** (*b*)
6. (*b*)	**7.** (*a*)	**8.** (*b*)	**9.** (*d*)	**10.** (*a*)
11. (*a*)	**12.** (*a*)	**13.** (*a*)	**14.** (*a*)	**15.** (*a*)
16. (*c*)	**17.** (*b*)	**18.** (*c*)	**19.** (*e*)	**20.** (*a*)
21. (*d*)	**22.** (*d*)	**23.** (*d*).		

THEORETICAL QUESTIONS

1. Define a thermodynamic system. Differentiate between open system, closed system and an isolated system.
2. How does a homogeneous system differ from a heterogeneous system ?
3. What do you mean by a pure substance ?
4. Explain the following terms :
   (*i*) State,   (*ii*) Process, and   (*iii*) Cycle.
5. Explain briefly zeroth law of thermodynamics.
6. What is a quasi-static process ?
7. What do you mean by 'reversible work' ?

UNSOLVED EXAMPLES

1. A cylindrical vessel of 60 cm diameter and 80 cm height contains 3.2 kg of a gas. The pressure measured with manometer indicates 60 cm of Hg above atmosphere when barometer reads 760 mm of Hg. Find :
  (*i*) the absolute pressure of the gas in the vessel in bar, and (*ii*) specific volume and density of gas.
                                                            [Ans. 1.81 bar ; 0.09 m³/kg ; 11.11 kg/m³**]**
2. A force of 1600 N is applied uniformly on a piston of 8 cm diameter. Determine the pressure on the piston.   **[Ans.** 319 kN/m²**]**
3. An oil of specific gravity 0.8 is contained in a tube to a depth of 80 cm. Determine the gauge pressure at this depth in kN/m².   **[Ans.** 6.275 kN/m²**]**
4. A vacuum recorded in the condenser of a steam power plant is 745 mm of Hg. Find the absolute pressure in the condenser in Pa. The barometer reading is 760 mm of Hg.   **[Ans.** 2262 Pa**]**
5. The pressure of gas in a pipe line is measured with a mercury manometer having one limb open to the atmosphere. If the difference in the height of mercury in the two limbs is 562 mm, calculate the gas

pressure. The barometer reads 761 mm Hg, the acceleration due to gravity is 9.79 m/s^2, and the density of mercury is 13,640 kg/m^3. **[Ans.** 1.77 bar]

6. A turbine is supplied with steam at a gauge pressure of 1.4 MPa. After expansion in the turbine the steam flows into a condenser which is maintained at a vacuum of 710 mm Hg. The barometric pressure is 772 mm Hg. Express the inlet and exhaust steam pressures in pascals (absolute). Take the density of mercury as 13600 kg/m^3. **[Ans.** 1.503 MPa ; 8.27 kPa]

7. Gas from a bottle of compressed helium is used to inflate an inelastic flexible balloon, originally folded completely flat to a volume of 0.5 m^3. If the barometer reads 760 mm Hg, what is the amount of work done upon the atmosphere by the balloon ? **[Ans.** 50.66 kJ]

8. A rubber balloon (flexible) is to be filled with hydrogen from a storage cylinder which contains hydrogen at 5 bar pressure until it has a volume of 1 m^3. The atmospheric pressure may be taken as 1.013×10^5 N/m^2. Determine the work done by the system comprising the hydrogen initially in the bottle.

[Ans. 101.3 kJ]

9. When the valve of the evacuated bottle is opened, atmospheric air rushes into it. If the atmospheric pressure is 101.325 kPa, and 1.2 m^3 of air (measured at atmospheric conditions) enters into the bottle, calculate the work done by the air. **[Ans.** – 60.8 kJ]

10. A pump forces 1.2 m^3/min of water horizontally from an open well to a closed tank where the pressure is 0.9 MPa. Compute the work the pump must do upon the water in an hour just to force the water into the tank against the pressure. **[Ans.** 15972 kJ]

11. 14.5 litres of gas at 172 MN/m^2 is expanded at constant pressure until its volume becomes 130.5 litres. Determine the work done by the gas. **[Ans.** 199.5 kJ]

12. 1 kg of a fluid is compressed reversibly according to a law $pv = 0.25$ where p is in bar and v is in m^3/kg. The final volume is 1/4 of the initial volume. Calculate the work done on the fluid and sketch the process on a p-v diagram. **[Ans.** 34660 Nm]

13. A gas having a volume of 0.05 m^3 and pressure of 6.9 bar expands reversibly in a cylinder behind a piston according to law $pv^{1.2}$ = constant until the volume is 0.08 m^3. Calculate the work done by the gas. Also sketch the process on a p-v diagram. **[Ans.** 15300 Nm]

14. A certain fluid at 10 bar is contained in a cylinder behind a piston, the initial volume being 0.05 m^3. Calculate the work done by the fluid when it expands reversibly, (*a*) At constant pressure to final volume of 0.2 m^3 ; (*b*) According to linear law to final volume of 0.2 m^3 and a final pressure of 2 bar ; (*c*) According to a law pv = constant to a final volume of 0.1 m^3 ; (*d*) According to law pv^3 = constant to a final volume of 0.06 m^3. Sketch all processes on p-v diagram.

[Ans. (*a*) 150000 ; (*b*) 90000 ; (*c*) 34700 ; (*d*) 7640 Nm]

15. A fluid undergoes the following processes :
 (*i*) Heated reversibly at a constant pressure of 1.05 bar until it has a specific volume of 0.1 m^3/kg.
 (*ii*) It is then compressed reversibly according to a law pv = constant to a pressure of 4.2 bar.
 (*iii*) It is then allowed to expand reversibly according to a law $pv^{1.3}$ = constant.
 (*iv*) Finally it is heated at constant volume back to initial conditions.
   The work done in the constant pressure process is 515 Nm and the mass of fluid present is 0.2 kg. Calculate the net work done on or by the fluid in the cycle and sketch the cycle on a p-v diagram.
[Ans. – 422 Nm]

2

First Law of Thermodynamics

2.1. INTERNAL ENERGY

It is the heat energy stored in a gas. If a certain amount of heat is supplied to a gas the result is that temperature of gas may increase or volume of gas may increase thereby doing some external work or both temperature and volume may increase ; but it will be decided by the conditions under which the gas is supplied heat. *If during heating of the gas the temperature increases its internal energy will also increase.*

Joule's law of internal energy states that internal energy of a perfect gas is a *function of temperature only.* In other words, internal energy of a gas is dependent on the temperature change only and is not affected by the change in pressure and volume.

We do not know how to find the absolute quantity of internal energy in any substance ; however, what is needed in engineering is the change of internal energy (ΔU).

2.2. LAW OF CONSERVATION OF ENERGY

In the early part of nineteenth century the scientists developed the concept of energy and hypothesis that it can be neither created nor destroyed ; this came to be known as the *law of the conservation of energy.* The first law of thermodynamics is merely one statement of this general law/principle with particular reference to heat energy and mechanical energy *i.e.* work.

2.3. FIRST LAW OF THERMODYNAMICS

It is observed that when a system is made to undergo a complete cycle then network is done *on* or *by* the system. Consider a cycle in which network is done by the system. Since energy cannot be created, this mechanical energy must have been supplied from some source of energy. Now the system has been returned to its initial state : Therefore, its *intrinsic* energy is unchanged, and

hence the mechanical energy has not been provided by the system itself. The only other energy involved in the cycle is the heat which was supplied and rejected in various processes. Hence, by the law of conservation of energy, the network done by the system is equal to the net heat supplied to the system. The First Law of Thermodynamics can, therefore, be stated as follows :

"When a system undergoes a thermodynamic cycle then the net heat supplied to the system from the surroundings is equal to network done by the system on its surroundings.

or

$$\oint dQ = \oint dW$$

where $\oint$ represents the sum for a complete cycle.

The first law of Thermodynamics *cannot be proved analytically, but experimental evidence has repeatedly confirmed its validity*, and since no phenomenon has been shown to contradict it, the first law is accepted as a *law of nature*. It may be remarked that no restriction was imposed which limited the application of first law to reversible energy transformation. Hence the first law applies to reversible as well as irreversible transformations : For a non-cyclic process, a more general formulation of first law of thermodynamics is required. A new concept which involves a term called *internal energy* fulfils this need.

— The First Law of Thermodynamics may also be stated as follows :

"Heat and work are mutually convertible but since energy can neither be created nor destroyed, the total energy associated with an energy conversion remains constant".

Or

— **"No machine can produce energy without corresponding expenditure of energy, *i.e.*, it is impossible to construct a perpetual motion machine of first kind".**

Fig. 2.1 shows the experiment for checking first law of thermodynamics.
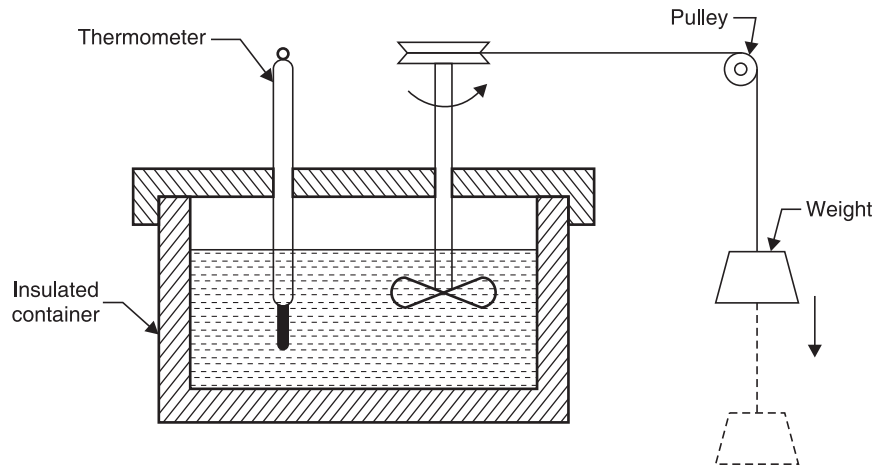
Fig. 2.1. Heat and work.

The work input to the paddle wheel is measured by the fall of weight, while the corresponding temperature rise of liquid in the insulated container is measured by the thermometer. It is already known to us from experiments on heat transfer that temperature rise can also be produced by heat transfer. The experiments show : (*i*) A definite quantity of work is always required to accomplish the same temperature rise obtained with a unit amount of heat. (*ii*) Regardless of whether the

temperature of liquid is raised by work transfer or heat transfer, the liquid can be returned by heat transfer in opposite direction to the identical state from which it started. The above results lead to the inference that *work and heat* are different forms of something more general, which is called *energy*.

— It can be stated as an invariable experience that whenever a physical system passes through a complete cycle the algebraic sum of the work transfers during the cycle $\oint dW$ bears a definite ratio to the algebraic sum of the heat transfers during the cycle, $\oint dQ$.

This may be expressed by the equation,

$$\oint dW = J \oint dQ \qquad \qquad \text{...(2.1)}$$

where J is the proportionality constant and is known as *Mechanical Equivalent of heat*. In S.I. units its value is unity, *i.e.*, 1 Nm/J.

2.4.   APPLICATION OF FIRST LAW TO A PROCESS

When a process is executed by a system, the *change in stored energy of the system is numerically equal to the net heat interactions minus the network interaction during the process.*

$\therefore \qquad \qquad E_2 - E_1 = Q - W$

$\therefore \qquad \qquad \Delta E = Q - W \qquad \qquad [\text{or} \quad Q = \Delta E + W]$

or $\qquad \qquad \displaystyle\int_1^2 d(Q - W) = \Delta E = E_2 - E_1 \qquad \qquad \text{...(2.2)}$

where E represents the *total internal energy.*

If the electric, magnetic and chemical energies are *absent* and changes in potential and kinetic energy for a *closed system* are neglected, the above equation can be written as

$$\int_1^2 d(Q - W) = \Delta U = U_2 - U_1 \qquad \qquad \text{...(2.3)}$$

$\therefore \qquad \qquad Q \ - W = \Delta U = U_2 - U_1 \qquad \qquad \text{...(2.4)}$

Generally, when heat is added to a system its temperature rises and external work is performed due to increase in volume of the system. *The rise in temperature is an indication of increase of internal energy.*

Heat *added* to the system will be considered as *positive* and the heat *removed* or *rejected*, from the system, as *negative*.

2.5.   ENERGY—A PROPERTY OF SYSTEM

Consider a system which changes its state from state 1 to state 2 by following the path L, and returns from state 2 to state 1 by following the path M (Fig. 2.2). So the system undergoes a cycle. Writing the first law for path L

$$Q_L = \Delta E_L + W_L \qquad \qquad \text{...(2.5)}$$

and for path M

$$Q_M = \Delta E_M + W_M \qquad \qquad \text{...(2.6)}$$

The processes L and M together constitute a cycle, for which

$$\oint dW = \oint dQ$$

$$W_L + W_M = Q_L + Q_M$$

or
$$Q_L - W_L = W_M - Q_M \qquad \qquad ...(2.7)$$

From equations (2.5), (2.6) and (2.7), it yields

$$\Delta E_L = - \Delta E_M \qquad \qquad ...(2.8)$$
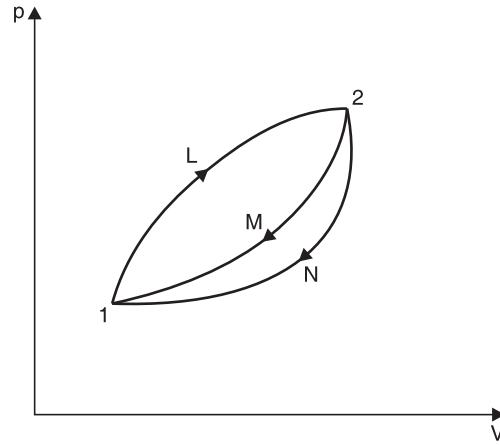
Fig. 2.2. Energy—a property of system.

Similarly, had the system returned from state 2 to state 1 by following the path N instead of path M

$$\Delta E_L = - \Delta E_N \qquad \qquad ...(2.9)$$

From equations (2.8) and (2.9),

$$\Delta E_M = \Delta E_N \qquad \qquad ...(2.10)$$

Thus, it is seen that the *change in energy between two states of a system is the same, whatever path the system may follow in undergoing that change of state.* If some arbitrary value of energy is assigned to state 2, the value of energy at state 1 is fixed independent of the path the system follows. Therefore, *energy has a definite value for every state of the system.* Hence, it is *a point function and a property of the system.*

2.6. PERPETUAL MOTION MACHINE OF THE FIRST KIND—PMM 1

— The first law of thermodynamics states the general principle of the conservation of energy. Energy is neither created nor destroyed, but only gets transformed from one form to another. *There can be no machine which would continuously supply mechanical work without some form of energy disappearing simultaneously* (Fig. 2.3). Such a *fictitious machine* is called a *perpetual motion machine of the first kind*, or in brief, PMM 1. A PMM 1 is thus *impossible.*
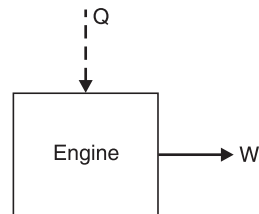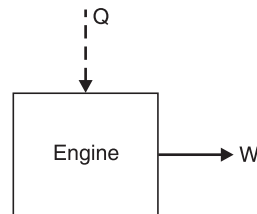
Fig. 2.3. A PPM 1. **Fig. 2.4.** The converse of PMM 1.

— The converse of the above statement is also true, *i.e.*, there can be no machine which would continuously consume work without some other form of energy appearing simultaneously (Fig. 2.4).

2.7. ENERGY OF AN ISOLATED SYSTEM

An isolated system is one in which there is no interaction of the system with the surroundings. For an isolated system,

$$dQ = 0, dW = 0$$

The first law of thermodynamics gives

$$dE = 0$$

or E = constant

The energy of an isolated system is always constant.

2.8. THE PERFECT GAS

2.8.1. The Characteristic Equation of State

— At temperatures that are considerably in excess of critical temperature of a fluid, and also at very low pressure, the vapour of fluid tends to obey the equation

$$\frac{pv}{T} = \text{constant} = R$$

In practice, no gas obeys this law rigidly, but many gases tend towards it.

An imaginary ideal gas which obeys this law is called a *perfect gas*, and the equation $\frac{pv}{T} = R$, is called the *characteristic equation of a state of a perfect gas.* The constant R is called the *gas constant.* Each perfect gas has a different gas constant.

Units of R are Nm/kg K or kJ/kg K.

Usually, the characteristic equation is written as

$$pv = RT \qquad\qquad\qquad\qquad ...(2.11)$$

or for m kg, occupying V m^3

$$pV = mRT \qquad\qquad\qquad\qquad ...(2.12)$$

— The characteristic equation in *another form*, can be derived by using kilogram-mole as a unit.

The *kilogram-mole* is defined as a quantity of a gas equivalent to M kg of the gas, where M is the molecular weight of the gas (*e.g.*, since the molecular weight of oxygen is 32, then 1 kg mole of oxygen is equivalent to 32 kg of oxygen).

As per definition of the kilogram-mole, for m kg of a gas, we have

$$m = nM \qquad\qquad\qquad\qquad ...(2.13)$$

where n = number of moles.

Note. Since the standard of mass is the kg, kilogram-mole will be written simply as mole.

Substituting for m from eqn. (2.13) in eqn. (2.12) gives

$$pV = nMRT$$

or $$MR = \frac{pV}{nT}$$

According to *Avogadro's hypothesis* the volume of 1 mole of any gas is the same as the volume of 1 mole of any other gas, when the gases are at the same temperature and pressure. Therefore,

$\dfrac{V}{n}$ is the same for all gases at the same value of p and T. That is the quantity $\dfrac{pV}{nT}$ is a *constant* for all gases. This constant is called *universal gas constant*, and is given the symbol, R_0.

i.e., $MR = R_0 = \dfrac{pV}{nT}$

or $pV = nR_0T$ ...(2.14)

Since $MR = R_0$, then

$$R = \dfrac{R_0}{M} \qquad\qquad\qquad ...(2.15)$$

It has been found experimentally that the volume of 1 mole of any prefect gas at 1 bar and 0°C is approximately 22.71 m^3.

Therefore from eqn. (2.14),

$$R_0 = \dfrac{pV}{nT} = \dfrac{1 \times 10^5 \times 22.71}{1 \times 273.15}$$

$$= 8314.3 \text{ Nm/mole K}$$

Using Eqn. (2.15), the gas constant for any gas can be found when the molecular weight is known.

Example. For oxygen which has a molecular weight of 32, the gas constant

$$R = \dfrac{R_0}{M} = \dfrac{8314}{32} = 259.8 \text{ Nm/kg K}.$$

2.8.2. Specific Heats

— The specific heat of a solid or liquid is usually defined as the *heat required to raise unit mass through one degree temperature rise.*

— For small quantities, we have

$$dQ = mcdT$$

where m = mass

c = specific heat

dT = temperature rise.

For a gas there are an infinite number of ways in which heat may be added between any two temperatures, and hence a *gas could have an infinite number of specific heats.* However, only two specific heats for gases are defined.

Specific heat at constant volume, c_v

and   *Specific heat at constant pressure, c_p.*

We have

$dQ = m\,c_p\,dT$ For a reversible non-flow process at *constant pressure*   ...(2.16)

and $dQ = m\,c_v\,dT$ For a reversible non-flow process at *constant volume*   ...(2.17)

The values of c_p and c_v, for a perfect gas, are constant for any one gas at all pressures and temperatures. Hence, integrating eqns. (2.16) and (2.17), we have

Flow of heat in a reversible constant pressure process

$$= mc_p\,(T_2 - T_1) \qquad\qquad\qquad ...(2.18)$$

Flow of heat in a reversible constant volume process

$$= mc_v\,(T_2 - T_1) \qquad\qquad\qquad ...(2.19)$$

In case of *real gases, c_p* and *c_v vary with temperature,* but a suitable *average value* may be used for most practical purposes.

2.8.3. Joule's Law

Joule's law states as follows :

"*The internal energy of a perfect gas is a function of the absolute temperature only.*"

i.e., $u = f(T)$

To evaluate this function let 1 kg of a perfect gas be heated at constant volume.

According to non-flow energy equation,

$$dQ = du + dW$$

$dW = 0$, since volume remains constant

∴ $dQ = du$

At constant volume for a perfect gas, from eqn. (2.17), for 1 kg

$$dQ = c_v dT$$

∴ $dQ = du = c_v dT$

and integrating $u = c_v T + K$, K being constant.

According to Joule's law $u = f(T)$, which means that internal energy varies linearly with absolute temperature. Internal energy can be made zero at any arbitrary reference temperature. For a perfect gas it can be assumed that $u = 0$ when $T = 0$, hence constant K is zero.

i.e.,   Internal energy, $u = c_v T$ for a perfect gas   ...(2.20)

or   For mass m, of a prefect gas

Internal energy, $U = mc_v T$ ...(2.21)

For a perfect gas, in any process between states 1 and 2, we have from eqn. (2.21)

Gain in internal energy,   ...(2.22)

$$U_2 - U_1 = mc_v (T_2 - T_1)$$

Eqn. (2.22) gives the gains of internal energy for a perfect gas between two states *for any process, reversible* or *irreversible.*

2.8.4. Relationship between Two Specific Heats

Consider a perfect gas being heated at constant pressure from T_1 to T_2.

According to non-flow equation,

$$Q = (U_2 - U_1) + W$$

Also for a perfect gas,

$$U_2 - U_1 = mc_v (T_2 - T_1)$$
$$Q = mc_v (T_2 - T_1) + W$$

In a constant pressure process, the work done by the fluid,

$$W = p(V_2 - V_1)$$

$$= mR(T_2 - T_1) \qquad \left[\begin{array}{l} \because \;\; p_1 V_1 = mRT_1 \\ \quad\; p_2 V_2 = mRT_2 \\ \quad\quad\; p_1 = p_2 = p \text{ in this case} \end{array} \right]$$

On substituting

$$Q = mc_v (T_2 - T_1) + mR (T_2 - T_1) = m(c_v + R) (T_2 - T_1)$$

But for a constant pressure process,

$$Q = mc_p (T_2 - T_1)$$

By equating the two expressions, we have

$$m(c_v + R)(T_2 - T_1) = mc_p(T_2 - T_1)$$

$\therefore$ $\qquad\qquad\qquad\qquad c_v + R = c_p$

or $\qquad\qquad\qquad\qquad\quad c_p - c_v = R$ ...(2.23)

Dividing both sides by c_v, we get

$$\frac{c_p}{c_v} - 1 = \frac{R}{c_v}$$

$\therefore$ $\qquad\qquad\qquad\qquad\quad c_v = \frac{R}{\gamma - 1}$ ...[2.23 (a)]

where $\gamma = c_p/c_v$

Similarly, dividing both sides by c_p, we get

$$c_p = \frac{\gamma R}{\gamma - 1}$$ ...[2.23 (b)]

$$\left[\begin{array}{l} \text{In M.K.S. units}: c_p - c_v = \dfrac{R}{J}\,; c_v = \dfrac{R}{J(\gamma - 1)}, c_p = \dfrac{\gamma R}{(\gamma - 1)J} \\ \text{In SI units the value of } J \text{ is unity.} \end{array}\right]$$

2.8.5. Enthalpy

— One of the fundamental quantities which occur invariably in thermodynamics is the sum of internal energy (u) and pressure volume product (pv). This sum is called **Enthalpy** (h).

i.e., $\qquad\qquad\qquad\qquad h = u + pv$ ...(2.24)

— The enthalpy of a fluid is the property of the fluid, since it consists of the sum of a property and the product of the two properties. Since enthalpy is a property like internal energy, pressure, specific volume and temperature, it can be introduced into any problem whether the process is a flow or a non-flow process.

The total enthalpy of mass, m, of a fluid can be

$$H = U + pV, \text{ where } H = mh.$$

For a **perfect gas,**

Referring eqn. (2.24),

$$\begin{aligned} h &= u + pv \\ &= c_v T + RT && [\because \quad pv = RT] \\ &= (c_v + R)T \\ &= c_p T && [\because \quad c_p = c_v + R] \end{aligned}$$

i.e., $\qquad\qquad\qquad\qquad h = c_p T$

and $\qquad\qquad\qquad\qquad H = mc_p T.$

(Note that, since it has been assumed that $u = 0$ at $T = 0$, then $h = 0$ at $T = 0$).

2.8.6. Ratio of Specific Heats

The ratio of specific heat at constant pressure to the specific heat at constant volume is given the symbol γ (gamma).

i.e., $\qquad\qquad\qquad\qquad \gamma = \frac{c_p}{c_v}$ ...(2.25)

Since $c_p = c_v + R$, it is clear that c_p must be *greater* than c_v for any perfect gas. It follows, therefore, that the ratio, $\frac{c_p}{c_v} = \gamma$ is *always greater than unity.*

In general, the approximate values of γ are as follows :

For *monoatomic* gases such as *argon, helium* = 1.6.

For *diatomic* gases such as *carbon monoxide, hydrogen, nitrogen* and *oxygen* = 1.4.

For triatomic gases such as *carbon dioxide* and *sulphur dioxide* = 1.3.

For some hydro-carbons the value of γ is quite low.

[*e.g.*, for ethane γ = 1.22, and for isobutane γ = 1.11]

2.9.   APPLICATION OF FIRST LAW OF THERMODYNAMICS TO A NON-FLOW OR CLOSED SYSTEM

(1) Reversible Constant Volume Process (v = constant)—*Isochoric process*

In a constant volume process the working substance is contained in a rigid vessel, hence the boundaries of the system are immovable and no work can be done on or by the system, other than paddle-wheel work input. It will be assumed that '*constant volume*' implies zero work unless stated otherwise.

Fig. 2.5 shows the system and states before and after the heat addition at constant volume.
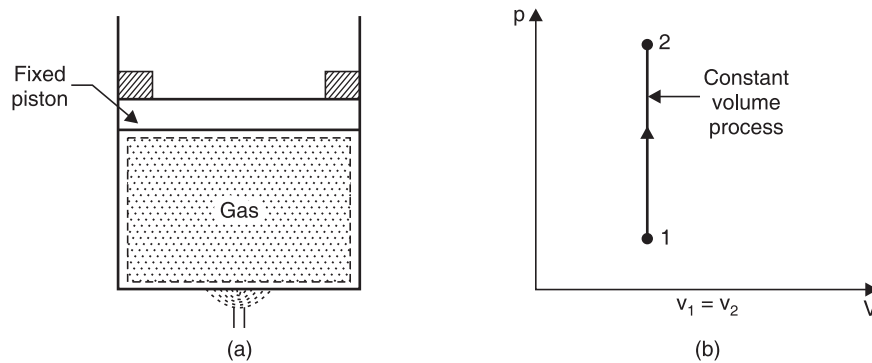
Fig. 2.5. Reversible constant volume process.

Considering mass of the working substance as *unity* and applying first law of thermodynamics to the process

$$Q = (u_2 - u_1) + W \qquad \qquad ...(2.26)$$

The work done $W = \int_1^2 pdv = 0$ as $dv = 0$.

∴ $Q = (u_2 - u_1) = c_v(T_2 - T_1)$ ...[2.27 (*a*)]

where c_v = specific heat at constant volume.

For mass, *m*, of working substance

$$Q = U_2 - U_1 = mc_v(T_2 - T_1) \qquad \qquad ...[2.27 (b)]$$
$$[\because \quad mu = U]$$

(2) Reversible Constant Pressure Process (p = constant)—*Isobaric process*

It can be seen from Fig. 2.5 (*b*) that when the boundary of the system is *inflexible* as in a constant volume process, then the pressure rises when heat is supplied. Hence for a constant pressure process, the boundary must move against an external resistance as heat is supplied ; for instance a gas [Fig. 2.6 (*a*)] in a cylinder behind a piston can be made to undergo a constant pressure process. Since the *piston is pushed through a certain distance* by the force exerted by the gas, then the work is done by the gas on its surroundings.

Fig. 2.6 shows the system and states before and after the heat addition at constant pressure.

Considering unit mass of working substance and applying first law of thermodynamics to the process
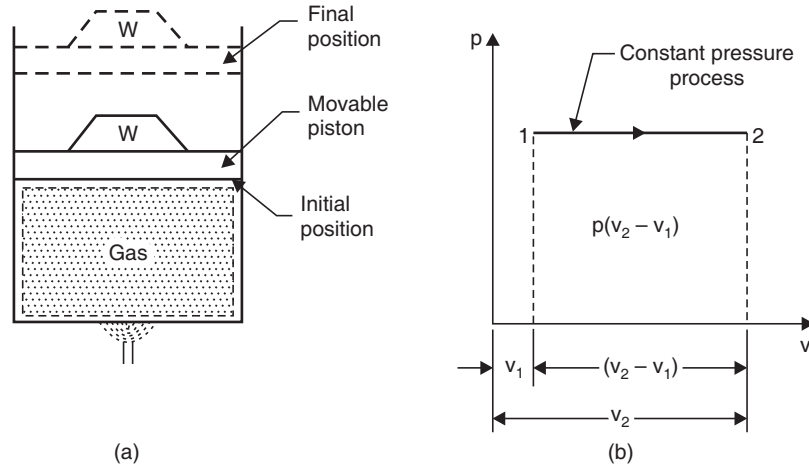
$$Q = (u_2 - u_1) + W$$

Fig. 2.6. Reversible constant pressure process.

The work done, $W = \int_1^2 pdv = p(v_2 - v_1)$

$\therefore$ $Q = (u_2 - u_1) + p(v_2 - v_1) = u_2 - u_1 + pv_2 - pv_1$

$= (u_2 + pv_2) - (u_1 + pv_1) = h_2 - h_1$ $[\because \quad h = u + pv]$

or $Q = h_2 - h_1 = c_p \, (T_2 - T_1)$ ...(2.28)

where h = enthalpy (specific)

c_p = specific heat at constant pressure.

For mass, m, of working substance

$$Q = H_2 - H_1 = mc_p \, (T_2 - T_1)$$ ...[2.28 (a)]

$[\because \quad mh = H]$

(3) Reversible Temperature or Isothermal Process (pv = constant, T = constant)

A *process at a constant temperature is called an isothermal process*. When a working substance in a cylinder behind a piston expands from a high pressure to a low pressure there is a tendency for the temperature to fall. In an isothermal expansion heat must be added continuously in order to keep the temperature at the initial value. Similarly in an isothermal compression heat must be removed from the working substance continuously during the process.

Fig. 2.7 shows the system and the states before and after the heat addition at constant temperature.

Considering unit mass of working substance and applying first law to the process

$$Q = (u_2 - u_1) + W$$
$$= c_v \, (T_2 - T_1) + W$$
$$= 0 + W \qquad\qquad [\because \quad T_2 = T_1]$$

The work done, $W = \int_1^2 pdv$
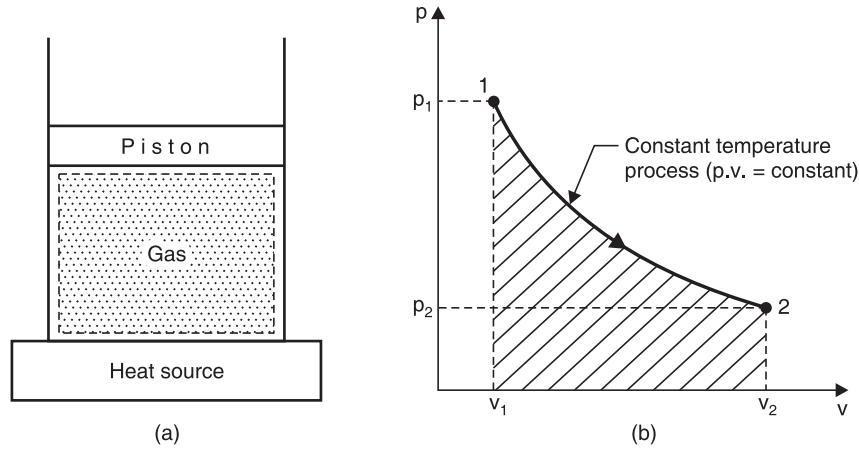
Fig. 2.7. Reversible isothermal process.

In this case pv = constant or $p = \dfrac{C}{v}$ (where C = constant)

$\therefore \qquad\qquad W = \displaystyle\int_{v_1}^{v_2} C\,\dfrac{dv}{v} = C\left[\log_e v\right]_{v_1}^{v_2} = C\log_e\dfrac{v_2}{v_1}$

The constant C can either be written as p_1v_1 or as p_2v_2, since

$\qquad\qquad p_1v_1 = p_2v_2$ = constant, C

i.e., $\qquad\qquad W = p_1v_1\log_e\dfrac{v_2}{v_1}$ per unit mass of working substance

or $\qquad\qquad W = p_2v_2\log_e\dfrac{v_2}{v_1}$ per unit mass of working substance

$\therefore \qquad\qquad Q = W = p_1v_1\log_e\dfrac{v_2}{v_1}$ ...(2.29)

For mass, m, of the working substance

$\qquad\qquad Q = p_1V_1\log_e\dfrac{V_2}{V_1}$ ...[2.29 (a)]

or $\qquad\qquad Q = p_1V_1\log_e\dfrac{p_1}{p_2}$ $\qquad\left[\because\ \dfrac{V_2}{V_1} = \dfrac{p_1}{p_2}\right]$ ...[2.29 (b)]

(4) Reversible Adiabatic Process (pv^γ = constant)

An **adiabatic process i**s *one in which no heat is transferred to or from the fluid during the process.* Such a process can be reversible or irreversible. The reversible adiabatic non-flow process will be considered in this section.

Considering unit mass of working substance and applying first law to the process

$\qquad\qquad Q = (u_2 - u_1) + W$

$\qquad\qquad O = (u_2 - u_1) + W$

or $\qquad\qquad W = (u_1 - u_2)$ for any adiabatic process ...(2.30)

Eqn. (2.30) is true for an adiabatic process whether the process is reversible or not. In an adiabatic expansion, the work done W by the fluid is at the expense of a reduction in the internal

energy of the fluid. Similarly in an adiabatic compression process all the work done on the fluid goes to increase the internal energy of the fluid.

For an adiabatic process to take place, perfect thermal insulation for the system must be available.

To derive the law pv^γ = constant :

To obtain a law relating p and v for a reversible adiabatic process let us consider the non-flow energy equation in differential form,

$$dQ = du + dW$$

For a reversible process

$$dW = pdv$$

$\therefore$ $\qquad\qquad\qquad dQ = du + pdv = 0$

(Since for an adiabatic process $Q = 0$)

Also for a perfect gas

$$pv = RT \quad \text{or} \quad p = \frac{RT}{v}$$

Hence substituting,

$$du + \frac{RTdv}{v} = 0$$

Also $\qquad\qquad\qquad u = c_v T \quad \text{or} \quad du = c_v dT$

$\therefore$ $\qquad\qquad c_v dT + \frac{RTdv}{v} = 0$

Dividing both sides by T, we get

$$c_v \frac{dT}{T} + \frac{Rdv}{v} = 0$$

Integrating

$$c_v \log_e T + R \log_e v = \text{constant}$$

Substituting $T = \dfrac{pv}{R}$

$$c_v \log_e \frac{pv}{R} + R \log_e v = \text{constant}$$

Dividing throughout both sides by c_v

$$\log_e \frac{pv}{R} + \frac{R}{c_v} \cdot \log_e v = \text{constant}$$

Again $\qquad\qquad\qquad c_v = \frac{R}{(\gamma - 1)} \quad \text{or} \quad \frac{R}{c_v} = \gamma - 1$

Hence substituting

$$\log_e \frac{pv}{R} + (\gamma - 1) \log_e v = \text{constant}$$

$\therefore$ $\qquad\qquad \log_e \frac{pv}{R} + \log_e v^{\gamma - 1} = \text{constant}$

$$\log_e \frac{pv \times v^{\gamma - 1}}{R} = \text{constant}$$

i.e., $\log_e \dfrac{pv^\gamma}{R} =$ constant

i.e., $\dfrac{pv^\gamma}{R} = e^{\text{constant}} =$ constant

or $pv^\gamma =$ constant   ...(2.31)

Expression for work W :

A reversible adiabatic process for a perfect gas is shown on a *p-v* diagram in Fig. 2.8 (*b*).
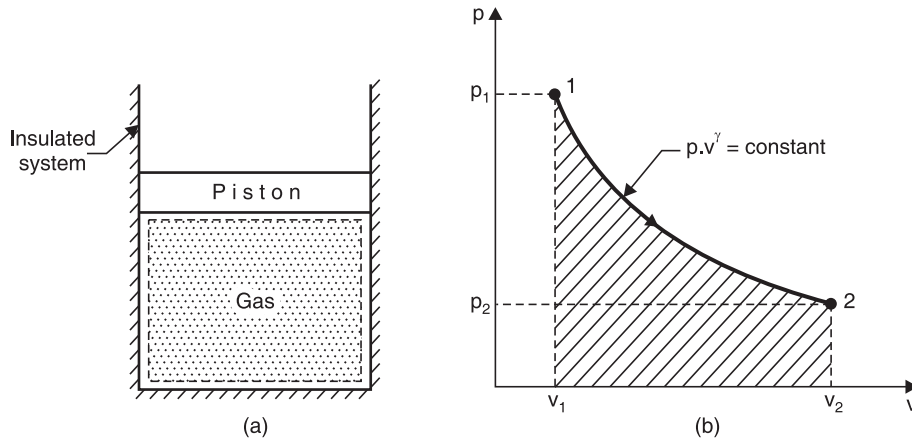
Fig. 2.8. Reversible adiabatic process.

The work done is given by the shaded area, and this area can be evaluated by integration.

i.e., $W = \displaystyle\int_{v_1}^{v_2} p\, dv$

Therefore, since $pv^\gamma =$ constant, C, then

$$W = \int_{v_1}^{v_2} C\, \frac{dv}{v^\gamma} \qquad\qquad \left[\because\ p = \frac{C}{v^\gamma}\right]$$

i.e., $W = C \displaystyle\int_{v_1}^{v_2} \frac{dv}{v^\gamma} = C \left.\frac{v^{-\gamma+1}}{-\gamma+1}\right|_{v_1}^{v_2}$

$$= C\left(\frac{v_2^{-\gamma+1} - v_1^{-\gamma+1}}{1-\gamma}\right) = C\left(\frac{v_1^{-\gamma+1} - v_2^{-\gamma+1}}{\gamma-1}\right)$$

The constant in this equation can be written as $p_1 v_1^\gamma$ or as $p_2 v_2^\gamma$. Hence,

$$W = \frac{p_1 v_1^\gamma v_1^{-\gamma+1} - p_2 v_2^\gamma v_2^{-\gamma+1}}{\gamma-1} = \frac{p_1 v_1 - p_2 v_2}{\gamma-1}$$

i.e., $W = \dfrac{p_1 v_1 - p_2 v_2}{\gamma-1}$ ...(2.32)

or $W = \dfrac{R(T_1 - T_2)}{\gamma-1}$ ...(2.33)

Relationship between T and v, and T and p :

By using equation $pv = RT$, the relationship between T and v, and T and p, may by derived as follows :

i.e.,
$$pv = RT$$

$\therefore$
$$p = \frac{RT}{v}$$

Putting this value in the equation $pv^\gamma = $ constant

$$\frac{RT}{v} \cdot v^\gamma = \text{constant}$$

i.e.,
$$Tv^{\gamma-1} = \text{constant} \qquad \qquad ...(2.34)$$

Also $v = \dfrac{RT}{p}$; hence substituting in equation $pv^\gamma = $ constant

$$p\left(\frac{RT}{p}\right)^\gamma = \text{constant}$$

$\therefore$
$$\frac{T^\gamma}{p^{\gamma-1}} = \text{constant}$$

or
$$\frac{T}{(p)^{\frac{\gamma-1}{\gamma}}} = \text{constant} \qquad \qquad ...(2.35)$$

Therefore, for a reversible adiabatic process for a perfect gas between states 1 and 2, we can write :

From eqn. (2.31),

$$p_1 v_1{}^\gamma = p_2 v_2{}^\gamma \quad \text{or} \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^\gamma \qquad \qquad ...(2.36)$$

From eqn. (2.34),

$$T_1 v_1{}^{\gamma-1} = T_2 v_2{}^{\gamma-1} \quad \text{or} \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} \qquad \qquad ...(2.37)$$

From eqn. (2.35),

$$\frac{T_1}{(p_1)^{\frac{\gamma-1}{\gamma}}} = \frac{T_2}{(p_2)^{\frac{\gamma-1}{\gamma}}} \quad \text{or} \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad \qquad ...(2.38)$$

From eqn. (2.30), the work done in an adiabatic process per kg of gas is given by $W = (u_1 - u_2)$. The gain in internal energy of a perfect gas is given by equation :

$$u_2 - u_1 = c_v (T_2 - T_1) \qquad \qquad \text{(for 1 kg)}$$

$\therefore$
$$W = c_v (T_1 - T_2)$$

Also, we know that

$$c_v = \frac{R}{\gamma - 1}$$

Hence substituting, we get

$$W = \frac{R(T_1 - T_2)}{\gamma - 1}$$

Using equation, $\qquad pv = RT$

$$W = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$$

This is the same expression obtained before as eqn. (2.32).

(5) Polytropic Reversible Process (pv^n = constant)

It is found that many processes in practice approximate to a reversible law of form pv^n = constant, where n is a constant. Both vapours and perfect gases obey this type of law closely in many non-flow processes. Such processes are *internally reversible*.

We know that for any reversible process,

$$W = \int p \, dv$$

For a process in pv^n = constant, we have

$$p = \frac{C}{v^n}, \text{ where } C \text{ is a constant}$$

$\therefore \qquad W = C \int_{v_1}^{v_2} \frac{dv}{v^n} = C \left| \frac{v^{-n+1}}{-n+1} \right| = C \left(\frac{v_2^{-n+1} - v_1^{-n+1}}{-n+1} \right)$

i.e., $\qquad W = C \left(\frac{v_1^{-n+1} - v_2^{-n+1}}{n-1} \right) = \frac{p_1 v_1^n v_1^{-n+1} - p_2 v_2^n v_2^{-n+1}}{n-1}$

(since the constant C, can be written as $p_1 v_1^n$ or as $p_2 v_2^n$)

i.e., Work done, $\quad W = \dfrac{p_1 v_1 - p_2 v_2}{n-1}$ $\qquad\qquad$ ...(2.39)

or $\qquad W = \dfrac{R(T_1 - T_2)}{n-1}$ $\qquad\qquad$ ...(2.40)

Eqn. (2.39) is true for any working substance undergoing a reversible polytropic process. It follows also that for any polytropic process, we can write

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2} \right)^n \qquad\qquad ...(2.41)$$

The following relations can be derived (following the same procedure as was done under reversible adiabatic process)

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{n-1} \qquad\qquad ...(2.42)$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \qquad\qquad ...(2.43)$$

Heat transfer during polytropic process (for perfect gas $pv = RT$) :

Using non-flow energy equation, the heat flow/transfer during the process can be found,

i.e., $\qquad Q = (u_2 - u_1) + W$

$\qquad\qquad = c_v(T_2 - T_1) + \dfrac{R(T_1 - T_2)}{n-1}$

i.e., $\qquad Q = \dfrac{R(T_1 - T_2)}{n-1} - c_v (T_1 - T_2)$

Also $$c_v = \frac{R}{(\gamma - 1)}$$

On substituting,

$$Q = \frac{R}{n-1} \, (T_1 - T_2) - \frac{R}{(\gamma - 1)} \, (T_1 - T_2)$$

i.e., $$Q = R(T_1 - T_2) \left(\frac{1}{n-1} - \frac{1}{\gamma - 1} \right)$$

$$= \frac{R(T_1 - T_2)\,(\gamma - 1 - n + 1)}{(\gamma - 1)\,(n - 1)} = \frac{R(T_1 - T_2)\,(\gamma - n)}{(\gamma - 1)\,(n - 1)}$$

$\therefore$ $$Q = \frac{(\gamma - n)}{(\gamma - 1)} \frac{R(T_1 - T_2)}{(n - 1)}$$

or $$Q = \left(\frac{\gamma - n}{\gamma - 1} \right) W \qquad \left[\because \ W = \frac{R(T_1 - T_2)}{(n - 1)} \right] \quad ...(2.44)$$

In a polytropic process, the *index n depends only on the heat and work quantities* during the process. The various processes considered earlier are special cases of polytropic process for a perfect gas. For example,

When $n = 0$ pv° = constant $\quad$ *i.e., p* = constant

When $n = \infty$ pv^∞ = constant

or $p^{1/\infty} v$ = constant, $\quad$ *i.e., v* = constant

When $n = 1$ $\quad$ pv = constant, $\quad$ *i.e., T* = constant

$\qquad$ [since $(pv)/T$ = constant for a perfect gas]

When $n = \gamma$ $\quad$ pv^γ = constant, *i.e.*, reversible adiabatic

This is illustrated on a *p-v* diagram in Fig. 2.9.

 (*i*) State 1 to state *A* is *constant pressure cooling* ($n = 0$).

(*ii*) State 1 to state *B* is *isothermal compression* ($n = 1$).
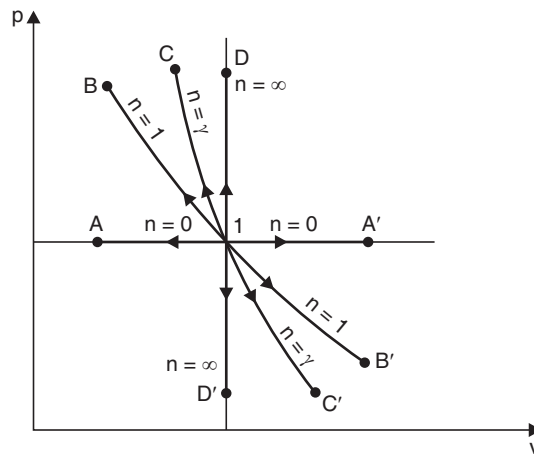
Fig. 2.9. Polytropic process.

(*iii*) State 1 to state C is *reversible adiabatic compression* ($n = \gamma$).

(*iv*) State 1 to state D is *constant volume heating* ($n = \infty$).

Similarly,

(*i*) State 1 to state A' is *constant pressure heating* ($n = 0$).

(*ii*) State 1 to state B' is *isothermal expansion* ($n = 1$).

(*iii*) State 1 to state C' is *reversible adiabatic expansion* ($n = \gamma$).

(*iv*) State 1 to state D' is *constant volume cooling* ($n = \infty$).

It may be noted that, since γ is always greater then unity, then process 1 to C must lie between processes 1 to B and 1 to D ; similarly, process 1 to C' must lie between processes 1 to B' and 1 to D'.

(6) Free Expansion

Consider two vessels 1 and 2 interconnected by a short pipe with a valve A, and perfectly thermally insulated (Fig. 2.10). Initially let the vessel 1 be filled with a fluid at a certain pressure, and let 2 be completely evacuated. When the valve A is opened the fluid in 1 will expand rapidly to fill both vessels 1 and 2. The pressure finally will be lower than the initial pressure in vessel 1. This is known as *free or unresisted expansion*. The process is *highly irreversible* ; since the fluid is eddying continuously during the process. Now applying first law of thermodynamics (or non-flow energy equation) between the initial and final states,

$$Q = (u_2 - u_1) + W$$
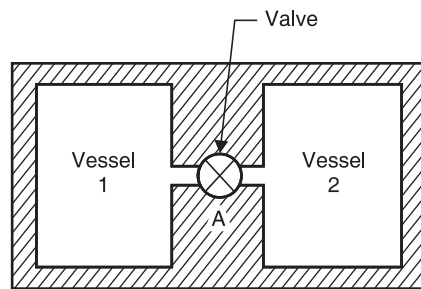
Fig. 2.10. Free expansion.

In this process, no work is done on or by the fluid, since the boundary of the system does not move. No heat flows to or from the fluid since the system is well lagged. The process is therefore, *adiabatic but irreversible*.

i.e., $u_2 - u_1 = 0$ or $u_2 = u_1$

In a free expansion, therefore, the internal energy initially equals the initial energy finally.

For a perfect gas,

$$u = c_v T$$

∴   For a free expansion of a perfect gas,

$$c_v T_1 = c_v T_2$$

i.e., $T_1 = T_2$

That is, for a perfect gas undergoing a free expansion, the initial temperature is equal to the final temperature.

Table 2.1 Summary of Processes for Perfect Gas (Unit mass)

Process	Index n	Heat added	$\int_1^2 pdv$	p, v, T relations	Specific heat, c
Constant pressure	$n = 0$	$c_p(T_2 - T_1)$	$p(v_2 - v_1)$	$\dfrac{T_2}{T_1} = \dfrac{v_2}{v_1}$	c_p
Constant volume	$n = \infty$	$c_v(T_2 - T_1)$	0	$\dfrac{T_1}{T_2} = \dfrac{p_1}{p_2}$	c_v
Constant temperature	$n = 1$	$p_1 v_1 \log_e \dfrac{v_2}{v_1}$	$p_1 v_1 \log_e \dfrac{v_2}{v_1}$	$p_1 v_1 = p_2 v_2$	∞
Reversible adiabatic	$n = \gamma$	0	$\dfrac{p_1 v_1 - p_2 v_2}{\gamma - 1}$	$p_1 v_1{}^\gamma = p_2 v_2{}^\gamma$ $\dfrac{T_2}{T_1} = \left(\dfrac{v_1}{v_2}\right)^{\gamma - 1}$ $= \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$	0
Polytropic	$n = n$	$= c_n(T_2 - T_1)$ $= c_v\left(\dfrac{\gamma - n}{1 - n}\right)$ $\times (T_2 - T_1)$ $= \dfrac{\gamma - n}{\gamma - 1} \times$ work done (non-flow)	$\dfrac{p_1 v_1 - p_2 v_2}{n - 1}$	$p_1 v_1{}^n = p_2 v_2{}^n$ $\dfrac{T_2}{T_1} = \left(\dfrac{v_1}{v_2}\right)^{n - 1}$ $= \left(\dfrac{p_2}{p_1}\right)^{\frac{n - 1}{n}}$	$c_n = c_v\left(\dfrac{\gamma - n}{1 - n}\right)$

Note. Equations must be used keeping dimensional consistence.

Example 2.1. *In an internal combustion engine, during the compression stroke the heat rejected to the cooling water is 50 kJ/kg and the work input is 100 kJ/kg.*

Calculate the change in internal energy of the working fluid stating whether it is a gain or loss.

Solution. Heat rejected to the cooling water, $Q = -50$ kJ/kg

(–ve sign since heat is rejected)

Work input, $W = -100$ kJ/kg

(–ve sign since work is supplied to the system)

Using the relation, $Q = (u_2 - u_1) + W$

$-50 = (u_2 - u_1) - 100$

or $u_2 - u_1 = -50 + 100 = 50$ kJ/kg

Hence, **gain in internal energy = 50 kJ/kg. (Ans.)**

Example 2.2. *In an air motor cylinder the compressed air has an internal energy of 450 kJ/kg at the beginning of the expansion and an internal energy of 220 kJ/kg after expansion. If the work done by the air during the expansion is 120 kJ/kg, calculate the heat flow to and from the cylinder.*

Solution. Internal energy at beginning of the expansion,

$$u_1 = 450 \text{ kJ/kg}$$

Internal energy after expansion,

$$u_2 = 220 \text{ kJ/kg}$$

Work done by the air during expansion,

$$W = 120 \text{ kJ/kg}$$

Heat flow, Q :

Using the relation, $Q = (u_2 - u_1) + W$

∴ $Q = (220 - 450) + 120$

$$= -230 + 120 = -110 \text{ kJ/kg}$$

Hence, **heat rejected by air = 110 kJ/kg. (Ans.)**

Example 2.3. *0.3 kg of nitrogen gas at 100 kPa and 40°C is contained in a cylinder. The piston is moved compressing nitrogen until the pressure becomes 1 MPa and temperature becomes 160°C. The work done during the process is 30 kJ.*

Calculate the heat transferred from the nitrogen to the surroundings.

$$c_v \text{ for nitrogen = 0.75 kJ/kg K}$$

Solution. Mass of nitrogen, $m = 0.3$ kg

Temperature before compression = 40°C or 313 K

Temperature after compression = 160°C or 433 K

The work done during the compression process, $W = -30$ kJ

According to first law of thermodynamics,

$$Q = \Delta U + W = (U_2 - U_1) + W$$
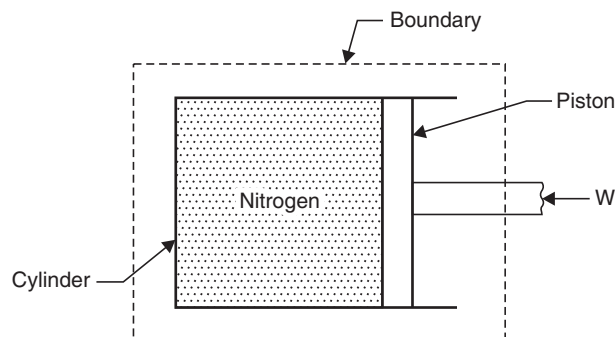
Fig. 2.11

$$= mc_v (T_2 - T_1) + W$$
$$= 0.3 \times 0.75(433 - 313) - 30 = -3 \text{ kJ}$$

Hence, **heat 'rejected' during the process = 3 kJ. (Ans.)**

Note. Work, *W* has been taken –ve because it has been supplied from outside.

Example 2.4. *When a stationary mass of gas was compressed without friction at constant pressure its initial state of 0.4 m³ and 0.105 MPa was found to change to final state of 0.20 m³ and 0.105 MPa. There was a transfer of 42.5 kJ of heat from the gas during the process.*

How much did the internal energy of the gas change ?

Solution.
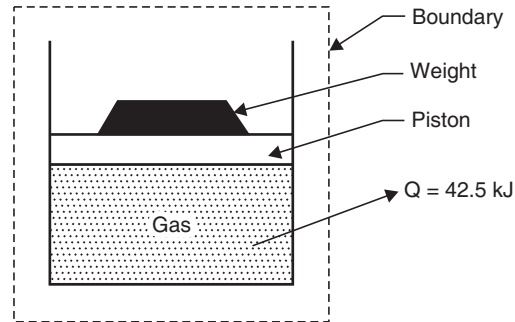
Fig. 2.12

Initial state

Pressure of gas, $\qquad p_1 = 0.105$ MPa

Volume of gas, $\qquad V_1 = 0.4$ m^3

Final state

Pressure of gas, $\qquad p_2 = 0.105$ MPa

Volume of gas, $\qquad V_2 = 0.20$ m^3

Process used : *constant pressure*

Heat transferred, $\qquad Q = -42.5$ kJ

(–ve sign indicates that heat is rejected)

Change in internal energy, $\Delta U = U_2 - U_1$:

First law for a stationary system in a process gives

$$Q = \Delta U + W$$

or $\qquad\qquad Q_{1-2} = (U_2 - U_1) + W_{1-2} \qquad\qquad\qquad ...(i)$

Here $\qquad\qquad W_{1-2} = \int_{V_1}^{V_2} p dV = p(V_2 - V_1)$

$$= 0.105(0.20 - 0.40) \text{ MJ} = -21 \text{ kJ}$$

$$[\because \quad 1 \text{ MJ} = 10^3 \text{ kJ}]$$

Substituting this value of W_{1-2} in eqn. (*i*), we get

$$-42.5 = (U_2 - U_1) - 21$$

$\therefore \qquad\qquad U_2 - U_1 = -42.5 + 21 = -21.5$ kJ

Hence **'decrease' in internal energy = 21.5 kJ. (Ans.)**

Example 2.5. *A container is divided into compartments by a partition. The container is completely insulated so that there is no heat transfer. One portion contains gas at temperature T_1 and pressure p_1 while the other portion also has the same gas but at temperature T_2 and pressure p_2.*

How will the first law of thermodynamics conclude the result if partition is removed ?

Solution. Refer to Fig. 2.13.

According to first law of thermodynamics,

$$\delta Q = \delta U + \delta W$$

When partition is removed, $\qquad \delta Q = 0$

$$\delta W = 0$$

$\therefore \qquad\qquad\qquad \delta U = 0.$
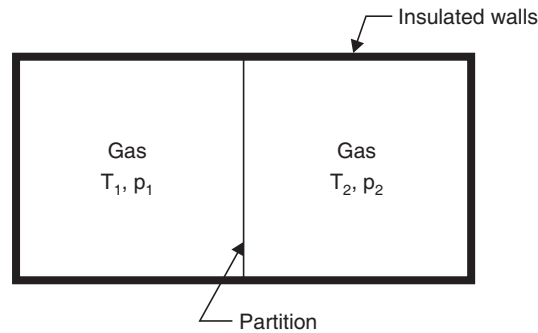
Fig. 2.13

Conclusion. *There is conservation of internal energy.*

Example 2.6. *Air enters a compressor at 10^5 Pa and 25°C having volume of 1.8 m^3/kg and is compressed to 5 × 10^5 Pa isothermally.*

Determine : (*i*) *Work done ;*

(*ii*) *Change in internal energy ; and*

(*iii*) *Heat transferred.*

Solution. Initial pressure of air, $p_1 = 10^5$ Pa

Initial temperature of air, $T_1 = 25 + 273 = 298$ K

Final pressure of air, $p_2 = 5 × 10^5$ Pa

Final temperature of air, $T_2 = T_1 = 298$ K (isothermal process)

Since, it is a closed steady state process, we can write down the first law of thermodynamics as,

$$Q = (u_2 - u_1) + W \ ......\text{per kg}$$

(*i*) *For isothermal process :*

$$W_{1-2} = \int_1^2 p.\, dv = p_1 v_1 \log_e \left(\frac{p_1}{p_2} \right)$$
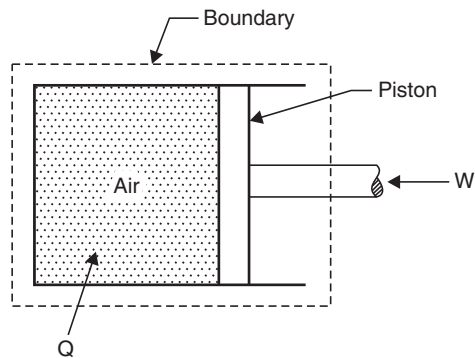
Fig. 2.14

As $p_1 v_1 = p_2 v_2$ for isothermal process

∴ $W_{1-2} = - 10^5 × 1.8 \log_e \left(\dfrac{1 × 10^5}{5 × 10^5} \right)$

$$= -2.897 \times 10^5 = -289.7 \text{ kJ/kg.}$$

(– ve sign indicates that the work is supplied to the air)

∴ **Work done on the air = 289.7 kJ/kg. (Ans.)**

(*ii*) Since temperature is constant,

∴ $\qquad u_2 - u_1 = 0$

∴ **Change in internal energy = zero. (Ans.)**

(*iii*) Again, $\qquad Q_{1-2} = (u_2 - u_1) + W$

$$= 0 + (-289.7) = -289.7 \text{ kJ}$$

(– ve sign indicates that heat is lost from the system to the surroundings)

∴ **Heat rejected = 289.7 kJ/kg. (Ans.)**

Example 2.7. *A cylinder containing the air comprises the system. Cycle is completed as follows* :

(*i*) *82000 Nm of work is done by the piston on the air during compression stroke and 45 kJ of heat are rejected to the surroundings.*

(*ii*) *During expansion stroke 100000 Nm of work is done by the air on the piston.*

Calculate the quantity of heat added to the system.

Solution. Refer to Fig. 2.15.

Compression stroke. Process 1–2 :

Work done by the piston *on the air,* $W_{1-2} = -82000 \text{ Nm} (= -82 \text{ kJ})$

Heat rejected to the system, $Q_{1-2} = -45 \text{ kJ}$

Now, $\qquad Q_{1-2} = (U_2 - U_1) + W$

$$-45 = (U_2 - U_1) + (-82)$$

∴ $\qquad (U_2 - U_1) = 37 \text{ kJ}$ $\qquad\qquad$ ...(*i*)

Expansion stroke. Process 2–1 :

Work done by air on the piston, $W_{2-1} = 100000 \text{ Nm} (= 100 \text{ kJ})$

Now, $\qquad Q_{2-1} = (U_1 - U_2) + W$

$$= -37 + 100 \text{ kJ} = 63 \text{ kJ}$$

Hence, **quantity of heat added to the system = 63 kJ. (Ans.)**
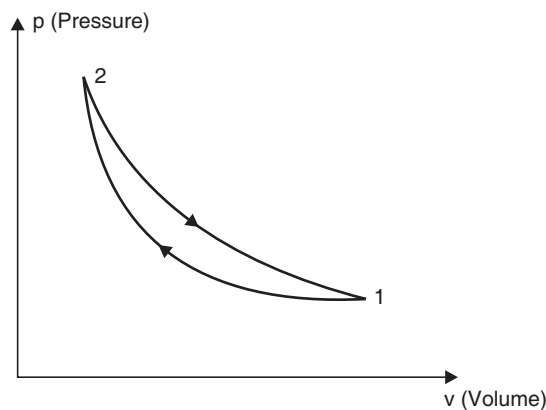
Fig. 2.15

☞ **Example 2.8.** *A tank containing air is stirred by a paddle wheel. The work input to the paddle wheel is 9000 kJ and the heat transferred to the surroundings from the tank is 3000 kJ.*

Determine : (i) Work done ;

(ii) Change in internal energy of the system.

Solution. Refer to Fig. 2.16.
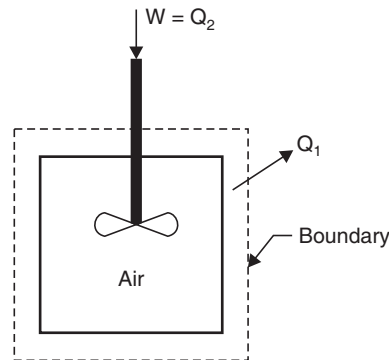
Fig. 2.16

Work input to the paddle wheel = 9000 kJ

Heat transferred to the surroundings from the tank = 3000 kJ

As it is a closed system, the first law of thermodynamics can be written as

$$U_1 - Q + W = U_2 \qquad\qquad\qquad ...(i)$$

The *work* enters into the tank in the form of energy only so this should be considered as *heat input*.

∴ $Q = Q_1 - Q_2$

                                           $= 3000 - 9000 = -6000$ kJ.

(*i*) Since volume does not change (being constant volume process)

∴   Work done, $W = 0$

Putting the value of $W = 0$ in eqn. (*i*), we get

(*ii*) $U_1 - (-6000) + 0 = U_2$

∴ $U_2 - U_1 = 6000$ kJ

Hence, **change in internal energy (increase) = 6000 kJ.   (Ans.)**

Example 2.9. *A stone of 20 kg mass and a tank containing 200 kg water comprise a system. The stone is 15 m above the water level initially. The stone and water are at the same temperature initially. If the stone falls into water, then determine ΔU, ΔPE, ΔKE, Q and W, when*

(*i*) *the stone is about to enter the water,*

(*ii*) *the stone has come to rest in the tank, and*

(*iii*) *the heat is transferred to the surroundings in such an amount that the stone and water come to their initial temperature.*

Solution. Refer to Fig. 2.17.

Mass of stone   = 20 kg

Mass of water in the tank   = 200 kg

Height of stone above water level   = 15 m
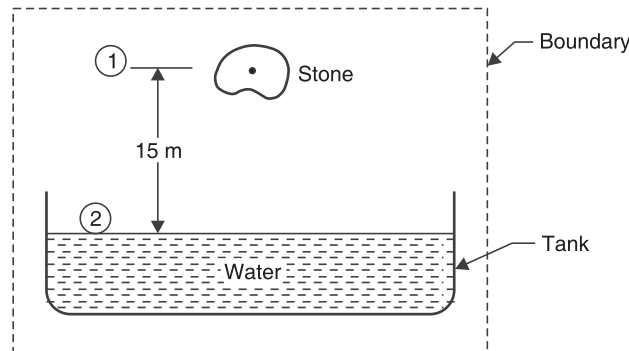
Fig. 2.17

Applying the first law of thermodynamics,

$$Q = (U_2 - U_1) + m \left[\frac{C_2^2 - C_1^2}{2} \right] + mg\,(Z_2 - Z_1) + W$$

$$= \Delta U + \Delta KE + \Delta PE + W$$

Here $\qquad Q$ = heat leaving the boundary.

(*i*) **When the stone is about to enter the water,**

$$Q = 0,\ W = 0,\ \Delta U = 0$$

$\because \qquad\qquad -\Delta KE = \Delta PE = mg\,(Z_2 - Z_1)$

$$= 20 \times 9.81\,(0 - 15) = -2943 \text{ J}$$

$\therefore \qquad\qquad \Delta KE = 2943 \text{ J}$

and $\qquad\qquad \Delta PE = -\textbf{2943 J.} \quad \textbf{(Ans.)}$

(*ii*) **When the stone dips into the tank and comes to rest**

$$Q = 0,\ W = 0,\ \Delta KE = 0$$

Substituting these values in equation (*i*), we get

$$0 = \Delta U + 0 + \Delta PE + 0$$

$\therefore \qquad\qquad \Delta U = -\Delta PE = -(-2943) = \textbf{2943 J.} \quad \textbf{(Ans.)}$

This shows that the internal energy (temperature) of the system increases.

(*iii*) **When the water and stone come to their initial temperature,**

$$W = 0,\ \Delta KE = 0$$

Substituting these values in equation (*i*), we get

$\therefore \qquad\qquad Q = -\Delta U = -\textbf{2943 J.} \quad \textbf{(Ans.)}$

The *negative* sign shows that the *heat is lost from the system to the surroundings.*

☞ **Example 2.10.** *When a system is taken from state l to state m, in Fig. 2.18, along path lqm, 168 kJ of heat flows into the system, and the system does 64 kJ of work :*

 (*i*) *How much will be the heat that flows into the system along path lnm if the work done is 21 kJ ?*

(*ii*) *When the system is returned from m to l along the curved path, the work done on the system is 42 kJ. Does the system absorb or liberate heat, and how much of the heat is absorbed or liberated ?*

(*iii*) *If $U_l = 0$ and $U_n = 84$ kJ, find the heat absorbed in the processes ln and nm.*

Solution. Refer to Fig. 2.18.
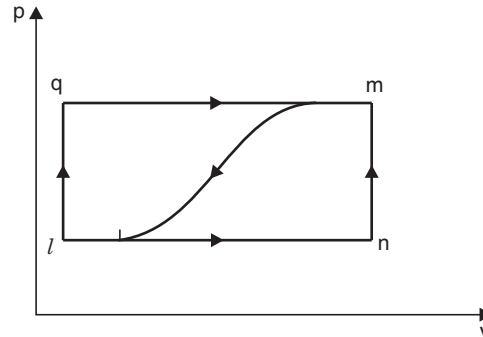
Fig. 2.18

$$Q_{l-q-m} = 168 \text{ kJ}$$
$$W_{l-q-m} = 64 \text{ kJ}$$

We have, $Q_{l-q-m} = (U_m - U_l) + W_{l-q-m}$
$$168 = (U_m - U_l) + 64$$

∴ $U_m - U_l = \textbf{104 kJ.}$ **(Ans.)**

(i) $Q_{l-n-m} = (U_m - U_l) + W_{l-n-m}$
$$= 104 + 21 = \textbf{125 kJ.}\quad\textbf{(Ans.)}$$

(ii) $Q_{m-l} = (U_l - U_m) + W_{m-l}$
$$= -104 + (-42) = \textbf{– 146 kJ.}\quad\textbf{(Ans.)}$$

The system liberates 146 kJ.

(iii) $W_{l-n-m} = W_{l-n} + W_{n-m} = W_{l-m} = 21 \text{ kJ}$
$$[\because\quad W_{n-m} = 0, \text{ since volume does not change.}]$$

∴ $Q_{l-n} = (U_n - U_l) + W_{l-n}$
$$= (84 - 0) + 21 = \textbf{105 kJ.}\quad\textbf{(Ans.)}$$

Now $Q_{l-m-n} = 125 \text{ kJ} = Q_{l-n} + Q_{n-m}$

∴ $Q_{n-m} = 125 - Q_{l-n}$
$$= 125 - 105 = \textbf{20 kJ. (Ans.)}$$

Example 2.11. *In a system, executing a non-flow process, the work and heat per degree change of temperature are given by*

$$\frac{dW}{dT} = 200 \text{ W-s/°C} \quad and \quad \frac{dQ}{dT} = 160 \text{ J/°C}$$

What will be the change of internal energy of the system when its temperature changes from $T_1 = 55°C$ *to* $T_2 = 95°C$ *?*

Solution. Initial temperature, $T_1 = 55°C$;   Final temperature, $T_2 = 95°C$

$$\frac{dW}{dT} = 200 \text{ W-s/°C} ; \quad \frac{dQ}{dT} = 160 \text{ J/°C.}$$

Change of internal energy :

Now, $\frac{dW}{dT} = 200 \text{ W-s/°C}$

$$\therefore \qquad\qquad W = \int_{T_1}^{T_2} 200 \, dT = \left| 200\,T \right|_{T_1}^{T_2} = \left| 200\,T \right|_{55}^{95}$$

$$= 200\,(95 - 55) = 8000 \text{ W-s} = 8000 \text{ J} \qquad [\because \quad 1 \text{ W-s} = 1 \text{ J}]$$

Also, $\qquad\qquad\qquad \dfrac{dQ}{dT} = 160 \text{ J/°C}$

$$\therefore \qquad\qquad Q = \int_{T_1}^{T_2} 160 \, dT = \left| 160\,T \right|_{T_1}^{T_2}$$

$$= \left| 160\,T \right|_{55}^{95} = 160\,(95 - 55) = 6400 \text{ J}$$

Applying the first law of thermodynamics to the given non-flow system,

$$Q = \Delta U + W$$

$$\therefore \qquad\qquad 6400 = \Delta U + 8000$$

or $\qquad\qquad\qquad \Delta U = -1600 \text{ J} = \textbf{– 1.6 kJ.} \quad \textbf{(Ans.)}$

The –ve sign indicates that there is *decrease* in internal energy.

Example 2.12. *A fluid system, contained in a piston and cylinder machine, passes through a complete cycle of four processes. The sum of all heat transferred during a cycle is – 340 kJ. The system completes 200 cycles per min.*

Complete the following table showing the method for each item, and compute the net rate of work output in kW.

Process	Q (kJ/min)	W(kJ/min)	ΔE(kJ/min)
1–2	0	4340	—
2–3	42000	0	—
3–4	– 4200	—	– 73200
4–1	—	—	—

Solution. Sum of all heat transferred during the cycle = – 340 kJ.

Number of cycles completed by the system = 200 cycles/min.

Process 1–2 :

$$Q = \Delta E + W$$

$$0 = \Delta E + 4340$$

$\therefore \qquad\qquad \Delta E = -4340 \text{ kJ/min.}$

Process 2–3 :

$$Q = \Delta E + W$$

$$42000 = \Delta E + 0$$

$$\Delta E = 42000 \text{ kJ/min.}$$

Process 3–4 :

$$Q = \Delta E + W$$

$$-4200 = -73200 + W$$

$\therefore \qquad\qquad W = 69000 \text{ kJ/min.}$

Process 4–1 :

$$\Sigma Q = -340 \text{ kJ}$$
$$\text{Cycle}$$

The system completes 200 cycles/min

$\therefore \qquad Q_{1-2} + Q_{2-3} + Q_{3-4} + Q_{4-1} = -340 \times 200 = -68000 \text{ kJ/min}$

or $\qquad 0 + 42000 + (-4200) + Q_{4-1} = -68000$

$\qquad\qquad\qquad Q_{4-1} = -105800 \text{ kJ/min}.$

Now, $\int dE = 0$, since cyclic integral of any property is zero.

$$\Delta E_{1-2} + \Delta E_{2-3} + \Delta E_{3-4} + \Delta E_{4-1} = 0$$
$$-4340 + 42000 + (-73200) + \Delta E_{4-1} = 0$$

$\therefore \qquad\qquad \Delta E_{4-1} = 35540 \text{ kJ/min}.$

$\therefore \qquad\qquad W_{4-1} = Q_{4-1} - \Delta E_{4-1}$

$\qquad\qquad\qquad\qquad = -105800 - 35540 = -141340 \text{ kJ/min}$

The completed table is given below :

Process	Q(kJ/min)	W(kJ/min)	ΔE(kJ/min)
1–2	0	4340	– 4340
2–3	42000	0	42000
3–4	– 4200	69000	– 73200
4–1	– 105800	– 141340	35540

Since $\qquad \underset{\text{cycle}}{\Sigma Q} = \underset{\text{cycle}}{\Sigma W}$

Rate of work output $\qquad = -68000 \text{ kJ/min} = -\dfrac{68000}{60} \text{ kJ/s or kW}$

$$= 1133.33 \text{ kW.} \quad \textbf{(Ans.)}$$

Example 2.13. *The power developed by a turbine in a certain steam plant is 1200 kW. The heat supplied to the steam in the boiler is 3360 kJ/kg, the heat rejected by the system to cooling water in the condenser is 2520 kJ/kg and the feed pump work required to pump the condensate back into the boiler is 6 kW.*

Calculate the steam flow round the cycle in kg/s.

Solution. The power developed by the turbine = 1200 kW

The heat supplied to the steam in the boiler = 3360 kJ/kg

The heat rejected by the system to cooling water = 2520 kJ/kg

Feed pump work = 6 kW
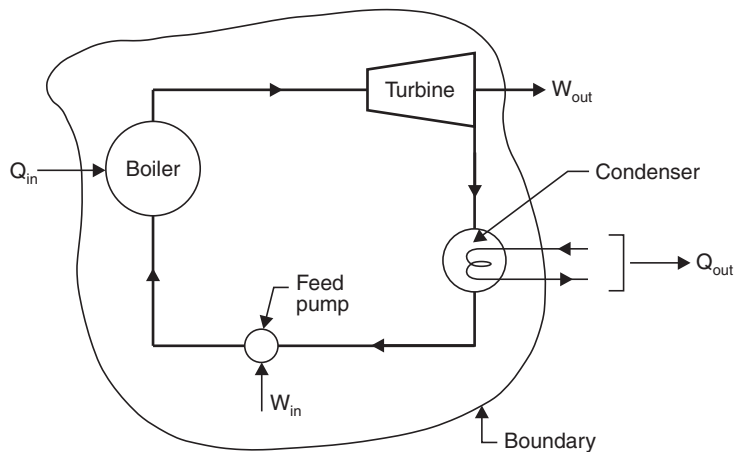
Fig. 2.19

Fig. 2.19 shows the cycle. A boundary is shown which encompasses the entire plant. Strictly, this boundary should be thought of as encompassing the working fluid only.

$$\oint dQ = 3360 - 2520 = 840 \text{ kJ/kg}$$

Let the system flow be in kg/s.

$$\therefore \qquad \oint dQ = 840 \ \dot{m} \ \text{ kJ/s}$$

$$\oint dW = 1200 - 6 = 1194 \text{ kJ/s}$$

But
$$\oint dQ = \oint dW$$

i.e.,
$$840 \ \dot{m} = 1194$$

$$\therefore \qquad \dot{m} = \frac{1194}{840} = 1.421 \text{ kg/s}$$

∴ **Steam flow round the cycle = 1.421 kg/s. (Ans.)**

Example 2.14. *A closed system of constant volume experiences a temperature rise of 25°C when a certain process occurs. The heat transferred in the process is 30 kJ. The specific heat at constant volume for the pure substance comprising the system is 1.2 kJ/kg °C, and the system contains 2.5 kg of this substance. Determine :*

(*i*) *The change in internal energy ;*

(*ii*) *The work done.*

Solution. Temperature rise, $(T_2 - T_1) = 25°C$

The heat transferred in the process, $Q = 30$ kJ

Specific heat at constant volume, $c_v = 1.2$ kJ/kg °C

Mass of the substance, $\qquad m = 2.5$ kg

Now, $\qquad\qquad\qquad \Delta U = m \int_{T_1}^{T_2} c_v \, dT$

$$= 2.5 \int_{T_1}^{T_2} 1.2 \, dT = 3.0 \times (T_2 - T_1)$$

$$= 3.0 \times 25 = \textbf{75 kJ}$$

Hence, **the change in internal energy is 75 kJ. (Ans.)**

According to the first law of thermodynamics,

$$Q = \Delta U + W$$

$\therefore \qquad\qquad\qquad 30 = 75 + W$

$\therefore \qquad\qquad\qquad W = 30 - 75 = -45$ kJ

Hence, **the work done = – 45 kJ. (Ans.)**

It may be observed that *even though the volume is constant, the work is not zero.* Clearly, the process is **irreversible.**

Example 2.15. *A system receives 50 kJ of heat while expanding with volume change of 0.14 m^3 against an atmosphere of 1.2 × 10^5 N/m^2. A mass of 90 kg in the surroundings is also lifted a distance of 5.5 metres.*

(*i*) *Find the change in energy of the system.*

(*ii*) *The system is returned to its initial volume by an adiabatic process which requires 110 kJ of work. Find the change in energy of the system.*

(*iii*) *For the combined processes of (i) and (ii) determine the change in energy of the system.*

Solution. Heat received by the system,

$$Q = 50 \text{ kJ}$$

Change in volume $\Delta V = 0.14 \text{ m}^3$

Pressure   = $1.2 \times 10^5 \text{ N/m}^2$

Mass lifted in the surroundings   = 90 kg

Distance through which lifted   = 5.5 m

Work done during adiabatic process = – 110 kJ.

(i) $Q = \Delta E + W$ ...(i)

Now, $W = p.\Delta V + W_{net}$

$$= \left(\frac{1.2 \times 10^5 \times 0.14}{1000} \right) + \left(\frac{90 \times 5.5 \times 9.8}{1000} \right) \text{ kJ}$$

$$= 16.8 + 4.85 = 21.65 \text{ kJ}$$

But [from (i)], $\Delta E = Q - W$

$$= 50 - 21.65 = \textbf{28.35 kJ.} \quad \textbf{(Ans.)}$$

(ii) Since the process is adiabatic,

$$Q = 0$$

and $\Delta E = - W$

$$= - (- 110) = \textbf{110 kJ.} \quad \textbf{(Ans.)}$$

(iii) Change in internal energy,

$$\Delta E = Q - W$$

$$= 50 - [(- 110) + 21.65] = \textbf{138.35 kJ.} \quad \textbf{(Ans.)}$$

☞ **Example 2.16.** *A fluid system undergoes a non-flow frictionless process following the pressure-volume relation as $p = \dfrac{5}{V} + 1.5$ where p is in bar and V is in m³. During the process the volume changes from 0.15 m³ to 0.05 m³ and the system rejects 45 kJ of heat. Determine :*

(i) Change in internal energy ;

(ii) Change in enthalpy.

Solution. Pressure-volume relation : $p = \dfrac{5}{V} + 1.5$

Initial volume, $V_1 = 0.15 \text{ m}^3$

Final volume, $V_2 = 0.05 \text{ m}^3$

Heat *rejected* by the system, $Q = - 45 \text{ kJ}$

Work done is given by,

$$W = \int_1^2 p.dV = \int_{V_1}^{V_2} \left(\frac{5}{V} + 1.5 \right) dV$$

$$= \int_{0.15}^{0.05} \left(\frac{5}{V} + 1.5 \right) dV = \left[5 \log_e \frac{V_2}{V_1} + 1.5(V_2 - V_1) \right]_{0.15}^{0.05} \times 10^5 \text{ Nm}$$

$$= 10^5 \left[5 \log_e \frac{0.05}{0.15} + 1.5(0.05 - 0.15) \right] = 10^5(- 5.49 - 0.15) \text{ Nm}$$

$$= - 5.64 \times 10^5 \text{ N-m} = - 5.64 \times 10^5 \text{ J} \quad [\because \quad 1 \text{ Nm} = 1 \text{ J}]$$

$$= - 564 \text{ kJ}.$$

(*i*) Applying the first law energy equation,

$$Q = \Delta U + W$$
$$- 45 = \Delta U + (- 564)$$

∴ $\Delta U = 519 \text{ kJ}$.   (Ans.)

This shows that the internal energy is **increased.**

(*ii*) **Change in enthalpy,**

$$\Delta H = \Delta U + \Delta (pV)$$
$$= 519 \times 10^3 + (p_2V_2 - p_1V_1)$$
$$p_1 = \frac{5}{V_1} + 1.5 = \frac{5}{0.15} + 1.5 = 34.83 \text{ bar}$$
$$= 34.83 \times 10^5 \text{ N/m}^2$$
$$p_2 = \frac{5}{V_2} + 1.5 = \frac{5}{0.05} + 1.5$$
$$= 101.5 \text{ bar} = 101.5 \times 10^5 \text{ N/m}^2$$

∴ $\Delta H = 519 \times 10^3 + (101.5 \times 10^5 \times 0.05 - 34.83 \times 10^5 \times 0.15)$
$$= 519 \times 10^3 + 10^3(507.5 - 522.45)$$
$$= 10^3(519 + 507.5 - 522.45) = 504 \text{ kJ}$$

∴   **Change in enthalpy = 504 kJ.**   (Ans.)

☞ **Example 2.17.** *The following equation gives the internal energy of a certain substance*

$$u = 3.64\ pv + 90$$

where u is kJ/kg, p is in kPa and v is in m³/kg.

A system composed of 3.5 kg of this substance expands from an initial pressure of 500 kPa and a volume of 0.25 m³ to a final pressure 100 kPa in a process in which pressure and volume are related by pv^{1.25} = constant.

 (*i*) *If the expansion is quasi-static, find Q, ΔU and W for the process.*

(*ii*) *In another process, the same system expands according to the same pressure-volume relationship as in part (i), and from the same initial state to the same final state as in part (i), but the heat transfer in this case is 32 kJ. Find the work transfer for this process.*

(*iii*) *Explain the difference in work transfer in parts (i) and (ii).*

Solution. Internal energy equation : $u = 3.64\ pv + 90$

Initial volume, $V_1 = 0.25 \text{ m}^3$

Initial pressure, $p_1 = 500 \text{ kPa}$

Final pressure, $p_2 = 100 \text{ kPa}$

Process : $pv^{1.25} = \text{constant.}$

(*i*) Now, $u = 3.64\ pv + 90$
$$\Delta u = u_2 - u_1$$
$$= 3.64\ (p_2v_2 - p_1v_1) \qquad \text{...per kg}$$

∴ $\Delta U = 3.64\ (p_2V_2 - p_1V_1) \qquad \text{...for 3.5 kg}$

Now, $p_1V_1^{1.25} = p_2V_2^{1.25}$

$$V_2 = V_1 \left(\frac{p_1}{p_2}\right)^{1/1.25} = 0.25 \left(\frac{500}{100}\right)^{1/1.25}$$

$$= 0.906 \text{ m}^3$$

$\therefore$ $\qquad$ $\Delta U = 3.64\,(100 \times 10^3 \times 0.906 - 500 \times 10^3 \times 0.25)\ \text{J}$ $\qquad$ $[\because\ \ 1\ \text{Pa} = 1\ \text{N/m}^2]$

$\qquad\qquad\qquad = 3.64 \times 10^5\,(0.906 - 5 \times 0.25)\ \text{J}$

$\qquad\qquad\qquad = -\,3.64 \times 10^5 \times 0.344\ \text{J} = -\,125.2\ \text{kJ}$

i.e., $\qquad\qquad$ **$\Delta U = -\,125.2$ kJ.   (Ans.)**

For a **quasi-static process**

$$W = \int p\,dV = \frac{p_1 V_1 - p_2 V_2}{n-1}$$

$$= \frac{(500 \times 10^3 \times 0.25 - 100 \times 10^3 \times 0.906)}{(1.25 - 1)} = \frac{125 - 90.6}{0.25}\ \text{kJ} = 137.6\ \text{kJ}$$

$\therefore$ $\qquad\qquad$ $Q = \Delta U + W$

$\qquad\qquad\qquad = -\,125.2 + 137.6 = 12.4\ \text{kJ}$

i.e., $\qquad\qquad$ **$Q = 12.4$ kJ.   (Ans.)**

(*ii*) Here $Q = 32$ kJ

Since the end states are the same, ΔU would remain the same as in (*i*)

$\therefore$ $\qquad\qquad$ $W = Q - \Delta U = 32 - (-\,125.2)$

$\qquad\qquad\qquad = \textbf{157.2 kJ.   (Ans.)}$

(*iii*) The work in (*ii*) is **not equal** to $\int p\,dV$ since the process is **not quasi-static.**

☞ **Example 2.18.** *The properties of a system, during a reversible constant pressure non-flow process at p = 1.6 bar, changed from v_1 = 0.3 m³/kg, T_1 = 20°C to v_2 = 0.55 m³/kg, T_2 = 260°C. The specific heat of the fluid is given by*

$$c_p = \left(1.5 + \frac{75}{T + 45}\right)\ kJ/kg°C\ where\ T\ is\ in\ °C.$$

Determine : (*i*) *Heat added/kg ;* $\qquad\qquad$ (*ii*) *Work done/kg ;*

(*iii*) *Change in internal energy/kg ;* $\qquad$ (*iv*) *Change in enthalpy/kg.*

Solution. Initial volume, $v_1 = 0.3$ m³/kg

Initial temperature, $\qquad T_1 = 20°C$

Final volume, $\qquad\qquad v_2 = 0.55$ m³/kg

Final temperature, $\qquad T_2 = 260°C$

Constant pressure, $\qquad p = 1.6$ bar

Specific heat at constant pressure, $c_p = \left(1.5 + \dfrac{75}{T + 45}\right)$ kJ/kg°C

(*i*) The **heat added** per kg of fluid is given by

$$Q = \int_{T_1}^{T_2} c_p\,dT = \int_{20}^{260} \left(1.5 + \frac{75}{T + 45}\right) dT$$

$$= \left| 1.5T + 75\log_e (T + 45) \right|_{20}^{260}$$

$$= 1.5(260 - 20) + 75 \times \log_e \left(\frac{260 + 45}{20 + 45}\right) = 475.94\ \text{kJ}$$

$\therefore$ **Heat added** $\qquad = \textbf{475.94 kJ/kg.   (Ans.)}$

(*ii*) The *work done* per kg of fluid is given by

$$W = \int_{v_1}^{v_2} p \, dv = p(v_2 - v_1) = 1.6 \times 10^5 (0.55 - 0.3) \text{ Nm}$$

$$= 40 \times 10^3 \text{ J} = 40 \text{ kJ}$$

∴   **Work done = 40 kJ/kg.   (Ans.)**

(*iii*) **Change in internal energy,**

$$\Delta u = Q - W = 475.94 - 40 = \textbf{435.94 kJ/kg. (Ans.)}$$

(*iv*) **Change in enthalpy,** (for **non-flow process**)

$$\Delta h = Q = \textbf{475.94 kJ/kg.   (Ans.)}$$

Example 2.19. *1 kg of gaseous* CO_2 *contained in a closed system undergoes a reversible process at constant pressure. During this process 42 kJ of internal energy is decreased. Determine the work done during the process.*

Take $c_p = 840$ *J/kg °C and* $c_v = 600$ *J/kg °C.*

Solution. Mass CO_2, $m = 1$ kg

Decrease in internal energy, $\Delta u = -42$ kJ $= -42 \times 10^3$ J

Specific heat at constant pressure, $c_p = 840$ J/kg°C

Specific heat at constant volume, $c_v = 600$ J/kg°C

Let, initial temperature of $CO_2 = T_1$

Final temperature of $CO_2 = T_2$

Now change in internal energy,

$$\Delta U = m \times c_v (T_2 - T_1)$$
$$-42 \times 10^3 = 1 \times 600(T_2 - T_1)$$

∴ $$T_2 - T_1 = -\frac{42 \times 10^3}{600} = -70°C$$

The heat supplied or rejected,

$$Q = mc_p(T_2 - T_1)$$
$$= 1 \times 840 \times (-70) = -58800 \text{ J or} -58.8 \text{ kJ}$$

Applying first law to the process,

$$Q = \Delta U + W$$
$$-58.8 = -42 + W \quad \text{or} \quad W = -16.8 \text{ kJ}$$

∴   **Work done during the process = – 16.8 kJ.   (Ans.)**

☞ **Example 2.20.** *A fluid is contained in a cylinder by a spring-loaded, frictionless piston so that the pressure in the fluid is a linear function of the volume (p = a + bV). The internal energy of the fluid is given by the following equation*

$$U = 42 + 3.6 \, pV$$

where U is in kJ, p in kPa, and V in cubic metre. If the fluid changes from an initial state of 190 kPa, 0.035 m³ to a final state of 420 kPa, 0.07 m³, with no work other than that done on the piston, find the direction and magnitude of the work and heat transfer.

Solution. Relation between pressure and volume, $p = a + bV$.

Equation of internal energy : $U = 42 + 3.6pV$

Initial pressure, $p_1 = 190$ kPa

Initial volume, $V_1 = 0.035$ m³

Final pressure, $p_2 = 420$ kPa

Final volume, $V_2 = 0.07$ m³

The change in internal energy of the fluid during the process

$$U_2 - U_1 = (42 + 3.6p_2V_2) - (42 + 3.6p_1V_1)$$
$$= 3.6(p_2V_2 - p_1V_1)$$
$$= 3.6(4.2 \times 10^5 \times 0.07 - 1.9 \times 10^5 \times 0.035) \text{ J}$$
$$= 360(4.2 \times 0.07 - 1.9 \times 0.035)\text{kJ}$$
$$= 81.9 \text{ kJ}$$

Now, $p = a + bV$

$$190 = a + b \times 0.035 \qquad\qquad ...(i)$$
$$420 = a + b \times 0.07 \qquad\qquad ...(ii)$$

Subtracting (i) from (ii), we get

$$230 = 0.035\, b \quad \text{or} \quad b = \frac{230}{0.035} = 6571 \text{ kN/m}^5$$

and $a = -40 \text{ kN/m}^2$

Work transfer involved during the process

$$W_{1-2} = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} (a + bV)\,dV = a(V_2 - V_1) + b\left(\frac{V_2^2 - V_1^2}{2}\right)$$

$$= (V_2 - V_1)\left[a + \frac{b}{2}(V_1 + V_2)\right]$$

$$= (0.07 - 0.035)\left[-40 \text{ kN/m}^2 + \frac{6571}{2} \text{ kN/m}^5 (0.035 + 0.07)\right] = 10.67 \text{ kJ}$$

∴ **Work done by the system = 10.67 kJ.   (Ans.)**

Heat transfer involved,

$$Q_{1-2} = (U_2 - U_1) + W_{1-2} = 81.9 + 10.67 = 92.57 \text{ kJ.}$$

92.57 kJ of heat flows into the system during the process. (Ans.)

Example 2.21. *90 kJ of heat are supplied to a system at a constant volume. The system rejects 95 kJ of heat at constant pressure and 18 kJ of work is done on it. The system is brought to original state by adiabatic process. Determine :*

(i) The adiabatic work ;

(ii) The values of internal energy at all end states if initial value is 105 kJ.

Solution. Refer Fig. 2.20.

Heat supplied at constant volume = 90 kJ.

Heat rejected at constant pressure = – 95 kJ

Work done *on* the system = – 18 kJ

Initial value of internal energy, $U_l = 105$ kJ

Process *l – m* (*constant volume*) :

$$W_{l-m} = 0$$
$$Q_{l-m} = 90 = U_m - U_l$$
∴ $U_m = U_l + 90 = 105 + 90$
$$= 195 \text{ kJ}$$

Process $m - n$ (*constant pressure*) :

$$Q_{m-n} = (U_n - U_m) + W_{m-n}$$
$$- 95 = (U_n - U_m) - 18$$

$\therefore$ $U_n - U_m = - 77 \text{ kJ}$
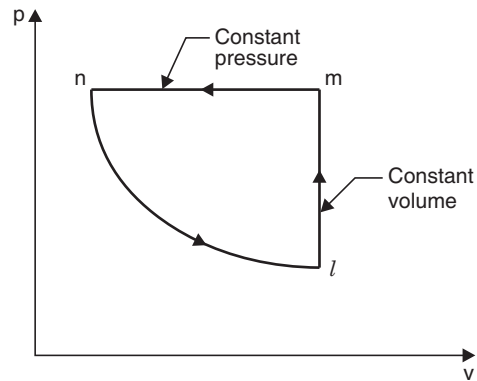
Fig. 2.20

$\therefore$ $U_n = 195 - 77 = 118 \text{ kJ}$

$Q_{n-l} = 0$ being adiabatic process

$\therefore$ $\oint \delta Q = 90 - 95 = - 5 \text{ kJ}$

and $\oint \delta W = - 18 + W_{n-l} = - 5$

$\therefore$ $W_{n-l} = - 5 + 18 = 13 \text{ kJ}$

Hence,   **$W_{n-l} = 13 \text{ kJ} ; U_l = 105 \text{ kJ} ; U_m = 195 \text{ kJ} ; U_n = 118 \text{ kJ}. \text{ (Ans.)}$**

Example 2.22. *A movable frictionless piston closes a fully insulated cylinder on one side and offers a constant resistance during its motion. A paddle is drawn into the cylinder and does work on the system.*

Prove that the paddle work is equal to change in enthalpy.

Solution. Refer Fig 2.21.
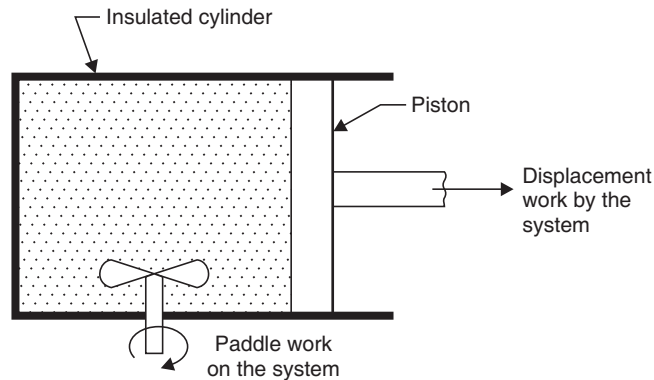
Fig. 2.21

$$Q = W_{paddle} = \Delta U + p\Delta V = \Delta U + \Delta(pV) = \Delta(U + pV) = \Delta H$$

Hence **paddle work is equal to change in enthalpy.   (Ans.)**

☞ **Example 2.23.** *0.2 m³ of air at 4 bar and 130°C is contained in a system. A reversible adiabatic expansion takes place till the pressure falls to 1.02 bar. The gas is then heated at constant pressure till enthalpy increases by 72.5 kJ. Calculate :*

(i) The work done ;

(ii) The index of expansion, if the above processes are replaced by a single reversible polytropic process giving the same work between the same initial and final states.

Take c_p = 1 kJ/kg K, c_v = 0.714 kJ/kg K.

Solution. Refer Fig. 2.22.
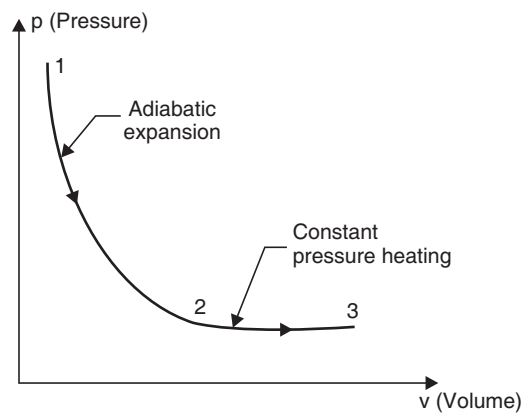
Fig. 2.22

Initial volume, V_1 = 0.2 m³
Initial pressure, p_1 = 4 bar = 4 × 10⁵ N/m²
Initial temperature, T_1 = 130 + 273 = 403 K
Final pressure after adiabatic expansion,
                            p_2 = 1.02 bar = 1.02 × 10⁵ N/m²
Increase in enthalpy during constant pressure process
                            = 72.5 kJ.

(i) **Work done :**

Process 1–2 : *Reversibe adiabatic process :*

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

$$V_2 = V_1 \left(\frac{p_1}{p_2} \right)^{\frac{1}{\gamma}}$$

Also $\gamma = \dfrac{c_p}{c_v} = \dfrac{1}{0.714} = 1.4$

∴ $V_2 = 0.2 \times \left(\dfrac{4 \times 10^5}{1.02 \times 10^5} \right)^{\frac{1}{1.4}} = 0.53$ m³

Also,
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$\therefore$
$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 403 \left(\frac{1.02 \times 10^5}{4 \times 10^5}\right)^{\frac{1.4-1}{1.4}} = 272.7 \text{ K}$$

Mass of the gas,

$$m = \frac{p_1 V_1}{R T_1} \qquad\qquad [\because \quad pV = mRT]$$

where $R = (c_p - c_v) = (1 - 0.714)$ kJ/kg K

$\qquad = 0.286$ kJ/kg K $= 286$ J/kg K or 286 Nm/kg K

$\therefore$
$$m = \frac{4 \times 10^5 \times 0.2}{286 \times 403} = 0.694 \text{ kg.}$$

Process 2–3. *Constant pressure* :

$$Q_{2-3} = m c_p (T_3 - T_2)$$
$$72.5 = 0.694 \times 1 \times (T_3 - 272.7)$$

$\therefore$
$$T_3 = \frac{72.5}{0.694} + 272.7 = 377 \text{ K}$$

Also,
$$\frac{V_2}{T_2} = \frac{V_3}{T_3}$$

or
$$\frac{0.53}{272.7} = \frac{V_3}{377}$$

$\therefore$
$$V_3 = \frac{0.53 \times 377}{272.7} = 0.732 \text{ m}^3$$

Work done by the path 1-2-3 is given by

$$W_{1-2-3} = W_{1-2} + W_{2-3}$$

$$= \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} + p_2(V_3 - V_2)$$

$$= \frac{4 \times 10^5 \times 0.2 - 1.02 \times 10^5 \times 0.53}{1.4 - 1} + 1.02 \times 10^5(0.732 - 0.53)$$

$$= \frac{10^5(4 \times 0.2 - 1.02 \times 0.53)}{0.4} + 1.02 \times 10^5(0.732 - 0.53)$$

$$= 64850 + 20604 = 85454 \text{ Nm or J}$$

Hence, **total work done = 85454 Nm or J.   (Ans.)**

(*ii*) **Index of expansion, n :**

If the work done by the polytropic process is the same,

$$W_{1-2-3} = W_{1-3} = \frac{p_1 V_1 - p_3 V_3}{n - 1}$$

$$85454 = \frac{4 \times 10^5 \times 0.2 - 1.02 \times 10^5 \times 0.732}{(n-1)} = \frac{5336}{n-1}$$

∴ $n = \dfrac{5336}{85454} + 1$

i.e., $n = 1.062$

Hence, **value of index = 1.062.   (Ans.)**

Example 2.24. *The following is the equation which connects u, p and v for several gases*

$$u = a + bpv$$

where a and b are constants. Prove that for a reversible adiabatic process,

$$pv^\gamma = constant, \; where \; \gamma = \frac{b+1}{b}.$$

Solution. Considering a unit mass.

For a reversible adiabatic process, first law gives

$$0 = du + pdv$$

∴ $\dfrac{du}{dv} = -p$ ...(*i*)

Also, $u = a + bpv$

∴ $\dfrac{du}{dv} = \dfrac{d(a + bpv)}{dv} = bv\dfrac{dp}{dv} + bp$

$$= b\left(p + v \cdot \frac{dp}{dv} \right)$$ ...(*ii*)

Equating (*i*) and (*ii*), we get

$$b\left(p + v \cdot \frac{dp}{dv} \right) = -p$$

$$bp + b \cdot v \cdot \frac{dp}{dv} = -p$$

$$bp + p + bv \cdot \frac{dp}{dv} = 0$$

$$p(b + 1) + bv \cdot \frac{dp}{dv} = 0$$

Multiplying both sides by $\dfrac{dv}{bpv}$, we get

$$\left(\frac{b+1}{b} \right)\frac{dv}{v} + \frac{dp}{p} = 0$$

or

$$\frac{dp}{p} + \left(\frac{b+1}{b} \right)\frac{dv}{v} = 0$$

$$d(\log_e p) + \left(\frac{b+1}{b} \right)d(\log_e v) = 0$$

Also, $\dfrac{b+1}{b} = \gamma$ (given)

$\therefore$ $\quad\quad\quad\quad\quad\quad\quad\quad d(\log_e p) + \gamma d \cdot (\log_e v) = 0$

Integrating, we get $\quad\quad\quad\quad\quad pv^\gamma = \text{constant.}$

Example 2.25. *A 15 cm diameter vertical cylinder, closed by a piston contains a combustible mixture at a temperature of 30°C. The piston is free to move and its weight is such that the mixture pressure is 3 bar. Upper surface of the piston is exposed to the atmosphere. The mixture is ignited. As the reaction proceeds, the piston moves slowly upwards and heat transfer to the surroundings takes place. When the reaction is complete and the contents have been reduced to the initial temperature of 30°C, it is found that the piston has moved upwards a distance of 8.5 cm and the magnitude of heat transfer is 4 kJ. Evaluate :*

(i) *The work* ;

(ii) *Decrease in internal energy of the system.*

Solution. Diameter of vertical cylinder, $\quad d = 15$ cm (or 0.15 m)

Temperature of combustible mixture $\quad\quad = 30°C$ (or 303 K)

Pressure of the mixture $\quad\quad\quad\quad\quad\quad = 3$ bar $= 3 \times 10^5$ N/m²

Upward displacement of the system $\quad\quad = 8.5$ cm (or 0.085 m)

Magnitude of heat transfer, $\quad\quad\quad\quad Q = -4$ kJ $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ ...(i)

(i) **Work done by the system,** $\quad W = \int pdv$

$$= 3 \times 10^5 \int dv \quad\quad [\because \quad p = \text{constant} = 3 \times 10^5 \text{ N/m}^2]$$

$$= 3 \times 10^5 \left[\frac{\pi}{4} \times (0.15)^2 \times 0.085 \right] \text{Nm}$$

$$= 450.62 \text{ Nm or J} = 0.4506 \text{ kJ}$$

$\therefore$ $\quad\quad\quad\quad\quad\quad\quad\quad W = \textbf{0.4506 kJ. (Ans.)}$

(ii) By first law of thermodynamics,

$$Q = \Delta U + W$$
$$- 4 = \Delta U + 0.4506$$

$\therefore$ $\quad\quad\quad\quad\quad\quad\quad\quad \Delta U = -4.4506 \text{ kJ}$

$\therefore$ **Decrease in internal energy = 4.4506 kJ. (Ans.)**

Example 2.26. *A house wife, on a warm summer day, decides to beat the heat by closing the windows and doors in the kitchen and opening the refrigerator door. At first she feels cool and refreshed, but after a while the effect begins to wear off.*

Evaluate the situation as it relates to First Law of Thermodynamics, considering the room including the refrigerator as the system.

Solution. Initially, the temperature of air in the room falls when it communicates with the cool refrigerator with its door open. This makes the house wife feel cool.

Considering the room and its contents as the system, and assuming the walls, windows and doors non-conducting, we find, $Q = 0$.

To operate the refrigerator, *electricity* is supplied from outside and hence *external work W is done on the system.*

Applying the first law to the system,

$$Q = \Delta U + W$$
$$0 = \Delta U + (-W)$$

$\therefore$ $\quad\quad\quad\quad\quad\quad\quad\quad \Delta U = W$

The right hand side is a positive figure indicating the increase in energy of the system with time. As the energy is increasing the temperature of air increases and hence the effect of coolness gradually begins to wear off.

It may be pointed out here that in this case the energy rise manifests itself in a rise in temperature.

☞ **Example 2.27.** *A cylinder contains 0.45 m³ of a gas at 1 × 10⁵ N/m² and 80°C. The gas is compressed to a volume of 0.13 m³, the final pressure being 5 × 10⁵ N/m². Determine :*

(i) The mass of gas ;

(ii) The value of index 'n' for compression ;

(iii) The increase in internal energy of the gas ;

(iv) The heat received or rejected by the gas during compression.

Take γ = 1.4, R = 294.2 J/kg°C.

Solution. Initial volume of gas, $V_1 = 0.45$ m³

Initial pressure of gas, $p_1 = 1 \times 10^5$ N/m²

Initial temperature, $T_1 = 80 + 273 = 353$ K

Final volume after compression, $V_2 = 0.13$ m³

The final pressure, $p_2 = 5 \times 10^5$ N/m².

(i) To find mass 'm' using the relation

$$m = \frac{p_1 V_1}{R T_1} = \frac{1 \times 10^5 \times 0.45}{294.2 \times 353} = \textbf{0.433 kg.}   \textbf{(Ans.)}$$

(ii) To find index 'n' using the relation

$$p_1 V_1{}^n = p_2 V_2{}^n$$

or

$$\left(\frac{V_1}{V_2}\right)^n = \frac{p_2}{p_1}$$

$$\left(\frac{0.45}{0.13}\right)^n = \left(\frac{5 \times 10^5}{1 \times 10^5}\right) = 5$$

or $(0.365)^n = 5$

Taking log on both sides, we get

$$n \log_e 3.46 = \log_e 5$$

$$n = \log_e 5 / \log_e 3.46 = \textbf{1.296.}   \textbf{(Ans.)}$$

(iii) In a polytropic process,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{0.45}{0.13}\right)^{1.296-1} = 1.444$$

∴ $T_2 = 353 \times 1.444 = 509.7$ K

Now, increase in internal energy,

$$\Delta U = m c_v (T_2 - T_1)$$

$$= 0.433 \times \frac{R}{(\gamma - 1)} (T_2 - T_1) \qquad \left[\because \quad c_v = \frac{R}{(\gamma - 1)}\right]$$

$$= 0.433 \times \frac{294.2}{(1.4 - 1)1000} (509.7 - 353)$$

$$= \textbf{49.9 kJ.}   \textbf{(Ans.)}$$

(iv) $Q = \Delta U + W$

Now, $W = \dfrac{p_1 V_1 - p_2 V_2}{n - 1} = \dfrac{mR(T_1 - T_2)}{n - 1}$

$$= \dfrac{0.433 \times 294.2(353 - 509.7)}{1.296 - 1}$$

$= -67438$ N-m   or -67438 J $= -67.44$ kJ

∴ $Q = 49.9 + (-67.44) = -17.54$ kJ

∴   **Heat rejected = 17.54 kJ.   (Ans.)**

Example 2.28.   *Air at 1.02 bar, 22°C, initially occupying a cylinder volume of 0.015 m³, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bar. Calculate :*

 (i) *The final temperature ;*

(ii) *The final volume ;*

(iii) *The work done.*   **(PTU, Jan. 2002)**

Solution. Initial pressure, p_1 = 1.02 bar

Initial temperature, T_1 = 22 + 273 = 295 K

Initial volume, V_1 = 0.015 m³

Final pressure, p_2 = 6.8 bar

Law of compression : $pv^\gamma = C$

 (i) **Final temperature :**

Using the relation,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{T_2}{295} = \left(\frac{6.8}{1.02} \right)^{\frac{1.4 - 1}{1.4}}$$ $[\because \quad \gamma$ for air = 1.4$]$

∴ $T_2 = 295 \left(\dfrac{6.8}{1.02} \right)^{\frac{1.4 - 1}{1.4}} = 507.24$ K

i.e.   **Final temperature** = 507.24 − 273 = **234.24°C.   (Ans.)**

(ii) **Final volume :**

Using the relation,

$$p_1 V_1{}^\gamma = p_2 V_2{}^\gamma$$

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1} \right)^\gamma \quad \text{or} \quad \frac{V_2}{V_1} = \left(\frac{p_1}{p_2} \right)^{\frac{1}{\gamma}}$$

∴ $V_2 = V_1 \times \left(\dfrac{p_1}{p_2} \right)^{\frac{1}{\gamma}} = 0.015 \times \left(\dfrac{1.02}{6.8} \right)^{\frac{1}{1.4}} = 0.00387$ m³

i.e.,   **Final volume = 0.00387 m³.   (Ans.)**

(*iii*) **Wond done :**

Now, work done *on the air*,

$$W = \frac{mR(T_1 - T_2)}{(\gamma - 1)} \qquad\qquad ...(i)$$

where m is the mass of air and is found by the following relation

$$pV = mRT$$

$$\therefore \qquad\qquad m = \frac{p_1 V_1}{RT_1} = \frac{1.02 \times 10^5 \times 0.015}{0.287 \times 10^3 \times 295} \qquad [\because \quad R \text{ for air} = 0.287 \times 10^3]$$

$$= 0.01807 \text{ kg}$$

$$\therefore \qquad\qquad W = \frac{0.01807 \times 0.287 \times 10^3 (295 - 507.24)}{(1.4 - 1)} = -2751 \text{ J or} -2.751 \text{ kJ}$$

i.e.,   **Work done = 2.751 kJ.   (Ans.)**

(–ve sign indicates that work is done on the air).

Example 2.29. *0.44 kg of air at 180°C expands adiabatically to three times its original volume and during the process, there is a fall in temperature to 15°C. The work done during the process is 52.5 kJ. Calculate c_p and c_v.*

Solution. Refer Fig. 2.23.
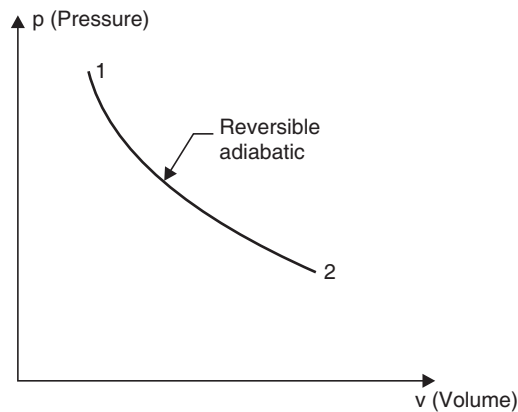
Fig. 2.23

Mass of air, m = 0.44 kg
Initial temperature, T_1 = 180 + 273 = 453 K

$$\text{Ratio} = \frac{V_2}{V_1} = 3$$

Final temperature, T_2 = 15 + 273 = 288 K
Work done during the process, W_{1-2} = 52.5 kJ

$$c_p = ?, c_v = ?$$

For adiabatic process, we have

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$$

$$\frac{288}{453} = \left(\frac{1}{3}\right)^{\gamma - 1} \quad \text{or} \quad 0.6357 = (0.333)^{\gamma - 1}$$

or   Taking log on both sides, we get
$$\log_e (0.6357) = (\gamma - 1) \log_e (0.333)$$
$$- 0.453 = (\gamma - 1) \times (- 1.0996)$$

$\therefore$ $\gamma = \dfrac{0.453}{1.0996} + 1 = 1.41$

Also, $\dfrac{c_p}{c_v} = \gamma = 1.41$

Work done during adiabatic process,

$$W_{1-2} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$\therefore$ $52.5 = \dfrac{0.44\,R\,(453 - 288)}{(1.41 - 1)}$

$\therefore$ $R = \dfrac{52.5\,(1.41 - 1)}{0.44\,(453 - 288)} = 0.296$

$\therefore$ $c_p - c_v = 0.296$ $[\because \quad R = c_p - c_v]$

Also $\dfrac{c_p}{c_v} = 1.41 \quad \text{or} \quad c_p = 1.41\,c_v$

$\therefore$ $1.41\,c_v - c_v = 0.296$

or $c_v = \textbf{0.722 kJ/kg K.} \quad \textbf{(Ans.)}$
and $c_p = \textbf{1.018 kJ/kg K.} \quad \textbf{(Ans.)}$

☞ **Example 2.30.** *1 kg of ethane (perfect) gas is compressed from 1.1 bar, 27°C according to a law* $pV^{1.3}$ *= constant, until the pressure is 6.6 bar. Calculate the heat flow to or from the cylinder walls.*

Given : molecular weight of ethane = 30, c_p *= 1.75 kJ/kg K.*

Solution. Mass of ethane gas, $m = 1$ kg

Initial pressure, $p_1 = 1.1$ bar
Initial temperature, $T_1 = 27 + 273 = 300$ K
Final pressure, $p_2 = 6.6$ bar
Law of compression, $pV^{1.3} = C$

Quantity of heat transferred, Q :

Now, characteristic gas constant,

$$R = \frac{\text{Universal gas constant } (R_0)}{\text{Molecular weight } (M)}$$

$$= \frac{8314}{30} = 277.13 \text{ Nm/kg K} = 277.31 \text{ J/kg K}$$

$$= 0.277 \text{ kJ/kg K}$$

Also $c_p - c_v = R$
$\therefore$ $c_v = c_p - R = 1.75 - 0.277 = 1.473 \text{ kJ/kg K}$

$$\gamma = \frac{c_p}{c_v} = \frac{1.75}{1.473} = 1.188$$

In case of a polytropic process,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{6.6}{1.1}\right)^{\frac{1.3-1}{1.3}} = 1.5119$$

∴ $T_2 = 300 \times 1.5119 = 453.6$ K

Now, work done, $W = \dfrac{R(T_1 - T_2)}{n-1} = \dfrac{0.277(300 - 453.6)}{1.3-1} = -141.8$ kJ/kg

To find heat flow, using the relation,

$$Q = \left(\frac{\gamma - n}{\gamma - 1}\right)W = \left(\frac{1.188 - 1.3}{1.188 - 1}\right) \times -141.8 = +84.5 \text{ kJ/kg}$$

i.e.,   **Heat supplied = 84.5 kJ/kg.   (Ans.)**

Example 2.31. *Fig 2.24 shows a cylinder of 8 cm inside diameter having a piston loaded with a spring (stiffness = 150 N/cm of compression). The initial pressure, volume and temperature of air in the cylinder are 3×10^5 N/m², 0.000045 m³ and 20°C respectively. Determine the amount of heat added to the system so that piston moves by 3.5 cm.*

Assume c_v = 0.71 kJ/kg K and R = 0.287 kJ/kg K.

Solution. Insider diameter of the cylinder
            = 8 cm

Stiffness of the spring, S = 150 N/cm

Initial pressure of air,
            $p_1 = 3 \times 10^5$ N/m² or 30 N/cm²

Initial volume of air,
            V_1 = 0.000045 m³ = 45 cm³

Initial temperature of air,
            T_1 = 20 + 273 = 293 K

Specific heat at constant volume,
            c_v = 0.71 kJ/kg K

Characteristic constant for air,
            R = 0.287 kJ/kg K

Refer Fig. 2.25.

Fig. 2.24

Let, oo = an arbitrary datum from which the
                position of the lower face of the
                piston is to be measured

      y = distance of the lower face of the piston

      $y = y_0$, when spring length is its free length

      p = pressure of air within the cylinder when $y = y_0$.

Now, force balance for the piston is given by
            $Ap = S(y - y_0)$ ...(*i*)

where A = the area of the piston, and

      S = stiffness of the spring.

With heat transfer to the air, let the pressure inside the cylinder increase by dp forcing the piston to move upward by distance dy. Now the force balance for the piston is
            $A(p + dp) = S(y + dy - y_0)$ ...(*ii*)
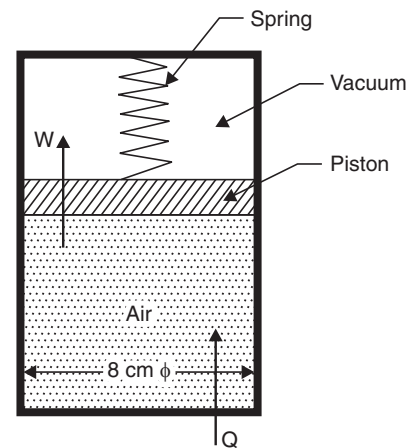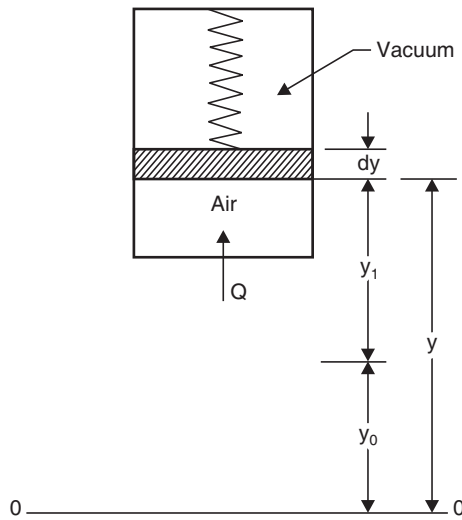
Fig. 2.25
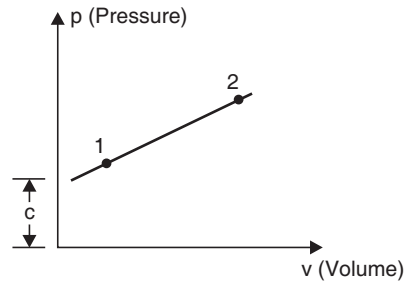
Fig. 2.26

From equations (*i*) and (*ii*), we have

$$Adp = Sdy \qquad \qquad ...(iii)$$

The increase in volume dV of the gas for the piston displacement is given by

$$dV = Ady \qquad \qquad ...(iv)$$

$$\therefore \qquad \qquad dp = \frac{S}{A^2}\,dy \qquad \qquad ...(v)$$

$$\therefore \qquad \qquad p = \frac{S}{A^2}\,V + C \qquad \qquad ...(vi)$$

The p-v relationship for the process is a straght line (Fig. 2.26) having a slope of $\dfrac{S}{A^2}$ and pressure axis intercept of C. The value of C can be found out from the knowledge of pressure and volume at any state point.

Now, substituting the values of p_1, V_1, A in equation (*vi*), we get

$$p = \frac{150}{\left(\frac{\pi}{4}\times 8^2\right)^2}\,V + C$$

or $\qquad \qquad \qquad p = 0.0594\,V + C \qquad \qquad ...(vii)$

where p is in N/cm^2 and V is in cm^3.

$$\therefore \qquad \qquad p_1 = 0.0594\,V_1 + C$$
$$30 = 0.0594 \times 45 + C$$
$$\therefore \qquad \qquad C = 27.33$$

Hence, p-v relationship for the process is,

$$p = 0.0594\,V + 27.33 \qquad \qquad ...(viii)$$

During the process the piston is moved by a distance of 4 cm.

This increases the volume of gas by

$$3.5 \times A^2 = 3.5 \times \left(\frac{\pi}{4} \times 8^2 \right) = 175.9 \text{ cm}^3$$

Hence, the final volume of air,

$$V_2 = 45 + 175.9 = 220.9 \text{ cm}^3$$

Substituting this value in equation (*viii*), we get

$$p(= p_2) = 0.0594 \times 220.9 + 27.33 = 40.45 \text{ N/cm}^2$$

The work done W during the process is given by

$$W = \int pdV = \int_{p_1}^{p_2} \frac{A^2}{S} \, pdp$$

$$= \frac{A^2}{S} \left(\frac{p_2^2 - p_1^2}{2} \right) = \frac{A^2}{S} \left(\frac{p_2 + p_1}{2} \right) \left(\frac{p_2 - p_1}{2} \right)$$

$$= \frac{A^2}{S} \left(\frac{p_2 + p_1}{2} \right) \frac{S}{A^2} (V_2 - V_1)$$

or

$$W = \left(\frac{p_2 + p_1}{2} \right)(V_2 - V_1) \qquad \qquad \qquad ...(ix)$$

$$= (\text{Mean pressure}) \times (\text{Change in volume})$$

$$W = \left(\frac{40.45 + 30}{2} \right) \times (220.9 - 45)$$

$$= 6196 \text{ Ncm} \quad \text{or} \quad 61.96 \text{ Nm}$$

It may be noted that work done does not cross the system boundary when spring and cylinder are considered a system.

Now, to find T_2, using the relation,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$\therefore$

$$T_2 = \frac{p_2 V_2 T_1}{p_1 V_1} = \frac{40.45 \times 220.9 \times 293}{30 \times 45} = 1939.3 \text{ K}$$

Also,

$$m = \frac{p_1 v_1}{R_1 T_1} = \frac{30 \times 45}{(0.287 \times 10^3) \times 293} = 0.0001605 \text{ kg}$$

Now, change in internal energy,

$$\Delta U = m \times c_v \times (T_2 - T_1)$$

$$= 0.0001605 \times 0.71 \times (1939.3 - 293) = 0.1876 \text{ kJ}$$

According to first law,

$$Q_{1-2} = \Delta U + W$$

$$= 0.1876 + 61.96 \times 10^{-3} = 0.2495 \text{ kJ}$$

$\therefore$ **Amount of heat added to the system = 0.2495 kJ.   (Ans.)**

2.10. APPLICATION OF FIRST LAW TO STEADY FLOW PROCESS

Steady Flow Energy Equation (S.F.E.E.)

In many practical problems, the rate at which the fluid flows through a machine or piece of apparatus is constant. This type of flow is called *steady flow*.

Assumptions

The following assumptions are made in the system analysis :

 (i) The mass flow through the system remains constant.

(ii) Fluid is uniform in composition.

(iii) The only interaction between the system and surroundings are work and heat.

(iv) The state of fluid at any point remains constant with time.

 (v) In the analysis only potential, kinetic and flow energies are considered.

Fig. 2.27 shows a schematic flow process for an open system. An open system is one in which both mass and energy may cross the boundaries. A wide interchange of energy may take place within an open system. Let the system be an automatic engine with the inlet manifold at the first state point and exhaust pipe as the second point. There would be an interchange of chemical energy in the fuel, kinetic energy of moving particles, internal energy of gas and heat transferred and shaft work within the system. From Fig. 2.27 it is obvious that if there is no variation of flow of mass or energy with time across the boundaries of the system the steady flow will prevail. The conditions may pass through the cyclic or non-cyclic changes within the system. As a result the mass entering the system equals the mass leaving, also energy entering the system equals energy leaving.
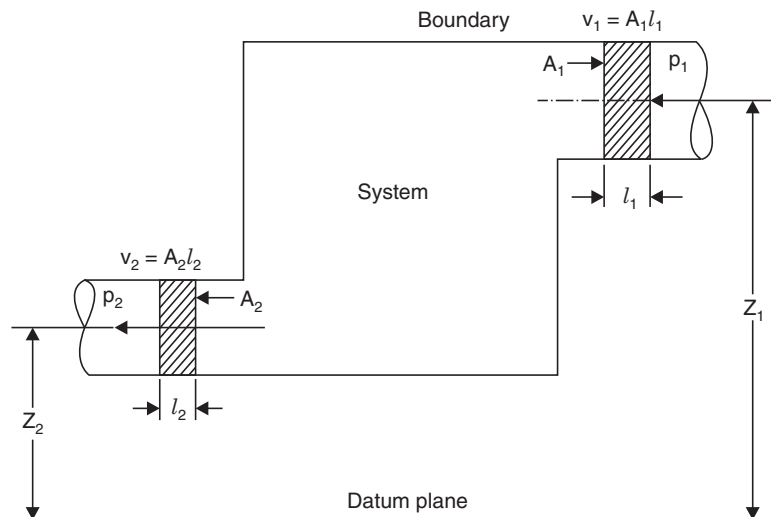
Fig. 2.27

The steady flow equation can be expressed as follows :

$$u_1 + \frac{C_1^{\,2}}{2} + Z_1 g + p_1 v_1 + Q = u_2 + \frac{C_2^{\,2}}{2} + Z_2 g + p_2 v_2 + W \qquad \text{...(2.45)}$$

$$(u_1 + p_1 v_1) + \frac{C_1^{\,2}}{2} + Z_1 g + Q = (u_2 + p_2 v_2) + \frac{C_2^{\,2}}{2} + Z_2 g + W$$

$$h_1 + \frac{C_1^{\,2}}{2} + Z_1 g + Q = h_2 + \frac{C_2^{\,2}}{2} + Z_2 g + W \qquad [\because \quad h = u + pv]$$

If Z_1 and Z_2 are neglected, we get

$$h_1 + \frac{C_1{}^2}{2} + Q = h_2 + \frac{C_2{}^2}{2} + W \qquad\qquad \text{...[2.45 (a)]}$$

where Q = heat supplied (or entering the boundary) per kg of fluid ;

$\quad\quad\quad W$ = work done by (or work coming out of the boundary) 1 kg of fluid ;

$\quad\quad\quad C$ = velocity of fluid ;

$\quad\quad\quad Z$ = height above datum ;

$\quad\quad\quad p$ = pressure of the fluid ;

$\quad\quad\quad u$ = internal energy per kg of fluid ;

$\quad\quad pv$ = energy required for 1 kg of fluid.

This equation is applicable to any medium in any steady flow. It is applicable not only to rotary machines such as certrifugal fans, pumps and compressors but also to reciprocating machines such as steam engines.

In a steady flow the rate of mass flow of fluid at any section is the same as at any other section. Consider any section of cross-sectional area A, where the fluid velocity is C, the rate of volume flow past the section is CA. Also, since mass flow is volume flow divided by specific volume.

$$\text{Mass flow rate, } m = \frac{CA}{v} \qquad\qquad \text{...(2.46)}$$

(where v = *specific volume* at the section)

This equation is known as the **continuity of mass equation**.

With reference to Fig. 2.27.

$$\therefore \qquad\qquad m = \frac{C_1 A_1}{v_1} = \frac{C_2 A_2}{v_2} \qquad\qquad \text{...[2.46 (a)]}$$

2.11. ENERGY RELATIONS FOR FLOW PROCESS

The energy equation (m kg of fluid) for a steady flow system is given as follows :

$$m\left(u_1 + \frac{C_1{}^2}{2} + Z_1 g + p_1 v_1 \right) + Q = m\left(u_2 + \frac{C_2{}^2}{2} + Z_2 g + p_2 v_2 \right) + W$$

i.e.
$$Q = m\left[(u_2 - u_1) + (Z_2 g - Z_1 g) + \left(\frac{C_2{}^2}{2} - \frac{C_1{}^2}{2} \right) + (p_2 v_2 - p_1 v_1) \right] + W$$

i.e.,
$$Q = m\left[(u_2 - u_1) + g(Z_2 - Z_1) + \left(\frac{C_2{}^2 - C_1{}^2}{2} \right) + (p_2 v_2 - p_1 v_1) \right] + W$$

$$= \Delta U + \Delta PE + \Delta KE + \Delta(pv) + W$$

where
$$\Delta U = m(u_2 - u_1)$$
$$\Delta PE = mg(Z_2 - Z_1)$$
$$\Delta KE = m\left(\frac{C_2{}^2 - C_1{}^2}{2} \right)$$
$$\Delta pv = m(p_2 v_2 - p_1 v_1)$$

$$\therefore \qquad Q - \Delta U = [\Delta PE + \Delta KE + \Delta(pv) + W] \qquad\qquad \text{...(2.47)}$$

For non-flow process,

$$Q = \Delta U + W = \Delta U + \int_1^2 p dV$$

i.e., $$Q - \Delta U = \int_1^2 p \cdot dV \qquad \qquad ...(2.48)$$

The internal energy is a function of temperature only and it is a point function. Therefore, for the same two temperatures, change in internal energy is the same whatever may be the process, non-flow, or steady flow, reversible or irreversible.

For the same value of Q transferred to non-flow and steady flow process and for the same temperature range, we can equate the values of eqns. (2.47) and (2.48) for $(Q - \Delta U)$.

$\therefore$ $$\int_1^2 p \cdot dV = \Delta PE + \Delta KE + \Delta(pV) + W \qquad \qquad ...(2.49)$$

where W = work transfer in flow process

and $\int_1^2 p \cdot dV$ = total change in mechanical energy of reversible steady flow process.

Property Relations for Energy Equations

We know that

$$h = u + pv$$

Differentiating above equation

$$dh = du + pdv + vdp$$

But $\qquad dQ = du + p.dv$ (as per first law applied to closed system)

or $\qquad du = dQ - p.dv$

Substituting this value of du in the above equation, we get

$$dh = dQ - p.dv + pdv + vdp$$
$$= dQ + vdp$$

$\therefore \qquad vdp = dh - dQ$

$\therefore$ $$-\int_1^2 vdp = Q - \Delta h \qquad \qquad ...(2.50)$$

where $-\int_1^2 vdp$ represents on a *p-v* diagram the area behind 1–2 as shown in Fig. 2.28 (*b*).

The eqn. (2.47) for a unit mass flow can be written as

$$dQ = d(PE) + d(KE) + du + d(pv) + dW$$

Substituting the value of $dQ = du + p.dv$ in the above equation, we get

$$du + pdv = d(PE) + d(KE) + du + pdv + vdp + dW$$

$\therefore \qquad -vdp = d(PE) + d(KE) + dW$

$\therefore$ $$-\int_1^2 vdp = \Delta PE + \Delta KE + W \qquad \qquad ...[2.50 \,(a)]$$

If $\Delta PE = 0$ (as in most of thermodynamic systems)

$$-\int_1^2 vdp = \Delta KE + W \qquad \qquad ...[2.50 \,(b)]$$

If $W = 0$, the area behind the curve represents ΔKE and if $\Delta KE = 0$, area behind the curve represents W which is shaft work.

$-\int_1^2 vdp$ is a *positive quantity and represents work done by the system.*

If $\Delta PE = 0$ and $W = 0$, then

$$-\int_1^2 vdp = \Delta KE, \text{ this is applicable in case of a } nozzle.$$

i.e.,
$$\int_1^2 vdp = \frac{C^2}{2} \text{ in the case of a nozzle.}$$

If $\Delta PE = 0$ and $\Delta KE = 0$, as in case of a *compressor*, $-\int_1^2 vdp = W$

or
$$W = \int_1^2 vdp \text{ in the case of a } compressor.$$

The integrals $\int_1^2 pdv$ and $\int_1^2 vdp$ are shown in Fig. 2.28 (*a*) and (*b*).
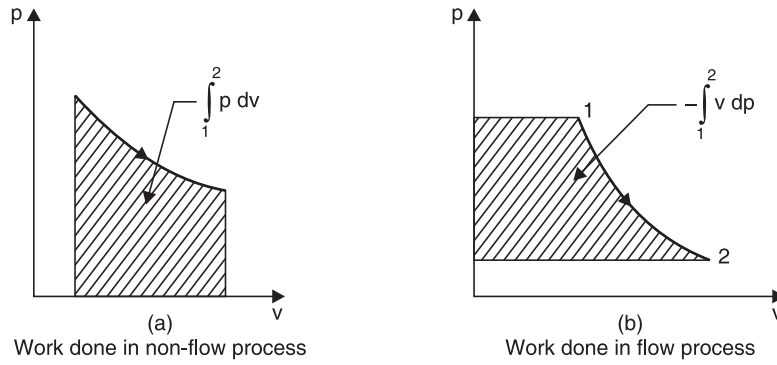
Fig. 2.28. Representation of work on *p-v* diagram.

The work done during non-flow process is given by

$$\int_1^2 pdv = Q - \Delta u \qquad\qquad ...[2.50\ (c)]$$

For isothermal process, we have
$$\Delta u = 0 \text{ and } \Delta h = 0.$$
Substituting these values in (equations) 2.50 and [2.50 (*c*)]

$$-\int_1^2 vdp = Q \text{ and } \int_1^2 pdv = Q$$

$$\therefore \qquad\qquad \int_1^2 pdv = -\int_1^2 vdp$$

The above equation indicates that the *area under both curves is same for an isothermal process.*

Note. In all the above equations '*v*' represents volume per unit mass as mass flow is considered unity.

Now let us find out expressions for work done for different flow processes as follows :

(*i*) **Steady flow constant pressure process :**

$$W = -\int_1^2 v \,.\, dp = 0 \qquad [\because \quad dp = 0] \qquad\qquad ...(2.51)$$

(*ii*) **Steady flow constant volume process :**

$$W = -\int_1^2 Vdp = -V(p_2 - p_1) = V(p_1 - p_2)$$

i.e., $\qquad\qquad W = V(p_1 - p_2)$ ...(2.52)

(*iii*) **Steady flow constant temperature process :**

The constant temperature process is represented by

$$pV = p_1V_1 = p_2V_2 = C \text{ (constant)}$$

$\therefore \qquad\qquad W = -\int_1^2 Vdp$

$$= -\int_1^2 \frac{C}{p} \, dp \qquad\qquad\qquad \left[\because \ V = \frac{C}{p}\right]$$

$$= -C\int_1^2 \frac{dp}{p} - C \,|\log_e p\,|_1^2$$

$$= -C \log_e \frac{p_2}{p_1} = C \log_e \frac{p_1}{p_2}$$

i.e., $\qquad\qquad W = p_1V_1 \log_e \left(\dfrac{p_1}{p_2}\right)$ ...(2.53)

Now substitituing the values of *W* in the equation (2.49), considering unit mass flow :

(*a*) The energy equation for *constant pressure flow process*

$$dQ = \Delta PE + \Delta KE + \Delta h$$
$$= \Delta h \text{ (if } \Delta PE = 0 \text{ and } \Delta KE = 0).$$

(*b*) The energy equation for constant volume flow process

$$dQ = -\int_1^2 vdp + \Delta PE + \Delta KE + \Delta u + pdv + vdp$$

$$= \Delta PE + \Delta KE + \Delta u \qquad \left[\because \ pdv = 0 \ \text{ and } \ v \,.\, dp = \int_1^2 vdp\right]$$

$\therefore \qquad\qquad dQ = \Delta u \text{ (if } \Delta PE = 0 \text{ and } \Delta KE = 0)$

2.12. ENGINEERING APPLICATIONS OF STEADY FLOW ENERGY EQUATION (S.F.E.E.)

2.12.1. Water Turbine

Refer Fig. 2.29. In a water turbine, water is supplied from a height. The potential energy of water is converted into kinetic energy when it enters into the turbine and part of it is converted into useful work which is used to generate electricity.

Considering centre of turbine shaft as *datum*, the energy equation can be written as follows :

$$\left(u_1 + p_1v_1 + Z_1g + \frac{C_1^2}{2}\right) + Q = \left(u_2 + p_2v_2 + Z_2g + \frac{C_2^2}{2}\right) + W$$

In this case,

$$Q = 0$$
$$\Delta u = u_2 - u_1 = 0$$
$\therefore \qquad\qquad v_1 = v_2 = v$
$$Z_2 = 0$$

$$\therefore \qquad \left(p_1 v + Z_1 g + \frac{C_1^{\,2}}{2} \right) = \left(p_2 v + Z_2 g + \frac{C_2^{\,2}}{2} \right) + W \qquad \qquad ...(2.54)$$

W is *positive* because work is done by the system (or work comes out of the boundary).
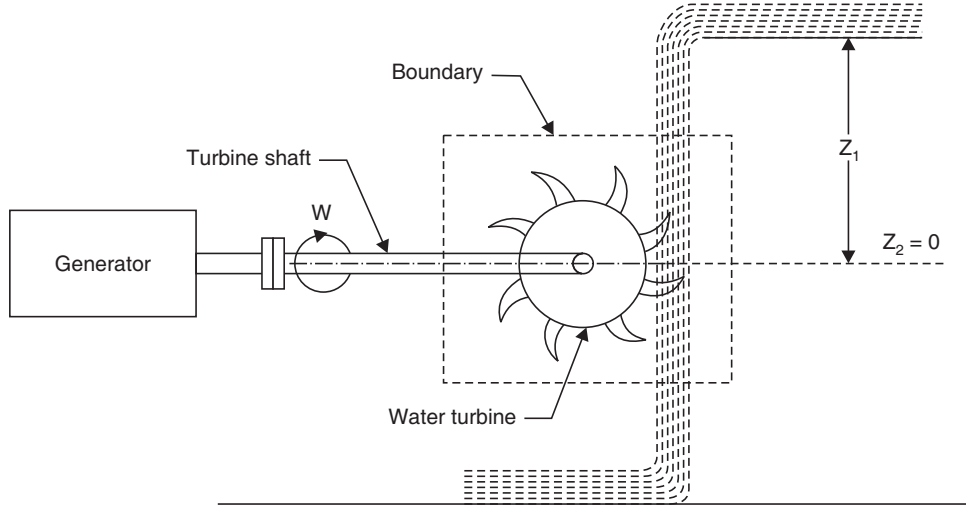
Fig. 2.29. Water turbine.

2.12.2. Steam or Gas Turbine

In a steam or gas turbine steam or gas is passed through the turbine and part of its energy is converted into work in the turbine. This output of the turbine runs a generator to produce electricity as shown in Fig. 2.30. The steam or gas leaves the turbine at lower pressure or temperature.

Applying energy equation to the system.

Here, $\qquad\qquad\qquad\qquad Z_1 = Z_2 \qquad (i.e., \Delta Z = 0)$

$$h_1 + \frac{C_1^{\,2}}{2} - Q = h_2 + \frac{C_2^{\,2}}{2} + W \qquad\qquad ...(2.55)$$
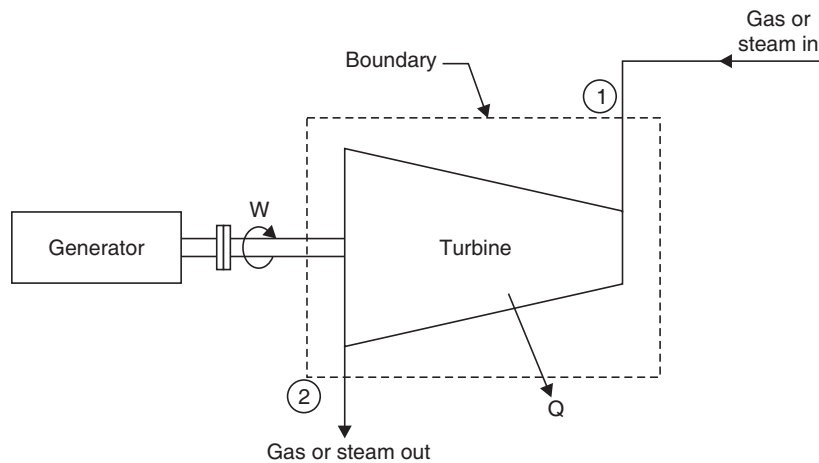
Fig. 2.30. Steam or gas turbine.

The sign of Q is *negative* because heat is *rejected* (or comes out of the boundary).

The sign of W is *positive* because work is done by the system (or work comes out of the boundary).

2.12.3. Centrifugal Water Pump

A centrifugal water pump draws water from a lower level and pumps to higher level as shown in Fig. 2.31. Work is required to run the pump and this may be supplied from an external source such as an electric motor or a diesel engine.
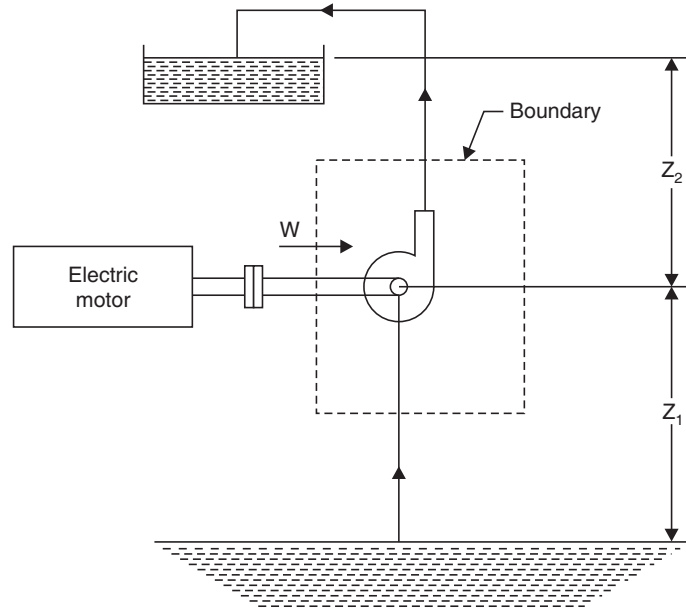
Fig. 2.31. Centrifugal water pump.

Here $Q = 0$ and $\Delta u = 0$ as there is no change in temperature of water ; $v_1 = v_2 = v$.

Applying the energy equation to the system

$$\text{or} \qquad p_1 v_1 + Z_1 g + \frac{C_1^2}{2} = p_2 v_2 + Z_2 g + \frac{C_2^2}{2} - W \qquad \qquad \text{...(2.56)}$$

The sign of W is *negative* because work is done on the system (or work enters the boundary).

2.12.4. Centrifugal Compressor

Refer Fig. 2.32. A centrifugal compressor compresses air and supplies the same at moderate pressure and in large quantity.

Applying energy equation to the system (Fig. 2.32)

$$\Delta Z = 0 \text{ (generally taken)}$$

$$\left(h_1 + \frac{C_1^2}{2} \right) - Q = \left(h_2 + \frac{C_2^2}{2} \right) - W$$

The Q is taken as *negative* as heat is *lost* from the system and W is taken as *negative* as work is *supplied* to the system.

$$\text{or} \qquad \left(h_1 + \frac{C_1^2}{2} \right) - Q = \left(h_2 + \frac{C_2^2}{2} \right) - W \qquad \qquad \text{...(2.57)}$$
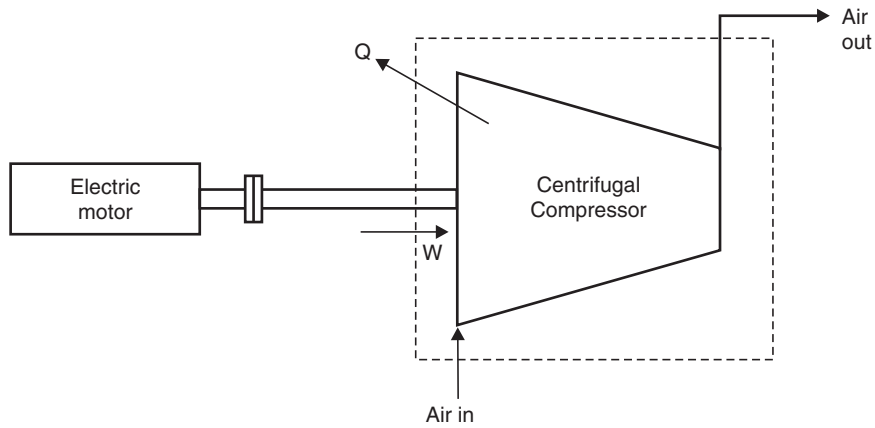
Fig. 2.32. Centrifugal compressor.

2.12.5. Reciprocating Compressor

Refer Fig. 2.33. The reciprocating compressor draws in air from atmosphere and supplies at a considerable higher pressure in small quantities (compared with centrifugal compressor). The reciprocating compressor can be considered as steady flow system *provided the control volume includes the receiver which reduces the fluctuations of flow considerably.*
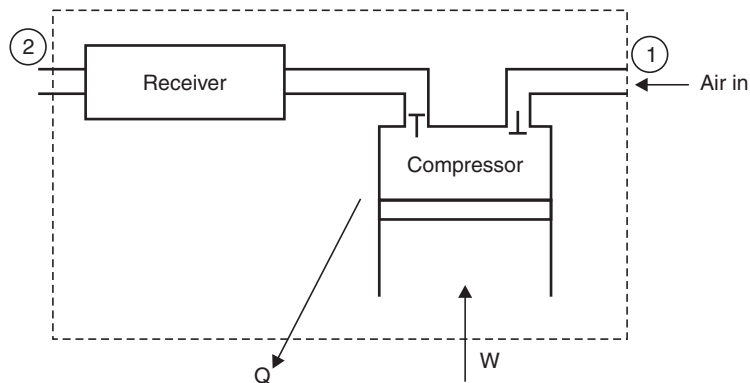
Fig. 2.33. Reciprocating compressor.

Applying energy equation to the system.

$\Delta PE = 0$ and $\Delta KE = 0$ since these changes are negligible compared with other energies.

$\therefore$ $\qquad\qquad\qquad\qquad\qquad h_1 - Q = h_2 - W$ $\qquad\qquad\qquad\qquad$ ...(2.58)

2.12.6. Boiler

A boiler transfers heat to the incoming water and generates the steam. The system is shown in Fig. 2.34.

For this system,

$$\Delta Z = 0 \quad \text{and} \quad \Delta\left(\frac{C^2}{2}\right) = 0$$

$W = 0$ since neither any work is developed nor absorbed.

Applying energy equation to the system

$\qquad\qquad\qquad h_1 + Q = h_2$ $\qquad\qquad\qquad\qquad\qquad\qquad$ ...(2.59)
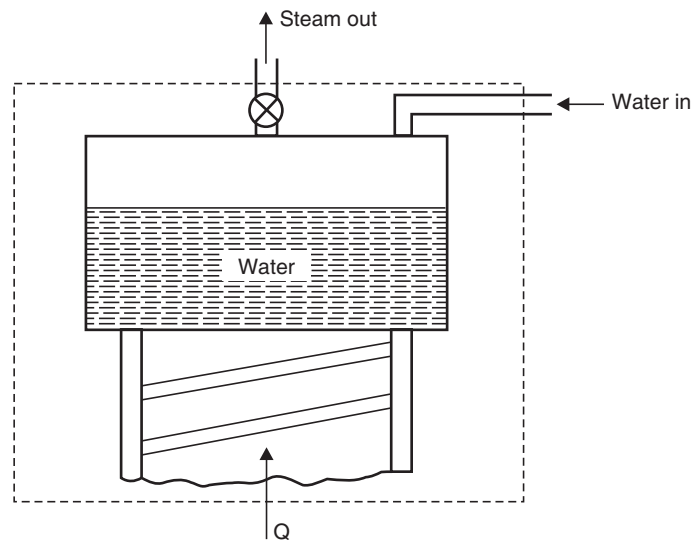
Fig. 2.34. Boiler.

2.12.7. Condenser

The condenser is used to condense the steam in case of steam power plant and condense the refrigerant vapour in the refrigeration system using water or air as cooling medium. Fig. 2.35 shows the system.

For this system :

$$\Delta PE = 0, \Delta KE = 0 \text{ (as their values are very small compared with enthalpies)}$$

$$W = 0 \text{ (since neither any work is developed nor absorbed)}$$

Using energy equation to steam flow

$$h_1 - Q = h_2 \qquad\qquad ...[2.60 \ (a)]$$

where Q = heat lost by 1 kg of steam passing through the condenser.
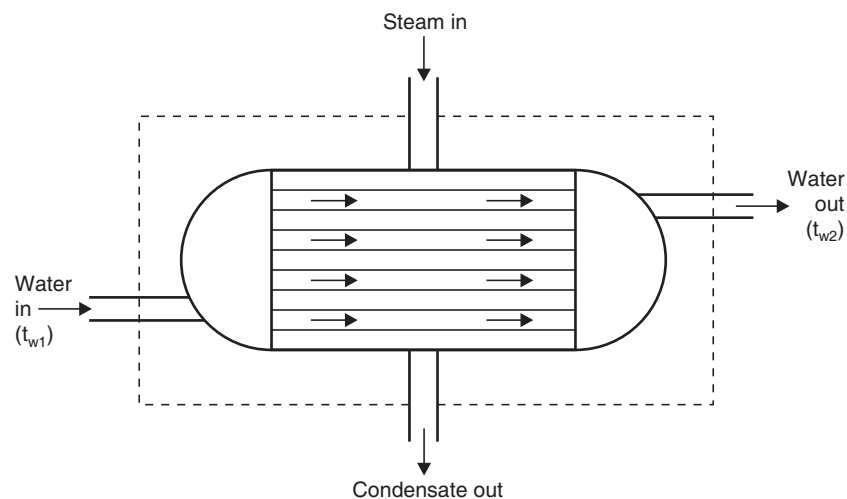
Fig. 2.35. Condenser.

Assuming there are no other heat interactions execpt the heat transfer between steam and water, then

Q = heat gained by water passing through the condenser

$$= m_w(h_{w2} - h_{w1}) = m_w c_w(t_{w2} - t_{w1})$$

Substituting this value of Q in Eqn. [2.60 (a)], we get

$$h_1 - h_2 = m_w(h_{w2} - h_{w1}) = m_w c_w(t_{w2} - t_{w1}) \qquad \qquad ...[2.60 (b)]$$

where, m_w = mass of cooling water passing through the condenser, and

c_w = specific heat of water.

2.12.8. Evaporator

An evaporator is an equipment used in refrigeration plant to carry heat from the refrigerator to maintain the low temperature. Here the refrigerant liquid is passed through the evaporator and it comes out as vapour, absorbing its latent heat from the surroundings of the evaporator. Fig. 2.36 shows the system. For this system
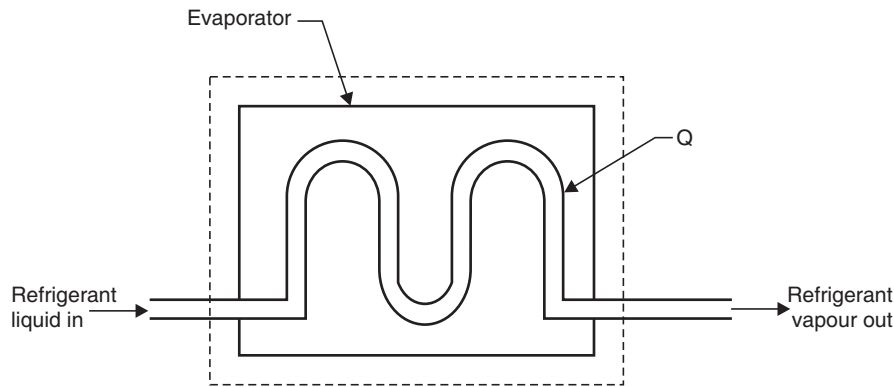
Fig. 2.36. Evaporator.

$$\Delta PE = 0, \Delta KE = 0$$

$$W = 0 \qquad \qquad [\because \quad \text{no work is absorbed or supplied}]$$

Applying the energy equation to the system

$$h_1 + Q = h_2 \qquad \qquad ...(2.61)$$

Q is taken as + ve because heat *flows from the surroundings to the system* as the temperature in the system is lower than the surroundings.

2.12.9. Steam Nozzle

In case of a nozzle as the enthalpy of the fluid decreases and pressure drops simultaneously the flow of fluid is *accelerated.* This is generally used to convert the part of the energy of steam into *kinetic energy of steam* supplied to the turbine.

Fig. 2.37 shows a commonly used convergent-divergent nozzle.

For this system,

$$\Delta PE = 0$$

$$W = 0$$

$$Q = 0$$

Applying the energy equation to the system,

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$
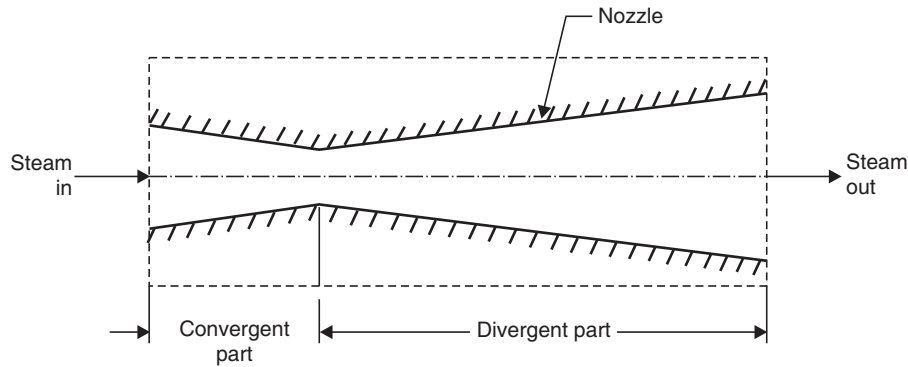
Fig. 2.37. Steam nozzle.

or
$$\frac{C_2{}^2}{2} - \frac{C_1{}^2}{2} = h_1 - h_2 \quad \text{or} \quad C_2{}^2 - C_1{}^2 = 2(h_1 - h_2)$$

or
$$C_2{}^2 = C_1{}^2 + 2(h_1 - h_2)$$

$$\therefore \qquad C_2 = \sqrt{C_1{}^2 + 2(h_1 - h_2)} \qquad\qquad ...(2.62)$$

where velocity C is in m/s and enthalpy h in joules.

If $\qquad\qquad C_1 << C_2$, then

$$C_2 = \sqrt{2(h_1 - h_2)} \qquad\qquad ...[2.63\ (a)]$$

$$\therefore \qquad C_2 = \sqrt{2\Delta h}\ . \qquad\qquad ...[2.63\ (b)]$$

2.13. THROTTLING PROCESS AND JOULE-THOMSON POROUS PLUG EXPERIMENT

Throttling process involves the *passage of a higher pressure fluid through a narrow constriction. The effect is the reduction in pressure and increase in volume.* This process is *adiabatic* as no heat flows from and to the system, but it is *not reversible*. It is *not an isentropic process*. The *entropy* of the fluid actually *increases*.

Such a process occurs in a flow through a *porous plug,* a *partially closed valve* and a *very narrow orifice.* The porous plug is shown in Fig. 2.38.
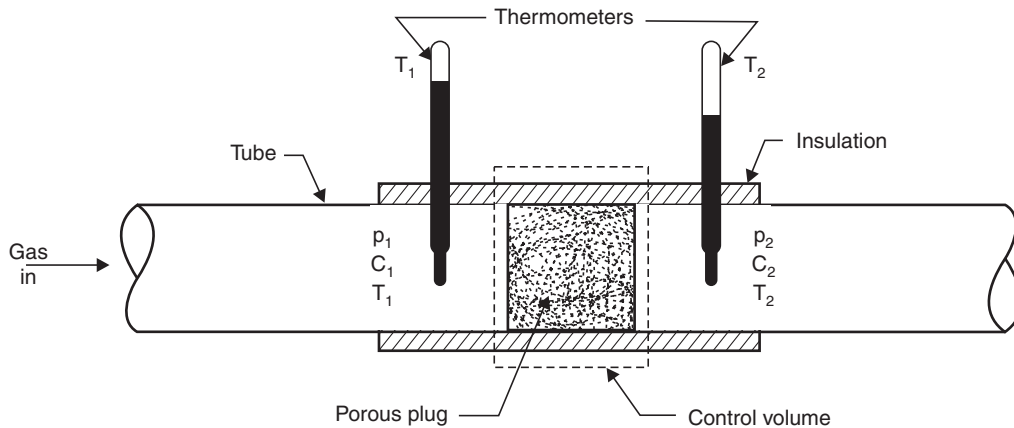
Fig. 2.38. The Joule-Thomson porous plug experiment.

In this system,

$$Q = 0 \qquad (\because \quad \text{system is isolated})$$
$$W = 0 \qquad (\because \quad \text{there is no work interaction})$$
$$\Delta PE = 0 \qquad (\because \quad \text{inlet and outlet are at the same level})$$
$$\Delta KE = 0 \qquad (\because \quad \text{kinetic energy does not change significantly})$$

Applying the energy equation to the system

$$h_1 = h_2$$

This shows that *enthalpy remains constant during adiabatic throttling process.*

The throttling process is commonly used for the following purposes :

(*i*) For determining the condition of steam (dryness fraction).

(*ii*) For controlling the speed of the turbine.

(*iii*) Used in refrigeration plant for reducing the pressure of the refrigerant before entry into the evaporator.

Throttling process frequently encountered in practice was investigated by Joule and Thomson (Lord Kelvin) in their famous *porous plug experiment* (Fig. 2.38). A stream of gas at pressure p_1 and temperature T_1 is forced continuously through a *porous plug* in a tube from which it emerges at a lower pressure p_2 and temperature T_2. The whole apparatus is *thermally insulated.*

In this process (as earlier stated)

$$h_1 = h_2$$

Whether the temperature and internal energy change in a throttling process depends on whether the fluid behaves as an ideal gas or not. Since the enthalpy of an ideal gas is a function of temperature alone, it follows that

$$T_1 = T_2 \text{ for (throttling process)}_{ideal\ gas} \qquad \qquad ...(2.64)$$

and, therefore, $u_1 = u_2$

For an *ideal gas,* therefore, the *throttling* process takes place at

(*i*) constant enthalpy,

(*ii*) constant temperature, and

(*iii*) constant internal energy.

The *enthalpy of a real gas is not a function of temperature alone.* In this case

$$T_1 \neq T_2 \qquad \qquad ...(2.65)$$

Also since the *pv* product may be different before and after throttling, the change in internal energy is not zero, as it is in *free expansion*, but is given by

$$u_2 - u_1 = p_1 v_1 - p_2 v_2 \qquad \qquad ...(2.66)$$

Joule-Thomson and Joule Co-efficients

When a real gas undergoes a throttling process a change in temperature takes place. Let us perform a series of the experiments on the same gas, keeping p_1 and T_1 constant, by varying the pressure downstream of the plug to various values p_2, p_3, p_4 etc. After throttling let T_1, T_2, T_3, T_4 etc. be the corresponding temperatures. Now if a graph is plotted between p and T (Fig. 2.39), a smooth curve drawn through these points will be a curve of *constant enthalpy* because $h_1 = h_2 = h_3 = h_4$ etc.

It may be noted that this curve does *not* represent the process executed by the gas in passing through the plug, since the process is *irreversible* and the gas does not pass through a sequence of equilibrium states.

The *slope* of a constant enthalpy line or a *p-T* diagram at a particular state may be *positive, zero* or *negative* value. The slope is called *Joule-Thomson co-efficient*, μ and is given by
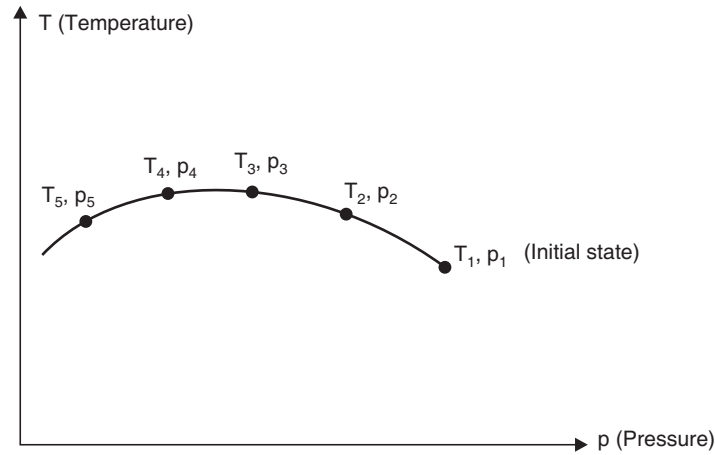
Fig. 2.39. Constant enthalpy curve.

$$\mu = \left(\frac{\partial T}{\partial p}\right)_h \qquad \qquad ...(2.67)$$

= 0 for ideal gas.

If we carry out other series of experiments similar to described above starting from different initial states, we can obtain a family of constant enthalpy curves as shown in Fig. 2.40. The states where $\mu = 0$ are called '*inversion states*' and locus of these states is called the *inversion curve*.
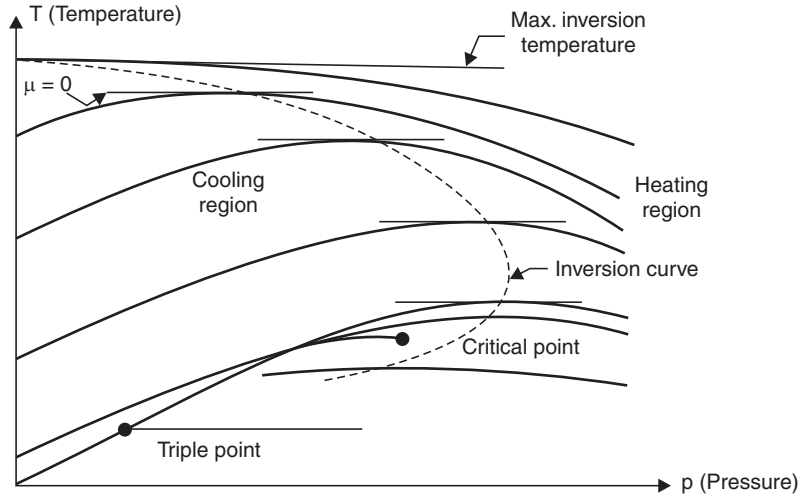
Fig. 2.40. Inversion curve.

The region inside the inversion curve is the cooling region since μ is *positive*, and temperature falls with fall in pressure.

The region outside the inversion curve is the heating region since μ is negative and temperature rises with fall in pressure.

Cooling can take place only if the initial temperature before throttling is below the *maximum inversion temperature*. This temperature is about $5\,T_c$.

The *maximum inversion temperatures* of some gases are given below :

(*i*) He = 24 K   (*ii*) H_2 = 195 K

(*iii*) Air = 603 K   (*iv*) N_2 = 261 K

(*v*) A = 732 K   (*vi*) CO_2 = 1500 K.

The free expansion is also a Joule process (not Joule-Thomson). The Joule co-efficient is defined by

$$\text{Joule co-efficient} = \eta = -\left(\frac{\partial T}{\partial v}\right)_u \qquad\qquad ...(2.68)$$

For free expansion of gases the experimental data obtained is limited. From the data available it appears that η is positive (*i.e.* cooling accompanies a fall in pressure or increase in specific volume).

Note. The throttling process is used in the liquification of gases where μ is positive.

STEADY FLOW SYSTEMS

Example 2.32. *10 kg of fluid per minute goes through a reversible steady flow process. The properties of fluid at the inlet are : p_1 = 1.5 bar, ρ_1 = 26 kg/m³, C_1 = 110 m/s and u_1 = 910 kJ/kg and at the exit are p_2 = 5.5 bar, ρ_2 = 5.5 kg/m³, C_2 = 190 m/s and u_2 = 710 kJ/kg. During the passage, the fluid rejects 55 kJ/s and rises through 55 metres. Determine :*

(*i*) *The change in enthalpy* (Δh) *;*

(*ii*) *Work done during the process* (W).

Solution. Flow of fluid   = 10 kg/min

Properties of fluid at the **inlet** :

Pressure, p_1 = 1.5 bar = 1.5 × 10⁵ N/m²

Density, ρ_1 = 26 kg/m³

Velocity, C_1 = 110 m/s

Internal energy, u_1 = 910 kJ/kg

Properties of the fluid at the **exit** :

Pressure, p_2 = 5.5 bar = 5.5 × 10⁵ N/m²

Density, ρ_2 = 5.5 kg/m³

Velocity, C_2 = 190 m/s

Internal energy, u_2 = 710 kJ/kg

Heat *rejected* by the fluid,

$$Q = 55 \text{ kJ/s}$$

Rise is elevation of fluid = 55 m.

(*i*) **The change in enthalpy,**

$$\Delta h = \Delta u + \Delta(pv) \qquad\qquad ...(i)$$

$$\Delta(pv) = \frac{p_2 v_2 - p_1 v_1}{1}$$

$$= \frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} = \frac{5.5 \times 10^5}{5.5} - \frac{1.5 \times 10^5}{26}$$

$$= 1 \times 10^5 - 0.0577 \times 10^5$$

$$= 10^5 \times 0.9423 \text{ Nm or J} = 94.23 \text{ kJ}$$

$$\Delta u = u_2 - u_1 = (710 - 910) = -200 \text{ kJ/kg}$$

Substituting the value in eqn. (*i*), we get

$$\Delta h = -200 + 94.23 = -\textbf{105.77 kJ/kg.} \quad \textbf{(Ans.)}$$
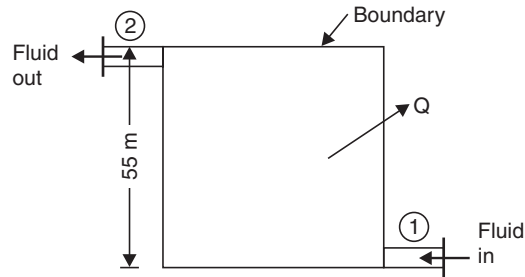
Fig. 2.41

(*ii*) The steady flow equation for unit mass flow can be written as

$$Q = \Delta KE + \Delta PE + \Delta h + W$$

where Q is the heat transfer per kg of fluid

$$Q = 55 \text{ kJ/s} = \frac{55 \text{ kJ/s}}{\frac{10}{60} \text{ kg/s}} = 55 \times 6 = 330 \text{ kJ/kg}$$

Now,

$$\Delta KE = \frac{C_2^2 - C_1^2}{2} = \frac{(190)^2 - (110)^2}{2} \text{ Nm or J} = 12000 \text{ J} \quad \text{or} \quad 12 \text{ kJ/kg}$$

$$\Delta PE = (Z_2 - Z_1)\, g = (55 - 0) \times 9.81 \text{ Nm or J} = 539.5 \text{ J} \quad \text{or} \quad \approx 0.54 \text{ kJ/kg}$$

Substituting the value in steady flow equation,

$$- 330 = 12 + 0.54 - 105.77 + W \quad \text{or} \quad W = - 236.77 \text{ kJ/kg.}$$

Work done per second $= - 236.77 \times \dfrac{10}{60} = - 39.46 \text{ kJ/s} = -$ **39.46 kW.** **(Ans.)**

Example 2.33. *In a gas turbine unit, the flow of gases through the turbine is 15 kg/s and the power developed by the turbine is 12000 kW. The enthalpies of gases at the inlet and outlet are 1260 kJ/kg and 400 kJ/kg respectively, and the velocity of gases at the inlet and outlet are 50 m/s and 110 m/s respectively. Calculate :*

(*i*) *The rate at which heat is rejected to the turbine, and*

(*ii*) *The area of the inlet pipe given that the specific volume of the gases at the inlet is 0.45 m³/kg.*

Solution. Rate of flow of gases, $\dot{m} = 15$ kg/s

Volume of gases at the inlet, $\quad v = 0.45$ m³/kg

Power developed by the turbine, $P = 12000$ kW

∴   Work done, $\qquad\qquad W = \dfrac{12000}{15} = 800$ kJ/kg

Enthalpy of gases at the inlet, $\quad h_1 = 1260$ kJ/kg

Enthalpy of gases at the oulet, $\quad h_2 = 400$ kJ/kg

Velocity of gases at the inlet, $\quad C_1 = 50$ m/s

Velocity of gases at the outlet, $\quad C_2 = 110$ m/s.

(*i*) **Heat rejected, Q :**

Using the flow equation,

$$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W \qquad\qquad\qquad ...(i) \quad [\because \quad Z_1 = Z_2]$$

Kinetic energy at inlet $= \dfrac{C_1^{\,2}}{2} = \dfrac{50^2}{2}$ m^2/s^2

$= \dfrac{50^2 \text{ kg m}^3}{2\,\text{s}^2\,\text{kg}}$

= 1250 Nm/kg

= 1.25 kJ/kg

Kinetic energy at oulet $= \dfrac{C_1^{\,2}}{2} = \dfrac{110^2}{2 \times 1000}$

= 6.05 kJ/kg

Gases out

Gases in

Gas turbine

W_{out}

Q_{out}

Boundary

Fig. 2.42

Substituting these values in equation (i), we get

$\qquad$ 1260 + 1.25 + Q = 400 + 6.05 + 800

$\therefore \qquad\qquad\qquad Q = -\,55.2$ kJ/kg

i.e. **Heat rejected** $= +\,55.2$ kJ/kg $= 55.2 \times 15$ kJ/s $=$ **828 kW.** **(Ans.)**

(ii) **Inlet area, A :**

Using the relation,

$$\dot{m} = \frac{CA}{v}$$

$\therefore \qquad\qquad A = \dfrac{v\dot{m}}{C} = \dfrac{0.45 \times 15}{50} =$ **0.135 m².** **(Ans.)**
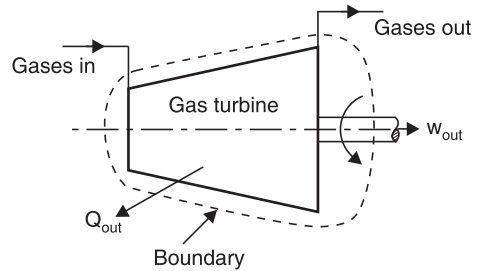
☞ **Example 2.34.** *In an air compressor air flows steadily at the rate of 0.5 kg / s through an air compressor. It enters the compressor at 6 m/s with a pressure of 1 bar and a specific volume of 0.85 m³/kg and leaves at 5 m/s with a pressure of 7 bar and a specific volume of 0.16 m³/kg. The internal energy of the air leaving is 90 kJ/kg greater than that of the air entering. Cooling water in a jacket surrounding the cylinder absorbs heat from the air at the rate of 60 kJ/s. Calculate :*

(i) *The power required to drive the compressor* ;

(ii) *The inlet and output pipe cross-sectional areas.*

Solution. Air flow rate through the compressor, $\dot{m}$ = 0.5 kg/s

Velocity of air at the inlet to compressor, $\qquad C_1$ = 6 m/s

Velocity of air at the outlet of compressor, $\qquad C_2$ = 5 m/s

Pressure of air at the inlet to the compressor, $\qquad p_1$ = 1 bar

Pressure of air at outlet to the compressor, $\qquad p_2$ = 7 bar

Specific volume of air at inlet to the compressor, v_1 = 0.85 m³/kg

Specific volume of air at outlet to the compressor, v_2 = 0.16 m³/kg

Difference of internal energy at the outlet and inlet of the compressor,

$\qquad\qquad (u_2 - u_1)$ = 90 kJ/kg

Heat rejected by air (to cooling water),

$$Q = -\,\frac{60}{0.5} = -\,120 \text{ kJ/kg.}$$
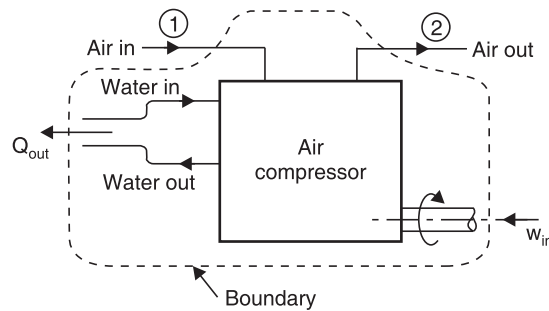
Fig. 2.43

(*i*) **Power required to drive the compressor :**

Using the steady flow energy equation,

$$u_1 + \frac{C_1^2}{2} + p_1 v_1 + Q = u_2 + \frac{C_2^2}{2} + p_2 v_2 + W$$

$\therefore$ $$W = (u_1 - u_2) + \left(\frac{C_1^2}{2} - \frac{C_2^2}{2}\right) + (p_1 v_1 - p_2 v_2) + Q$$

$$= -90 + \frac{1}{1000}\left(\frac{6^2}{2} - \frac{5^2}{2}\right) + \frac{10^5}{1000}(1 \times 0.85 - 7 \times 0.16) + (-120)$$

$$= -90 + 0.0055 - 27 - 120 = -237 \text{ kJ/kg (app.)}.$$

(Note that the change in kinetic energy is negligibly small in comparison with the other terms).

i.e.,   Work input required = 237 kJ/kg = 237 × 0.5 kJ/s = 118.5 kW

Hence, **power required to drive the compressor = 118.5 kW.   (Ans.)**

(*ii*) **Inlet and outlet pipe cross-sectional areas, A_1 and A_2 :**

Using the relation,

$$\dot{m} = \frac{CA}{v}$$

$\therefore$ $$A_1 = \frac{\dot{m} v_1}{C_1} = \frac{0.5 \times 0.85}{6} \text{ m}^2 = 0.0708 \text{ m}^2$$

i.e.,   Inlet pipe cross-sectional area, A_1 = **0.0708 m².   (Ans.)**

Again, $$A_2 = \frac{\dot{m} v_2}{C_2} = \frac{0.5 \times 0.16}{5} \text{ m}^2 = 0.016 \text{ m}^2.$$

i.e.,   Outlet pipe cross-sectional area, A_2 = **0.016 m².   (Ans.)**

Note. In this example, the steady flow energy equation has been used, despite the fact the compression consists of : suction of air ; compression in a closed cylinder ; and discharge of air. The steady flow equation can be used because the cycle of processes takes place many times in a minute, and therefore, average effect is steady flow of air through the machine.

Example 2.35. *In a steam plant, 1 kg of water per second is supplied to the boiler. The enthalpy and velocity of water entering the boiler are 800 kJ/kg and 5 m / s. The water receives 2200 kJ/kg of heat in the boiler at constant pressure. The steam after passing through the turbine comes out with a velocity of 50 m/s, and its enthalpy is 2520 kJ/kg. The inlet is 4 m above the turbine exit. Assuming the heat losses from the boiler and the turbine to the surroundings are 20 kJ/s, calculate the power developed by the turbine. Consider the boiler and turbine as a single system.*

Solution. Enthalpy of water entering the boiler, $h_1 = 800$ kJ/kg
Velocity of water entering the boiler, $C_1 = 5$ m/s
Enthalpy of steam at the outlet of the turbine, $h_2 = 2520$ kJ/kg
Velocity of steam at the outlet of the turbine, $C_2 = 50$ m/s
Elevation difference, $(Z_1 - Z_2) = 4$ m
Net heat added to the water in the boiler, $h_1 = 2200 - 20 = 2180$ kJ/kg
Power developed by the turbine :
Using the flow equation,

$$h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W$$

$\therefore$
$$W = (h_1 - h_2) + \left(\frac{C_1^2}{2} - \frac{C_2^2}{2}\right) + (Z_1 - Z_2)\, g + Q$$

$$= (800 - 2520) + \frac{1}{1000}\left[\frac{5^2}{2} - \frac{50^2}{2}\right] + \frac{4 \times 9.81}{1000} + 2180$$

$$= -1720 + \frac{1}{1000}(12.5 - 1250) + \frac{39.24}{1000} + 2180$$
$$= -1720 - 1.2375 + 0.03924 + 2180$$
$$= 458.8 \text{ kJ/kg} = 458.8 \text{ kJ/s} = 458.8 \text{ kW}$$

Hence, **power developed by the turbine = 458.8 kW.   (Ans.)**

Example 2.36. *A turbine, operating under steady-flow conditions, receives 4500 kg of steam per hour. The steam enters the turbine at a velocity of 2800 m/min, an elevation of 5.5 m and a specific enthalpy of 2800 kJ/kg. It leaves the turbine at a velocity of 5600 m/min, an elevation of 1.5 m and a specific enthalpy of 2300 kJ/kg. Heat losses from the turbine to the surroundings amout to 16000 kJ/h.*

Determine the power output of the turbine.

Solution. Quantity of steam supplied to the turbine, $m = 4500$ kg/h
Steam velocity at the entrance to the turbine, $C_1 = 2800$ m/min
Elevation at the entrance, $Z_1 = 5.5$ m
Specific enthalpy at the entrance, $h_1 = 2800$ kJ/g
Steam velocity at the exit, $C_2 = 5600$ m/min
Elevation at the exit, $Z_2 = 1.5$ m
Specific enthalpy at the exit, $h_2 = 2300$ kJ/kg
Heat losses from the turbine to the surroundings, $Q = -16000$ kJ/h
Applying the steady flow energy equation at entry (1) and exit (2)

$$m\left(h_1 + \frac{C_1^2}{2} + Z_1 g\right) + Q = m\left(h_2 + \frac{C_2^2}{2} + Z_2 g\right) + W$$

$\therefore$
$$Q - W = m\left[(h_2 - h_1) + \left(\frac{C_2^2 - C_1^2}{2}\right) + (Z_2 - Z_1)g\right]$$

or
$$-\frac{16000}{3600} - W = \frac{4500}{3600}\left[(2300 - 2800) + \left\{\frac{\left(\frac{5600}{60}\right)^2 - \left(\frac{2800}{60}\right)^2}{2 \times 1000}\right\} + \frac{(1.5 - 5.5) \times 9.81}{1000}\right]$$

or $\qquad -4.44 - W = 1.25 (500 + 3.26 - 0.039)$ or $W = 633.44$ kJ/s

∴ **Power output of the turbine = 633.44 kW. (Ans.)**

Example 2.37. *Steam at a 6.87 bar, 205°C, enters in an insulated nozzle with a velocity of 50 m/s. It leaves at a pressure of 1.37 bar and a velocity of 500 m/s.*

Determine the final enthalpy of steam.

Solution. Pressure of steam at the entrance, $p_1 = 6.87$ bar

The velocity with which steam enters the nozzle, $C_1 = 50$ m/s

Pressure of steam at the exit, $p_2 = 1.37$ bar

Velocity of steam at the exit, $C_2 = 500$ m/s.
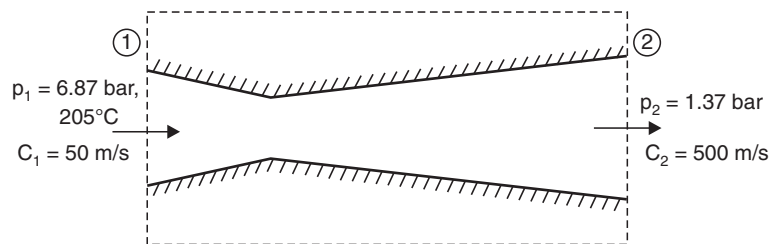
Fig. 2.44

The steady flow energy equation is given by

$$h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W \qquad \text{...(i)}$$

Considering the nozzle as an open system, it is evident that :
— there is *no work transfer* across the boundary of the system (*i.e., W = 0*)
— there is *no heat transfer* because the nozzle is insulated (*i.e., Q = 0*).
— the change in potential energy is negligible since there is no significant difference in elevation between the entrance and exit of the nozzle [*i.e.* $(Z_2 - Z_1) g = 0$].

Thus equation (*i*) reduces to

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

∴ $\qquad (h_2 - h_1) + \dfrac{C_2^2 - C_1^2}{2} = 0$

From steam table corresponding to 6.87 bar, $\quad h_1 = 2850$ kJ/kg

∴ $\qquad (h_2 - 2850) + \dfrac{(500)^2 - (50)^2}{2 \times 1000} = 0$

or $\qquad\qquad h_2 - 2850 + 123.75 = 0$ or $h_2 = 2726.25$ kJ

Hence final enthalpy of steam = 2726.25 kJ. (Ans.)

Example 2.38. *The working fluid, in a steady flow process flows at a rate of 220 kg / min. The fluid rejects 100 kJ/s passing through the sytsem. The conditions of the fluid at inlet and outlet are given as : $C_1 = 320$ m/s, $p_1 = 6.0$ bar, $u_1 = 2000$ kJ/kg, $v_1 = 0.36$ m³/kg and $C_2 = 140$ m / s, $p_2 = 1.2$ bar, $u_2 = 1400$ kJ/kg, $v_2 = 1.3$ m³/kg. The suffix 1 indicates the condition at inlet and 2 indicates at outlet of the system.*

Determine the power capacity of the system in MW.

The change in potential energy may be neglected.

Solution. Refer Fig. 2.45.
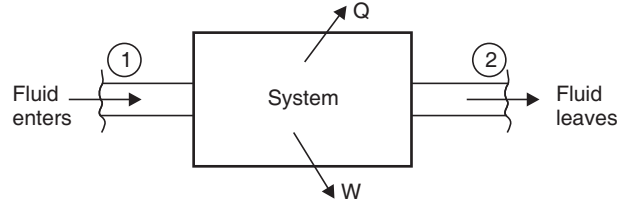
Fig. 2.45

Conditions of the fluid at point 1 :

Velocity, $C_1 = 320$ m/s

Pressure, $p_1 = 6.0$ bar $= 6 \times 10^5$ N/m^2

Internal energy, $u_1 = 2000$ kJ/kg

Specific volume, $v_1 = 0.36$ m^3/kg.

Conditions of the fluid at point 2 :

Velocity, $C_2 = 140$ m/s

Pressure, $p_2 = 1.2$ bar $= 1.2 \times 10^5$ N/m^2

Internal energy, $u_2 = 1400$ kJ/kg

Specific volume, $v_2 = 1.3$ m^3/kg

Heat rejected by the fluid, $Q = 100$ kJ/s (–).

Power capacity of the system :

Applying the energy equation at '1' and '2', we get

$$m\left[u_1 + p_1 v_1 + \frac{C_1{}^2}{2} + Z_1 g\right] \pm Q = m\left[u_1 + p_2 v_2 + \frac{C_2{}^2}{2} + Z_2 g\right] \pm W$$

Taking –ve sign for Q as the system *rejects* heat and +ve sign for W as the system *develops work*.

$$m\left[u_1 + p_1 v_1 + \frac{C_1{}^2}{2} + Z_1 g\right] - Q = m\left[u_2 + p_2 v_2 + \frac{C_2{}^2}{2} + Z_2 g\right] + W$$

$$\therefore \qquad W = m\left[(u_1 - u_2) + (p_1 v_1 - p_2 v_2) + \left(\frac{C_1{}^2 - C_2{}^2}{2}\right)\right] - Q. \qquad [\because \quad (Z_1 - Z_2)\, g = 0]$$

In the above equation :

— the mass flow is in kg/s

— velocity in m/s

— internal energy in J/kg

— pressure in N/m^2

— specific volume m^3/kg

— the value of Q is in J/s

Then the unit of W will be J/s.

$$\therefore \quad W = \frac{220}{60}\left[(2000 - 1400) \times 10^3 + 10^5(6 \times 0.36 - 1.2 \times 1.3) + \left(\frac{320^2 - 140^2}{2}\right)\right] - 100 \times 10^3$$

$$= \frac{220}{60} \ [600 \times 10^3 + 10^5 \times 0.6 + 41.4 \times 10^3] - 100 \times 10^3$$

$$= \frac{220}{60} \ [600 \times 10^3 + 60 \times 10^3 + 41.4 \times 10^3] - 100 \times 10^3$$

$$= 10^3 \times 2471.8 \ \text{J/s} \qquad\qquad [\because \quad 1 \ \text{kJ} = 10^3 \ \text{J}]$$

$$= 2471.8 \ \text{kJ/s or kW} = 2.4718 \ \text{MW}$$

Hence **power capacity of the system = 2.4718 MW.   (Ans.)**

☞ **Example 2.39.** *A stream of gases at 7.5 bar, 750°C and 140 m/s is passed through a turbine of a jet engine. The stream comes out of the turbine at 2.0 bar, 550°C and 280 m/s. The process may be assumed adiabatic. The enthalpies of gas at the entry and exit of the turbine are 950 kJ / kg and 650 kJ/kg of gas respectively.*

Determine the capacity of the turbine if the gas flow is 5 kg/s.

Solution. Refer Fig. 2.46.
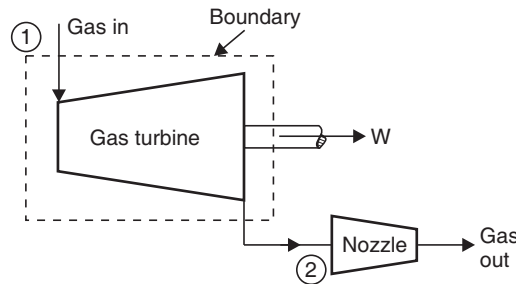
Fig. 2.46

Conditions at '1' :

Pressure, p_1 = 7.5 bar = 7.5 × 10^5 N/m², 750°C

Velocity, C_1 = 140 m/s

Enthalpy, h_1 = 950 kJ/kg

Conditions at '2' :

Pressure, p_2 = 2.0 bar = 2 × 10^5 N/m², 550°C

Velocity, C_2 = 280 m/s

Enthalpy, h_2 = 650 kJ/kg

Gas flow, m = 5 kg/s.

Capacity of the turbine :

Considering the flow of gas as 1 kg and neglecting the change in potential energy, we can write the steady flow energy equation for the turbine as

$$h_1 + \frac{C_1^{\ 2}}{2} \pm Q = h_2 + \frac{C_2^{\ 2}}{2} \pm W$$

Q = 0 as the system is adiabatic and W should be taken as +ve since it develops work.

$$\therefore \qquad\qquad h_1 + \frac{C_1^{\ 2}}{2} = h_2 + \frac{C_2^{\ 2}}{2} + W$$

$$\therefore \qquad\qquad W = (h_1 - h_2) + \frac{C_1^{\ 2} - C_2^{\ 2}}{2} = 10^3 \ (950 - 650) + \frac{140^2 - 280^2}{2}$$

$$= 10^3 \times 300 - 29.4 \times 10^3$$

$$= 270.6 \times 10^3 \text{ J/kg} = 270.6 \text{ kJ/kg}.$$

Power capacity of the turbine

$$= m W = 5 \times 270.6 = 1353 \text{ kJ/s}$$

$$= \textbf{1353 kW.} \quad \textbf{(Ans.)}$$

Example 2.40. *12 kg of air per minute is delivered by a centrifugal air compressor. The inlet and outlet conditions of air are C_1 = 12 m/s, p_1 = 1 bar, v_1 = 0.5 m³/kg and C_2 = 90 m/s, p_2 = 8 bar, v_2 = 0.14 m³/kg. The increase in enthalpy of air passing through the compressor is 150 kJ/kg and heat loss to the surroundings is 700 kJ/min.*

Find : (i) Motor power required to drive the compressor ;

(ii) Ratio of inlet to outlet pipe diameter.

Assume that inlet and discharge lines are at the same level.

Solution. Quantity of air delivered by the compressor, $m = \dfrac{12}{60}$ = 0.2 kg/s.
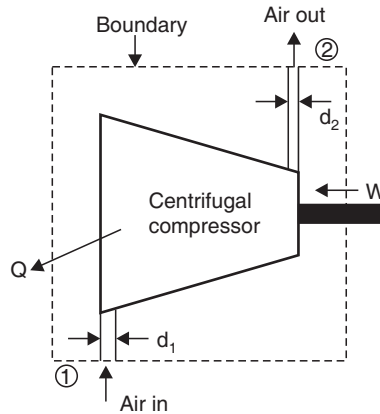
Fig. 2.47

Conditions of air at the inlet 1 :

Velocity, $\qquad\qquad\qquad C_1$ = 12 m/s

Pressure, $\qquad\qquad\qquad p_1$ = 1 bar = 1 × 10⁵ N/m²

Specific volume, $\qquad\quad v_1$ = 0.5 m³/kg

Conditions of air at the outlet 2 :

Velocity, $\qquad\qquad\qquad C_2$ = 90 m/s

Pressure, $\qquad\qquad\qquad p_2$ = 8 bar = 8 × 10⁵ N/m²

Specific volume, $\qquad\quad v_2$ = 0.14 m³/kg

Increase in enthalpy of air passing through the compressor,

$$(h_2 - h_1) = 150 \text{ kJ/kg}$$

Heat lost to the surroundings,

$$Q = -700 \text{ kJ/min} = -11.67 \text{ kJ/s}.$$

(*i*) **Motor power required to drive the compressor :**

Applying energy equation to the system,

$$m \left(h_1 + \frac{C_1^{\,2}}{2} + Z_1 g \right) + Q = m \left(h_2 + \frac{C_2^{\,2}}{2} + Z_2 g \right) + W$$

Now, $Z_1 = Z_2$ (given)

$\therefore$ $m \left(h_1 + \dfrac{C_1{}^2}{2} \right) + Q = m \left(h_2 + \dfrac{C_2{}^2}{2} \right) + W$

$$W = m \left[(h_1 - h_2) + \frac{C_1{}^2 - C_2{}^2}{2} \right] + Q$$

$$= 0.2 \left[-150 + \frac{12^2 - 90^2}{2 \times 1000} \right] + (-11.67)$$

$$= -42.46 \text{ kJ/s} = -42.46 \text{ kW}$$

$\therefore$ **Motor power required (or work done on the air) = 42.46 kW.   (Ans.)**

(ii) **Ratio of inlet to outlet pipe diameter, $\dfrac{\mathbf{d_1}}{\mathbf{d_2}}$:**

The mass of air passing through the compressor is given by

$$m = \frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2}$$

$\therefore$ $\dfrac{A_1}{A_2} = \dfrac{C_2}{C_1} \times \dfrac{v_1}{v_2} = \dfrac{90}{12} \times \dfrac{0.5}{0.14} = 26.78$

$\therefore$ $\left(\dfrac{d_1}{d_2} \right)^2 = 26.78$ or $\dfrac{d_1}{d_2} = 5.175$

Hence ratio of inlet to outlet pipe diameter = 5.175.   (Ans.)

Example 2.41. *In a test of water cooled air compressor, it is found that the shaft work required to drive the compressor is 175 kJ/kg of air delivered and the enthalpy of air leaving is 70 kJ/kg greater than that entering and that the increase in enthalpy of circulating water is 92 kJ/kg.*

Compute the amount of heat transfer to the atmosphere from the compressor per kg of air.

Solution. Refer to Fig. 2.48.

Shaft work required to drive the compressor, $W = -175$ kJ/kg

Increase in enthalpy of air passing through the compressor, $(h_2 - h_1) = 70$ kJ/kg

Increase in enthalpy of circulating water, $Q_{\text{water}} = -92$ kJ/kg

Amount of heat transferred to atmosphere, $Q_{\text{atm.}} = ?$

Applying steady-flow energy equation at '1' and '2', we get

$$h_1 + \frac{C_1{}^2}{2} + Z_1 g + Q = h_2 + \frac{C_2{}^2}{2} + Z_2 g + W$$

or $Q = (h_2 - h_1) + \left(\dfrac{C_2{}^2 - C_1{}^2}{2} \right) + (Z_2 - Z_1) g + W$

Assuming change in P.E. and K.E. to be negligible.

$\therefore$ $Q = (h_2 - h_1) + W = 70 + (-175) = -105$ kJ

But $Q = Q_{\text{atm}} + Q_{\text{water}}$ or $-105 = Q_{\text{atm}} + (-92)$

$\therefore$ $Q_{\text{atm}} = -13$ kJ/kg.

Thus heat transferred to atmosphere = 13 kJ/kg.   (Ans.)
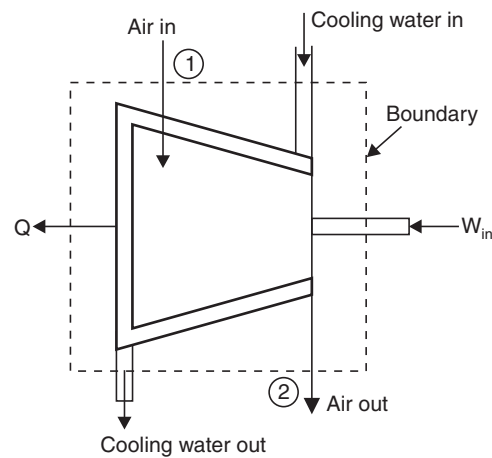
Fig. 2.48

☞ **Example 2.42.** *At the inlet to a certain nozzle the enthalpy of fluid passing is 2800 kJ/kg, and the velocity is 50 m/s. At the discharge end the enthalpy is 2600 kJ/kg. The nozzle is horizontal and there is negligible heat loss from it.*

(i) Find the velocity at exit of the nozzle.

(ii) If the inlet area is 900 cm² and the specific volume at inlet is 0.187 m³/kg, find the mass flow rate.

(iii) If the specific volume at the nozzle exit is 0.498 m³/kg, find the exit area of nozzle.

Solution. Refer Fig. 2.49.
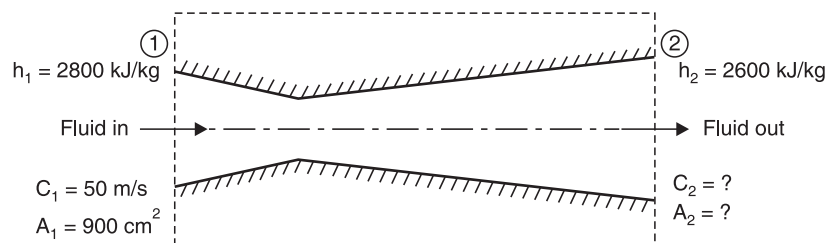
Fig. 2.49

Conditions of fluid at inlet (1) :

Enthalpy,	$h_1 = 2800$ kJ/kg
Velocity,	$C_1 = 50$ m/s
Area,	$A_1 = 900$ cm² $= 900 \times 10^{-4}$ m²
Specific volume,	$v_1 = 0.187$ m³/kg

Conditions of fluid at exit (2) :

Enthalpy,	$h_2 = 2600$ kJ/kg
Specific volume,	$v_2 = 0.498$ m³/kJ
Area,	$A_2 = ?$
Mass flow rate,	$\dot{m} = ?$

(i) **Velocity at exit of the nozzle, C_2 :**

Applying energy equation at '1' and '2', we get

$$h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W$$

Here $Q = 0, W = 0, Z_1 = Z_2$

$\therefore$ $h_1 + \dfrac{C_1^2}{2} = h_2 + \dfrac{C_2^2}{2}$

$$\frac{C_2^2}{2} = (h_1 - h_2) + \frac{C_1^2}{2}$$

$$= (2800 - 2600) \times 1000 + \frac{50^2}{2} = 201250 \text{ Nm}$$

$\therefore$ $C_2^2 = 402500$

$\therefore$ **C_2 = 634.4 m/s.   (Ans.)**

(ii) **Mass flow rate $\dot{m}$:**

By continuity equation,

$$\dot{m} = \frac{AC}{v} = \frac{A_1 C_1}{v_1} = \frac{900 \times 10^{-4} \times 50}{0.187} \text{ kg/s} = 24.06 \text{ kg/s}$$

$\therefore$ **Mass flow rate   = 24.06 kg/s.   (Ans.)**

(iii) **Area at the exit, A_2 :**

Now, $\dot{m} = \dfrac{A_2 C_2}{v_2}$

$$24.06 = \frac{A_2 \times 634.4}{0.498}$$

$\therefore$ $A_2 = \dfrac{24.06 \times 0.498}{634.4} = 0.018887 \text{ m}^2 = 188.87 \text{ cm}^2$

Hence, **area at the exit   = 188.87 cm^2.   (Ans.)**

Example 2.43. *In one of the sections of the heating plant in which there are no pumps enters a steady flow of water at a temperature of 50°C and a pressure of 3 bar (h = 240 kJ/kg). The water leaves the section at a temperature of 35°C and at a pressure of 2.5 bar (h = 192 kJ/kg). The exit pipe is 20 m above the entry pipe.*

Assuming change in kinetic energy to be negligible, evaluate the heat transfer from the water per kg of water flowing.

Solution. Refer Fig. 2.50.

Enthalpy at '1', h_1 = 240 kJ/kg

Enthalpy at '2', h_2 = 192 kJ/kg

                     $Z_2 - Z_1$ = 20 m

Applying steady flow energy equation,

$$h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W$$

$$Q = (h_2 - h_1) + \left(\frac{C_2^2 - C_1^2}{2} \right) + (Z_2 - Z_1)\, g + W$$
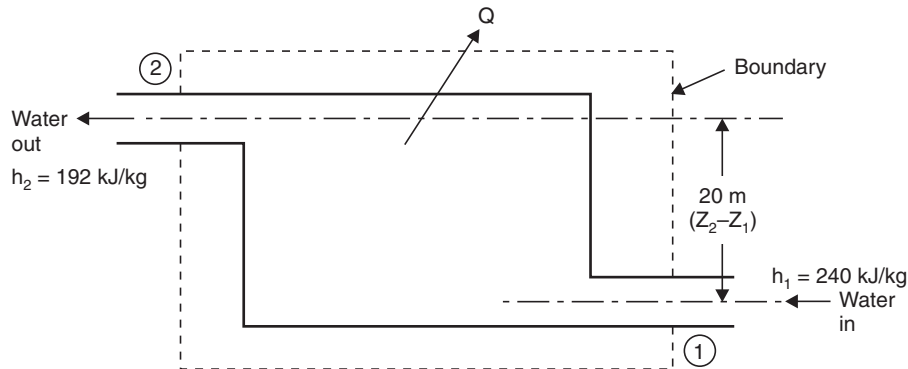
Fig. 2.50

Here $W = 0$ (no pumps)

$$\frac{C_2^{\,2} - C_1^{\,2}}{2} = 0$$ (given)

∴ $Q = (192 - 240) + \dfrac{20 \times 9.81}{1000} = -47.8 \text{ kJ/kg}$

∴ **Heat transfer from water/kg = 47.8 kJ/kg.   (Ans.)**

Example 2.44. *The gas leaving the turbine of a gas turbine jet engine flows steadily into the engine jet pipe at a temperature of 900°C, a pressure of 2 bar and a velocity of 300 m/s relative to the pipe. Gas leaves the pipe at a temperature of 820°C and a pressure of 1.1 bar. Heat transfer from the gas is negligible. Using the following data evaluate the relative velocity of gas leaving the jet pipe. For the gas at t = 820°C, h = 800 kJ/kg and at 910°C, 915 kJ/kg.*

Solution. Pressure at entry to the engine jet pipe, $p_1 = 2$ bar

Velocity relative to the pipe, $C_1 = 300$ m/s

Heat transfer from gas, $Q = 0$

At temperature, $t_1 = 910°C$, $h_1 = 915$ kJ/kg

At temperature, $t_2 = 820°C$, $h_2 = 800$ kJ/kg

Relative velocity of gas leaving the jet pipe, $C_2 = ?$

Steady flow energy equation is given by :

$$h_1 + \frac{C_1^{\,2}}{2} + Z_1 g + Q = h_2 + \frac{C_2^{\,2}}{2} + Z_2 g + W$$

$$Q = 0$$

$$W = 0$$

$$Z_1 = Z_2 \quad \text{(assumed)}$$

$$h_1 + \frac{C_1^{\,2}}{2} = h_2 + \frac{C_2^{\,2}}{2}$$

$$\frac{C_2^{\,2}}{2} = (h_1 - h_2) + \frac{C_1^{\,2}}{2} = (915 - 800) \times 1000 + \frac{300^2}{2}$$

∴ $C_2^{\,2} = 320000 \quad \text{or} \quad C_2 = 565.7$ m/s.

Hence relative velocity of gas leaving the jet pipe = 565.7 m/s.   (Ans.)

Example 2.45. *A centrifugal pump delivers 50 kg of water per second. The inlet and outlet pressures are 1 bar and 4.2 bar respectively. The suction is 2.2 m below the centre of the pump and*

delivery is 8.5 m above the centre of the pump. The suction and delivery pipe diameters are 20 cm and 10 cm respectively.

Determine the capacity of the electric motor to run the pump. **(PTU, Jan. 2002)**

Solution. Refer to Fig. 2.51.
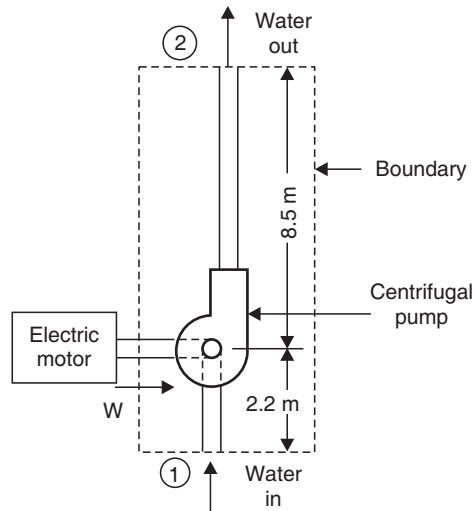
Fig. 2.51

Quantity of water delivered by the pump, $m_w = 50$ kg/s

Inlet pressure, $p_1 = 1$ bar $= 1 \times 10^5$ N/m²

Outlet pressure, $p_2 = 4.2$ bar $= 4.2 \times 10^5$ N/m²

Suction-below the centre of the pump = 2.2 m

Delivery-above the centre of the pump = 8.5 m

Diameter of suction pipe, $d_1 = 20$ cm = 0.2 m

Diameter of delivery pipe, $d_2 = 10$ cm = 0.1 m

Capacity of electric motor :

Steady flow energy equation is given by

$$m_w \left(u_1 + p_1 v_1 + \frac{C_1^2}{2} + Z_1 g \right) + Q = m_w \left(u_2 + p_2 v_2 + \frac{C_2^2}{2} + Z_2 g \right) + W \qquad ...(i)$$

Considering the datum from suction 1, as shown

$$Z_1 = 0, Z_2 = 8.5 + 2.2 = 10.7 \text{ m}$$
$$u_2 - u_1 = 0 ; \quad Q = 0$$

Thus eqn. (*i*) reduces to

$$W = m_w \left[(p_1 v_1 - p_2 v_2) + (Z_1 - Z_2)g + \left(\frac{C_1^2 - C_2^2}{2} \right) \right] \qquad ...(ii)$$

As water is incompressible fluid.

$$\therefore \qquad v_2 = v_1 = v = \frac{1}{\rho} = \frac{1}{1000}$$

The mass flow through inlet and exit pipe is given by

$$m_w = \frac{\pi}{4} \times d_1{}^2 \times C_1 \times \rho = \frac{\pi}{4} \times d_2{}^2 \times C_2 \times \rho \text{ as } \rho_1 = \rho_2 = \rho \text{ (for water)}$$

$\therefore$ $$50 = \frac{\pi}{4} \times (0.2)^2 \times C_1 \times 1000$$

$\therefore$ $$C_1 = \frac{50 \times 4}{\pi \times (0.2)^2 \times 1000} = 1.59 \text{ m/s}$$

and $$C_2 = \frac{50 \times 4}{\pi \times (0.1)^2 \times 1000} = 6.37 \text{ m/s}$$

Substituting the values in eqn. (*ii*) we get

$$W = 50 \left[\left(1 \times 10^5 \times \frac{1}{1000} - 4.2 \times 10^5 \times \frac{1}{1000} \right) + (0 - 10.7) \times 9.81 + \left(\frac{1.59^2 - 6.37^2}{2} \right) \right]$$

$$= 50[-320 - 104.96 - 19.02]$$

$$= 22199 \text{ J/s} \quad \text{or} \quad 22.199 \text{ kJ/s} \simeq 22.2 \text{ kW}.$$

Hence capacity of electric motor = 22.2 kW.   (Ans.)

Example 2.46. *During flight, the air speed of a turbojet engine is 250 m/s. Ambient air temperature is – 14°C. Gas temperature at outlet of nozzle is 610°C. Corresponding enthalpy values for air and gas are respectively 250 and 900 kJ/kg. Fuel air ratio is 0.0180. Chemical energy of fuel is 45 MJ/kg. Owing to incomplete combustion 6% of chemical energy is not released in the reaction. Heat loss from the engine is 21 kJ/kg of air.*

Calculate the velocity of the exhaust jet.

Solution. Refer Fig. 2.52.
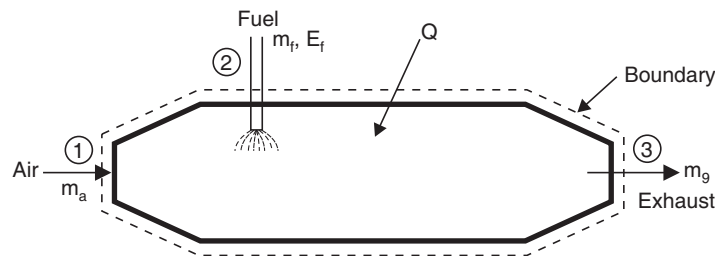
Fig. 2.52

Air speed of turbojet engine, $C_a = 250$ m/s
Ambient air temperature $= -14°C$
Gas temperature at outlet of nozzle $= 610°C$
Enthalpy of air, $h_a = 250$ kJ/kg
Enthalpy of gas, $h_g = 900$ kJ/kg
Fuel air ratio $= 0.0180$

[If, mass of air, $m_a = 1$ kg, then mass of fuel, $m_f = 0.018$ kg
and mass of gas = 1 + 0.018 = 1.018 kg]

Chemical energy of the fuel = 45 MJ/kg.

Heat loss from the engine, $Q = 21$ kJ/kg of air

Velocity of the exhaust gas jet, C_g :

Energy equation for turbojet engine is given by,

$$m_a \left(h_a + \frac{C_a^{\;2}}{2} \right) + m_f E_f + Q = m_g \left(h_g + \frac{C_g^{\;2}}{2} + E_g \right)$$

$$1 \left(250 + \frac{250^2}{2 \times 1000} \right) + 0.018 \times 45 \times 10^3 + (-21)$$

$$= 1.018 \left[900 + \frac{C_g^{\;2}}{2 \times 1000} + 0.06 \times \frac{0.018}{1.018} \times 45 \times 10^3 \right]$$

$$281.25 + 810 - 21 = 1.018 \left(900 + \frac{C_g^{\;2}}{2000} + 47.74 \right)$$

$$1070.25 = 1.018 \left(947.74 + \frac{C_g^{\;2}}{2000} \right)$$

$$\therefore \qquad\qquad C_g = 455.16 \text{ m/s}$$

Hence, **velocity of exhaust gas jet = 455.16 m/s.   (Ans.)**

☞**Example 2.47.** *Air at a temperature of 20°C passes through a heat exchanger at a velocity of 40 m/s where its temperature is raised to 820°C. It then enters a turbine with same velocity of 40 m/s and expands till the temperature falls to 620°C. On leaving the turbine, the air is taken at a velocity of 55 m/s to a nozzle where it expands until the temperature has fallen to 510°C. If the air flow rate is 2.5 kg/s, calculate :*

(i) Rate of heat transfer to the air in the heat exchanger ;

(ii) The power output from the turbine assuming no heat loss ;

(iii) The velocity at exit from the nozzle, assuming no heat loss.

Take the enthalpy of air as $h = c_p t$, where c_p is the specific heat equal to 1.005 kJ/kg°C and t the temperature.

Solution. Refer Fig. 2.53.

Temperature of air, $t_1 = 20°C$

Velocity of air, $\qquad C_1 = 40$ m/s.

Temperature of air after passing the heat exchanger, $t_2 = 820°C$
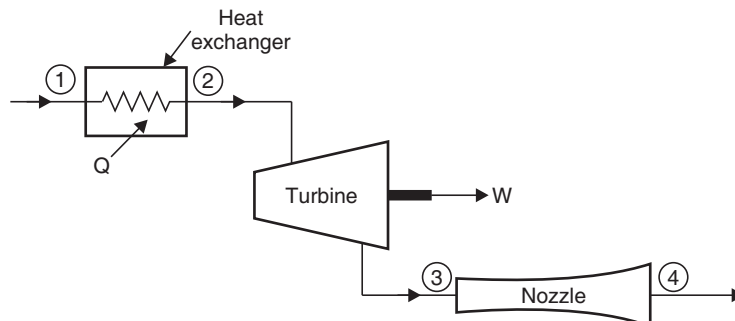
Fig. 2.53

Velocity of air at entry to the turbine, C_2 = 40 m/s

Temperature of air after leaving the turbine, t_3 = 620°C

Velocity of air at entry to nozzle, C_3 = 55 m/s

Temperature of air after expansion through the nozzle, t_4 = 510°C

Air flow rate, $\dot{m}$ = 2.5 kg/s.

(*i*) **Heat exchanger :**

Rate of heat transfer :

Energy equation is given as

$$m\left(h_1 + \frac{C_1{}^2}{2} + Z_1 g\right) + Q_{1-2} = m\left(h_2 + \frac{C_2{}^2}{2} + Z_2 g\right) + W_{1-2}$$

Here, $Z_1 = Z_2,\ C_1,\ C_2 = 0,\ W_{1-2} = 0$

∴ $mh_1 + Q_{1-2} = mh_2$

or $Q_{1-2} = m(h_2 - h_1)$

$$= mc_p\,(t_2 - t_1) = 2.5 \times 1.005\,(820 - 20) = 2010 \text{ kJ/s.}$$

Hence, **rate of heat transfer = 2010 kJ/s.   (Ans.)**

(*ii*) **Turbine :**

Power output of turbine :

Energy equation for turbine gives

$$m\left(h_2 + \frac{C_2{}^2}{2}\right) = m\left(h_3 + \frac{C_3{}^2}{2}\right) + W_{2-3} \qquad\qquad [\because\ Q_{2-3} = 0,\ Z_1 = Z_2]$$

∴ $$W_{2-3} = m\left(h_2 + \frac{C_2{}^2}{2}\right) - m\left(h_3 + \frac{C_3{}^2}{2}\right)$$

$$= m\left[(h_2 - h_3) + \left(\frac{C_2{}^2 - C_3{}^2}{2}\right)\right]$$

$$= m\left[c_p(t_2 - t_3) + \frac{C_2{}^2 - C_3{}^2}{2}\right]$$

$$= 2.5\left[1.005\,(820 - 620) + \frac{(40)^2 - (55)^2}{2 \times 1000}\right]$$

$$= 2.5\,[201 + 0.7125] = 504.3 \text{ kJ/s or } 504.3 \text{ kW}$$

Hence, **power output of turbine = 504.3 kW.   (Ans.)**

(*iii*) **Nozzle :**

Velocity at exit from the nozzle :

Energy equation for nozzle gives,

$$h_3 + \frac{C_3{}^2}{2} = h_4 + \frac{C_4{}^2}{2} \qquad\qquad [\because\ W_{3-4} = 0,\ Q_{3-4} = 0,\ Z_1 = Z_2]$$

$$\frac{C_4{}^2}{2} = (h_3 - h_4) + \frac{C_3{}^2}{2} = c_p(t_3 - t_4) + \frac{C_3{}^2}{2}$$

$$= 1.005(620 - 510) + \frac{55^2}{2 \times 1000} = 112.062 \times 10^3 \text{ J}$$

$\therefore \qquad\qquad\qquad\qquad\qquad C_4 = 473.4 \text{ m/s.}$

Hence, **velocity at exit from the nozzle = 473.4 m/s.   (Ans.)**

2.14. HEATING-COOLING AND EXPANSION OF VAPOURS

The basic energy equations for non-flow and flow processes are *also valid for vapours*.

$\therefore$ When $\Delta KE = 0$ and $\Delta PE = 0$

$\qquad\qquad\qquad dQ = du + p \cdot dv \qquad$ ......for *non-flow process.*

$\qquad\qquad\qquad dQ = dh - v \cdot dp \qquad$ ......for *flow process.*

The various processes using vapour are discussed below :

1. Constant Volume Heating or Cooling. The constant volume heating process is represented on *p-v*, *T-s* and *h-s* diagram as shown in Fig. 2.54 (*a*), (*b*), (*c*) respectively. It is assumed that the steam is in wet condition before heating at pressure p_1 becomes superheated after heating and pressure increases from p_1 to p_2.

Since the mass of steam, m, remains constant during the heating process,

$\therefore \qquad\qquad\qquad m = \dfrac{V}{x_1 v_{g_1}} = \dfrac{V}{v_{sup_2}}, \qquad$ where V is the total constant volume of steam
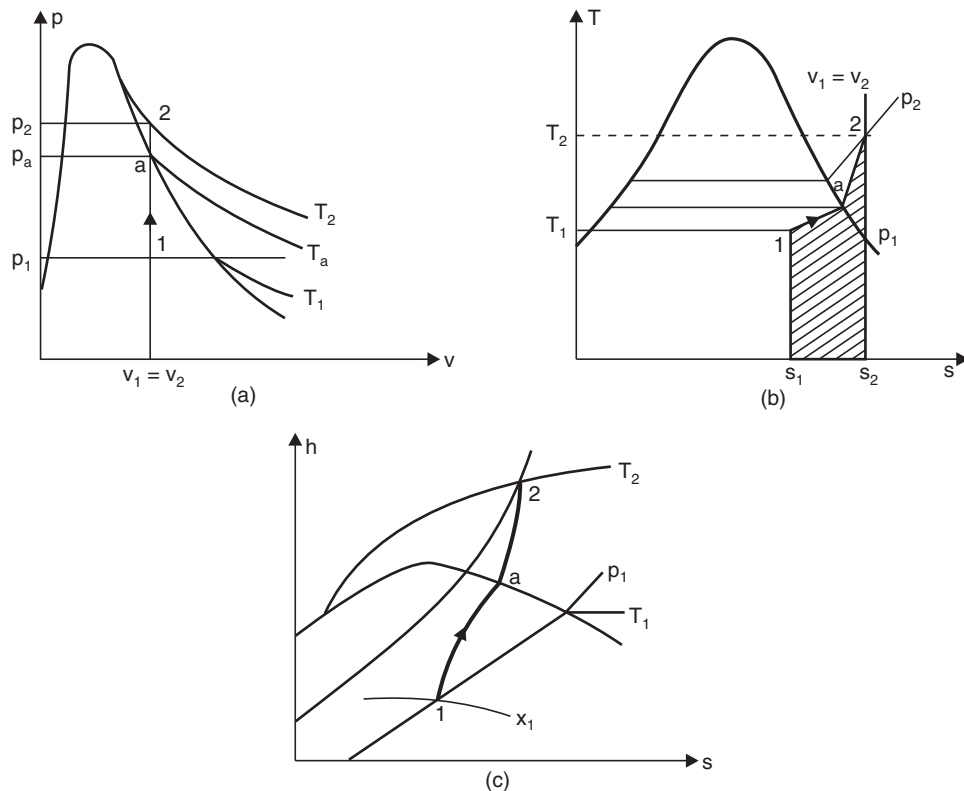
Fig. 2.54. Constant volume process.

Also
$$\frac{v_{sup_2}}{v_{g_2}} = \frac{T_{sup_2}}{T_{s_2}}$$

$\therefore$
$$\frac{T_{sup_2}}{T_{s_2}} = \frac{x_1 v_{g_1}}{v_{g_2}} \qquad \qquad ...(2.69)$$

v_{g_2} and T_{s_2} can be found from the steam tables corresponding to pressure p_2 and then T_{sup2} can be calculated by using the above equation. When the final condition is known, the change in all other properties can be found easily.

If after cooling, the condition of steam remains wet, then the mass fraction be obtained as follows :

$$\frac{V}{x_1 v_{g_1}} = \frac{V}{x_2 v_{g_2}}$$

$$x_2 = \frac{x_1 v_{g_1}}{v_{g_2}} \qquad \qquad ...(2.70)$$

where v_{g_2} can be found from the steam tables corresponding to pressure p_2.

Applying the first law of thermodynamics, we have

$$Q = \Delta u + \int_1^2 p \cdot dv$$

$$= \Delta u \text{ as } \int_1^2 p\, dv = 0$$

$$= u_2 - u_1$$

i.e.,
$$Q_1 = [h_2 - p_2 v_{sup_2}] - [h_1 - p_1(x_1 v_{g_1})] \qquad \qquad ...(2.71)$$

In case the condition of steam remains wet after heating, then

$$Q = (u_2 - u_1) = [h_2 - p_2(x_2 v_{g_2})] - [h_1 - p_1(x_1 v_{g_1})] \qquad ...(2.72)$$

In the *cooling process,* the same equations are used except that the suffixes 1, 2 are interchanged.

☞**Example 2.48.** *A rigid cylinder of volume 0.028 m³ contains steam at 80 bar and 350°C. The cylinder is cooled until the pressure is 50 bar. Calculate :*

(*i*) *The state of steam after cooling ;*

(*ii*) *The amount of heat rejected by the steam.*

Solution. Volume of rigid cylinder = 0.028 m³

Pressure of steam before cooling, p_1 = 80 bar

Temperature of steam before cooling = 350°C

Pressure of steam after cooling, p_2 = 50 bar

Steam at 80 bar and 350°C is in a superheated state, and the specific volume from tables is 0.02995 m³/kg. Hence the mass of steam in the cylinder is given by

$$m = \frac{0.028}{0.02995} = 0.935 \text{ kg}$$

Internal energy at state 1, (*80 bar, 350°C*),

$$u_1 = h_1 - p_1 v_1$$

$$= 2987.3 - \frac{80 \times 10^5 \times 0.02995}{10^3} \quad \text{or} \quad u_1 = 2747.7 \text{ kJ/kg}.$$

(*i*) **State of steam after cooling :**

At state 2, $p_2 = 50$ bar and $v_2 = 0.02995$ m³/kg, therefore, steam is wet, and dryness fraction is given by,

$$x_2 = \frac{v_2}{v_{g_2}} = \frac{0.02995}{0.0394} = 0.76.$$

(*ii*) **Heat rejected by the steam :**

Internal energy at state 2 (50 bar),

$$u_2 = (1 - x_2)\, u_{f_2} + x_2 u_{g_2}$$

$$= (1 - 0.76) \times 1149 + 0.76 \times 2597 = 2249.48 \text{ kJ/kg}$$

At constant volume, $\quad Q = U_2 - U_1 = m(u_2 - u_1)$

$$= 0.935(2249.48 - 2747.7) = -465.5 \text{ kJ}$$

i.e.,   **Heat rejected**   **= 465.5 kJ.   (Ans.)**

Fig. 2.55 shows the process drawn on *T-s* diagram, the shaded area representing the heat rejected by the system.
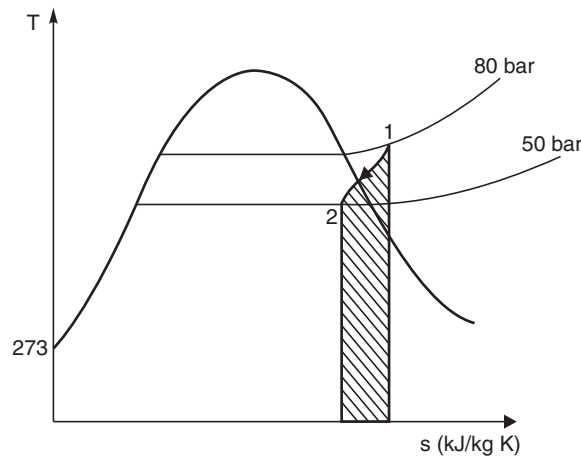
Fig. 2.55

2. Constant pressure Heating or Cooling. Fig. 2.56 (*a*), (*b*) and (*c*) shows the constant pressure heating process on *p-v*, *T-s* and *h-s* diagrams respectively.

Generation of steam in the boilers is an example of constant pressure heating.

Applying first law of thermodynamics, we have

$$Q = \Delta u + \int_1^2 p \cdot dv$$

$$= (u_2 - u_1) + p(v_2 - v_1) = (u_2 + pv_2) - (u_1 + pv_1) = h_2 - h_1$$

If the initial condition of steam is wet and final condition is superheated, then

$$Q = (u_2 + p\, v_{sup_2}) - (u_1 + p \cdot x_1\, v_{g_1})$$

$$= (h_2 - h_1) \qquad\qquad\qquad \text{...(2.73)}$$

here h_1 and h_2 are the actual enthalpies of steam per kg before and after heating.
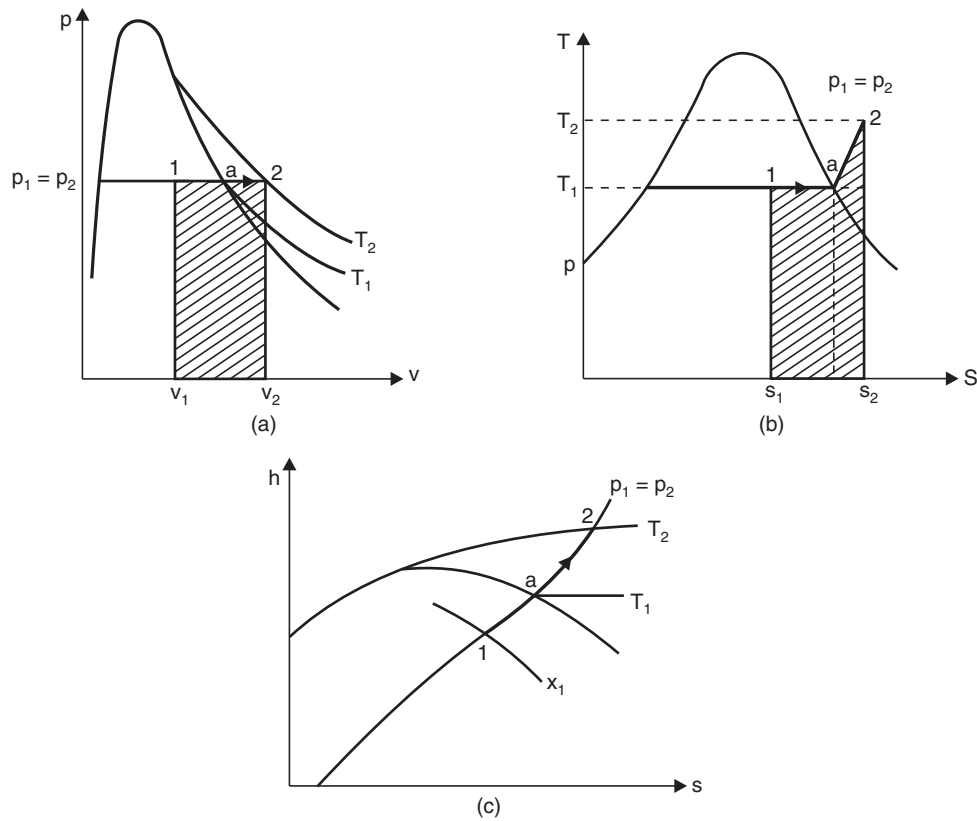
Fig. 2.56. Constant pressure process.

The heat added during the constant pressure process is equal to the change in enthalpy of steam during the process. When the steam is wet before heating and becomes superheated after heating the work done,

$$W = p \ (v_{sup_2} - x_1 v_{g_1}) \hspace{3cm} ...(2.74)$$

Example 2.49. *0.08 kg of dry steam is heated at a constant pressure of 2 bar until the volume occupied is 0.10528 m³. Calculate :*

 (*i*) *Heat supplied ;*

(*ii*) *Work done.*

Solution. Mass of steam, m = 0.08 kg

Pressure of steam, p = 2 bar

Volume occupied after heating = 0.10528 m³

Initially the steam is dry saturated at 2 bar, hence

$$h_1 = h_g \text{ (at 2 bar)} = 2706.3 \text{ kJ/kg}$$

Finally the steam is at 2 bar and the specific volume is given by

$$v_2 = \frac{0.10528}{0.08} = 1.316 \text{ m}^3\text{/kg}$$

Hence the steam is *superheated finally* (since the value of v_g at 2 bar = 0.885 m³/kg). From superheat tables at 2 bar and 1.316 m³/kg the temperature of steam is 300°C, and the enthalpy, h_2 = 3071.8 kJ/kg.
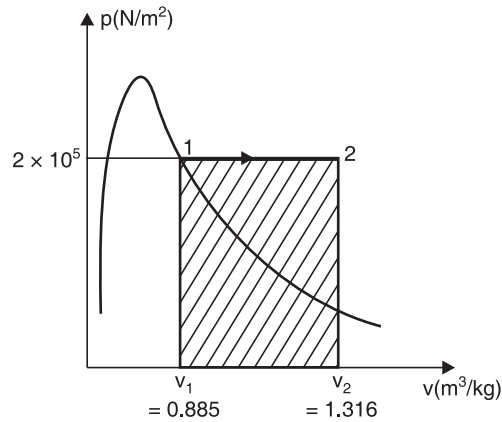
Fig. 2.57

(*i*) **Heat supplied :**

Heat supplied,

$$Q = H_2 - H_1 = m(h_2 - h_1)$$
$$= 0.08(3071.8 - 2706.3)$$
$$= \textbf{29.24 kJ.} \quad \textbf{(Ans.)}$$

(*ii*) **Work done :**

The process is shown on a *p-v* diagram in Fig. 2.57. The work done is given by the shaded area

i.e.,

$$W = p(v_2 - v_1) \text{ Nm/kg}$$

Here

$$v_1 = v_g \text{ at 2 bar} = 0.885 \text{ m}^3\text{/kg}$$

and

$$v_2 = 1.316 \text{ m}^3\text{/kg}$$

∴

$$W = 2 \times 10^5 (1.316 - 0.885) = 2 \times 10^5 \times 0.431 \text{ Nm/kg}$$

Now work done by the total mass of steam (0.08 kg) present

$$= 0.08 \times 2 \times 10^5 \times 0.431 \times 10^{-3} \text{ kJ}$$
$$= \textbf{6.896 kJ.} \quad \textbf{(Ans.)}$$

Example 2.50. *1 kg of steam at 8 bar, entropy 6.55 kJ/kg K, is heated reversibly at constant pressure until the temperature is 200°C. Calculate the heat supplied, and show on a T-s diagram the area which represents the heat flow.*

Solution. Mass of steam, $m = 1$ kg

Pressure of steam, $p = 8$ bar

Entropy of steam (at 8 bar), $s = 6.55$ kJ/kg K

Temperature after heating $= 200°C$

At 8 bar, $s_g = 6.66$ kJ/kg K, hence steam is *wet*, since the actual entropy, s, is less than s_g.

To find the dryness fraction x_1, using the relation,

$$s_1 = s_{f_1} + x_1 s_{fg_1}$$
$$6.55 = 2.0457 + x_1 \times 4.6139$$

∴

$$x_1 = \frac{6.55 - 2.0457}{4.6139} = 0.976$$

Now, *initial enthalpy* (at 8 bar),

$$h_1 = h_{f_1} + x_1 h_{fg_1}$$
$$= 720.9 + 0.976 \times 2046.5 = 2718.28 \text{ kJ/kg}$$
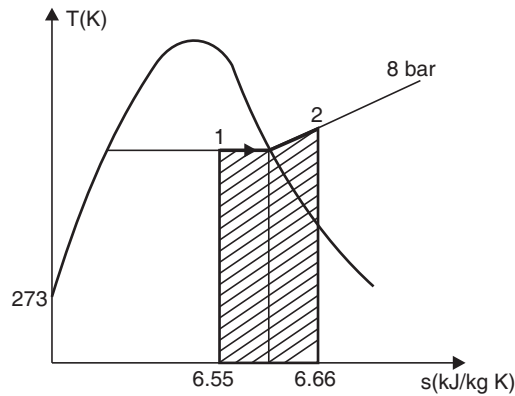
Fig. 2.58

Final enthalpy, h_2 : At state 2 the steam is at 200°C at 8 bar and is therefore, superheated. From superheated tables, $h_2 = 2839.3$ kJ/kg

Now, $Q = h_2 - h_1 = 2839.3 - 2718.28 = 121.02$ kJ/kg

i.e.,   **Heat supplied   = 121.02 kJ/kg.   (Ans.)**

The *T-s* diagram showing the process is given in Fig. 2.58, the shaded area representing the heat flow.

3. Constant Temperature or Isothermal Expansion. Fig. 2.59 (*a*), (*b*) and (*c*) shows the constant temperature or isothermal expansion on *p-v, T-s* and *h-s* diagrams respectively.

In the wet region, the constant temperature process is also a constant pressure process during evaporation and as well as condensation. When the steam becomes saturated it behaves like a gas and constant temperature process in superheated region becomes hyperbolic (*pv* = constant).

When the wet steam is heated at constant temperature till it becomes *dry and saturated,* then the heat transfer (*Q*) is given by :

$$Q = h_2 - h_1$$

and work done, $$W = p_1(v_{g_2} - x_1 v_{g_1})$$

$$= p \, v_{g_1} (1 - x_1)$$

$[\because \quad v_{g_2} = v_{g_1}$ as pressure remains constant during this process]

This process is limited to *wet region only*.

Hyperbolic process (*pv* = constant) is also an isothermal process in the superheat region as the steam behaves like a gas in this region. The work done during the hyperbolic expansion in a *non-flow system* is given by

$$W = \int_1^2 p \, dv = \int_1^2 \frac{C}{v} \, dv = C \log_e \left(\frac{v_2}{v_1} \right)$$

$$= p_1 v_1 \log_e \left(\frac{v_2}{v_1} \right) \qquad \qquad ...(2.75)$$

where v_1 and v_2 are the specific volumes of steam before and after expansion.
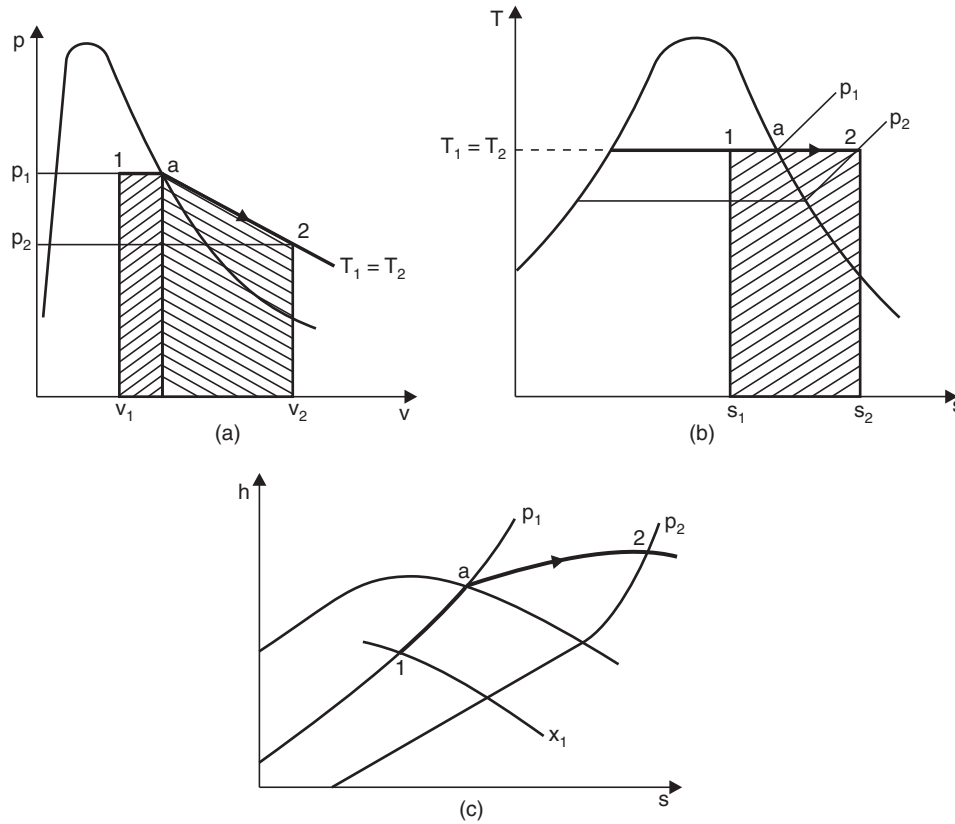
Fig. 2.59. Constant temperature or isothermal expansion.

Applying first law of energy equation,

$$Q = \Delta u + \int_1^2 p \cdot dv$$

$$= (u_2 - u_1) + p_1 v_1 \log_e \left(\frac{v_2}{v_1}\right)$$

$$= (h_2 - p_2 v_2) - (h_1 - p_1 v_1) + p_1 v_1 \log_e \left(\frac{v_2}{v_1}\right)$$

Since $\qquad p_1 v_1 = p_2 v_2$

$\therefore \qquad Q = (h_2 - h_1) + p_1 v_1 \log_e \dfrac{v_2}{v_1}$ ...(2.76)

Example 2.51. *Steam at 7 bar and dryness fraction 0.95 expands in a cylinder behind a piston isothermally and reversibly to a pressure of 1.5 bar. The heat supplied during the process is found to be 420 kJ / kg. Calculate per kg :*

(*i*) *The change of internal energy ;* (*ii*) *The change of enthalpy ;*

(*iii*) *The work done.*

Solution. Initial pressure of steam, $p_1 = 7$ bar $= 7 \times 10^5$ N/m^2

Final pressure of steam, $p_2 = 1.5$ bar $= 1.5 \times 10^5$ N/m^2

Heat supplied during the process, Q = 420 kJ/kg.

The process is shown in Fig. 2.60. The saturation temperature corresponding to 7 bar is 165°C. Therefore, the steam is *superheated at the state 2*.
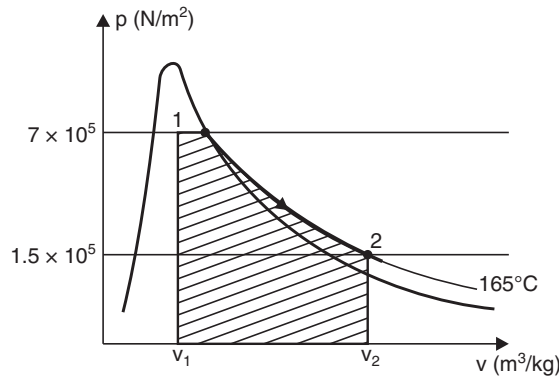
Fig. 2.60

(*i*) **Change of internal energy :**

The internal energy at state 1 is found by using the relation :

$$u_1 = (1 - x) u_f + xu_g$$
$$= (1 - 0.95) 696 + (0.95 \times 2573)$$

$\therefore$ $\quad\quad\quad\quad\quad\quad\quad\quad u_1 = 2479.15$ kJ/kg

Interpolating from superheat tables at 1.5 bar and 165°C, we have

$$u_2 = 2580 + \frac{15}{50} (2856 - 2580)$$
$$= 2602.8 \text{ kJ/kg}$$

$\therefore$ **Gain in internal energy,**

$$u_2 - u_1 = 2602.8 - 2479.15 = \textbf{123.65 kJ/kg.} \quad \textbf{(Ans.)}$$

(*ii*) **Change of enthalpy :**

Enthalpy at state 1 (7 bar),

$$h_1 = h_{f_1} + x_1 h_{fg_1}$$

At 7 bar. $\quad\quad\quad\quad h_f = 697.1$ kJ/kg and $h_{fg} = 2064.9$ kJ/kg

$\therefore$ $\quad\quad\quad\quad\quad\quad h_1 = 697.1 + 0.95 \times 2064.9 = 2658.75$ kJ/kg

Interpolating from superheat tables at 1.5 bar and 165°C, we have

$$h_2 = 2772.6 + \frac{15}{50} (2872.9 - 2772.6) = 2802.69 \text{ kJ/kg}$$

$\therefore$ Change of enthalpy

$$= h_2 - h_1 = 2802.69 - 2658.75 = \textbf{143.94 kJ/kg.} \quad \textbf{(Ans.)}$$

(*iii*) **Work done :**

From non-flow energy equation,

$$Q = (u_2 - u_1) + W$$

$\therefore$ $\quad\quad\quad\quad W = Q - (u_2 - u_1) = 420 - 123.65 = 296.35$ kJ/kg

i.e., **Work done by the steam = 296.35 kJ/kg. (Ans.)**

Note. The work done is also given by the area on the Fig. 2.60 $\left(\int_{v_1}^{v_2} pdv\right)$, this can only be evaluated graphically.

Example 2.52. *In a steam engine cylinder the steam expands from 5.5 bar to 0.75 bar according to the hyperbolic law, pv = constant. If the steam is initially dry and saturated, calculate per kg of steam :*

(*i*) *Work done ;*

(*ii*) *Heat flow to or from the cylinder walls.*

Solution. Initial pressure of steam, p_1 = 5.5 bar = 5.5×10^5 N/m^2

Initial condition of steam, x_1 = 1

Final pressure of steam, p_2 = 0.75 bar = 0.75 bar × 10^5 N/m^2

At 5.5 bar, $v_1 = v_g$ = 0.3427 m^3/kg

Also $p_1 v_1 = p_2 v_2$

∴ $v_2 = \dfrac{p_1 v_1}{p_2} = \dfrac{5.5 \times 0.3427}{0.75}$ = 2.513 m^3/kg

At 0.75 bar, v_g = 2.217 m^3/kg.

Since $v_2 > v_g$ (at 0.75 bar), therefore, the steam is *superheated* at state 2.

Interpolating from superheat tables at 0.75 bar, we have

$$u_2 = 2510 + \left(\frac{2.513 - 2.271}{2.588 - 2.271}\right)(2585 - 2510)$$

$$= 2510 + \frac{0.242}{0.317} \times 75 = 2567.25 \text{ kJ/kg.}$$

For dry saturated steam at 5.5 bar

$$u_1 = u_g = 2565 \text{ kJ/kg}$$

Hence, gain in internal energy

$$= u_2 - u_1 = 2567.25 - 2565 = 2.25 \text{ kJ/kg}$$

The process is shown on a *p-v* diagram in Fig. 2.61, the shaded area representing the work done.
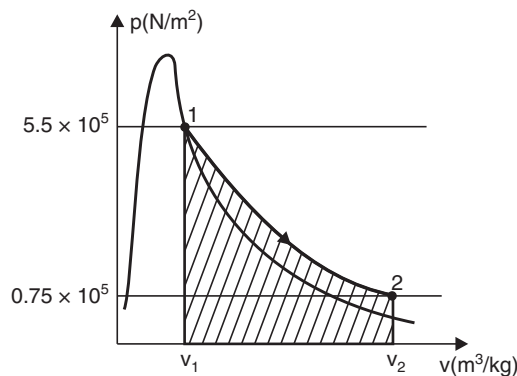
Fig. 2.61

Now,
$$W = \int_{v_1}^{v_2} p \; dv$$

$$= \int_{v_1}^{v_2} \left(\frac{\text{constant}}{v}\right) dv \qquad \left[\because \quad pv = \text{constant and } p = \frac{\text{constant}}{v}\right]$$

$$= \text{constant} \left[\log_e v\right]_{v_1}^{v_2}$$

The constant is either $p_1 v_1$ or $p_2 v_2$

i.e.,
$$W = 5.5 \times 10^5 \times 0.3427 \times \log_e \frac{p_1}{p_2} \qquad \left[\because \quad p_1 v_1 = p_2 v_2 \text{ or } \frac{v_2}{v_1} = \frac{p_1}{p_2}\right]$$

$$= 5.5 \times 10^5 \times 0.3427 \times \log_e \left(\frac{5.5}{0.75}\right) = 375543 \text{ Nm/kg}.$$

Using non-flow energy equation, we get
$$Q = (u_2 - u_1) + W$$

$$= 2.25 + \frac{375543}{10^3} = 377.79 \text{ say } 378 \text{ kJ/kg}$$

i.e., **Heat supplied**   **= 378 kJ/kg.   (Ans.)**

Example 2.53. *Dry saturated steam at 100 bar expands isothermally and reversibly to a pressure of 10 bar. Calculate per kg of steam :*

(*i*) *The heat supplied ;*

(*ii*) *The work done.*

Solution. Initial pressure of steam, p_1 = 100 bar

Final pressure of steam, p_2 = 10 bar

The process is shown in Fig. 2.62, the shaded area representing the heat supplied.
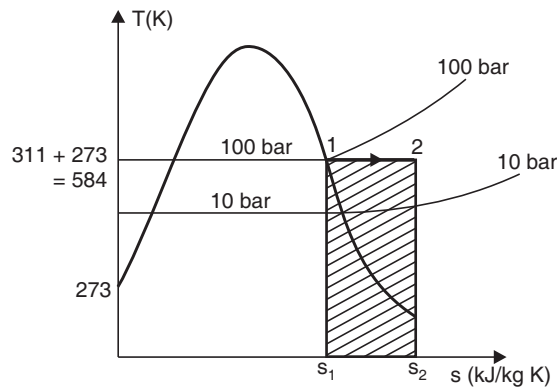
Fig. 2.62

At 100 bar, dry saturated : From steam tables,

$$s_1 = s_g = 5.619 \text{ kJ/kg K and } t_{s_1} = 311°C$$

At 10 bar and 311°C the steam is superheated, hence interpolating

$$s_2 = 7.124 + \left(\frac{311 - 300}{350 - 300}\right)(7.301 - 7.124) \quad \text{or} \quad s_2 = 7.163 \text{ kJ/kg K}.$$

(*i*) **Heat supplied :**

Now, heat supplied, Q = shaded area = $T(s_2 - s_1)$

$$= 584(7.163 - 5.619) = \textbf{901.7 kJ/kg.} \textbf{(Ans.)}$$

(*ii*) **Work done :**

To find work done, applying non-flow energy equation,

$$Q = (u_2 - u_1) + W$$

or $W = Q - (u_2 - u_1)$

From steam tables at 100 bar, dry saturated,

$$u_1 = u_g = 2545 \text{ kJ/kg}$$

At 10 bar 311°C, interpolating,

$$u_2 = 2794 + \left(\frac{311 - 300}{350 - 300}\right)(2875 - 2794)$$

i.e., $u_2 = 2811.8 \text{ kJ/kg}$

Then, $W = Q - (u_2 - u_1)$

$$= 901.7 - (2811.8 - 2545) = 634.9 \text{ kJ/kg}$$

Hence, **work done by the steam = 634.9 kJ/kg.   (Ans.)**

4. Reversible Adiabatic or Isentropic Process. Fig. 2.63 (*a*), (*b*) and (*c*) shows the isentropic process on *p-v*, *T-s* and *h-s* diagrams respectively.
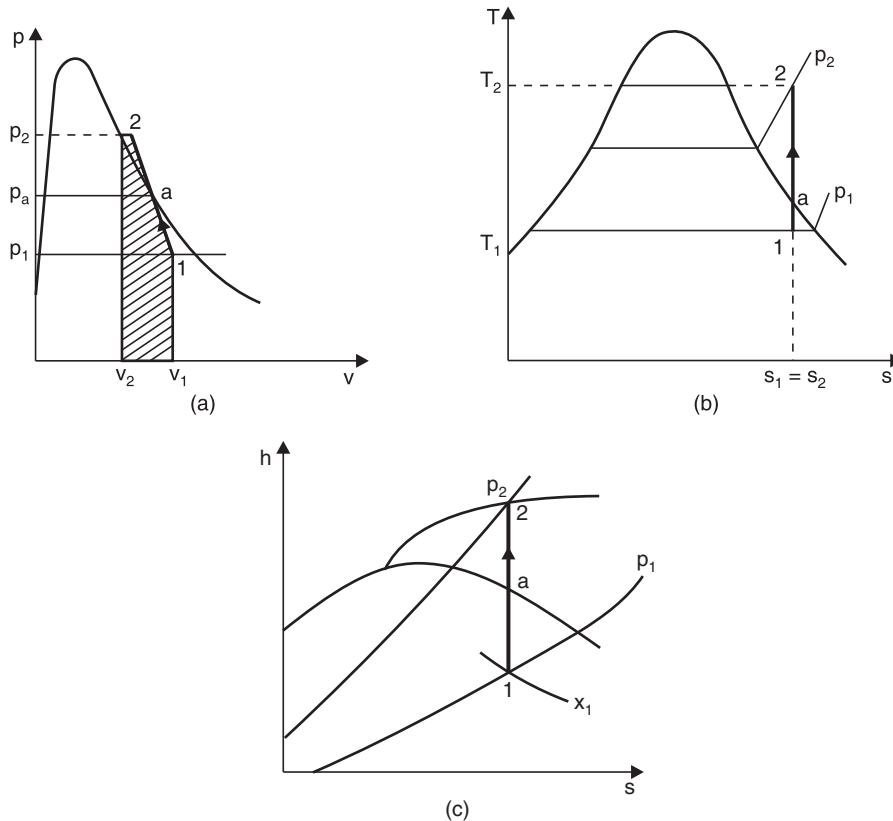
Fig. 2.63. Reversible adiabatic or isentropic process.

Let us consider that the process is non-flow reversible adiabatic. Now applying first law energy equation, we have

$$Q = \Delta u + \int_1^2 p \cdot dv = (u_2 - u_1) + W$$

As for adiabatic process, $Q = 0$

∴ $W = (u_1 - u_2)$ ...(2.77)

In case the process is steady flow reversible adiabatic, then first law energy equation can be written as

$$u_1 + p_1 v_1 + Q = u_2 + p_2 v_2 + W$$

where v_1 and v_2 are the specific volumes of steam before and after executing the process.

∴ $h_1 + 0 = h_2 + W$ ($\because \quad Q = 0$)

∴ $W = (h_1 - h_2)$ ...(2.78)

☞**Example 2.54.** *1 kg of steam at 120 bar and 400°C expands reversibly in a perfectly thermally insulated cylinder behind a piston until the pressure is 38 bar and the steam is then dry saturated. Calculate the work done by the steam.*

Solution. Mass of steam, $m = 1$ kg

Initial pressure of steam, $p_1 = 120$ bar $= 120 \times 10^5$ N/m^2

Initial temperature of steam, $t_1 = 400°C.$

Final pressure of steam, $p_2 = 38$ bar

From superheat tables, at *120 bar and 400°C*

$$h_1 = 3051.3 \text{ kJ/kg and } v_1 = 0.02108 \text{ m}^3\text{/kg}$$

Now, using the equation :

$$u = h - pv$$

∴ $u_1 = 3051.3 - \dfrac{120 \times 10^5 \times 0.02108}{10^3} = 2798.34$ kJ/kg

Also, $u_1 = u_g$ at 38 bar $= 2602$ kJ/kg.

Since the cylinder is perfectly thermally insulated then no heat flows to or from the steam during the expansion, the process therefore is adiabatic.

∴   **Work done by the steam,** $W = u_1 - u_2$

$$= 2798.34 - 2602 = \textbf{196.34 kJ/kg.}\quad \textbf{(Ans.)}$$

The process is shown on *p-v* diagram in Fig. 2.64, the shaded area representing the work done.
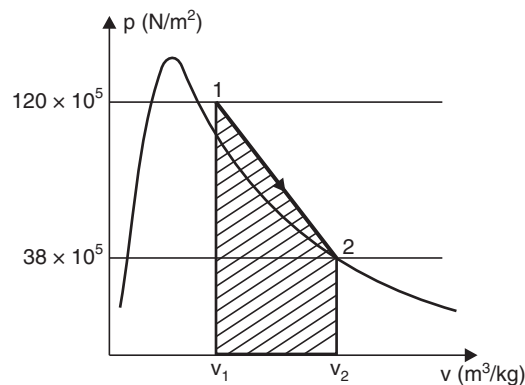
Fig. 2.64

5. Polytropic process. In this process, the steam follows the law pv^n = constant. This process on p-v, T-s and h-s diagrams is shown in Fig. 2.65 (a), (b) and (c).
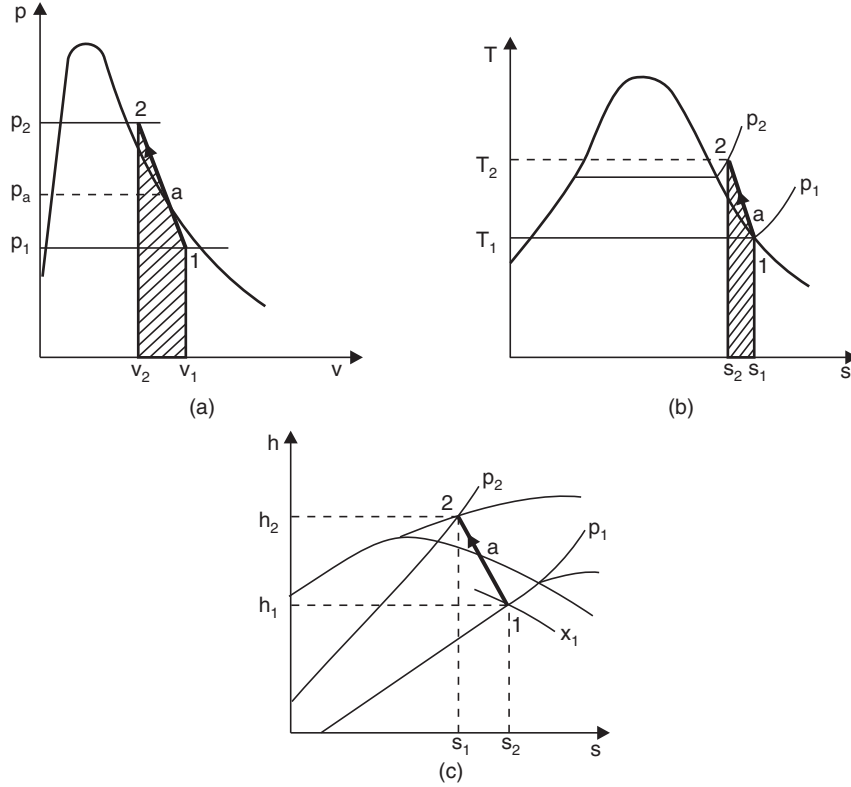
Fig. 2.65. Polytropic process.

The work done during this process is given by

$$W = \left(\frac{p_1 v_1 - p_2 v_2}{n-1} \right) \text{Nm/kg}$$

Applying the first law energy equation to non-flow process, we have

$$Q = \Delta u + W$$

$$= (u_2 - u_1) + \left(\frac{p_1 v_1 - p_2 v_2}{n-1} \right)$$

$$= (h_2 - p_2 v_2) - (h_1 - p_1 v_1) + \left(\frac{p_1 v_1 - p_2 v_2}{n-1} \right)$$

$$= (h_2 - h_1) + (p_1 v_1 - p_2 v_2) \left(1 + \frac{1}{n-1} \right)$$

$$= (h_2 - h_1) + \frac{n}{n-1} (p_1 v_1 - p_2 v_2) \qquad \qquad ...(2.79)$$

In adiabatic process $Q = 0$ if $\Delta s \neq 0$ then the process behaves like *adiabatic process* and *not isentropic*. Such a process with steam will be a particular case of the law pv^n = constant. The index n in this case will be that particular index which will satisfy the condition :

$$Q = 0$$

$$\therefore \qquad\qquad 0 = \Delta u + W$$

$$\therefore \qquad\qquad W = -\Delta u = -(u_2 - u_1) = (u_1 - u_2)$$

i.e., $\qquad\qquad W = (u_1 - u_2)$ ...(2.80)

Adiabatic process (not reversible) is also a polytropic process with an index n. The appropriate value of n for adiabatic compression of steam is

$$n = 1.13 \text{ for wet steam}$$

$$n = 1.3 \text{ for superheated steam}$$

When the initial condition and end condition are *both in wet region* then $p_1 v_1{}^n = p_2 v_2{}^n$ reduces to :

$$p_1 (x_1 v_{g_1})^n = p_2 (x_2 v_{g_2})^n$$

As p_1, x_1, n and p_2 are specified the value of x_2 can be calculated.

When the end condition is *superheated*, then

$$p_1 (x_1 v_{g_1})^n = p_2 (v_{\text{sup}_2})^n.$$

Solving for v_2, then using

$$\frac{v_2}{v_{g_2}} = \frac{T_{\text{sup}}}{T_{s_2}}$$

T_{sup} can be calculated. Knowing T_{s_2} and T_{sup} all properties at the end condition can be calculated.

☞**Example 2.55.** *In a steam engine the steam at the beginning of the expansion process is at 7 bar, dryness fraction 0.98 and expansion follows the law $pv^{1.1}$ = constant, down to a pressure of 0.34 bar. Calculate per kg of steam :*

(i) The work done during expansion ;

(ii) The heat flow to or from the cylinder walls during the expansion.

Solution. Initial pressure of steam, p_1 = 7 bar = 7×10^5 N/m²

Dryness fraction, $\qquad x_1 = 0.98$

Law of expansion, $\qquad pv^{1.1}$ = constant

Final pressure of steam, $\ p_2$ = 0.34 bar = 0.34×10^5 N/m².

At 7 bar : $\qquad\qquad v_g = 0.273$ m³/kg

$\therefore \qquad\qquad v_1 = x_1 v_g = 0.98 \times 0.273 = 0.267$ m³/kg

Also, $\qquad\qquad p_1 v_1{}^n = p_2 v_2{}^n$

i.e., $\qquad\qquad \dfrac{v_2}{v_1} = \left(\dfrac{p_1}{p_2}\right)^{1/n}$

$\therefore \qquad\qquad \dfrac{u_2}{0.267} = \left(\dfrac{7}{0.34}\right)^{\frac{1}{1.1}}$ or $\quad v_2 = 0.267 \left(\dfrac{7}{0.34}\right)^{\frac{1}{1.1}} = 4.174$ m³/kg.

(i) **Work done by the steam during the process :**

$$W = \frac{p_1 v_1 - p_2 v_2}{n-1} = \frac{7 \times 10^5 \times 0.267 - 0.34 \times 10^5 \times 4.174}{(1.1 - 1)}$$
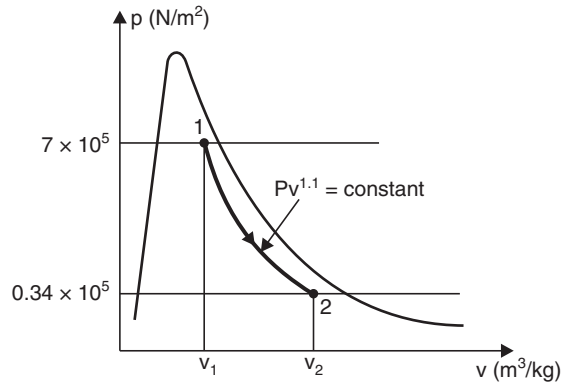
Fig. 2.66

$$= \frac{10^5}{0.1} (1.869 - 1.419) = 10^5 \times 4.5 \text{ Nm/kg}$$

i.e., **Work done** $= \dfrac{10^5 \times 4.5}{10^3} = $ **450 kJ/kg. (Ans.)**

At 0.34 bar : $v_g = 4.65$ m³/kg, therefore, steam is *wet* at state 2 (since $v_2 < v_g$).

Now, $v_2 = x_2 v_g$, where $x_2 = $ dryness fraction at pressure p_2 (0.34 bar)

$$4.174 = x_2 \times 4.65 \qquad \text{or} \qquad x_2 = \frac{4.174}{4.65} = 0.897$$

The expansion is shown on a *p-v* diagram in Fig. 2.66, the area under 1-2 represents the work done per kg of steam.

(*ii*) **Heat transferred :**

Internal energy of steam at initial state 1 per kg,

$$u_1 = (1 - x_1) u_f + x_1 u_g = (1 - 0.98) \, 696 + 0.98 \times 2573 = 2535.46 \text{ kJ/kg}$$

Internal energy of steam at final state 2 per kg,

$$u_2 = (1 - x_2) \, u_f + x_2 u_g$$
$$= (1 - 0.897) \, 302 + 0.897 \times 2472 = 2248.49 \text{ kJ/kg}$$

Using the non-flow energy equation,

$$Q = (u_2 - u_1) + W$$
$$= (2248.49 - 2535.46) + 450 = 163.03 \text{ kJ/kg}$$

i.e., **Heat supplied** $= $ **163.03 kJ/kg. (Ans.)**

Example 2.56. *Steam enters a steam turbine at a pressure of 15 bar and 350°C with a velocity of 60 m/s. The steam leaves the turbine at 1.2 bar and with a velocity of 180 m/s. Assuming the process to be reversible adiabatic, determine the work done per kg of steam flow through the turbine.*

Neglect the change in potential energy.

Solution. Initial pressure of steam, $p_1 = 15$ bar

Initial temperature of steam, $t_1 = t_{sup} = 350$°C

Initial velocity of steam, $C_1 = 60$ m/s

Final pressure, $p_2 = 1.2$ bar

Final velocity, $C_2 = 180$ m/s

Process of expansion : *Reversible adiabatic*
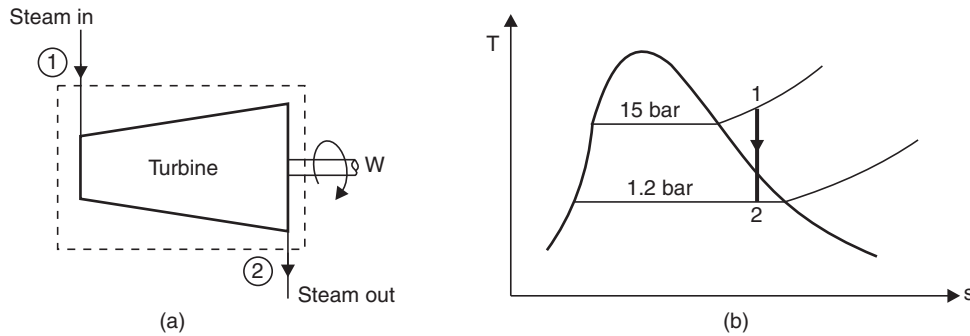
Fig. 2.67

As the process is reversible adiabatic, it will be represented by a vertical line on T-s diagram by 1-2 as it is also a constant entropy process.

The condition at point '2' can be calculated by equating the entropy at point '1' and point '2',

i.e., $s_1 = s_2$ .....per kg of steam

$$7.102 = s_{f_2} + x_2(s_{g_2} - s_{f_2})$$

$$= 1.3609 + x_2(7.2984 - 1.3609)$$

∴ $$x_2 = \frac{7.102 - 1.3609}{7.2984 - 1.3609} = 0.967$$

$$h_2 = h_{f_2} + x_2 h_{f_{g_2}} = 439.4 + 0.967 \times 2244.1 = 2609.44 \text{ kJ/kg}$$

$$h_1 \text{ (at 15 bar and 350°C)} = 3147.5 \text{ kJ/kg}$$

Applying the first law energy equation for steady flow process,

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} + W$$

i.e., $$W = (h_1 - h_2) + \left(\frac{C_1^2 - C_2^2}{2}\right)$$

$$= 3147.5 - 2609.44 + \left(\frac{60^2 - 180^2}{2 \times 10^3}\right)$$

$$= 3147.5 - 2609.44 - 14.4 = 523.66 \text{ kJ/kg.}$$

Hence **work done per kg of steam = 523.66 kJ/kg.   (Ans.)**

Example 2.57. *Steam at 10 bar and 200 °C enters a convergent divergent nozzle with a velocity of 60 m/s and leaves at 1.5 bar and with a velocity of 650 m/s. Assuming that there is no heat loss, determine the quality of the steam leaving the nozzle.*

Solution. Initial pressure of steam, p_1 = 10 bar

Initial temperature of steam, $t_1 = t_{sup} = 200°C$

Initial velocity, $C_1 = 60$ m/s

Final velocity, $C_2 = 650$ m/s

Final pressure, $p_2 = 1.5$ bar

Heat loss   = nil

Quality of steam at the outlet :

It is a steady-state non-work developing system. Applying the steady flow energy equation to the process, we get

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} \qquad\qquad (\because \quad Q = 0,\ W = 0)$$

$$\therefore \qquad\qquad h_2 = h_1 + \left(\frac{C_1^2 - C_2^2}{2} \right)$$

At 10 bar, 250°C : $\qquad h_1 = 2827.9$ kJ/kg (from steam tables)

$$\therefore \qquad\qquad h_2 = 2827.9 + \left[\frac{60^2 - 650^2}{2 \times 10^3} \right] = 2618.45 \text{ kJ/kg}$$
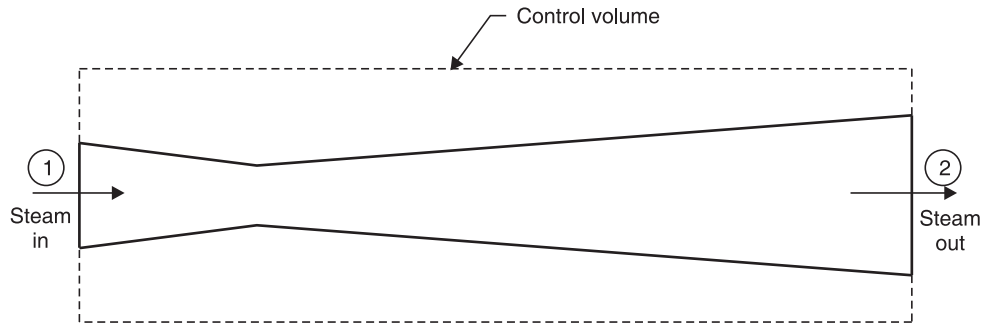
Fig. 2.68

As the enthalpy and pressure of steam at the exit of the nozzle are known, we can find out quality of steam,

$$h_{g_2} \text{ (at 1.5 bar)} = 2693.4 \text{ kJ/kg}$$

As $h_2 < h_{g_2}$, the steam is *wet*.

The enthalpy of wet steam is given by

$$h_2 = h_{f_2} + x_2\, h_{f_{g_2}}$$

$$2618.45 = 467.1 + x_2 \times 2226.2$$

$$\therefore \qquad\qquad x_2 = \frac{2618.45 - 467.1}{2226.2} = 0.966.$$

Hence **the condition of steam leaving the nozzle is 96.6% dry. (Ans.)**

6. Throttling. A flow of fluid is said to be throttled when there is some *restriction to the flow*, when the velocities before and after the restriction are either equal or negligibly small, and when there is a *negligible heat loss to the surroundings*.

The restriction to the flow can be :

 (*i*) partly open valve

 (*ii*) an orifice or

 (*iii*) any other sudden reduction in the cross-section of the flow.

An example of throttling is shown in Fig. 2.69. It is represented on *T-s* and *h-s* diagrams as shown in Figs. 2.70 and 2.71 respectively. The fluid (say steam) flowing steadily along a well-lagged pipe, passes through an orifice at section *X*. Since the pipe is well-lagged it can be assumed that no heat flows to or from the fluid.
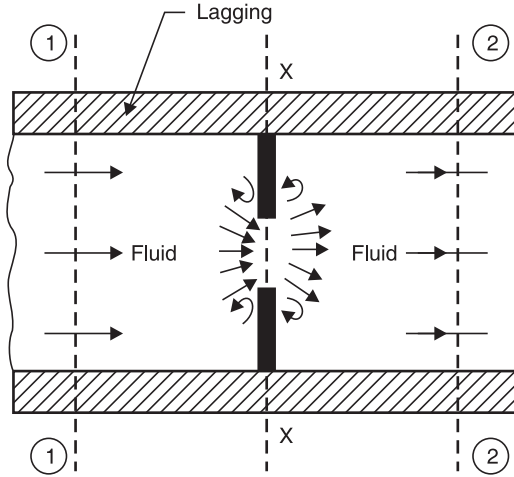
Fig. 2.69. Throttling.
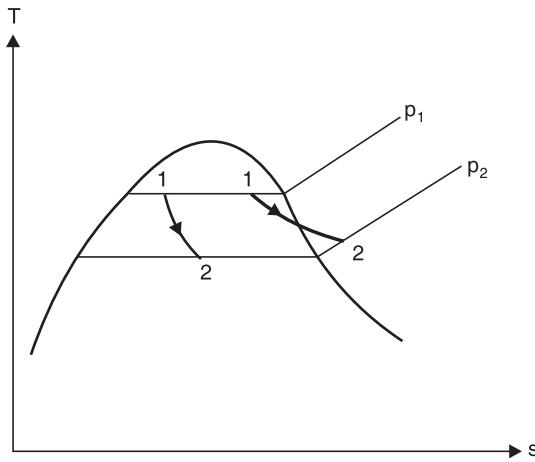
Fig. 2.70. *T-s* diagram.
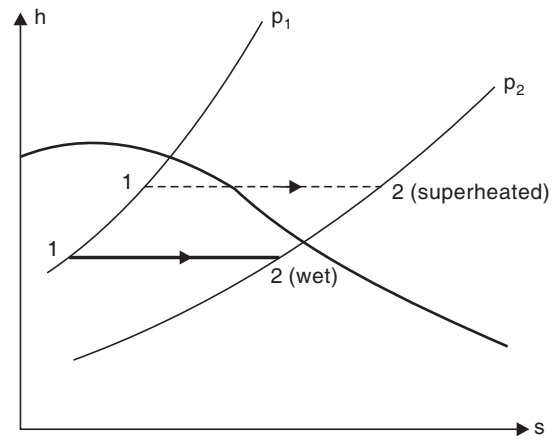
Fig. 2.71. *h-s* diagram.

Applying flow equation between any two sections of the flow, we have

$$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W$$

Now since $Q = 0$, and $W = 0$, then

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

When the velocities C_1 and C_2 are small, or when C_1 is very nearly equal to C_2, then the K.E. terms may be neglected.

Then $\qquad\qquad\qquad\qquad\qquad\qquad h_1 = h_2$ $\qquad\qquad\qquad\qquad\qquad\qquad$ ...(2.81)

i.e., For a throttling process :

$\qquad\qquad\qquad$ *Initial enthalpy = Final enthalpy.*

$\qquad$ *The process is adiabatic but highly irreversible because of the eddying of the fluid around the orifice at X.* Between sections 1 and *X* the enthalpy drops and K.E. increases as the fluid accelerates through the orifice. Between sections *X* and 2 the *enthalpy increases as K.E. is destroyed by fluid eddies.*

$\qquad$ During throttling *pressure always falls.*

$\qquad$ The throttling process is *used* for the following *purposes* :

$\qquad$ 1. To *determine the dryness fraction of steam.*

$\qquad$ 2. To *control the speed of the engine and turbine.*

$\qquad$ 3. To *reduce* the pressure and temperature of the liquid refrigerant from the condenser condition to evaporator condition in a *refrigeration system.*

$\qquad$ ☞**Example 2.58.** *Steam at 18 bar is throttled to 1 bar and the temperature after throttling is found to be 150°C. Calculate the initial dryness fraction of the steam.*

$\qquad$ **Solution.** Pressure of steam before throttling, $p_1 = 18$ bar

$\qquad$ Pressure of steam after throttling = 1 bar

$\qquad$ Temperature after throttling = 150°C

$\qquad$ **Initial dryness fraction, x_1 :**

$\qquad$ From superheat tables at **1 bar and 150°C,** we have

$\qquad\qquad\qquad\qquad\qquad\qquad h_2 = 2776.4$ kJ/kg

$\qquad$ Then for throttling $\qquad\qquad h_1 = h_2 = 2776.4$

$\qquad$ But $\qquad\qquad\qquad\qquad\qquad h_1 = h_{f_1} + x_1 h_{f_{g_1}}$

$\qquad$ **At 18 bar :** $\qquad\qquad\qquad h_f = 884.6$ kJ/kg, $\quad h_{f_g} = 1910.3$ kJ/kg

$\qquad$ ∴ $\qquad\qquad\qquad\qquad\qquad 2776.4 = 884.6 + x_1 \times 1910.3$

or $\qquad\qquad\qquad\qquad\qquad\qquad x_1 = \dfrac{2776.4 - 884.6}{1910.3} = 0.99$

i.e., $\qquad$ **Initial, dryness fraction = 0.99.   (Ans.)**

$\qquad$ The process is shown on a *p-v* diagram in Fig. 2.72. States 1 and 2 are fixed, but the intermediate states are indeterminate ; the process must be drawn dotted, as shown. No work is done during the process, and the area under the line 1-2 is **not equal to work done.**
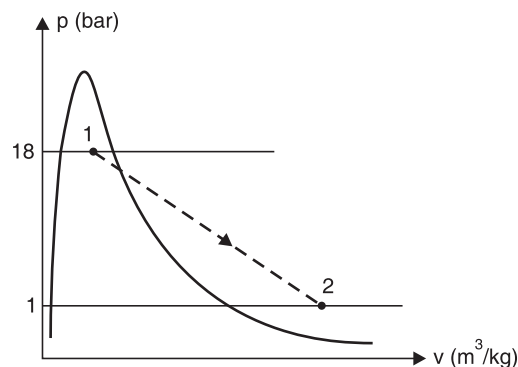
Fig. 2.72

Example 2.59. *Steam at 10 bar and 0.9 dryness fraction is throttled to a pressure of 2 bar. Determine the exit condition of steam using* **Mollier chart.**

Solution. Refer Fig. 2.73.
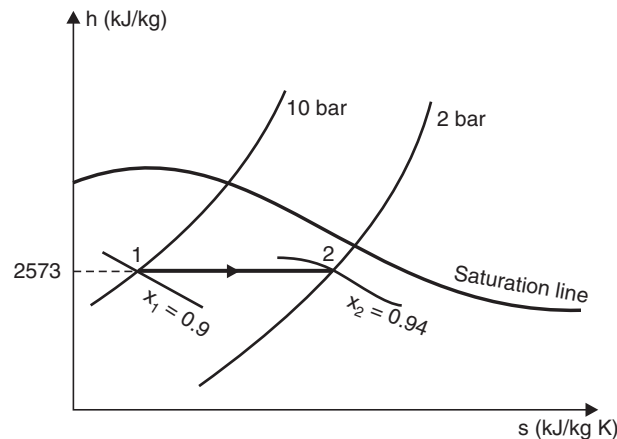
Fig. 2.73

Locate the point '1' at an intersection of the 10 bar pressure line and 0.9 dryness fraction line.

Throttling is a constant enthalpy line so draw a line parallel to *X*-axis till it cuts the 2 bar line and locate the point 2. The dryness fraction of steam at point 2 is 0.94.

(The total enthalpy before and after throttling = 2573 kJ/kg)

Hence **exit condition of steam = 0.94.   (Ans.)**

Note. This process occurs during the control of flow of steam supplied to a turbine to take care of the varying load.

☞**Example 2.60.** *Steam initially at a pressure of 15 bar and 0.95 dryness expands isentropically to 7.5 bar and is then throttled until it is just dry. Determine per kg of steam* :

 (*i*) *Change in entropy* ;

(*ii*) *Change in enthalpy* ;

(*iii*) *Change in internal energy.*

Using : (*a*) *Steam tables*

(*b*) *Mollier chart.*

Is the entire process reversible ? *Justify your statement.*

Solution. (*a*) **Using steam tables**

Condition 1 : 15 bar, 0.95 dryness

$$h_{f_1} = 844.7 \text{ kJ/kg} \; ; \; t_{s_1} = 198.3°\text{C}, \; s_{f_1} = 2.3145 \text{ kJ/kg K},$$

$$s_{g_1} = 6.4406 \text{ kJ/kg K}, \; v_{g_1} = 0.132 \text{ m}^3\text{/kg}$$

$$h_1 = h_{f_1} + x_1 h_{f_{g1}} = 844.7 + 0.95 \times 1945.2 = 2692.64 \text{ kJ/kg}$$

$$s_1 = s_{f_1} + x_1 (s_{g_1} - s_{f_1}) = 2.3145 + 0.95(6.4406 - 2.3145) = 6.2343 \text{ kJ/kg K}.$$

Condition 2 : 7.5 bar

$$h_{f_2} = 709.3 \text{ kJ/kg}, \ t_{s_2} = 167.7°C, \ h_{f_{g_2}} = 2055.55 \text{ kJ/kg}, \ s_{f_2} = 2.0195 \text{ kJ/kg K}$$

$$s_{g_2} = 6.6816 \text{ kJ/kg K}, \ v_{g_2} = 0.255 \text{ m}^3/\text{kg}.$$

Considering **isentropic expansion 1-2 :**

(*i*) **Change in entropy** = 0

i.e.,   Entropy at 1 = entropy at 2

$$\therefore \qquad\qquad\qquad\qquad s_1 = s_2$$

$$6.2343 = s_{f_2} + x_2 \, (s_{g_2} - s_{f_2})$$

$$= 2.0195 + x_2(6.6816 - 2.0195)$$

$$\therefore \qquad\qquad\qquad x_2 = \frac{6.2343 - 2.0195}{6.6816 - 2.0195} = 0.9$$

Now, enthalpy at point 2,

$$h_2 = h_{f_2} + x_2 \, h_{f_{g_2}} = 709.3 + 0.9 \times 2055.55 = 2559.29 \text{ kJ/kg}.$$

(*ii*) **Change in enthalpy** = $h_2 - h_1$

$$= 2559.29 - 2692.64 = - \textbf{133.35 kJ/kg.} \quad \textbf{(Ans.)}$$

(–ve sign indicates *decrease*).

(*iii*) **Change in internal energy :**

Internal energy at point 1,

$$u_1 = h_1 - p_1 x_1 v_{g_1}$$

$$= 2692.64 - 15 \times 10^5 \times 0.95 \times 0.132 \times 10^{-3} = 2504.54 \text{ kJ/kg}$$

Internal energy at point 2,

$$u_2 = h_2 - p_2 x_2 v_{g_2}$$

$$= 2559.29 - 7.5 \times 10^5 \times 0.9 \times 0.255 \times 10^{-3} = 2387.16 \text{ kJ/kg}$$

$\therefore$ Change in internal energy

$$= u_2 - u_1 = 2387.16 - 2504.54 = - 117.38 \text{ kJ/kg}$$

(–ve sign indicates *decrease*)

Considering the **throttling expansion 2-3 :**

Entropy at point 2,

$$s_2 = (s_1) = 6.2343 \text{ kJ/kg K}$$

Entropy at point 3,

$$s_3 = s_{f_3} + x_3 \, (s_{g_3} - s_{f_3})$$

The pressure at point 3 can be read from *h-s* chart (p_3 = 0.06 bar) and the corresponding values of s_{f_3} and $s_{f_{g_3}}$ from steam tables.

Condition 3. At 0.06 bar, $x_3 = 1$. From steam tables,

$$s_{f_3} = 0.521 \text{ kJ/kg K}, \ s_{g_3} = 8.330 \text{ kJ/kg K}$$

$\therefore \qquad\qquad\qquad s_3 = 0.521 + 1 \times (8.330 - 0.521) = 8.330 \text{ kJ/kg K}$

Change in entropy $\qquad = s_3 - s_2$

$$= 8.330 - 6.2343 = 2.0957 \text{ kJ/kg K}$$

Change in enthalpy $= 0$

i.e., $\qquad\qquad\qquad\qquad h_2 = h_3$

Change in internal energy $= 0$

i.e., $\qquad\qquad\qquad\qquad u_3 = u_2$

Combining the results obtained from isentropic and throttling expansion, we get during the entire process.

(*i*) **Change in entropy = 2.0957 kJ/kg K (increase).   (Ans.)**

(*ii*) **Change in enthalpy = 133.35 kJ/kg K (decrease).   (Ans.)**

(*iii*) **Change in internal energy = 117.38 kJ/kg (decrease).   (Ans.)**

Only the expansion of steam from point 1 to 2 (*i.e., isentropic expansion*) is *reversible* because of unresisted flow whereas the expansion from point 2 to point 3 (*i.e., throttling expansion*) is *irreversible* because of frictional resistance to flow. *Increase of entropy also shows that expansion from point 2 to point 3 is irreversible.*

(*b*) **Using Mollier chart.**

Refer Fig. 2.74.

Locate point 1 at an intersection of 15 bar pressure line and 0.95 dryness fraction line.

Draw vertical line from point 1 intersecting 7.5 bar pressure line at point 2. Line 1-2 represents *isentropic expansion.*

From point 2 draw a horizontal line intersecting at the saturation line at point 3. Line 2-3 then represents *throttling expansion.*

From *Mollier chart* :

$$h_1 = 2692 \text{ kJ/kg}, \qquad\qquad h_2 = 2560 \text{ kJ/kg}$$
$$s_1 = s_2 = 6.23 \text{ kJ/kg}, \qquad s_3 = 8.3 \text{ kJ/kg K}$$

$\therefore$ (*i*) **Change in entropy** $\quad = s_3 - (s_1 \text{ or } s_2)$

$\qquad\qquad\qquad\qquad\qquad = 8.3 - 6.23 = $ **2.07 kJ/kg K (increase).   (Ans.)**

(*ii*) **Change in enthalpy** $\qquad = h_2 \text{ (or } h_3) - h_1$

$\qquad\qquad\qquad\qquad\qquad = 2560 - 2692 = -132 \text{ kJ/kg}$

$\qquad\qquad\qquad\qquad\qquad = $ **132 kJ/kg (decrease).   (Ans.)**

(*iii*) **Change in internal energy**
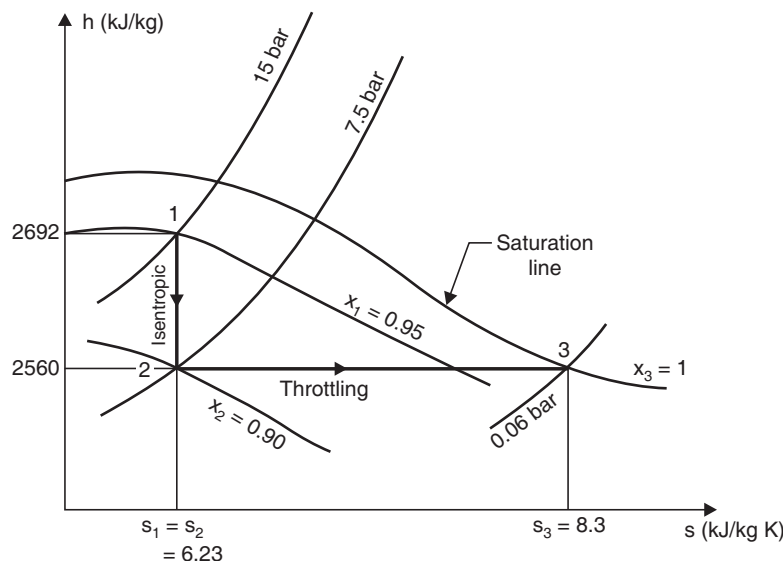
Fig. 2.74

2.15. UNSTEADY FLOW PROCESSES

In engineering practice, the variable flow process applications are as common as the steady flow process. The *rate of energy and mass transfer into and out of the control volume are not same in the case of an unstable* (or variable or transient) *flow process.*

Following two cases only will be discussed :

1. Filling a tank.

2. Emptying a tank or tank discharge.

1. Filling a tank :

Let m_1 = initial mass of fluid ;

p_1 = initial pressure ;

v_1 = initial specific volume ;

T_1 = initial temperature ;

u_1 = initial specific internal energy ;

and m_2 = final mass of fluid ;

p_2 = final pressure ;

v_2 = final specific volume ;

T_2 = final temperature ;

u_2 = final specific internal energy ;

Also, let p' = entering fluid pressure ;

v' = entering fluid specific volume ;

T' = entering fluid temperature ;

C' = entering fluid velocity ;

u' = entering specific internal energy of fluid ;

h' = entering specific enthalpy of fluid.
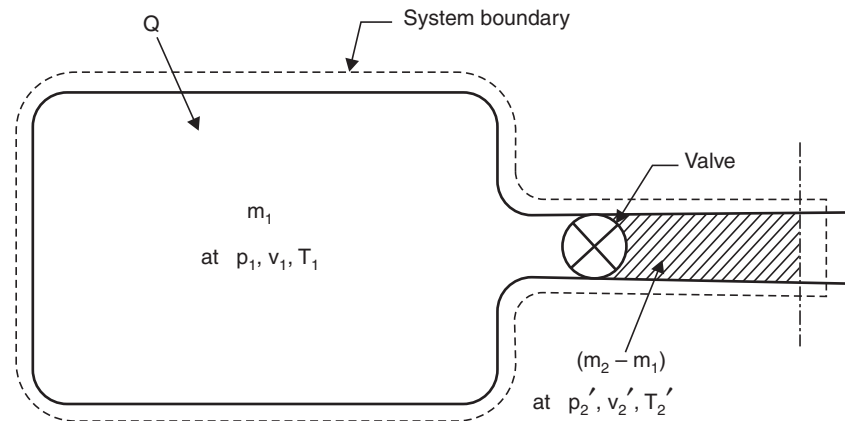
Fig. 2.75

The quantity of fluid entering

$$= m_2 - m_1$$

Energy of entering fluid

$$= (m_2 - m_1) \left(u' + p'v' + \frac{C'^2}{2} \right) \qquad \qquad ...(2.82)$$

$$= (m_2 - m_1)\left(h' + \frac{C'^2}{2}\right) \qquad \qquad ...(2.83)$$

If Q = heat transferred into the control volume, we have

$$(m_2 - m_1)\left(h' + \frac{C'^2}{2}\right) + Q = m_2\,u_2 - m_1\,u_1 \qquad \qquad ...(2.84)$$

When the tank is fully insulated and thus no heat transfer takes place

$$Q = 0$$

and

$$(m_2 - m_1)\left(h' + \frac{C'^2}{2}\right) = m_2 u_2 - m_1\,u_1 \qquad \qquad ...(2.85)$$

Also, if the tank is empty initially and fully insulated for heat transfer,

$$m_1 = 0$$

Thus

$$h' + \frac{C'^2}{2} = u_2 \qquad \qquad ...(2.86)$$

Also, if kinetic energy in the pipe line is neglected

$$h' = u_2 \qquad \qquad ...(2.87)$$

2. **Emptying a tank :**

Analogous to the filling of the tank, the equation can be written as

$$(m_1 - m_2)\left(h' + \frac{C'^2}{2}\right) - Q = m_1 u_1 - m_2 u_2 \qquad \qquad ...(2.88)$$

where h' = specific enthalpy of leaving fluid, and

C' = velocity of leaving fluid.

For fully emptying the tank and no heat transfer and negligible exit velocity

$$h' = u_1 \qquad \qquad ...(2.89)$$

Example 2.61. *An air receiver of volume 5.5 m³ contains air at 16 bar and 42°C. A valve is opened and some air is allowed to blow out to atmosphere. The pressure of the air in the receiver drops rapidly to 12 bar when the valve is then closed.*

Calculate the mass of air which has left the receiver.

Solution. Initial volume of air, V_1 = 5.5 m³

Initial pressure of air, p_1 = 16 bar

Initial temperature of air, T_1 = 42 + 273 = 315 K

Final volume of air, $V_2 = V_1$ = 5.5 m³

Final pressure of air, p_2 = 12 bar

Mass of air which left the receiver :

Mass of air in the *initial* condition,

$$m_1 = \frac{p_1 V_1}{RT_1} = \frac{16 \times 10^5 \times 5.5}{(0.287 \times 10^3) \times 315} = 97.34 \text{ kg.}$$

Assuming that the mass in the receiver undergoes a reversible adiabatic process, then

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{T_2}{315} = \left(\frac{12}{16}\right)^{\frac{1.4-1}{1.4}} = \left(\frac{12}{16}\right)^{0.286} \quad \text{or} \quad T_2 = 315 \times \left(\frac{12}{16}\right)^{0.286} = 290 \text{ K}$$

Now mass of air in the receiver in *final* condition,

$$m_2 = \frac{p_2 V_2}{R T_2} = \frac{12 \times 10^5 \times 5.5}{(0.287 \times 10^3) \times 290} = 79.3 \text{ kg.}$$

∴ **Mass of air which left the receiver,**

$$m = m_1 - m_2 = 97.34 - 79.3 = \textbf{18.04 kg.} \quad \textbf{(Ans.)}$$

Example 2.62. *A 1.6 m³ tank is filled with air at a pressure of 5 bar and a temperature of 100°C. The air is then let off to the atmosphere through a valve. Assuming no heat transfer, determine the work obtainable by utilising the kinetic energy of the discharge air to run a frictionless turbine.*

Take : Atmospheric pressure = 1 bar ;

c_p *for air = 1 kJ/kg K ;*

c_v *for air = 0.711 kJ/kg K.*

Solution. Initial volume of air, $V_1 = 1.6 \text{ m}^3$

Initial pressure of air, $p_1 = 5 \text{ bar} = 5 \times 10^5 \text{ N/m}^2$

Initial temperature of air, $T_1 = 100 + 273 = 373 \text{ K}$

Final pressure of air, $p_2 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$

Now, initial quantity of air in the tank before discharge,

$$m_1 = \frac{p_1 V_1}{R T_1} = \frac{5 \times 10^5 \times 1.6}{(0.287 \times 10^3) \times 373} = 7.47 \text{ kg.}$$

Assuming that system undergoes a reversible adiabatic expansion

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

where T_2 is the final temperature of air in the tank.

∴ $$\frac{T_2}{373} = \left(\frac{1}{5}\right)^{\frac{1.4-1}{1.4}} = 0.631$$

$$T_2 = 373 \times 0.631 = 235.4 \text{ K} \qquad\qquad (i.e., \text{ finally in the line})$$

The final quantity of air remaining in the tank is

$$m_2 = \frac{p_2 V_2}{R T_2} = \frac{1 \times 10^5 \times 1.6}{(0.287 \times 10^3) \times 235.4} = 2.368 \text{ kg.}$$

With $Q = 0$, kinetic energy is found from,

$$(m_1 - m_2)\left(h' + \frac{C'^2}{2}\right) = m_1 u_1 - m_2 u_2$$

or $$(m_1 - m_2) h' + (m_1 - m_2)\frac{C'^2}{2} = m_1 u_1 - m_2 u_2$$

∴   Kinetic energy,

$$(m_1 - m_2)\frac{C'^2}{2} = (m_1 u_1 - m_2 u_2) - (m_1 - m_2) h'$$

$$= m_1 c_v T_1 - m_2 c_v T_2 - (m_1 - m_2)\, c_p T_2$$
$$= 7.47 \times 0.771 \times 373 - 2.368 \times 0.711 \times 235.4 - (7.47 - 2.368) \times 1 \times 235.4$$
$$= 2148.24 - 396.33 - 1201 = \textbf{550.9 kJ.} \quad \textbf{(Ans.)}$$

Example 2.63. *A frictionless piston is free to move in a closed cylinder. Initially there is 0.035 m³ of oxygen at 4.5 bar, 60°C on one side of the piston and 0.07 m³ of methane at 4.5 bar and – 12°C on the other side. The cylinder walls and piston may be regarded as perfect thermal insulators but the oxygen may be heated electrically. Heating takes place so that the volume of oxygen doubles. Find* :

  (*i*) *Final state condition* ;

 (*ii*) *Work done by the piston* ;

(*iii*) *Heat transferred to oxygen.*

Treat both gases as perfect and take :

For oxygen c_p = *0.88 kJ/kg K, R = 0.24 kJ/kg K*

For methane c_p = *1.92 kJ/kg K, R = 0.496 kJ/kg K.*

Solution. For oxygen :

Initial volume, $V_1 = 0.035$ m³

Initial pressure, $p_1 = 4.5$ bar

Initial temperature, $T_1 = 60 + 273 = 333$ K

For methane :

Initial volume, $V_1 = 0.07$ m³

Final volume, $V_2 = 0.035$ m³

Initial pressure, $p_1 = 4.5$ bar

Initial temperature of methane,

                   $T_1 = -12 + 273 = 261$ K.

For Methane :

$$c_p = R\frac{\gamma}{\gamma - 1} \qquad \text{or} \qquad 1.92 = 0.496 \left(\frac{\gamma}{\gamma - 1} \right)$$

or $\dfrac{1.92}{0.496} = \dfrac{\gamma}{\gamma - 1}$ or $1.92\,(\gamma - 1) = 0.496\,\gamma$

∴ $\gamma = \dfrac{1.92}{(1.92 - 0.496)} = 1.348$ say 1.35

For Oxygen :

$$c_v = c_p - R = 0.88 - 0.24 = 0.64 \text{ kJ/kg K.}$$

(*i*) According to problem ; for *methane*

          pV^γ = constant holds good

∴ $p_1 V_1^\gamma = p_2 V_2^\gamma$

$$p_2 = p_1 \cdot \left(\frac{V_1}{V_2} \right)^\gamma = 4.5\,(2)^{1.35} = \textbf{11.47 bar.} \quad \textbf{(Ans.)}$$

Also, $\dfrac{p_1 V_1}{T_1} = \dfrac{p_2 V_2}{T_2}$

or $T_2 = \dfrac{p_2 V_2 T_1}{p_1 V_1} = \dfrac{11.47 \times 0.035 \times 261}{4.5 \times 0.07} = \textbf{332.6 K.} \quad \textbf{(Ans.)}$

$$\therefore \quad \text{Work done} = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} = \frac{4.5 \times 10^5 \times 0.07 - 11.47 \times 10^5 \times 0.035}{(1.35 - 1)} \text{ J}$$

$$= \frac{10^5 (4.5 \times 0.07 - 11.47 \times 0.035)}{0.35 \times 1000} = \text{ kJ}$$

$$= -24.7 \text{ kJ } (\textit{done on the methane})$$

(ii) **The piston will be in virtual equilibrium and hence zero work is effected by the piston. (Ans.)**

(iii) Work done by oxygen = work done on methane and expansion of oxygen is effected in the system

$$\therefore \quad W_{oxygen} = +24.7 \text{ kJ}$$

and $$Q = (U_2 - U_1) + W$$

$$\text{Amount of oxygen present} = \frac{p_1 V_1}{R T_1} = \frac{4.5 \times 10^5 \times 0.035}{0.24 \times 1000 \times 333} = 0.197 \text{ kg}$$

and $$T_2 = \frac{p_2 V_2}{p_1 V_1} \times T_1 = \frac{11.47 \times 0.07 \times 333}{4.5 \times 0.035} = \mathbf{1697.5 \text{ K}}. \quad \textbf{(Ans.)}$$

(As the piston is free, the final pressure of oxygen and methane will be **same**).

$$\therefore \quad Q = (U_2 - U_1) + W$$
$$= mc_v (T_2 - T_1) + W$$
$$= 0.197 \times 0.64 (1697.5 - 333) + 24.7 = \mathbf{196.7 \text{ kJ}}. \quad \textbf{(Ans.)}$$

HIGHLIGHTS

1. *Internal energy* is the heat energy stored in a gas. The internal energy of a perfect gas is a function of *temperature* only.

2. First law of thermodynamics states :
   — Heat and work are mutually convertible but since energy can neither be created nor destroyed, the total energy associated with an energy conversion remains constant.

   Or

   — No machine can produce energy without corresponding expenditure of energy, *i.e.* it is impossible to construct a perpetual motion machine of first kind.

   First law can be expressed as follows :
   $$Q = \Delta E + W$$
   $$Q = \Delta U + W \qquad \text{... if electric, magnetic, chemical energies are absent and changes in potential and kinetic energies are neglected.}$$

3. There can be no machine which would continuously supply mechanical work without some form of energy disappearing simultaneously. Such a fictitious machine is called a perpetual motion machine of the first kind, or in brief, PMM1. A PMM1 is thus impossible.

4. The energy of an isolated system is always constant.

5. In case of
   (i) **Reversible constant volume process** (v = constant)
   $$\Delta u = c_v (T_2 - T_1) \; ; \; W = 0 \; ; \; Q = c_v (T_2 - T_1)$$
   (ii) **Reversible constant pressure process** (p = constant)
   $$\Delta u = c_v (T_2 - T_1) \; ; \; W = p(v_2 - v_1) \; ; \; Q = c_p (T_2 - T_1)$$
   (iii) **Reversible temperature or isothermal process** (pv = constant)
   $$\Delta u = 0, \; W = p_1 V_1 \log_e r, \; Q = W$$
   where r = expansion or compression ratio.

(*iv*) **Reversible adiabatic process** (pv^γ = constant)

$$\pm \Delta u = \mp W = \frac{R(T_1 - T_2)}{\gamma - 1} ; Q = 0 ; \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

(*v*) **Polytropic reversible process** (pv^n = constant)

$$\Delta u = c_v (T_2 - T_1) ; W = \frac{R(T_1 - T_2)}{n - 1} ; Q = \Delta u + W ;$$

and $$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n - 1} = \left(\frac{p_2}{p_1}\right)^{\frac{n - 1}{n}} \text{ and } \left(\frac{\gamma - n}{n - 1}\right) \times W.$$

6. Steady flow equation can be expressed as follows :

$$u_1 + \frac{C_1^2}{2} Z_1 g + p_1 v_1 + Q = u_2 + \frac{C_2^2}{2} + Z_2 g + p_2 v_2 + W \qquad \text{...(i)}$$

or $$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W, \text{ neglecting } Z_1 \text{ and } Z_2 \qquad \text{...(ii)}$$

where Q = heat supplied per kg of fluid ; W = work done by 1 kg of fluid ;

C = velocity of fluid ; Z = height above datum ;

p = pressure of the fluid ; u = internal energy per kg of fluid ;

pv = energy required per kg of fluid.

This equation is applicable to any medium in any steady flow.

7. During adiabatic *throttling process* enthalpy remains constant. The slope of a constant enthalpy line on a *p-T* diagram is called Joule-Thomson co-efficient, μ.

8. In unsteady-flow processes, the rates at which mass and energy enter the control volume may not be the same as the rate of flow of mass and energy moving out of the control volume. The filling of a tank is an example of unsteady flow process.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. If all the variables of a stream are independent of time it is said to be in
    (*a*) steady flow   (*b*) unsteady flow
    (*c*) uniform flow   (*d*) closed flow
    (*e*) constant flow.

2. A control volume refers to
    (*a*) a fixed region in space   (*b*) a specified mass
    (*c*) an isolated system   (*d*) a reversible process only
    (*e*) a closed system.

3. Internal energy of a perfect gas depends on
    (*a*) temperature, specific heats and pressure   (*b*) temperature, specific heats and enthalpy
    (*c*) temperature, specific heats and entropy   (*d*) temperature only.

4. In reversible polytropic process
    (*a*) true heat transfer occurs   (*b*) the entropy remains constant
    (*c*) the enthalpy remains constant   (*d*) the internal energy remains constant
    (*e*) the temperature remains constant.

5. An isentropic process is always
    (*a*) irreversible and adiabatic   (*b*) reversible and isothermal
    (*c*) frictionless and irreversible   (*d*) reversible and adiabatic
    (*e*) none of the above.

6. The net work done per kg of gas in a polytropic process is equal to

(a) $p_1 v_1 \log_e \dfrac{v_2}{v_1}$

(b) $p_1 (v_1 - v_2)$

(c) $p_2 \left(v_2 - \dfrac{v_1}{v_2} \right)$

(d) $\dfrac{p_1 v_1 - p_2 v_2}{n - 1}$

(e) $\dfrac{p_2 v_1 - p_2 v_2}{n - 1}$.

7. Steady flow occurs when
   (a) conditions do not change with time at any point
   (b) conditions are the same at adjacent points at any instant
   (c) conditions change steadily with the time
   (d) $\left(\dfrac{\partial v}{\partial t} \right)$ is constant.

8. A reversible process requires that
   (a) there be no heat transfer
   (b) Newton's law of viscosity be satisfied
   (c) temperature of system and surroundings be equal
   (d) there be no viscous or coloumb friction in the system
   (e) heat transfer occurs from surroundings to system only.

9. The first law of thermodynamics for steady flow
   (a) accounts for all energy entering and leaving a control volume
   (b) is an energy balance for a specified mass of fluid
   (c) is an expression of the conservation of linear momentum
   (d) is primarily concerned with heat transfer.
   (e) is restricted in its application to perfect gases.

10. The characteristic equation of gases $pV = mRT$ holds good for
    (a) monoatomic gases
    (b) diatomic gas
    (c) real gases
    (d) ideal gases
    (e) mixture of gases.

11. A gas which obeys kinetic theory perfectly is known as
    (a) monoatomic gas
    (b) diatomic gas
    (c) real gas
    (d) pure gas
    (e) perfect gas.

12. Work done in a free expansion process is
    (a) zero
    (b) minimum
    (c) maximum
    (d) positive
    (e) negative.

13. Which of the following is not a property of the system?
    (a) Temperature
    (b) Pressure
    (c) Specific volume
    (d) Heat
    (e) None of the above.

14. In the polytropic process equation pv^n = constant, if $n = 0$, the process is termed as
    (a) constant volume
    (b) constant pressure
    (c) constant temperature
    (d) adiabatic
    (e) isothermal.

15. In the polytropic process equation pv^n = constant, if n is infinitely large, the process is termed as
    (*a*) constant volume   (*b*) constant pressure
    (*c*) constant temperature   (*d*) adiabatic
    (*e*) isothermal.

16. The processes or systems that do not involve heat are called
    (*a*) isothermal processes   (*b*) equilibrium processes
    (*c*) thermal processes   (*d*) steady processes
    (*e*) adiabatic processes.

17. In a reversible adiabatic process the ratio (T_1/T_2) is equal to

    (*a*) $\left(\dfrac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$ (*b*) $\left(\dfrac{v_1}{v_2}\right)^{\frac{\gamma-1}{\gamma}}$

    (*c*) $\left(v_1 v_2\right)^{\frac{\gamma-1}{2\gamma}}$ (*d*) $\left(\dfrac{v_2}{v_1}\right)^{\gamma}$.

18. In isothermal process
    (*a*) temperature increases gradually   (*b*) volume remains constant
    (*c*) pressure remains constant   (*d*) enthalpy change is maximum
    (*e*) change in internal energy is zero.

19. During throttling process
    (*a*) internal energy does not change   (*b*) pressure does not change
    (*c*) entropy does not change   (*d*) enthalpy does not change
    (*e*) volume change is negligible.

20. When a gas is to be stored, the type of compression that would be ideal is
    (*a*) isothermal   (*b*) adiabatic
    (*c*) polytropic   (*d*) constant volume
    (*e*) none of the above.

21. If a process can be stopped at any stage and reversed so that the system and surroundings are exactly restored to their initial states, it is known as
    (*a*) adiabatic process   (*b*) isothermal process
    (*c*) ideal process   (*d*) frictionless process
    (*e*) energyless process.

22. The state of a substance whose evaporation from its liquid state is complete, is known as
    (*a*) vapour   (*b*) perfect gas
    (*c*) air   (*d*) steam.

23. In SI units, the value of the universal gas constant is
    (*a*) 0.8314 J/mole/K   (*b*) 8.314 J/mole/K
    (*c*) 83.14 J/mole/K   (*d*) 831.4 J/mole/K
    (*e*) 8314 J/mole/K.

24. When a gas is heated at constant pressure, the heat supplied
    (*a*) increases the internal energy of the gas
    (*b*) increases the temperature of the gas
    (*c*) does some external work during expansion
    (*d*) both (*b*) and (*c*)
    (*e*) none of the above.

25. The gas constant (R) is equal to the
    (*a*) sum of two specific heats   (*b*) difference of two specific heats
    (*c*) product of two specific heats   (*d*) ratio of two specific heats.

26. The heat absorbed or rejected during a polytropic process is

(a) $\left(\dfrac{\gamma - n}{\gamma - 1}\right) \times$ work done

(b) $\left(\dfrac{\gamma - n}{\gamma - 1}\right)^2 \times$ work done

(c) $\left(\dfrac{\gamma - n}{\gamma - 1}\right)^{1/2} \times$ work done

(d) $\left(\dfrac{\gamma - n}{\gamma - 1}\right)^3 \times$ work done.

Answers

1. (a)	2. (a)	3. (d)	4. (a)	5. (d)
6. (d)	7. (a)	8. (d)	9. (a)	10. (c)
11. (e)	12. (a)	13. (d)	14. (b)	15. (a)
16. (e)	17. (a)	18. (e)	19. (d)	20. (a)
21. (c)	22. (b)	23. (e)	24. (d)	25. (b)
26. (a).				

<div style="text-align:center">

THEORETICAL QUESTIONS

</div>

1. Define 'internal energy' and prove that it is a property of a system.
2. Explain the First Law of Thermodynamics as referred to closed systems undergoing a cyclic change.
3. State the First Law of Thermodynamics and prove that for a non-flow process, it leads to the energy equation $Q = \Delta U + W$.
4. What is the mechanical equivalent of heat? Write down its value when heat is expressed in kJ and work is expressed in Nm.
5. What do you mean by "Perpetual motion machine of first kind-PMM 1" ?
6. Why only in a constant pressure non-flow process, the enthalpy change is equal to heat transfer ?
7. Prove that the rate of change of heat interchange per unit change of volume when gas is compressed or expanded is given by $\dfrac{\gamma - n}{\gamma - 1} \times \dfrac{pdv}{J}$.
8. Write down the general energy equation for steady flow system and simplify when applied for the following systems :
   (i) Centrifugal water pump
   (ii) Reciprocating air compressor
   (iii) Steam nozzle
   (iv) Steam turbine
   (v) Gas turbine.
9. Explain clearly the difference between a non-flow and a steady flow process.
10. For isothermal flow and non-flow steady processes, prove that

$$\int_1^2 pdv = -\int_1^2 v \cdot dp$$

Also state the assumptions made.

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UNSOLVED EXAMPLES

</div>

Closed Systems

1. In a cyclic process, heat transfers are + 14.7 kJ, – 25.2 kJ, – 3.56 kJ and + 31.5 kJ. What is the net work for this cyclic process? [**Ans.** 17.34 kJ]
2. A domestic refrigerator is loaded with food and the door closed. During a certain period the machine consumes 1 kWh of energy and the internal energy of the system drops by 5000 kJ. Find the net heat transferred in the system. [**Ans.** – 8.6 MJ]
3. 1.5 kg of liquid having a constant specific heat of 2.5 kJ/kg°C is stirred in a *well-insulated* chamber causing the temperature to rise by 15°C. Find :

    (*i*) Change in internal energy, and

    (*ii*) Work done for the process.   [**Ans.** (*i*) 56.25 kJ ; (*ii*) $W = -56.25$ kJ]

4. A system is composed of a stone having a mass of 10 kg and a bucket containing 100 kg of water. Initially the stone and water are at the same temperature, the stone then falls into the water. Determine ΔU, ΔKE, ΔPE, ΔQ and ΔW for the following cases :

    (*i*) At the instant the stone is about to enter the water.

    (*ii*) Just after the stone comes to rest in the bucket.

                [**Ans.** (*i*) $\Delta Q = \Delta W = \Delta E = 0$, $\Delta KE = 4.184$ kJ, $\Delta PE = -4.184$ kJ ;

                 (*ii*) $\Delta Q = 0$, $\Delta W = 0$, $\Delta KE = 0$, $\Delta U = +4.184$ kJ, $\Delta PE = -4.184$ kJ]

5. A closed system of constant volume experiences a temperature rise of 20°C when a certain process occurs. The heat transferred in the process is 18 kJ. The specific heat at constant volume for the pure substance comprising the system is 1.2 kJ/kg°C, and the system contains 2 kg of this substance. Determine the change in internal energy and the work done.   [**Ans.** $\Delta U = 48$ kJ ; $W = -30$ kJ]

6. A stationary mass of gas is compressed without friction from an initial state of 2 m³ and 2×10^5 N/m² to a final state of 1 m³ and 2×10^5 N/m², the pressure remaining the same. There is a transfer of 360 kJ of heat from the gas during the process. How much does the internal energy of the gas change?

                                      [**Ans.** $\Delta U = -160$ kJ]

7. The internal energy of a certain substance is given by the following equation :

$$u = pv + 84$$

    where u is given in kJ/kg, p is in kPa and v is in m³/kg.

    A system composed of 3 kg of this substance expands from an initial pressure of 500 kPa and a volume of 0.22 m³ to a final pressure 100 kPa in a process in which pressure and volume are related by $pv^{1.2} = $ constant.

    (*i*) If the expansion is quasi-static, find Q, ΔU and W for the process.

    (*ii*) In another process the same system expands according to the same pressure-volume relationship as in part (*i*) and from the same initial state to the same final state as in part (*i*) but the heat transfer in this case is 30 kJ. Find the work transfer for this process.

    (*iii*) Explain the difference in work transfer in parts (*i*) and (*ii*).

                    [**Ans.** (*i*) $\Delta U = -91$ kJ, $W = 127.5$ kJ, $Q = 36.5$ kJ ; (*ii*) $W = 121$ kJ ;

            (*iii*) The work in (*ii*) is not equal to $\int pdV$ since the process is not quasi-static.]

8. A fluid is contained in a cylinder by a spring-loaded, frictionless piston so that the pressure in the fluid is linear function of the volume ($p = a + bV$). The internal energy of the fluid is given by the following equation $U = 34 + 3.15 \, pV$

    where U is in kJ, p in kPa and V in cubic metre. If the fluid changes from an initial state of 170 kPa, 0.03 m³ to a final state of 400 kPa, 0.06 m³, with no work other than that done on the piston, find the direction and magnitude of the work and heat transfer.

            [**Ans.** $W_{1-2} = 10.35$ kJ ; $Q_{1-2} = 69.85$ kJ (heat flows into the system during the process)]

9. A piston cylinder arrangement has a gas in the cylinder space. During a constant pressure expansion to a larger volume the work effect for the gas are 1.6 kJ, the heat added to the gas and cylinder arrangement is 3.2 kJ and the friction between the piston and cylinder wall amounts to 0.24 kJ. Determine the change in internal energy of the entire apparatus (gas, cylinder, piston).

                                            [**Ans.** 1.84 kJ]

10. A system receives 42 kJ of heat while expanding with volume change of 0.123 m³ against an atmosphere of 12 N/cm². A mass of 80 kg in the surroundings is also lifted a distance of 6 metres.

    (*i*) Find the change in energy of the system.

    (*ii*) The system is returned to its initial volume by an adiabatic process which requires 100 kJ of work. Find the change in energy of system.

    (*iii*) Determine the total change in energy of the system.

                          [**Ans.** (*i*) 22.54 kJ ; (*ii*) 100 kJ ; (*iii*) 122.54 kJ]

11. A thermally insulated battery is being discharged at atmospheric pressure and constant volume. During a 1 hour test it is found that a current of 50 A and 2 V flows while the temperature increases from 20°C to 32.5°C. Find the change in internal energy of the cell during the period of operation.

                                      [**Ans.** -36×10^4 J]

12. In a certain steam plant the turbine develops 1000 kW. The heat supplied to the steam in the boiler is 2800 kJ/kg, the heat received by the system from cooling water in the condenser is 2100 kJ/kg and the feed pump work required to pump the condensate back into the boiler is 5 kW. Calculate the steam flow round the cycle in kg/s.   [**Ans.** 1.421 kg/s]

13. In the compression stroke of an internal-combustion engine the heat rejected to the cooling water is 45 kJ/kg and the work input is 90 kJ/kg. Calculate the change in internal energy of the working fluid stating whether it is a gain or a loss.   [**Ans.** 45 kJ/kg (gain)]

14. 85 kJ of heat are supplied to a system at constant volume. The system rejects 90 kJ of heat at constant pressure and 20 kJ of work is done on it. The system is brought to its original state by adiabatic process. Determine the adiabatic work. Determine also the value of internal energy at all end states if initial value is 100 kJ.   [**Ans.** $W = 15$ kJ ; $U_1 = 100$ kJ, $U_2 = 185$ kJ ; $U_3 = 115$ kJ]

15. A closed system undergoes a reversible process at a constant pressure process of 3.5 bar and its volume changes from 0.15 m³ to 0.06 m³. 25 kJ of heat is rejected by the system during the process. Determine the change in internal energy of the system.   [**Ans.** 6.5 kJ (increase)]

16. An air compressor takes in air at 10^5 Pa and 27°C having volume of 1.5 m³/kg and compresses it to 4.5×10^5 Pa. Find the work done, heat transfer and change in internal energy if the compression is isothermal.   [**Ans.** -225 kJ ; -225 kJ ; $\Delta U = 0$]

17. A cylinder fitted with piston contains 0.2 kg of N_2 at 100 kPa and 30°C. The piston is moved compressing N_2 until the pressure becomes 1 MPa and temperature becomes 150°C. The work done during the process is 20 kJ. Determine the heat transferred from N_2 to the surroundings. Take $c_v = 0.75$ kJ/kg K for N_2.   [**Ans.** -2 kJ]

18. A closed system consisting of 1 kg of gaseous CO_2 undergoes a reversible process at constant pressure causing a decrease of 30 kJ in internal energy. Determine the work done during the process. Take $c_p = 840$ J/kg °C and $c_v = 600$ J/kg °C.   [**Ans.** -12 kJ]

19. The specific heat at constant pressure of 1 kg fluid undergoing a non-flow constant pressure process is given by

$$c_p = \left[2.5 + \frac{40}{T + 20} \right] \text{ kJ/kg °C}$$

where T is in °C.

The pressure during the process is maintained at 2 bar and volume changes from 1 m³ to 1.8 m³ and temperature changes from 50°C to 450°C. Determine :

(*i*) Heat added   (*ii*) Work done

(*iii*) Change in internal energy   (*iv*) Change in enthalpy.

[**Ans.** (*i*) 1076 kJ ; (*ii*) 160 kJ ; (*iii*) 916 kJ ; (*iv*) 1076 kJ]

20. One kg of nitrogen (molecular weight 28) is compressed reversibly and isothermally from 1.01 bar, 20°C to 4.2 bar. Calculate the work done and the heat flow during the process. Assume nitrogen to be a perfect gas.   [**Ans.** $W = 124$ kJ/kg ; $Q = -124$ kJ/kg]

21. Air at 1.02 bar, 22°C, initially occupying a cylinder volume of 0.015 m³, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bar. Calculate :

(*i*) The final temperature   (*ii*) The final volume

(*iii*) The work done on the mass of air in the cylinder.

[**Ans.** (*i*) 234.5°C ; (*ii*) 0.00388 m³ ; (*iii*) 2.76 kJ]

22. One kg of a perfect gas is compressed from 1.1 bar, 27°C according to a law $pv^{1.3} = $ constant, until the pressure is 6.6 bar. Calculate the heat flow to or from the cylinder walls,

(*i*) When the gas is ethane (molecular weight 30), which has

$c_p = 1.75$ kJ/kg K.

(*ii*) When the gas is argon (molecular weight 40), which has

$c_p = 0.515$ kJ/kg K.   [**Ans.** (*i*) 84.5 kJ/kg ; (*ii*) -59.4 kJ/kg]

23. One kg of air at 1 bar, 15°C is compressed reversibly and adiabatically to a pressure of 4 bar. Calculate the final temperature and the work done on the air.   [**Ans.** 155°C ; 100.5 kJ/kg]

24. A certain perfect gas is compressed reversibly from 1 bar, 17°C to a pressure of 5 bar in a perfectly thermally insulated cylinder, the final temperature being 77°C. The work done on the gas during the compression is 45 kJ/kg. Calculate γ, c_v, R and the molecular weight of the gas.

[**Ans.** 1.132 ; 0.75 kJ/kg K ; 0.099 kJ/kg K ; 84]

25. One kg of air at 1.02 bar, 20°C is compressed reversibly according to a law $pv^{1.3}$ = constant, to a pressure of 5.5 bar. Calculate the work done on the air and heat flow to or from the cylinder walls during the compression.   [**Ans.** 133.5 kJ/kg ; – 33.38 kJ/kg]

26. 0.05 kg of carbon dioxide (molecular weight 44), occupying a volume of 0.03 m³ at 1.025 bar, is compressed reversibly until the pressure is 6.15 bar. Calculate final temperature, the work done on the CO_2, the heat flow to or from the cylinder walls,

  (*i*) When the process is according to law $pv^{1.4}$ = constant,

 (*ii*) When the process is isothermal,

(*iii*) When the process takes place in a perfectly thermally insulated cylinder.

Assume CO_2 to be a perfect gas, and take γ = 1.3.

[**Ans.** 270°C ; 5.138 kJ ; 1.713 kJ ; 52.6°C ; 5.51 kJ ; – 5.51 kJ ; 219°C ; 5.25 kJ ; 0 kJ]

27. Oxygen (molecular weight 32) is compressed reversibly and polytropically in a cylinder from 1.05 bar, 15°C to 4.2 bar in such a way that one-third of the work input is rejected as heat to the cylinder walls. Calculate the final temperature of the oxygen.

Assume oxygen to be a perfect gas and take c_v = 0.649 kJ/kg K.   [**Ans.** 113°C]

28. A cylinder contains 0.5 m³ of a gas at 1×10^5 N/m² and 90°C. The gas is compressed to a volume of 0.125 m³, the final pressure being 6×10^5 N/m². Determine :

  (*i*) The mass of gas.

 (*ii*) The value of index '*n*' for compression.

(*iii*) The increase in internal energy of gas.

(*iv*) The heat received or rejected by the gas during compression.

(γ = 1.4, R = 294.2 Nm/kg°C).   [**Ans.** (*i*) 0.468 kg ; (*ii*) 1.292 ; (*iii*) 62.7 kJ ; (*iv*) – 22.67 kJ]

Steady Flow Systems

29. 12 kg of a fluid per minute goes through a reversible steady flow process. The properties of fluid at the inlet are p_1 = 1.4 bar, ρ_1 = 25 kg/m³, C_1 = 120 m/s and u_1 = 920 kJ/kg and at the exit are p_2 = 5.6 bar, ρ_2 = 5 kg/m³, C_2 = 180 m/s and u_2 = 720 kJ/kg. During the passage, the fluid rejects 60 kJ/s and rises through 60 metres. Determine : (*i*) the change in enthalpy (Δ*h*) and (*ii*) work done during the process (*W*).   [**Ans.** (*i*) Δ*h* = – 93.6 kJ/kg ; (*ii*) *W* = – 44.2 kW]

30. In the turbine of a gas turbine unit the gases' flow through the turbine is 17 kg/s and the power developed by the turbine is 14000 kW. The enthalpies of the gases at inlet and outlet are 1200 kJ/kg and 360 kJ/kg respectively, and the velocities of the gases at inlet and outlet are 60 m/s and 150 m/s respectively. Calculate the rate at which the heat is rejected from the turbine. Find also the area of the inlet pipe given that the specific volume of the gases at inlet is 0.5 m³/kg.

[**Ans.** 119.3 kW (heat rejected) ; 0.142 m³]

31. Air flows steadily at the rate of 0.4 kg/s through an air compressor, entering at 6 m/s with a pressure of 1 bar and a specific volume of 0.85 m³/kg, and leaving at 4.5 m/s with a pressure of 6.9 bar and a specific volume of 0.16 m³/kg. The internal energy of air leaving is 88 kJ/kg greater than that of the air entering. Cooling water in a jacket surrounding the cylinder absorbs heat from the air at the rate of 59 kJ/s. Calculate the power required to drive the compressor and the inlet and outlet pipe cross-sectional areas.   [**Ans.** 104.4 kW ; 0.057 m² ; 0.014 m²]

32. A turbine operating under steady flow conditions receives steam at the following state : pressure 13.8 bar ; specific volume 0.143 m³/kg ; internal energy 2590 kJ/kg ; velocity 30 m/s. The state of the steam leaving the turbine is : pressure 0.35 bar ; specific volume 4.37 m³/kg ; internal energy 2360 kJ/kg ; velocity 90 m/s. Heat is lost to the surroundings at the rate of 0.25 kJ/s. If the rate of steam flow is 0.38 kg/s, what is the power developed by the turbine ?   [**Ans.** 102.8 kW]

33. A nozzle is a device for increasing the velocity of a steadily flowing stream of fluid. At the inlet to a certain nozzle the enthalpy of the fluid is 3025 kJ/kg and the velocity is 60 m/s. At the exit from the nozzle the enthalpy is 2790 kJ/kg. The nozzle is horizontal and there is negligible heat loss from it.

  (*i*) Find the velocity at the nozzle exit.

 (*ii*) If the inlet area is 0.1 m² and specific volume at inlet is 0.19 m³/kg, find the rate of flow of fluid.

(*iii*) If the specific volume at the nozzle exit is 0.5 m³/kg, find the exit area of the nozzle.

[**Ans.** (*i*) 688 m/s ; (*ii*) 31.6 kg/s ; (*iii*) 0.0229 m²]

34. A gas flows steadily through a rotary compressor. The gas enters the compressor at a temperature of 16°C, a pressure of 100 kPa, and an enthalpy of 391.2 kJ/kg. The gas leaves the compressor at a temperature of 245°C, a pressure of 0.6 MPa and an enthalpy of 534.5 kJ/kg. There is no heat transfer to or from the gas as it flows through the compressor.

  (*i*) Evaluate the external work done per unit mass of gas assuming the gas velocities at entry and exit to be negligible.

(*ii*) Evaluate the external work done per unit mass of gas when the gas velocity at entry is 80 m/s and that at exit is 160 m/s.   [**Ans.** 143.3 kJ/kg 152.9 kJ/kg]

35. A turbine, operating under steady-flow conditions, receives 5,000 kg of steam per hour. The steam enters the turbine at a velocity of 3,000 m/min, an elevation of 5 m and a specific enthalpy of 2787 kJ/kg. It leaves the turbine at a velocity of 6000 m/min, an elevation of 1 m and a specific enthalpy of 2259 kJ/kg. Heat losses from the turbine to the surroundings amount to 16736 kJ/h.

Determine the power output of the turbine.   [**Ans.** 723 kW]

36. In a steady flow process, the working fluid flows at a rate of 240 kg/min. The fluid rejects 120 kJ/s passing through the system. The conditions of fluid at inlet and outlet are given as : C_1 = 300 m/s, p_1 = 6.2 bar, u_1 = 2100 kJ/kg, v_1 = 0.37 m^3/kg and C_2 = 150 m/s, p_2 = 1.3 bar, u_2 = 1500 kJ/kg, v_2 = 1.2 m^3/kg. The suffix 1 indicates the conditions at inlet and 2 indicates at outlet of the system. Neglecting the change in potential energy, determine the power capacity of the system in MW.

          [**Ans.** 2.7086 MW]

37. Steam enters a turbine at 20 m/s and specific enthalpy of 3000 kJ/kg and leaves the turbine at 40 m/s and specific enthalpy of 2500 kJ/kg. Heat lost to the surroundings is 25 kJ/kg of steam as the steam passes through the turbine. If the steam flow rate is 360000 kg/h, determine the output from the turbine in MW.   [**Ans.** 47.44 MW]

38. A stream of gases at 7.5 bar, 800°C and 150 m/s is passed through a turbine of a jet engine. The stream comes out of the turbine at 2.0 bar, 600°C and 300 m/s. The process may be assumed adiabatic. The enthalpies of gas at the entry and exit of the turbine are 960 kJ/kg and 700 kJ/kg gas respectively.

Determine the capacity of the turbine if the gas flow is 4 kg/s.   [**Ans.** 905 kW]

39. In a steam power plant 1.5 kg of water is supplied per second to the boiler. The enthalpy and velocity of water entering into the boiler are 800 kJ/kg and 10 m/s. Heat at the rate of 2200 kJ per kg of water is supplied to the water. The steam after passing through the turbine comes out with a velocity of 50 m/s and enthalpy of 2520 kJ/kg. The boiler inlet is 5 m above the turbine exit. The heat loss from the boiler is 1800 kJ/min and from the turbine 600 kJ/min.

Determine the power capacity of the turbine, considering boiler and turbine as single unit.

          [**Ans.** 678 kW]

40. 15 kg of air per minute is delivered by a centrifugal compressor. The inlet and outlet conditions of air are : C_1 = 10 m/s, p_1 = 1 bar, v_1 = 0.5 m^3/kg and C_2 = 80 m/s, p_2 = 7 bar, v_2 = 0.15 m^3/kg. The increase in enthalpy of air passing through the compressor is 160 kJ/kg, and heat loss to the surroundings is 720 kJ/min. Assuming that inlet and discharge lines are at the same level, find :

  (*i*) Motor power required to drive the compressor.

(*ii*) Ratio of inlet to outlet pipe diameter. $\left[\textbf{Ans.}\ (i)\ 52.78\ \text{kW}\ ; (ii)\ \dfrac{d_1}{d_2} = 5.16\right]$

41. A centrifugal air compressor used in gas turbine receives air at 100 kPa and 300 K and it discharges air at 400 kPa and 500 K. The velocity of air leaving the compressor is 100 m/s. Neglecting the velocity at the entry of the compressor, determine the power required to drive the compressor if the mass flow rate is 15 kg/s.

Take c_p (air) = 1 kJ/kg K and assume that there is no heat transfer from the compressor to the surroundings.   [**Ans.** 3075 kW]

42. In a water cooled compressor 0.5 kg of air is compressed per second. A shaft input of 60 kW is required to run the compressor. Heat lost to the cooling water is 30 per cent of input and 10 per cent of the input is lost in bearings and other frictional effects. Air enters the compressor at 1 bar and 20°C. Neglecting the changes in kinetic energy and potential energy, determine the exit air temperature. Take c_p = 1 kJ/kg°C air.

Consider steady flow process.   [**Ans.** 92°C]

43. Steam at 7 bar and 200°C enters an insulated convergent divergent nozzle with a velocity of 60 m/s. It leaves the nozzle at a pressure of 1.4 bar and enthalpy of 2600 kJ/kg.
   Determine the velocity of the steam at exit.   [**Ans.** 701 m/s]

44. A petrol engine develops 50 kW brake power. The fuel and air flow rates are 10 kg and 107 kg/h. The temperature of fuel-air mixture entering the engine is 20°C and temperature of gases leaving the engine is 500°C. The heat transfer rate from the engine to the jacket cooling water is 50 kJ/s and that to the surroundings is 10 kJ/s.
   Evaluate the increase in the specific enthalpy of the mixture as it flow through the engine.
                                                                        [**Ans.** – 110 kJ/s]

45. A compressor takes air at 100 kN/m^2 and delivers the same at 550 kN/m^2. The compressor discharges 16 m^3 of free air per minute. The densities of air at inlet and exit are 1.25 kg/m^3 and 5 kg/m^3. The power of the motor driving the compressor is 40 kW. The heat lost to the cooling water circulated around the compressor is 30 kJ/kg of air passing through the compressor.
   Neglecting changes in P.E. and K.E. determine the change in specific internal energy.
                                                                        [**Ans.** 60 kJ/kg]

46. A centrifugal pump operating under steady flow conditions delivers 3000 kg of water per minute at 20°C. The suction pressure is 0.8 bar and delivery pressure is 3 bar. The suction pipe diameter is 15 cm and discharge pipe diameter is 10 cm. Find the capacity of the drive motor.
   Neglect the change in internal energy and assume that the suction and discharge are at same level.
                                                                        [**Ans.** 11.8 kW]

47. 60 kg of water is delivered by a centrifugal pump per second. The inlet and outlet pressures are 1 bar and 4 bar respectively. The suction is 2 m below the centre of the pump and delivery is 8 m above the centre of the pump. Determine the capacity of the electric motor to run the pump. The suction and delivery pipe diameters are 20 cm and 10 cm respectively.   [**Ans.** 27.15 kW]

48. The air speed of a turbojet engine in flight is 270 m/s. Ambient air temperature is – 15°C. Gas temperature outlet of the nozzle is 600°C. Corresponding enthalpy values for air and gas are respectively 260 and 912 kJ/kg. Fuel air ratio is 0.0190. Chemical energy of the fuel is 44.5 MJ/kg. Owing to incomplete combustion 5% of the chemical energy is not released in the reaction. Heat loss from the engine is 21 kJ/kg of air.
   Calculate the velocity of exhaust jet.   [**Ans.** 560 m/s]

49. Air at a temperature of 15°C passes through a heat exchanger at a velocity of 30 m/s where its temperature is raised to 800°C. It then enters a turbine with the same velocity of 30 m/s and expands until the temperature falls to 650°C. On leaving the turbine, the air is taken at a velocity of 60 m/s to a nozzle where it expands until the temperature has fallen to 500°C. If the air flow rate is 2 kg/s, calculate (*i*) the rate of heat transfer to the air, (*ii*) the power output from the turbine assuming no heat loss, and (*iii*) the velocity at exit from nozzle, assuming no heat loss.
   Take the enthalpy of air as $h = c_p t$, where c_p is the specific heat equal to 1.005 kJ/kg°C and t the temperature.   [**Ans.** 1580 kJ/s ; 298.8 kW ; 554 m/s]

Vapour (Steam)

50. 0.05 kg of steam is heated at a constant pressure of 2 bar until the volume occupied is 0.0658 m^3. Calculate the heat supplied and work done.   [**Ans.** 18.25 kJ ; 4.304 kJ]

51. Steam at 7 bar and dryness fraction 0.9 expands in a cylinder behind a piston isothermally and reversibly to a pressure of 1.5 bar. Calculate the change of internal energy and the change of enthalpy per kg of steam. The heat supplied during the process is found to be 400 kJ/kg. Calculate the work done per kg of steam.   [**Ans.** 217.5 kJ/kg (gain) ; 245.7 kJ/kg ; 182.5 kJ/kg]

52. 1 kg of steam at 100 bar and 375°C expands reversibly in a perfectly thermally insulated cylinder behind a piston until pressure is 38 bar and the steam is then saturated.
   Calculate the work done by the steam.   [**Ans.** 169.7 kJ/kg]

53. In a steam engine the steam at the beginning of the expansion process is at 7 bar, dryness fraction 0.95, and the expansion follows the law $pv^{1.1}$ = constant, down to a pressure of 0.34 bar. Calculate the work done per kg of steam during the expansion, and the heat flow per kg of steam to or from the cylinder walls during the expansion.   [**Ans.** 436 kJ/kg ; 155.6 kJ/kg (heat supplied)]

54. Steam at 19 bar is throttled to 1 bar and the temperature after throttling is found to be 150°C. Calculate the initial dryness fraction of the steam.   [**Ans.** 0.989]

55. 1 kg of steam at 7 bar, entropy 6.5 kJ/kg K, is heated reversibly at constant pressure until the temperature is 250°C. Calculate the heat supplied, and show on a *T-s* diagram the area which represents the heat flow. [**Ans.** 283 kJ/kg]

56. 1 kg of steam at 20 bar, dryness fraction 0.9, is heated reversibly at constant pressure to a temperature of 300°C.

Calculate the heat supplied and change of entropy and show the process on a *T-s* diagram, indicating the area which represents the heat flow. [**Ans.** 415 kJ/kg ; 0.8173 kJ/kg K]

57. Steam at 0.05 bar, 100°C is to be condensed completely by a reversible constant pressure process.

Calculate the heat to be removed per kg of steam and the change of entropy. Sketch the process on a *T-s* diagram and shade in the area which represents the heat flow. [**Ans.** 2550 kJ/kg ; 8.292 kJ/kg K]

58. 0.05 kg of steam at 10 bar, dryness fraction 0.84, is heated reversibly in a rigid vessel until the pressure is 20 bar.

Calculate the change of entropy and the heat supplied. Show the area which represents the heat supplied on a *T-s* diagram. [**Ans.** 0.0704 kJ/kg K ; 36.85 kJ]

59. 1 kg of steam undergoes a reversible isothermal process from 20 bar and 250°C to a pressure of 30 bar. Calculate the heat flow, stating whether it is supplied or rejected and sketch the process on a *T-s* diagram. [**Ans.** – 135 kJ/kg]

60. Steam at 5 bar, 250°C, expands isentropically to a pressure of 0.7 bar. Calculate the final condition of steam. [**Ans.** 0.967]

61. Steam expands reversibly in a cylinder behind a piston from 6 bar dry saturated, to a pressure of 0.65 bar. Assuming that the cylinder is perfectly thermally insulated, calculate the work done during the expansion per kg of steam. Sketch the process on a *T-s* diagram. [**Ans.** 323.8 kJ/kg]

62. A steam engine receives steam at 4 bar, dryness fraction 0.8, and expands it according to a law $pv^{1.05}$ = constant to a condenser pressure of 1 bar. Calculate the change of entropy per kg of steam during the expansion, and sketch the process on a *T-s* diagram. [**Ans.** 0.381 kJ/kg K]

63. Steam at 15 bar is throttled to 1 bar and a temperature of 150°C. Calculate the initial dryness fraction and the change of entropy. Sketch the process on a *T-s* diagram and state the assumptions made in the throttling process. [**Ans.** 0.992, 1.202 kJ/kg K]

64. Steam enters a turbine at 70 bar, 500°C and leaves at 2 bar in a dry saturated state. Calculate the isentropic efficiency and effectiveness of the process. Neglect changes of kinetic and potential energy and assume that the process is adiabatic.

The atmospheric temperature is 17°C. [**Ans.** 84.4% ; 88%]

65. Steam at 10 bar and 250°C expands until the pressure becomes 2.75 bar. The dryness fraction of the steam at the end of expansion is 0.95. Determine the change in internal energy. [**Ans.** – 273 kJ/kg]

66. Calculate the quantity of heat required to form 2.5 kg of dry steam at 11 bar from water at 20°C. Also determine the amount of heat removed at constant pressure to cause the steam to become 0.95 dry. Calculate the specific volume at the respective conditions.

[**Ans.** 6740 kJ ; 250 kJ ; 0.1775 m³/kg ; 0.167 m³/kg]

67. Steam at 10 bar and 0.95 dryness is available. Determine the final condition of steam in each of the following operations :

  (*i*) 160 kJ of heat is removed at constant pressure ;

 (*ii*) It is cooled at constant volume till the temperature inside falls to 140°C.

 (*iii*) Steam expands isentropically in a steam turbine developing 300 kJ of work per kg of steam when the exit pressure of steam is 0.5 bar. [**Ans.** (*i*) 0.874 ; (*ii*) 0.367 ; (*iii*) 0.882]

68. Calculate the internal energy of 0.3 m³ of steam at 4 bar and 0.95 dryness. If this steam is superheated at constant pressure through 30°C, determine the heat added and change in internal energy.

[**Ans.** 2451 kJ/kg ; 119 kJ ; 107.5 kJ/kg]

69. 1 kg of water at 30°C and 1 bar is heated at constant pressure until it becomes saturated vapour. Determine the change in volume, and internal energy during the process.

[**Ans.** 1.694 m³/kg (app.) ; 2380.6 kJ/kg]

70. Water is supplied to the boiler at 15 bar and 80°C and steam is generated at the same pressure at 0.9 dryness. Determine the heat supplied to the steam in passing through the boiler and change in entropy. [**Ans.** 2260.5 kJ/kg ; 4.92 kJ/kg K]

71. A cylindrical vessel of 5 m^3 capacity contains wet steam at 1 bar. The volume of vapour and liquid in the vessel are 4.95 m^3 and 0.05 m^3 respectively. Heat is transferred to the vessel until the vessel is filled with saturated vapour. Determine the heat transfer during the process.   [**Ans.** 104.93 MJ]

72. A closed vessel of 0.5 m^3 capacity contains dry saturated steam at 3.5 bar. The vessel is cooled until the pressure is reduced to 2 bar. Calculate :
  (*i*) The mass of steam in the vessel,
  (*ii*) Final dryness fraction of the steam, and
  (*iii*) The amount of heat transferred during the process.   [**Ans.** (*i*) 0.955 kg ; (*ii*) 0.582 ; (*iii*) – 828 kJ]

73. A closed vessel of 0.3 m^3 capacity contains steam at 8 bar and 200°C ;
   (*i*) Determine the mass of the steam in the vessel.
   (*ii*) The vessel is cooled till the steam becomes just dry and saturated. What will be the pressure of the steam in the vessel at this stage ?
  (*iii*) The vessel is further cooled till the temperature drops to 158.85°C. Determine the pressure and condition of the steam.   [**Ans.** (*i*) 1.2 kg ; (*ii*) 7.362 bar ; (*iii*) 6 bar, 0.826]

74. 0.5 kg of steam at 4 bar is contained in a cylinder fitted with a piston. The initial volume of steam is 0.1 m^3. Heat is transferred to the steam at constant pressure until the temperature becomes 300°C. Determine the heat transfer and work done during the process.   [**Ans.** 771 kJ ; 91 kJ]

75. A quantity of steam at 13 bar and 0.8 dryness occupies 0.1 m^3. Determine the heat supplied to raise the temperature of the steam to 250°C at constant pressure and percentage of this heat which appears as external work. Take specific heat for superheated steam as 2.2 kJ/kg K.   [**Ans.** 423 kJ/kg ; 15.3%]

76. A certain quantity of dry and saturated steam at 1.5 bar occupies initially a volume of 2.32 m^3. It is compressed until the volume is halved :
   (*i*) Isothermally,
  (*ii*) As per the law pv = constant, determine the final condition of steam in each case.
  Also determine the heat rejected during the isothermal compression process.
                                                                    [**Ans.** (*i*) 0.5, 2226.5 kJ ; (*ii*) 0.956]

77. Steam enters a turbine at a pressure of 10 bar and 300°C with a velocity of 50 m/s. The steam leaves the turbine at 1.5 bar and with a velocity of 200 m/s. Assuming the process to be reversible adiabatic and neglecting the change in potential energy, determine the work done per kg of steam flow through the turbine.   [**Ans.** 375.55 kJ/kg]

78. Steam at 10 bar and 300°C passing through a convergent divergent nozzle expands reversibly and adiabatically till the pressure falls to 2 bar. If the velocity of steam entering into the nozzle is 50 m/s, determine the exit velocity of the steam.   [**Ans.** 832 m/s]

Unsteady Flow Processes

79. An air receiver of volume 6 m^3 contains air at 15 bar and 40.5°C. A valve is opened and some air is allowed to blow out to atmosphere. The pressure of the air in the receiver drops rapidly to 12 bar when the valve is then closed. Calculate the mass of air which has left the receiver.   [**Ans.** 14.7 kg]

80. The internal energy of air is given, at ordinary temperatures, by $u = u_0 + 0.718t$
  where u is in kJ/kg, u_0 is any arbitrary value of u at 0°C, kJ/kg and t is temperature in °C.
  Also for air, $pv = 0.287 (t + 273)$ where p is in kPa and v is in m^3/kg.
   (*i*) An evacuated bottle is fitted with a valve through which air from the atmosphere, at 760 mm Hg and 25°C, is allowed to flow slowly to fill the bottle. If no heat is transferred to or from the air in the bottle, what will its temperature be when the pressure in the bottle reaches 760 mm Hg ?
  (*ii*) If the bottle initially contains 0.03 m^3 of air at 400 mm Hg and 25°C, what will the temperature be when the pressure in the bottle reaches 760 mm of Hg ?   [**Ans.** (*i*) 144.2°C ; (*ii*) 71.6°C]

3

Second Law of Thermodynamics and Entropy

> **3.1. Limitations of first law of thermodynamics and introduction to second law. 3.2. Performance of heat engine and reversed heat engine. 3.3. Reversible processes. 3.4. Statements of second law of thermodynamics**—Clausius statement—Kelvin-Planck statement—Equivalence of Clausius statement to the Kelvin-Planck statement. **3.5. Perpetual motion machine of the second kind. 3.6. Thermodynamic temperature. 3.7. Clausius inequality. 3.8. Carnot cycle. 3.9. Carnot's theorem. 3.10. Corollary of Carnot's theorem. 3.11. Efficiency of the reversible heat engine. 3.12. Entropy**—Introduction—Extropy—a property of a system—Change of entropy in a reversible process. **3.13. Entropy and irreversibility. 3.14. Change in entropy of the universe. 3.15. Temperature entropy diagram. 3.16. Characteristics of entropy. 3.17. Entropy changes for a closed system**—General case for change of entropy of a gas—Heating a gas at constant volume—Heating a gas at constant pressure—Isothermal process—Adiabatic process—Polytropic process—Approximation for heat absorbed. **3.18. Entropy changes for an open system. 3.19. The third law of thermodynamics**—Highlights—Objective Type Questions—Theoretical Questions—Unsolved Examples.

3.1. LIMITATIONS OF FIRST LAW OF THERMODYNAMICS AND INTRODUCTION TO SECOND LAW

It has been observed that *energy can flow* from a system in the form of *heat* or *work*. The first law of thermodynamics sets no limit to the amount of the total energy of a system which can be caused to flow out as work. A limit is imposed, however, as a result of the principle enunciated in the second law of thermodynamics which states that heat will flow naturally from one energy reservoir to another at a lower temperature, but not in opposite direction without assistance. This is very important because a heat engine operates between two energy reservoirs at different temperatures.

Further the first law of thermodynamics *establishes equivalence between the quantity of heat used and the mechanical work but does not specify the conditions under which conversion of heat into work is possible, neither the direction in which heat transfer can take place*. This gap has been *bridged* by the second law of thermodynamics.

3.2. PERFORMANCE OF HEAT ENGINE AND REVERSED HEAT ENGINE

Refer Fig. 3.1 (*a*). A *heat engine* is used to produce the maximum work transfer from a given positive heat transfer. The measure of success is called the *thermal efficiency* of the engine and is defined by the ratio :

Thermal efficiency, $\qquad \eta_{th} = \dfrac{\dot{W}}{Q_1}$ $\qquad\qquad$ ...(3.1)

where $\quad W$ = net work transfer from the engine, and

$\qquad Q_1$ = heat transfer to engine.

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For a *reversed heat engine* [Fig. 3.1 (*b*)] acting as a *refrigerator* when the purpose is to achieve the maximum heat transfer from the cold reservoir, the measure of success is called the co-efficient of performance (C.O.P.). It is defined by the ratio :

$$\text{Co-efficient of performance, (C.O.P.)}_{ref.} = \frac{Q_2}{W} \qquad \qquad ...(3.2)$$

where Q_2 = heat transfer *from cold reservoir*

W = the net work transfer to the refrigerator.

For a **reversed heat engine** [Fig. 3.1 (*b*)] acting as a *heat pump*, the measure of success is again called the *co-efficient of performance*. It is defined by the ratio :

$$\text{Co-efficient of performance, (C.O.P.)}_{heat\ pump} = \frac{Q_1}{W} \qquad \qquad ...(3.3)$$

where Q_1 = heat transfer *to hot reservoir*

W = net work transfer to the heat pump.

(a) Heat engine   (b) Heat pump or refrigerator
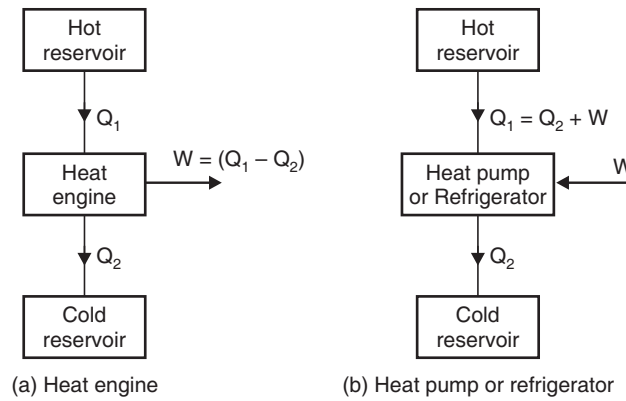
Fig. 3.1

In all the above three cases application of the first law gives the relation $Q_1 - Q_2 = W$, and this can be used to rewrite the expressions for thermal efficiency and co-efficient of performance solely in terms of the heat transfers.

$$\eta_{th} = \frac{Q_1 - Q_2}{Q_1} \qquad \qquad ...(3.4)$$

$$(C.O.P.)_{ref} = \frac{Q_2}{Q_1 - Q_2} \qquad \qquad ...(3.5)$$

$$(C.O.P.)_{heat\ pump} = \frac{Q_1}{Q_1 - Q_2} \qquad \qquad ...(3.6)$$

It may be seen that η_{th} is *always less than unity* and $(C.O.P.)_{heat\ pump}$ is *always greater than unity*.

3.3.   REVERSIBLE PROCESSES

A reversible process should fulfil the following *conditions* :

1. The process should not involve friction of any kind.

2. Heat transfer should not take place with finite temperature difference.

    3. The energy transfer as heat and work during the forward process should be identically equal to energy transfer as heat and work during the reversal of the process.

    4. There should be no free or unrestricted expansion.

    5. There should be no mixing of the fluids.

    6. The process must proceed in a series of equilibrium states.

Some examples of *ideal reversible processes* are :

  (*i*) Frictionless adiabatic expansion or compression ;

 (*ii*) Frictionless isothermal expansion or compression ;

(*iii*) Condensation and boiling of liquids.

Some examples of *irreversible processes are* :

  (*i*) Combustion process ;   (*ii*) Mixing of two fluids

(*iii*) All processes involving friction ;   (*iv*) Flow of electric current through a resistance

 (*v*) Heat flow from a higher temperature to lower temperature.

    Reversible processes are preferred because the devices which produce work such as engines and turbines, reversible process of the working fluid delivers *more work* than the corresponding irreversible processes. Also in case of fans, compressors, refrigerators and pumps *less power input* is required when *reversible processes* are used in place of corresponding irreversible ones.

    In thermodynamic analysis concept of reversibility, though hypothetical, is very important because a reversible process is the *most efficient process*. Only reversible processes can be truely represented on property diagrams. Thermodynamic reversibility can only be approached but can *never* be achieved. Thus the main task of the engineer is to design the system which will evolve approximate reversible processes.

3.4. STATEMENTS OF SECOND LAW OF THERMODYNAMICS

    The second law of thermodynamics has been enunciated meticulously by Clausius, Kelvin and Planck in slightly different words although both statements are basically identical. Each statement is based on an *irreversible process*. The *first considers transformation of heat between two thermal reservoirs* while the *second considers the transformation of heat into work*.

3.4.1. Clausius Statement

    "*It is impossible for a self acting machine working in a cyclic process unaided by any external agency, to convey heat from a body at a lower temperature to a body at a highter temperature*".

    In other words, heat of, itself, cannot flow from a colder to a hotter body.

3.4.2. Kelvin-Planck Statement

    "*It is impossible to construct an engine, which while operating in a cycle produces no other effect except to extract heat from a single reservoir and do equivalent amount of work*".

    Although the Clausius and Kelvin-Planck statements appear to be different, they are really equivalent in the sense that a *violation of either statement implies violation of other*.

3.4.3. Equivalence of Clausius Statement to the Kelvin-Planck Statement

    Refer Fig. 3.2. Consider a higher temperature reservoir T_1 and low temperature reservoir T_2. Fig. 3.2 shows a heat pump which requires no work and transfers an amount of Q_2 from a low temperature to a higher temperature reservoir (in violation of the Clausius statement). Let an amount of heat Q_1 (greater than Q_2) be transferred from high temperature reservoir to heat engine which devolops a net work, $W = Q_1 - Q_2$ and rejects Q_2 to the low temperature reservoir. Since there is no heat interaction with the low temperature, it can be eliminated. The combined system of the heat engine and heat pump acts then like a heat engine exchanging heat with a single reservoir, which is the violation of the Kelvin-Planck statement.
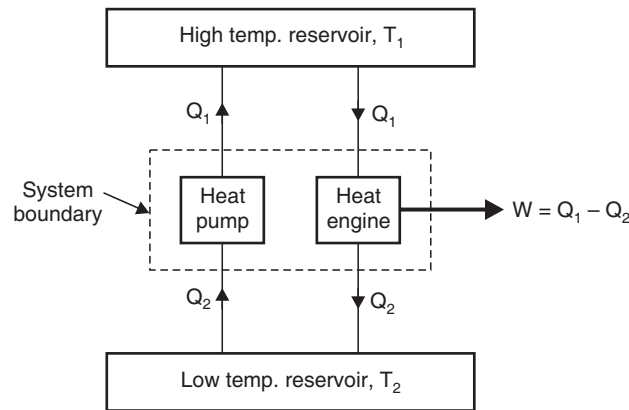
Fig. 3.2. Equivalence of Clausius statement to Kelvin-Planck statement.

3.5. PERPETUAL MOTION MACHINE OF THE SECOND KIND (PMM2)

— A machine which voilates the first law of thermodynamics is called the perpetual motion machine of the first kind (PMM1). Such a machine creates its own energy from nothing and *does not exist*.

— Without violating the first law, a machine can be imagined which would continuously absorb heat from a single thermal *reservoir and would convert this heat completely into work. The efficiency* of such a machine would be *100 per cent*. This machine is called the *perpetual motion machine of the second kind (PMM2)*.

Fig. 3.3 shows the perpetual motion machine of the second kind. *A machine of this kind will evidently violate the second law of thermodynamics*.
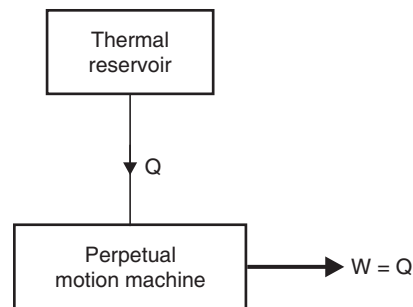
Fig. 3.3. Perpetual motion machine of second kind (PMM2).

3.6. THERMODYNAMIC TEMPERATURE

Take the case of reversible heat engine operating between two reservoirs. Its thermal efficiency is given by (3.4),

$$\eta_{th} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

The temperature of a reservoir remains uniform and fixed irrespective of heat transfer. This means that reservoir has only one property defining its state and the heat transfer from a reservoir

is some function of that property, *temperature*. Thus $Q = \phi(K)$, where K is the temperature of reservoir. The choice of the function is universally accepted to be such that the relation,

$$\frac{Q_1}{Q_2} = \frac{\phi(K_1)}{\phi(K_2)} \quad \text{becomes} \quad \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \qquad \qquad ...(3.7)$$

where T_1 and T_2 are the thermodynamic temperatures of the reservoirs. Zero thermodynamic temperature (that temperature to which T_2 tends, as the heat transfer Q_2 tends to zero) has never been attained and *one form of third law of thermodynamics is the statement* :

"The temperature of a system cannot be reduced to zero in a finite number of processes."

After establishing the concept of a zero thermodynamic temperature, a reference reservoir is chosen and assigned a numerical value of temperature. Any other thermodynamic temperature may now be defined in terms of reference value and the heat transfers that would occur with reversible engine,

$$T = T_{ref} \cdot \frac{Q}{Q_{ref}} \qquad \qquad ...(3.8)$$

The determination of thermodynamic temperature cannot be made in this way as it is not possible to build a reversible engine. Temperatures are determined by the application of thermodynamic relations to other measurements.

The SI unit of thermodynamic temperature is the kelvin (K). The relation between thermodynamic temperature and celsius scale, which is in common use is :

Thermodynamic temperature = Celsius temperature + 273.16°.

The kelvin unit of thermodynamic temperature is the fraction $\dfrac{1}{273.16}$ of thermodynamic temperature of 'Triple point' of water.

3.7. CLAUSIUS INEQUALITY

When a reversible engine uses more than two reservoirs the third or higher numbered reservoirs will not be equal in temperature to the original two. Consideration of expression for efficiency of the engine indicates that for maximum efficiency, all the heat transfer should take place at maximum or minimum reservoir temperatures. Any intermediate reservoir used will, therefore, lower the efficiency of the heat engine. Practical engine cycles often involve continuous changes of temperature during heat transfer. A relationship among processes in which these sort of changes occur is necessary. The ideal approach to a cycle in which temperature continually changes is to consider the system to be in communication with a large number of reservoirs in procession. Each reservoir is considered to have a temperature differing by a small amount from the previous one. In such a model it is possible to imagine that each reservoir is replaced by a reversible heat engine in communication with standard reservoirs at same temperature T_0. Fig. 3.4 shows one example to this substitution.

The system to which the heat transfer is effected is neither concerned with the source of energy it receives nor with the method of transfer, save that it must be reversible. Associated with the small heat transfer dQ to the original system is a small work transfer dW and for this system the first law gives

$$\sum_{cycle} (\delta Q - \delta W) = 0 \qquad \qquad ...(3.9)$$
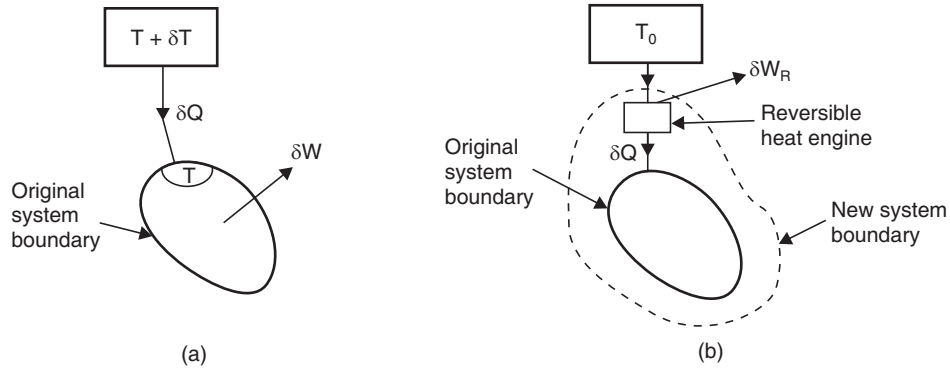
Fig. 3.4. The clausius inequality.

Now consider the engine replacing the reservoirs and apply the second law to the new system in Fig. 3.4 (*b*). If the new system is not a perpetual motion machine of second kind, no positive work transfer is possible with a single reservoir.

Therefore,

$$\sum_{\text{cycle}} (\delta W - \delta W_R) \le 0 \qquad \qquad ...(3.10)$$

But by the definition of thermodynamic temperature in equation (3.8)

$$\frac{\delta W_R}{\delta Q} = \frac{\delta Q_0 - \delta Q}{\delta Q} = \frac{T_0 - T}{T} \qquad \qquad ...(3.11)$$

and by combination of equations (3.9), (3.10) and (3.11)

$$T_0 \sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) \le 0 \text{ but } T_0 \ne 0 \text{ and therefore ;}$$

$$\sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) \le 0 \qquad \qquad ...(3.12)$$

This is known as **Clausius inequality**.

Let us now consider the case of a reversible engine for which

$$\sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) \le 0,$$

reverse the engine and for the reversible heat pump obtained it is possible to develop the expression,

$$- \sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) \le 0$$

The *negative sign indicates that the heat transfers have all reversed in direction when the engine was reversed*. This means that for the same machine we have two relations which are only satisfied if in the reversible case,

$$\sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) \le 0 \qquad \qquad ...(3.13)$$

For a reversible case, as the number of reservoirs used tends to infinity, the limiting value of the summation will be

$$\sum_{\text{cycle}} \left(\frac{\delta Q}{T} \right) = 0$$

In words, the Clausius inequality may be expressed as follows :

"*When a system performs a reversible cycle, then*

$$\sum_{\text{cycle}} \left(\frac{\delta Q}{T} \right) = 0,$$

but when the cycle is not reversible

$$\sum_{\text{cycle}} \left(\frac{\delta Q}{T} \right) < 0".$$

3.8. CARNOT CYCLE

The cycle was first suggested by a French engineer Sadi Carnot in 1824 which works on reversible cycle and is known as *Carnot cycle*.

Any fluid may be used to operate the Carnot cycle (Fig. 3.5) which is performed in an engine cylinder the head of which is supposed alternatively to be perfect conductor or a perfect insulator of a heat. Heat is caused to flow into the cylinder by the application of high temperature energy source to the cylinder head during expansion, and to flow from the cylinder by the application of a lower temperature energy source to the head during compression.
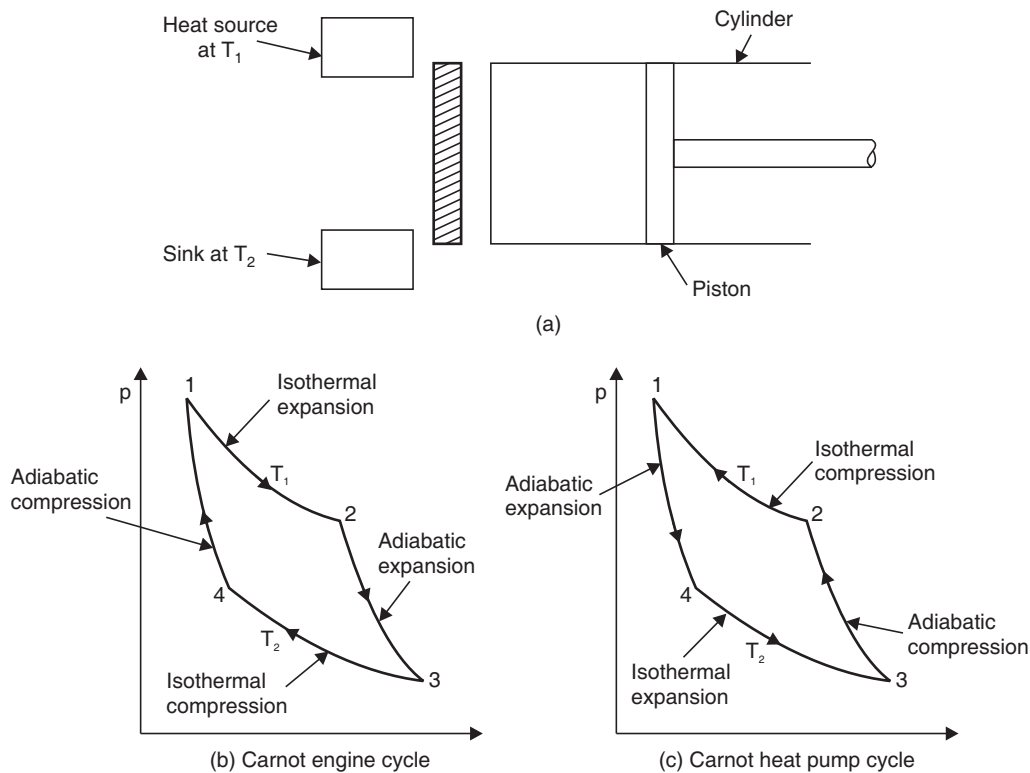
Fig. 3.5

The *assumptions* made for describing the working of the Carnot engine are as follows :

(*i*) The piston moving in a cylinder does not develop any friction during motion.

(*ii*) The walls of piston and cylinder are considered as perfect insulators of heat.

(*iii*) The cylinder head is so arranged that it can be a perfect heat conductor or perfect heat insulator.

(*iv*) The transfer of heat does not affect the temperature of source or sink.

(*v*) Working medium is a perfect gas and has constant specific heat.

(*vi*) Compression and expansion are reversible.

Following are the *four stages* of Carnot cycle :

Stage 1. (Process 1-2). Hot energy source is applied. Heat Q_1 is taken in whilst the fluid expands isothermally and reversibly at constant high temperature T_1.

Stage 2. (Process 2-3). The cylinder becomes a perfect insulator so that no heat flow takes place. The fluid expands adiabatically and reversibly whilst temperature falls from T_1 to T_2.

Stage 3. (Process 3-4). Cold energy source is applied. Heat Q_2 flows from the fluid whilst it is compressed isothermally and reversibly at constant lower temperature T_2.

Stage 4. (Process 4-1). Cylinder head becomes a perfect insulator so that no heat flow occurs. The compression is continued adiabatically and reversibly during which temperature is raised from T_2 to T_1.

The work delivered from the system during the cycle is represented by the enclosed area of the cycle. Again for a closed cycle, according to first law of the thermodynamics the work obtained is equal to the difference between the heat supplied by the source (Q_1) and the heat rejected to the sink (Q_2).

$$\therefore \qquad\qquad W = Q_1 - Q_2$$

Also, thermal efficiency, $\eta_{th} = \dfrac{\text{work done}}{\text{heat supplied by the source}} = \dfrac{Q_1 - Q_2}{Q_1}$

$$= 1 - \frac{Q_2}{Q_1}\left(= 1 - \frac{T_2}{T_1}\right) \qquad \begin{bmatrix} \because & Q_1 = mc_p T_1 \\ & Q_2 = mc_p\, T_2 \\ \text{where } m = \text{mass of fluid} \end{bmatrix}$$

Such an engine since it consists entirely of reversible processes, can operate in the reverse direction so that it follows the cycle shown in Fig. 3.5 (*b*) and operates as a **heat pump.** Q_2 is being taken in at the lower temperature T_2 during the isothermal expansion (process 4-3) and heat Q_1 is being rejected at the upper temperature T_1 (process 2-1). Work W will be needed to drive the pump. Again, the enclosed area represents this work which is exactly equal to that flowing from it when used as engine.

The Carnot cycle *cannot be performed in practice* because of the following **reasons** :

1. It is imposible to perform a frictionless process.

2. It is impossible to transfer the heat without temperature potential.

3. Isothermal process can be achieved only if the piston moves very slowly to allow heat transfer so that the temperature remains contant. Adiabatic process can be achieved only if the piston moves as fast as possible so that the heat transfer is negligible due to very short time available. The isothermal and adiabatic processes take place during the same stroke therefore the piston has to move very slowly for part of the stroke and it has to move very fast during remaining stroke. This variation of motion of the piston during the same stroke is not possible.

3.9. CARNOT'S THEOREM

"It states that of all engines operating between a given constant temperature source and a given constant temperature sink, none has a higher efficiency than a reversible engine".

Refer Fig. 3.6.
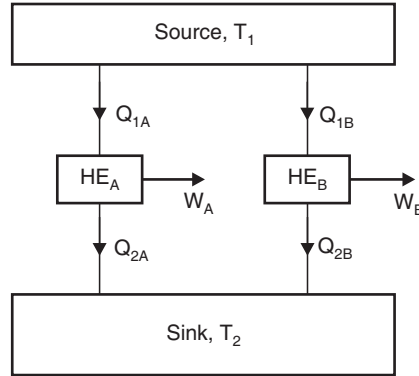
Fig. 3.6. Two cyclic heat engines HE_A and HE_B operating between the same source and sink, of which HE_B is reversible.

HE_A and HE_B are the two engines operating between the given source at temperature T_1 and the given sink at temperature T_2.

Let HE_A be *any* heat engine and HE_B be *any reversible* heat engine. We have to prove that efficiency of HE_B is more than that of HE_A. Let us assume that $\eta_A > \eta_B$. Let the rates of working of the engines be such that

$$Q_{1A} = Q_{1B} = Q_1$$

Since

$$\eta_A > \eta_B$$

$$\frac{W_A}{Q_{1A}} > \frac{W_B}{Q_{1B}}$$

$$\therefore \qquad W_A > W_B$$

Now, let HE_B be reversed. Since HE_B is a reversible heat engine, the magnitudes of heat and work transfer quantities will remain the same, but their directions will be reversed, as shown in Fig. 3.7. Since $W_A > W_B$, some part of W_A (equal to W_B) may be fed to drive the reversed heat engine
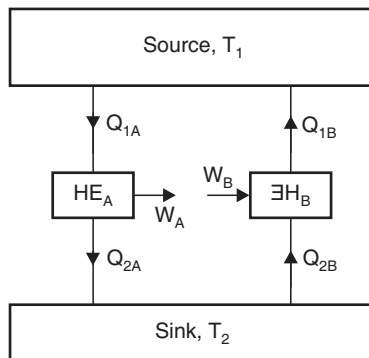
Fig. 3.7. HE_B is reversed.
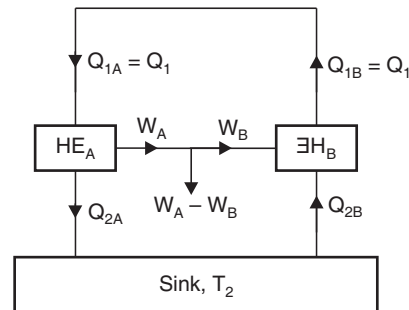
Fig. 3.8. HE_A and $\exists H_B$ together violate the Kelvin-Planck statement.

$\exists H_B$. Since $Q_{1A} = Q_{1B} = Q_1$, the heat discharged by $\exists H_B$ may be supplied to HE_A. The source may, therefore, be eliminated (Fig. 3.8). The net result is that HE_A and $\exists H_B$ together constitute a heat engine which, operating in a cycle produces net work $W_A - W_B$ while exchanging heat with a single reservoir at T_2. This violates the Kelvin-Planck statement of the second law. Hence the assumption that $\eta_A > \eta_B$ is wrong.

$$\therefore \qquad \qquad \eta_B \geq \eta_A.$$

3.10. COROLLARY OF CARNOT'S THEOREM

"The efficiency of all reversible heat engines operating between the same temperature levels is the same".

Refer Fig. 3.6. Let both the heat engines HE_A and HE_B be *reversible*. Let us assume $\eta_A > \eta_B$. Similar to the procedure outlined in the Article 3.9, if HE_B is reversed to run say, as a heat pump using some part of the work output (W_A) of engine HE_A, we see that the combined system of heat pump HE_B and engine HE_A, becomes a *PMM2*. So η_A cannot be greater than η_B. Similary, if we assume $\eta_B > \eta_A$ and reverse the engine HE_A, we observe that η_B cannot be greater than η_A

$$\therefore \qquad \qquad \eta_A = \eta_B.$$

Since the efficiencies of all reversible engines operating between the same heat reservoirs are the same, *the efficiency of a reversible engine is independent of the nature or amount of the working substance undergoing the cycle.*

3.11. EFFICIENCY OF THE REVERSIBLE HEAT ENGINE

The efficiency of a reversible heat engine in which heat is received solely at T_1 is found to be

$$\eta_{rev.} = \eta_{max} = 1 - \left(\frac{Q_2}{Q_1}\right)_{rev.} = 1 - \frac{T_2}{T_1}$$

or
$$\eta_{rev.} = \frac{T_1 - T_2}{T_1}$$

From the above expression, it may be noted that as T_2 decreases and T_1 increases, the efficiency of the reversible cycle increases.

Since η is always less than unity, T_2 is always greater than zero and +ve.

The C.O.P. of a refrigerator is given by

$$(\text{C.O.P.})_{ref.} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\dfrac{Q_1}{Q_2} - 1}$$

For a *reversible refrigerator*, using

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$(\text{C.O.P.})_{rev.} = \frac{1}{\dfrac{T_1}{T_2} - 1}$$

$$\therefore \qquad [(\text{C.O.P.})_{ref.}]_{rev.} = \frac{T_2}{T_1 - T_2} \qquad \qquad \qquad ...(3.14)$$

Similarly, for a *reversible heat pump*

$$[(\text{C.O.P.})_{heat\ pump}]_{rev.} = \frac{T_1}{T_1 - T_2} \qquad \qquad ...(3.15)$$

Example 3.1. *A heat engine receives heat at the rate of 1500 kJ/min and gives an output of 8.2 kW. Determine :*

(*i*) *The thermal efficiency* ;

(*ii*) *The rate of heat rejection.*

Solution. Heat received by the heat engine,

$$Q_1 = 1500 \text{ kJ/min}$$

$$= \frac{1500 \times 1000}{60} = 25 \times 10^3 \text{ J/s}$$

Work output, $W = 8.2 \text{ kW} = 8.2 \times 10^3 \text{ W}$ or   J/s.

(*i*) **Thermal efficiency,** $\eta_{th} = \dfrac{W}{Q_1}$

$$= \frac{8.2 \times 10^3}{25 \times 10^3} = 0.328 = 32.8\%$$

Hence, **thermal efficiency = 32.8%.   (Ans.)**

(*ii*) **Rate of heat rejection,**

$$Q_2 = Q_1 - W = 25 \times 10^3 - 8.2 \times 10^3$$
$$= 16.8 \times 10^3 \text{ J/s} = 16.8 \text{ kJ/s}$$

Hence, **the rate of heat rejection = 16.8 kJ/s.   (Ans.)**

Fig. 3.9

☞**Example 3.2.** *During a process a system receives 30 kJ of heat from a reservoir and does 60 kJ of work. Is it possible to reach initial state by an adiabatic process ?*

Solution. Heat received by the system = 30 kJ

Work done = 60 kJ
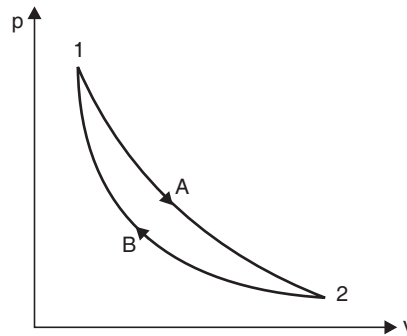
Fig. 3.10

Process 1-2 : By first law of thermodynamics

$$Q_{1-2} = (U_2 - U_1) + W_{1-2}$$
$$30 = (U_2 - U_1) + 60 \quad \therefore \quad (U_2 - U_1) = -30 \text{ kJ}.$$
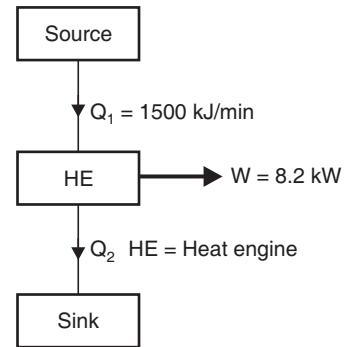
Process 2-1 : By first law of thermodynamics

$$Q_{2-1} = (U_1 - U_2) + W_{2-1}$$

∴ $0 = 30 + W_{2-1}$ ∴ $W_{2-1} = - 30$ kJ.

Thus 30 kJ work has to be done *on the system* to restore it to original state, by adiabatic process.

Example 3.3. *Find the co-efficient of performance and heat transfer rate in the condenser of a refrigerator in kJ/h which has a refrigeration capacity of 12000 kJ/h when power input is 0.75 kW.*

Solution. Refer Fig. 3.11.

Refrigeration capacity, $Q_2 = 12000$ kJ/h

Power input, $W = 0.75$ kW (= 0.75 × 60 × 60 kJ/h)

Co-efficient of performance, C.O.P. :

Heat transfer rate :

$$(C.O.P.)_{refrigerator} = \frac{\text{heat absorbed at lower temperature}}{\text{work input}}$$

∴ $C.O.P. = \dfrac{Q_2}{W} = \dfrac{12000}{0.75 \times 60 \times 60} = 4.44$

Hence **C.O.P. = 4.44. (Ans.)**

Hence transfer rate in condenser = Q_1

According to the first law

$$Q_1 = Q_2 + W = 12000 + 0.75 \times 60 \times 60 = 14700 \text{ kJ/h}$$

Hence, **heat transfer rate = 14700 kJ/h. (Ans.)**

Fig. 3.11

Example 3.4. *A domestic food refrigerator maintains a temperature of – 12°C. The ambient air temperature is 35°C. If heat leaks into the freezer at the continuous rate of 2 kJ/s determine the least power necessary to pump this heat out continuously.*

Solution. Freezer temperature,

$$T_2 = - 12 + 273 = 261 \text{ K}$$

Ambient air temperature,

$$T_1 = 35 + 273 = 308 \text{ K}$$

Rate of heat leakage into the freezer = 2 kJ/s

Least power required to pump the heat :

The refrigerator cycle removes heat from the freezer at the same rate at which heat leaks into it (Fig. 3.12).

For minimum power requirement

$$\frac{Q_2}{T_2} = \frac{Q_1}{T_1}$$

∴ $Q_1 = \dfrac{Q_2}{T_2} \times T_1 = \dfrac{2}{261} \times 308 = 2.36$ kJ/s

∴ $W = Q_1 - Q_2$

$= 2.36 - 2 = 0.36$ kJ/s = 0.36 kW

Fig. 3.12

Hence, **least power required to pump the heat continuously**

$$= \textbf{0.36 kW. (Ans.)}$$
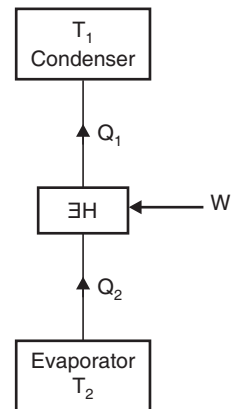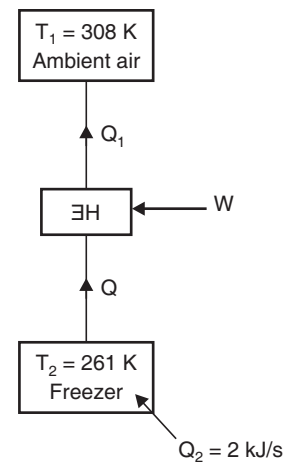
Example 3.5. *A house requires 2×10^5 kJ/h for heating in winter. Heat pump is used to absorb heat from cold air outside in winter and send heat to the house. Work required to operate the heat pump is 3×10^4 kJ/h. Determine :*

   (*i*) *Heat abstracted from outside ;*

  (*ii*) *Co-efficient of performance.*

   Solution. (*i*) Heat requirement of the house, Q_1 (or heat rejected)

$$= 2 \times 10^5 \text{ kJ/h}$$

   Work required to operate the heat pump,

$$W = 3 \times 10^4 \text{ kJ/h}$$

   Now, $Q_1 = W + Q_2$

where Q_2 is the heat abstracted from outside.

   $\therefore$ $2 \times 10^5 = 3 \times 10^4 + Q_2$

   Thus $Q_2 = 2 \times 10^5 - 3 \times 10^4$

$$= 200000 - 30000 = 170000 \text{ kJ/h}$$

   Hence, **heat abstracted from outside = 170000 kJ/h.   (Ans.)**

   (*ii*) $(\text{C.O.P.})_{heat\ pump} = \dfrac{Q_1}{Q_1 - Q_2}$

$$= \dfrac{2 \times 10^5}{2 \times 10^5 - 170000} = 6.66$$

   Hence, **co-efficient of performance = 6.66.   (Ans.)**

   Note. If the heat requirements of the house were the same but this amount of heat had to be abstracted from the house and rejected out, *i.e. cooling of the house in summer*, we have

$$(\text{C.O.P.})_{refrigerator} = \dfrac{Q_1}{Q_1 - Q_2} = \dfrac{Q_2}{W}$$

$$= \dfrac{170000}{3 \times 10^4} = \textbf{5.66}$$

   Thus the same device has two values of C.O.P. depending upon the objective.

   Example 3.6. *What is the highest possible theoretical efficiency of a heat engine operating with a hot reservoir of furnace gases at 2100°C when the cooling water available is at 15°C ?*

   Solution. Temperature of furnace gases, $T_1 = 2100 + 273 = 2373$ K

   Temperature of cooling water, $T_2 = 15 + 273 = 288$ K

   Now, $\eta_{max} (= \eta_{carnot}) = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{288}{2373} = \textbf{0.878 or 87.8\%.   (Ans.)}$

   Note. It should be noted that a system in practice operating between similar temperatures (*e.g.* a steam generating plant) would have a thermal efficiency of about 30%. The discrepency is *due to irreversibility in the actual plant*, and *also because of deviations from the ideal Carnot cycle made for various practical reasons.*

   Example 3.7. *A Carnot cycle operates between source and sink temperatures of 250°C and – 15°C. If the system receives 90 kJ from the source, find :*

   (*i*) *Efficiency of the system ;*   (*ii*) *The net work transfer ;*

  (*iii*) *Heat rejected to sink.*

Solution. Temperature of source, $T_1 = 250 + 273 = 523$ K

Temperature of sink, $T_2 = -15 + 273 = 258$ K

Heat received by the system, $Q_1 = 90$ kJ

(i) $$\eta_{\textbf{carnot}} = 1 - \frac{T_2}{T_1} = 1 - \frac{258}{523} = 0.506 = \textbf{50.6\%. \quad (Ans.)}$$

(ii) **The net work transfer,** $\quad W = \eta_{carnot} \times Q_1 \qquad\qquad \left[\because \quad \eta_{carnot} = \frac{W}{Q_1} \right]$

$$= 0.506 \times 90 = \textbf{45.54 kJ. \quad (Ans.)}$$

(iii) **Heat rejected to the sink,** $Q_2 = Q_1 - W \qquad\qquad\qquad [\because \quad W = Q_1 - Q_2]$

$$= 90 - 45.54 = \textbf{44.46 kJ. \quad (Ans.)}$$

Example 3.8. *An inventor claims that his engine has the following specifications* :

Temperature limits ...... *750°C and 25°C*

Power developed ...... *75 kW*

Fuel burned per hour ...... *3.9 kg*

Heating value of the fuel ...... *74500 kJ / kg*

State whether his claim is valid or not.

Solution. Temperature of source, $T_1 = 750 + 273 = 1023$ K

Temperature of sink, $T_2 = 25 + 273 = 298$ K

We know that the thermal efficiency of Carnot cycle is the maximum between the specified temperature limits.

Now, $$\eta_{carnot} = 1 - \frac{T_2}{T_1} = 1 - \frac{298}{1023} = 0.7086 \quad \text{or} \quad 70.86\%$$

The actual thermal efficiency claimed,

$$\eta_{thermal} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{75 \times 1000 \times 60 \times 60}{3.9 \times 74500 \times 1000} = 0.9292 \quad \text{or} \quad 92.92\%.$$

Since $\eta_{thermal} > \eta_{carnot}$, therefore claim of the inventor is **not valid** (or possible). **(Ans.)**

Example 3.9. *A cyclic heat engine operates between a source temperature of 1000°C and a sink temperature of 40°C. Find the least rate of heat rejection per kW net output of the engine.*

Solution. Temperature of source,

$$T_1 = 1000 + 273 = 1273 \text{ K}$$

Temperature of sink,

$$T_2 = 40 + 273 = 313 \text{ K}$$

Least rate of heat rejection per kW net output :

For a reversible heat engine, the rate of heat rejection will be minimum (Fig. 3.13)

$$\eta_{max} = \eta_{rev.} = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{313}{1273} = 0.754$$

Now $\quad \dfrac{W_{net}}{Q_1} = \eta_{max} = 0.754$
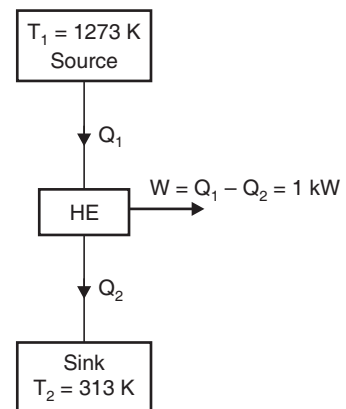
Fig. 3.13

$\therefore$ $Q_1 = \dfrac{W_{net}}{0.754} = \dfrac{1}{0.754} = 1.326$ kW

Now $Q_2 = Q_1 - W_{net} = 1.326 - 1$
$= 0.326$ kW

Hence, **the least rate of heat rejection**
= 0.326 kW.   (Ans.)

Example 3.10. *A fish freezing plant requires 40 tons of refrigeration. The freezing temperature is – 35°C while the ambient temperature is 30°C. If the performance of the plant is 20% of the theoretical reversed Carnot cycle working within the same temperature limits, calculate the power required.*

Given : 1 ton of refrigeration = 210 kJ/min.

Solution. Cooling required = 40 tons = 40 × 210
$= 8400$ kJ/min

Ambient temperature, $T_1 = 30 + 273 = 303$ K
Freezing temperature, $T_2 = -35 + 273 = 238$ K
Performance of plant $= 20\%$ of the theoretical reversed Carnot cycle

$$(\text{C.O.P.})_{refrigerator} = \frac{T_2}{T_1 - T_2} = \frac{238}{303 - 238} = 3.66$$

$\therefore$ Actual C.O.P = $0.20 \times 3.66 = 0.732$

Now work needed to produce cooling of 40 tons is calculated as follows :

$$(\text{C.O.P.})_{actual} = \frac{\text{cooling reqd.}}{\text{work needed}}$$

$$0.732 = \frac{8400}{W} \quad \text{or} \quad W = \frac{8400}{0.732} \text{ kJ/min} = 191.25 \text{ kJ/s} = 191.25 \text{ kW}$$

Hence, **power required = 191.25 kW.   (Ans.)**

Example 3.11. *Source 1 can supply energy at the rate of 12000 kJ/min at 320°C. A second source 2 can supply energy at the rate of 120000 kJ/min at 70°C. Which source (1 or 2) would you choose to supply energy to an ideal reversible heat engine that is to produce large amount of power if the temperature of the surroundings is 35°C ?*

Solution. Source 1 :
Rate of supply of energy = 12000 kJ/min
Temperature, $T_1 = 320 + 273 = 593$ K.

Source 2 :
Rate of supply of energy = 120000 kJ/min
Temperature, $T_1 = 70 + 273 = 343$ K
Temperature of the surroundings, $T_2 = 35°C + 273 = 308$ K

Let the Carnot engine be working in the two cases with the two source temperatures and the single sink temperature. The efficiency of the cycle will be given by :

$$\eta_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{308}{593} = 0.4806 \quad \text{or} \quad 48.06\%$$

$$\eta_2 = 1 - \frac{T_2}{T_1} = 1 - \frac{308}{343} = 0.102 \quad \text{or} \quad 10.2\%$$

∴   The work delivered in the two cases is given by

$$W_1 = 12000 \times 0.4806 = 5767.2 \text{ kJ/min}$$

and $$W_2 = 120000 \times 0.102 = 12240 \text{ kJ/min}.$$

Thus, choose **source 2**.   **(Ans.)**

Note. The source 2 is selected even though *efficiency* in this case is *lower*, because the *criterion for selection is the larger output.*

☞**Example 3.12.** *A reversible heat engine operates between two reservoirs at temperatures 700°C and 50°C. The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 50°C and – 25°C. The heat transfer to the engine is 2500 kJ and the net work output of the combined engine refrigerator plant is 400 kJ.*

(i) Determine the heat transfer to the refrigerant and the net heat transfer to the resevoir at 50°C ;

(ii) Reconsider (i) given that the efficiency of the heat engine and the C.O.P. of the refrigerator are each 45 per cent of their maximum possible values.

Solution. Refer Fig. 3.14.
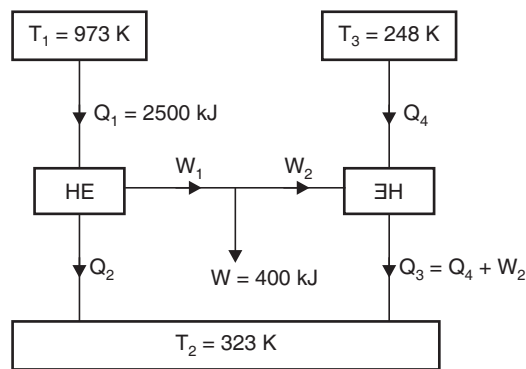
Fig. 3.14

Temperature, $T_1 = 700 + 273 = 973$ K
Temperature, $T_2 = 50 + 273 = 323$ K
Temperature, $T_3 = -25 + 273 = 248$ K
The heat transfer to the heat engine, $Q_1 = 2500$ kJ
The network output of the combined engine refrigerator plant,

$$W = W_1 - W_2 = 400 \text{ kJ}.$$

(*i*) Maximum efficiency of the heat engine cycle is given by

$$\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{323}{973} = 0.668$$

Again, $$\frac{W_1}{Q_1} = 0.668$$

∴ $$W_1 = 0.668 \times 2500 = 1670 \text{ kJ}$$

$$(\text{C.O.P.})_{max} = \frac{T_3}{T_2 - T_3} = \frac{248}{323 - 248} = 3.306$$

Also,   C.O.P. $= \dfrac{Q_4}{W_2} = 3.306$

Since, $W_1 - W_2 = W = 400$ kJ
                          $W_2 = W_1 - W = 1670 - 400 = 1270$ kJ
∴ $Q_4 = 3.306 \times 1270 = 4198.6$ kJ
                          $Q_3 = Q_4 + W_2 = 4198.6 + 1270 = 5468.6$ kJ
                          $Q_2 = Q_1 - W_1 = 2500 - 1670 = 830$ kJ.

Heat rejection to the 50°C reservoir

                          $= Q_2 + Q_3 = 830 + 5468.6 =$ **6298.6 kJ.**   **(Ans.)**

(*ii*) Efficiency of actual heat engine cycle,

                          $\eta = 0.45 \, \eta_{max} = 0.45 \times 0.668 = 0.3$
∴ $W_1 = \eta \times Q_1 = 0.3 \times 2500 = 750$ kJ
∴ $W_2 = 750 - 400 = 350$ kJ

C.O.P. of the actual refrigerator cycle,

                          C.O.P. $= \dfrac{Q_4}{W_2} = 0.45 \times 3.306 = 1.48$

∴ $Q_4 = 350 \times 1.48 =$ **518 kJ.**   **(Ans.)**
                          $Q_3 = 518 + 350 = 868$ kJ
                          $Q_2 = 2500 - 750 = 1750$ kJ

Heat rejected to 50°C reservoir

                          $= Q_2 + Q_3 = 1750 + 868 =$ **2618 kJ.**   **(Ans.)**

☞**Example 3.13.** (*i*) *A reversible heat pump is used to maintain a temperature of 0°C in a refrigerator when it rejects the heat to the surroundings at 25°C. If the heat removal rate from the refrigerator is 1440 kJ/min, determine the C.O.P. of the machine and work input required.*

(*ii*) *If the required input to run the pump is developed by a reversible engine which receives heat at 380°C and rejects heat to atmosphere, then determine the overall C.O.P. of the system.*

(PTU)

Solution. Refer Fig. 3.15 (*a*).
 (*i*) Temperature, $T_1 = 25 + 273 = 298$ K
     Temperature, $T_2 = 0 + 273 = 273$ K

(a) Single system   (b) Combined system
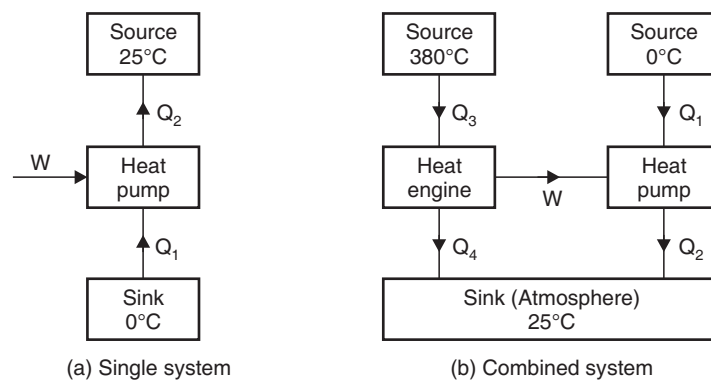
Fig. 3.15

Heat removal rate from the refrigerator, $Q_1 = 1440$ kJ/min = 24 kJ/s

Now, co-efficient of performance, for reversible heat pump,

$$\textbf{(C.O.P.)}_{\textbf{heat pump}} = \frac{T_1}{T_1 - T_2} = \frac{298}{(298 - 273)} = \textbf{11.92.} \quad \textbf{(Ans.)}$$

$$\therefore \qquad\qquad (\text{C.O.P.})_{\text{ref.}} = \frac{T_2}{T_1 - T_2} = \frac{273}{298 - 273} = 10.92$$

Now, $\qquad\qquad\qquad 10.92 = \dfrac{Q_1}{W} = \dfrac{24}{W}$

$$\therefore \qquad\qquad\qquad W = 2.2 \text{ kW}$$

i.e., $\qquad$ **Work input required = 2.2 kW.   (Ans.)**

$$Q_2 = Q_1 + W = 24 + 2.2 = 26.2 \text{ kJ/s}$$

(*ii*) Refer Fig. 3.15 (*b*).

The overall C.O.P. is given by,

$$\text{C.O.P.} = \frac{\text{Heat removed from the refrigerator}}{\text{Heat supplied from the source}}$$

$$= \frac{Q_1}{Q_3} \qquad\qquad\qquad\qquad\qquad\qquad ...(i)$$

For the reversible engine, we can write

$$\frac{Q_3}{T_3} = \frac{Q_4}{T_4}$$

or $\qquad\qquad\qquad\qquad \dfrac{Q_4 + W}{T_3} = \dfrac{Q_4}{T_4}$

or $\qquad\qquad\qquad\qquad \dfrac{Q_4 + 2.2}{(380 + 273)} = \dfrac{Q_4}{(25 + 273)}$

or $\qquad\qquad\qquad\qquad \dfrac{Q_4 + 2.2}{653} = \dfrac{Q_4}{298}$

or $\qquad\qquad\qquad\qquad 298(Q_4 + 2.2) = 653\, Q_4$

or $\qquad\qquad\qquad\qquad Q_4(653 - 298) = 298 \times 2.2$

or $\qquad\qquad\qquad\qquad Q_4 = \dfrac{298 \times 2.2}{(653 - 298)} = 1.847 \text{ kJ/s}$

$$\therefore \qquad\qquad\qquad Q_3 = Q_4 + W = 1.847 + 2.2 = 4.047 \text{ kJ/s}$$

Substituting this value in eqn. (*i*), we get

$$\textbf{C.O.P.} = \frac{24}{4.047} = \textbf{5.93.} \quad \textbf{(Ans.)}$$

If the purpose of the system is to supply the heat to the sink at 25°C, then

$$\textbf{Overall C.O.P.} = \frac{Q_2 + Q_4}{Q_3} = \frac{26.2 + 1.847}{4.047} = \textbf{6.93.} \quad \textbf{(Ans.)}$$

Example 3.14. *An ice plant working on a reversed Carnot cycle heat pump produces 15 tonnes of ice per day. The ice is formed from water at 0°C and the formed ice is maintained at 0°C. The heat is rejected to the atmosphere at 25°C. The heat pump used to run the ice plant is coupled to a Carnot*

engine which absorbs heat from a source which is maintained at 220°C by burning liquid fuel of 44500 kJ/kg calorific value and rejects the heat to the atmosphere. Determine :

  (*i*) *Power developed by the engine ;*

(*ii*) *Fuel consumed per hour.*

Take enthalpy of fusion of ice = 334.5 kJ/kg.

Solution. (*i*) Fig. 3.16 shows the arrangement of the system.

Amount of ice produced per day = 15 tonnes.
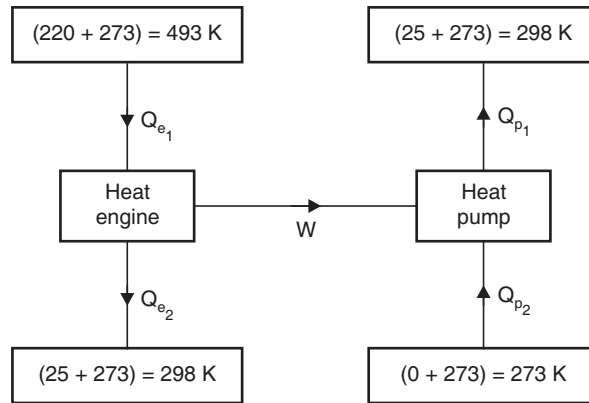
Fig. 3.16

∴   The amount of heat removed by the heat pump,

$$Q_{p_2} = \frac{15 \times 1000 \times 334.5}{24 \times 60} = 3484.4 \text{ kJ/min}$$

C.O.P. of the heat pump $= \dfrac{Q_{p_2}}{W} = \dfrac{273}{298 - 273}$.

∴ $W = Q_{p_2} \times \dfrac{298 - 273}{273} = 3484.4 \times \dfrac{25}{273} = 319.08 \text{ kJ/min}$

This work must be developed by the Carnot engine,

$$W = \frac{319.08}{60} = 5.3 \text{ kJ/s} = 5.3 \text{ kW}$$

Thus **power developed by the engine = 5.3 kW.**   **(Ans.)**

(*ii*) The efficiency of Carnot engine is given by

$$\eta_{carnot} = \frac{W}{Q_{e_1}} = 1 - \frac{298}{493} = 0.396$$

∴ $Q_{e_1} = \dfrac{W}{0.396} = \dfrac{5.3}{0.396} = 13.38 \text{ kJ/s}$

∴ Q_{e_1} (*per hour*) = 13.38 × 60 × 60 = 48168 kJ

∴ **Quantity of fuel consumed/hour**

$$= \frac{48168}{44500} = \textbf{1.082 kg/h.} \quad \textbf{(Ans.)}$$

Example 3.15. *Two Carnot engines work in series between the source and sink temperatures of 550 K and 350 K. If both engines develop equal power determine the intermediate temperature.*

Solution. Fig. 3.17 shows the arrangement of the system.

Temperature of the source, $T_1 = 550$ K

Temperature of the sink, $T_3 = 350$ K

Intermediate temperature, T_2 :

The efficiencies of the engines HE_1 and HE_2 are given by

$$\eta_1 = \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1} = \frac{W}{Q_2 + W} \qquad \qquad ...(i)$$

$$\eta_2 = \frac{W}{Q_2} = \frac{T_2 - T_3}{T_2} = \frac{W}{Q_3 + W} \qquad \qquad ...(ii)$$

From eqn. (*i*), we get

$$W = (Q_2 + W)\left(\frac{T_1 - T_2}{T_1}\right)$$

$$\therefore \qquad W\left[1 - \left(\frac{T_1 - T_2}{T_1}\right)\right] = Q_2\left(\frac{T_1 - T_2}{T_1}\right)$$

$$\therefore \qquad W\left(\frac{T_2}{T_1}\right) = Q_2\left(\frac{T_1 - T_2}{T_1}\right)$$

$$\therefore \qquad W = Q_2\left(\frac{T_1 - T_2}{T_1}\right) \qquad \qquad ...(iii)$$

From eqn. (*ii*), we get

$$W = Q_2\left(\frac{T_2 - T_3}{T_2}\right) \qquad \qquad ...(iv)$$

Now from eqns. (*iii*) and (*iv*), we get

$$T_1 - T_2 = T_2 - T_3$$
$$2T_2 = T_1 + T_3 = 550 + 350$$
$$\therefore \qquad T_2 = 450 \text{ K}$$

Hence **intermediate temperature = 450 K.   (Ans.)**

Example 3.16. *A Carnot heat engine draws heat from a reservoir at temperature T_1 and rejects heat to another reservoir at temperature T_3. The Carnot forward cycle engine drives a Carnot reversed cycle engine or Carnot refrigerator which absorbs heat from reservoir at temperature T_2 and rejects heat to a reservoir at temperature T_3. If the high temperature T_1 = 600 K and low temperature T_2 = 300 K, determine :*

(i) The temperature T_3 such that heat supplied to engine Q_1 is equal to the heat absorbed by refrigerator Q_2.

(ii) The efficiency of Carnot engine and C.O.P. of Carnot refrigerator.

Solution. Refer Fig. 3.18.

Temperature, $T_1 = 600$ K

Temperature, $T_2 = 300$ K

Efficiency of Carnot engine,

Source
550 K

T_1

Q_1

HE$_1$ → W

T_2

Q_2

HE$_2$ → W

T_3

Q_3

Sink
350 K
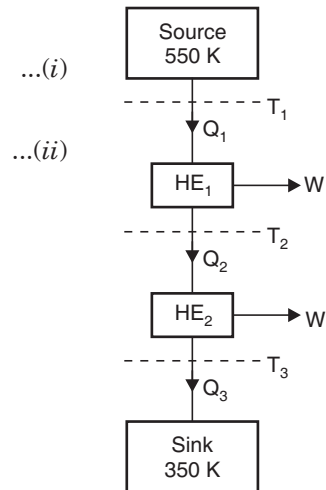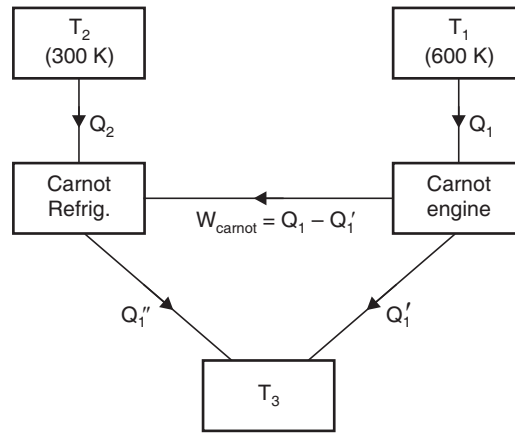
Fig. 3.17

Fig. 3.18

$$\eta_{carnot\ engine} = \frac{Q_1 - Q_1'}{Q_1} = \frac{T_1 - T_3}{T_1}$$

$$= \frac{\text{Work of Carnot engine}}{\text{Heat supplied to the Carnot engine}} = \frac{W_{carnot}}{Q_1}$$

or $$W_{carnot} = Q_1\left(\frac{T_1 - T_3}{T_1}\right) \qquad \qquad ...(i)$$

Also $\quad$ C.O.P.$_{(carnot\ refrigerator)} = \dfrac{Q_2}{Q_1'' - Q_2} = \dfrac{T_2}{T_3 - T_2}$

$$= \frac{\text{Heat absorbed}}{W_{carnot}} = \frac{Q_2}{W_{carnot}}$$

or $$W_{carnot} = Q_2\left(\frac{T_3 - T_2}{T_2}\right) \qquad \qquad ...(ii)$$

(*i*) **Temperature, T$_3$:**

From eqns. (*i*) and (*ii*), we get

$$Q_1\left(\frac{T_1 - T_3}{T_1}\right) = Q_2\left(\frac{T_3 - T_2}{T_2}\right)$$

∴ $$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}\left(\frac{T_1 - T_3}{T_3 - T_2}\right)$$

or $$\frac{Q_2}{Q_1} = 1 = \frac{300}{600}\left(\frac{600 - T_3}{T_3 - 300}\right)$$

or $$600 - T_3 = 2(T_3 - 300)$$

$$600 - T_3 = 2T_3 - 600 \quad \text{or} \quad T_3 = 400 \text{ K}$$

Hence, $\quad$ **temperature, T$_3$ = 400 K.   (Ans.)**

(*ii*) **Efficiency of Carnot engine,**

$$\eta_{carnot\ engine} = \frac{T_1 - T_3}{T_1} = \frac{600 - 400}{600} = 0.3333 = \textbf{33.33\%.} \quad \textbf{(Ans.)}$$

$$\text{C.O.P.}_{refrigerator} = \frac{T_2}{T_3 - T_2} = \frac{300}{400 - 300} = \textbf{3.} \quad \textbf{(Ans.)}$$

CLAUSIUS INEQUALITY

☞**Example 3.17.** *300 kJ/s of heat is supplied at a constant fixed temperature of 290°C to a heat engine. The heat rejection takes place at 8.5°C. The following results were obtained :*

(*i*) *215 kJ/s are rejected.*

(*ii*) *150 kJ/s are rejected.*

(*iii*) *75 kJ/s are rejected.*

Classify which of the result report a reversible cycle or irreversible cycle or impossible results.

Solution. Heat supplied at 290°C = 300 kJ/s

Heat rejected at 8.5°C : (*i*) 215 kJ/s, (*ii*) 150 kJ/s, (*iii*) 75 kJ/s.

Applying *Clausius inequality* to the cycle or process. Thus,

(*i*)
$$\sum_{cycle} \frac{\delta Q}{T} = \frac{300}{290 + 273} - \frac{215}{8.5 + 273}$$

$$= 0.5328 - 0.7637 = -0.2309 < 0.$$

∴ **Cycle is irreversible. (Ans.)**

(*ii*)
$$\sum_{cycle} \frac{\delta Q}{T} = \frac{300}{290 + 273} - \frac{150}{8.5 + 273}$$

$$= 0.5328 - 0.5328 = 0$$

∴ **Cycle is reversible. (Ans.)**

(*iii*)
$$\sum_{cycle} \frac{\delta Q}{T} = \frac{300}{290 + 273} - \frac{75}{8.5 + 273}$$

$$= 0.5328 - 0.2664 = 0.2664 > 0.$$

This **cycle is impossible** by second law of thermodynamics, *i.e. Clausius inequality*.   **(Ans.)**

Example 3.18. *A steam power plant operates between boiler temperature of 160°C and condenser temperature of 50°C. Water enters the boiler as saturated liquid and steam leaves the boiler as saturated vapour. Verify the Clausius inequality for the cycle.*

Given : Enthalpy of water entering boiler = 687 kJ/kg.

Enthalpy of steam leaving boiler = 2760 kJ/kg

Condenser pressure = 0.124 × 10⁵ N/m².

Solution. Boiler temperature, T_1 = 160 + 273 = 433 K

Condenser temperature, T_2 = 50 + 273 = 323 K

From steam tables :

Enthalpy of water entering boiler, h_{f_1} = 687 kJ/kg

Enthalpy of steam leaving boiler, h_2 = 2760 kJ/kg

Condenser pressure = 0.124×10^5 N/m^2

Boiler pressure = 6.18×10^5 N/m^2 ......(corresponding to 160°C)

Enthalpy of vapour leaving the turbine, h_3 = 2160 kJ/kg

  (assuming isentropic expansion)

Enthalpy of water leaving the condenser, hf_4 = 209 kJ/kg

Now Q_{boiler}, $Q_1 = h_2 - h_{f_1} = 2760 - 687 = 2073$ kJ/kg

and $Q_{condenser}$, $Q_2 = h_{f_4} - h_3 = 209 - 2160 = -1951$ kJ/kg

$\therefore$ $$\sum_{cycle} \frac{\delta Q}{T} = \frac{Q_1}{T_1} + \frac{Q_2}{T_2} = \frac{2073}{433} + \left(\frac{-1951}{323}\right)$$

$$= -1.25 \text{ kJ/kg K}$$

$$< 0. \qquad ...... \textbf{Proved.}$$

☞**Example 3.19.** *In a power plant cycle, the temperature range is 164°C to 51°C, the upper temperature being maintained in the boiler where heat is received and the lower temperature being maintained in the condenser where heat is rejected. All other processes in the steady flow cycle are adiabatic. The specific enthalpies at various points are given in Fig. 3.19.*

Verify the Clausius Inequality.
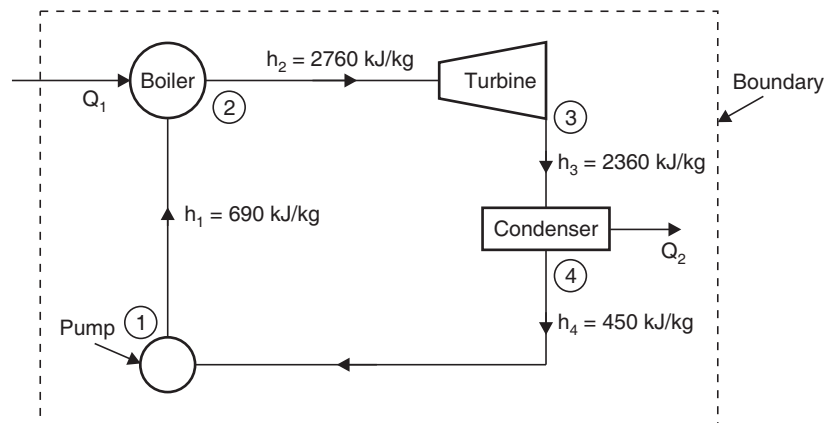
Fig. 3.19

Solution. Temperature maintained in boiler, T_1 = 164 + 273 = 437 K

Temperature maintained in condenser, T_2 = 51 + 273 = 324 K

Heat transferred in the boiler per kg of fluid,

$$Q_1 = h_2 - h_1 = 2760 - 690 = 2070 \text{ kJ/kg}$$

Heat transferred out at the condenser per kg of fluid,

$$Q_2 = h_4 - h_3 = 450 - 2360 = -1910 \text{ kJ/kg}$$

Since there is no transfer of heat at any other point, we have per kg

$$\sum_{cycle} \frac{\delta Q}{T} = \frac{Q_1}{T_1} + \frac{Q_2}{T_2} = \frac{2070}{437} + \left(\frac{-1910}{324}\right)$$

$$= 4.737 - 5.895$$

$$= -1.158 \text{ kJ/kg K} < 0.$$

The Clausius Inequality is proved. The steady flow cycle is obviously irreversible.

If the cycle is reversible between the same temperature limits and the heat supplied at higher temperature is same, the heat rejected can be calculated as follows :

$$\eta_{\text{reversible}} = 1 - \frac{T_2}{T_1} = 1 - \frac{324}{437} = 0.2586 \text{ or } 25.86\%$$

∴   Heat rejected per kg is given by

$$Q_2 = (1 - 0.2586) \times Q_1 = (1 - 0.2586) \times 2070 = 1534.7 \text{ kJ/kg}$$

$$\sum_{\text{cycle}} \frac{\delta Q}{T} = \frac{2070}{437} - \frac{1534.7}{324} = 4.73 - 4.73 = 0$$

i.e.,

$$\sum_{\text{cycle}} \frac{\delta Q}{T} = \frac{Q_{added}}{T_{source}} - \frac{Q_{rejected}}{T_{sink}} = 0$$

Thus **Clausius Equality sign for a reversible engine is verified.**

3.12. ENTROPY

3.12.1. Introduction

In heat engine theory, the term entropy plays a vital role and leads to important results which by other methods can be obtained much more laboriously.

It may be noted that all heat is not equally valuable for converting into work. Heat that is supplied to a substance at high temperature has a greater possibility of conversion into work than heat supplied to a substance at a lower temperature.

"*Entropy is a function of a quantity of heat which shows the possibility of conversion of that heat into work. The increase in entropy is small when heat is added at a high temperature and is greater when heat addition is made at a lower temperature. Thus for maximum entropy, there is minimum availability for conversion into work and for minimum entropy there is maximum availability for conversion into work.*"

3.12.2. Entropy—a Property of a System

Refer Fig. 3.20. Let us consider a system undergoing a reversible process from state 1 to state 2 along path *L* and then from state 2 to the original state 1 along path *M*. Applying the Clausius theorem to this reversible cyclic process, we have

$$\oint_R \frac{\delta Q}{T} = 0$$

(where the subscript designates a reversible cycle)

Hence when the system passes through the cycle 1-*L*-2-*M*-1, we have

$$\int_{1(L)}^{2} \frac{\delta Q}{T} + \int_{2(M)}^{1} \frac{\delta Q}{T} = 0 \qquad \qquad ...(3.16)$$

Now consider another reversible cycle in which the system changes from state 1 to state 2 along path *L*, but returns from state 2 to the original state 1 along a different path *N*. For this reversible cyclic process, we have

$$\int_{1(L)}^{2} \frac{\delta Q}{T} + \int_{2(N)}^{1} \frac{\delta Q}{T} = 0 \qquad \qquad ...(3.17)$$

Subtracting equation (3.17) from equation (3.16), we have

$$\int_{2(M)}^{1} \frac{\delta Q}{T} - \int_{2(N)}^{1} \frac{\delta Q}{T} = 0$$
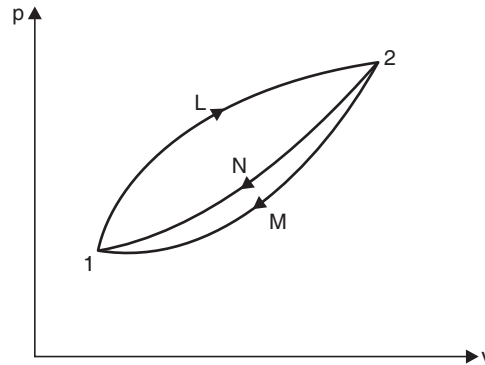
Fig. 3.20. Reversible cyclic process between two fixed end states.

or
$$\int_1^{2(M)} \frac{\delta Q}{T} = \int_1^{2(N)} \frac{\delta Q}{T}$$

As no restriction is imposed on paths L and M, except that they must be reversible, the

quantity $\dfrac{\delta Q}{T}$ *is a function of the initial and final states of the system and is independent of the path*

of the process. Hence it represents a property of the system. This property is known as the **"entropy".**

3.12.3. Change of Entropy in a Reversible Process

Refer Fig. 3.20.

Let S_1 = entropy at the initial state 1, and

          S_2 = entropy at the final state 2.

Then, the change in entropy of a system, as it undergoes a change from state 1 to 2, becomes

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_R \qquad \qquad ...(3.18)$$

Lastly, if the two equilibrium states 1 and 2 are infinitesimal near to each other, the integral sign may be omitted and $S_2 - S_1$ becomes equal to dS.

Hence equation (3.18) may be written as :

$$dS = \left(\frac{\delta Q}{T}\right)_R \qquad \qquad ...(3.19)$$

where dS is an exact differential.

Thus, from equation (3.19), we find that the change of entropy in a reversible process is

equal to $\dfrac{\delta Q}{T}$. *This is the mathematical formulation of the second law of thermodynamics.*

Equation (3.19) indicates that when an *inexact* differential δQ is divided by an integrating factor T during a reversible process, it becomes an *exact differential*.

The *third law of thermodynamics* states "*When a system is at zero absolute temperature, the entropy of system is zero*".

It is clear from the above law that the absolute value of entropy corresponding to a given

state of the system could be determined by integrating $\left(\dfrac{\delta Q}{T}\right)_R$ between the state at absolute zero

and the given state. *Zero entropy, however, means the absence of all molecular, atomic, electronic and nuclear disorders.*

As it is not practicable to get data at zero absolute temperature, the change in entropy is calculated either between two known states or by selecting some convenient point at which the entropy is given an arbitrary value of zero. For steam, the reference point at which the entropy is given an arbitrary value of zero is 0°C and for refrigerants like ammonia, Freon-12, carbon dioxide etc. the reference point is – 40°C, at which the entropy it taken as zero.

Thus, in practice we can *determine the change in entropy and not the absolute value of entropy.*

3.13. ENTROPY AND IRREVERSIBILITY

We know that change in entropy in a *reversible process* is equal to $\left(\dfrac{\delta Q}{T}\right)_R$ (equation 3.19). Let us now find the change in entropy in an *irreversible process.*
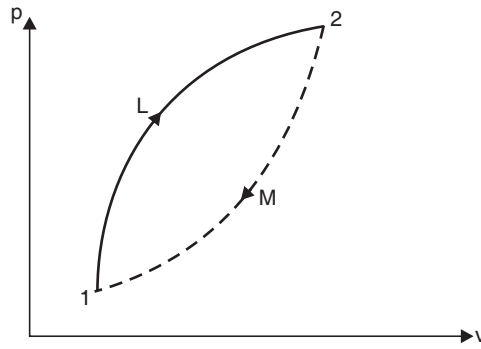
Fig. 3.21. Entropy change for an irreversible process.

Consider a closed system undergoing a change from state 1 to state 2 by a reversible process 1-*L*-2 and returns from state 2 to the initial state 1 by an irreversible process 2-*M*-1 as shown in Fig. 3.21 on the thermodynamic coordinates, pressure and volume.

Since entropy is a thermodynamic property, we can write

$$\oint dS = \int_{1(L)}^{2} (dS)_R \int_{2(M)}^{1} (dS)_I = 0 \qquad \qquad ...(3.20)$$

(Subscript *I* represents the irreversible process).

Now for a reversible process, from equation (3.19), we have

$$\int_{1(L)}^{2} (dS)_R = \int_{1(L)}^{2} \left(\frac{\delta Q}{T}\right)_R \qquad \qquad ...(3.21)$$

Substituting the value of $\int_{1(L)}^{2} (dS)_R$ in equation (3.20), we get

$$\int_{1(L)}^{2} \left(\frac{\delta Q}{T}\right)_R + \int_{2(M)}^{1} (dS)_I = 0 \qquad \qquad ...(3.22)$$

Again, since in equation (3.20) the processes 1-*L*-2 and 2-*M*-1 together form an irreversible cycle, applying Clausius equality to this expression, we get

$$\oint \frac{\delta Q}{T} = \int_{1(L)}^{2} \left(\frac{\delta Q}{T}\right)_R + \int_{2(M)}^{1} \left(\frac{\delta Q}{T}\right)_I < 0 \qquad \qquad ...(3.23)$$

Now subtracting equation (3.23) from equation (3.22), we get

$$\int_{2(M)}^{1} (dS)_I > \int_{2(M)}^{1} \left(\frac{\delta Q}{T}\right)_I$$

which for infinitesimal changes in states can be written as :

$$(dS)_I > \int \left(\frac{\delta Q}{T}\right)_I \qquad\qquad ...(3.24)$$

Equation (3.24) states that the *change in entropy in an irreversible process is greater than*

$\dfrac{\delta Q}{T}$.

Combining equations (3.23) and (3.24), we can write the equation in the general form as :

$$dS \geq \frac{\delta Q}{T} \qquad\qquad ...(3.25)$$

where *equality sign stands for the reversible process and inequality sign stands for the irreversible process.*

It may be noted here that the *effect of irreversibility is always to increase the entropy of the system.*

Let us now consider an *isolated system.* We know that in an isolated system, matter, work or heat cannot cross the boundary of the system. Hence according to first law of thermodynamics, the internal energy of the system will remain constant.

Since for an isolated system, $\delta Q = 0$, from equation (3.25), we get

$$(dS)_{isolated} \geq 0 \qquad\qquad ...(3.26)$$

Equation (3.26) states that the *entropy of an isolated system either increases or remains constant.* This is a corollary of the second law. It explains the *principle of increase in entropy.*

3.14. CHANGE IN ENTROPY OF THE UNIVERSE

We know that the entropy of an isolated system either increase or remains constant, *i.e.*

$$(dS)_{isolated} \geq 0$$

By including any system and its surrounding within a single boundary, as shown in Fig. 3.22, an isolated system can be formed. The combination of the system and the surroundings within a single boundary is sometimes called the **universe.** Hence, applying the principle of increase in entropy, we get

$$(dS)_{universe} \geq 0$$

where $(dS)_{universe} = (dS)_{system} + (dS)_{surroundings}$.

In the combined closed system consider that a quantity of heat δQ is transferred from the system at temperature T to the surroundings at temperature T_0. Applying eqn. (3.24) to this process, we can write

$$(dS)_{system} > - \frac{\delta Q}{T}$$

(–ve sign indicates that heat is transferred from the system).

Similarly, since an amount of heat δQ is absorbed by the surroundings, for a reversible process, we can write

$$(dS)_{surroundings} = \frac{\delta Q}{T_0}$$
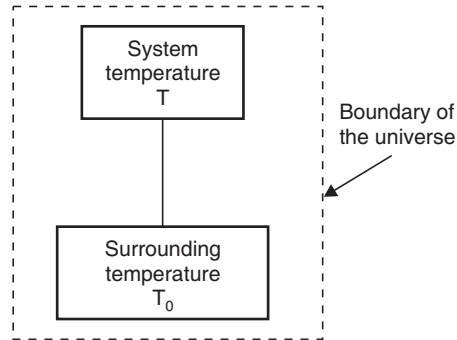
Fig. 3.22. Entropy change of universe.

Hence, the total change in entropy for the combined system

$$(dS)_{system} + (dS)_{surroundings} \geq - \frac{\delta Q}{T} + \frac{\delta Q}{T_0}$$

or
$$(dS)_{universe} \geq dQ \left(- \frac{1}{T} + \frac{1}{T_0} \right)$$

The same result can be obtained in the case of an open system.

For both closed and open systems, we can write

$$(dS)_{universe} \geq 0 \qquad\qquad\qquad\qquad\qquad ...(3.27)$$

Eqn. (3.27) states that the process involving the interaction of a system and the surroundings takes place only if the net entropy of the combined system increases or in the limit remains constant. *Since all natural processes are irreversible, the entropy is increasing continually.*

The entropy attains its maximum value when the system reaches a stable equilibrium state from a non-equilibrium state. This is the state of maximum disorder and is one of maximum thermodynamic probability.

3.15. TEMPERATURE ENTROPY DIAGRAM

If entropy is plotted horizontally and absolute temperature vertically the diagram so obtained is called *temperature entropy (T-s)* diagram. Such a diagram is shown in Fig. 3.23. If working fluid receives a small amount of heat dQ in an elementary portion ab of an operation AB when temperature is T, and if dQ is represented by the shaded area of which T is the mean ordinate, the width of the

figure must be $\dfrac{dQ}{T}$. This is called '*increment of entropy*' and is denoted by dS. The total heat

received by the operation will be given by the area under the curve AB and $(S_B - S_A)$ will be corresponding increase of entropy.

From above we conclude that :

Entropy change, $dS = \dfrac{\text{Heat change } (Q)}{\text{Absolute temperature } (T)}$.

"*Entropy may also be defined as the thermal property of a substance which remains constant when substance is expanded or compressed adiabatically in a cylinder*".

Note. 's' stands for specific entropy whereas 'S' means total entropy (*i.e.*, $S = ms$).
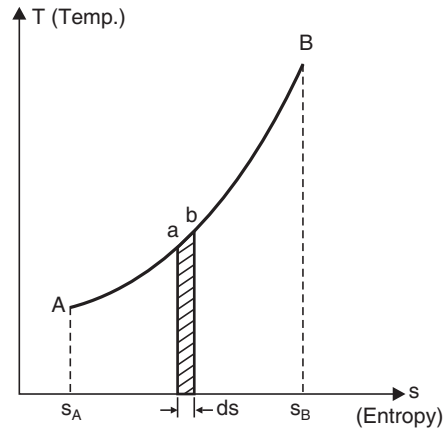
Fig. 3.23. Temperature entropy diagram.

3.16. CHARACTERISTICS OF ENTROPY

The characteristics of entropy in a summarised form are given below :
1. It increases when heat is supplied irrespective of the fact whether temperature changes or not.
2. It decrease when heat is removed whether temperature changes or not.
3. It remains unchanged in all adiabatic frictionless processes.
4. It increases if temperature of heat is lowered without work being done as in a throttling process.

3.17. ENTROPY CHANGES FOR A CLOSED SYSTEM

3.17.1. General Case for Change of Entropy of a Gas

Let 1 kg of gas at a pressure p_1, volume v_1, absolute temperature T_1 and entropy s_1, be heated such that its final pressure, volume, absolute temperature and entropy are p_2, v_2, T_2 and s_2 respectively. Then by law of conservation of energy,

$$dQ = du + dW$$

where dQ = small change of heat,

du = small internal energy, and

dW = small change of work done (pdv).

Now $\qquad dQ = c_v dT + pdv$

Dividing both sides by T, we get

$$\frac{dQ}{T} = \frac{c_v dT}{T} + \frac{pdv}{T}$$

But $\qquad \dfrac{dQ}{T} = ds$

and as $\qquad pv = RT$

$\therefore \qquad \dfrac{p}{T} = \dfrac{R}{v}$

Hence $\qquad ds = \dfrac{c_v dT}{T} = R\dfrac{dv}{v}$

Integrating both sides, we get

$$\int_{s_1}^{s_2} ds = c_v \int_{T_1}^{T_2} \frac{dT}{T} + R \int_{v_1}^{v_2} \frac{dv}{v}$$

or

$$(s_2 - s_1) = c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1} \qquad \qquad ...(3.28)$$

This expression can be reproduced in the following way :

According to the gas equation, we have

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

or

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \times \frac{v_2}{v_1}$$

Substituting the value of $\dfrac{T_2}{T_1}$ in eqn. (3.28), we get

$$s_2 - s_1 = c_v \log_e \frac{p_2}{p_1} \times \frac{v_2}{v_1} + R \log_e \frac{v_2}{v_1}$$

$$= c_v \log_e \frac{p_2}{p_1} + c_v \log_e \frac{v_2}{v_1} + R \log_e \frac{v_2}{v_1}$$

$$= c_v \log_e \frac{p_2}{p_1} + (c_v + R) \log_e \frac{v_2}{v_1}$$

$$= c_v \log_e \frac{p_2}{p_1} + c_p \log_e \frac{v_2}{v_1}$$

$$\therefore \qquad s_2 - s_1 = c_v \log_e \frac{p_2}{p_1} + c_p \log_e \frac{v_2}{v_1} \qquad \qquad ...(3.29)$$

Again, from gas equation,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad \frac{v_2}{v_1} = \frac{p_1}{p_2} \times \frac{T_2}{T_1}$$

Putting the value of $\dfrac{v_2}{v_1}$ in equation (3.28), we get

$$(s_2 - s_1) = c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{p_1}{p_2} \times \frac{T_2}{T_1}$$

$$= c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{p_1}{p_2} + R \log_e \frac{T_2}{T_1}$$

$$= (c_v + R) \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1}$$

$$= c_p \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1}$$

$$\therefore \qquad s_2 - s_1 = c_p \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1} . \qquad \qquad ...(3.30)$$

3.17.2. Heating a Gas at Constant Volume

Refer to Fig. 3.24. Let 1 kg of gas be heated at constant volume and let the change in entropy and absolute temperature be from s_1 to s_2 and T_1 to T_2 respectively.
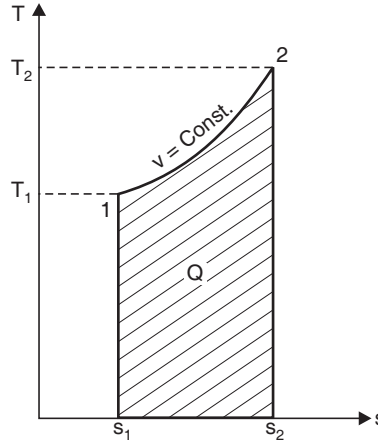
Fig. 3.24. T-s diagram : Constant volume process.

Then
$$Q = c_v(T_2 - T_1)$$

Differentiating to find small increment of heat dQ corresponding to small rise in temperature dT.

$$dQ = c_v dT$$

Dividing both sides by T, we get

$$\frac{dQ}{T} = c_v \cdot \frac{dT}{T}$$

or
$$ds = c_v \cdot \frac{dT}{T}$$

Integrating both sides, we get

$$\int_{s_1}^{s_2} ds = c_v \int_{T_1}^{T_2} \frac{dT}{T}$$

or
$$s_2 - s_1 = c_v \log_e \frac{T_2}{T_1} \qquad \qquad \text{...(3.31)}$$

3.17.3. Heating a Gas at Constant Pressure

Refer to Fig. 3.25. Let 1 kg of gas be heated at constant pressure, so that its absolute temperature changes from T_1 to T_2 and entropy s_1 to s_2.

Then
$$Q = c_p(T_2 - T_1).$$

Differentiating to find small increase in heat, dQ of this gas when the temperature rise is dT.

$$dQ = c_p \cdot dT$$

Dividing both sides by T, we get

$$\frac{dQ}{T} = c_p \cdot \frac{dT}{T}$$
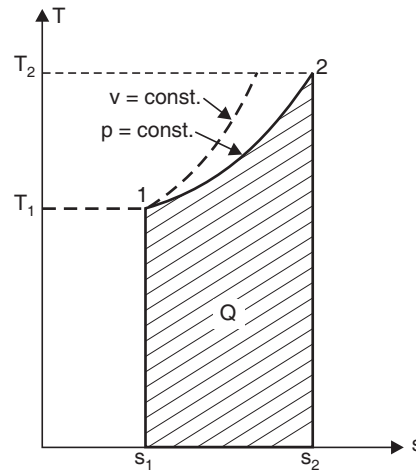
Fig. 3.25. *T-s* diagram : Constant pressure process.

or
$$ds = c_p \cdot \frac{dT}{T}$$

Integrating both sides, we get

$$\int_{s_1}^{s_2} ds = c_p \int_{T_1}^{T_2} \frac{dT}{T}$$

$$s_2 - s_1 = c_p \log_e \frac{T_2}{T_1} \qquad \qquad ...(3.32)$$

3.17.4. Isothermal Process

An isothermal expansion 1-2 at constant temperature T is shown in Fig. 3.26.
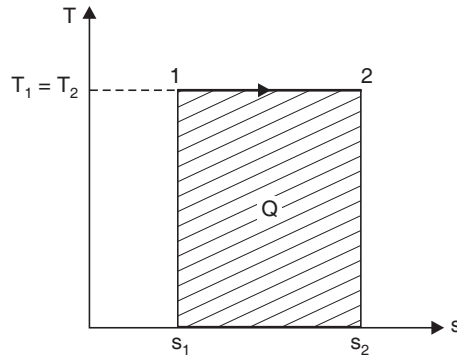
Fig. 3.26. *T-s* diagram : Isothermal process.

Entropy changes from s_1 to s_2 when gas absorbs heat during expansion. The heat taken by the gas is given by the area under the line 1-2 which also represents the work done during expansion. In other words, $Q = W$.

But
$$Q = \int_{s_1}^{s_2} T ds = T(s_2 - s_1)$$

and $\qquad W = p_1 v_1 \log_e \dfrac{v_2}{v_1} = RT_1 \log_e \dfrac{v_2}{v_1}$ per kg of gas $\qquad [\because \quad p_1 v_1 = RT_1]$

$\therefore \qquad\qquad T(s_2 - s_1) = RT_1 \log_e \dfrac{v_2}{v_1}$

or $\qquad\qquad s_2 - s_1 = R \log_e \dfrac{v_2}{v_1}$ $\qquad\qquad [\because \quad T_1 = T_2 = T]$ $\qquad$ ...(3.33)

3.17.5. Adiabatic Process (Reversible)

During an adiabatic process as heat is neither supplied nor rejected,

$$dQ = 0$$

or $\qquad\qquad \dfrac{dQ}{T} = 0$

or $\qquad\qquad ds = 0$ $\qquad\qquad\qquad\qquad\qquad\qquad$ ...(3.34)
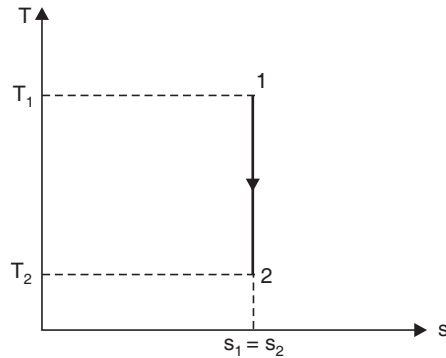
Fig. 3.27. *T-s* diagram : Adiabatic process.

This shows that there is no change in entropy and hence it is known as *isentropic process*.

Fig. 3.27 represents an adiabatic process. It is a vertical line (1-2) and therefore area under this line is nil ; hence heat supplied or rejected and entropy change is zero.

3.17.6. Polytropic Process

Refer to Fig. 3.28.
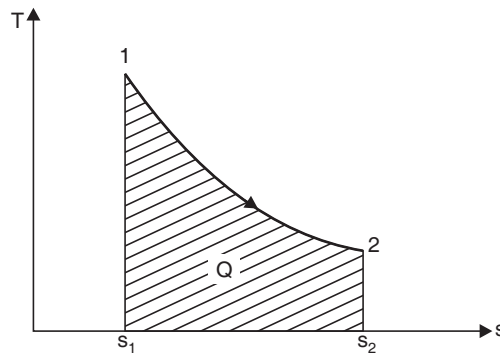
Fig. 3.28. *T-s* diagram : Polytropic process.

The expression for 'entropy change' in polytropic process (pv^n = constant) can be obtained from eqn. (3.28)

i.e.,
$$s_2 - s_1 = c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1}$$

Also
$$p_1 v_1{}^n = p_2 v_2{}^n$$

or
$$\frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^n \qquad \qquad \qquad \qquad \qquad ...(i)$$

Again, as
$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

or
$$\frac{p_1}{p_2} = \frac{v_2}{v_1} \times \frac{T_1}{T_2} \qquad \qquad \qquad \qquad \qquad ...(ii)$$

From (*i*) and (*ii*), we get
$$\left(\frac{v_2}{v_1}\right)^n = \frac{v_2}{v_1} \times \frac{T_1}{T_2}$$

or
$$\left(\frac{v_2}{v_1}\right)^{n-1} = \frac{T_1}{T_2}$$

or
$$\frac{v_2}{v_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$$

Substituting the value of $\dfrac{v_2}{v_1}$ in eqn. (3.28), we get

$$s_2 - s_1 = c_v \log_e \frac{T_2}{T_1} + R \log_e \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}} = c_v \log_e \frac{T_2}{T_1} + R\left(\frac{1}{n-1}\right) \log_e \frac{T_1}{T_2}$$

$$= c_v \log_e \frac{T_2}{T_1} - R\left(\frac{1}{n-1}\right)\log_e \frac{T_2}{T_1}$$

$$= c_v \log_e \frac{T_2}{T_1} - (c_p - c_v) \times \left(\frac{1}{n-1}\right)\log_e \frac{T_2}{T_1} \qquad [\because \quad R = c_p - c_v]$$

$$= c_v \log_e \frac{T_2}{T_1} - (\gamma \cdot c_v - c_v) \times \left(\frac{1}{n-1}\right)\log_e \frac{T_2}{T_1} \qquad [\because \quad c_p = \gamma \cdot c_v]$$

$$= c_v \left[1 - \left(\frac{\gamma - 1}{n-1}\right)\right]\log_e \frac{T_2}{T_1} = c_v \left[\frac{(n-1) - (\gamma - 1)}{(n-1)}\right]\log_e \frac{T_2}{T_1}$$

$$= c_v \left(\frac{n - 1 - \gamma + 1)}{n-1}\right)\log_e \frac{T_2}{T_1}$$

$$= c_v \cdot \left(\frac{n - \gamma}{n-1}\right)\log_e \frac{T_2}{T_1} \text{ per kg of gas}$$

$\therefore$
$$s_2 - s_1 = c_v \left(\frac{n - \gamma}{n-1}\right)\log_e \frac{T_2}{T_1} \text{ per kg of gas.} \qquad \qquad ...(3.35)$$

3.17.7. Approximation for Heat Absorbed

The curve LM shown in Fig. 3.29 is obtained by heating 1 kg of gas from initial state L to final state M. Let temperature during heating increases from T_1 to T_2. Then heat absorbed by the gas will be given by the area (shown shaded) under curve LM.
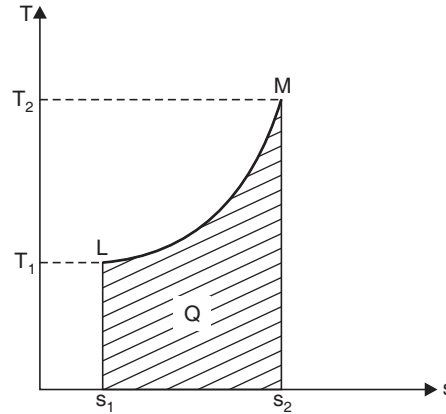
Fig. 3.29

As the curve on T-s diagram which represents the heating of the gas, usually has very slight curvature, it can be assumed a straight line for a small temperature range. Then,

Heat absorbed = area under the curve LM

$$= (s_2 - s_1)\left(\frac{T_1 + T_2}{2}\right) \qquad\qquad ...(3.36)$$

In other words, heat absorbed *approximately* equals the product of change of entropy and mean absolute temperature.

Table 3.1. Summary of Formulae

S. No.	Process	Change of entropy (per kg)
1.	General case	(i) $c_v \log_e \dfrac{T_2}{T_1} + R \log_e \dfrac{v_2}{v_1}$ (in terms of T and v)
		(ii) $c_v \log_e \dfrac{p_2}{p_1} + c_v \log_e \dfrac{v_2}{v_1}$ (in terms of p and v)
		(iii) $c_p \log_e \dfrac{T_2}{T_1} - R \log_e \dfrac{p_2}{p_1}$ (in terms of T and p)
2.	Constant volume	$c_v \log_e \dfrac{T_2}{T_1}$
3.	Constant pressure	$c_p \log_e \dfrac{T_2}{T_1}$
4.	Isothermal	$R \log_e \dfrac{v_2}{v_1}$
5.	Adiabatic	Zero
6.	Polytropic	$c_v \left(\dfrac{n - \gamma}{n - 1}\right) \log_e \dfrac{T_2}{T_1}$

3.18. ENTROPY CHANGES FOR AN OPEN SYSTEM

In an open system, as compared with closed system, there is additional change of entropy due to the mass crosssing the boundaries of the system. *The net change of entropy of a system due to mass transport is equal to the difference between the product of the mass and its specific entropy at the inlet and at the outlet of the system.* Therefore, the total change of entropy of the system during a small interval is given by

$$dS \geq \frac{dQ}{T_0} + \Sigma s_i \cdot dm_i - \Sigma s_0 \cdot dm_0$$

where T_0 = Temperature of the surroundings

s_i = Specific entropy at the inlet

s_0 = Specific entropy at the outlet

dm_i = Mass entering the system

dm_0 = Mass leaving the system.

(Subscripts i and 0 refer to inlet and outlet conditions)

The above equation in general form can be written as

$$dS \geq \frac{dQ}{T_0} + \Sigma s \cdot dm \qquad \qquad ...(3.37)$$

In equation (3.37) entropy flow *into* the system is considered *positive* and entropy *out-flow* is considered *negative*. The *equality sign is applicable to reversible process* in which the heat interactions and mass transport to and from the system is accomplished reversibly. The *inequality sign is applicable to irreversible processes.*

If equation (3.37) is divided by dt, then it becomes a rate equation and is written as :

$$\frac{dS}{dt} \geq \frac{1}{T_0} \cdot \frac{dQ}{dt} + \Sigma s \cdot \frac{dm}{dt} \qquad \qquad ...(3.38)$$

In a steady-state, steady flow process, the rate of change of entropy of the system $\left(\dfrac{dS}{dt}\right)$ becomes zero.

$\therefore$
$$O \geq \frac{1}{T_0} \frac{dQ}{dt} + \Sigma s \cdot \frac{dm}{dt}$$

or
$$\frac{1}{T_0} \dot{Q} + \Sigma s \cdot \dot{m} \leq 0 \qquad \qquad ...(3.39)$$

where $\dot{Q} = \dfrac{dQ}{dt}$

and $\dot{m} = \dfrac{dm}{dt}$.

For *adiabatic steady flow process,* $\dot{Q} = 0$

$$\Sigma s \cdot \dot{m} \leq 0 \qquad \qquad ...(3.40)$$

If the process is *reversible adiabatic*, then

$$\Sigma s \cdot \dot{m} = 0. \qquad \qquad ...(3.41)$$

ENTROPY

Example 3.20. *An iron cube at a temperature of 400°C is dropped into an insulated bath containing 10 kg water at 25°C. The water finally reaches a temperature of 50°C at steady state. Given that the specific heat of water is equal to 4186 J/kg K. Find the entropy changes for the iron cube and the water. Is the process reversible ? If so why ?*

Solution. *Given* : Temperature of iron cube = 400°C = 673 K

Temperature of water = 25°C = 298 K

Mass of water = 10 kg

Temperature of water and cube after equilibrium = 50°C = 323 K

Specific heat of water, c_{pw} = 4186 J/kg K

Entropy changes for the iron cube and the water :

Is the process reversible ?

Now,   Heat lost by iron cube = Heat gained by water

$$m_i c_{pi}(673 - 323) = m_w c_{pw}(323 - 298)$$
$$= 10 \times 4186 \,(323 - 298)$$

∴ $m_i c_{pi} = \dfrac{10 \times 4186(323 - 298)}{(673 - 323)} = 2990$

where m_i = Mass of iron, kg ;

      c_{pi} = Specific heat of iron, J/kg K

Entropy of iron at 673 K $= m_i c_{pi} \ln\left(\dfrac{673}{273}\right) = 2990 \ln\left(\dfrac{673}{273}\right)$

                                $= 2697.8$ J/K   [Taking 0°C as datum]

Entropy of water at 298 K $= m_w c_{pw} \ln\left(\dfrac{298}{273}\right)$

                                $= 10 \times 4186 \ln\left(\dfrac{298}{273}\right) = 3667.8$ J/K

Entropy of iron at 323 K $= 2990 \times \ln\left(\dfrac{323}{273}\right) = 502.8$ J/K

Entropy water at 323 K $= 10 \times 4186 \ln\left(\dfrac{323}{273}\right) = 7040.04$ J/K

Changer in entropy of iron $= 502.8 - 2697.8 = -2195$ J/K

Change in entropy of water $= 7040.04 - 3667.8 = 3372.24$ J/K

Net change in entropy $= 3372.24 - 2195 = 1177.24$ J/K

Since $\Delta S > 0$ hence the process is **irreversible.**   **(Ans.)**

Example 3.21. *An ideal gas is heated from temperature T_1 to T_2 by keeping its volume constant. The gas is expanded back to its initial temperature according to the law pv^n = constant. If the entropy changes in the two processes are equal, find the value of n is terms of the adiabatic index γ.*

Solution. Change in entropy during constant volume process

$$= mc_v \ln\left(\frac{T_2}{T_1}\right) \qquad\qquad\qquad ...(i)$$

Change in entropy during polytropic process (pv^n = constant)

$$= mc_v \left(\frac{\gamma - n}{n - 1} \right) \ln \left(\frac{T_2}{T_1} \right) \qquad \qquad ...(ii)$$

For the same entropy, equating (*i*) and (*ii*), we have

$$\frac{\gamma - n}{n - 1} = 1 \quad \text{or} \quad (\gamma - n) = (n - 1) \quad \text{or} \quad 2n = \gamma + 1$$

$$\therefore \qquad \qquad \mathbf{n = \frac{\gamma + 1}{2}}. \quad \textbf{(Ans.)}$$

3.19. THE THIRD LAW OF THERMODYNAMICS

The third law of thermodynamics is stated is follow :

"*The entropy of all perfect crystalline solids is zero at absolute zero temperature*".

- The third law of thermodynamics, often referred to as **Nernst law**, provides the basis for the calculation of absolute entropies of substances.

According to this law, if the entropy is zero at $T = 0$, the absolute entropy $s_{ab.}$ of a substance at any temperature T and pressence p is expressed by the expression

$$s_{ab.} = \int_0^{T_s = T_{f_1}} c_{ps} \frac{dT}{T} + \frac{h_{sf}}{T_s} + \int_{T_s}^{T_{f_2} = T_g} c_{pf} \frac{dT}{T} + \frac{h_{fg}}{T_g} \int_{T_g}^{T} c_{pg} \frac{dT}{T} \qquad ...(3.42)$$

where $\qquad \qquad T_s = T_{f_1} = T_{sf} = T_{sat}$ ... for fusion,

$\qquad \qquad \qquad T_{f_2} = T_g = T_{fg} = T_{sat}$ ... for vaporisation

$\quad c_{ps}, c_{pf}, c_{pg}$ = constant pressure specific heats for solids, liquids and gas,

$\qquad h_{sf}, h_{fg}$ = latent heats of fusion and vaporisation.

Thus by putting $s = 0$ at $T = 0$, one may integrate zero kelvin and standard state of 298.15 K and 1 atm., and find the entropy difference.

Further, it can be shown that the entropy of a crystalline substance at $T = 0$ is not a function

of pressure, *viz.*, $\left(\dfrac{\partial s}{\partial p} \right)_{T = 0} = 0$

However, at temperatures above absolute zero, the entropy is a function of pressure also. The absolute entropy of a substance at 1 atm pressure can be calculated using eqn. (3.42) ; for pressures different from 1 atm, necessary corrections have to be applied.

Example 3.22. *Air at 20°C and 1.05 bar occupies 0.025 m³. The air is heated at constant volume until the pressure is 4.5 bar, and then cooled at constant pressure back to original temperature. Calculate :*

(i) The net heat flow from the air.

(ii) The net entropy change.

Sketch the process on T-s diagram.

Solution. The processes are shown on a *T-s* diagram in Fig. 3.30.

For air :

Temperature, $\qquad T_1 = 20 + 273 = 293$ K

Volume, $\qquad \qquad V_1 = V_3 = 0.025$ m³

Pressure, $\qquad \qquad p_1 = 1.05$ bar = 1.05×10^5 N/m²

Pressure, $p_2 = 4.5$ bar $= 4.5 \times 10^5$ N/m^2.
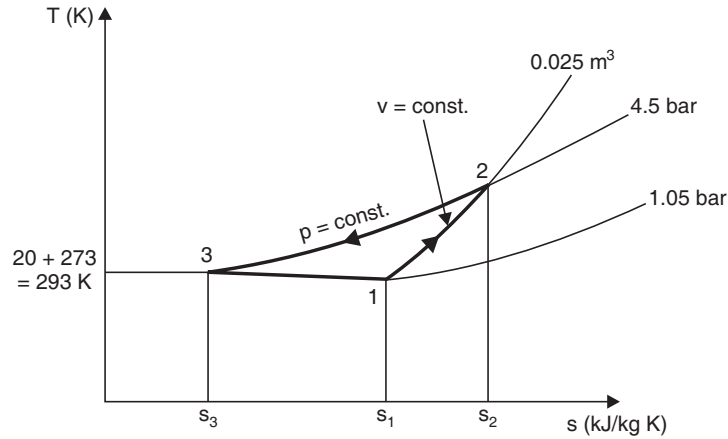
Fig. 3.30

(i) **Net heat flow :**

For a perfect gas (corresponding to point 1 of air),

$$m = \frac{p_1 V_1}{RT_1} = \frac{1.05 \times 10^5 \times 0.025}{0.287 \times 10^3 \times 293} = 0.0312 \text{ kg}$$

For a perfect gas at *constant volume*,

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

$$\frac{1.05}{293} = \frac{4.5}{T_2} \quad \text{or} \quad T_2 = \frac{4.5 \times 293}{1.05} = 1255.7 \text{ K}.$$

At **constant volume,**

$$Q = mc_v(T_2 - T_1) = 0.0312 \times 0.718 \,(1255.7 - 293)$$

i.e., $\quad Q_{1-2} = 21.56$ kJ.

Also, at **constant pressure,**

$$Q = m \times c_p \times (T_3 - T_2) = 0.0312 \times 1.005 \,(293 - 1255.7)$$

i.e., $\quad Q_{2-3} = -30.18$ kJ

∴ **Net heat flow** $= Q_{1-2} + Q_{2-3} = 21.56 + (-30.18) = -8.62$ kJ

i.e., **Heat rejected = 8.62 kJ. (Ans.)**

(ii) **Net entropy change :**

Referring to Fig. 3.30.

Net decrease in entropy,

$$S_1 - S_2 = (S_2 - S_3) - (S_2 - S_1)$$

At **constant pressure**, $dQ = mc_p \, dT$, hence

$$m(s_2 - s_3) = \int_{293}^{1255.7} \frac{mc_p \, dT}{T}$$

$$= 0.0312 \times 1.005 \times \log_e \frac{1255.7}{293}$$

i.e., $\quad S_2 - S_3 = 0.0456$ kJ/K

At **constant volume**, $dQ = mc_v \, dT$, hence

$$m(s_2 - s_1) = \int_{293}^{1255.7} \frac{mc_v \, dT}{T}$$

$$= 0.0312 \times 0.718 \times \log_e \frac{1255.7}{293} = 0.0326 \text{ kJ/K}$$

i.e., $S_2 - S_1 = 0.0326$ kJ/K

∴ $m(s_1 - s_3) = S_1 - S_3 = (S_2 - S_3) - (S_2 - S_1)$

$$= 0.0456 - 0.0326 = 0.013 \text{ kJ/K}$$

Hence, **decrease in entropy = 0.013 kJ/K.   (Ans.)**

Note that since *entropy is a property*, the decrease in entropy is given by $S_1 - S_3$, is *independent of the process undergone between states 1 and 3.*

Example 3.23. *0.04 m^3 of nitrogen contained in a cylinder behind a piston is initially at 1.05 bar and 15°C. The gas is compressed isothermally and reversibly until the pressure is 4.8 bar. Calculate :*

   (*i*) *The change of entropy ;*

  (*ii*) *The heat flow ;*

 (*iii*) *The work done ;*

   Sketch the process on a p-v and T-s diagram.

   Assume nitrogen to act as a perfect gas. Molecular weight of nitrogen = 28.

   Solution. Refer Fig. 3.31.

Initial pressure, $p_1 = 1.05$ bar $= 1.05 \times 10^5$ N/m^2

Initial volume, $V_1 = 0.04$ m^3

Temperature, $T_1 = 15 + 273 = 288$ K

Final pressure, $p_2 = 4.8$ bar $= 4.8 \times 10^5$ N/m^2

Final temperature, $T_2 = T_1 = 288$ K.

The process is shown on a *p-v* and a *T-s* diagram in Fig. 3.31 (*a*) and 3.31 (*b*) respectively. The shaded area in Fig. 3.31 (*a*) represents **work input**, and the shaded area on Fig. 3.31 (*b*) represents **heat rejected.**

Characteristic gas constant,

$$R = \frac{\text{Universal gas constant } R_0}{\text{Molecular weight, } M} = \frac{8314}{28} = 297 \text{ Nm/kg K}$$

Now, using characteristic gas equation (to find mass '*m*' of nitrogen)

$$p_1 V_1 = mRT_1$$

$$m = \frac{p_1 V_1}{RT_1} = \frac{1.05 \times 10^5 \times 0.04}{297 \times 288} = 0.0491 \text{ kg}$$

   (*i*) **The change of entropy,**

$$S_2 - S_1 = mR \log_e \frac{p_1}{p_2}$$

$$= 0.0491 \times \frac{297}{10^3} \log_e \left(\frac{1.05}{4.8}\right)$$

i.e., $S_2 - S_1 = -0.02216$ kJ/K.

∴   **Decrease in entropy, $S_1 - S_2 = 0.02216$ kJ/K.   (Ans.)**
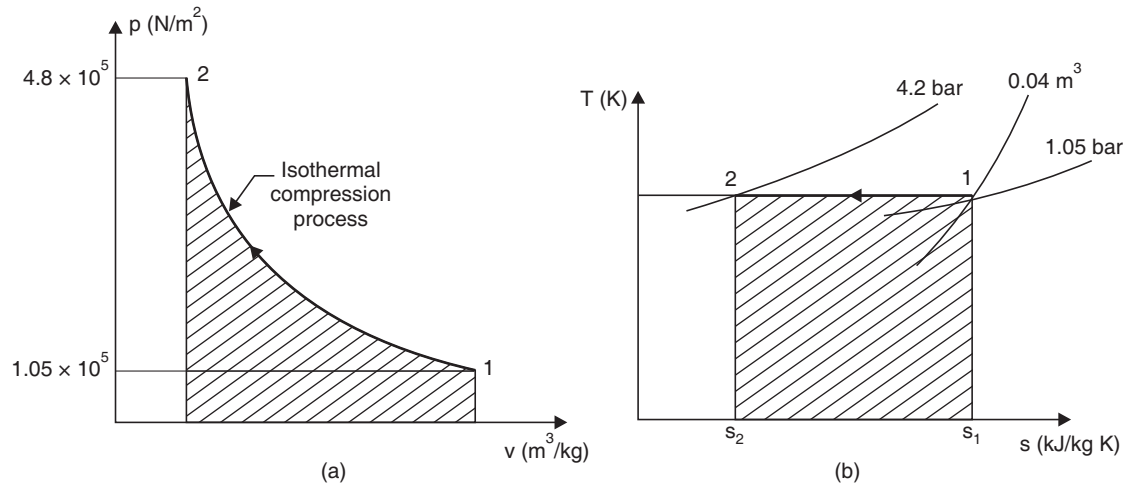
Fig. 3.31

(*ii*) **Heat rejected** = Shaded area on Fig. 3.31 (*b*)

$$= T(S_1 - S_2) = 288 \times 0.02216 = \textbf{6.382 kJ.} \quad \textbf{(Ans.)}$$

(*iii*) For an isothermal process for a perfect gas,

$$W = Q = 6.382 \text{ kJ}$$

Hence, **the work done on air = 6.382 kJ.   (Ans.)**

Example 3.24. *1 kg of gas enclosed in an isolated box of volume* v_1, *temperature* T_1 *and pressure* p_1 *is allowed to expand freely till volume increases to* $v_2 = 2v_1$.

Determine the change in entropy.

Take R for gas as 287 kJ/kg K.

Solution. During the process of free expansion in an isolated box,

$$\Delta U = 0, \ W = 0 \text{ and } Q = \Delta U + W = 0$$

The process is represented by dotted line on *p-v* diagram as shown in Fig. 3.32 (*a*) where $v_2 = 2v_1$.

To calculate the entropy change, assume that the irreversible free expansion process is replaced by a reversible isothermal process as temperature in free expansion remains constant, in such a way that the volume increases to double of its original as shown in Fig. 3.32 (*b*). As the work is developed by the system and heat is given to the system at constant temperature, during isothermal reversible system then as per first law of thermodynamics :

Fig. 3.32

$$\Delta U = 0, \ Q = W$$

i.e.,
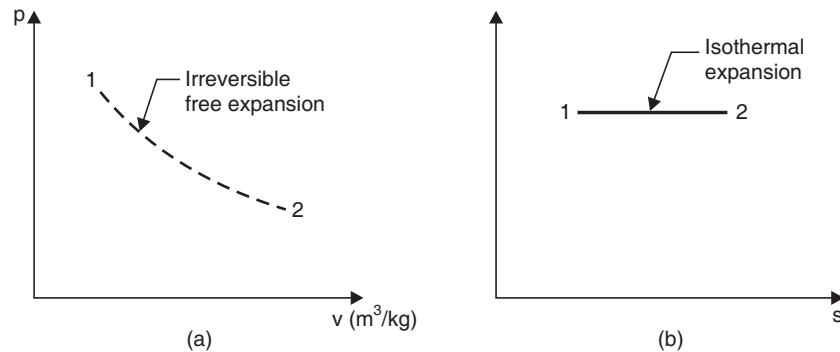
$$Q = \int_{v_1}^{v_2} p \cdot dv$$

$$= \int_{v_1}^{v_2} \frac{RT}{v} \cdot dv \qquad\qquad \left[\because \ pv = RT \text{ and } p = \frac{RT}{v} \right]$$

$$= RT \log_e \frac{v_2}{v_1}$$

$$\therefore \qquad\qquad \frac{Q}{T} = R \log_e \frac{v_2}{v_1}$$

But this is the expression for change in entropy of the system. Entropy being the property of the system, its change is same whether it is reversible or irreversible process.

∴   For the given process

$$\Delta s = R \log_e \left(\frac{v_2}{v_1} \right)$$

$$= 287 \log_e (2) \qquad\qquad [\because \ v_2 = 2v_1 \text{ (given)}]$$

$$= 198.9 \text{ kJ/kg K}$$

Hence **change in entropy = 198.9 kJ/kg K.   (Ans.)**

Example 3.25. *0.04 kg of carbon dioxide (molecular weight = 44) is compressed from 1 bar, 20°C, until the pressure is 9 bar, and the volume is then 0.003 m³. Calculate the change of entropy.*

Take c_p for carbon dioxide as 0.88 kJ/kg K, and assume carbon dioxide to be a perfect gas.

Solution. Mass of carbon dioxide, m = 0.04 kg

Molecular weight, M = 44

Initial pressure, p_1 = 1 bar = 1 × 10⁵ N/m²

Initial temperature, T_1 = 20 + 273 = 293 K

Final pressure, p_2 = 9 bar

Final volume, V_2 = 0.003 m³

c_p for carbon dioxide = 0.88 kJ/kg K

Change of entropy :
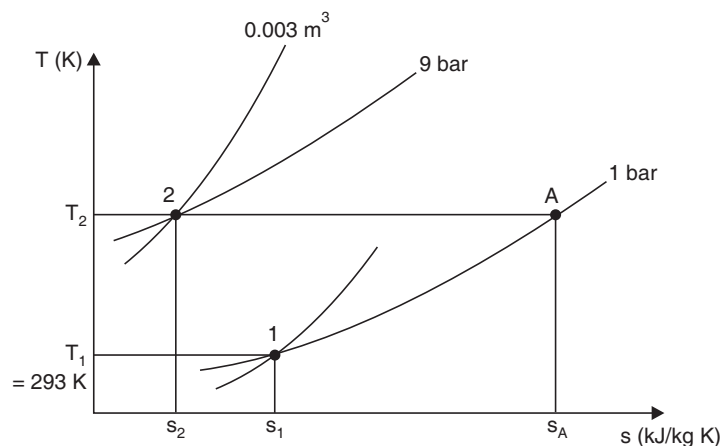
Fig. 3.33

Characteristics gas constant,

$$R = \frac{R_0}{M} = \frac{8314}{44} = 189 \text{ Nm/kg K}$$

To find T_2, using the relation,

$$p_2 V_2 = m R T_2$$

$$\therefore \qquad T_2 = \frac{p_2 V_2}{mR} = \frac{9 \times 10^5 \times 0.003}{0.04 \times 189} = 357 \text{ K}$$

Now

$$s_A - s_2 = R \log_e \frac{p_2}{p_1} = \frac{189}{10^3} \log_e \left(\frac{9}{1}\right)$$

$$= 0.4153 \text{ kJ/kg K}$$

Also at constant pressure from 1 to A

$$s_A - s_1 = c_p \log_e \frac{T_2}{T_1} = 0.88 \log_e \left(\frac{357}{293}\right)$$

$$= 0.1738 \text{ kJ/kg K}$$

Then

$$(s_1 - s_2) = (s_A - s_2) - (s_A - s_1)$$

$$= 0.4153 - 0.1738 = 0.2415 \text{ kJ/kg K}$$

Hence for 0.04 kg of carbon dioxide **decrease in entropy,**

$$S_1 - S_2 = m(s_1 - s_2) = 0.04 \times 0.2415$$

$$= \textbf{0.00966 kJ/K.} \quad \textbf{(Ans.)}$$

Note. In short, the change of entropy can be found by using the following relation :

$$(s_2 - s_1) = c_p \log_e \frac{T_2}{T_1} - R \log_e \frac{p_2}{p_1} = 0.88 \log_e \left(\frac{357}{293}\right) - \frac{189}{10^3} \log_e \left(\frac{9}{1}\right)$$

$$= 0.1738 - 0.4153 = -0.2415 \text{ kJ/kg K}$$

$$\therefore \qquad S_2 - S_1 = m(s_2 - s_1) = 0.04 \times (-0.2415)$$

$$= -0.00966 \text{ kJ/K}$$

(–ve sign means *decrease* in entropy)

or

$$S_1 - S_2 = 0.00966 \text{ kJ/K}.$$

Example 3.26. *Calculate the change of entropy of 1 kg of air expanding polytropically in a cylinder behind a piston from 7 bar and 600°C to 1.05 bar. The index of expansion is 1.25.*

Solution. The process is shown on a *T-s* diagram in Fig. 3.34.

Initial pressure, $p_1 = 7$ bar $= 7 \times 10^5$ N/m^2

Initial temperature, $T_1 = 600 + 273 = 873$ K

Final pressure, $p_2 = 1.05$ bar $= 1.05 \times 10^5$ N/m^2

Index of expansion, $n = 1.25$

Mass of air $= 1$ kg

To find T_2, using the relation,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$\therefore \qquad \frac{T_2}{873} = \left(\frac{1.05}{7}\right)^{\frac{1.25-1}{1.25}}$$
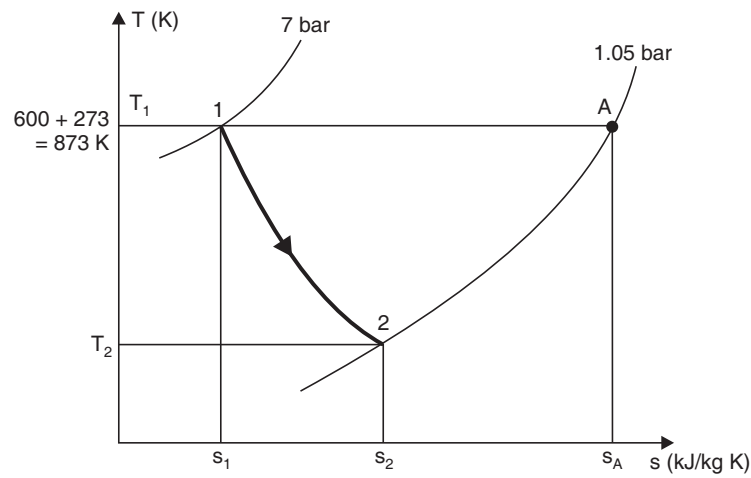
Fig. 3.34

or
$$T_2 = 873 \times \left(\frac{1.05}{7}\right)^{\frac{0.25}{1.25}} = 873 \times (0.15)^{0.2} = 597.3 \text{ K.}$$

Now replace the process 1 to 2 by processes, 1 to A and A to 2.

Then at *constant temperature* from 1 to A,

$$s_A - s_1 = R \log_e \frac{v_2}{v_1} = R \log_e \frac{p_1}{p_2} = 0.287 \log_e \left(\frac{7}{1.05}\right) = 0.544 \text{ kJ/kg K.}$$
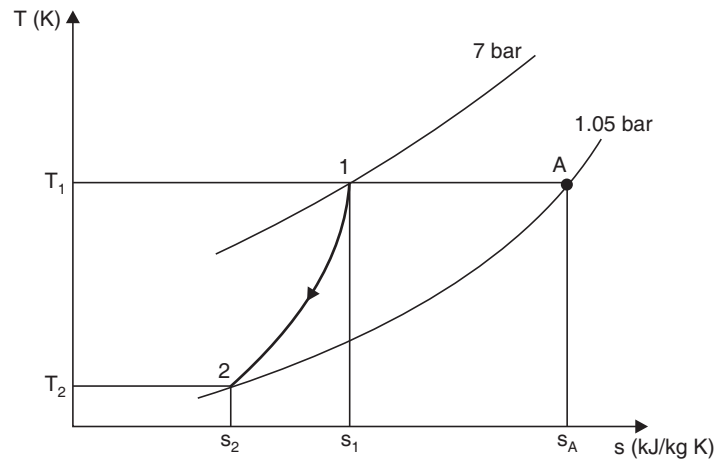
Fig. 3.35

At *constant pressure* from A to 2

$$s_A - s_2 = c_p \log_e \frac{T_1}{T_2} = 1.005 \log_e \frac{873}{597.3} = 0.3814 \text{ kJ/kg K}$$

Then $s_2 - s_1 = 0.544 - 0.3814 = 0.1626$ kJ/kg K

i.e.,   **Increase in entropy = 0.1626 kJ/kg K.   (Ans.)**

Note that if in this problem $s_A - s_2$ happened to be greater than $s_A - s_1$, this would mean that s_1 was greater than s_2, and the process should appear as shown in Fig. 3.35.

Note. The change of entropy can also be found by using the following relation :

$$s_2 - s_1 = c_v \left(\frac{n - \gamma}{n - 1}\right) \log_e \frac{T_2}{T_1}$$

$$= 0.718 \left(\frac{1.25 - 1.399}{1.25 - 1}\right) \log_e \left(\frac{597.3}{873}\right) \qquad \left[\because \quad \gamma = \frac{c_p}{c_v} = \frac{1.005}{0.718} = 1.399\right]$$

$$= 0.718 \times (-0.596) \times (-0.3795) = 0.1626 \text{ kJ/kg K (increase)}.$$

Example 3.27. *In an air turbine the air expands from 7 bar and 460°C to 1.012 bar and 160°C. The heat loss from the turbine can be assumed to be negligible.*

(*i*) *Show that the process is irreversible* ;

(*ii*) *Calculate the change of entropy per kg of air.*

Solution. Refer to Fig. 3.36.

Initial pressure, $p_1 = 7 \text{ bar} = 7 \times 10^5 \text{ N/m}^2$

Initial temperature, $T_1 = 460 + 273 = 733 \text{ K}$
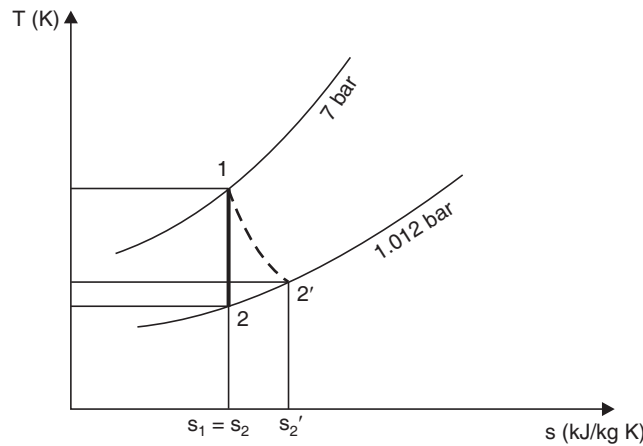
Fig. 3.36

Final pressure, $p_2 = 1.012 \text{ bar} = 1.012 \times 10^5 \text{ N/m}^2$

Final temperature, $T_2 = 160 + 273 = 433 \text{ K}$

(*i*) **To prove that the process is irreversible :**

Since the heat loss is negligible, the process is **adiabatic.**

For a reversible adiabatic process for a perfect gas, using the following equation :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{T_2}{733} = \left(\frac{1.012}{7}\right)^{\left(\frac{1.4 - 1}{1.4}\right)}$$

$\therefore \qquad T_2 = 733 \times \left(\frac{1.012}{7}\right)^{\frac{0.4}{1.4}} = 733 \times (0.1446)^{0.286} = 421.6 \text{ K}$

$$= 421.6 - 273 = 148.6°C.$$

But the actual temperature is 160°C at the pressure of 1.012 bar, hence the process is irreversible. **Proved.**

(*ii*) **Change of entropy per kg of air :**

The change of entropy $s_2' - s_1$, can be found by considering a reversible constant pressure process between 2 and 2′.

$$\therefore \qquad s_2' - s_2 = c_p \log_e \frac{T_2'}{T_2} = 1.005 \log_e \frac{433}{421.6} = 0.02681 \text{ kJ/kg K}$$

i.e., **Increase of entropy, $s_2' - s_1$ = 0.02681 kJ/kg K.   (Ans.)**

☞**Example 3.28.** *A fluid undergoes a reversible adiabatic compression from 4 bar, 0.3 m³ to 0.08 m³ according to the law, pv^{1.25} = constant.*

Determine : (*i*) *Change in enthalpy* ;   (*ii*) *Change in internal energy* ;

(*iii*) *Change in entropy* ;   (*iv*) *Heat transfer* ;

(*v*) *Work transfer.*

Solution. Refer to Fig. 3.37.
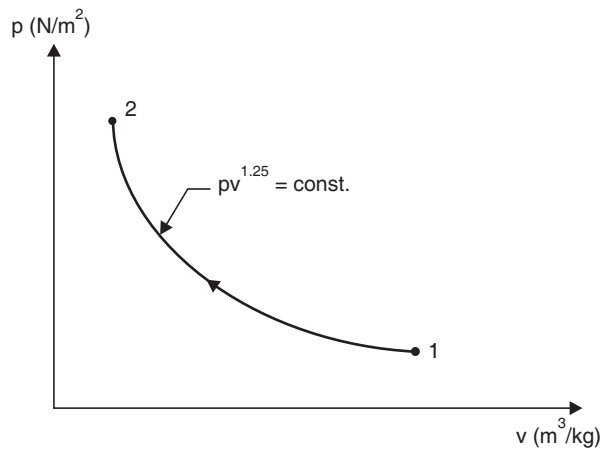
Fig. 3.37

Initial volume, $V_1 = 0.3 \text{ m}^3$
Initial pressure, $p_1 = 4 \text{ bar} = 4 \times 10^5 \text{ N/m}^2$
Final volume, $V_2 = 0.08 \text{ m}^3$
Law of compression : $pv^{1.25} = \text{constant.}$
For reversible adiabatic process,

$$p_1 V_1^n = p_2 V_2^n$$

or

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2} \right)^n$$

$$\therefore \qquad p_2 = p_1 \times \left(\frac{V_1}{V_2} \right)^n = 4 \times \left(\frac{0.3}{0.08} \right)^{1.25} = 20.87 \text{ bar.}$$

(*i*) **Change in enthalpy, $H_2 - H_1$:**

We know that, $\displaystyle\int_{H_1}^{H_2} dH = \int_{p_1}^{p_2} V dp$ ...(*i*)

Also $$p_1 V_1{}^n = p V^n$$

$\therefore$ $$V = \left(\frac{p_1 V_1{}^n}{p} \right)^{1/n}$$

Substituting this value of V in eqn. (i), we get

$$\int_{H_1}^{H_2} dH = \int_{p_1}^{p_2} \left[\left(\frac{p_1 V_1{}^n}{p} \right) \right]^{1/n} dp$$

$$= (p_1 V_1{}^n)^{1/n} \int_{p_1}^{p_2} p^{-1/n}\, dp$$

$$= (p_1 V_1{}^n)^{1/n} \left[\frac{p^{-\frac{1}{n}+1}}{-\frac{1}{n}+1} \right]_{p_1}^{p_2}$$

$$= [(p_1 V_1{}^n)]^{1/n} \left[\frac{p_2^{\left(1-\frac{1}{n}\right)} - p_1^{\left(1-\frac{1}{n}\right)}}{1-\frac{1}{n}} \right]$$

$$= (p_1 V_1{}^n)^{1/n} \times \frac{n}{(n-1)} \left[p_2^{\left(1-\frac{1}{n}\right)} - p_1^{\left(1-\frac{1}{n}\right)} \right]$$

$$= \frac{n(p_2 V_2 - p_1 V_1)}{n-1} \qquad\qquad [\because\ p_1 V_1{}^n = p_2 V_2{}^n]$$

$$= \frac{1.25}{(1.25-1)\times 10^3}\, [20.87 \times 10^5 \times 0.08 - 4 \times 10^5 \times 0.3] \text{ kJ}$$

$$= \frac{1.25}{0.25 \times 10^3} \times 10^5\, (20.87 \times 0.08 - 4 \times 0.3) \text{ kJ} = 234.8 \text{ kJ}.$$

Hence, **change in enthalpy = 234.8 kJ.   (Ans.)**

(ii) **Change in internal energy, $U_2 - U_1$:**

$$H_2 - H_1 = (U_2 + p_2 V_2) - (U_1 + p_1 V_1)$$
$$= (U_2 - U_1) + (p_2 V_2 - p_1 V_1)$$

$\therefore$ $$U_2 - U_1 = (H_2 - H_1) - (p_2 V_2 - p_1 V_1)$$

$$= 234.8 - \left(\frac{20.87 \times 10^5 \times 0.08 - 4 \times 10^5 \times 0.3}{10^3} \right) \text{ kJ}$$

$$= 234.8 - 46.96 = 187.84 \text{ kJ}.$$

Hence, **change in internal energy = 187.84 kJ.   (Ans.)**

(iii) **Change in entropy, $S_2 - S_1 = 0$.   (Ans.)**

(iv) **Heat transfer, $Q_{1-2} = 0$.   (Ans.)**

(v) **Work transfer, W_{1-2} :**

$$Q_{1-2} = (U_2 - U_1) + W_{1-2}$$

$\therefore$ $$W_{1-2} = Q_{1-2} - (U_2 - U_1)$$
$$= 0 - 187.84 = -187.84 \text{ kJ}$$

Hence, **work done on the fluid = 187.84 kJ.   (Ans.)**

☞**Example 3.29.** *An insulated cylinder of volume capacity 4 m³ contains 20 kg of nitrogen.*
Paddle work is done on the gas by stirring it till the pressure in the vessel gets increased from 4 bar
to 8 bar. Determine :

　(*i*) *Change in internal energy* ;

　(*ii*) *Work done* ;

　(*iii*) *Heat transferred ; and*

　(*iv*) *Change in entropy.*

　Take for nitrogen : c_p = 1.04 kJ / kg K, and c_v = 0.7432 kJ / kg K.

　Solution. Pressure,　　　p_1 = 4 bar = 4 × 10⁵ N/m²

　Pressure,　　　　　　　　p_2 = 8 bar = 8 × 10⁵ N/m²

　Volume,　　　　　　　$V_1 = V_2$ = 4 m³

and it is constant for both end states.

　Now,　　　　　　　　　　$\dfrac{p_1}{T_1} = \dfrac{p_2}{T_2}$

or　　　　　　　　　　　　$\dfrac{T_2}{T_1} = \dfrac{p_2}{p_1} = \dfrac{8 \times 10^5}{4 \times 10^5} = 2$

　Also,　　　　　　　　　$R = c_p - c_v = 1.04 - 0.7432 = 0.2968$ kJ/kg K.

　The mass of the gas in the cylinder is given by

$$m = \frac{pV}{RT} \text{ or } mT = \frac{pV}{R}$$

∴　　　　　　　　$mT_1 = \dfrac{p_1 V_1}{R} = \dfrac{4 \times 10^5 \times 4}{0.2968 \times 1000} = 5390.8$ kg K

and　　　　　　　$mT_2 = \dfrac{p_2 V_2}{R} = \dfrac{8 \times 10^5 \times 4}{0.2968 \times 1000} = 10781.6$ kg K.

　(*i*) **Change in internal energy,**

$$\Delta U = (U_2 - U_1)$$
$$= mc_v (T_2 - T_1) = c_v (mT_2 - mT_1)$$
$$= 0.7432 (10781.6 - 5390.8) = \textbf{4006.4 kJ.} \quad \textbf{(Ans.)}$$

　(*ii*) **Work done, W**

　Energy in the form of paddle work crosses into the system, but there is no change in system
boundary or *pdv* work is absent. No heat is transferred to the system. We have

　　　　　　　　　$Q_{1-2} = (U_2 - U_1) + W_{1-2}$

But　　　　　　　$Q_{1-2} = 0$

∴　　　　　　　　$W_{1-2} = -(U_2 - U_1) = -$ **4006.4 kJ　or　kN-m.　(Ans.)**

　(*iii*) **Heat transferred,** Q_{1-2} = **0.　(Ans.)**

　(*iv*) **Change in entropy,**

$$S_2 - S_1 = mc_v \log_e \frac{T_2}{T_1}$$

　For constant volume process

$$= 20 \times 0.7432 \log_e 2 = \textbf{10.3 kJ/K.} \quad \textbf{(Ans.)}$$

Example 3.30. *A certain gas has a specific heat at constant volume of 1.25 kJ/kg K. When it is expanded reversibly and adiabatically from a specific volume of 0.0624 m³/kg and a temperature of 530 K to a specific volume of 0.186 m³/kg its temperature falls by 165 K. When it is expanded into an evacuated space from the same initial condition to the same final specific volume its temperature falls only by 25 K.*

Find the change in entropy in each of the adiabatic processes.

Solution. Refer to Fig. 3.38.
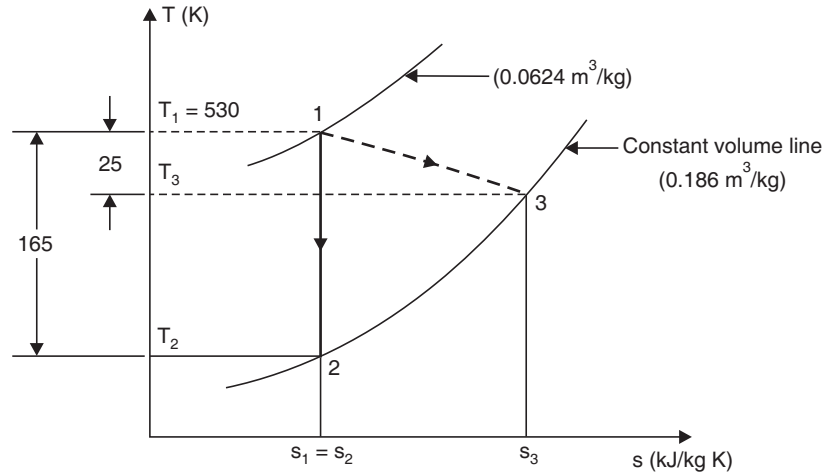
Fig. 3.38

Specific heat of gas at constant volume, $c_v = 1.25$ kJ/kg K
Initial specific volume, $v_1 = 0.0624$ m³/kg
Initial temperature, $T_1 = 530$ K
Final specific volume, $v_2 = 0.186$ m³/kg
Temperature fall when expanded reversibly and adiabatically = 165 K
Temperature fall when expanded into an evacuated space = 25 K

Change in entropy :

Path 1-2 : Reversible adiabatic process.

Change in entropy, $(s_2 - s_1) = 0$.

Path 1-3 : Adiabatic process such that $v_3 = 0.186$ m³/kg $(= v_2)$

(States 2 and 3 lie on the same constant volume line on T-s diagram)

and $T_1 - T_3 = 25$ K.

Change in entropy during this adiabatic process = $s_3 - s_1$.

To calculate $(s_3 - s_1)$ a reversible path has to be selected joining 3 and 1.

This is achieved by selecting the reversible adiabatic path 1-2 and the reversible constant volume process 2-3.

$$s_3 - s_1 = (s_3 - s_2) + (s_2 - s_1)$$
$$= (s_3 - s_2) + 0 = (s_3 - s_2)$$
$$= c_v \log_e \frac{T_3}{T_2} = 1.25 \log_e \left(\frac{530 - 25}{530 - 165} \right)$$
$$= 1.25 \log_e \left(\frac{505}{365} \right) = \textbf{0.4058 kJ/kg K.} \quad \textbf{(Ans.)}$$

Example 3.31. *A heat pump operates between two identical bodies which are at temperature T_1 and cools one of the bodies to a temperature T_2 ($T_2 < T_1$). Prove that for this operation the minimum work required by the heat pump is given by*

$$W = c_p \left(\frac{T_1^{\,2}}{T_2^{\,2}} + T_2 - 2T_1 \right)$$

where c_p is the specific heat which is same for both the bodies.

Solution. The arrangement is shown in Fig. 3.39.

For the minimum work absorbed by the heat pump, the heat pump must be reversed Carnot cycle engine and the required condition is :

$$\oint \left(\frac{dQ}{T} \right) = 0$$

For infinitely small changes, we can write

$$c_p \, \frac{dT_1}{T_1} + c_p \, \frac{dT_2}{T_2} = 0$$

If $T_1{}'$ is the final temperature of the high level reservoir, then the above equation can be written as :

$$c_p \int_{T_1}^{T_1{}'} \left(\frac{dT_1}{T_1} \right) + c_p \int_{T_1}^{T_2} \left(\frac{dT_2}{T_2} \right) = 0$$

∴ $$\log_e \left(\frac{T_1{}'}{T_1} \right) + \log_e \left(\frac{T_2}{T_1} \right) = 0 = \log_e (1)$$

∴ $$\log_e \left(\frac{T_1{}' T_2}{T_1{}^2} \right) = \log_e (1)$$

∴ $$T_1{}' = \left(\frac{T_1{}^2}{T_2} \right)$$

Now the work given to the heat pump

= heat rejected at higher level temperature

– heat picked up at lower level temperature

∴ $$W = c_p \int_{T_1}^{T_1{}'} (dT) - c_p \int_{T_2}^{T_1} dT$$

$$= c_p \, [(T_1{}' - T_1) - (T_1 - T_2)] = c_p \, (T_1{}' + T_2 - 2T_1)$$

Now substituting the value of $T_1{}'$ in the above equation in terms of T_1 and T_2

$$W = c_p \left[\frac{T_1{}^2}{T_2} + T_2 - 2T_1 \right]. \quad \textbf{Proved.}$$
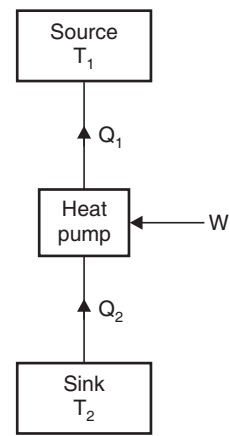
☞**Example 3.32.** *The connections of a reversible engine to three sources at 500 K, 400 K and 300 K are shown in Fig. 3.40. It draws 1500 kJ/min of energy from the source at 800 K and develops 200 kJ/min of work.*

(*i*) *Determine the heat interactions with the other two sources of heat.*

(*ii*) *Evaluate the entropy change due to each heat interaction with the engine.*

(*iii*) *Total entropy change during the cycle.*

Source T_1

↑ Q_1

Heat pump ← W

↑ Q_2

Sink T_2

Fig. 3.39
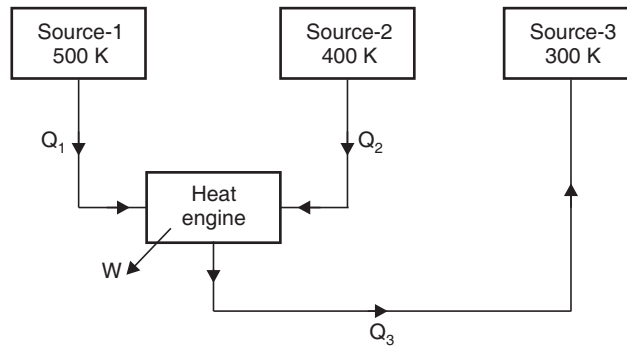
Fig. 3.40

Solution. Refer to Fig. 3.40.

Temperature of source 1 = 500 K

Temperature of source 2 = 400 K

Temperature of source 3 = 300 K

Heat energy drawn from source 1, Q_1 = 1500 kJ/min

Work developed, W = 200 kJ/min.

(*i*) The direction of heat flow from source 1 is known as given in the problem. Assume that the quantities of heats Q_2 and Q_3 are taken from heat sources and their directions are arbitrarily chosen.

For the cyclic operation of the engine

$$\oint \left(\frac{dQ}{T} \right) = 0$$

∴
$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} - \frac{Q_3}{T_3} = 0$$

and
$$Q_1 + Q_2 - Q_3 = W$$

∴
$$\frac{1500}{500} + \frac{Q_2}{400} - \frac{Q_3}{300} = 0 \qquad \qquad ...(i)$$

and
$$1500 + Q_2 - Q_3 = 200 \qquad \qquad ...(ii)$$

Solving eqns. (*i*) and (*ii*), we get

$$Q_2 = -1600 \text{ kJ/min and } Q_3 = -300 \text{ kJ/min.}$$

The above values indicate that the direction of Q_2 and Q_3 are reversed. Since Q_2 should be +ve and Q_3 also must be +ve but both are –ve therefore, their assumed directions should be reversed. The arrangement is shown in Fig. 3.41.

(*ii*) *Entropy change of source 1* $\quad = \dfrac{-Q_1}{T_1} = \dfrac{-1500}{500} = -\textbf{3 kJ/K.}$ **(Ans.)**

Entropy change of sink 2 $\quad = \dfrac{Q_2}{T_2} = \dfrac{1600}{400} = \textbf{4 kJ/K.}$ **(Ans.)**

Entropy change of source 3 $\quad = \dfrac{-Q_3}{T_3} = \dfrac{-300}{300} = -\textbf{1 kJ/K.}$ **(Ans.)**

(*iii*) *Net change of the entropy* $\quad = -3 + 4 - 1 = 0$
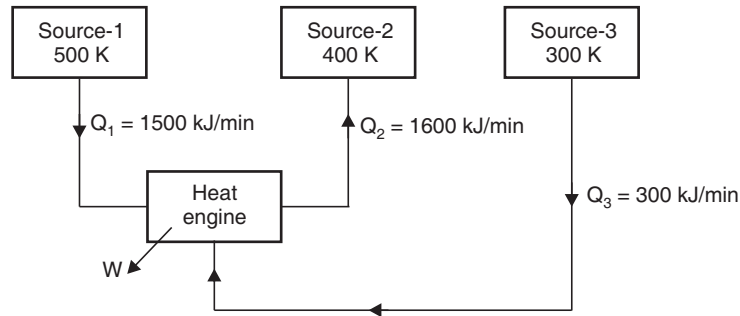
Fig. 3.41

As *the cycle is completed, the net change in entropy must be zero because entropy is a property.*

It may be observed from the new arrangement that the engine takes heat from source 1 and source 3 and rejects to source 2, only *i.e.*, the equipment does both *a heat engine and a heat pump function.*

Example 3.33. *The heat capacity of a system at constant volume is given by*

$$C_v = ZT^2$$

where Z = 0.045 J/K³

A system is originally at 250 K, and a thermal reservoir at 125 K is available. Determine the maximum amount of work that can be recovered as the system is cooled down to the temperature of the reservoir.

Solution. Refer to Fig. 3.42.

Initial temperature of the system, $T_1 = 250$ K

Temperature of the reservoir, $\quad T_2 = 125$ K

Heat capacity of the system at constant volume is

$$C_v = ZT^2 = 0.045\, T^2$$

$$\left[\begin{array}{l} \text{The product of mass and specific heat } (m_c) \text{ is called the} \\ \text{heat capacity of the substance. The capital letter } C,\ C_p \\ \text{or } C_v \text{ is used for heat capacity} \end{array} \right]$$

Heat removed from the system

$$Q_1 = \int_{T_1}^{T_2} C_v\, dT = \int_{250}^{125} 0.045T^2\ dT$$

$$= 0.045 \left[\frac{T^3}{3} \right]_{250}^{125} = \frac{0.045}{3}\ (125^3 - 250^3)$$

$$= -205.08 \times 10^3 \text{ J}$$

$$(\Delta S)_{system} = \int_{250}^{125} C_v\, \frac{dT}{T} = \int_{250}^{125} 0.045T^2\, \frac{dT}{T}$$

$$= 0.045 \int_{250}^{125} T dT = 0.045 \left[\frac{T^2}{2} \right]_{250}^{125}$$

$$= \frac{0.045}{2} (125^2 - 250^2) = -1054.7 \text{ J/K}$$
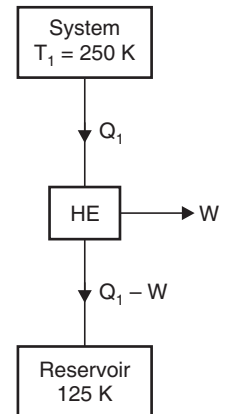
Fig. 3.42

$$(\Delta S)_{\text{reservoir}} = \frac{Q_1 - W}{T_{\text{reservoir}}} = \frac{205.08 \times 10^3 - W}{125} \text{ J/K}$$

$$(\Delta S)_{\text{working fluid in } HE} = 0$$

$$\therefore \quad (\Delta S)_{\text{universe}} = (\Delta S)_{\text{system}} + (\Delta S)_{\text{reservoir}} = -1054.7 + \frac{205.08 \times 10^3 - W}{125}$$

Since $(\Delta S)_{\text{universe}} \geq 0$

$$\therefore \quad -1054.7 + \frac{205.08 \times 10^3 - W}{125} \geq 0$$

or $\quad -1054.7 + 1640.64 - \dfrac{W}{125} \geq 0 \quad$ or $\quad 585.94 - \dfrac{W}{125} \geq 0$

or $\quad 585.94 \geq \dfrac{W}{125} \quad$ or $\quad \dfrac{W}{125} \leq 585.94$

i.e., $\quad W_{\text{(max.)}} = 585.94 \times 125 = $ **73.24 kJ.** **(Ans.)**

Example 3.34. *In an insulated duct air is flowing steadily. The pressure and temperature measurements of the air at two stations A and B are given below :*

Station	Pressure	Temperature
A	*140 kPa*	*60°C*
B	*110 kPa*	*15°C*

Establish the direction of the flow of the air in the duct.

For air assume that :

$$c_p = 1.005 \text{ kJ/kg K}$$

$$h = c_p\, T \text{ and } \frac{v}{T} = \frac{0.287}{p}$$

where p, v and T are pressure (in kPa), volume (in m³/kg) and temperature (in K) respectively.

Solution. From property relation,

$$T\,ds = dh - v\,dp$$

$$ds = \frac{dh}{T} - \frac{v\,dp}{T}$$

For two states at *A* and *B* the entropy changes of the system is given by

$$\int_{s_A}^{s_B} ds = \int_{T_A}^{T_B} \frac{c_p\, dT}{T} - \int_{P_A}^{P_B} 0.287\, \frac{dp}{p}$$

$$\therefore \quad s_B - s_A = 1.005 \log_e \frac{T_B}{T_A} - 0.287 \log_e \frac{110}{140}$$

$$= 1.005 \log_e \left(\frac{15 + 273}{60 + 273} \right) - 0.287 \log_e \frac{110}{140}$$

$$= -0.1459 + 0.0692 = -0.0767 \text{ kJ/kg K}$$

$$(\Delta s)_{\text{system}} = -0.0767 \text{ kJ/kg K}$$

Since the duct is insulated, $(\Delta s)_{\text{surroundings}} = 0$

$$\therefore \quad (\Delta s)_{\text{universe}} = -0.0767 \text{ kJ/kg K}.$$

This is impossible.

So, **the flow is from B to A.** **(Ans.)**

Example 3.35. *3 kg of water at 80°C is mixed with 4 kg of water at 15°C in an isolated system. Calculate the change of entropy due to mixing process.*

Solution. Fig. 3.43 shows the isolated system before mixing. When barrier is removed, the water from two compartments mix each other. Let t_m is the final equilibrium temperature after mixing.
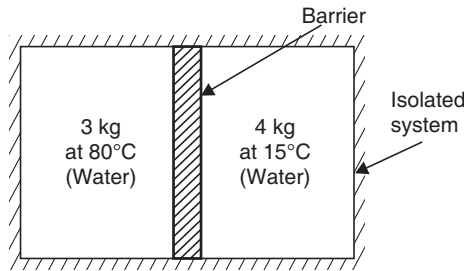
Fig. 3.43

Applying first law of thermodynamics to the isolated system :

Total energy before mixing = Total energy after mixing

$\therefore$ $\qquad\qquad\qquad 3c_{pw} (80 - 0) + 4c_{pw} (15 - 0) = 7 c_{pw} (t_m - 0)$

$\qquad\qquad\qquad\qquad\qquad\qquad$ [c_{pw} = specific heat of water at constant pressure]

or $\qquad\qquad\qquad 240c_{pw} + 60c_{pw} = 7c_{pw} t_m$

or $\qquad\qquad\qquad\qquad 240 + 60 = 7 t_m$

$\therefore$ $\qquad\qquad\qquad\qquad\qquad t_m = \dfrac{300}{7} = 42.85°C$

Initial entropy of the system,

$$= 3c_{pw} \log_e \left(\frac{80 + 273}{273} \right) + 4c_{pw} \log_e \left(\frac{15 + 273}{273} \right)$$

$$= 0.7709c_{pw} + 0.2139 \, c_{pw} = 0.9848 \, c_{pw}$$

Final entropy of the system

$$= (3 + 4) \, c_{pw} \log_e \left(\frac{42.85 + 273}{273} \right) = 1.0205 \, c_{pw}$$

Net change in entropy,

$\qquad\qquad \Delta S$ = Final entropy – Initial entropy

$\qquad\qquad\qquad = 1.0205c_{pw} - 0.9848 \, c_{pw} = 0.0357 \, c_{pw}$

$\qquad\qquad\qquad = 0.0357 \times 4.187$ kJ/K $\qquad\qquad$ [$\because \quad c_{pw}$ = 4.187 kJ/kg K]

$\qquad\qquad\qquad = 0.1495$ kJ/K

Hence, **net change in entropy = 0.1495 kJ/K.   (Ans.)**

Example 3.36. *A mass 'm' of fluid at temperature T_1 is mixed with an equal mass of the same fluid at T_2. Prove that the resultant change of entropy of the universe is* $2 \, mc \, \dfrac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}$ *and also prove that it is always positive.*

Solution. Mean temperature of the mixture = $\left(\dfrac{T_1 + T_2}{2} \right)$.

Thus change in entropy is given by

$$\Delta S = mc \int_{T_1}^{(T_1 + T_2)/2} \frac{dT}{T} - mc \int_{(T_1 + T_2)/2}^{T_2} \frac{dT}{T}$$

$$= mc \log_e \left(\frac{T_1 + T_2}{2T_1} \right) - mc \log_e \left(\frac{2T_2}{T_1 + T_2} \right)$$

$$= mc \log_e \left(\frac{T_1 + T_2}{2T_1} \right) + mc \log_e \left(\frac{T_1 + T_2}{2T_2} \right) = mc \log_e \frac{(T_1 + T_2)^2}{4T_1 T_2}$$

$$= mc \log_e \left[\frac{(T_1 + T_2)}{2\sqrt{T_1 T_2}} \right]^2 = 2 \, mc \log_e \left[\frac{T_1 + T_2}{2\sqrt{T_1 T_2}} \right]$$

$$= 2 \, mc \log_e \left[\frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}} \right]$$

i.e.,   **Resultant change of entropy of universe**

$$= \textbf{2 mc log}_\textbf{e} \left[\frac{\textbf{(T}_\textbf{1} + \textbf{T}_\textbf{2}\textbf{)/2}}{\sqrt{\textbf{T}_\textbf{1} \textbf{T}_\textbf{2}}} \right] \qquad \textbf{...Proved}$$

The arithmetic mean $\dfrac{T_1 + T_2}{2}$ is greater than geometric mean $\sqrt{T_1 T_2}$.

Therefore, $\log_e \dfrac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}$ is +ve.

Thus, **the entropy of the universe increases.**

☞**Example 3.37.** (*a*) *One kg of water at 0°C is brought into contact with a heat reservoir at 90°C. When the water has reached 90°C, find* :

 (*i*) *Entropy change of water* ;

(*ii*) *Entropy change of the heat reservoir* ;

(*iii*) *Entropy change of the universe.*

(*b*) *If water is heated from 0°C to 90°C by first bringing it in contact with a reservoir at 40°C and then with a reservoir at 90°C, what will the entropy change of the universe be* ?

(*c*) *Explain how water might be heated from 0°C to 90°C with almost no change in the entropy of the universe.*

Solution. Mass of water, m = 1 kg

Temperature of water, T_1 = 0 + 273 = 273 K

Temperature of the heat reservoir, T_2 = 90 + 273 = 363 K.

(*a*) Refer to Fig. 3.44. Water is being heated through a finite temperature difference. The entropy of water would increase and that of the reservoir would decrease so that net entropy change of the water (system) and the reservoir together would be positive definite. *Water is being heated, irreversibly,* and to find the entropy change of water, we have to assume a reversible path between the end states, which are at equilibrium.

Reservoir
90 + 273 = 363 K

Q

System
0 + 273
= 273 K
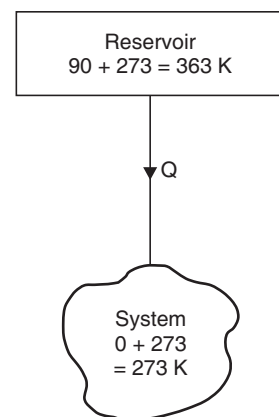
Fig. 3.44

(*i*) *Entropy of water,*

$$(\Delta s)_{water} = \int_{T_1}^{T_2} \frac{mcdT}{T} = mc \log_e \frac{T_2}{T_1}$$

(c = specific heat of water)

$$= 1 \times 4.187 \times \log_e \frac{363}{273}$$

$$= \textbf{1.193 kJ/kg K.} \quad \textbf{(Ans.)}$$

(*ii*) The temperature of the reservoir remains *constant* irrespective of the amount of the heat withdrawn.

Amount of heat absorbed by the system from the reservoir,

$$Q = 1 \times 4.187 \times (363 - 273) = 376.8 \text{ kJ}$$

∴   *Entropy change of the reservoir,*

$$(\Delta s)_{reservoir} = - \frac{Q}{T} = - \frac{376.8}{363} = -\textbf{1.038 kJ/K.} \quad \textbf{(Ans.)}$$

(–ve sign indicates decrease in entropy).

(*iii*) *Entropy change of the universe,*

$$(\Delta s)_{universe} = (\Delta s)_{water \, (or \, system)} + (\Delta s)_{reservoir}$$

$$= 1.193 + (- 1.038) = \textbf{0.155 kJ/K.} \quad \textbf{(Ans.)}$$

(*b*) The heating of water is being carried out in two stages, first from 0°C (273 K) to 40°C (*i.e.*, 313 K) by bringing in contact with a reservoir at 40°C (313 K), and then from 40°C (313 K) to 90°C (363 K) by bringing in contact with a second reservoir at 90°C (363 K).

$$(\Delta s)_{water} = \int_{273}^{313} mc \frac{dT}{T} + \int_{313}^{363} mc \frac{dT}{T} = 1 \times 4.187 \left(\log_e \frac{313}{273} + \log_e \frac{363}{313} \right)$$

$$= 4.187 \, (0.1367 + 0.1482) = 1.1928 \text{ kJ/K.}$$

$$(\Delta s)_{res. \, I} = - \frac{1 \times 4.187 \times (313 - 273)}{313} = - 0.535 \text{ kJ/K}$$

$$(\Delta s)_{res. \, II} = - \frac{1 \times 4.187 \, (363 - 313)}{363} = - 0.576 \text{ kJ/K}$$

∴ $$(\Delta s)_{univ.} = (\Delta s)_{water} + (\Delta s)_{res. \, I} + (\Delta s)_{res. \, II}$$

$$= 1.1928 + (- 0.535) + (- 0.576) = 0.0818 \text{ kJ/K.}$$

i.e.,   **Entropy change of universe = 0.0818 kJ/K.   (Ans.)**

(*c*) The entropy change of universe would be *less and less*, if the water is heated in *more and more stages*, by bringing the water in contact successively with *more and more heat reservoirs,* each succeeding reservoir being at a *higher temperature* than the preceding one. When water is heated in infinite steps, by bringing in contact with an infinite number of reservoirs in succession, so that at any instant the temperature difference between the water and the reservoir in contact is infinitesimally small, then the entropy change of the universe would be *zero* and the water would be *reversibly* heated.

Example 3.38. *1 kg of ice at – 5°C is exposed to the atmosphere which is at 25°C. The ice melts and comes into thermal equilibrium.*

(*i*) *Determine the entropy increase of the universe.*

(*ii*) *What is the minimum amount of work necessary to convert the water back into ice at – 5°C ?*

Take : c_p of ice = 2.093 kJ/kg°C

Latent heat of fusion of ice = 333.33 kJ/kg.

Solution. Mass of ice, $m = 1$ kg

Temperature of ice $= -5°C (= -5 + 273 = 268$ K)

Temperature of atmosphere $= 25°C (= 25 + 273 = 298$ K)

Heat absorbed by ice from the atmosphere (Fig. 3.45)

  = heat absorbed in solid phase

                + latent heat + heat absorbed in liquid phase

  $= 1 \times 2.093 \times [0 - (-5)] + 1 \times 333.33 + 1 \times 4.187 \times (25 - 0)$

  $= 10.46 + 333.33 + 104.67 = 448.46$ kJ.

(*i*) **Entropy increase of the universe,** $(\Delta s)_{universe}$:

Entropy change of the atmosphere,

$$(\Delta s)_{atm.} = -\frac{Q}{T} = -\frac{448.46}{298} = -1.5049 \text{ kJ/K}$$

Entropy change of system (ice) as it gets heated from $-5°C$ to $0°C$

$$(\Delta s_{\text{I}})_{system} = \int_{268}^{273} mc_p \frac{dT}{T} = 1 \times 2.093 \log_e \frac{273}{268} = 0.0386 \text{ kJ/K}$$

Entropy change of the system as ice melts at $0°C$ to become water at $0°C$.

$$(\Delta s_{\text{II}})_{system} = \frac{333.33}{273} = 1.2209 \text{ kJ/K}$$

Entropy change of water as it gets heated from $0°C$ to $25°C$

$$(\Delta s_{\text{III}})_{system} = \int_{273}^{298} mc_p \frac{dT}{T} = 1 \times 4.187 \log_e \left(\frac{298}{273}\right) = 0.3668 \text{ kJ/K}$$

Total entropy change of ice as it melts into water

$$(\Delta s)_{total} = \Delta s_{\text{I}} + \Delta s_{\text{II}} + \Delta s_{\text{III}}$$
$$= 0.0386 + 1.2209 + 0.3668 = 1.6263 \text{ kJ/K}$$

Then temperature-entropy diagram for the system as ice at $-5°C$ converts to water at $25°C$ is shown in Fig. 3.46.

  ∴ **Entropy increase of the universe,**

$$(\Delta s)_{univ.} = (\Delta s)_{system} + (\Delta s)_{atm.}$$
$$= 1.6263 + (-1.5049) = \textbf{0.1214 kJ/K.}\quad \textbf{(Ans.)}$$

(*ii*) **Minimum amount of work necessary to convert the water back into ice at –5°C, W_{min} :**

To convert 1 kg of water at $25°C$ to ice at $-5°C$, 448.46 kJ of heat have to be removed from it, and the system has to be brought from state 4 to state 1 (Fig. 3.46). A refrigerator cycle, as shown in Fig. 3.47, is assumed to accomplish this. The entropy change of the system would be the same, *i.e.*, $s_4 - s_1$, with the only difference that its sign will be negative, because heat is removed from the system (Fig. 3.46).

$$(\Delta s)_{system} = s_1 - s_4 \text{ (negative)}$$

The entropy change of the working fluid in the refrigerator would be zero, since it is operating in a cycle, *i.e.*,

$$(\Delta s)_{refrigerator} = 0.$$

The entropy change of the atmosphere (*positive*),

$$(\Delta s)_{atm.} = \frac{Q + W}{T}$$

(Figure, top right):

Atmosphere 25°C

↓ Q

$-5°C$ Ice
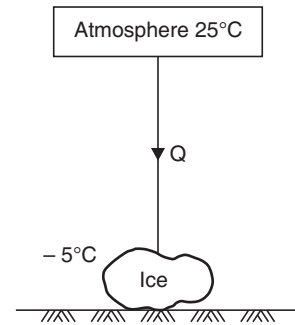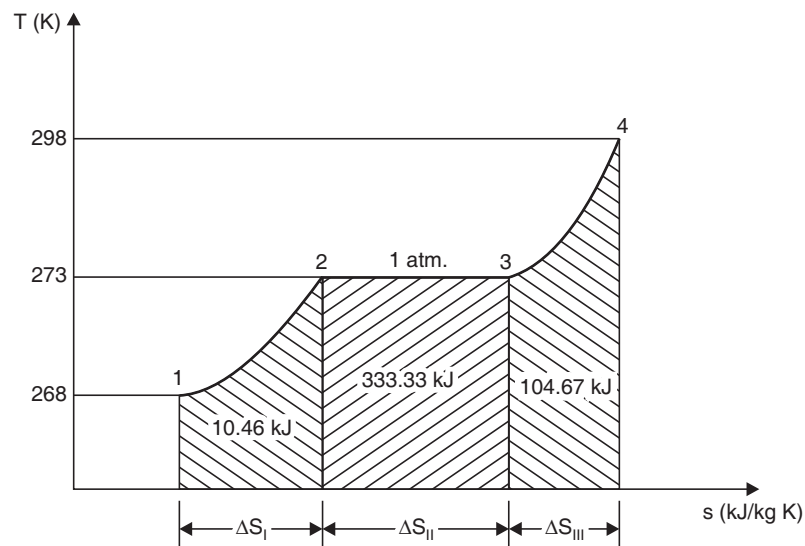
Fig. 3.45

Fig. 3.46

∴   Entropy change of the universe

$$(\Delta s)_{univ.} = (\Delta s)_{system} + (\Delta s)_{refrigerator} + (\Delta s)_{atm.}$$

$$= (s_1 - s_4) + 0 + \frac{Q + W}{T} = (s_1 - s_4) + \frac{Q + W}{T}$$
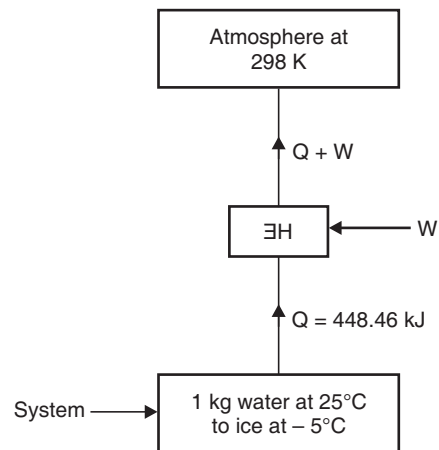
Fig. 3.47

By the principle of increase of entropy

$$(\Delta s)_{univ.\ or\ isolated\ system} \geq 0$$

∴ $$\left[(s_1 - s_4) + \frac{Q + W}{T} \right] \geq 0$$

∴ $$\frac{Q + W}{T} \geq (s_4 - s_1)$$

$$W \geq T\,(s_4 - s_1) - Q$$

$$W_{(min)} = T (s_4 - s_1) - Q$$

Here
$$Q = 448.46 \text{ kJ}$$
$$T = 298 \text{ K}$$
$$s_4 - s_1 = 1.6263 \text{ kJ/K}$$

$\therefore$ $\quad$ $\mathbf{W_{(min)}} = 298 \times 1.6263 - 448.46 = \mathbf{36.17 \text{ kJ.}}$ $\quad$ **(Ans.)**

HIGHLIGHTS

---

1. **Clausius statement :**
   "It is impossible for a self-acting machine working in a cyclic process, unaided by any external agency, to convey heat from a body at a lower temperature to a body at a higher temperature."
   Kelvin-Planck statement :
   "It is impossible to construct an engine, which while operating in a cycle produces no other effect except to extract heat from a single reservoir and do equivalent amount of work".
   Although above statements of second law of thermodynamic appear to be different, they are really equivalent in the sense that violation of either statement implies violation of other.

2. **Perpetual motion machine of second kind** (*PMM*2) is that imaginary machine which would continuously absorb heat from a single thermal reservoir and convert this heat completely into work. The efficiency of such a machine would be 100%.

3. Clausius inequality is given by,

$$\sum_{\text{Cycle}} \left(\frac{\delta Q}{T} \right) \leq 0$$

   "When a system performs a reversible cycle, then

$$\sum_{\text{Cycle}} \left(\frac{\delta Q}{T} \right) = 0,$$

   but when the cycle is not reversible

$$\sum_{\text{Cycle}} \left(\frac{\delta Q}{T} \right) < 0."$$

4. **'Entropy'** is a function of a quantity of heat which shows the possibility of conversion of that heat into work. The increase in entropy is small when heat is added at a high temperature and is greater when heat addition is made at lower temperature. Thus for maximum entropy, there is a minimum availability for conversion into work and for minimum entropy there is maximum availability for conversion into work.

5. **Entropy changes for a closed system (per kg) :**
   (*i*) *General case* :

   (*a*) $c_v \log_e \dfrac{T_2}{T_1} + R \log_e \dfrac{v_2}{v_1}$ (in terms of T and v)

   (*b*) $c_v \log_e \dfrac{p_2}{p_1} + c_p \log_e \dfrac{v_2}{v_1}$ (in terms of p and v)

   (*c*) $c_p \log_e \dfrac{T_2}{T_1} - R \log_e \dfrac{p_2}{p_1}$ > (in terms of T and p)

   (*ii*) *Constant volume* : $c_v \log_e \dfrac{T_2}{T_1}$ $\qquad\qquad$ (*iii*) *Constant pressure* : $c_p \log_e \dfrac{T_2}{T_1}$

   (*iv*) *Isothermal* : $R \log_e \dfrac{v_2}{v_1}$ $\qquad\qquad$ (*v*) *Adiabatic* : zero

(*vi*) *Polytropic* : $\qquad c_v\left(\dfrac{n-\gamma}{n-1}\right)\log_e\dfrac{T_2}{T_1}$.

6. Entropy change for an open system

$$dS \geq \dfrac{dQ}{T_0} + \Sigma s_i\, .\, dm_i - \Sigma s_0\, .\, dm_0$$

where T_0 = temperature of the surroundings

Subscripts i and 0 refer to inlet and outlet conditions.

<div style="border:1px solid">

OBJECTIVE TYPE QUESTIONS

</div>

Choose the Correct Answer :

1. Second law of thermodynamics defines
    (*a*) heat   (*b*) work
    (*c*) enthalpy   (*d*) entropy
    (*e*) internal energy.

2. For a reversible adiabatic process, the change in entropy is
    (*a*) zero   (*b*) minimum
    (*c*) maximum   (*d*) infinite
    (*e*) unity.

3. For any reversible process, the change in entropy of the system and surroundings is
    (*a*) zero   (*b*) unity
    (*c*) negative   (*d*) positive
    (*e*) infinite.

4. For any irreversible process the net entropy change is
    (*a*) zero   (*b*) positive
    (*c*) negative   (*d*) infinite
    (*e*) unity.

5. The processes of a Carnot cycle are
    (*a*) two adiabatic and two constant volume
    (*b*) one constant volume and one constant pressure and two isentropics
    (*c*) two adiabatics and two isothermals
    (*d*) two constant volumes and two isothermals
    (*e*) two isothermals and two isentropics.

6. Isentropic flow is
    (*a*) irreversible adiabatic flow   (*b*) ideal fluid flow
    (*c*) perfect gas flow   (*d*) frictionless reversible flow
    (*e*) reversible adiabatic flow.

7. In a Carnot engine, when the working substance gives heat to the sink
    (*a*) the temperature of the sink increases
    (*b*) the temperature of the sink remains the same
    (*c*) the temperature of the source decreases
    (*d*) the temperatures of both the sink and the source decrease
    (*e*) changes depend on the operating conditions.

8. If the temperature of the source is increased, the efficiency of the Carnot engine
    (*a*) decreases   (*b*) increases
    (*c*) does not change   (*d*) will be equal to the efficiency of a practical engine
    (*e*) depends on other factors.

9. The efficiency of an ideal Carnot engine depends on
    (*a*) working substance   (*b*) on the temperature of the source only

    (*c*) on the temperature of the sink only

    (*d*) on the temperatures of both the source and the sink

    (*e*) on the construction of engine.

10. The efficiency of a Carnot engine using an ideal gas as the working substance is

    (*a*) $\dfrac{T_1 - T_2}{T_1}$ (*b*) $\dfrac{T_1}{T_1 - T_2}$

    (*c*) $\dfrac{T_1\, T_2}{T_1 - T_2}$ (*d*) $\dfrac{T_1 - T_2}{T_1 T_2}$

    (*e*) $\dfrac{T_2(T_1 - T_2)}{T_1(T_1 + T_2)}$.

11. In a reversible cycle, the entropy of the system

    (*a*) increases   (*b*) decreases

    (*c*) does not change   (*d*) first increases and then decreases

    (*e*) depends on the properties of working substance.

12. A frictionless heat engine can be 100% efficient only if its exhaust temperature is

    (*a*) equal to its input temperature   (*b*) less than its input temperature

    (*c*) 0°C   (*d*) 0°K

    (*e*) – 100°C.

13. Kelvin-Planck's law deals with

    (*a*) conservation of energy   (*b*) conservation of heat

    (*c*) conservation of mass   (*d*) conversion of heat into work

    (*e*) conversion of work into heat.

14. Which of the following statements is correct according to Clausius statement of second law of thermodynamics ?

    (*a*) It is impossible to transfer heat from a body at a lower temperature to a body at a higher temperature

    (*b*) It is impossible to transfer heat from a body at a lower temperature to a body at a higher temperature, without the aid of an external source

    (*c*) It is possible to transfer heat from a body at a lower temperature to a body at a higher temperature by using refrigeration cycle

    (*d*) None of the above.

15. According to Kelvin-Planck's statement of second law of thermodynamics

    (*a*) It is impossible to construct an engine working on a cyclic process, whose sole purpose is to convert heat energy into work

    (*b*) It is possible to construct an engine working on a cyclic process, whose sole purpose is to convert the heat energy into work

    (*c*) It is impossible to construct a device which while working in a cyclic process produces no effect other than the transfer of heat from a colder body to a hotter body

    (*d*) When two dissimilar metals are heated at one end and cooled at the other, the e.m.f. developed is proportional to the difference of their temperatures at the two end

    (*e*) None of the above.

16. The property of a working substance which increases or decreases as the heat is supplied or removed in a reversible manner is known as

    (*a*) enthalpy   (*b*) internal energy

    (*c*) entropy   (*d*) external energy.

17. The entropy may be expressed as a function of

    (*a*) pressure and temperature   (*b*) temperature and volume

    (*c*) heat and work   (*d*) all of the above

    (*e*) none of the above.

18. The change of entropy, when heat is absorbed by the gas is

    (*a*) positive   (*b*) negative

    (*c*) positive or negative.

19. Which of the following statements is *correct* ?
    (*a*) The increase in entropy is obtained from a given quantity of heat at a low temperature
    (*b*) The change in entropy may be regarded as a measure of the rate of the availability of heat for transformation into work
    (*c*) The entropy represents the maximum amount of work obtainable per degree drop in temperature
    (*d*) All of the above.

20. The condition for the reversibility of a cycle is
    (*a*) the pressure and temperature of working substance must not differ, appreciably from those of the surroundings at any stage in the process
    (*b*) all the processes taking place in the cycle of operation, must be extremely slow
    (*c*) the working parts of the engine must be friction free
    (*d*) there should be no loss of energy during the cycle of operation
    (*e*) all of the above.

21. In an irreversible process there is a
    (*a*) loss of heat   (*b*) no loss of work
    (*c*) gain of heat   (*d*) no gain of heat.

22. The main cause for the irreversibility is
    (*a*) mechanical and fluid friction   (*b*) unrestricted expansion
    (*c*) heat transfer with a finite temperature difference
    (*d*) all of the above.

23. The efficiency of the Carnot cycle may be increased by
    (*a*) increasing the highest temperature   (*b*) decreasing the highest temperature
    (*c*) increasing the lowest temperature   (*d*) decreasing the lowest temperature
    (*e*) keeping the lowest temperature constant.

24. Which of the following is the correct statement ?
    (*a*) All the reversible engines have the same efficiency
    (*b*) All the reversible and irreversible engines have the same efficiency
    (*c*) Irreversible engines have maximum efficiency
    (*d*) All engines are designed as reversible in order to obtain maximum efficiency.

Answers

1. (*d*)	**2.** (*a*)	**3.** (*a*)	**4.** (*b*)	**5.** (*e*)
6. (*e*)	**7.** (*b*)	**8.** (*b*)	**9.** (*d*)	**10.** (*a*)
11. (*c*)	**12.** (*d*)	**13.** (*d*)	**14.** (*b*)	**15.** (*e*)
16. (*c*)	**17.** (*a*)	**18.** (*a*)	**19.** (*d*)	**20.** (*e*)
21. (*a*)	**22.** (*d*)	**23.** (*d*)	**24.** (*a*).	

THEORETICAL QUESTIONS

1. State the limitations of first law of thermodynamics.
2. What is the difference between a heat engine and a reversed heat engine ?
3. Enumerate the conditions which must be fulfilled by a reversible process. Give some examples of ideal reversible processes.
4. What is an irreversible process ? Give some examples of irreversible processes.
5. Give the following statements of second law of thermodynamics.
   (*i*) Clausius statement
   (*ii*) Kelvin-Planck statement.
6. Define heat engine, refrigerator and heat pump.
7. What is the perpetual motion machine of the second kind ?
8. What do you mean by 'Thermodynamic temperature' ?
9. What do you mean by 'Clausius inequality' ?

10. Describe the working of a Carnot cycle.
11. Derive an expression for the efficiency of the reversible heat engine.
12. What do you mean by the term 'Entropy' ?
13. What are the characteristics of entropy ?
14. Prove that entropy is a property of a system.
15. Derive an expression for the change in entropy of the universe.
16. What is a temperature entropy diagram ?
17. Derive expressions for entropy changes for a closed system in the following cases :
    (*i*) General case for change of entropy of a gas
   (*ii*) Heating a gas at constant volume
  (*iii*) Heating a gas at constant pressure
   (*iv*) Polytropic process.
18. Give an expression for entropy changes for an open system.

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UNSOLVED EXAMPLES

</div>

Heat Engines and Reversible Heat Engines

1. A cyclic heat engine operates between a source temperature of 800°C and a sink temperature of 30°C. What is the least rate of heat rejection per kW net output of the engine ? [**Ans.** 0.392 kW]

2. A domestic food freezer maintains a temperature of – 15°C. The ambient air temperature is 30°C. If heat leaks into the freezer at the continuous rate of 1.75 kJ/s what is the least power necessary to pump this heat out continuously ? [**Ans.** 0.31 kW]

3. A reversible heat engine operates between two reservoirs at temperatures of 600°C and 40°C. The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 40°C and – 20°C. The heat transfer to the heat engine is 2000 kJ and the net work output for the combined engine refrigerator is 360 kJ. (*i*) Calculate the heat transfer to the refrigerant and the net heat transfer to the reservoir at 40°C. (*ii*) Reconsider (*i*) given that the efficiency of the heat engine and the C.O.P. of the refrigerator are each 40 per cent of their maximum possible values.
   [**Ans.** Heat rejection to 40°C reservoir (*i*) 5539 kJ ; (*ii*) 1899.6 kJ]

4. A heat engine is supplied heat at the rate of 1700 kJ/min and gives an output of 9 kW. Determine the thermal efficiency and the rate of heat rejection. [**Ans.** 31.76% ; 9.333 kJ/s]

5. What is the highest possible theoretical efficiency of a heat engine operating with a hot reservoir of furnace gases at 2000°C when the cooling water available is at 10°C ? [**Ans.** 87.54%]

6. A Carnot cycle operates between source and sink temperatures of 260°C and – 17.8°C. If the system receives 100 kJ from the source, find (*i*) efficiency of the system, (*ii*) the net work transfer, (*iii*) heat rejected to the sink. [**Ans.** (*i*) 52.2% ; (*ii*) 52.2 kJ ; (*iii*) 47.8 kJ]

7. Source *A* can supply energy at a rate of 11,000 kJ/min at 320°C. A second source *B* can supply energy at a rate of 110,000 kJ/min at 68°C. Which source *A* or *B*, would you choose to supply energy to an ideal reversible engine that is to produce large amount of power if the temperature of the surroundings is 40°C ? [**Ans.** Source *B*]

8. A fish freezing plant requires 50 tons of refrigeration. The freezing temperature is – 40°C while the ambient temperature is 35°C. If the performance of the plant is 15% of the theoretical reversed Carnot cycle working within the same temperature limits, calculate the power required. [**Ans.** 375 kW]

Clausius Inequality

9. A heat engine is supplied with 278 kJ/s of heat at a constant fixed temperature of 283°C and the heat rejection takes place at 5°C. The following results were reported : (*i*) 208 kJ/s are rejected, (*ii*) 139 kJ/s are rejected, (*iii*) 70 kJ/s are rejected.
   Classify which of the results report a reversible cycle or irreversible cycle or impossible results.
   [**Ans.** (*i*) Irreversible ; (*ii*) Reversible ; (*iii*) Impossible]

Entropy

10. Air at 15°C and 1.05 bar occupies 0.02 m³. The air is heated at constant volume until the pressure is 4.2 bar, and then cooled at constant pressure back to the original temperature. Calculate the net heat flow to or from the air and the net entropy change.

    Sketch the process on a T-s diagram.

    [**Ans.** Heat rejected = 6.3 kJ, decrease in entropy of air = 0.0101 kJ/K]

11. 0.03 m³ of nitrogen contained in a cylinder behind a piston is initially at 1.05 bar and 15°C. The gas is compressed isothermally and reversibly until the pressure is 4.2 bar. Calculate the change of entropy, the heat flow, and the work done, and sketch the process on a p-v and T-s diagram. Assume nitrogen to act as a perfect gas. Molecular weight of nitrogen = 28.

    [**Ans.** 0.01516 kJ/K (decrease) ; 4.37 kJ (heat rejected) ; 4.37 kJ]

12. Calculate the change of entropy of 1 kg of air expanding polytropically in a cylinder behind a piston from 6.3 bar and 550°C to 1.05 bar. The index of expansion is 1.3.   [**Ans.** 0.1 kJ/kg K (increase)]

13. 0.05 kg of carbon dioxide (molecular weight = 44) is compressed from 1 bar, 15°C, until the pressure is 8.3 bar, and the volume is then 0.004 m³. Calculate the change of entropy. Take c_p for carbon dioxide as 0.88 kJ/kg K, and assume carbon dioxide to be a perfect gas.   [**Ans.** 0.0113 kJ/K (decrease)]

14. In an air turbine the air expands from 6.8 bar and 430°C to 1.013 bar and 150°C. The heat loss from the turbine can be assumed to be negligible. Show that the process is irreversible, and calculate the change of entropy per kg of air.   [**Ans.** 0.0355 kJ/kg K (increase)]

15. (*i*) One kg of water at 273 K is brought into contact with a heat reservoir at 373 K. When the water has reached 373 K, find the entropy change of the water of the heat reservoir, and of the universe.

    (*ii*) If water is heated from 273 K to 373 K by first bringing it in contact with a reservoir at 323 K and then with reservoir at 373 K, what will the entropy change of the universe be ?

    [**Ans.** (*i*) 0.183 kJ/K ; (*ii*) 0.098 kJ/K]

16. One kg of ice at – 5°C is exposed to the atmosphere which is at 20°C. The ice melts and comes into thermal equilibrium with the atmosphere.

    (*i*) Determine the entropy increase of the universe.

    (*ii*) What is the minimum amount of work necessary to convert the water back into ice at – 5°C ? c_p of ice is 2.093 kJ/kg °C and the latent heat of fusion of ice is 333.3 kJ/kg.

    [**Ans.** (*i*) 0.0949 kJ/K (increase) (*ii*) 28.5 kJ]

17. A system has a heat capacity at constant volume $C_v = AT^2$, where A = 0.042 J/K³. The system is originally at 200 K and a thermal reservoir at 100 K is available. What is the maximum amount of work that can be recovered as the system is cooled down to the temperature of the reservoir ?

    [**Ans.** 35 kJ]

18. A fluid undergoes a reversible adiabatic compression from 0.5 MPa, 0.2 m³ to 0.05 m³ according to the law, $pv^{1.3}$ = constant. Determine the change in enthalpy, internal energy and entropy, and the heat transfer and work transfer during the process.   [**Ans.** 223.3 kJ ; 171.77 kJ, ; zero ; zero ; – 171.77 kJ]

19. A rigid cylinder containing 0.006 m³ of nitrogen (molecular weight 28) at 1.04 bar, 15°C, is heated reversibly until the temperature is 90°C. Calculate the change of entropy and the heat supplied. Sketch the process on T-s diagram. Take the isentropic index, γ, for nitrogen as 1.4, and assume that nitrogen is a perfect gas.   [**Ans.** 0.00125 kJ/K ; 0.407 kJ]

20. 1 m³ of air is heated reversibly at constant pressure from 15°C to 300°C, and is then cooled reversibly at constant volume back to the initial temperature. The initial pressure is 1.03 bar. Calculate the net heat flow and overall change of entropy, and sketch the process on a T-s diagram.

    [**Ans.** 101.5 kJ ; 0.246 kJ/K]

21. 1 kg of air is allowed to expand reversibly in a cylinder behind a piston in such a way that the temperature remains constant at 260°C while the volume is doubled. The piston is then moved in, and heat is rejected by the air reversibly at constant pressure until the volume is the same as it was initially. Calculate the net heat flow and the overall change of entropy. Sketch the processes on a T-s diagram.   [**Ans.** – 161.9 kJ/kg ; – 0.497 kJ/kg K]

22. 1 kg of air at 1.013 bar, 17°C, is compressed according to a law $pv^{1.3}$ = constant, until the pressure is 5 bar. Calculate the change of entropy and sketch the process on a T-s diagram, indicating the area, which represents the heat flow.   [**Ans.** – 0.0885 kJ/kg K]

23. 0.06 m^3 of ethane (molecular weight 30), at 6.9 bar and 60°C, is allowed to expand isentropically in a cylinder behind a piston to a pressure of 1.05 bar and a temperature of 107°C. Calculate γ, R, c_p, c_v for ethane, and calculate the work done during the expansion. Assume ethane to be a perfect gas. The same mass of ethane at 1.05 bar, 107°C, is compressed to 6.9 bar according to a law $pv^{1.4}$ = constant. Calculate the final temperature of the ethane and the heat flow to or from the cylinder walls during the compression. Calculate also the change of entropy during the compression, and sketch both processes on a p-v and a T-s diagram.

[**Ans.** 1.219 ; 0.277 kJ/kg K ; 1.542 kJ/kg K ; 1.265 kJ/kg K ; 54.2 kJ ; 378°C ; 43.4 kJ ; 0.0867 kJ/K]

24. In a reversible process the rate of heat transfer to the system per unit temperature rise is given by

$\dfrac{dQ}{dT}$ = 0.5 kJ/°C. Find the change in entropy of the system if its temperature rises from 500 K to 800 K. [**Ans.** 0.235 kJ/kg°C]

25. In a reversible cycle 100 kJ of heat is received at 500 K ; then an adiabatic expansion occurs to 400 K, at which temperature 50 kJ of heat is received, then a further adiabatic expansion to 300 K at which temperature 100 kJ of heat is rejected :

(*i*) Find the change in entropy which occurs as the system is restored to its initial state in the remainder of the cycle.

(*ii*) If during the remainder of the cycle heat is transferred only at 400 K, how much heat is transferred and in what direction ? [**Ans.** 0.008 kJ/K ; + 3.2 kJ]

26. 1 kg of air is compressed according to the law $pv^{1.25}$ = constant from 1.03 bar and 15°C to 16.45 bar. Calculate the change in entropy. [**Ans.** 0.255 kJ/kg K]

27. A quantity of gas (mean molecular weight 36.2) is compressed according to the law pv^n = constant, the initial pressure and volume being 1.03 bar and 0.98 m^3 respectively. The temperature at the start of compression is 17°C and at the end it is 115°C. The amount of heat rejected during compression is 3.78 kJ, c_p = 0.92. Calculate :

(*i*) Value of n,   (*ii*) Final pressure,

(*iii*) Change in entropy.   [**Ans.** (*i*) 1.33 ; (*ii*) 1.107 bar ; (*iii*) 0.228 kJ/kg K]

PART B

4

Gas Power Cycles

4.1. DEFINITION OF A CYCLE

A **cycle** is defined as a *repeated series of operations occurring in a certain order*. It may be repeated by repeating the processes in the same order. The cycle may be of imaginary perfect engine or actual engine. The former is called **ideal cycle** and the latter **actual cycle**. In ideal cycle all accidental heat losses are prevented and the working substance is assumed to behave like a perfect working substance.

4.2. AIR STANDARD EFFICIENCY

To compare the effects of different cycles, it is of paramount importance that the effect of the calorific value of the fuel is altogether eliminated and this can be achieved by considering air (which is assumed behave as a perfect gas) as the working substance in the engine cylinder. *The efficiency of engine using air as the working medium is known as an* "**Air standard efficiency**". This efficiency is oftenly called **ideal efficiency**.

The actual efficiency of a cycle is always less than the air-standard efficiency of that cycle under ideal conditions. This is taken into account by introducing a new term "**Relative efficiency**" which is defined as :

$$\eta_{\text{relative}} = \frac{\text{actual thermal efficiency}}{\text{air standard efficiency}} \qquad ...(4.1)$$

The analysis of all air standard cycles is based upon the following *assumptions* :

Assumptions :

1. The gas in the engine cylinder is a perfect gas *i.e.*, it obeys the gas laws and has constant specific heats.

2. The physical constants of the gas in the cylinder are the same as those of air at moderate temperatures *i.e.*, the molecular weight of cylinder gas is 29.

$$c_p = 1.005 \text{ kJ/kg-K}, \qquad c_v = 0.718 \text{ kJ/kg-K}.$$

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3. The compression and expansion processes are adiabatic and they take place without internal friction, *i.e.*, these processes are isentropic.

4. No chemical reaction takes place in the cylinder. Heat is supplied or rejected by bringing a hot body or a cold body in contact with cylinder at appropriate points during the process.

5. The cycle is considered closed with the same 'air' always remaining in the cylinder to repeat the cycle.

4.3.   THE CARNOT CYCLE

This cycle has the *highest possible efficiency* and consists of four simple operations namely,

(*a*) Isothermal expansion

(*b*) Adiabatic expansion

(*c*) Isothermal compression

(*d*) Adiabatic compression.

The condition of the Carnot cycle may be imagined to occur in the following way :

One kg of air is enclosed in the cylinder which (except at the end) is made of perfect non-conducting material. A source of heat '*H*' is supposed to provide unlimited quantity of heat, non-conducting cover '*C*' and a sump '*S*' which is of infinite capacity so that its temperature remains unchanged irrespective of the fact how much heat is supplied to it. The temperature of source H is T_1 and the same is of the working substance. The working substance while rejecting heat to sump '*S*' has the temperature. T_2 *i.e.*, the same as that of sump S.

Following are the *four stages* of the Carnot cycle. Refer Fig. 4.1 (*a*).
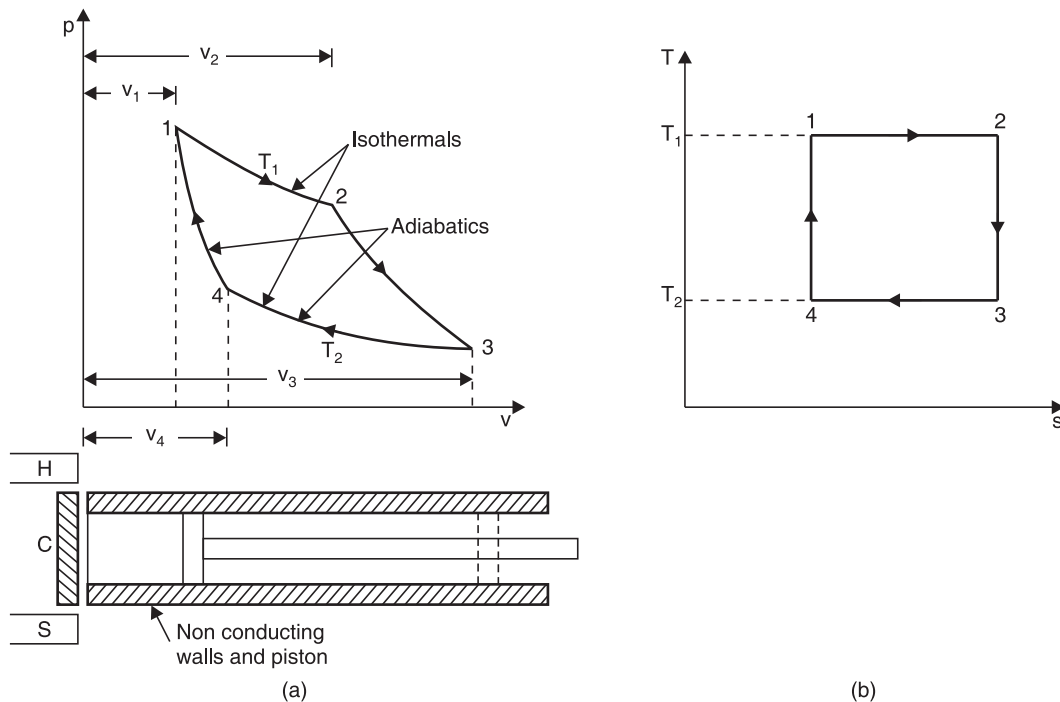
Fig. 4.1

Stage (1). Line 1-2 [Fig. 4.1 (*a*)] represents the isothermal expansion which takes place at temperature T_1 when source of heat H is applied to the end of cylinder. Heat supplied in this case is given by $RT_1 \log_e r$ and where r is the ratio of expansion.

Stage (2). Line 2-3 represents the application of non-conducting cover to the end of the cylinder. This is followed by the adiabatic expansion and the temperature falls from T_1 to T_2.

Stage (3). Line 3-4 represents the isothermal compression which takes place when sump 'S' is applied to the end of cylinder. Heat is rejected during this operation whose value is given by $RT_2 \log_e r$ where r is the ratio of compression.

Stage (4). Line 4-1 represents repeated application of non-conducting cover and adiabatic compression due to which temperature increases from T_2 to T_1.

It may be noted that ratio of expansion during isothermal 1-2 and ratio of compression during isothermal 3-4 must be equal to get a closed cycle.

Fig. 4.1 (*b*) represents the Carnot cycle on *T-s* coordinates.

Now according to law of conservation of energy,

Heat supplied   = work done + heat rejected

Work done   = heat supplied – heat rejected

$$= RT_1 . \log_e r - RT_2 \log_e r$$

Efficiency of cycle $$= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{R \log_e r \,(T_1 - T_2)}{RT_1 . \log_e r}$$

$$= \frac{T_1 - T_2}{T_1} \qquad\qquad ...(4.2)$$

From this equation, it is quite obvious that if temperature T_2 decreases efficiency increases and it becomes 100% if T_2 becomes absolute zero which, of course is impossible to attain. Further more *it is not possible to produce an engine that should work on Carnot's cycle as it would necessitate the piston to travel very slowly during first portion of the forward stroke (isothermal expansion) and to travel more quickly during the remainder of the stroke (adiabatic expansion) which however is not practicable.*

Example 4.1. *A Carnot engine working between 400°C and 40°C produces 130 kJ of work.* *Determine* :

(*i*) *The engine thermal efficiency.*

(*ii*) *The entropy changes during heat rejection process.*

(*iii*) *The heat added.*

Solution. Temperature, $T_1 = T_2 = 400 + 273 = 673$ K

Temperature, $T_3 = T_4 = 40 + 273 = 313$ K

Work produced, $W = 130$ kJ.

(*i*) **Engine thermal efficiency,**

$$\eta_{th.} = \frac{673 - 313}{673} = \textbf{0.535 or 53.5\%.} \quad \textbf{(Ans.)}$$

(*ii*) **Heat added :**

$$\eta_{th.} = \frac{\text{work done}}{\text{heat added}}$$

i.e., $$0.535 = \frac{130}{\text{heat added}}$$

∴   Heat added $= \dfrac{130}{0.535} = \textbf{243 kJ.} \quad \textbf{(Ans.)}$

(*iii*) **Entropy change during the heat rejection process, ($s_3 - s_4$) :**

Heat rejected   = heat added – work done

                                     = 243 – 130 = 113 kJ
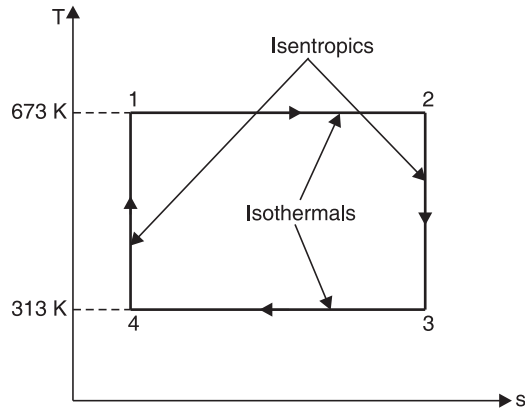
Fig. 4.2

Heat rejected   = $T_3(s_3 - s_4) = 113$

∴ $(s_3 - s_4) = \dfrac{113}{T_3} = \dfrac{113}{313}$ **= 0.361 kJ/K.   (Ans.)**

Example 4.2. *0.5 kg of air (ideal gas) executes a Carnot power cycle having a thermal efficiency of 50 per cent. The heat transfer to the air during the isothermal expansion is 40 kJ. At the beginning of the isothermal expansion the pressure is 7 bar and the volume is 0.12 m³. Determine :*

 (*i*) *The maximum and minimum temperatures for the cycle in K ;*

(*ii*) *The volume at the end of isothermal expansion in m³ ;*

(*iii*) *The heat transfer for each of the four processes in kJ.*

For air c_v = 0.721 kJ/kg K, and c_p = 1.008 kJ/kg K.

Solution. Refer to Fig. 4.3.

Given : m = 0.5 kg ; η_{th} = 50% ; Heat transferred during isothermal expansion = 40 kJ ;

        p_1 = 7 bar, V_1 = 0.12 m³ ; c_v = 0.721 kJ/kg K ; c_p = 1.008 kJ/kg K.

p-V diagram   T-s diagram
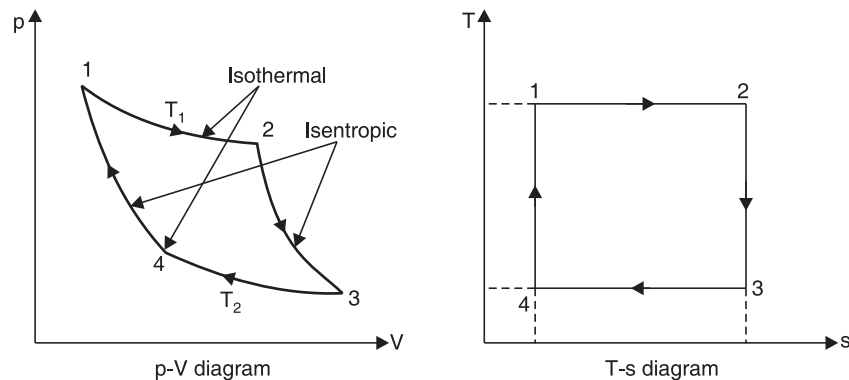
Fig. 4.3. Carnot cycle.

(i) **The maximum and minimum temperatures, T_1, T_2 :**

$$p_1 V_1 = mRT_1$$
$$7 \times 10^5 \times 0.12 = 0.5 \times 287 \times T_1$$

∴   Maximum temperature, $\mathbf{T_1} = \dfrac{7 \times 10^5 \times 0.12}{0.5 \times 287} = \textbf{585.4 K.}$ **(Ans.)**

$$\eta_{cycle} = \dfrac{T_1 - T_2}{T_1} \quad \Rightarrow \quad 0.5 = \dfrac{584.5 - T_2}{585.4}$$

∴   Minimum temperature, $T_2 = 585.4 - 0.5 \times 585.4 = \textbf{292.7 K.}$ **(Ans.)**

(ii) **The volume at the end of isothermal expansion V_2 :**

Heat transferred during isothermal expansion

$$= p_1 V_1 \ln(r) = mRT_1 \ln\left(\dfrac{V_2}{V_1}\right) = 40 \times 10^3 \qquad \qquad \text{...(Given)}$$

or $0.5 \times 287 \times 585.4 \ \ln\left(\dfrac{V_2}{0.12}\right) = 40 \times 10^3$

or $\ln\left(\dfrac{V_2}{0.12}\right) = \dfrac{40 \times 10^3}{0.5 \times 287 \times 585.4} = 0.476$

∴ $\mathbf{V_2} = 0.12 \times (e)^{0.476} = \textbf{0.193 m}^3.$ **(Ans.)**

(iii) **The heat transfer for each of the four processes :**

Process	Classification	Heat transfer
1–2	Isothermal expansion	40 kJ
2–3	Adiabatic reversible expansion	zero
3–4	Isothermal compression	– 40 kJ
4–1	Adiabatic reversible compression	zero. **(Ans.)**

Example 4.3. *In a Carnot cycle, the maximum pressure and temperature are limited to 18 bar and 410°C. The ratio of isentropic compression is 6 and isothermal expansion is 1.5. Assuming the volume of the air at the beginning of isothermal expansion as 0.18 m³, determine :*

(i) *The temperature and pressures at main points in the cycle.*

(ii) *Change in entropy during isothermal expansion.*

(iii) *Mean thermal efficiency of the cycle.*

(iv) *Mean effective pressure of the cycle.*

(v) *The theoretical power if there are 210 working cycles per minute.*

Solution. Refer to Fig. 4.4.

Maximum pressure, $p_1 = 18$ bar

Maximum temperature, $T_1 = (T_2) = 410 + 273 = 683$ K

Ratio of isentropic (or adiabatic) compression, $\dfrac{V_4}{V_1} = 6$

Ratio of isothermal expansion, $\dfrac{V_2}{V_1} = 1.5.$

Volume of the air at the beginning of isothermal expansion,

$$V_1 = 0.18 \text{ m}^3.$$
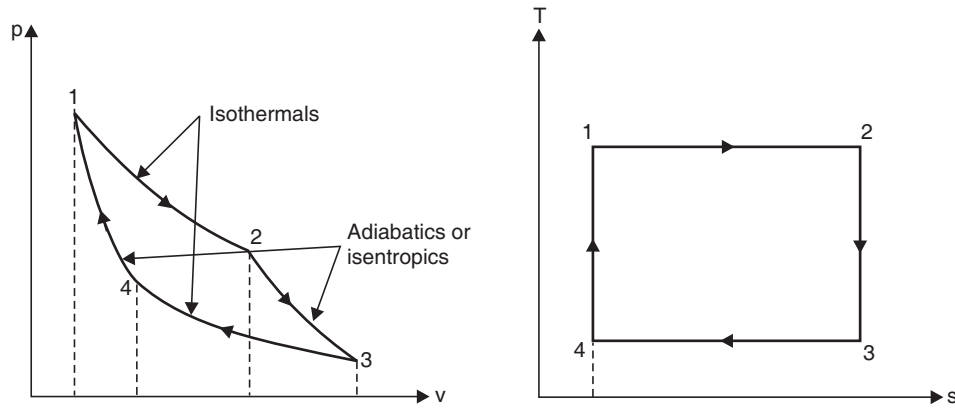
Fig. 4.4

(*i*) **Temperatures and pressures at the main points in the cycle :**

For the *isentropic process* 4-1

$$\frac{T_1}{T_4} = \left(\frac{V_4}{V_1}\right)^{\gamma - 1} = (6)^{1.4 - 1} = (6)^{0.4} = 2.05$$

$\therefore \qquad\qquad T_4 = \dfrac{T_1}{2.05} = \dfrac{683}{2.05} = 333.2 \text{ K} = T_3$

Also, $\qquad\qquad \dfrac{p_1}{p_4} = \left(\dfrac{V_4}{V_1}\right)^{\gamma} = (6)^{1.4} = 12.29$

$\therefore \qquad\qquad p_4 = \dfrac{p_1}{12.29} = \dfrac{18}{12.29} = 1.46 \text{ bar}$

For the *isothermal process* 1-2

$$p_1 V_1 = p_2 V_2$$

$$p_2 = \frac{p_1 V_1}{V_2} = \frac{18}{1.5} = 12 \text{ bar}$$

For isentropic process 2-3, we have

$$p_2 V_2{}^{\gamma} = p_3 V_3{}^{\gamma}$$

$$p_3 = p_2 \times \left(\frac{V_2}{V_3}\right)^{\gamma} = 12 \times \left(\frac{V_1}{V_4}\right)^{\gamma} \qquad\qquad\qquad \left[\because \ \frac{V_4}{V_1} = \frac{V_3}{V_2}\right]$$

$$= 12 \times \left(\frac{1}{6}\right)^{1.4} = \textbf{0.97 bar.} \quad \textbf{(Ans.)}$$

Hence, $\qquad\qquad \begin{array}{ll} p_1 = 18 \text{ bar} & T_1 = T_2 = \textbf{683 K} \\ p_2 = 12 \text{ bar} & \\ p_3 = 0.97 \text{ bar} & T_3 = T_4 = \textbf{333.2 K} \\ p_4 = 1.46 \text{ bar} & \end{array} \Bigg\} \quad \textbf{(Ans.)}$

(*ii*) **Change in entropy :**

Change in entropy during isothermal expansion,

$$S_2 - S_1 = \frac{p_1 V_1}{T_1} \log_e \left(\frac{V_2}{V_1} \right)$$

$$= \frac{18 \times 10^5 \times 0.18}{10^3 \times 683} \log_e (1.5) = \textbf{0.192 kJ/K.} \quad \textbf{(Ans.)}$$

(*iii*) **Mean thermal efficiency of the cycle :**

Heat supplied, $\quad Q_s = p_1 V_1 \log_e \left(\frac{V_2}{V_1} \right)$

$$= T_1 (S_2 - S_1)$$
$$= 683 \times 0.192 = 131.1 \text{ kJ}$$

Heat rejected, $\quad Q_r = p_4 V_4 \log_e \left(\frac{V_3}{V_4} \right)$

$\quad = T_4 (S_3 - S_4)$ because increase in entropy during heat addition is equal to decrease in entropy during heat rejection.

$\therefore \quad\quad Q_r = 333.2 \times 0.192 = 63.97 \text{ kJ}$

$\therefore$ *Efficiency,* $\quad \eta = \dfrac{Q_s - Q_r}{Q_s} = 1 - \dfrac{Q_r}{Q_s}$

$$= 1 - \frac{63.97}{131.1} = \textbf{0.512} \text{ or } \textbf{51.2\%.} \quad \textbf{(Ans.)}$$

(*iv*) **Mean effective pressure of the cycle, p_m :**

The mean effective pressure of the cycle is given by

$$p_m = \frac{\text{work done per cycle}}{\text{stroke volume}}$$

$$\frac{V_3}{V_1} = 6 \times 1.5 = 9$$

Stroke volume, $V_s = V_3 - V_1 = 9V_1 - V_1 = 8V_1 = 8 \times 0.18 = 1.44 \text{ m}^3$

$\therefore \quad\quad p_m = \dfrac{(Q_s - Q_r) \times J}{V_s} = \dfrac{(Q_s - Q_r) \times 1}{V_s}$ $\quad\quad (\because J = 1)$

$$= \frac{(131.1 - 63.97) \times 10^3}{1.44 \times 10^5} = \textbf{0.466 bar.} \quad \textbf{(Ans.)}$$

(*v*) **Power of the engine, P :**

Power of the engine working on this cycle is given by

$$P = (131.1 - 63.97) \times (210/60) = \textbf{234.9 kW.} \quad \textbf{(Ans.)}$$

Example 4.4. *A reversible engine converts one-sixth of the heat input into work. When the temperature of the sink is reduced by 70°C, its efficiency is doubled. Find the temperature of the source and the sink.*

Solution. Let $\quad\quad T_1$ = temperature of the source (*K*)

$\quad\quad\quad\quad\quad\quad T_2$ = temperature of the sink (*K*)

First case :

$$\frac{T_1 - T_2}{T_1} = \frac{1}{6} \qquad i.e., \qquad 6T_1 - 6T_2 = T_1$$

or $\qquad\qquad\qquad 5T_1 = 6T_2 \quad$ or $\qquad T_1 = 1.2T_2 \qquad\qquad\qquad ...(i)$

Second case :

$$\frac{T_1 - [T_2 - (70 + 273)]}{T_1} = \frac{1}{3}$$

$$\frac{T_1 - T_2 + 343}{T_1} = \frac{1}{3}$$

$$3T_1 - 3T_2 + 1029 = T_1$$

$$2T_1 = 3T_2 - 1029$$

$$2 \times (1.2T_2) = 3T_2 - 1029 \qquad\qquad\qquad (\because \quad T_1 = 1.2T_2)$$

$$2.4T_2 = 3T_2 - 1029$$

or $\qquad\qquad\qquad 0.6T_2 = 1029$

$\therefore \qquad\qquad\qquad T_2 = \dfrac{1029}{0.6} = \textbf{1715 K} \quad \textbf{or} \quad \textbf{1442°C.} \quad \textbf{(Ans.)}$

and $\qquad\qquad\qquad T_1 = 1.2 \times 1715 = \textbf{2058 K} \quad \textbf{or} \quad \textbf{1785°C.} \quad \textbf{(Ans.)}$

Example 4.5. *An inventor claims that a new heat cycle will develop 0.4 kW for a heat addition of 32.5 kJ/min. The temperature of heat source is 1990 K and that of sink is 850 K. Is his claim possible ?*

Solution. Temperature of heat source, $\qquad T_1 = 1990$ K

Temperature of sink, $\qquad\qquad\qquad\qquad T_2 = 850$ K

Heat supplied, $\qquad\qquad\qquad\qquad\qquad = 32.5$ kJ/min

Power developed by the engine, $\qquad P = 0.4$ kW

The most efficient engine is one that works on Carnot cycle

$$\eta_{carnot} = \frac{T_1 - T_2}{T_1} = \frac{1990 - 850}{1990} = 0.573 \text{ or } 57.3\%$$

Also, thermal efficiency of the engine,

$$\eta_{th.} = \frac{\text{work done}}{\text{heat supplied}} = \frac{0.4}{(32.5/60)} = \frac{0.4 \times 60}{32.5}$$

$$= 0.738 \text{ or } 73.8\%$$

which is not feasible as no engine can be more efficient than that working on Carnot cycle.

Hence, claim of the inventor is **not true.** **(Ans.)**

Example 4.6. *An ideal engine operates on the Carnot cycle using a perfects gas as the working fluid. The ratio of the greatest to the least volume is fixed and is x : 1, the lower temperature of the cycle is also fixed, but the volume compression ratio 'r' of the reversible adiabatic compression is variable. The ratio of the specific heats is γ.*

Show that if the work done in the cycle is a maximum then,

$$(\gamma - 1) \log_e \frac{x}{r} + \frac{1}{r^{\gamma - 1}} - 1 = 0 \,.$$

Solution. Refer to Fig. 4.1.

$$\frac{V_3}{V_1} = x$$

$$\frac{V_4}{V_1} = r$$

During isothermals, since compression ratio = expansion ratio

$$\therefore \qquad \frac{V_3}{V_4} = \frac{V_2}{V_1}$$

Also $\qquad \dfrac{V_3}{V_4} = \dfrac{V_3}{V_1} \times \dfrac{V_1}{V_4} = x \times \dfrac{1}{r} = \dfrac{x}{r}$

Work done per kg of the gas

$$= \text{heat supplied} - \text{heat rejected} = RT_1 \log_e \frac{x}{r} - RT_2 \log_e \frac{x}{r}$$

$$= R(T_1 - T_2) \log_e \frac{x}{r} = RT_2 \left(\frac{T_1}{T_2} - 1\right) \log_e \frac{x}{r}$$

But $\qquad \dfrac{T_1}{T_2} = \left(\dfrac{V_4}{V_1}\right)^{\gamma - 1} = (r)^{\gamma - 1}$

$\therefore$ Work done per kg of the gas,

$$W = RT_2 (r^{\gamma - 1} - 1) \log_e \frac{x}{r}$$

Differentiating W w.r.t. 'r' and equating to zero

$$\frac{dW}{dr} = RT_2 \left[(r^{\gamma - 1} - 1) \left\{ \frac{r}{x} \times (- xr^{-2}) \right\} + \log_e \frac{x}{r} \{(\gamma - 1) r^{\gamma - 2}\} \right] = 0$$

or $\qquad (r^{\gamma - 1} - 1)\left(-\dfrac{1}{r}\right) + (\gamma - 1) \times r^{\gamma - 2} \log_e \dfrac{x}{r} = 0$

or $\qquad - r^{\gamma - 2} + \dfrac{1}{r} + r^{\gamma - 2} (\gamma - 1) \log_e \dfrac{x}{r} = 0$

or $\qquad r^{\gamma - 2} \left\{ -1 + \dfrac{1}{r \cdot r^{\gamma - 2}} + (\gamma - 1) \log_e \dfrac{x}{r} \right\} = 0$

or $\qquad -1 + \dfrac{1}{r \cdot r^{\gamma - 2}} + (\gamma - 1) \log_e \dfrac{x}{r} = 0$

$$(\gamma - 1) \log_e \frac{x}{r} + \frac{1}{r^{\gamma - 1}} - 1 = 0. \quad \textbf{Proved.}$$

4.4. CONSTANT VOLUME OR OTTO CYCLE

This cycle is so named as it was conceived by 'Otto'. On this cycle, petrol, gas and many types of oil engines work. It is the standard of comparison for internal combustion engines.

Fig. 4.5 (*a*) and (*b*) shows the theoretical *p-V* diagram and *T-s* diagrams of this cycle respectively.

The point 1 represents that cylinder is full of air with volume v_1, pressure p_1 and absolute temperature T_1.

Line 1-2 represents the adiabatic compression of air due to which p_1, v_1 and T_1 change to p_2, v_2 and T_2 respectively.

Line 2-3 shows the supply of heat to the air at constant volume so that p_2 and T_2 change to p_3 and T_3 (v_3 being the same as v_2).

Line 3-4 represents the adiabatic expansion of the air. During expansion p_3, v_3 and T_3 change to a final value of p_4, V_4 or V_1 and T_4 respectively.

Line 4-1 shows the rejection of heat by air at constant volume till original state (point 1) reaches.

Consider 1 kg of air (working substance).

Heat supplied at constant volume $= c_v(T_3 - T_2)$.

(a)   (b)
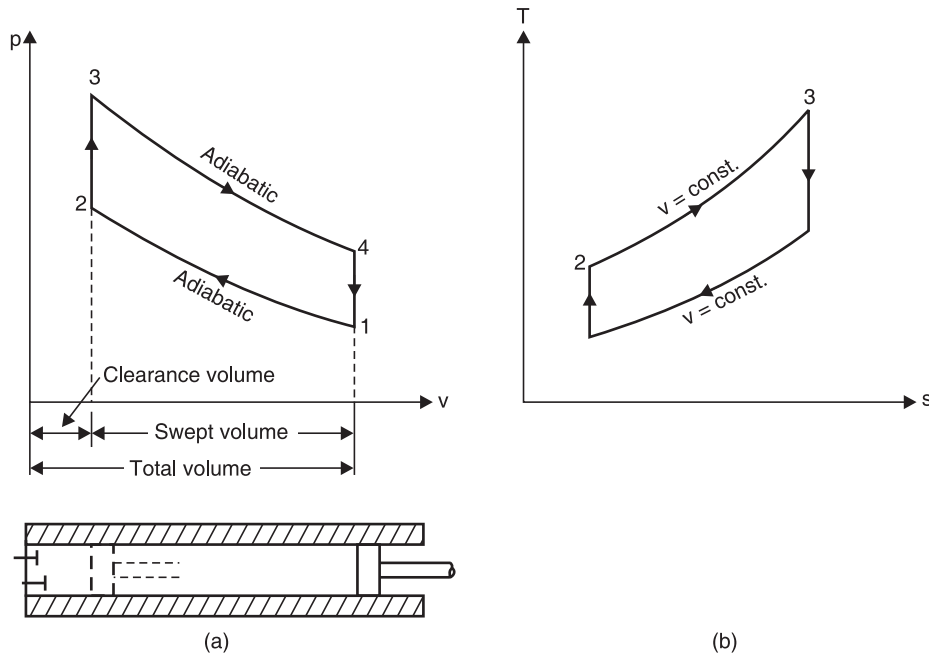
Fig. 4.5

Heat rejected at constant volume $= c_v(T_4 - T_1)$.

But, work done   = heat supplied − heat rejected

$$= c_v(T_3 - T_2) - c_v(T_4 - T_1)$$

∴   Efficiency $= \dfrac{\text{work done}}{\text{heat supplied}} = \dfrac{c_v(T_3 - T_2) - c_v(T_4 - T_1)}{c_v(T_3 - T_2)}$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2} \qquad\qquad\qquad ...(i)$$

Let compression ratio, $r_c\ (= r) = \dfrac{v_1}{v_2}$

and expansion ratio, $r_e\ (= r) = \dfrac{v_4}{v_3}$

(These two ratios are same in this cycle)

As $\dfrac{T_2}{T_1} = \left(\dfrac{v_1}{v_2}\right)^{\gamma - 1}$

Then, $T_2 = T_1 \cdot (r)^{\gamma-1}$

Similarly, $\dfrac{T_3}{T_4} = \left(\dfrac{v_4}{v_3}\right)^{\gamma-1}$

or $T_3 = T_4 \cdot (r)^{\gamma-1}$

Inserting the values of T_2 and T_3 in equation (*i*), we get

$$\eta_{\text{otto}} = 1 - \frac{T_4 - T_1}{T_4 \cdot (r)^{\gamma-1} - T_1 \cdot (r)^{\gamma-1}} = 1 - \frac{T_4 - T_1}{r^{\gamma-1}(T_4 - T_1)}$$

$$= 1 - \frac{1}{(r)^{\gamma-1}} \qquad\qquad ...(4.3)$$

This expression is known as the air standard efficiency of the Otto cycle.

It is clear from the above expression that efficiency increases with the increase in the value of r, which means we can have maximum efficiency by increasing r to a considerable extent, but *due to practical difficulties its value is limited to about 7.*

The *net work done per kg* in the Otto cycle can also be expressed in terms of *pv*. If p is expressed in bar *i.e.*, 10^5 N/m^2, then work done

$$W = \left(\frac{p_3 v_3 - p_4 v_4}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1}\right) \times 10^2 \text{ kJ} \qquad\qquad ...(4.4)$$

Also $\dfrac{p_3}{p_4} = r^\gamma = \dfrac{p_2}{p_1}$

$\therefore$ $\dfrac{p_3}{p_2} = \dfrac{p_4}{p_1} = r_p$

where r_p stands for *pressure ratio*

and $v_1 = r v_2 = v_4 = r v_3$ $\left[\because\ \dfrac{v_1}{v_2} = \dfrac{v_4}{v_3} = r\right]$

$\therefore$ $W = \dfrac{1}{\gamma - 1}\left[p_4 v_4\left(\dfrac{p_3 v_3}{p_4 v_4} - 1\right) - p_1 v_1\left(\dfrac{p_2 v_2}{p_1 v_1} - 1\right)\right]$

$$= \frac{1}{\gamma - 1}\left[p_4 v_4\left(\frac{p_3}{p_4 r} - 1\right) - p_1 v_1\left(\frac{p_2}{p_1 r} - 1\right)\right]$$

$$= \frac{v_1}{\gamma - 1}\left[p_4 (r^{\gamma-1} - 1) - p_1 (r^{\gamma-1} - 1)\right]$$

$$= \frac{v_1}{\gamma - 1}\left[(r^{\gamma-1} - 1)(p_4 - p_1)\right]$$

$$= \frac{p_1 v_1}{\gamma - 1}\left[(r^{\gamma-1} - 1)(r_p - 1)\right] \qquad\qquad ...[4.4\ (a)]$$

Mean effective pressure (p_m) is given by :

$$p_m = \left[\left(\frac{p_3 v_3 - p_4 v_4}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1}\right) \div (v_1 - v_2)\right] \text{bar} \qquad\qquad ...(4.5)$$

Also
$$p_m = \frac{\left[\dfrac{p_1 v_1}{\gamma - 1}(r^{\gamma-1} - 1)(r_p - 1)\right]}{(v_1 - v_2)}$$

$$= \frac{\dfrac{p_1 v_1}{\gamma - 1}[(r^{\gamma-1} - 1)(r_p - 1)]}{(v_1 - v_2)} = \frac{\dfrac{p_1 v_1}{\gamma - 1}[(r^{\gamma-1} - 1)(r_p - 1)]}{v_1 - \dfrac{v_1}{r}}$$

$$= \frac{\dfrac{p_1 v_1}{\gamma - 1}[(r^{\gamma-1} - 1)(r_p - 1)]}{v_1\left(\dfrac{r-1}{r}\right)}$$

i.e.,
$$\mathbf{p_m} = \frac{p_1 r[(r^{\gamma-1} - 1)(r_p - 1)]}{(\gamma - 1)(r - 1)}.$$
 ...(4.6)

Example 4.7. *The efficiency of an Otto cycle is 60% and γ = 1.5. What is the compression ratio ?*

Solution. Efficiency of Otto cycle, η = 60%

Ratio of specific heats,   γ = 1.5

Compression ratio,   r = ?

Efficiency of Otto cycle is given by

$$\eta_{Otto} = 1 - \frac{1}{(r)^{\gamma-1}}$$

$$0.6 = 1 - \frac{1}{(r)^{1.5-1}}$$

or
$$\frac{1}{(r)^{0.5}} = 0.4 \quad \text{or} \quad (r)^{0.5} = \frac{1}{0.4} = 2.5 \text{ or } r = 6.25$$

Hence, *compression ratio* = **6.25.**   **(Ans.)**

Example 4.8. *The minimum pressure and temperature in an Otto cycle are 100 kPa and 27°C. The amount of heat added to the air per cycle is 1500 kJ/kg.*

(i) Determine the pressures and temperatures at all points of the air standard Otto cycle.

(ii) Also calculate the specific work and thermal efficiency of the cycle for a compression ratio of 8 : 1.

Take for air : c_v = 0.72 kJ/kg K, and γ = 1.4.

Solution. Refer to Fig. 4.6.

Given : p_1 = 100 kPa = 10^5 N/m² or 1 bar ; T_1 = 27 + 273 = 300 K ;

Heat added = 1500 kJ/kg ; r = 8 : 1 ; c_v = 0.72 kJ/kg ; γ = 1.4.

Consider 1 kg of air :

(i) **Pressures and temperatures at all points :**

Adiabatic compression process 1-2

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (8)^{1.4-1} = 2.297$$

$\therefore$ $T_2 = 300 \times 2.297 = \textbf{689.1 K.}$ **(Ans.)**

Also, $p_1 v_1{}^\gamma = p_2 v_2{}^\gamma$

or $\dfrac{p_2}{p_1} = \left(\dfrac{v_1}{v_2}\right)^\gamma = (8)^{1.4} = 18.379$

$\therefore$ $\textbf{p}_2 = 1 \times 18.379 = \textbf{18.379 bar.}$ **(Ans.)**

Constant volume process 2-3

Heat added during the process,

$$c_v(T_3 - T_2) = 1500$$

or $0.72(T_3 - 689.1) = 1500$

or $T_3 = \dfrac{1500}{0.72} + 689.1 = \textbf{2772.4 K.}$ **(Ans.)**

Also, $\dfrac{p_2}{T_2} = \dfrac{p_3}{T_3}$ $\Rightarrow$ $\textbf{p}_3 = \dfrac{p_2 T_3}{T_2} = \dfrac{18.379 \times 2772.4}{689.1} = \textbf{73.94 bar.}$ **(Ans.)**

Adiabatic expansion process 3-4 :

$$\dfrac{T_3}{T_4} = \left(\dfrac{v_4}{v_3}\right)^{\gamma-1} = (r)^{\gamma-1} = (8)^{1.4-1} = 2.297$$

$\therefore$ $T_4 = \dfrac{T_3}{2.297} = \dfrac{2772.4}{2.297} = \textbf{1206.9 K.}$ **(Ans.)**

Also, $p_3 v_3{}^\gamma = p_4 v_4{}^\gamma \Rightarrow \textbf{p}_4 = p_3 \times \left(\dfrac{v_3}{v_4}\right)^\gamma = 73.94 \times \left(\dfrac{1}{8}\right)^{1.4} = \textbf{4.023 bar.}$ **(Ans.)**

(ii) **Specific work and thermal efficiency :**

Specific work   = Heat added – heat rejected

= $c_v(T_3 - T_2) - c_v(T_4 - T_1) = c_v[(T_3 - T_2) - (T_4 - T_1)]$

= $0.72\,[(2772.4 - 689.1) - (1206.9 - 300)] = \textbf{847 kJ/kg.}$ **(Ans.)**

Thermal efficiency, $\eta_{th} = 1 - \dfrac{1}{(r)^{\gamma-1}} = 1 - \dfrac{1}{(8)^{1.4-1}} = \textbf{0.5647}$ or **56.47%.** **(Ans.)**

Example 4.9. *An air standard otto cycle has a volumetric compression ratio of 6, the lowest cycle pressure of 0.1 MPa and operates between temperature limits of 27°C and 1569°C.*

(i) Calculate the temperature and pressure after the isentropic expansion (ratio of specific heats = 1.4).

(ii) Since it is observed that values in (i) are well above the lowest cycle operating conditions, the expansion process was allowed to continue down to a pressure of 0.1 MPa. Which process is required to complete the cycle ? Name the cycle so obtained.

(iii) Determine by what percentage the cycle efficiency has been improved.

Solution. Refer to Fig. 4.7.

Given : $\dfrac{v_1}{v_2} = \dfrac{v_4}{v_3} = r = 6$; $p_1 = 0.1$ MPa $= 1$ bar ; $T_1 = 27 + 273 = 300$ K ;

$T_3 = 1569 + 273 = 1842$ K ; $\gamma = 1.4$.
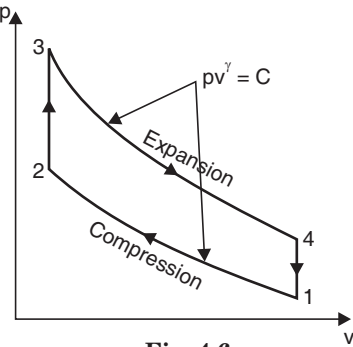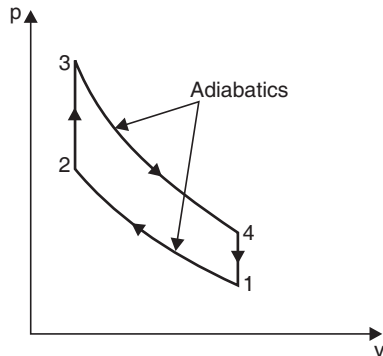
Fig. 4.7
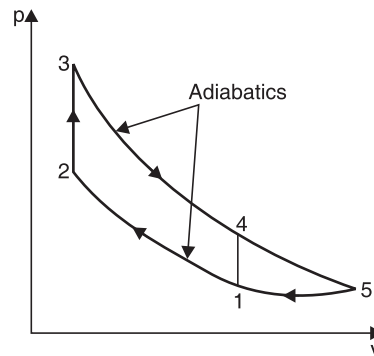
Fig. 4.8. Atkinson cycle.

(*i*) **Temperature and pressure after the isentropic expansion, T_4, p_4 :**

Consider 1 kg of air :

For the compression process 1-2

$$p_1 v_1^{\gamma} = p_2 v_2^{\gamma} \quad \Rightarrow \quad p_2 = p_1 \times \left(\frac{v_1}{v_2}\right)^{\gamma} = 1 \times (6)^{1.4} = 12.3 \text{ bar}$$

Also,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (6)^{1.4-1} = 2.048$$

∴ $T_2 = 300 \times 2.048 = 614.4 \text{ K}$

For the constant volume process 2-3 :

$$\frac{p_2}{T_2} = \frac{p_3}{T_3} \quad \Rightarrow \quad p_3 = \frac{p_2 T_3}{T_2} = 12.3 \times \frac{1842}{614.4} = 36.9 \text{ bar.}$$

For the expansion process 3-4 :

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = (6)^{1.4-1} = 2.048$$

∴ $$\mathbf{T_4} = \frac{T_3}{2.048} = \frac{1842}{2.048} = \mathbf{900.} \quad \textbf{(Ans.)}$$

Also

$$p_3 v_3^{\gamma} = p_4 v_4^{\gamma} \quad \Rightarrow \quad p_4 = p_3 \times \left(\frac{v_3}{v_4}\right)^{\gamma}$$

or

$$\mathbf{p_4} = 36.9 \times \left(\frac{1}{6}\right)^{1.4} = \mathbf{3 \text{ bar.}} \quad \textbf{(Ans.)}$$

(*ii*) **Process required to complete the cycle :**

Process required to complete the cycle is the *constant pressure* scavenging.

The cycle is called **Atkinson cycle** (Refer Fig. 4.8).

(*iii*) **Percentage improvement/increase in efficiency :**

$$\eta_{\text{otto}} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(6)^{1.4-1}} = \mathbf{0.5116} \quad \textbf{or} \quad \mathbf{51.16\%.} \quad \textbf{(Ans.)}$$

$$\eta_{\text{Atkinson}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}}$$

$$= \frac{c_v(T_3 - T_2) - c_p(T_5 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{c_p(T_5 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{\gamma(T_5 - T_1)}{(T_3 - T_2)}$$

Now, $\qquad \dfrac{T_5}{T_3} = \left(\dfrac{p_5}{p_3}\right)^{\frac{\gamma - 1}{\gamma}}$ or $T_5 = 1842 \times \left(\dfrac{1.0}{36.9}\right)^{\frac{1.4 - 1}{1.4}} = 657$ K

∴ $\qquad \eta_{\text{Atkinson}} = 1 - \dfrac{1.4\,(657 - 300)}{(1842 - 614.4)} = \mathbf{0.592}$ or **59.29%**

∴ **Improvement in efficiency** = 59.29 − 51.16 = **8.13%. (Ans.)**

Example 4.10. *An engine of 250 mm bore and 375 mm stroke works on Otto cycle. The clearance volume is 0.00263 m³. The initial pressure and temperature are 1 bar and 50°C. If the maximum pressure is limited to 25 bar, find the following* :

(*i*) *The air standard efficiency of the cycle.*

(*ii*) *The mean effective pressure for the cycle.*

Assume the ideal conditions.

Solution. Bore of the engine, $\qquad D = 250$ mm = 0.25 m

Stroke of the engine, $\qquad L = 375$ mm = 0.375 m

Clearance volume, $\qquad V_c = 0.00263$ m³

Initial pressure, $\qquad p_1 = 1$ bar

Initial temperature, $\qquad T_1 = 50 + 273 = 323$ K

Maximum pressure, $\qquad p_3 = 25$ bar

Swept volume, $\qquad V_s = \pi/4\ D^2 L = \pi/4 \times 0.25^2 \times 0.375 = 0.0184$ m³

Compression ratio, $\qquad r = \dfrac{V_s + V_c}{V_c} = \dfrac{0.0184 + 0.00263}{0.00263} = 8.$
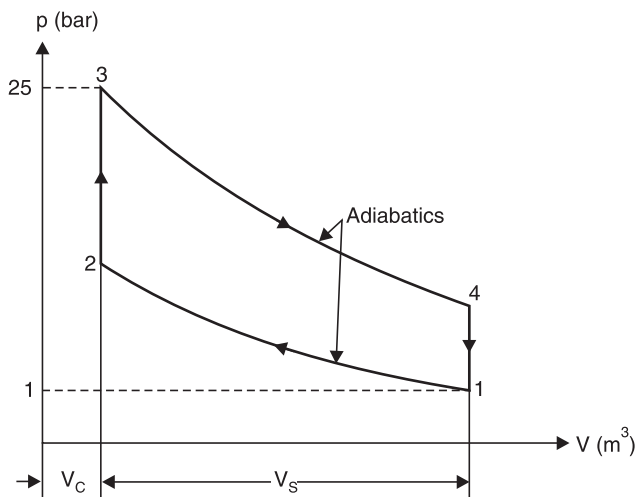
Fig. 4.9

(i) **Air standard efficiency :**

The air standard efficiency of Otto cycle is given by

$$\eta_{Otto} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(8)^{1.4-1}} = 1 - \frac{1}{(8)^{0.4}}$$

$$= 1 - 0.435 = \textbf{0.565 or 56.5\%}\ \ \textbf{(Ans.)}$$

(ii) **Mean effective pressure, p_m :**

For adiabatic (or isentropic) process 1-2

$$p_1 V_1{}^\gamma = p_2 V_2{}^\gamma$$

or $\qquad\qquad p_2 = p_1 \left(\dfrac{V_1}{V_2}\right)^\gamma = 1 \times (r)^{1.4} = 1 \times (8)^{1.4} = 18.38$ bar

∴ Pressure ratio, $\qquad r_p = \dfrac{p_3}{p_2} = \dfrac{25}{18.38} = 1.36$

The mean effective pressure is given by

$$p_m = \frac{p_1 r\,[(r^{\gamma-1} - 1)(r_p - 1)]}{(\gamma - 1)(r - 1)} = \frac{1 \times 8\,[\{(8)^{1.4-1} - 1\}(1.36 - 1)]}{(1.4 - 1)(8 - 1)}$$

$$= \frac{8(2.297 - 1)(0.36)}{0.4 \times 7} = 1.334 \text{ bar}$$

Hence, *mean effective pressure* = **1.334 bar.**   **(Ans.)**

Example 4.11. *A certain quantity of air at a pressure of 1 bar and temperature of 70°C is compressed adiabatically until the pressure is 7 bar in Otto cycle engine. 465 kJ of heat per kg of air is now added at constant volume. Determine* :

(i) *Compression ratio of the engine.*

(ii) *Temperature at the end of compression.*

(iii) *Temperature at the end of heat addition.*

Take for air c_p = 1.0 kJ/kg K, c_v = 0.706 kJ/kg K.

Show each operation on p-V and T-s diagrams.

Solution. Refer to Fig 4.10.
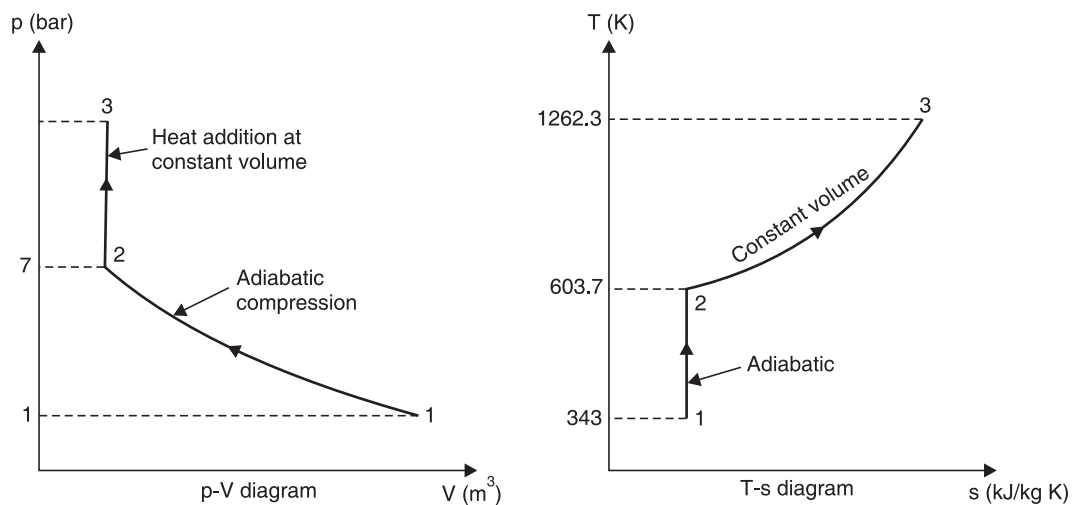
Fig. 4.10

Initial pressure, p_1 = 1 bar

Initial temperature, T_1 = 70 + 273 = 343 K

Pressure after adiabatic compression, p_2 = 7 bar

Heat addition at constant volume, Q_s = 465 kJ/kg of air

Specific heat at constant pressure, c_p = 1.0 kJ/kg K

Specific heat at constant volume, c_v = 0.706 kJ/kg K

$$\therefore \qquad \gamma = \frac{c_p}{c_v} = \frac{1.0}{0.706} = 1.41$$

(*i*) **Compression ratio of engine, r :**

According to adiabatic compression 1-2

$$p_1 V_1{}^\gamma = p_2 V_2{}^\gamma$$

or $$\left(\frac{V_1}{V_2}\right)^\gamma = \frac{p_2}{p_1}$$

or $$(r)^\gamma = \frac{p_2}{p_1} \qquad\qquad\qquad \left(\because \ \frac{V_1}{V_2} = r\right)$$

or $$r = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{7}{1}\right)^{\frac{1}{1.41}} = (7)^{0.709} = 3.97$$

Hence, *compression ratio of the engine* = **3.97.**   **(Ans.)**

(*ii*) **Temperature at the end of compression, T_2 :**

In case of adiabatic compression 1-2,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (3.97)^{1.41-1} = 1.76$$

$$\therefore \qquad\qquad T_2 = 1.76 \ T_1 = 1.76 \times 343 = 603.7 \text{ K or } 330.7°C$$

Hence, *temperature at the end of compression* = **330.7°C.**   **(Ans.)**

(*iii*) **Temperature at the end of heat addition, T_3 :**

According to constant volume heating operation 2-3

$$Q_s = c_v(T_3 - T_2) = 465$$

$$0.706 \ (T_3 - 603.7) = 465$$

or $$T_3 - 603.7 = \frac{465}{0.706}$$

or $$T_3 = \frac{465}{0.706} + 603.7 = 1262.3 \text{ K or } 989.3°C$$

Hence, *temperature at the end of heat addition* = **989.3°C.**   **(Ans.)**

Example 4.12. *In a constant volume 'Otto cycle', the pressure at the end of compression is 15 times that at the start, the temperature of air at the beginning of compression is 38°C and maximum temperature attained in the cycle is 1950°C. Determine :*

(*i*) *Compression ratio.*

(*ii*) *Thermal efficiency of the cycle.*

(*iii*) *Work done.*

Take γ for air = 1.4.

Solution. Refer to Fig. 4.11.

Initial temperature, $T_1 = 38 + 273 = 311$ K
Maximum temperature, $T_3 = 1950 + 273 = 2223$ K.

(*i*) **Compression ratio, r = ?**

For adiabatic compression 1-2,

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma} \quad \text{or} \quad \left(\frac{V_1}{V_2}\right)^{\gamma} = \frac{p_2}{p_1}$$

But $\dfrac{p_2}{p_1} = 15$ ...(given)

∴ $(r)^{\gamma} = 15$ $\left[\because\ r = \dfrac{V_1}{V_2}\right]$

or $(r)^{1.4} = 15$ or $r = (15)^{\frac{1}{1.4}} = (15)^{0.714} = 6.9$

Hence, *compression ratio* = **6.9.**   (**Ans.**)
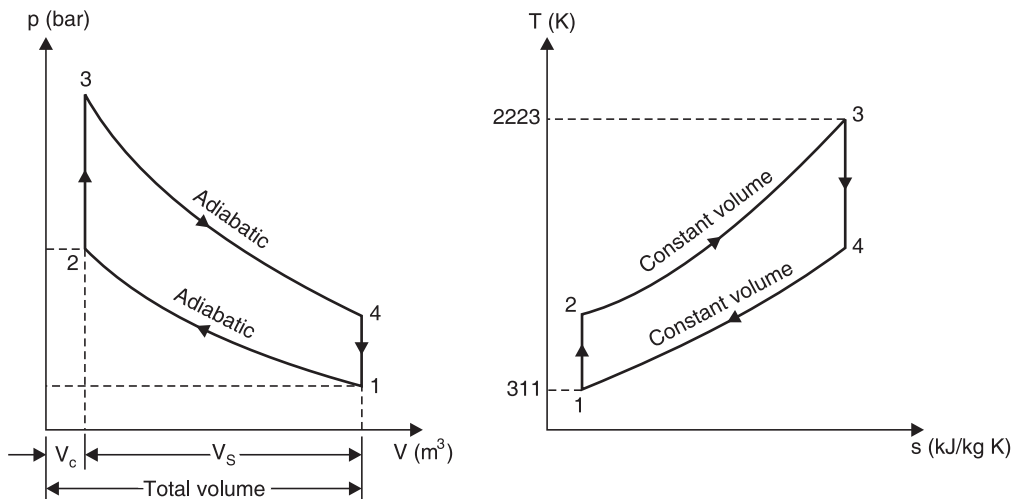
Fig. 4.11

(*ii*) **Thermal efficiency :**

Thermal efficiency, $\eta_{\text{th.}} = 1 - \dfrac{1}{(r)^{\gamma-1}} = 1 - \dfrac{1}{(6.9)^{1.4-1}}$

$$= 0.538 \text{ or } 53.8\%. \quad (\textbf{Ans.})$$

(*iii*) **Work done :**

Again, for adiabatic compression

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= (r)^{\gamma-1} = (6.9)^{1.4-1} = (6.9)^{0.4} = 2.16$$

or $T_2 = T_1 \times 2.16 = 311 \times 2.16 = 671.7$ K or 398.7°C

For adiabatic expansion process 3-4

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = (r)^{\gamma-1} = (6.9)^{0.4} = 2.16$$

or $$T_4 = \frac{T_3}{2.16} = \frac{2223}{2.16} = 1029 \text{ K or } 756°C$$

Heat supplied per kg of air

$$= c_v(T_3 - T_2) = 0.717(2223 - 671.7)$$
$$= 1112.3 \text{ kJ/kg or air}$$

$$\left[c_v = \frac{R}{\gamma - 1} = \frac{0.287}{1.4 - 1} = 0.717 \,\text{kJ/kg K} \right]$$

Heat rejected per kg of air

$$= c_v(T_4 - T_1) = 0.717(1029 - 311)$$
$$= 514.8 \text{ kJ/kg of air}$$

∴   *Work done* = heat supplied – heat rejected

$$= 1112.3 - 514.8$$
$$= \textbf{597.5 kJ or 597500 Nm.}   \textbf{(Ans.)}$$

☞ **Example 4.13.** *An engine working on Otto cycle has a volume of 0.45 m³, pressure 1 bar and temperature 30°C at the beginning of compression stroke. At the end of compression stroke, the pressure is 11 bar. 210 kJ of heat is added at constant volume. Determine* :

 (*i*) *Pressures, temperatures and volumes at salient points in the cycle.*

(*ii*) *Percentage clearance.*

(*iii*) *Efficiency.*

(*iv*) *Mean effective pressure.*

(*v*) *Ideal power developed by the engine if the number of working cycles per minute is 210.*

Assume the cycle is reversible.

**Solution.** Refer to Fig. 4.12.

Volume, $V_1 = 0.45 \text{ m}^3$

Initial pressure, $p_1 = 1$ bar

Initial temperature, $T_1 = 30 + 273 = 303$ K

Pressure at the end of compression stroke, $p_2 = 11$ bar

Heat added at constant volume = 210 kJ

Number of working cycles/min. = 210.

 (*i*) **Pressures, temperatures and volumes at salient points :**

For adiabatic compression 1-2

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

or $$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (r)^{\gamma}$$

or $$r = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{11}{1}\right)^{\frac{1}{1.4}} = (11)^{0.714} = 5.5$$

Also $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (5.5)^{1.4-1} = 1.977 \simeq 1.98$$

∴ $$T_2 = T_1 \times 1.98 = 303 \times 1.98 = \textbf{600 K.}   \textbf{(Ans.)}$$
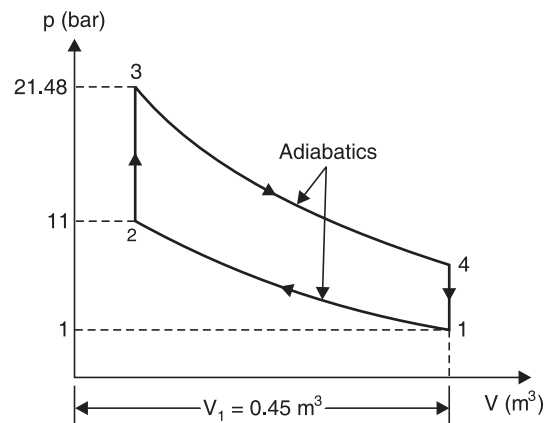
Fig. 4.12

Applying gas laws to points 1 and 2

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$\therefore$ $V_2 = \frac{T_2}{T_1} \times \frac{p_1}{p_2} \times V_1 = \frac{600 \times 1 \times 0.45}{303 \times 11} = \textbf{0.081 m}^3.$ **(Ans.)**

The heat supplied during the process 2-3 is given by :

$$Q_s = m c_v (T_3 - T_2)$$

where $m = \dfrac{p_1 V_1}{R T_1} = \dfrac{1 \times 10^5 \times 0.45}{287 \times 303} = 0.517$ kg

$\therefore$ $210 = 0.517 \times 0.71 (T_3 - 600)$

or $T_3 = \dfrac{210}{0.517 \times 0.71} + 600 = \textbf{1172 K.}$ **(Ans.)**

For the constant volume process 2-3

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

$\therefore$ $p_3 = \dfrac{T_3}{T_2} \times p_2 = \dfrac{1172}{600} \times 11 = \textbf{21.48 bar.}$ **(Ans.)**

$V_3 = V_2 = \textbf{0.081 m}^3.$ **(Ans.)**

For the adiabatic (or isentropic) process 3-4

$$p_3 V_3{}^\gamma = p_4 V_4{}^\gamma$$

$$p_4 = p_3 \times \left(\frac{V_3}{V_4}\right)^\gamma = p_3 \times \left(\frac{1}{r}\right)^\gamma$$

$$= 21.48 \times \left(\frac{1}{5.5}\right)^{1.4} = \textbf{1.97 bar.} \quad \textbf{(Ans.)}$$

Also $\dfrac{T_4}{T_3} = \left(\dfrac{V_3}{V_4}\right)^{\gamma - 1} = \left(\dfrac{1}{r}\right)^{\gamma - 1} = \left(\dfrac{1}{5.5}\right)^{1.4 - 1} = 0.505$

$\therefore$ $T_4 = 0.505, T_3 = 0.505 \times 1172 = \textbf{591.8 K.}$ **(Ans.)**

$V_4 = V_1 = \textbf{0.45 m}^3.$ **(Ans.)**

(*ii*) **Percentage clearance :**

Percentage clearance

$$= \frac{V_c}{V_s} = \frac{V_2}{V_1 - V_2} \times 100 = \frac{0.081}{0.45 - 0.081} \times 100$$

$$= \textbf{21.95\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Efficiency :**

The heat rejected per cycle is given by

$$Q_r = m c_v (T_4 - T_1)$$
$$= 0.517 \times 0.71 (591.8 - 303) = 106 \text{ kJ}$$

The air-standard efficiency of the cycle is given by

$$\eta_{\text{otto}} = \frac{Q_s - Q_r}{Q_s} = \frac{210 - 106}{210} = \textbf{0.495 or 49.5\%.} \quad \textbf{(Ans.)}$$

$$\left[\begin{array}{l} \text{Alternatively :} \\ \eta_{\text{otto}} = 1 - \dfrac{1}{(r)^{\gamma-1}} = 1 - \dfrac{1}{(5.5)^{1.4-1}} = \textbf{0.495 or 49.5\%.} \quad \textbf{(Ans.)} \end{array}\right]$$

(iv) **Mean effective pressure, p_m :**

The mean effective pressure is given by

$$p_m = \frac{W\,(\text{work done})}{V_s\,(\text{swept volume})} = \frac{Q_s - Q_r}{(V_1 - V_2)}$$

$$= \frac{(210 - 106)\times 10^3}{(0.45 - 0.081)\times 10^5} = \textbf{2.818 bar.} \quad \textbf{(Ans.)}$$

(v) **Power developed, P :**

Power developed, P = work done per second

= work done per cycle × number of cycles per second

= (210 − 106) × (210/60) = **364 kW.**   **(Ans.)**

Example 4.14. (a) *Show that the compression ratio for the maximum work to be done per kg of air in an Otto cycle between upper and lower limits of absolute temperatures T_3 and T_1 is given by*

$$r = \left(\frac{T_3}{T_1}\right)^{1/2\,(\gamma-1)}$$

(b) *Determine the air-standard efficiency of the cycle when the cycle develops maximum work with the temperature limits of 310 K and 1220 K and working fluid is air. What will be the percentage change in efficiency if helium is used as working fluid instead of air ? The cycle operates between the same temperature limits for maximum work development.*

Consider that all conditions are ideal.

Solution. Refer to Fig. 4.13.

(a) The work done per kg of fluid in the cycle is given by

$$W = Q_s - Q_r = c_v(T_3 - T_2) - c_v(T_4 - T_1)$$

But $\dfrac{T_2}{T_1} = \left(\dfrac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1}$

∴ $T_2 = T_1 . (r)^{\gamma-1}$ ...(i)

Similarly, $T_3 = T_4 . (r)^{\gamma-1}$ ...(ii)

∴ $W = c_v\left[T_3 - T_1 . (r)^{\gamma-1} - \dfrac{T_3}{(r)^{\gamma-1}} + T_1\right]$

...(iii)

This expression is a function of r when T_3 and T_1 are fixed. The value of W will be maximum when

$$\frac{dW}{dr} = 0.$$

∴ $\dfrac{dW}{dr} = -T_1 . (\gamma-1)(r)^{\gamma-2} - T_3(1-\gamma)(r)^{-\gamma} = 0$

or $T_3(r)^{-\gamma} = T_1(r)^{\gamma-2}$
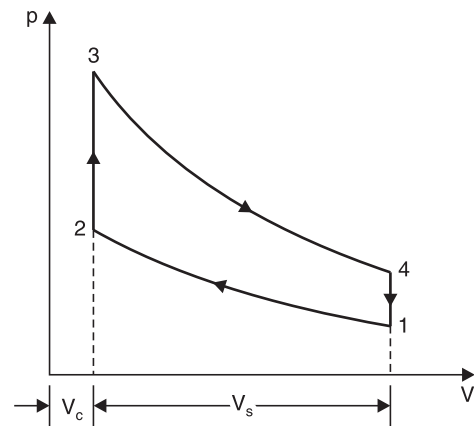
Fig. 4.13

or $$\frac{T_3}{T_1} = (r)^{2(\gamma-1)}$$

$\therefore$ $$r = \left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)}. \quad \textbf{Proved.}$$

(b) **Change in efficiency :**

For *air* $\gamma = 1.4$

$\therefore$ $$r = \left(\frac{T_3}{T_1}\right)^{1/2(1.4-1)} = \left(\frac{1220}{310}\right)^{1/0.8} = 5.54$$

The air-standard efficiency is given by

$$\eta_{\text{Otto}} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(5.54)^{1.4-1}} = 0.495 \text{ or } 49.5\%. \quad \textbf{(Ans.)}$$

If *helium* is used, then the values of

$$c_p = 5.22 \text{ and } c_v = 3.13$$

$\therefore$ $$\gamma = \frac{c_p}{c_v} = \frac{5.22}{3.13} = 1.67$$

The compression ratio for maximum work for the temperature limits T_1 and T_3 is given by *b*.

$$r = \left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)} = \left(\frac{1220}{310}\right)^{1/2(1.67-1)} = 2.77$$

The air-standard efficiency is given by

$$\eta_{\text{Otto}} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(2.77)^{1.67-1}} = 0.495 \text{ or } 49.5\%.$$

Hence, change in efficiency is nil.   (Ans.)

Example 4.15. (a) *An engine working on Otto cycle, in which the salient points are 1, 2, 3 and 4, has upper and lower temperature limits T_3 and T_1. If the maximum work per kg of air is to be done, show that the intermediate temperature is given by*

$$T_2 = T_4 = \sqrt{T_1 T_3}.$$

(b) *If an engine works on Otto cycle between temperature limits 1450 K and 310 K, find the maximum power developed by the engine assuming the circulation of air per minute as 0.38 kg.*

Solution. (a) Refer to Fig. 4.13 (Example 4.14).

Using the equation (iii) of example 4.14.

$$W = c_v \left[T_3 - T_1 \cdot (r)^{\gamma-1} - \frac{T_3}{(r)^{\gamma-1}} + T_1 \right]$$

and differentiating W w.r.t. r and equating to zero

$$r = \left(\frac{T_3}{T_1}\right)^{1/2(\gamma-1)}$$

$$T_2 = T_1(r)^{\gamma-1} \quad \text{and} \quad T_4 = T_3/(r)^{\gamma-1}$$

Substituting the value of r in the above equation

$$T_2 = T_1 \left[\left(\frac{T_3}{T_1} \right)^{1/2(\gamma-1)} \right]^{\gamma-1} = T_1 \left(\frac{T_3}{T_1} \right)^{1/2} = \sqrt{T_1 T_3}$$

Similarly,

$$T_4 = \frac{T_3}{\left[\left(\dfrac{T_3}{T_1} \right)^{1/2(\gamma-1)} \right]^{\gamma-1}} = \frac{T_3}{\left(\dfrac{T_3}{T_1} \right)^{1/2}} = \sqrt{T_3 T_1}$$

$\therefore$ $\qquad\qquad\qquad T_2 = T_4 = \sqrt{T_1 T_3}.$ **Proved.**

(*b*) **Power developed, P = ?**

$$\left. \begin{array}{l} T_1 = 310 \text{ K} \\ T_3 = 1450 \text{ K} \\ m = 0.38 \text{ kg} \end{array} \right\} \quad \text{...(given)}$$

Work done $\qquad W = c_v[(T_3 - T_2) - (T_4 - T_1)]$

$$T_2 = T_4 = \sqrt{T_1 T_3} = \sqrt{310 \times 1450} = 670.4 \text{ K}$$

$\therefore \qquad\qquad W = 0.71 [(1450 - 670.4) - (670.4 - 310)]$

$\qquad\qquad\qquad = 0.71(779.6 - 360.4) = 297.6 \text{ kJ/kg}$

Work done per second $= 297.6 \times (0.38/60) = 1.88 \text{ kJ/s}$

Hence, **power developed, P = 1.88 kW. (Ans.)**

Example 4.16. *For the same compression ratio, show that the efficiency of Otto cycle is greater than that of Diesel cycle.*

Solution. Refer to Fig. 4.14.

We know that $\qquad \eta_{\text{otto}} = 1 - \dfrac{1}{(r)^{\gamma-1}}$

and $\qquad\qquad \eta_{\text{diesel}} = 1 - \dfrac{1}{(r)^{\gamma-1}} \times \dfrac{1}{\gamma} \left\{ \dfrac{p^\gamma - 1}{p - 1} \right\}$

As the compression ratio is same, $\dfrac{V_1}{V_2} = \dfrac{V_1'}{V_2'} = r$
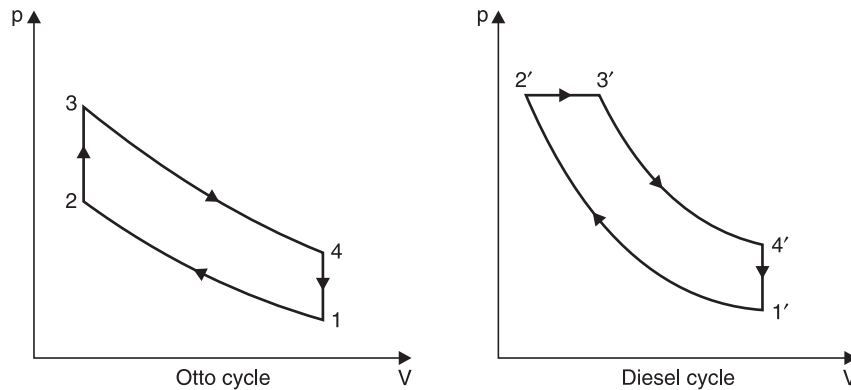
Fig. 4.14

If $\dfrac{V_4'}{V_3'} = r_1$, then cut off ratio, $\rho = \dfrac{V_3'}{V_2'} = \dfrac{r}{r_1}$

Putting the value of ρ in η_{diesel}, we get

$$\eta_{\text{diesel}} = 1 - \frac{1}{(r)^{\gamma-1}} \times \frac{1}{\gamma} \left[\frac{\left(\dfrac{r}{r_1}\right)^{\gamma} - 1}{\dfrac{r}{r_1} - 1} \right]$$

From above equation, we observe $\dfrac{r}{r_1} > 1$

Let $r_1 = r - \delta$, where δ is a small quantity.

Then

$$\frac{r}{r_1} = \frac{r}{r - \delta} = \frac{r}{r\left(1 - \dfrac{\delta}{r}\right)} = \left(1 - \frac{\delta}{r}\right)^{-1} = 1 + \frac{\delta}{r} + \frac{\delta^2}{r^2} + \frac{\delta^3}{r^3} + \dots$$

and

$$\left(\frac{r}{r_1}\right)^{\gamma} = \frac{r^{\gamma}}{r^{\gamma}\left(1 - \dfrac{\delta}{r}\right)^{\gamma}} = \left(1 - \frac{\delta}{r}\right)^{-\gamma} = 1 + \frac{\gamma\delta}{r} + \frac{\gamma(\gamma+1)}{2!} \cdot \frac{\delta^2}{r^2} + \dots$$

$\therefore$

$$\eta_{\text{diesel}} = 1 - \frac{1}{(r)^{\gamma-1}} \times \frac{1}{\gamma} \left[\frac{\dfrac{\gamma \cdot \delta}{r} + \dfrac{\gamma(\gamma+1)}{2!} \cdot \dfrac{\delta^2}{r^2} + \dots}{\dfrac{\delta}{r} + \dfrac{\delta^2}{r^2} + \dots} \right]$$

$$= 1 - \frac{1}{(r)^{\gamma-1}} \left[\frac{\dfrac{\delta}{r} + \dfrac{\gamma+1}{2} \cdot \dfrac{\delta^2}{r^2} + \dots}{\dfrac{\delta}{r} + \dfrac{\delta^2}{r^2} + \dots} \right]$$

The ratio inside the bracket is greater than 1 since the co-efficients of terms δ^2/r^2 is greater than 1 in the numerator. Its means that something more is subtracted in case of diesel cycle than in Otto cycle.

Hence, *for same compression ratio* $\eta_{otto} > \eta_{diesel}$.

4.5. CONSTANT PRESSURE OR DIESEL CYCLE

This cycle was introduced by Dr. R. Diesel in 1897. It differs from Otto cycle, in, that heat is supplied at constant pressure instead of at constant volume. Fig. 4.15 (*a* and *b*) shows the *p*-*V* and *T*-*s* diagrams of this cycle respectively.

This cycle comprises of the following operations :

 (*i*) 1-2......Adiabatic compression.

(*ii*) 2-3......Addition of heat at constant pressure.

(*iii*) 3-4......Adiabatic expansion.

(*iv*) 4-1......Rejection of heat at constant volume.
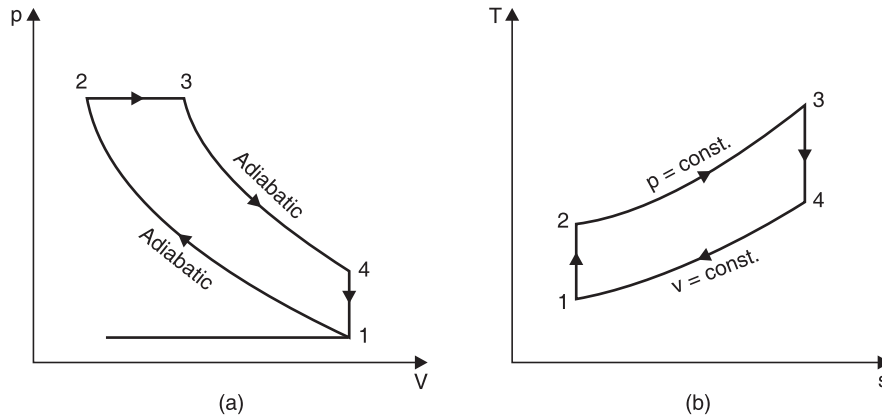
Fig. 4.15

Point 1 represents that the cylinder is full of air. Let p_1, V_1 and T_1 be the corresponding pressure, volume and absolute temperature. The piston then compresses the air adiabatically (*i.e.,* pV^γ = constant) till the values become p_2, V_2 and T_2 respectively (at the end of the stroke) at point 2. Heat is then added from a hot body at a constant pressure. During this addition of heat let volume increases from V_2 to V_3 and temperature T_2 to T_3, corresponding to point 3. This point (3) is called the *point of cut off*. The air then expands adiabatically to the conditions p_4, V_4 and T_4 respectively corresponding to point 4. Finally, the air rejects the heat to the cold body at constant volume till the point 1 where it returns to its original state.

Consider 1 kg of air.

Heat supplied at constant pressure = $c_p(T_3 - T_2)$

Heat rejected at constant volume   = $c_v(T_4 - T_1)$

Work done   = heat supplied – heat rejected

$$= c_p(T_3 - T_2) - c_v(T_4 - T_1)$$

∴ $\eta_{diesel} = \dfrac{\text{work done}}{\text{heat supplied}}$

$$= \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$= 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)} \qquad \qquad ...(i) \qquad \left[\because \ \frac{c_p}{c_v} = \gamma\right]$$

Let compression ratio, $r = \dfrac{v_1}{v_2}$

and cut off ratio, $\rho = \dfrac{v_3}{v_2}$ *i.e.,* $\dfrac{\text{volume at cut off}}{\text{clearance volume}}$

Now, during adiabatic compression 1-2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (r)^{\gamma - 1}$$

or $T_2 = T_1 \cdot (r)^{\gamma - 1}$

During constant pressure process 2-3,

$$\frac{T_3}{T_2} = \frac{v_3}{v_2} = \rho$$

or $\qquad\qquad\qquad T_3 = \rho \cdot T_2 = \rho \cdot T_1 \cdot (r)^{\gamma-1}$

During adiabatic expansion 3-4

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1}$$

$$= \left(\frac{r}{\rho}\right)^{\gamma-1} \qquad\qquad \left(\because \ \ \frac{v_4}{v_3} = \frac{v_1}{v_3} = \frac{v_1}{v_2} \times \frac{v_2}{v_3} = \frac{r}{\rho}\right)$$

$\therefore \qquad\qquad T_4 = \dfrac{T_3}{\left(\dfrac{r}{\rho}\right)^{\gamma-1}} = \dfrac{\rho \cdot T_1 (r)^{\gamma-1}}{\left(\dfrac{r}{\rho}\right)^{\gamma-1}} = T_1 \cdot \rho^{\gamma}$

By inserting values of T_2, T_3 and T_4 in equation (*i*), we get

$$\eta_{\text{diesel}} = 1 \ \frac{(T_1 \cdot \rho^{\gamma} - T_1)}{(\rho \cdot T_1 \cdot (r)^{\gamma-1} - T_1 \cdot (r)^{\gamma-1})} = 1 - \frac{(\rho^{\gamma} - 1)}{\gamma (r)^{\gamma-1} (\rho - 1)}$$

or $\qquad\qquad \eta_{\text{diesel}} = 1 - \dfrac{1}{\gamma\,(r)^{\gamma-1}}\left[\dfrac{\rho^{\gamma} - 1}{\rho - 1}\right] \qquad\qquad\qquad\qquad …(4.7)$

It may be observed that equation (4.7) for efficiency of diesel cycle is different from that of the Otto cycle only in bracketed factor. This factor is always greater than unity, because $\rho > 1$. Hence, *for a given compression ratio, the Otto cycle is more efficient.*

The *net work* for diesel cycle can be expressed in terms of *pv* as follows :

$$W = p_2(v_3 - v_2) + \frac{p_3 v_3 - p_4 v_4}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1}$$

$$= p_2 (\rho v_2 - v_2) + \frac{p_3\,\rho v_2 - p_4 r v_2}{\gamma - 1} - \frac{p_2 v_2 - p_1 r v_2}{\gamma - 1} \quad \left[\begin{array}{l} \because \quad \dfrac{v_3}{v_2} = \rho \ \therefore\ v_3 = \rho v_2 \\[2mm] \text{and}\ \ \dfrac{v_1}{v_2} = r \ \therefore\ v_1 = r v_2 \\[2mm] \text{But}\quad v_4 = v_1 \\ \therefore\qquad v_4 = r v_2 \end{array}\right]$$

$$= p_2 v_2 (\rho - 1) + \frac{p_3\,\rho\,v_2 - p_4 r v_2}{\gamma - 1} - \frac{p_2 v_2 - p_1 r v_2}{\gamma - 1}$$

$$= \frac{v_2[p_2 (\rho - 1)(\gamma - 1) + p_3 \rho - p_4 r - (p_2 - p_1 r)]}{\gamma - 1}$$

$$= \frac{v_2\left[p_2 (\rho - 1)(\gamma - 1) + p_3\left(\rho - \dfrac{p_4 r}{p_3}\right) - p_2\left(1 - \dfrac{p_1 r}{p_2}\right)\right]}{\gamma - 1}$$

$$= \frac{p_2 v_2[(\rho - 1)(\gamma - 1) + \rho - \rho^{\gamma} \cdot r^{1-\gamma} - (1 - r^{1-\gamma})]}{\gamma - 1} \quad \left[\begin{array}{l} \because \quad \dfrac{p_4}{p_3} = \left(\dfrac{v_3}{v_4}\right)^{\gamma} \\[3mm] \qquad = \left(\dfrac{\rho}{r}\right)^{\gamma} = \rho^{\gamma}\, r^{-\gamma} \end{array}\right]$$

$$= \frac{p_1 v_1 r^{\gamma-1}[(\rho - 1)(\gamma - 1) + \rho - \rho^\gamma \, r^{1-\gamma} - (1 - r^{1-\gamma})]}{\gamma - 1}$$

$$\left[\because \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^\gamma \text{ or } p_2 = p_1 \cdot r^\gamma \text{ and } \frac{v_1}{v_2} = r \text{ or } v_2 = v_1 r^{-1} \right]$$

$$= \frac{p_1 v_1 r^{\gamma-1}[\gamma(\rho - 1) - r^{1-\gamma}(\rho^\gamma - 1)]}{\gamma - 1} \qquad \qquad ...(4.8)$$

Mean effective pressure p_m is given by :

$$p_m = \frac{p_1 v_1 r^{\gamma-1}[\gamma(\rho - 1) - r^{1-\gamma}(\rho^\gamma - 1)]}{(\gamma - 1)v_1\left(\dfrac{r - 1}{r}\right)}$$

or $\qquad \qquad \mathbf{p_m} = \dfrac{p_1 r^\gamma[\gamma(\rho - 1) - r^{1-\gamma}(\rho^\gamma - 1)]}{(\gamma - 1)(r - 1)} \qquad \qquad ...(4.9)$

Example 4.17. *A diesel engine has a compression ratio of 15 and heat addition at constant pressure takes place at 6% of stroke. Find the air standard efficiency of the engine.*

Take γ for air as 1.4.

Solution. Refer to Fig. 4.16.

Compression ratio, $r \left(= \dfrac{V_1}{V_2}\right) = 15$

γ for air = 1.4

Air standard efficiency of diesel cycle is given by

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma(r)^{\gamma-1}}\left[\frac{\rho^\gamma - 1}{\rho - 1}\right] ...(i)$$

where ρ = cut off ratio $= \dfrac{V_3}{V_2}$

But $\qquad V_3 - V_2 = \dfrac{6}{100} V_s$

$\qquad \qquad \qquad$ (V_s = stroke volume)

$\qquad \qquad \qquad = 0.06 \, (V_1 - V_2)$

$\qquad \qquad \qquad = 0.06 \, (15 \, V_2 - V_2)$

$\qquad \qquad \qquad = 0.84 \, V_2$

or $\qquad \qquad V_3 = 1.84 \, V_2$

$\therefore \qquad \qquad \rho = \dfrac{V_3}{V_2} = \dfrac{1.84 \, V_2}{V_2} = 1.84$

Putting the value in eqn. (*i*), we get

$$\eta_{\text{diesel}} = 1 - \frac{1}{1.4 \, (15)^{1.4-1}}\left[\frac{(1.84)^{1.4} - 1}{1.84 - 1}\right]$$

$$= 1 - 0.2417 \times 1.605 = \mathbf{0.612 \text{ or } 61.2\%.} \quad \textbf{(Ans.)}$$
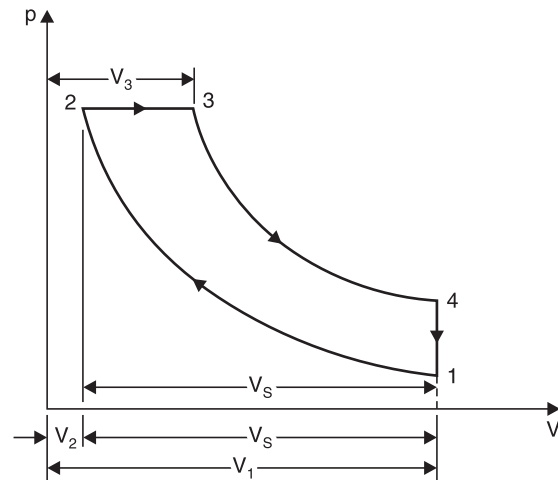
Fig. 4.16

Example 4.18. *The stroke and cylinder diameter of a compression ignition engine are 250 mm and 150 mm respectively. If the clearance volume is 0.0004 m³ and fuel injection takes place at constant pressure for 5 per cent of the stroke determine the efficiency of the engine. Assume the engine working on the diesel cycle.*

Solution. Refer to Fig. 4.16.

Length of stroke, L = 250 mm = 0.25 m

Diameter of cylinder, D = 150 mm = 0.15 m

Clearance volume, V_2 = 0.0004 m³

Swept volume, V_s = π/4 D^2L = π/4 × 0.15² × 0.25 = 0.004418 m³

Total cylinder volume   = swept volume + clearance volume

                          = 0.004418 + 0.0004 = 0.004818 m³

Volume at point of cut off, $V_3 = V_2 + \dfrac{5}{100} V_s$

$$= 0.0004 + \frac{5}{100} \times 0.004418 = 0.000621 \text{ m}^3$$

∴   Cut off ratio, $\rho = \dfrac{V_3}{V_2} = \dfrac{0.000621}{0.0004} = 1.55$

Compression ratio, $r = \dfrac{V_1}{V_2} = \dfrac{V_s + V_2}{V_2} = \dfrac{0.004418 + 0.0004}{0.0004}$

                          = 12.04

Hence, $\eta_{\text{diesel}} = 1 - \dfrac{1}{\gamma(r)^{\gamma-1}}\left[\dfrac{\rho^\gamma - 1}{\rho - 1}\right] = 1 - \dfrac{1}{1.4 \times (12.04)^{1.4-1}}\left[\dfrac{(1.55)^{1.4} - 1}{1.55 - 1}\right]$

                          = 1 − 0.264 × 1.54 = **0.593 or 59.3%.   (Ans.)**

Example 4.19. *Calculate the percentage loss in the ideal efficiency of a diesel engine with compression ratio 14 if the fuel cut off is delayed from 5% to 8%.*

Solution. Let the clearance volume (V_2) be unity.

Then, compression ratio, r = 14

Now, when the fuel is cut off at 5%, we have

$$\frac{\rho - 1}{r - 1} = \frac{5}{100}$$

or $\dfrac{\rho - 1}{14 - 1} = 0.05$

or $\rho - 1$ = 13 × 0.05 = 0.65

∴ ρ = 1.65

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma(r)^{\gamma-1}}\left[\frac{\rho^\gamma - 1}{\rho - 1}\right] = 1 - \frac{1}{1.4\,(14)^{1.4-1}}\left[\frac{(1.65)^{1.4} - 1}{1.65 - 1}\right]$$

                          = 1 − 0.248 × 1.563 = 0.612 or 61.2%

When the fuel is cut off at 8%, we have

$$\frac{\rho - 1}{r - 1} = \frac{8}{100}$$

or $\dfrac{\rho - 1}{14 - 1} = \dfrac{8}{100} = 0.08$

$\therefore$ $\rho = 1 + 1.04 = 2.04$

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma(r)^{\gamma-1}} \left[\frac{\rho^\gamma - 1}{\rho - 1}\right] = 1 - \frac{1}{1.4\,(14)^{1.4-1}} \left[\frac{(2.04)^{1.4} - 1}{2.04 - 1}\right]$$

$$= 1 - 0.248 \times 1.647 = \textbf{0.591 or 59.1\%.} \quad \textbf{(Ans.)}$$

Hence, percentage loss in efficiency due to delay in fuel cut off

$$= 61.2 - 59.1 = \textbf{2.1\%.} \quad \textbf{(Ans.)}$$

Example 4.20. *The mean effective pressure of a Diesel cycle is 7.5 bar and compression ratio is 12.5. Find the percentage cut off of the cycle if its initial pressure is 1 bar.*

Solution. Mean effective pressure, $p_m = 7.5$ bar

Compression ratio, $r = 12.5$

Initial pressure, $p_1 = 1$ bar

Refer Fig. 4.15.

The mean effective pressure is given by

$$p_m = \frac{p_1 r^\gamma [\gamma(\rho - 1) - r^{1-\gamma}\,(\rho^\gamma - 1)]}{(\gamma - 1)(r - 1)}$$

$$7.5 = \frac{1 \times (12.5)^{1.4}[1.4(\rho - 1) - (12.5)^{1-1.4}\,(\rho^{1.4} - 1)]}{(1.4 - 1)(12.5 - 1)}$$

$$7.5 = \frac{34.33\,[1.4\rho - 1.4 - 0.364\rho^{1.4} + 0.364]}{4.6}$$

$$7.5 = 7.46\,(1.4\,\rho - 1.036 - 0.364\,\rho^{1.4})$$

$$1.005 = 1.4\,\rho - 1.036 - 0.364\,\rho^{1.4}$$

or $2.04 = 1.4\,\rho - 0.364\,\rho^{1.4}$

or $0.346\,\rho^{1.4} - 1.4\,\rho + 2.04 = 0$

Solving by trial and error method, we get

$$\rho = 2.24$$

$\therefore$ $\%\ cut\ off = \dfrac{\rho - 1}{r - 1} \times 100 = \dfrac{2.24 - 1}{12.5 - 1} \times 100 = \textbf{10.78\%.} \quad \textbf{(Ans.)}$

Example 4.21. *An engine with 200 mm cylinder diameter and 300 mm stroke works on theoretical Diesel cycle. The initial pressure and temperature of air used are 1 bar and 27°C. The cut off is 8% of the stroke. Determine :*

(i) Pressures and temperatures at all salient points.

(ii) Theoretical air standard efficiency.

(iii) Mean effective pressure.

(iv) Power of the engine if the working cycles per minute are 380.

Assume that compression ratio is 15 and working fluid is air.

Consider all conditions to be ideal.

Solution. Refer to Fig. 4.17.

Cylinder diameter, $D = 200$ mm or 0.2 m

Stroke length, $L = 300$ mm or 0.3 m

Initial pressure, $p_1 = 1.0$ bar

Initial temperature, $T_1 = 27 + 273 = 300$ K

Cut off $= \dfrac{8}{100}\,V_s = 0.08\,V_s$

(*i*) **Pressures and temperatures at salient points :**

Now, stroke volume, $V_s = \pi/4 \, D^2 L = \pi/4 \times 0.2^2 \times 0.3 = 0.00942$ m³

$$V_1 = V_s + V_c = V_s + \frac{V_s}{r-1} \qquad\qquad \left[\because \; V_c = \frac{V_s}{r-1}\right]$$

$$= V_s\left(1 + \frac{1}{r-1}\right) = \frac{r}{r-1} \times V_s$$
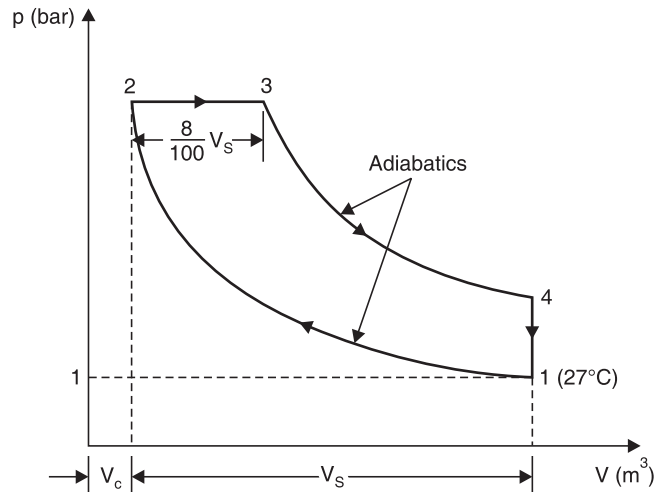
Fig. 4.17

i.e., $V_1 = \dfrac{15}{15-1} \times V_s = \dfrac{15}{14} \times 0.00942 = \textbf{0.0101 m}^3.$ **(Ans.)**

Mass of the air in the cylinder can be calculated by using the gas equation

$$p_1 V_1 = m R T_1$$

$$m = \frac{p_1 V_1}{R T_1} = \frac{1 \times 10^5 \times 0.0101}{287 \times 300} = 0.0117 \text{ kg/cycle}$$

For the adiabatic (or isentropic) process 1-2

$$p_1 V_1{}^\gamma = p_2 V_2{}^\gamma$$

or $\dfrac{p_2}{p_1} = \left(\dfrac{V_1}{V_2}\right)^\gamma = (r)^\gamma$

$\therefore$ $p_2 = p_1 \cdot (r)^\gamma = 1 \times (15)^{1.4} = \textbf{44.31 bar.}$ **(Ans.)**

Also, $\dfrac{T_2}{T_1} = \left(\dfrac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (15)^{1.4-1} = 2.954$

$\therefore$ $T_2 = T_1 \times 2.954 = 300 \times 2.954 = \textbf{886.2 K.}$ **(Ans.)**

$$V_2 = V_c = \frac{V_s}{r-1} = \frac{0.00942}{15-1} = \textbf{0.0006728 m}^3. \quad \textbf{(Ans.)}$$

$$p_2 = p_3 = \textbf{44.31 bar.} \quad \textbf{(Ans.)}$$

% cut off ratio $= \dfrac{\rho-1}{r-1}$

$$\frac{8}{100} = \frac{\rho - 1}{15 - 1}$$

i.e., $\qquad\qquad \rho = 0.08 \times 14 + 1 = 2.12$

$\therefore \qquad\qquad V_3 = \rho\, V_2 = 2.12 \times 0.0006728 =$ **0.001426 m³.** **(Ans.)**

$$\left[\begin{array}{l} V_3 \text{ can also be calculated as follows :} \\ V_3 = 0.08\ V_s + V_c = 0.08 \times 0.00942 + 0.0006728 = 0.001426\ \text{m}^3 \end{array} \right]$$

For the constant pressure process 2-3

$$\frac{V_3}{T_3} = \frac{V_2}{T_2}$$

$\therefore \qquad\qquad T_3 = T_2 \times \dfrac{V_3}{V_2} = 886.2 \times \dfrac{0.001426}{0.0006728} =$ **1878.3 K.** **(Ans.)**

For the isentropic process 3-4

$$p_3\, V_3{}^\gamma = p_4 V_4{}^\gamma$$

$$p_4 = p_3 \times \left(\frac{V_3}{V_4}\right)^\gamma = p_3 \times \frac{1}{(7.07)^{1.4}}$$

$$= \frac{44.31}{(7.07)^{1.4}}$$

$$= \textbf{2.866 bar.} \quad \textbf{(Ans.)}$$

$$\left[\begin{array}{l} \because\ \dfrac{V_4}{V_3} = \dfrac{V_4}{V_2} \times \dfrac{V_2}{V_3} \\[2mm] \qquad = \dfrac{V_1}{V_2} \times \dfrac{V_2}{V_3} \\[2mm] \qquad = \dfrac{r}{\rho},\ \because\ V_4 = V_1 \\[2mm] \qquad = \dfrac{15}{2.12} = 7.07 \end{array} \right]$$

Also, $\qquad\qquad \dfrac{T_4}{T_3} = \left(\dfrac{V_3}{V_4}\right)^{\gamma - 1} = \left(\dfrac{1}{7.07}\right)^{1.4 - 1} = 0.457$

$\therefore \qquad\qquad T_4 = T_3 \times 0.457 = 1878.3 \times 0.457 =$ **858.38 K.** **(Ans.)**

$\qquad\qquad V_4 = V_1 =$ **0.0101 m³.** **(Ans.)**

(*ii*) **Theoretical air standard efficiency :**

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma (r)^{\gamma - 1}} \left[\frac{\rho^\gamma - 1}{\rho - 1} \right] = 1 - \frac{1}{1.4\,(15)^{1.4-1}} \left[\frac{(2.12)^{1.4} - 1}{2.12 - 1} \right]$$

$$= 1 - 0.2418 \times 1.663 = \textbf{0.598 or 59.8\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Mean effective pressure, p_m :**

Mean effective pressure of Diesel cycle is given by

$$p_m = \frac{p_1 (r)^\gamma [\gamma(\rho - 1) - r^{1-\gamma}\,(\rho^\gamma - 1)]}{(\gamma - 1)(r - 1)}$$

$$= \frac{1 \times (15)^{1.4}[1.4(2.12 - 1) - (15)^{1-1.4}\,(2.12^{1.4} - 1)]}{(1.4 - 1)(15 - 1)}$$

$$= \frac{44.31\,[1.568 - 0.338 \times 1.863]}{0.4 \times 14} = \textbf{7.424 bar.} \quad \textbf{(Ans.)}$$

(*iv*) **Power of the engine, P:**

Work done per cycle

$$= p_m V_s = \frac{7.424 \times 10^5 \times 0.00942}{10^3} = 6.99 \text{ kJ/cycle}$$

Work done per second

$$= \text{work done per cycle} \times \text{no. of cycles per second}$$

$$= 6.99 \times 380/60 = 44.27 \text{ kJ/s} = 44.27 \text{ kW}$$

Hence, *power of the engine* = **44.27 kW.   (Ans.)**

Example 4.22. *The volume ratios of compression and expansion for a diesel engine as measured from an indicator diagram are 15.3 and 7.5 respectively. The pressure and temperature at the beginning of the compression are 1 bar and 27°C.*

Assuming an ideal engine, determine the mean effective pressure, the ratio of maximum pressure to mean effective pressure and cycle efficiency.

Also find the fuel consumption per kWh if the indicated thermal efficiency is 0.5 of ideal efficiency, mechanical efficiency is 0.8 and the calorific value of oil 42000 kJ/kg.

Assume for air : $c_p = 1.005 \ kJ/kg \ K$; $c_v = 0.718 \ kJ/kg \ K$, $\gamma = 1.4$.

Solution. Refer Fig. 4.18.

Given : $\dfrac{V_1}{V_2} = 15.3$; $\dfrac{V_4}{V_3} = 7.5$ η ; $p_1 = 1$ bar ;

$\qquad\qquad T_1 = 27 + 273 = 300$ K ;

$\qquad\qquad \eta_{th(I)} = 0.5 \times \eta_{\text{air-standard}}$; $\eta_{\text{mech.}} = 0.8$,

$\qquad\qquad C = 42000$ kJ/kg.

The cycle is shown in Fig. 4.18, the subscripts denote the respective points in the cycle.

Mean effective pressure, p_m :

$$p_m = \frac{\text{work done by the cycle}}{\text{swept volume}}$$

Work done = heat added – heat rejected

Heat added = $mc_p \, (T_3 - T_2)$, and

Heat rejected = $mc_v \, (T_4 - T_1)$

Fig. 4.18. Diesel cycle.

Now assume air as a perfect gas and mass of oil in the air-fuel mixture is negligible and is not taken into an account.

Process 1-2 is an adiabatic compression process, thus

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \quad \text{or} \quad T_2 = T_1 \times \left(\frac{V_1}{V_2}\right)^{1.4-1} \qquad\qquad \text{(Since } \gamma = 1.4\text{)}$$

or $\qquad\qquad\qquad T_2 = 300 \times (15.3)^{0.4} = 893.3$ K

Also $\qquad\qquad p_1 V_1^{\gamma} = p_2 V_2^{\gamma} \quad \Rightarrow \quad p_2 = p_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times (15.3)^{1.4} = 45.56$ bar

Process 2-3 is a constant pressure process, hence

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \quad \Rightarrow \quad T_3 = \frac{V_3 T_2}{V_2} = 2.04 \times 893.3 = 1822.3 \text{ K.}$$

Assume that the volume at point (V_2) is 1 m³.

Thus the mass of air involved in the process,

$$m = \frac{p_2 V_2}{RT_2} = \frac{45.56 \times 10^5 \times 1}{287 \times 893.3} = 17.77 \text{ kg.}$$

$$\left[\because \quad \frac{V_4}{V_3} = \frac{V_1}{V_3} = \frac{V_1}{V_2} \times \frac{V_2}{V_3} \right.$$
$$\left. \text{or} \quad \frac{V_3}{V_2} = \frac{V_1}{V_2} \times \frac{V_3}{V_4} = \frac{15.3}{7.5} = 2.04 \right]$$

Process 3-4 is an adiabatic expansion process, then

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{1}{7.5}\right)^{1.4-1} = 0.4466$$

or $\qquad\qquad T_4 = 1822.3 \times 0.4466 = 813.8 \text{ K}$

$\therefore$ Work done $= mc_p(T_3 - T_2) - mc_v(T_4 - T_1)$

$\qquad\qquad\qquad = 17.77 \, [1.005(1822.3 - 893.3) - 0.718(813.8 - 300)] = 10035 \text{ kJ}$

$\therefore \qquad\qquad \mathbf{p_m} = \dfrac{\text{work done}}{\text{swept volume}} = \dfrac{10035}{(V_1 - V_2)} = \dfrac{10035}{(15.3 \, V_2 - V_2)}$

$\qquad\qquad\qquad = \dfrac{10035}{14.3} = 701.7 \text{ kN/m}^2 = \mathbf{7.017 \text{ bar.}}$ **(Ans.)** $(\because \quad V_2 = 1 \text{ m}^3 \text{ assumed})$

Ratio of maximum pressure to mean effective pressure

$\qquad\qquad\qquad = \dfrac{p_2}{p_m} = \dfrac{45.66}{7.017} = \mathbf{6.49.}$ **(Ans.)**

Cycle efficiency, η_{cycle} **:**

$\qquad\qquad \eta_{\text{cycle}} = \dfrac{\text{work done}}{\text{heat supplied}}$

$\qquad\qquad\qquad = \dfrac{10035}{mc_p(T_3 - T_2)} = \dfrac{10035}{17.77 \times 1.005(1822.3 - 893.3)} = \mathbf{0.6048}$ or $\mathbf{60.48\%.}$ **(Ans.)**

Fuel consumption per kWh ; $\mathbf{m_f}$

$\qquad\qquad \eta_{\text{th}(I)} = 0.5 \, \eta_{\text{cycle}} = 0.5 \times 0.6048 = 0.3024$ or 30.24%

$\qquad\qquad \eta_{\text{th}(B)} = 0.3024 \times 0.8 = 0.242$

Also, $\qquad \eta_{\text{th}(B)} = \dfrac{B.P.}{m_f \times C} = \dfrac{1}{\dfrac{m_f}{3600} \times 42000} = \dfrac{3600}{m_f \times 42000}$ or $0.242 = \dfrac{3600}{m_f \times 42000}$

or $\qquad\qquad \mathbf{m_f} = \dfrac{3600}{0.242 \times 42000} = \mathbf{0.354 \text{ kg/kWh.}}$ **(Ans.)**

4.6. DUAL COMBUSTION CYCLE

This cycle (also called the *limited pressure cycle or mixed cycle*) is a combination of Otto and Diesel cycles, in a way, that heat is added partly at constant volume and partly at constant pressure ; *the advantage of which is that more time is available to fuel (which is injected into the engine cylinder before the end of compression stroke) for combustion. Because of lagging characteristics of fuel this cycle is invariably used for diesel and hot spot ignition engines.*

The dual combustion cycle (Fig. 4.19) consists of the following operations :

  (*i*) 1-2—Adiabatic compression

 (*ii*) 2-3—Addition of heat at constant volume

(*iii*) 3-4—Addition of heat at constant pressure

 (*iv*) 4-5—Adiabatic expansion

  (*v*) 5-1—Rejection of heat at constant volume.

Consider 1 kg of air.

Total heat supplied $\qquad$ = heat supplied during the operation 2-3 + heat supplied during the operation 3-4

$\qquad\qquad\qquad = c_v(T_3 - T_2) + c_p(T_4 - T_3)$

Heat rejected during operation 5-1 $= c_v(T_5 - T_1)$

Work done $\qquad\qquad$ = heat supplied – heat rejected

$\qquad\qquad\qquad = c_v(T_3 - T_2) + c_p(T_4 - T_3) - c_v(T_5 - T_1)$
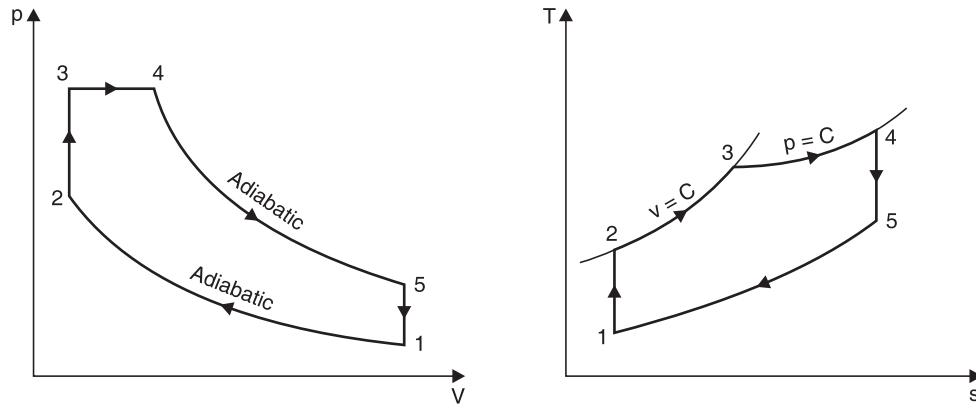
Fig. 4.19

$$\eta_{dual} = \frac{\text{word done}}{\text{heat supplied}} = \frac{c_v(T_3 - T_2) + c_p(T_4 - T_3) - c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$

$$= 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$

$$= 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)} \qquad \qquad ...(i) \quad \left(\because \quad \gamma = \frac{c_p}{c_v} \right)$$

Compression ratio, $\qquad r = \dfrac{v_1}{v_2}$

During adiabatic compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{\gamma-1} = (r)^{\gamma-1} \qquad \qquad ...(ii)$$

During constant volume heating process

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

or $\qquad\qquad\qquad\quad \dfrac{T_3}{T_2} = \dfrac{p_3}{p_2} = \beta$, where β is known as **pressure or explosion ratio**.

or $\qquad\qquad\qquad\quad T_2 = \dfrac{T_3}{\beta} \qquad\qquad\qquad\qquad\qquad\qquad\qquad ...(iii)$

During adiabatic expansion process,

$$\frac{T_4}{T_5} = \left(\frac{v_5}{v_4} \right)^{\gamma-1}$$

$$= \left(\frac{r}{\rho} \right)^{\gamma-1} \qquad\qquad\qquad\qquad\qquad ...(iv)$$

$$\left(\because \quad \frac{v_5}{v_4} = \frac{v_1}{v_4} = \frac{v_1}{v_2} \times \frac{v_2}{v_4} = \frac{v_1}{v_2} \times \frac{v_3}{v_4} = \frac{r}{\rho}, \rho \text{ being the cut off ratio} \right)$$

During constant pressure heating process,

$$\frac{v_3}{T_3} = \frac{v_4}{T_4}$$

$$T_4 = T_3 \, \frac{v_4}{v_3} = \rho \, T_3 \qquad\qquad ...(v)$$

Putting the value of T_4 in the equation (iv), we get

$$\frac{\rho T_3}{T_5} = \left(\frac{r}{\rho}\right)^{\gamma-1}$$

or

$$T_5 = \rho. \, T_3. \left(\frac{\rho}{r}\right)^{\gamma-1}$$

Putting the value of T_2 in equation (ii), we get

$$\frac{\dfrac{T_3}{\beta}}{T_1} = (r)^{\gamma-1}$$

$$T_1 = \frac{T_3}{\beta} \cdot \frac{1}{(r)^{\gamma-1}}$$

Now inserting the values of T_1, T_2, T_4 and T_5 in equation (i), we get

$$\eta_{\text{dual}} = 1 - \frac{\left[\rho . T_3\left(\dfrac{\rho}{r}\right)^{\gamma-1} - \dfrac{T_3}{\beta} \cdot \dfrac{1}{(r)^{\gamma-1}}\right]}{\left[\left(T_3 - \dfrac{T_3}{\beta}\right) + \gamma(\rho T_3 - T_3)\right]} = 1 - \frac{\dfrac{1}{(r)^{\gamma-1}}\left(\rho^\gamma - \dfrac{1}{\beta}\right)}{\left(1 - \dfrac{1}{\beta}\right) + \gamma(\rho - 1)}$$

i.e.,

$$\eta_{\text{dual}} = 1 - \frac{1}{(r)^{\gamma-1}} \cdot \frac{(\beta . \rho^\gamma - 1)}{[(\beta - 1) + \beta_\gamma(\rho - 1)]} \qquad\qquad ...(4.10)$$

Work done is given by,

$$W = p_3(v_4 - v_3) + \frac{p_4 v_4 - p_5 v_5}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1}$$

$$= p_3 v_3(\rho - 1) + \frac{(p_4 \rho v_3 - p_5 r v_3) - (p_2 v_3 - p_1 r v_3)}{\gamma - 1}$$

$$= \frac{p_3 v_3(\rho - 1)(\gamma - 1) + p_4 v_3\left(\rho - \dfrac{p_5}{p_4} r\right) - p_2 v_3\left(1 - \dfrac{p_1}{p_2} r\right)}{\gamma - 1}$$

Also

$$\frac{p_5}{p_4} = \left(\frac{v_4}{v_5}\right)^\gamma = \left(\frac{\rho}{r}\right)^\gamma$$

and

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^\gamma = r^\gamma$$

also,

$$p_3 = p_4, \, v_2 = v_3, \, v_5 = v_1$$

$\therefore$

$$W = \frac{v_3[p_3(\rho - 1)(\gamma - 1) + p_3\,(\rho - \rho^\gamma r^{1-\gamma}) - p_2\,(1 - r^{1-\gamma})]}{(\gamma - 1)}$$

$$= \frac{p_2 v_2[\beta(\rho - 1)(\gamma - 1) + \beta\,(\rho - \rho^\gamma r^{1-\gamma}) - (1 - r^{1-\gamma})]}{(\gamma - 1)}$$

$$= \frac{p_1 (r)^\gamma\, v_1\, /\, r\, [\beta\gamma\,(\rho - 1) + (\beta - 1) - r^{1-\gamma}\,(\beta\rho^{\gamma-1})]}{\gamma - 1}$$

$$= \frac{p_1 v_1 r^{\gamma-1}\, [\beta\gamma\,(\rho - 1) + (\beta - 1)] - r^{\gamma-1}\,(\beta\rho^{\gamma-1})]}{\gamma - 1} \qquad \qquad ...(4.11)$$

Mean effective pressure (p_m) is given by,

$$p_m = \frac{W}{v_1 - v_2} = \frac{W}{v_1 \left(\dfrac{r-1}{r}\right)} = \frac{p_1 v_1\, [r^{1-\gamma}\beta\gamma\,(\rho - 1) + (\beta - 1)] - r^{1-\gamma}\,(\beta\rho^\gamma - 1)]}{(\gamma - 1)\, v_1 \left(\dfrac{r-1}{r}\right)}$$

$$\mathbf{p_m} = \frac{p_1 (r)^\gamma [\beta(\rho - 1) + (\beta - 1) - r^{1-\gamma}(\beta\rho^\gamma - 1)]}{(\gamma - 1)\,(r - 1)} \qquad \qquad ...(4.12)$$

Example 4.23. *The swept volume of a diesel engine working on dual cycle is 0.0053 m³ and clearance volume is 0.00035 m³. The maximum pressure is 65 bar. Fuel injection ends at 5 per cent of the stroke. The temperature at the start of the compression are 80°C and 0.9 bar. Determine the air standard efficiency of the cycle. Take γ for air = 1.4.*

Solution. Refer to Fig. 4.20.

Swept volume, $V_s = 0.0053$ m³

Clearance volume, $V_c = V_3 = V_2 = 0.00035$ m³

Maximum pressure, $p_3 = p_4 = 65$ bar

Initial temperature, $T_1 = 80 + 273 = 353$ K

Initial pressure, $p_1 = 0.9$ bar

η_{dual} :

The efficiency of a dual combustion cycle is given by

$$\eta_{\text{dual}} = 1 - \frac{1}{(r)^{\gamma-1}} \left[\frac{\beta \cdot \rho^\gamma - 1}{(\beta - 1) + \beta\gamma(\rho - 1)} \right]$$
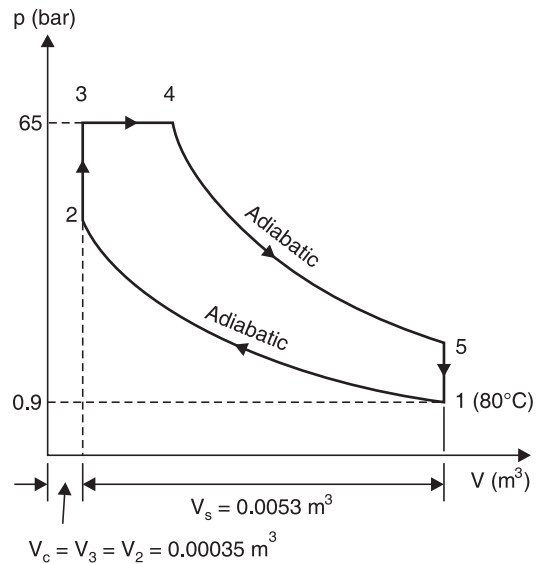
Fig. 4.20

Compression ratio, $\quad r = \dfrac{V_1}{V_2} = \dfrac{V_s + V_c}{V_c} = \dfrac{0.0053 + 0.00035}{0.00035}$

$$= 16.14 \qquad\qquad [\because\ V_2 = V_c = \text{clearance volume}]$$

Cut off ratio, $\quad \rho = \dfrac{V_4}{V_3} = \dfrac{\dfrac{5}{100} V_s + V_3}{V_3} = \dfrac{0.05 V_s + V_c}{V_c} \qquad\qquad (\because\ V_2 = V_3 = V_c)$

$$= \frac{0.05 \times 0.0053 + 0.00035}{0.00035} = 1.757 \text{ say } 1.76$$

Also during the compression operation 1-2

$$p_1 V_1{}^\gamma = p_2 V_2{}^\gamma$$

or $\qquad\qquad \dfrac{p_2}{p_1} = \left(\dfrac{V_1}{V_2}\right)^\gamma = (16.15)^{1.4} = 49.14$

or $\qquad\qquad p_2 = p_1 \times 49.14 = 0.9 \times 49.14 = 44.22$ bar

Pressure or explosion ratio,

$$\beta = \frac{p_3}{p_2} = \frac{65}{44.22} = 1.47$$

Putting the value of r, ρ and β in equation (i), we get

$$\eta_{dual} = 1 - \frac{1}{(16.14)^{1.4-1}}\left[\frac{1.47 \times (1.76)^{1.4} - 1}{(1.47 - 1) + 1.47 \times 1.4\,(1.76 - 1)}\right]$$

$$= 7 - 0.328 \left[\frac{3.243 - 1}{0.47 + 1.564}\right] = \mathbf{0.6383\ or\ 63.83\%.\quad (Ans.)}$$

Example 4.24. *An oil engine working on the dual combustion cycle has a compression ratio 14 and the explosion ratio obtained from an indicator card is 1.4. If the cut off occurs at 6 per cent of stroke, find the ideal efficiency. Take γ for air = 1.4.*

Solution. Refer to Fig. 4.19.

Compression ratio, $r = 14$

Explosion ratio, $\beta = 1.4$

If ρ is the cut off ratio, then

$$\frac{\rho - 1}{r - 1} = \frac{6}{100} \quad \text{or} \quad \frac{\rho - 1}{14 - 1} = 0.06$$

∴ $\rho = 1.78$

Ideal efficiency is given by

$$\eta_{ideal\ or\ dual} = 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{(\beta \rho^{\gamma} - 1)}{(\beta - 1) + \beta\gamma(\rho - 1)}\right]$$

$$= 1 - \frac{1}{(14)^{1.4-1}}\left[\frac{1.4 \times (1.78)^{1.4} - 1}{(1.4 - 1) + 1.4 \times 1.4\,(1.78 - 1)}\right]$$

$$= 1 - 0.348 \left[\frac{3.138 - 1}{0.4 + 1.528}\right] = \mathbf{0.614\ or\ 61.4\%.\quad (Ans.)}$$

Example 4.25. *The compression ratio for a single cylinder engine operating on dual cycle is 9. The maximum pressure in the cylinder is limited to 60 bar. The pressure and temperature of the air at the beginning of the cycle are 1 bar and 30°C. Heat is added during constant pressure process upto 4 per cent of the stroke. Assuming the cylinder diameter and stroke length as 250 mm and 300 mm respectively, determine :*

(i) The air standard efficiency of the cycle.

(ii) The power developed if the number of working cycles are 3 per second.

Take for air $c_v = 0.71$ and : $c_p = 1.0$.

Solution. Refer Fig. 4.21.
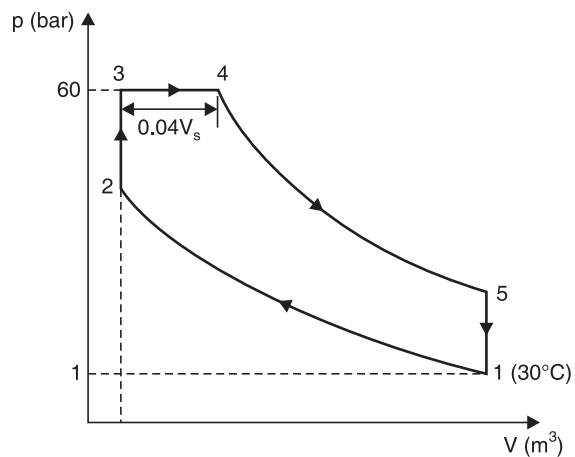
Fig. 4.21

Cylinder diameter, $D = 250$ mm = 0.25 m
Compression ratio, $r = 9$
Stroke length, $L = 300$ mm = 0.3 m
Initial pressure, $p_1 = 1$ bar
Initial temperature, $T_1 = 30 + 273 = 303$ K
Maximum pressure, $p_3 = p_4 = 60$ bar
Cut off   = 4% of stroke volume
Number of working cycles/sec. = 3.

(*i*) **Air standard efficiency = ?**

Now, swept volume, $V_s = \pi/4\ D^2 L = \pi/4 \times 0.25^2 \times 0.3$
$$= 0.0147 \text{ m}^3$$

Also, compression ratio, $r = \dfrac{V_s + V_c}{V_c}$

i.e., $9 = \dfrac{0.0147 + V_c}{V_c}$

$\therefore$ $V_c = \dfrac{0.0147}{8} = 0.0018 \text{ m}^3$

$\therefore$ $V_1 = V_s + V_c = 0.0147 + 0.0018 = 0.0165 \text{ m}^3$

For the adiabatic (or isentropic) process 1-2,
$$p_1 V_1^\gamma = p_2 V_2^\gamma$$

$$p_2 = p_1 \times \left(\dfrac{V_1}{V_2}\right)^\gamma = 1 \times (r)^\gamma = 1 \times (9)^{1.4} = 21.67 \text{ bar}$$

Also, $\dfrac{T_2}{T_1} = \left(\dfrac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (9)^{1.4-1} = (9)^{0.4} = 2.408$

$\therefore$ $T_2 = T_1 \times 2.408 = 303 \times 2.408 = 729.6$ K

For the constant volume process 2-3,
$$\dfrac{T_3}{p_3} = \dfrac{T_2}{p_2}$$

$\therefore$ $T_3 = T_2 \cdot \dfrac{p_3}{p_2} = 729.6 \times \dfrac{60}{21.67} = 2020$ K

Also, $\dfrac{\rho - 1}{r - 1} = \dfrac{4}{100}$ or   0.04

$\therefore$ $\dfrac{\rho - 1}{9 - 1} = 0.04$ or $\rho = 1.32$

For the constant pressure process 3-4,
$$\dfrac{V_4}{T_4} = \dfrac{V_3}{T_3}$$

or $\dfrac{T_4}{T_3} = \dfrac{V_4}{V_3} = \rho$

$\therefore$ $T_4 = T_3 \times \rho = 2020 \times 1.32 = 2666.4$ K

Also expansion ratio, $\dfrac{V_5}{V_4} = \dfrac{V_5}{V_2} \times \dfrac{V_2}{V_4} = \dfrac{V_1}{V_2} \times \dfrac{V_3}{V_4}$ $[\because \quad V_5 = V_1 \text{ and } V_2 = V_3]$

$$= \dfrac{r}{\rho}$$

For adiabatic process 4-5,

$$\dfrac{T_5}{T_4} = \left(\dfrac{V_4}{V_5}\right)^{\gamma-1} = \left(\dfrac{\rho}{r}\right)^{\gamma-1}$$

$\therefore$ $T_5 = T_4 \times \left(\dfrac{\rho}{r}\right)^{\gamma-1} = 2666.4 \times \left(\dfrac{1.32}{9}\right)^{1.4-1} = 1237 \text{ K}$

Also $p_4 V_4{}^\gamma = p_5 V_5{}^\gamma$

$$p_5 = p_4 \cdot \left(\dfrac{V_4}{V_5}\right)^\gamma = 60 \times \left(\dfrac{r}{\rho}\right)^\gamma = 60 \times \left(\dfrac{1.32}{9}\right)^{1.4} = 4.08 \text{ bar}$$

Heat supplied, $Q_s = c_v(T_3 - T_2) + c_P(T_4 - T_3)$
$\qquad\qquad\qquad = 0.71\,(2020 - 729.6) + 1.0\,(2666.4 - 2020) = 1562.58 \text{ kJ/kg}$

Heat rejected, $Q_r = c_v\,(T_5 - T_1)$
$\qquad\qquad\qquad = 0.71\,(1237 - 303) = 663.14 \text{ kJ/kg}$

$$\eta_{\text{air-standard}} = \dfrac{Q_s - Q_r}{Q_s} = \dfrac{1562.58 - 663.14}{1562.58} = \mathbf{0.5756 \text{ or } 57.56\%}. \quad \textbf{(Ans.)}$$

(*ii*) **Power developed by the engine, P = ?**

Mass of air in the cycle is given by

$$m = \dfrac{p_1 V_1}{R T_1} = \dfrac{1 \times 10^5 \times 0.0165}{287 \times 303} = 0.0189 \text{ kg}$$

$\therefore$ Work done per cycle $= m(Q_s - Q_r)$
$\qquad\qquad\qquad\qquad\quad = 0.0189\,(1562.58 - 663.14) = 16.999 \text{ kJ}$

Power developed $=$ Work done per cycle $\times$ no. of cycles per second
$\qquad\qquad\qquad\qquad\quad = 16.999 \times 3 = \mathbf{50.99 \text{ say } 51 \text{ kW}}. \quad \textbf{(Ans.)}$

☞**Example 4.26.** *In an engine working on Dual cycle, the temperature and pressure at the beginning of the cycle are 90°C and 1 bar respectively. The compression ratio is 9. The maximum pressure is limited to 68 bar and total heat supplied per kg of air is 1750 kJ. Determine :*

(i) Pressure and temperatures at all salient points

(ii) Air standard efficiency

(iii) Mean effective pressure.

Solution. Refer Fig. 4.22.

Initial pressure, $p_1 = 1 \text{ bar}$
Initial temperature, $T_1 = 90 + 273 = 363 \text{ K}$
Compression ratio, $r = 9$
Maximum pressure, $p_3 = p_4 = 68 \text{ bar}$
Total heat supplied $= 1750 \text{ kJ/kg}$
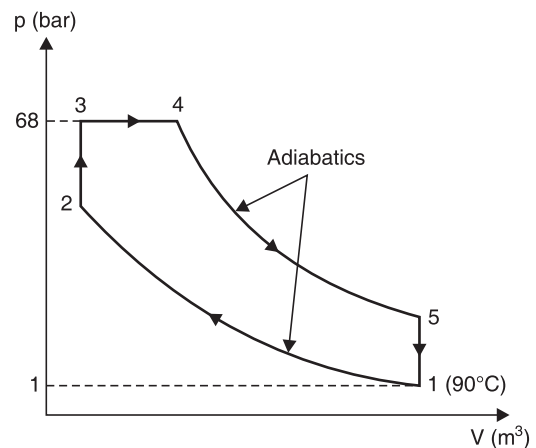
Fig. 4.22

(*i*) **Pressures and temperatures at salient points :**

— For the *isentropic process* 1-2,

$$p_1 V_1{}^\gamma = p_2 V_2{}^\gamma$$

$$p_2 = p_1 \times \left(\frac{V_1}{V_2}\right)^\gamma = 1 \times (r)^\gamma = 1 \times (9)^{1.4}$$

$$= \textbf{21.67 bar.} \quad \textbf{(Ans.)}$$

Also, $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (9)^{1.4-1} = 2.408$$

∴ $$T_2 = T_1 \times 2.408 = 363 \times 2.408 = \textbf{874.1 K.} \quad \textbf{(Ans.)}$$

$$p_3 = p_4 = \textbf{68 bar.} \quad \textbf{(Ans.)}$$

— For the *constant volume process* 2-3,

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

∴ $$T_3 = T_2 \times \frac{p_3}{p_2} = 874.1 \times \frac{68}{21.67} = \textbf{2742.9 K.} \quad \textbf{(Ans.)}$$

Heat added at constant volume

$$= c_v\,(T_3 - T_2) = 0.71\,(2742.9 - 874.1) = 1326.8 \text{ kJ/kg}$$

∴ Heat added at constant pressure

$$= \text{total heat added} - \text{heat added at constant volume}$$

$$= 1750 - 1326.8 = 423.2 \text{ kJ/kg}$$

∴ $$c_p(T_4 - T_3) = 423.2$$

or $$1.0(T_4 - 2742.9) = 423.2$$

∴ $$T_4 = \textbf{3166 K.} \quad \textbf{(Ans.)}$$

For *constant pressure process* 3-4,

$$\rho = \frac{V_4}{V_3} = \frac{T_4}{T_3} = \frac{3166}{2742.9} = 1.15$$

— For *adiabatic (or isentropic) process* 4-5,

$$\frac{V_5}{V_4} = \frac{V_5}{V_2} \times \frac{V_2}{V_4} = \frac{V_1}{V_2} \times \frac{V_3}{V_4} = \frac{r}{\rho} \qquad \left(\because \ \rho = \frac{V_4}{V_3}\right)$$

Also $$p_4 V_4{}^\gamma = p_5 V_5{}^\gamma$$

∴ $$p_5 = p_4 \times \left(\frac{V_4}{V_5}\right)^\gamma = 68 \times \left(\frac{\rho}{r}\right)^\gamma = 68 \times \left(\frac{1.15}{9}\right)^{1.4}$$

$$= \textbf{3.81 bar.} \quad \textbf{(Ans.)}$$

Again, $$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\gamma-1} = \left(\frac{\rho}{r}\right)^{\gamma-1} = \left(\frac{1.15}{9}\right)^{1.4-1} = 0.439$$

∴ $$T_5 = T_4 \times 0.439 = 3166 \times 0.439 = \textbf{1389.8 K.} \quad \textbf{(Ans.)}$$

(*ii*) **Air standard efficiency :**

Heat rejected during constant volume process 5-1,

$$Q_r = C_v(T_5 - T_1) = 0.71(1389.8 - 363)$$

$$= 729 \text{ kJ/kg}$$

$$\therefore \qquad \eta_{\text{air-standard}} = \frac{\text{work done}}{\text{heat supplied}} = \frac{Q_s - Q_r}{Q_s}$$

$$= \frac{1750 - 729}{1750} = 0.5834 \text{ or } 58.34\%. \quad \textbf{(Ans.)}$$

(*iii*) **Mean effective pressure, p_m :**

Mean effective pressure is given by

$$p_m = \frac{\text{work done per cycle}}{\text{stroke volume}}$$

or

$$p_m = \frac{1}{V_s}\left[p_3(V_4 - V_3) + \frac{p_4 V_4 - p_5 V_5}{\gamma - 1} - \frac{p_2 V_2 - p_1 V_1}{\gamma - 1} \right]$$

$$V_1 = V_5 = r\, V_c,\ V_2 = V_3 = V_c,\ V_4 = \rho V_c,$$

$$V_s = (r - 1)\, V_c \qquad\qquad \left[\begin{array}{l} \because\quad r = \dfrac{V_s + V_c}{V_c} = 1 + \dfrac{V_s}{V_c} \\[2mm] \therefore\quad V_s = (\gamma - 1)\, V_c \end{array} \right]$$

$$\therefore \qquad p_m = \frac{1}{(r-1)V_c}\left[p_3(\rho V_c - V_c) + \frac{p_4 \rho V_c - p_5 \times r V_c}{\gamma - 1} - \frac{p_2 V_c - p_1 r V_c}{\gamma - 1} \right]$$

$$r = 9,\ \rho = 1.15,\ \gamma = 1.4$$

$$p_1 = 1 \text{ bar},\ p_2 = 21.67 \text{ bar},\ p_3 = p_4 = 68 \text{ bar},\ p_5 = 3.81 \text{ bar}$$

Substituting the above values in the above equation, we get

$$p_m = \frac{1}{(9-1)}\left[68(1.15 - 1) + \frac{68 \times 1.15 \times 3.81 \times 9}{1.4 - 1} - \frac{21.67 - 9}{1.4 - 1} \right]$$

$$= \tfrac{1}{8}\ (10.2 + 109.77 - 31.67) = 11.04 \text{ bar}$$

Hence, *mean effective pressure* = **11.04 bar. (Ans.)**

Example 4.27. *An I.C. engine operating on the dual cycle (limited pressure cycle) the temperature of the working fluid (air) at the beginning of compression is 27°C. The ratio of the maximum and minimum pressures of the cycle is 70 and compression ratio is 15. The amounts of heat added at constant volume and at constant pressure are equal. Compute the air standard thermal efficiency of the cycle. State three main reasons why the actual thermal efficiency is different from the theoretical value. Take γ for air = 1.4.*

Solution. Refer to Fig. 4.23.

Given : $T_1 = 27 + 273 = 300$ K ; $\dfrac{p_3}{p_1} = 70,\ \dfrac{v_1}{v_2} = \dfrac{v_1}{v_3} = 15$

Air standard efficiency, $\eta_{\text{air-standard}}$:

Consider 1 kg of air :

Adiabatic compression process 1-2 :

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (15)^{1.4-1} = 2.954$$

$$\therefore \qquad T_2 = 300 \times 2.954 = 886.2 \text{ K}$$

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = (15)^{1.4} \quad \Rightarrow \quad p_2 = 44.3 p_1$$

Constant pressure process 2-3 :

$$\frac{p_2}{T_2} = \frac{p_3}{T_3} \qquad \text{or} \qquad T_3 = T_2 \times \frac{p_3}{p_2}$$

$$= 886.2 \times \frac{70 \, p_1}{44.3 \, p_1} = 1400 \text{ K}$$

Also, heat added at constant volume = heat added at constant pressure   ...(Given)

or $c_v(T_3 - T_2) = c_p(T_4 - T_3)$

or $T_3 - T_2 = \gamma(T_4 - T_3)$

or $T_4 = T_3 + \dfrac{T_3 - T_2}{\gamma}$

$$= 1400 + \frac{1400 - 886.2}{1.4} = 1767 \text{ K}.$$

Constant volume process 3-4 :

$$\frac{v_3}{T_3} = \frac{v_4}{T_4} \quad \Rightarrow \quad \frac{v_4}{v_3} = \frac{T_4}{T_3} = \frac{1767}{1400} = 1.26$$

Also, $\dfrac{v_4}{v_3} = \dfrac{v_4}{(v_1/15)} = 1.26 \quad$ or $\quad v_4 = 0.084 v_1$

Also, $v_5 = v_1$

Fig. 4.23. Dual cycle.

Adiabatic expansion process 4-5 :

$$\frac{T_4}{T_5} = \left(\frac{v_5}{v_4}\right)^{\gamma-1} = \left(\frac{v_1}{0.084 \, v_1}\right)^{1.4-1} = 2.69$$

∴ $T_5 = \dfrac{T_4}{2.69} = \dfrac{1767}{2.69} = 656.9$ K.

∴ $\eta_{\text{air-standard}} = \dfrac{\text{Work done}}{\text{Heat supplied}} = \dfrac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}}$

$$= 1 - \frac{\text{Heat rejected}}{\text{Heat supplied}}$$

$$= 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

$$= 1 - \frac{(656.9 - 300)}{(1400 - 886.2) + 1.4(1767 - 1400)} = \mathbf{0.653} \text{ or } \mathbf{65.3\%. \ (Ans.)}$$

Reasons for actual thermal efficiency being different from the theoretical value :

1. In theoretical cycle working substance is taken *air* whereas in actual cycle *air with fuel acts as working substance*.

2. The fuel combustion phenomenon and associated problems like dissociation of gases, dilution of charge during suction stroke, etc. have *not* been taken into account.

3. Effect of variable specific heat, heat loss through cylinder walls, inlet and exhaust velocities of air/gas etc. have *not* been taken into account.

Example 4.28. *A Diesel engine working on a dual combustion cycle has a stroke volume of 0.0085 m³ and a compression ratio 15 : 1. The fuel has a calorific value of 43890 kJ/kg. At the end of suction, the air is at 1 bar and 100°C. The maximum pressure in the cycle is 65 bar and air fuel ratio is 21 : 1. Find for ideal cycle the thermal efficiency. Assume $c_p = 1.0$ and $c_v = 0.71$.*
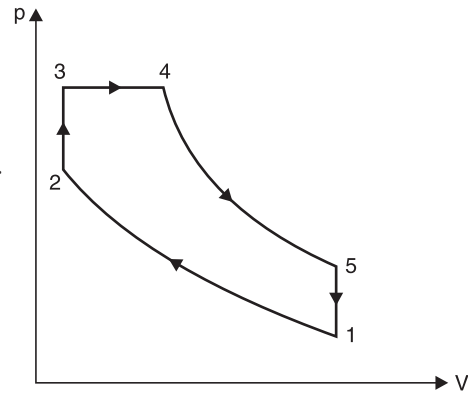
Solution. Refer to Fig. 4.24.

Initial temperature,	T_1 = 100 + 273 = 373 K	
Initial pressure,	p_1 = 1 bar	
Maximum pressure in the cycle,	$p_3 = p_4$ = 65 bar	
Stroke volume,	V_s = 0.0085 m³	
Air-fuel ratio	= 21 : 1	
Compression ratio,	r = 15 : 1	
Calorific value of fuel,	C = 43890 kJ/kg	
	c_p = 1.0, c_v = 0.71	
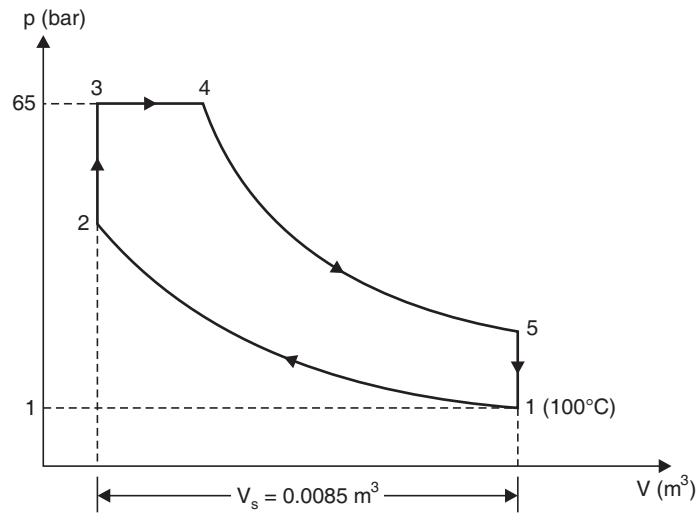
Fig. 4.24

Thermal efficiency :

$$V_s = V_1 - V_2 = 0.0085$$

and as
$$r = \frac{V_1}{V_2} = 15, \text{ then } V_1 = 15V_2$$

∴
$$15V_2 - V_2 = 0.0085$$

or
$$14V_2 = 0.0085$$

or
$$V_2 = V_3 = V_c = \frac{0.0085}{14} = 0.0006 \text{ m}^3$$

or
$$V_1 = 15V_2 = 15 \times 0.0006 = 0.009 \text{ m}^3$$

— For adiabatic compression process 1-2,

$$p_1 V_1^\gamma = p_2 V_2^\gamma$$

or
$$p_2 = p_1 \cdot \left(\frac{V_1}{V_2}\right)^\gamma = 1 \times (15)^{1.41} \qquad \left[\gamma = \frac{c_p}{c_v} = \frac{1.0}{0.71} = 1.41\right]$$

$$= 45.5 \text{ bar}$$

Also,
$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (15)^{1.41-1} = 3.04$$

$\therefore$ $T_2 = T_1 \times 3.04 = 373 \times 3.04 = 1134$ K or 861°C

— For constant volume process 2-3,

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

or
$$T_3 = T_2 \times \frac{p_3}{p_2} = 1134 \times \frac{65}{45.5} = 1620 \text{ K or } 1347°C$$

According to characteristic equation of gas,

$$p_1 V_1 = m R T_1$$

$\therefore$
$$m = \frac{p_1 V_1}{R T_1} = \frac{1 \times 10^5 \times 0.009}{287 \times 373} = 0.0084 \text{ kg (air)}$$

Heat added during constant volume process 2-3,

$$= m \times c_v \, (T_3 - T_2)$$
$$= 0.0084 \times 0.71 \, (1620 - 1134)$$
$$= 2.898 \text{ kJ}$$

Amount of fuel added during the constant volume process 2-3,

$$= \frac{2.898}{43890} = 0.000066 \text{ kg}$$

Also as air-fuel ratio is 21 : 1.

$\therefore$ Total amount of fuel added $= \dfrac{0.0084}{21} = 0.0004$ kg

Quantity of fuel added during the process 3-4,

$$= 0.0004 - 0.000066 = 0.000334 \text{ kg}$$

$\therefore$ *Heat added during the constant pressure operation 3-4*

$$= 0.000334 \times 43890 = 14.66 \text{ kJ}$$

But $(0.0084 + 0.0004) \, c_p \, (T_4 - T_3) = 14.66$

or
$$0.0088 \times 1.0 \, (T_4 - 1620) = 14.66$$

$\therefore$
$$T_4 = \frac{14.66}{0.0088} + 1620 = 3286 \text{ K or } 3013°C$$

Again for operation 3-4,

$$\frac{V_3}{T_3} = \frac{V_4}{T_4}$$

or
$$V_4 = \frac{V_3 T_4}{T_3} = \frac{0.0006 \times 3286}{1620} = 0.001217 \text{ m}^3$$

For adiabatic expansion operation 4-5,

$$\frac{T_4}{T_5} = \left(\frac{V_5}{V_4} \right)^{\gamma - 1} = \left(\frac{0.009}{0.001217} \right)^{1.41 - 1} = 2.27$$

or
$$T_5 = \frac{T_4}{2.27} = \frac{3286}{2.27} = 1447.5 \text{ K or } 1174.5°C$$

Heat rejected during constant volume process 5-1,

$$= m \, c_v \, (T_5 - T_1)$$
$$= (0.00854 + 0.0004) \times 0.71 \, (1447.5 - 373) = 6.713 \text{ kJ}$$

Work done = heat supplied – heat rejected

$$= (2.898 + 14.66) - 6.713 = 10.845 \text{ kJ}$$

∴   *Thermal efficiency,*

$$\eta_{th} = \frac{\text{work done}}{\text{heat supplied}} = \frac{10.845}{(2.898 + 14.66)}$$

= 0.6176 or 61.76%.   (Ans.)

Example 4.29. *The compression ratio and expansion ratio of an oil engine working on the dual cycle are 9 and 5 respectively. The initial pressure and temperature of the air are 1 bar and 30°C. The heat liberated at constant pressure is twice the heat liberated at constant volume. The expansion and compression follow the law $pV^{1.25}$ = constant. Determine :*

 (*i*) *Pressures and temperatures at all salient points.*

(*ii*) *Mean effective pressure of the cycle.*

(*iii*) *Efficiency of the cycle.*

 (*iv*) *Power of the engine if working cycles per second are 8.*

 Assume : Cylinder bore = 250 mm and stroke length = 400 mm.

 Solution. Refer Fig. 4.25.
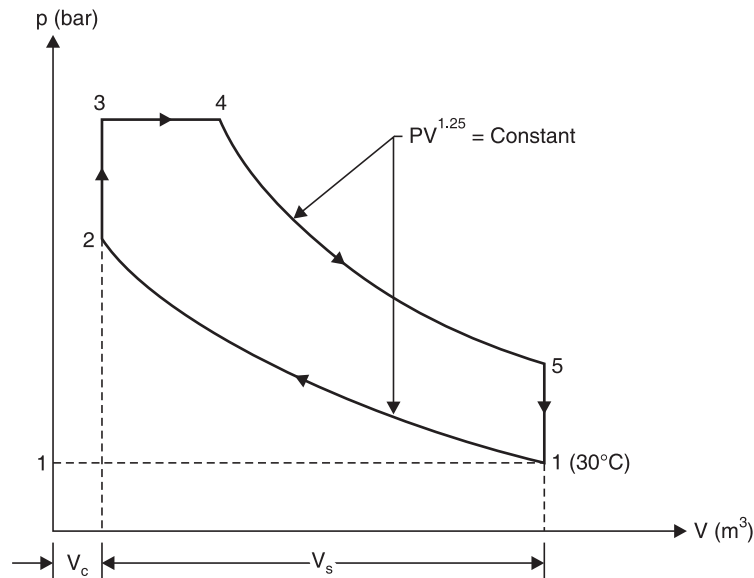
Fig. 4.25

Initial temperature, T_1 = 30 + 273 = 303 K

Initial pressure, p_1 = 1 bar

Compression and expansion law,

$$pV^{1.25} = \text{constant}$$

Compression ratio, r_c = 9

Expansion ratio, r_e = 5

Number of cycles/sec   = 8

Cylinder diameter, D = 250 mm = 0.25 m

Stroke length, L = 400 mm = 0.4 m

Heat liberated at constant pressure

$$= 2 \times \text{heat liberated at constant volume}$$

(*i*) **Pressure and temperatures at all salient points :**

— For compression process 1-2,

$$p_1 V_1{}^n = p_2 V_2{}^n$$

$\therefore$ $$p_2 = p_1 \times \left(\frac{V_1}{V_2}\right)^n = 1 \times (9)^{1.25} = \textbf{15.59 bar.} \quad \textbf{(Ans.)}$$

A1so,
$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = (9)^{1.25 - 1} = 1.732$$

$\therefore$
$$T_2 = T_1 \times 1.732 = 303 \times 1.732$$
$$= \textbf{524.8 K or 251.8°C.} \quad \textbf{(Ans.)}$$

Also, $\qquad c_p(T_4 - T_3) = 2 \times c_v(T_3 - T_2)$ (given) $\hspace{2cm}$ ...(*i*)

For constant pressure process 3-4,

$$\frac{T_4}{T_3} = \frac{V_4}{V_3} = \rho = \frac{\text{compression ratio } (r_c)}{\text{expansion ratio } (r_e)}$$

$$= \frac{9}{5} = 1.8$$

$$T_4 = 1.8 \, T_3$$

$$\left[\begin{array}{l} \dfrac{V_5}{V_4}(r_e) = \dfrac{V_5}{V_3} \times \dfrac{V_3}{V_4} \\[2mm] \qquad = \dfrac{V_1}{V_3} \times \dfrac{1}{\rho} \\[2mm] \qquad = \dfrac{V_1}{V_2} \times \dfrac{1}{\rho} = \dfrac{r_c}{\rho} \\[2mm] \therefore \quad \rho = \dfrac{r_c}{\dfrac{V_5}{V_4}} = \dfrac{r_c}{r_e} \end{array}\right]$$

Substituting the values of T_2 and T_4 in the equation (*i*), we get

$$1.0(1.8T_3 - T_3) = 2 \times 0.71(T_3 - 524.8)$$
$$0.8T_3 = 1.42(T_3 - 524.8)$$
$$0.8T_3 = 1.42T_3 - 745.2$$

$\therefore$
$$0.62T_3 = 745.2$$
$$T_3 = \textbf{1201.9 K or 928.9°C.} \quad \textbf{(Ans.)}$$

Also,
$$\frac{p_3}{T_3} = \frac{p_2}{T_2} \qquad \text{...... for process 3-2}$$

$\therefore$
$$p_3 = p_2 \times \frac{T_3}{T_2} = 15.59 \times \frac{1201.9}{524.8} = \textbf{35.7 bar.} \quad \textbf{(Ans.)}$$

$$p_4 = p_3 = \textbf{35.7 bar.} \quad \textbf{(Ans.)}$$
$$T_4 = 1.8T_3 = 1.8 \times 1201.9 = \textbf{2163.4 K or 1890.4°C.} \quad \textbf{(Ans.)}$$

— For expansion process 4-5,

$$p_4 V_4{}^n = p_5 V_5{}^n$$

$$p_5 = p_4 \times \left(\frac{V_4}{V_5}\right)^n = p_4 \times \frac{1}{(r_e)^n} = \frac{35.7}{(5)^{1.25}} = \textbf{4.77 bar.} \quad \textbf{(Ans.)}$$

Also
$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{n-1} = \frac{1}{(r_e)^{n-1}} = \frac{1}{(5)^{1.25-1}} = 0.668$$

$\therefore$
$$T_5 = T_4 \times 0.668 = 2163.4 \times 0.668 = \textbf{1445 K or 1172°C.} \quad \textbf{(Ans.)}$$

(*ii*) **Mean effective pressure, p_m :**

Mean effective pressure is given by

$$p_m = \frac{1}{V_s}\left[p_3(V_4 - V_3) + \frac{p_4V_4 - p_5V_5}{n - 1} - \frac{p_2V_2 - p_1V_1}{n - 1} \right]$$

$$= \frac{1}{(r_c - 1)}\left[p_3(\rho - 1) + \frac{p_4\rho - p_5r_c}{n - 1} - \frac{p_2 - p_1r_c}{n - 1} \right]$$

Now, $r_c = \rho$, $\rho = 1.8$, $n = 1.25$, $p_1 = 1$ bar, $p_2 = 15.59$ bar, $p_3 = 35.7$ bar, $p_4 = 35.7$ bar, $p_5 = 4.77$ bar

∴ $$p_m = \frac{1}{(9 - 1)}\left[35.7\,(1.8 - 1) + \frac{35.7 \times 1.8 - 4.77 \times 9}{1.25 - 1} - \frac{15.59 - 1 \times 9}{1.25 - 1} \right]$$

$$= \frac{1}{8}\,[28.56 + 85.32 - 26.36] = 10.94 \text{ bar}$$

Hence, *mean effective pressure* = **10.94 bar.**   **(Ans.)**

(*iii*) **Efficiency of the cycle :**

Work done per cycle is given by

$$W = p_m V_s$$

here $V_s = \pi/4 D^2 L = \pi/4 \times 0.25^2 \times 0.4 = 0.0196 \text{ m}^3$

∴ $$W = \frac{10.94 \times 10^5 \times 0.0196}{1000} = 21.44 \text{ kJ/cycle}$$

Heat supplied per cycle $= mQ_s$

where m is the mass of air per cycle which is given by

$$m = \frac{p_1V_1}{RT_1}$$

where $V_1 = V_s + V_c = \dfrac{r_c}{r_c - 1}\,V_s$

$$\left[\begin{array}{l} \qquad r_c = \dfrac{V_s + V_c}{V_c} = 1 + \dfrac{V_s}{V_c} \\[2mm] \text{or} \quad V_c = \dfrac{V_s}{r_c - 1} \\[2mm] \therefore \quad V_1 = V_s + \dfrac{V_s}{r_c - 1} = V_s\left(1 + \dfrac{1}{r_c - 1}\right) = \dfrac{r_c}{r_c - 1}\,V_s \end{array} \right]$$

$$= \frac{9}{9 - 1} \times 0.0196 = 0.02205 \text{ m}^3$$

∴ $$m = \frac{1 \times 10^5 \times 0.02205}{287 \times 303} = 0.02535 \text{ kg/cycle}$$

∴   Heat supplied per cycle

$$= mQ_s = 0.02535[c_v(T_3 - T_2) + c_p(T_4 - T_3)]$$
$$= 0.02535[0.71(1201.9 - 524.8) + 1.0(2163.4 - 1201.9)]$$
$$= 36.56 \text{ kJ/cycle}$$

$$Efficiency = \frac{\text{work done per cycle}}{\text{heat supplied per cycle}} = \frac{21.44}{36.56} = \textbf{0.5864 or 58.64\%.} \quad \textbf{(Ans.)}$$

(*iv*) **Power of the engine, P = ?**

Power of the engine,   P = work done per second

= work done per cycle × no. of cycles/sec.

= 21.44 × 8 = **171.52 kW.**   **(Ans.)**

4.7. COMPARISON OF OTTO, DIESEL AND DUAL COMBUSTION CYCLES

Following are the important variable factors which are used as a basis for comparison of the cycles :

— Compression ratio

— Maximum pressure

— Heat supplied

— Heat rejected

— Net work

Some of the above mentioned variables are fixed when the performance of Otto, Diesel and dual combustion cycles is to be compared.

4.7.1. Efficiency Versus Compression Ratio

Fig. 4.26 shows the comparison for the air standard efficiencies of the Otto, Diesel and Dual combustion cycles at various compression ratios and with given cut off ratio for the Diesel and Dual combustion cycles. It is evident from the Fig. 4.26 that the air standard efficiencies increase with the increase in the compression ratio. *For a given compression ratio otto cycle is the most efficient while the Diesel cycle is the least efficient.* ($\eta_{\text{otto}} > \eta_{\text{dual}} > \eta_{\text{diesel}}$).

Fig. 4.26. Comparison of efficiency at various compression ratio.

Note. The maximum compression ratio for the petrol engine is limited by detonation. In their respective ratio ranges, the Diesel cycle is more efficient than the Otto cycle.

4.7.2. For the Same Compression Ratio and the Same Heat Input

A comparison of the cycles (Otto, Diesel and Dual) on the *p-v* and *T-s* diagrams for the *same compression ratio and heat supplied* is shown in Fig. 4.27.
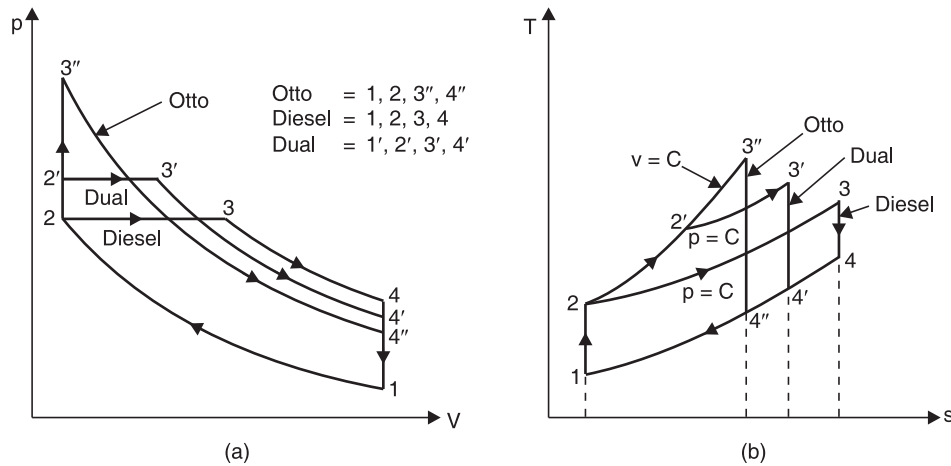
Fig. 4.27. (*a*) *p-V* diagram, (*b*) *T-s* diagram.

We know that,
$$\eta = 1 - \frac{\text{heat rejected}}{\text{heat supplied}} \qquad ...(4.13)$$

Since, all the cycles reject their heat at the same specific volume, process line from state 4 to 1, the quantity of *heat rejected from each cycle is represented by the appropriate area under the line 4 to 1 on the T-s diagram*. As is evident from the eqn. (4.13) the cycle which has the least heat rejected will have the highest efficiency. Thus, Otto cycle is the most efficient and Diesel cycle is the least efficient of the three cycles.

i.e., $\qquad \eta_{\text{otto}} > \eta_{\text{dual}} > \eta_{\text{diesel}}.$

4.7.3. For Constant Maximum Pressure and Heat Supplied

Fig. 4.28 shows the Otto and Diesel cycles on *p-V* and *T-s* diagrams for constant maximum pressure and heat input respectively.
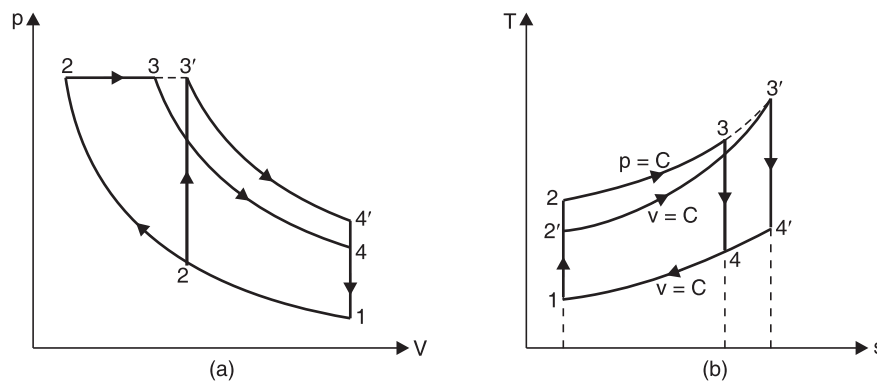
Fig. 4.28. (*a*) *p-V* diagram, (*b*) *T-s* diagram.

— For the maximum pressure the points 3 and 3′ must lie on a constant pressure line.

— On *T-s* diagram the heat rejected from the Diesel cycle is represented by the area under the line 4 to 1 and this area is less than the Otto cycle area under the curve 4′ to 1 ; hence the *Diesel cycle is more efficient than the Otto cycle for the condition of maximum pressure and heat supplied*.

Example 4.30. *With the help of p-V and T-s diagrams compare the cold air standard otto, diesel and dual combustion cycles for same maximum pressure and maximum temperature.*

Solution. Refer to Fig. 4.29 (*a*) and (*b*).

The air-standard Otto, Dual and Diesel cycles are drawn on common *p-V* and *T-s* diagrams for the same maximum pressure and maximum temperature, for the purpose of comparison.

Otto 1-2-3-4-1, Dual 1-2′-3′-3-4-1, Diesel 1-2″-3-4-1 [Fig. 4.29 (*a*)].

Slope of constant volume lines on *T-s* diagram is higher than that of constant pressure lines. [Fig. 4.29 (*b*)].
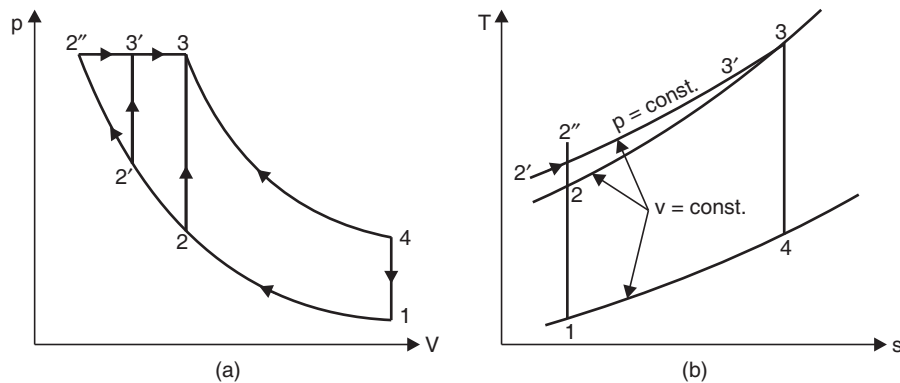
Fig. 4.29

Here the otto cycle must be limited to a low compression ratio (*r*) to fulfil the condition that point 3 (same maximum pressure and temperature) is to be a common state for all the three cycles.

The construction of cycles on *T-s* diagram proves that for the given conditions the heat rejected is same for all the three cycles (area under process line 4-1). Since, by definition,

$$\eta = 1 - \frac{\text{heat rejected}, Q_r}{\text{heat supplied}, Q_s} = 1 - \frac{\text{Const}}{Q_s}$$

the cycle, with greater heat addition will be more efficiency. From the *T-s* diagram.

$$Q_{s(\text{diesel})} = \text{area under 2″-3}$$
$$Q_{s(\text{dual})} = \text{area under 2′-3′-3}$$
$$Q_{s(\text{otto})} = \text{area under 2-3}.$$

It can be seen that, $Q_{s(\text{diesel})} > Q_{s(\text{dual})} > Q_{s(\text{otto})}$

and thus, $\eta_{\text{diesel}} > \eta_{\text{dual}} > \eta_{\text{otto}}$.

4.8. ATKINSON CYCLE

This cycle consists of two adiabatics, a constant volume and a constant pressure process.

p-V diagram of this cycle is shown in Fig. 4.30. It consists of the following four operations :

 (*i*) 1-2—Heat rejection at constant pressure

(*ii*) 2-3—Adiabatic compression

(*iii*) 3-4—Addition of heat at constant volume

(*iv*) 4-1—Adiabatic expansion.

Considering 1 kg of air

Compression ratio $\qquad = \dfrac{V_2}{V_3} = \alpha$

Expansion ratio $\qquad = \dfrac{V_1}{V_4} = r$

Heat supplied at constant volume $= c_v(T_4 - T_3)$

Heat rejected $\qquad = c_v(T_1 - T_2)$

Work done $\qquad =$ heat supplied

$\qquad\qquad$ – heat rejected

$\qquad\qquad = c_v(T_4 - T_3) - c_v(T_1 - T_2)$

$$\eta = \frac{\text{work done}}{\text{heat supplied}} = \frac{c_v(T_4 - T_3) - c_p(T_1 - T_2)}{c_v(T_4 - T_3)}$$

$$= 1 - \gamma \cdot \frac{(T_1 - T_2)}{(T_4 - T_3)} \qquad\qquad\qquad ...(i)$$

During adiabatic compression 2-3

$$\frac{T_3}{T_2} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} = (\alpha)^{\gamma-1}$$

or $\qquad\qquad T_3 = T_2\,(\alpha)^{\gamma-1} \qquad\qquad\qquad ...(ii)$

During constant pressure operation 1-2

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

or $\qquad\qquad \dfrac{T_2}{T_1} = \dfrac{V_2}{V_1} = \dfrac{\alpha}{r} \qquad\qquad\qquad ...(iii)$

$$\left(\frac{V_2}{V_1} = \frac{V_2}{V_3} \times \frac{V_3}{V_1} = \frac{V_2}{V_3} \times \frac{V_4}{V_1} = \frac{\alpha}{r}\right)$$

During adiabatic expansion 4-1

$$\frac{T_4}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} = (r)^{\gamma-1}$$

$$T_1 = \frac{T_4}{(r)^{\gamma-1}} \qquad\qquad\qquad ...(iv)$$

Putting the value of T_1 in equation (*iii*), we get

$$T_2 = \frac{T_4}{(r)^{\gamma-1}} \cdot \frac{\alpha}{r}$$

$$= \frac{\alpha T_4}{r^{\gamma}} \qquad\qquad\qquad ...(v)$$

Substituting the value of T_2 in equation (*ii*), we get

$$T_3 = \frac{\alpha T_4}{r^{\gamma}}\,(\alpha)^{\gamma-1} = \left(\frac{\alpha}{r}\right)^{\gamma} \cdot T_4$$
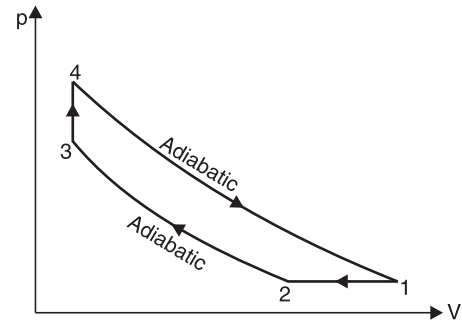
Fig. 4.30

Finally putting the values of T_1, T_2 and T_3 in equation (i), we get

$$\eta = 1 - \gamma \left(\frac{\dfrac{T_4}{r^{\gamma-1}} - \dfrac{\alpha T_4}{(r)^\gamma}}{T_4 - \left(\dfrac{\alpha}{r}\right)^\gamma . T_4} \right) = 1 - \gamma \left(\frac{r - \alpha}{r^\gamma - \alpha^\gamma} \right)$$

Hence, air standard efficiency $= 1 - \gamma . \left(\dfrac{r - \alpha}{r^\gamma - \alpha^\gamma} \right)$ ...(4.14)

Example 4.31. (*Atkinson cycle*). *A perfect gas undergoes a cycle which consists of the following processes taken in order :*

(*a*) *Heat rejection at constant pressure.*

(*b*) *Adiabatic compression from 1 bar and 27°C to 4 bar.*

(*c*) *Heat addition at constant volume to a final pressure of 16 bar.*

(*d*) *Adiabatic expansion to 1 bar.*

Calculate : (*i*) *Work done/kg of gas*

(*ii*) *Efficiency of the cycle.*

Take : $c_p = 0.92$, $c_v = 0.75$.

Fig. 4.31

Solution. Refer to Fig. 4.31.

Pressure, $p_2 = p_1 = 1$ bar

Temperature, $T_2 = 27 + 273 = 300$ K

Pressure after adiabatic compression, $p_3 = 4$ bar

Final pressure after heat addition, $p_4 = 16$ bar

For *adiabatic compression 2-3,*

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \qquad\qquad \left[\gamma = \frac{c_p}{c_v} = \frac{0.92}{0.75} = 1.22\right]$$

$$= \left(\frac{4}{1}\right)^{\frac{1.22-1}{1.22}} = 1.284$$

$\therefore$ $T_3 = T_2 \times 1.284 = 300 \times 1.284 = 385.2$ K or 112.2°C

— For *constant volume process 3-4,*

$$\frac{p_4}{T_4} = \frac{p_3}{T_3}$$

$$T_4 = \frac{p_4 T_3}{p_3} = \frac{16 \times 385.2}{4} = 1540.8 \text{ K} \quad \text{or} \quad 1267.8°C$$

— For *adiabatic expansion process 4-1,*

$$\frac{T_4}{T_1} = \left(\frac{p_4}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{16}{1}\right)^{\frac{1.22-1}{1.22}} = 1.648$$
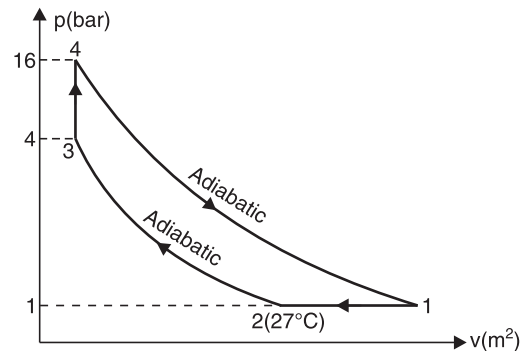
or
$$T_1 = \frac{T_4}{1.648} = \frac{1540.8}{1.648} = 934.9 \text{ K} \quad \text{or} \quad 661.9°C.$$

(i) **Work done per kg of gas, W** = ?

Heat supplied
$$= c_v (T_4 - T_3)$$
$$= 0.75 (1540.8 - 385.2) = 866.7 \text{ kJ/kg}$$

Heat rejected $= c_p (T_1 - T_2) = 0.92(934.9 - 300) = 584.1 \text{ kJ/kg}$

Work done / kg of gas, W = heat supplied – heat rejected
$$= 866.7 - 584.1 = 282.6 \text{ kJ/kg}$$
$$= \textbf{282600 Nm/kg.} \quad \textbf{(Ans.)}$$

(ii) **Efficiency of the cycle** = ?

Efficiency,
$$\eta = \frac{\text{work done}}{\text{heat supplied}}$$
$$= \frac{282.6}{866.7} = \textbf{0.326} \quad \text{or} \quad \textbf{32.6}\%. \quad \textbf{(Ans.)}$$

4.9. ERICSSON CYCLE

It is so named as it was invented by Ericsson. Fig. 4.32 shows *p-V* diagram of this cycle.

It comprises of the following operations :

(i) 1-2—Rejection of heat at constant pressure

(ii) 2-3—Isothermal compression

(iii) 3-4—Addition of heat at constant pressure

(iv) 4-1—Isothermal expansion.

Considering 1 kg of air.

Volume ratio, $\quad r = \dfrac{V_2}{V_3} = \dfrac{V_1}{V_4}$

Fig. 4.32

Heat supplied to air from an external source
$$= \text{heat supplied during the isothermal expansion 4-1}$$
$$= RT_1 \log_e r$$

Heated rejected by air to an external source
$$= RT_2 . \log_e r$$

Work done $= \text{heat supplied} - \text{heat rejected}$
$$= RT_1 . \log_e r - RT_2 . \log_e r = R \log_e r (T_1 - T_2)$$

$$\eta = \frac{\text{work done}}{\text{heat supplied}} = \frac{R \log_e r (T_1 - T_2)}{RT_1 . \log_e r}$$

$$= \frac{T_1 - T_2}{T_1} \qquad\qquad\qquad ...(4.15)$$

which is the same as Carnot cycle.

4.10. GAS TURBINE CYCLE—BRAYTON CYCLE

4.10.1. Ideal Brayton cycle

Brayton cycle is a constant pressure cycle for a perfect gas. It is also called **Joule cycle.** The heat transfers are achieved in reversible constant pressure heat exchangers. An ideal gas
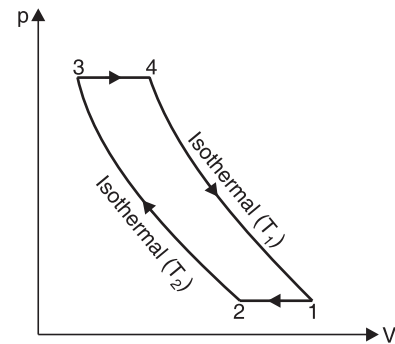
turbine plant would perform the processes that make up a Brayton cycle. The cycle is shown in Fig. 4.33 (*a*) and it is represented on *p-V* and *T-s* diagrams as shown in Fig. 4.33 (*b, c*).

The various operations are as follows :

Operation 1-2 : The air is compressed isentropically from the lower pressure p_1 to the upper pressure p_2, the temperature rising from T_1 to T_2. No heat flow occurs.

Operation 2-3 : Heat flows into the system increasing the volume from V_2 to V_3 and temperature from T_2 to T_3 whilst the pressure remains constant at p_2. Heat received = mc_p $(T_3 - T_2)$.
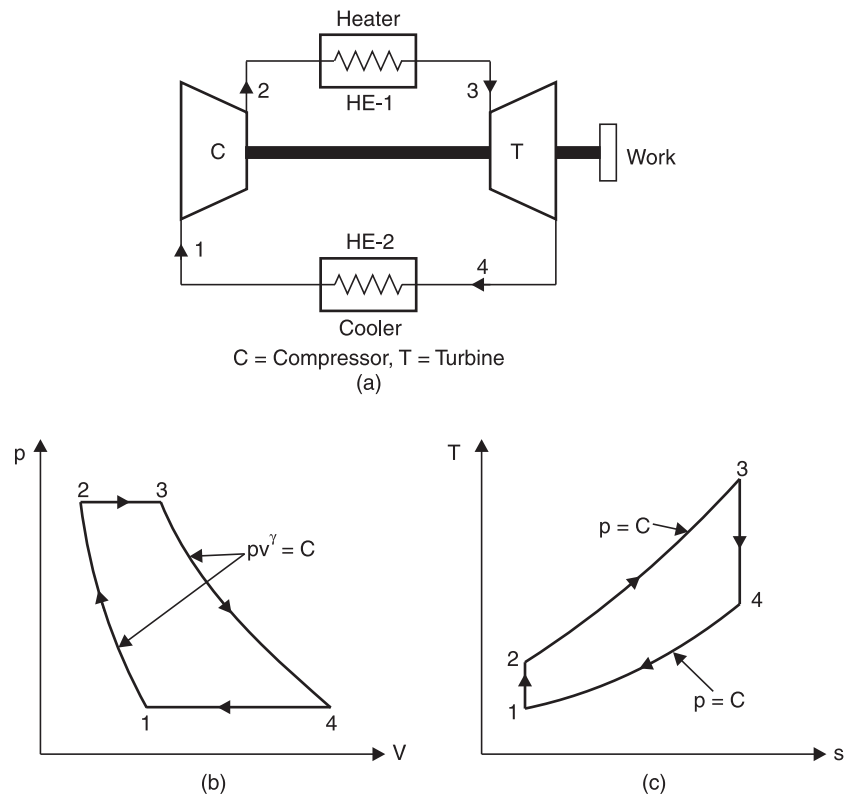
Fig. 4.33. Brayton cycle : (*a*) Basic components of a gas turbine power plant
(*b*) *p-V* diagram (*c*) *T-s* diagram.

Operation 3-4 : The air is expanded isentropically from p_2 to p_1, the temperature falling from T_3 to T_4. No heat flow occurs.

Operation 4-1 : Heat is rejected from the system as the volume decreases from V_4 to V_1 and the temperature from T_4 to T_1 whilst the pressure remains constant at p_1. Heat rejected = mc_p $(T_4 - T_1)$.

$$\eta_{air\text{-}standard} = \frac{\text{work done}}{\text{heat received}}$$

$$= \frac{\text{heat received/cycle} - \text{heat rejected/cycle}}{\text{heat received/cycle}}$$

$$= \frac{mc_p(T_3 - T_2) - mc_p(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Now, from isentropic expansion

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1 \, (r_p)^{\frac{\gamma-1}{\gamma}}, \text{ where } r_p = \text{pressure ratio.}$$

Similarly $\quad \dfrac{T_3}{T_4} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$

or $\qquad\qquad T_3 = T_4 \, (r_p)^{\frac{\gamma-1}{\gamma}}$

$$\therefore \quad \eta_{\text{air-standard}} = 1 - \frac{T_4 - T_1}{T_4(r_p)^{\frac{\gamma-1}{\gamma}} - T_1(r_p)^{\frac{\gamma-1}{\gamma}}}$$

$$= 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} \qquad\qquad \text{...(4.16)}$$

Fig. 4.34. Effect of pressure ratio on the efficiency of Brayton cycle.

The eqn. (4.16) shows that the *efficiency of the ideal joule cycle increases with the pressure ratio. The absolute limit of upper pressure is determined by the limiting temperature of the material of the turbine at the point at which this temperature is reached by the compression process alone, no further heating of the gas in the combustion chamber would be permissible and the work of expansion would ideally just balance the work of compression so that no excess work would be available for external use.*

4.10.2. Pressure Ratio for Maximum Work

Now we shall prove that the *pressure ratio for maximum work is a function of the limiting temperature ratio.*

Work output during the cycle

$$= \text{heat received/cycle} - \text{heat rejected/cycle}$$
$$= mc_p \, (T_3 - T_2) - mc_p \, (T_4 - T_1)$$
$$= mc_p \, (T_3 - T_4) - mc_p \, (T_2 - T_1)$$
$$= mc_p \left[T_3 \left(1 - \frac{T_4}{T_3} \right) - T_1 \left(\frac{T_2}{T_1} - 1 \right) \right]$$

In case of a given turbine the minimum temperature T_1 and the maximum temperature T_3 are prescribed, T_1 being the temperature of the atmosphere and T_3 the maximum temperature which the metals of turbine would withstand. Consider the specific heat at constant pressure c_p to be constant. Then,

Since, $\qquad\qquad \dfrac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} = \dfrac{T_2}{T_1}$

Using the constant $\quad 'z' = \dfrac{\gamma-1}{\gamma}$,

We have, work output/cycle

$$W = K \left[T_3 \left(1 - \frac{1}{r_p{}^z} \right) - T_1(r_p{}^z - 1) \right]$$

Differentiating with respect to r_p

$$\frac{dW}{dr_p} = K\left[T_3 \times \frac{z}{r_p^{(z+1)}} - T_1 z r_p^{(z-1)}\right] = 0 \text{ for a maximum}$$

$\therefore \qquad \dfrac{zT_3}{r_p^{(z+1)}} = T_1 z\, (r_p)^{(z-1)}$

$\therefore \qquad r_p^{2z} = \dfrac{T_3}{T_1}$

$\therefore \qquad r_p = (T_3/T_1)^{1/2z} \quad i.e., \quad r_p = (T_3/T_1)^{\frac{\gamma}{2(\gamma-1)}}$ ...(4.17)

Thus, the *pressure ratio for maximum work is a function of the limiting temperature ratio.*

4.10.3. Work Ratio

Work ratio is defined as the *ratio of net work output to the work done by the turbine.*

$\therefore \qquad$ Work ratio $= \dfrac{W_T - W_C}{W_T}$

$$\left[\begin{array}{l}\text{where } W_T = \text{work obtained from this turbine,} \\ \text{and} \quad W_C = \text{work supplied to the compressor}\end{array}\right]$$

$$= \frac{mc_p(T_3 - T_4) - mc_p(T_2 - T_1)}{mc_p(T_3 - T_4)} = 1 - \frac{T_2 - T_1}{T_3 - T_4}$$

$$= 1 - \frac{T_1}{T_3}\left[\frac{(r_p)^{\frac{\gamma-1}{\gamma}} - 1}{1 - \dfrac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}}\right] = 1 - \frac{T_1}{T_3}(r_p)^{\frac{\gamma-1}{\gamma}}$$ ...(4.18)

4.10.4. Open Cycle Gas Turbine—Actual Brayton Cycle

Refer to Fig. 4.35. The fundamental gas turbine unit is one operating on the open cycle in which a rotary compressor and a turbine are mounted on a common shaft. Air is drawn into the compressor and after compression passes to a combustion chamber. Energy is supplied in the combustion chamber by spraying fuel into the air stream, and the resulting hot gases expand through the turbine to the atmosphere. In order to achieve net work output from the unit, the turbine must develop more gross work output than is required to drive the compressor and to overcome mechanical losses in the drive. The products of combustion coming out from the turbine are exhausted to the atmosphere as they cannot be used any more. The working fluids (air and fuel) must be replaced continuously as they are exhausted into the atmosphere.

If pressure loss in the combustion chamber is neglected, this cycle may be drawn on a *T-s* diagram as shown in Fig. 4.36.

Line 1–2′ represents : *irreversible adiabatic compression.*

Line 2′–3 represents : *constant pressure heat supply in the combustion chamber.*

Line 3–4′ represents : *irreversible adiabatic expansion.*

Line 1–2 represents : *ideal isentropic compression.*

Line 3–4 represents : *ideal isentropic expansion.*
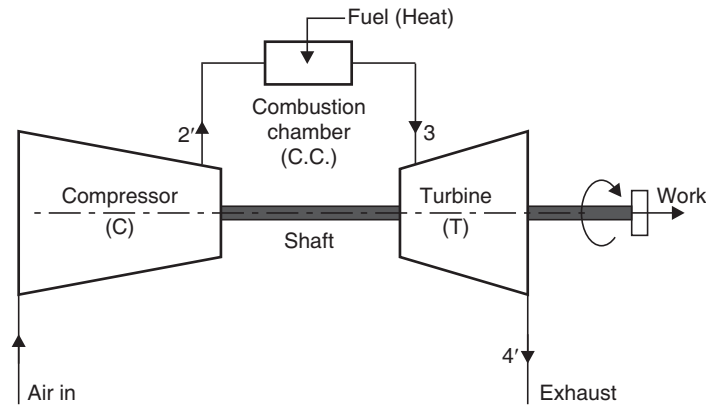
Fig 4.35. Open cycle gas turbine.

Assuming change in kinetic energy between the various points in the cycle to be negligibly small compared with enthalpy changes and then applying the flow equation to each part of cycle, for unit mass, we have

Work input (compressor) = $c_p\,(T_2' - T_1)$

Heat supplied (combustion chamber)

$$= c_p\,(T_3 - T_2')$$

Work output (turbine) $= c_p\,(T_3 - T_4')$

∴   Net work output = work output – work input

$$= c_p\,(T_3 - T_4') - c_p\,(T_2' - T_1)$$

$$\eta_{\text{Thermal}} = \frac{\text{net work output}}{\text{heat supplied}}$$

$$= \frac{c_p(T_3 - T_4') - c_p(T_2' - T_1)}{c_p(T_3 - T_2')}$$
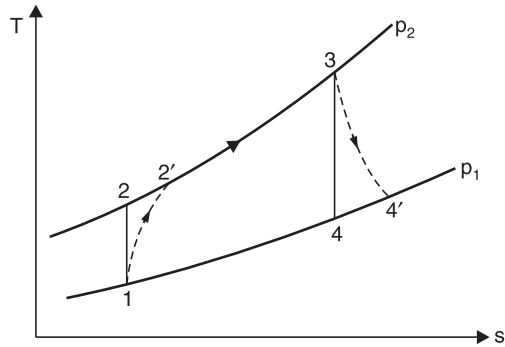
Fig. 4.36

Compressor isentropic efficiency, $\eta_{\text{comp}} = \dfrac{\text{work input required in isentropic compression}}{\text{actual work required}}$

$$= \frac{c_p(T_2 - T_1)}{c_p(T_2' - T_1)} = \frac{T_2 - T_1}{T_2' - T_1} \qquad \qquad ...(4.19)$$

Turbine isentropic efficiency, $\eta_{\text{turbine}} = \dfrac{\text{actual work output}}{\text{isentropic work output}}$

$$= \frac{c_p(T_3 - T_4')}{c_p(T_3 - T_4)} = \frac{T_3 - T_4'}{T_3 - T_4} \qquad \qquad ...(4.20)$$

Note. With the variation in temperature, the value of the specific heat of a real gas varies, and also in the open cycle, the specific heat of the gases in the combustion chamber and in turbine is different from that in the compressor because fuel has been added and a chemical change has taken place. Curves showing the variation of c_p with temperature and air/fuel ratio can be used, and a suitable mean value of c_p and hence γ can be found out. It is usual in gas turbine practice to assume fixed mean value of c_p and γ for the expansion process, and fixed mean values of c_p and γ for the compression process. In an open cycle gas turbine unit the mass flow of gases in turbine is greater than that in compressor due to mass of fuel burned, but it is possible to neglect mass of fuel, since the air/fuel ratios used are large. Also, in many cases, air is bled from the

compressor for cooling purposes, or in the case of air-craft at high altitudes, bled air is used for de-icing and cabin air conditioning. This amount of air bled is approximately the same as the mass of fuel injected therein.

4.10.5. Methods for Improvement of Thermal Efficiency of Open Cycle Gas Turbine Plant

The following methods are employed to increase the specific output and thermal efficiency of the plant :

1. Intercooling   2. Reheating   3. Regeneration.

1. **Intercooling.** A compressor in a gas turbine cycle utilises the major percentage of power developed by the gas turbine. The work required by the compressor can be reduced by compressing the air in two stages and incorporating an intercooler between the two as shown in Fig. 4.37. The corresponding *T-s* diagram for the unit is shown in Fig. 4.38. The actual processes take place as follows :
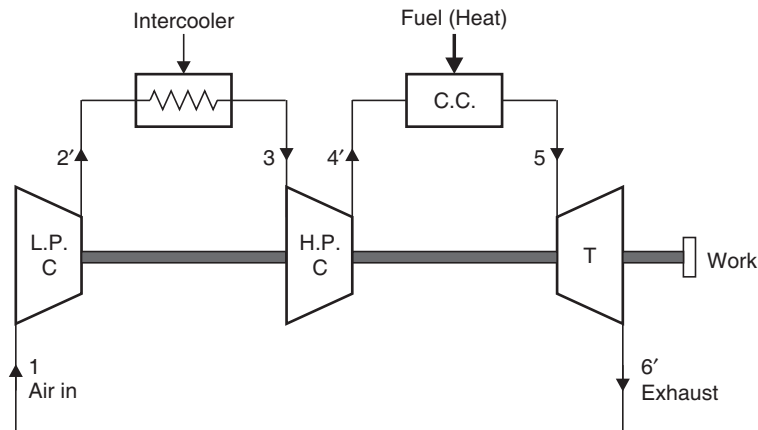
Fig. 4.37. Turbine plant with intercooler.

1–2′   ... *L.P.* (*Low pressure*) *compression*

2′–3   ... *Intercooling*

3–4′   ... H.P. (High pressure) compression

4′–5′   ... C.C. (Combustion chamber)-heating

5–6′   ... *T* (Turbine)-expansion

The ideal cycle for this arrangement is 1-2-3-4-5-6 ; the compression process without intercooling is shown as 1-*L*′ in the actual case, and 1-*L* in the ideal isentropic case.

Now,

Work input (with intercooling)

$$= c_p(T_2{}' - T_1) + c_p(T_4{}' - T_3) \quad ...(4.21)$$

Work input (without intercooling)

$$= c_p(T_L{}' - T_1) = c_p(T_2{}' - T_1) + c_p(T_L{}' - T_2{}') \\ ...(4.22)$$

By comparing equation (4.28) with equation (4.21) it can be observed that the *work input with*
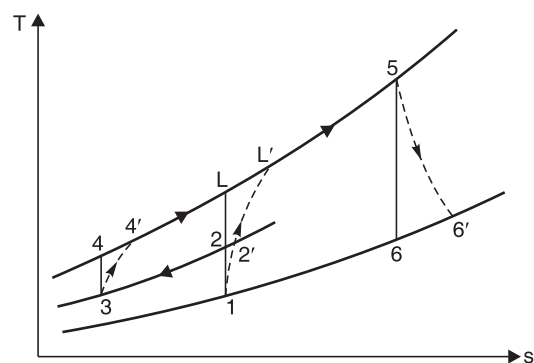
Fig. 4.38. *T-s* diagram for the unit.

intercooling is less than the work input with no intercooling, when c_p $(T_4' - T_3)$ is *less* than $c_p(T_L' - T_2')$. This is so if it is assumed that isentropic efficiencies of the two compressors, operating separately, are each equal to the isentropic efficiency of the single compressor which would be required if no intercooling were used. Then $(T_4' - T_3) < (T_L' - T_2')$ since the pressure lines diverge on the *T-s* diagram from left to the right.

Again, work ratio

$$= \frac{\text{net work output}}{\text{gross work output}}$$

$$= \frac{\text{work of expansion} - \text{work of compression}}{\text{work of expansion}}$$

From this we may conclude that *when the compressor work input is reduced then the work ratio is increased*.

However the heat supplied in the combustion chamber when intercooling is used in the cycle, is given by,

Heat supplied *with intercooling* $= c_p(T_5 - T_4')$

Also the heat supplied when intercooling is not used, with the same maximum cycle temperature T_5, is given by

Heat supplied *without intercooling* $= c_p$ $(T_5 - T_L')$

Thus, the *heat supplied when intercooling is used is greater than with no intercooling. Although the net work output is increased by intercooling it is found in general that the increase in heat to be supplied causes the thermal efficiency to decrease.* When intercooling is used a supply of cooling water must be readily available. The additional bulk of the unit may offset the advantage to be gained by increasing the work ratio.

2. **Reheating.** The output of a gas turbine can be amply improved by expanding the gases in two stages with a reheater between the two as shown in Fig. 4.39. The H.P. turbine drives the compressor and the L.P. turbine provides the useful power output. The corresponding *T-s* diagram is shown in Fig. 4.40. The line 4'-L' represents the expansion in the L.P. turbine if reheating is not employed.

Neglecting mechanical losses the work output of the H.P. turbine must be exactly equal to the work input required for the compressor

i.e., c_{pa} $(T_2' - T_1) = c_{pg}$ $(T_3 - T_4')$

The work output (net output) of L.P. turbine is given by,

Net work output (with reheating) $= c_{pg}$ $(T_5 - L_6')$

and   Net work output (without reheating)

$$= c_{pg}\ (T_4' - T_L')$$
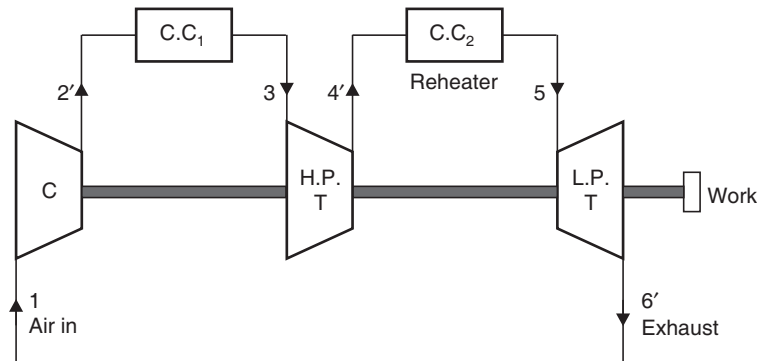
Fig. 4.39. Gas turbine with reheater.
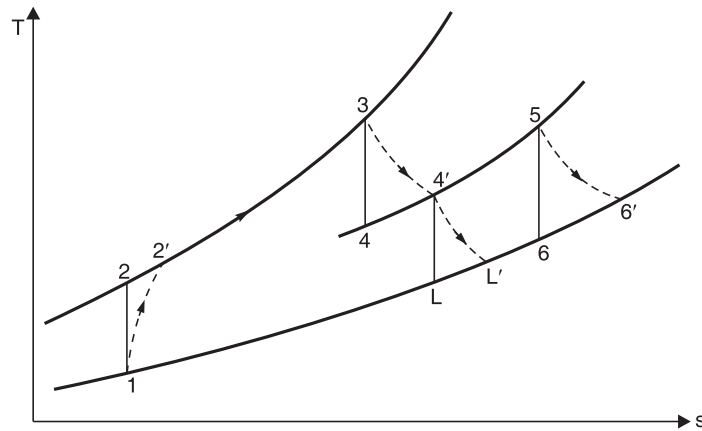
Fig. 4.40. *T-s* diagram for the unit.

Since the pressure lines diverge to the right on *T-s* diagram it can be seen that the temperature difference $(T_5 - T_6{}')$ is always *greater* than $(T_4{}' - T_L{}')$, so that *reheating increases the net work output.*

Although net work is increased by reheating the heat to be supplied is also increased, and the net effect *can be to reduce the thermal efficiency*

Heat supplied $= c_{pg}\,(T_3 - T_2{}') + c_{pg}\,(T_5 - T_4{}').$

Note. c_{pa} and c_{pg} stand for specific heats of air and gas respectively at constant pressure.

3. **Regeneration.** The exhaust gases from a gas turbine carry a large quantity of heat with them since their temperature is far above the ambient temperature. They can be used to heat the air coming from the compressor thereby reducing the mass of fuel supplied in the combustion chamber. Fig. 4.41 shows a gas turbine plant with a regenerator. The corresponding *T-s* diagram is shown in Fig. 4.42. 2′-3 represents the heat flow into the compressed air during its passage through the heat exchanger and 3-4 represents the heat taken in from the combustion of fuel. Point 6 represents the temperature of exhaust gases at discharge from the heat exchanger. The maximum
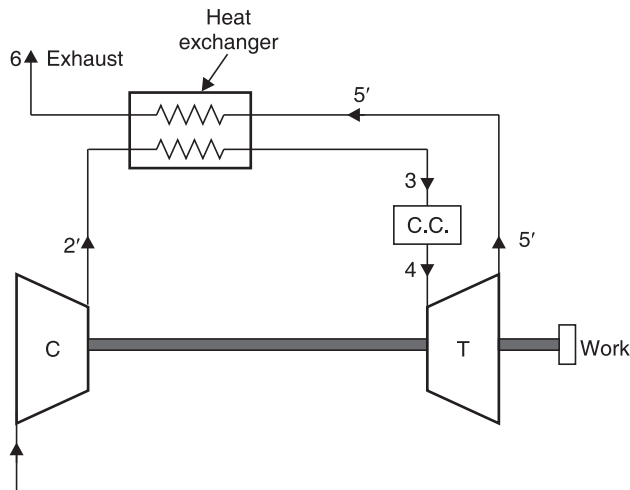
Fig. 4.41. Gas turbine with regenerator.
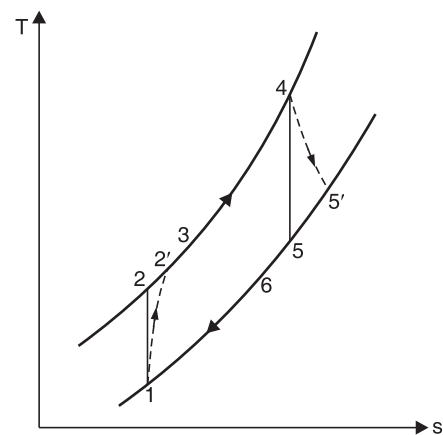
Fig. 4.42. *T-s* diagram for the unit.

temperature to which the air could be heated in the heat exchanger is ideally that of exhaust gases, but less than this is obtained in practice because a temperature gradient must exist for an unassisted transfer of energy. The effectiveness of the heat exchanger is given by :

Effectiveness,

$$\varepsilon = \frac{\text{increase in enthalpy per kg of air}}{\text{available increase in enthalpy per kg of air}}$$

$$= \frac{(T_3 - T_2{}')}{(T_5{}' - T_2{}')} \qquad \qquad ...(4.23)$$

(assuming c_{pa} and c_{pg} to be equal)

A heat exchanger is usually used in large gas turbine units for marine propulsion or industrial power.

4.10.6. Effect of Operating Variables on Thermal Efficiency

The thermal efficiency of *actual open cycle* depends on the following thermodynamic variables :

 (*i*) Pressure ratio

(*ii*) Turbine inlet temperature (T_3)

(*iii*) Compressor inlet temperature (T_1)

(*iv*) Efficiency of the turbine (η_{turbine})

(*v*) Efficiency of the compressor (η_{comp}).

Effect of Turbine Inlet-Temperature and Pressure ratio. If the permissible turbine inlet-temperature (with the other variables being constant) of an *open cycle gas turbine power plant* is increased its *thermal efficiency* is *amply improved*. A practical limitation to increasing the turbine inlet temperature, however, is the ability of the material available for the turbine blading to *with- stand the high rotative and thermal stresses.*

Refer Fig. 4.43. For *a given turbine inlet-temperature, as the pressure ratio increases, the heat supplied as well as the heat rejected are reduced. But the ratio of change of heat supplied is not the same as the ratio of change heat rejected.* As a consequence, there *exists an optimum pressure ratio producing maximum thermal efficiency for a given turbine inlet temperature.*

As the pressure ratio increases, the thermal efficiency also increases until it becomes maximum and then it drops off with a further increase in pressure ratio (Fig. 4.44). Further, as the turbine inlet-temperature increases, the peaks of the curves flatten out giving a greater range of ratios of pressure optimum efficiency.

Following particulars are worthnoting :

Gas temperatures	Efficiency (gas turbine)
550 to 600°C	20 to 22%
900 to 1000°C	32 to 35%
Above 1300°C	more than 50%

Effect of turbine and compressor efficiencies :

Refer Fig. 4.45. The thermal efficiency of the actual gas turbine cycle is very sensitive to variations in the efficiencies of the compressor and turbine. There is a particular pressure ratio at which maximum efficiencies occur. For lower efficiencies, the peak of the thermal efficiency occurs at lower pressure ratios and *vice-versa*.
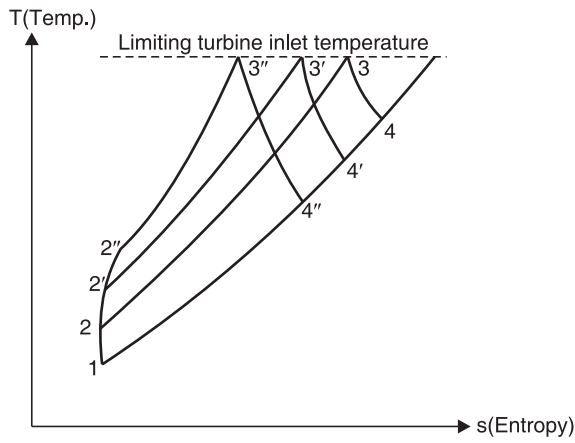
Fig. 4.43
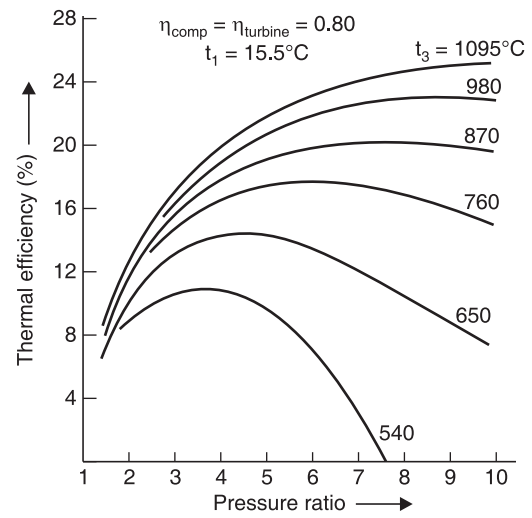
Fig. 4.44. Effect of pressure ratio and turbine inlet-temperature.
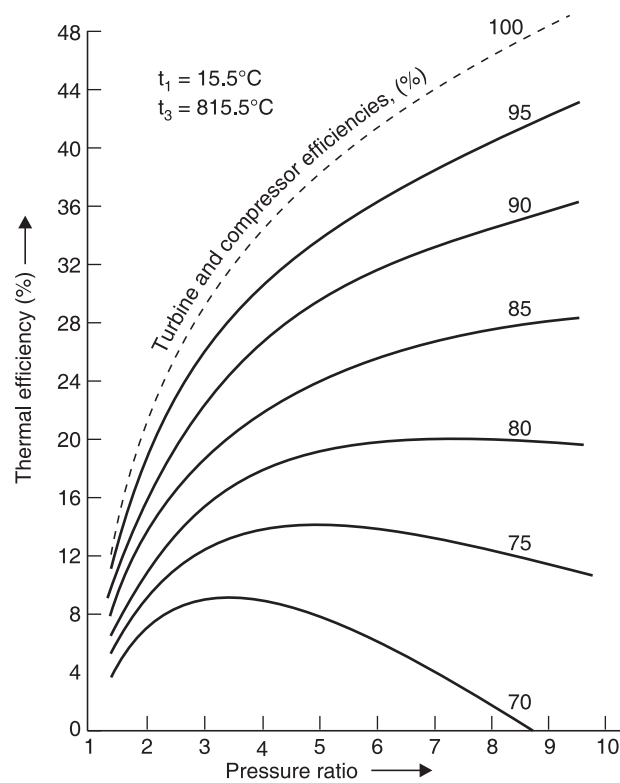
Fig. 4.45. Effect of component efficiency.
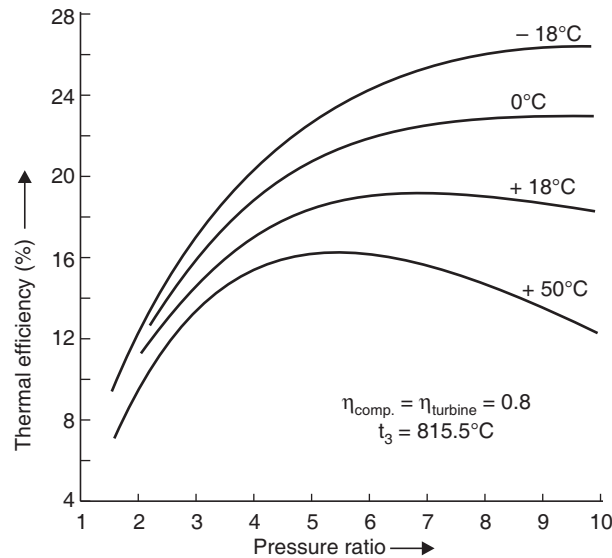
Fig. 4.46. Effect of compressor inlet temperature.

Effect of compressor Inlet-Temperature :

Refer Fig. 4.46. *With the decrease in the compressor inlet-temperature there is increase in thermal efficiency of the plant.* Also the peaks of thermal efficiency occurs at high pressure ratios and the curves become flatter giving thermal efficiency over a wider pressure ratio range.

4.10.7. Closed Cycle Gas Turbine

Fig. 4.47 shows an arrangement of closed cycle stationary gas turbine plant in which air is continuously circulated. This ensures that the air is not polluted by the addition of combustion waste product, since the heating of air is carried out in the form of heat exchanger shown in the diagram as air heater. The air exhausted from the power turbine is cooled before readmission to L.P. compressor. The various operations as indicated on T-s diagram (Fig. 4.48) are as follows :

Operation 1–2′ :   Air is compressed from p_1 to p_x in the L.P. compressor.

Operation 2′–3 :   Air is cooled in the intercoolar at constant pressure p_x.

Operation 3–4′ :   Air is compressed in the H.P. compressor from p_x to p_2.

Operation 4′–5 :   High pressure air is heated at constant pressure by exhaust gases from power turbine in the heat exchanger to T_5.

Operation 5–6 :   High pressure is air further heated at constant pressure to the maximum temperature T_6 by an air heater (through external combustion).

Operation 6–7′ :   The air is expanded in the H.P. turbine from p_2 to p_x producing work to drive the compressor.

Operation 7′–8 :   Exhaust air from the H.P. turbine is heated at constant pressure in the air heater (through external combustion) to the maximum temperature T_8 ($= T_6$).

Operation 8–9′ :   The air is expanded in the L.P. turbine from p_x to p_1, producing energy for a flow of work externally.

Operation 9′–10 :   Air from L.P. turbine is passed to the heat exchanger where energy is transferred to the air delivered from the H.P. compressor. The temperature of air leaving the heat exchanger and entering the cooler is T_{10}.

Operation 10–11 :   Air cooled to T_1 by the cooler before entering the L.P. compressor.
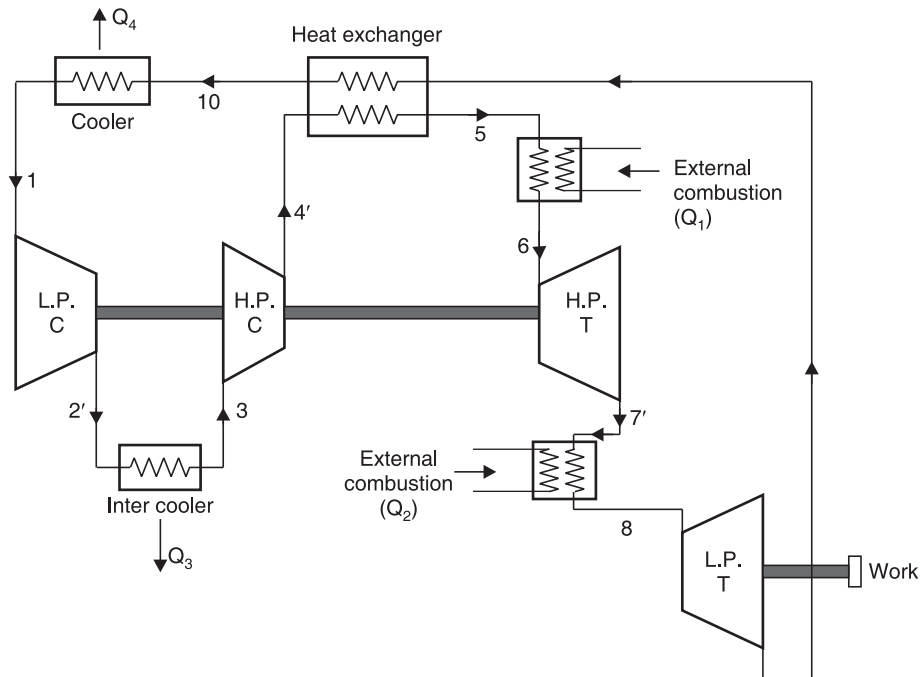
Fig. 4.47. Closed cycle gas turbine plant.
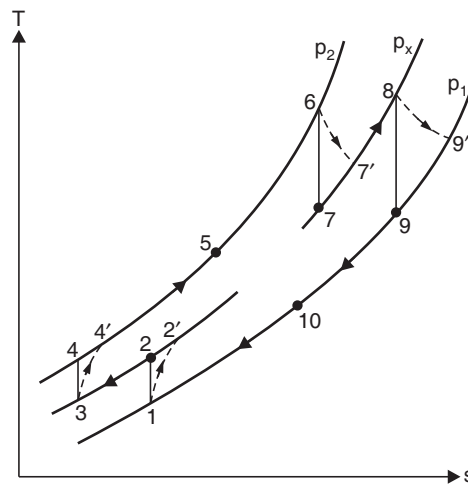
Fig. 4.48. *T-s* diagram for the plant.

The energy balance for the whole plant is as follows :

$$Q_1 + Q_2 - Q_3 - Q_4 = W$$

In a closed cycle plant, in practice, the control of power output is achieved by varying the mass flow by the use of a reservoir in the circuit. The *reservoir maintains the design pressure and temperature and therefore achieves an approximately constant level of efficiency for varying* loads. In this cycle, since it is closed, *gases other than air with favourable properties can be used* ; furthermore, it is possible to burn solid fuels in the combustion heaters. *The major factor responsible*

for inefficiency in this cycle is the large irreversible temperature drop which occurs in the air heaters between the furnace and circulating gas.

Note 1. In closed cycle gas turbines, although air has been extensively used, the use of 'helium' which though of a lower density, has been inviting the attention of manufacturers for its use, for large output gas turbine units. *The specific heat of helium at constant pressure is about 'five times' that of air, therefore for each kg mass flow the heat drop and hence energy dealt with in helium machines is nearly five times of those in case of air.* The *surface area of the heat exchanger for helium can be kept as low as 1/3 of that required for gas turbine plant using air as working medium.* For the same temperature ratio and for the plants of the same output the *cross-sectional area required for helium is much less than that for air.* It may therefore be concluded that the *size of helium unit is considerably small comparatively.*

2. Some gas turbine plants work on a combination of two cycles the open cycle and the closed cycle. Such a combination is called the *semi-closed cycle. Here a part of the working fluid is confined within the plant and another part flows from and to atmosphere.*

4.10.8. Gas Turbine Fuels

The various fuels used in gas turbines are enumerated and discussed below :

1. Gaseous fuels
2. Liquid fuels
3. Solid fuels

1. **Gaseous Fuels.** *Natural gas is the ideal fuel for gas turbines*, but this is not available everywhere.

Blast furnace and *producer gases* may also be used for gas turbine power plants.

2. **Liquid Fuels.** Liquid fuels of petroleum origin such as distillate oils or residual oils are most commonly used for gas turbine plant. The essential qualities of these fuels include *proper volatility, viscosity and calorific volume*. At the same time it *should be* free *from any contents of moisture* and *suspended impurities that would log the small passages of the nozzles and damage valves and plungers of the fuel pumps.*

Minerals like *sodium, vanadium* and *calcium* prove *very harmful* for the *turbine blading* as these build deposits or corrode the blades. The sodium in ash should be less than 30% of the vanadium content as otherwise the ratio tends to be critical. The actual sodium content may be between 5 ppm to 10 ppm (part per million). If the vanadium is over 2 ppm, the magnesium in ash tends to become critical. *It is necessary that the magnesium in ash is at least three times the quantity of vanadium.* The content of calcium and lead should not be over 10 ppm and 5 ppm respectively.

Sodium is removed from residual oils by mixing with 5% of water and then double centrifuging when sodium leaves with water. Magnesium is added to the washed oil in the form of epsom salts, before the oil is sent into the combustor. This checks the corrosive action of vanadium. Residual oils burn with less ease than distillate oils and the latter are often used to start the unit from cold, after which the residual oils are fed in the combustor. In cold conditions residual oils need to be preheated.

3. **Solid Fuels.** The use of solid fuels such as coal in pulverised form in gas turbines presents several difficulties most of which have been only partially overcome yet. The pulverising plant for coal is gas turbines applications is much lighter and small than its counterpart in steam generators. *Introduction of fuel in the combustion chamber of a gas turbine is required to be done against a high pressure whereas the pressure in the furnace of a steam plant is atmospheric.* Furthermore, the *degree of completeness of combustion in gas turbine applications has to be very high as otherwise soot and dust in gas would deposit on the turbine blading.*

Some practical applications of solid fuel burning in turbine combustors have been commercially, made available in recent years. In one such design finely crushed coal is used instead of pulverised fuel. This fuel is carried in stream of air tangentially into one end of a cylindrical

furnace while gas comes out at the centre of opposite end. As the fuel particles roll around the circumference of the furnace they are burnt and a high temperature of about 1650°C is maintained which causes the mineral matter of fuel to be converted into a liquid slag. The slag covers the walls of the furnace and runs out through a top hole in the bottom. The result is that fly-ash is reduced to a very small content in the gases. In *another design* a regenerator is used to transfer the heat to air, the combustion chamber being located on the outlet of the turbine, and the combustion is carried out in the turbine exhaust stream. The advantage is that only clean air is handled by the turbine.

Example 4.32. *An isentropic air turbine is used to supply 0.1 kg/s of air at 0.1 MN/m^2 and at 285 K to a cabin. The pressure at inlet to the turbine is 0.4 MN/m^2. Determine the temperature at turbine inlet and the power developed by the turbine. Assume c_p = 1.0 kJ/kg K.*

Solution. *Given :* $\dot{m}_a$ = 0.1 kg/s ;

p_1 = 0.1 MN/m^2 = 1 bar ; T_4 = 285 K ;

p_2 = 0.4 MN/m^2 = 4 bar ; c_p = 1.0 kJ/kg K.

Temperature at turbine inlet, T$_3$:

$$\frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}} = 1.486$$

∴ T_3 = 285 × 1.486 = **423.5 K.   (Ans.)**

Power developed, P :

$$P = \dot{m}_a c_p\,(T_3 - T_4)$$
$$= 0.1 \times 1.0\,(423.5 - 285)$$
$$= \textbf{13.85 kW.   (Ans.)}$$
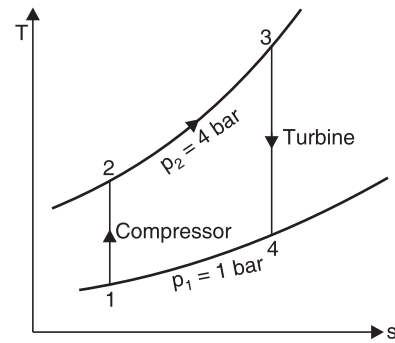
Fig. 4.49

Example 4.33. *A gas turbine is supplied with gas at 5 bar and 1000 K and expands it adiabatically to 1 bar. The mean specific heat at constant pressure and constant volume are 1.0425 kJ/kg K and 0.7662 kJ/kg K respectively.*

(i) Draw the temperature-entropy diagram to represent the processes of the simple gas turbine system.

(ii) Calculate the power developed in kW per kg of gas per second and the exhaust gas temperature.

Solution. *Given :* p_1 = 1 bar ; p_2 = 5 bar ; T_3 = 1000 K ; c_p = 1.0425 kJ/kg K ;

c_v = 0.7662 kJ/kg K ; $\gamma = \dfrac{c_p}{c_v} = \dfrac{1.0425}{0.7662}$ = 1.36

(*i*) **Temperature-entropy diagram :**

Temperature-entropy diagram representing the processes of the simple gas turbine system is shown in Fig. 4.50.

(*ii*) **Power required**

$$\frac{T_4}{T_3} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{5}\right)^{\frac{1.36-1}{1.36}} = 0.653$$

∴ T_4 = 1000 × 0.653 = 653 K

Power developed per kg of gas per second

$$= c_p(T_3 - T_4)$$
$$= 1.0425(1000 - 653)$$
$$= \textbf{361.7 kW.   (Ans.)}$$
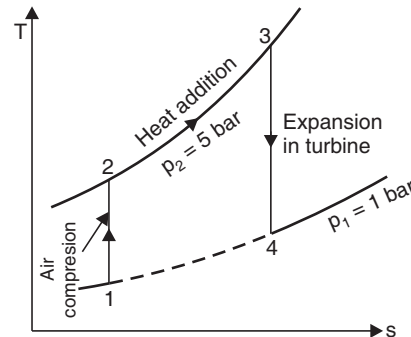
Fig. 4.50

Example 4.34. *Air enters the compressor of a gas turbine plant operating on Brayton cycle at 1 bar, 27°C. The pressure ratio in the cycle is 6. If $W_T = 2.5\ W_C$, where W_T and W_C are the turbine and compressor work respectively, calculate the maximum temperature and the cycle efficiency.*

Solution. *Given :* $p_1 = 1$ bar ; $T_1 = 27 + 273 = 300$ K ; $\dfrac{p_2}{p_1} = 6$; $W_T = 2.5\ W_C$

Maximum temperature, T_3 :

Now,
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.4-1}{1.4}}$$

$$= (6)^{\frac{0.4}{1.4}} = 1.668$$

∴ $T_2 = 300 \times 1.668 = 500.4$ K

Also,
$$\frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{0.4}{1.4}} = 1.668$$

or $T_3 = 1.668 T_4$

Now, compressor work, $W_C = mc_p(T_2 - T_1)$,
and, turbine work, $W_T = mc_p(T_3 - T_4)$
Since, $W_T = 2.5\ W_C$ (Given)
∴ $mc_p(T_3 - T_4) = 2.5 \times mc_p(T_2 - T_1)$

$$\left(T_3 - \frac{T_3}{1.668}\right) = 2.5(500.4 - 300)$$

or $T_3\left(1 - \dfrac{1}{1.668}\right) = 501$

∴ **$T_3 = 1251$ K.   (Ans.)**

Fig. 4.51

Cycle efficiency, η_{cycle} :

$$\eta_{cycle} = \frac{W_T - W_C}{mc_p(T_3 - T_2)} = \frac{mc_p(T_3 - T_4) - mc_p(T_2 - T_1)}{mc_p(T_3 - T_2)}$$

$$= \frac{\left(1251 - \dfrac{1251}{1.668}\right) - (500.4 - 300)}{(1251 - 500.4)} = \frac{501 - 200.4}{750.4}$$

= 0.40   or   40%.   (Ans.)

Example 4.35. *A closed cycle ideal gas turbine plant operates between temperature limits of 800°C and 30°C and produces a power of 100 kW. The plant is designed such that there is no need for a regenerator. A fuel of calorific 45000 kJ/kg is used. Calculate the mass flow rate of air through the plant and rate of fuel consumption.*

Assume $c_p = 1$ kJ/kg K and $\gamma = 1.4$.

Solution. *Given :* $T_1 = 30 + 273 = 303$ K ; $T_3 = 800 + 273 = 1073$ K ; $C = 45000$ kJ/kg ;
$c_p = 1$ kJ/kg K ; $\gamma = 1.4$; $W_{turbine} - W_{compressor} = 100$ kW.

m_a, m_f :

Since, no regenerator is used we can assume the turbine expands the gases upto T_4 in such a way that the exhaust gas temperature from the turbine is equal to the temperature of air coming out of the compressor, *i.e.*, $T_2 = T_4$.
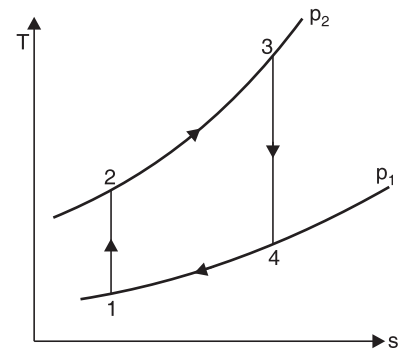
$$\frac{p_2}{p_1} = \frac{p_3}{p_4}; \ \frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

and $\qquad\qquad \dfrac{p_3}{p_4} = \left(\dfrac{T_3}{T_4}\right)^{\frac{\gamma}{\gamma-1}}$

$\therefore\qquad\qquad \dfrac{T_2}{T_1} = \dfrac{T_3}{T_4} = \dfrac{T_3}{T_2}$

$\qquad\qquad\qquad (\because \quad T_2 = T_4 \ ...... \ \text{assumed})$

or $\qquad\qquad T_2^2 = T_1 T_3 \quad \text{or} \quad T_2 = \sqrt{T_1 T_3}$

or $\qquad\qquad T_2 = \sqrt{303 \times 1073} = 570.2 \ \text{K}$

Now, $\qquad\qquad W_{\text{turbine}} - W_{\text{compressor}} = \dot{m}_f \times C \times \eta$

or $\qquad\qquad 100 = \dot{m}_f \times 45000 \times \left[1 - \dfrac{T_4 - T_1}{T_3 - T_2}\right]$

$\qquad\qquad\qquad = \dot{m}_f \times 45000 \left[1 - \dfrac{570.2 - 303}{1073 - 570.2}\right] = \dot{m}_f \times 21085.9$

or $\qquad\qquad \mathbf{\dot{m}_f} = \dfrac{100}{21085.9} = \mathbf{4.74 \times 10^{-3} \ kg/s. \ \ (Ans.)}$

Again, $W_{\text{turbine}} - W_{\text{compressor}} = 100 \ \text{kW}$

$\qquad\qquad (\dot{m}_a + \dot{m}_f) \ (T_3 - T_4) - \dot{m}_a \times 1 \times (T_2 - T_1) = 100$

or $\qquad (\dot{m}_a + 0.00474)(1073 - 570.2) - \dot{m}_a \ (570.2 - 303) = 100$

or $\qquad\qquad (\dot{m}_a + 0.00474) \times 502.8 - 267.2 \ \dot{m}_a = 100$

or $\qquad\qquad 502.8 \ \dot{m}_a + 2.383 - 267.2 \ \dot{m}_a = 100$

or $\qquad\qquad\qquad 235.6 \ \dot{m}_a = 97.617$

$\therefore \qquad\qquad\qquad\qquad \mathbf{\dot{m}_a = 0.414 \ kg/s. \ \ (Ans.)}$

Example 4.36. *In an air-standard regenerative gas turbine cycle the pressure ratio is 5. Air enters the compressor at 1 bar, 300 K and leaves at 490 K. The maximum temperature in the cycle is 1000 K. Calculate the cycle efficiency, given that the efficiency of the regenerator and the adiabatic efficiency of the turbine are each 80%. Assume for air, the ratio of specific heats is 1.4. Also, show the cycle on a T-s diagram.*

Solution. *Given :* $p_1 = 1 \ \text{bar} \ ; \ T_1 = 300 \ \text{K}, \ T_2' = 490 \ \text{K} \ ; \ T_3 = 1000 \ \text{K} \ ; \ \dfrac{p_2}{p_1} = 5,$

$\qquad \eta_{\text{turbine}} = 80\%, \ \varepsilon = 80\% = 0.8 \ ; \ \gamma = 1.4.$

Now, $\qquad\qquad \dfrac{T_3}{T_4} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (5)^{\frac{1.4-1}{1.4}} = 1.5838$

$\therefore \qquad\qquad T_4 = \dfrac{T_3}{1.5838} = \dfrac{1000}{1.5838} = 631.4 \ \text{K}$

Also, $\qquad \eta_{\text{turbine}} = \dfrac{T_3 - T_4'}{T_3 - T_4} \quad \text{or} \quad 0.8 = \dfrac{1000 - T_4'}{1000 - 631.4}$

$\therefore \qquad\qquad T_4' = 1000 - 0.8(1000 - 631.4) = 705 \ \text{K}$
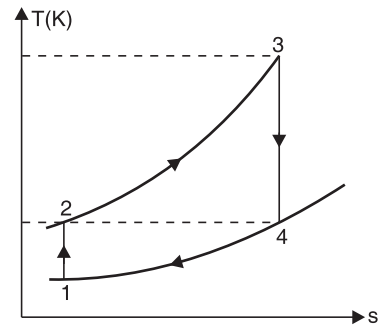
Fig. 4.52

Efficiency of heat exchanger,

$$\varepsilon = \frac{T_5 - T_2'}{T_4' - T_2'}$$

or

$$0.8 = \frac{T_5 - 490}{705 - 490}$$

∴ $T_5 = 0.8(705 - 490) + 490 = 662$ K

Work consumed by compressor

$$= c_p(T_2' - T_1)$$
$$= 1.005(490 - 300) = 190.9 \text{ kJ/kg}$$

Work done by turbine $= c_p(T_3 - T_4')$
$$= 1.005(1000 - 705) = 296.5 \text{ kJ/kg}.$$

Heat supplied $= c_p(T_3 - T_5)$
$$= 1.005(1000 - 662) = 339.7 \text{ kJ/kg}$$

∴ Cycle efficiency,

$$\eta_{\text{cycle}} = \frac{\text{Net work}}{\text{Heat supplied}}$$

$$= \frac{\text{Turbine work} - \text{Compressor work}}{\text{Heat supplied}}$$

$$= \frac{296.5 - 190.9}{339.7} = \textbf{0.31} \quad \text{or} \quad \textbf{31\%. \quad (Ans.)}$$

Fig. 4.53

Example 4.37. *Find the required air-fuel ratio in a gas turbine whose turbine and compressor efficiencies are 85% and 80%, respectively. Maximum cycle temperature is 875°C. The working fluid can be taken as air ($c_p = 1.0 \text{ kJ/kg K}, \gamma = 1.4$), which enters the compressor at 1 bar and 27°C. The pressure ratio is 4. The fuel used has calorific value of 42000 kJ/kg. There is a loss of 10% of calorific value in the combustion chamber.*

Solution. $\eta_{\text{turbine}} = 85\%$; $\eta_{\text{compressor}} = 80\%$; $T_3 = 273 + 875 = 1148$ K,

$T_1 = 27 + 273 = 300$ K ; $c_p = 1.0$ kJ/kg K ; $\gamma = 1.4$, $p_1 = 1$ bar,

$p_2 = 4$ bar (Since pressure ratio is 4) ; $C = 42000$ kJ/kg K,

$\eta_{c.c.} = 90\%$ (since loss in the combustion chamber is 10%) :

For isentropic compressor 1-2 :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{1.4-1}{1.4}} = 1.486$$

∴ $T_2 = 300 \times 1.486 = 445.8$ K

$$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1}$$

or

$$0.8 = \frac{445.8 - 300}{T_2' - 300}$$

or

$$T_2' = \frac{445.8 - 300}{0.8} + 300 = 482.2 \text{ K}$$

Fig. 4.54

Now, heat supplied by the fuel = heat taken by the burning gases

$$0.9 \times m_f \times C = (m_a + m_f) \times c_p \times (T_3 - T_2')$$
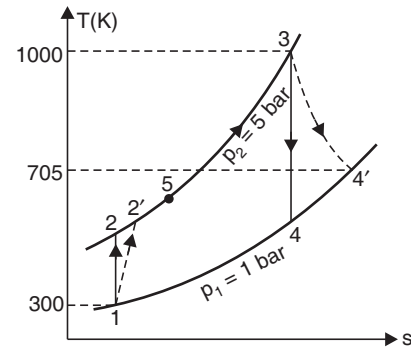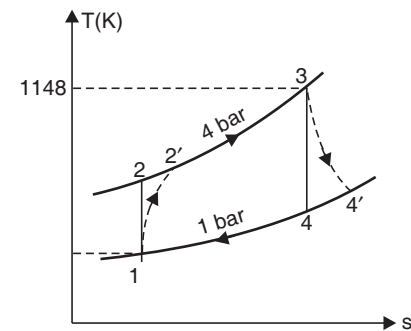
$$\therefore \qquad C = \left(\frac{m_a + m_f}{m_f}\right) \times \frac{c_p(T_3 - T_2')}{0.9} = \left(\frac{m_a}{m_f} + 1\right) \times \frac{c_p(T_3 - T_2')}{0.9}$$

or

$$42000 = \left(\frac{m_a}{m_f} + 1\right) \times \frac{1.00(1148 - 482.2)}{0.9} = 739.78 \left(\frac{m_a}{m_f} + 1\right)$$

$$\therefore \qquad \frac{m_a}{m_f} = \frac{42000}{739.78} - 1 = 55.77 \text{ say } 56$$

$\therefore$ **A/F ratio = 56 : 1.　(Ans.)**

Example 4.38. *A gas turbine unit receives air at 1 bar and 300 K and compresses it adiabatically to 6.2 bar. The compressor efficiency is 88%. The fuel has a heating value of 44186 kJ/kg and the fuel air ratio is 0.017 kJ/kg of air. The turbine internal efficiency is 90%. Calculate the work of turbine and compressor per kg of air compressed and thermal efficiency.*

For products of combustion, c_p = 1.147 kJ/kg K and γ = 1.333.

Solution. *Given :* $p_1 (= p_4)$ = 1 bar, T_1 = 300 K ; $p_2 (= p_3)$ = 6.2 bar ; $\eta_{compressor}$ = 88% ;

C = 44186 kJ/kg ; fuel-air ratio = 0.017 kJ/kg of air,

$\eta_{turbine}$ = 90%, C_p = 1.147 kJ/kg K ; γ = 1.333.

For isentropic compression process 1-2 :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{6.2}{1}\right)^{1.4} = 1.684$$

$$\therefore \qquad T_2 = 300 \times 1.684 = 505.2 \text{ K}$$

Now, $\qquad \eta_{compressor} = \dfrac{T_2 - T_1}{T_2' - T_1}$

$$0.88 = \frac{505.2 - 300}{T_2' - 300}$$

$$T_2' = \left(\frac{505.2 - 300}{0.88} + 300\right) = 533.2 \text{ K}$$
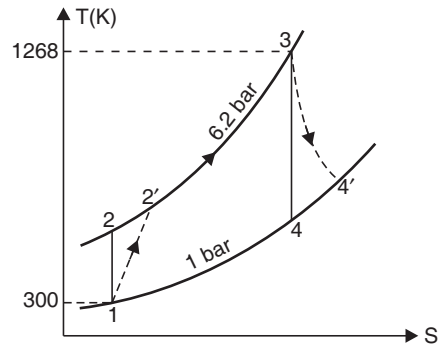
Fig. 4.55

Heat supplied

$$= (m_a + m_f) \times c_p(T_3 - T_2') = m_f \times C$$

or

$$\left(1 + \frac{m_f}{m_a}\right) \times c_p(T_3 - T_2') = \frac{m_f}{m_a} \times C$$

or

$$(1 + 0.017) \times 1.005(T_3 - 533.2)$$
$$= 0.017 \times 44186$$

$$\therefore \qquad T_3 = \frac{0.017 \times 44186}{(1 + 0.017) \times 1.005} + 533.2 = 1268 \text{ K}$$

For isentropic expansion process 3-4 :

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{1}{6.2}\right)^{\frac{1.333 - 1}{1.333}} = 0.634$$

$$\therefore \qquad T_4 = 1268 \times 0.634 = 803.9 \text{ K} \qquad\qquad (\because \quad \gamma_g = 1.333 \ldots.. \text{ Given})$$

Now, $\qquad \eta_{turbine} = \dfrac{T_3 - T_4'}{T_3 - T_4}$

$$0.9 = \frac{1268 - T_4{}'}{1268 - 803.9}$$

∴ $T_4{}' = 1268 - 0.9(1268 - 803.9) = 850.3 \text{ K}$

$W_{\text{compressor}} = c_p(T_2{}' - T_1) = 1.005(533.2 - 300) = 234.4 \text{ kJ/kg}$

$W_{\text{turbine}} = c_{pg}(T_3 - T_4{}') = 1.147(1268 - 850.3) = 479.1 \text{ kJ/kg}$

$\text{Network} = W_{\text{turbine}} - W_{\text{compressor}}$

$= 479.1 - 234.4 = 244.7 \text{ kJ/kg}$

Heat supplied per kg of air = 0.017 × 44186 = 751.2 kJ/kg

∴   Thermal efficiency, $\eta_{\text{th}} = \dfrac{\text{Net work}}{\text{Heat supplied}}$

$= \dfrac{244.7}{751.2} = \mathbf{0.3257}$ or   **32.57%.   (Ans.)**

Example 4.39. *The air enters the compressor of an open cycle constant pressure gas turbine at a pressure of 1 bar and temperature of 20°C. The pressure of the air after compression is 4 bar. The isentropic efficiencies of compressor and turbine are 80% and 85% respectively. The air-fuel ratio used is 90 : 1. If flow rate of air is 3.0 kg/s, find :*

(i) Power developed.

(ii) Thermal efficiency of the cycle.

Assume $c_p = 1.0$ kJ/kg K and $\gamma = 1.4$ for air and gases

Calorific value of fuel = 41800 kJ/kg.

Solution. $p_1 = 1$ bar, $T_1 = 20 + 273$ = 293 K

$p_2 = 4$ bar, $\eta_{\text{compressor}} = 80\%$, $\eta_{\text{turbine}} = 85\%$

Air-fuel ratio   = 90 : 1,   Air flow rate, $m_a = 3.0$ kg/s

(*i*) **Power developed, P :**

Refer Fig. 4.56 (*b*)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}} = 1.486$$

∴ $T_2 = (20 + 273) \times 1.486 = 435.4 \text{ K}$

Fig. 4.56

$$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2{}' - T_1}$$

$$0.8 = \frac{435.4 - 293}{T_2{}' - 293}$$

$$\therefore \qquad T_2{}' = \frac{435.4 - 293}{0.8} + 293 = 471 \text{ K}$$

Heat supplied by fuel $\qquad$ = heat taken by burning gases

$$m_f \times C = (m_a + m_f)\, c_p (T_3 - T_2{}')$$

(where m_a = mass of air, m_f = mass of fuel)

$$\therefore \qquad C = \left(\frac{m_a}{m_f} + 1\right) c_p\, (T_3 - T_2{}')$$

$$\therefore \qquad 41800 = (90 + 1) \times 1.0 \times (T_3 - 471)$$

i.e., $\qquad T_3 = \dfrac{41800}{91} + 471 = 930 \text{ K}$

Again, $\qquad \dfrac{T_4}{T_3} = \left(\dfrac{p_4}{p_3}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\dfrac{1}{4}\right)^{\frac{0.4}{1.4}} = 0.672$

$$\therefore \qquad T_4 = 930 \times 0.672 = 624.9 \text{ K}$$

$$\eta_{\text{turbine}} = \frac{T_3 - T_4{}'}{T_3 - T_4}$$

$$0.85 = \frac{930 - T_4{}'}{930 - 824.9}$$

$$\therefore \qquad T_4{}' = 930 - 0.85\,(930 - 624.9) = 670.6 \text{ K}$$

$$W_{\text{turbine}} = m_g \times c_p \times (T_3 - T_4{}')$$

(where m_g is the mass of hot gases formed per kg of air)

$$\therefore \qquad W_{\text{turbine}} = \left(\frac{90 + 1}{90}\right) \times 1.0 \times (930 - 670.6)$$

$$= 262.28 \text{ kJ/kg of air.}$$

$$W_{\text{compressor}} = m_a \times c_p \times (T_2{}' - T_1) = 1 \times 1.0 \times (471 - 293)$$
$$= 178 \text{ kJ/kg of air}$$

$$W_{\text{net}} = W_{\text{turbine}} - W_{\text{compressor}}$$
$$= 262.28 - 178 = 84.28 \text{ kJ/kg of air.}$$

Hence, *power developed, P* = 84.28 × 3 = **252.84 kW/kg of air.** **(Ans.)**

(*ii*) **Thermal efficiency of cycle,** $\eta_{\textbf{thermal}}$:

Heat supplied per kg of air passing through combustion chamber

$$= \frac{1}{90} \times 41800 = 464.44 \text{ kJ/kg of air}$$

$$\therefore \qquad \eta_{\text{thermal}} = \frac{\text{work output}}{\text{heat supplied}}$$

$$= \frac{84.28}{464.44} = \textbf{0.1814} \quad \text{or} \quad \textbf{18.14\%.} \quad \textbf{(Ans.)}$$

Example 4.40. *A gas turbine unit has a pressure ratio of 6 : 1 and maximum cycle temperature of 610°C. The isentropic efficiencies of the compressor and turbine are 0.80 and 0.82 respectively. Calculate the power output in kilowatts of an electric generator geared to the turbine when the air enters the compressor at 15°C at the rate of 16 kg/s.*

Take $c_p = 1.005$ kJ/kg K and $\gamma = 1.4$ for the compression process, and take $c_p = 1.11$ kJ/kg K and $\gamma = 1.333$ for the expansion process.

Solution. $T_1 = 15 + 273 = 288$ K, $T_3 = 610 + 273 = 883$ K, $\dfrac{p_2}{p_1} = 6$,

$\eta_{compressor} = 0.80$, $\eta_{turbine} = 0.82$,

Air flow rate = 16 kg/s

For compression process : $c_p = 1.005$ kJ/kg K, $\gamma = 1.4$

For expansion process : $c_p = 1.11$ kJ/kg K, $\gamma = 1.333$
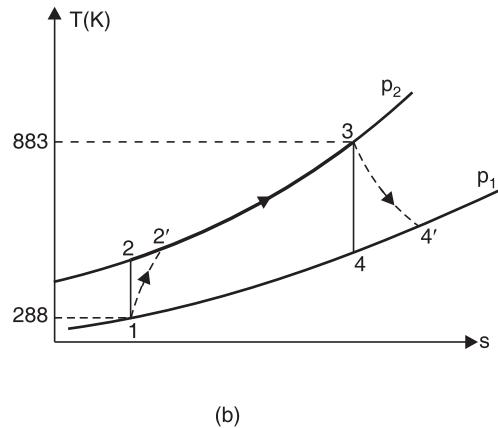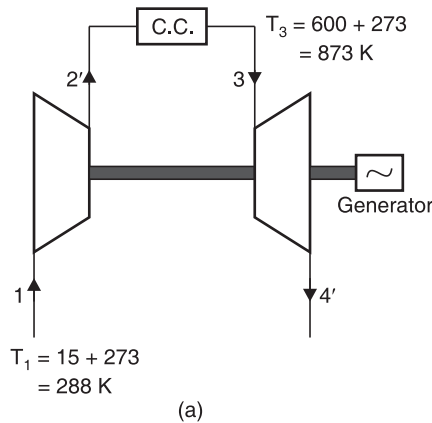
Fig. 4.57

In order to evaluate the net work output it is necessary to calculate temperatures $T_2{'}$ and $T_4{'}$. To calculate we must first calculate T_2 and then use the isentropic efficiency.

For an isentropic process,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.4-1}{1.4}} = 1.67$$

$\therefore$ $T_2 = 288 \times 1.67 = 481$ K

Also, $\eta_{compressor} = \dfrac{T_2 - T_1}{T_2{'} - T_1}$

$$0.8 = \frac{481 - 288}{T_2{'} - T_1}$$

$\therefore$ $T_2{'} = \dfrac{481 - 288}{0.8} + 288 = 529$ K

Similarly for the turbine,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.333-1}{1.333}} = 1.565$$

$$\therefore \qquad\qquad T_4 = \frac{T_3}{1.565} = \frac{883}{1.565} = 564 \text{ K}$$

Also, $\qquad\qquad \eta_{\text{turbine}} = \frac{T_3 - T_4{}'}{T_3 - T_4} = \frac{883 - T_4{}'}{883 - 564}$

$$\therefore \qquad\qquad 0.82 = \frac{883 - T_4{}'}{883 - 564}$$

$\therefore \qquad\qquad T_4{}' = 883 - 0.82 \,(883 - 564) = 621.4 \text{ K}$

Hence,

Compressor work input, $\quad W_{\text{compressor}} = c_p (T_2{}' - T_1)$
$$= 1.005 \,(529 - 288) = 242.2 \text{ kJ/kg}$$

Turbine work output,

$$W_{\text{turbine}} = c_p \,(T_3 - T_4{}')$$
$$= 1.11 \,(883 - 621.4) = 290.4 \text{ kJ/kg}.$$

$\therefore \quad$ Net work output, $\qquad W_{\text{net}} = W_{\text{turbine}} - W_{\text{compressor}}$
$$= 290.4 - 242.2 = 48.2 \text{ kJ/kg}$$

Power in kilowatts $\qquad = 48.2 \times 16 = \textbf{771.2 kW.} \quad \textbf{(Ans.)}$

Example 4.41. *Calculate the thermal efficiency and work ratio of the plant in example 4.26, assuming that c_p for the combustion process is 1.11 kJ/kg K.*

Solution. Heat supplied $= c_p \,(T_3 - T_2{}')$
$$= 1.11 \,(883 - 529) = 392.9 \text{ kJ/kg}$$

$$\eta_{\text{thermal}} = \frac{\text{net work output}}{\text{heat supplied}} = \frac{48.2}{392.9}$$

$$= \textbf{0.1226} \quad \text{or} \quad \textbf{12.26\%.} \quad \textbf{(Ans.)}$$

Now, $\qquad Work\ ratio = \dfrac{\text{net work output}}{\text{gross work output}} = \dfrac{48.2}{W_{\text{turbine}}} = \dfrac{48.2}{290.4} = \textbf{0.166.} \quad \textbf{(Ans.)}$

Example 4.42. *In a constant pressure open cycle gas turbine air enters at 1 bar and 20°C and leaves the compressor at 5 bar. Using the following data : Temperature of gases entering the turbine = 680°C, pressure loss in the combustion chamber = 0.1 bar, $\eta_{compressor}$ = 85%, $\eta_{turbine}$ = 80%, $\eta_{combustion}$ = 85%, γ = 1.4 and c_p = 1.024 kJ/kg K for air and gas, find*

(i) The quantity of air circulation if the plant develops 1065 kW.

(ii) Heat supplied per kg of air circulation.

(iii) The thermal efficiency of the cycle.

Mass of the fuel may be neglected.

Solution. $\qquad\qquad p_1 = 1 \text{ bar}, p_2 = 5 \text{ bar}, p_3 = 5 - 0.1 = 4.9 \text{ bar}, p_4 = 1 \text{ bar}$
$$T_1 = 20 + 273 = 293 \text{ K}, T_3 = 680 + 273 = 953 \text{ K}$$
$$\eta_{\text{compressor}} = 85\%, \eta_{\text{turbine}} = 80\%, \eta_{\text{combustion}} = 85\%.$$

For air and gases : $\qquad c_p = 1.024 \text{ kJ/kg K}, γ = 1.4$

Power developed by the plant, $P = 1065 \text{ kW}$

(i) **The quantity of air circulation, m_a = ?**

For isentropic compression 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{γ-1}{γ}} = \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = 1.584$$

$\therefore \qquad\qquad T_2 = 293 \times 1.584 = 464 \text{ K}$
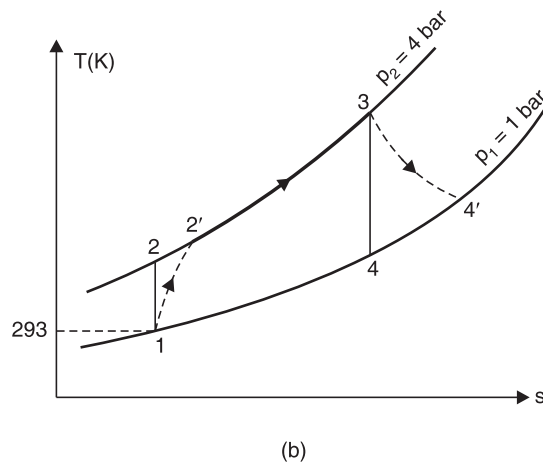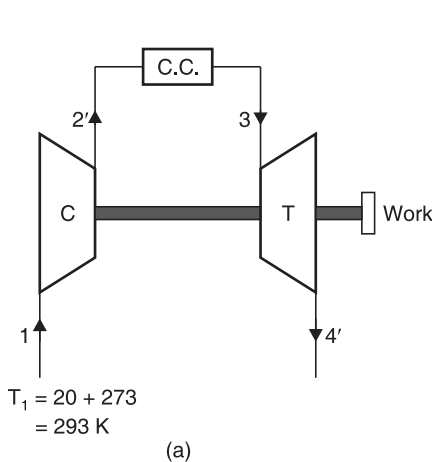
Now, $\eta_{\text{compressor}} = \dfrac{T_2 - T_1}{T_2{}' - T_1}$ *i.e.,* $0.85 = \dfrac{464 - 293}{T_2{}' - 293}$

$\therefore$ $T_2{}' = \dfrac{464 - 293}{0.85} + 293 = 494\ \text{K}$

For isentropic expansion process 3-4,

$$\dfrac{T_4}{T_3} = \left(\dfrac{p_4}{p_3}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\dfrac{1}{4.9}\right)^{\frac{1.4 - 1}{1.4}} = 0.635$$

$\therefore$ $T_4 = 953 \times 0.635 = 605\ \text{K}$

Now, $\eta_{\text{turbine}} = \dfrac{T_3 - T_4{}'}{T_3 - T_4}$

$$0.8 = \dfrac{953 - T_4{}'}{953 - 605}$$

$\therefore$ $T_4{}' = 953 - 0.8\,(953 - 605) = 674.6\ \text{K}$

$W_{\text{compressor}} = c_p\,(T_2{}' - T_1) = 1.024\,(494 - 293) = 205.8\ \text{kJ/kg}$

$W_{\text{turbine}} = c_p\,(T_3 - T_4{}') = 1.024\,(953 - 674.6) = 285.1\ \text{kJ/kg.}$

$\therefore$ $W_{\text{net}} = W_{\text{turbine}} - W_{\text{compressor}} = 285.1 - 205.8 = 79.3\ \text{kJ/kg of air}$

If the mass of air is flowing is m_a kg/s, the power developed by the plant is given by

$$P = m_a \times W_{\text{net}}\ \text{kW}$$
$$1065 = m_a \times 79.3$$

$\therefore$ $m_a = \dfrac{1065}{79.3} = 13.43\ \text{kg.}$

i.e.,   **Quantity of air circulation = 13.43 kg.   (Ans.)**

(*ii*) **Heat supplied per kg of air circulation :**

Actual heat supplied per kg of air circulation

$$= \dfrac{c_p(T_3 - T_2{}')}{\eta_{\text{combustion}}} = \dfrac{1.024(953 - 494)}{0.85} = 552.9\ \text{kJ/kg.}$$

(*iii*) **Thermal efficiency of the cycle, η_{thermal} :**

$$\eta_{\text{thermal}} = \dfrac{\text{work output}}{\text{heat supplied}} = \dfrac{79.3}{552.9} = \mathbf{0.1434\ or\ 14.34\%.}\quad \textbf{(Ans.)}$$

Example 4.43. *In a gas turbine the compressor is driven by the high pressure turbine. The exhaust from the high pressure turbine goes to a free low pressure turbine which runs the load. The air flow rate is 20 kg/s and the minimum and maximum temperatures are respectively 300 K and 1000 K. The compressor pressure ratio is 4. Calculate the pressure ratio of the low pressure turbine and the temperature of exhaust gases from the unit. The compressor and turbine are isentropic. C_p of air and exhaust gases = 1 kJ/kg K and γ = 1.4.*

Solution. *Given :* $\dot{m}_a$ = 20 kg/s ; T_1 = 300 K ; T_3 = 1000 K ; $\dfrac{p_2}{p_1}$ = 4 ; c_p = 1 kJ/kg K ; γ = 1.4.

Pressure of low pressure turbine, $\dfrac{\mathbf{p_4}}{\mathbf{p_5}}$:
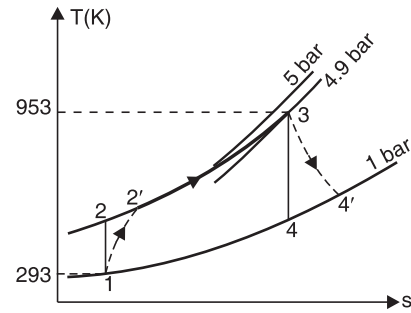
Since the compressor is driven by high pressure turbine,

$\therefore$ $\qquad$ $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{0.4}{1.4}} = 1.486$

or $\qquad$ $T_2 = 300 \times 1.486 = 445.8$ K

Also, $\quad \dot{m}_a c_p(T_2 - T_1) = \dot{m}_a (T_3 - T_4)$

$\qquad\qquad\qquad\qquad$ (Neglecting mass of fuel)

or $\qquad\qquad T_2 - T_1 = T_3 - T_4$

$\qquad\qquad 445.8 - 300 = 1000 - T_4, \quad$ or $\quad T_4 = 854.2$ K

For process 3-4 :

$$\dfrac{T_3}{T_4} = \left(\dfrac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{or} \quad \dfrac{p_3}{p_4} = \left(\dfrac{T_3}{T_4}\right)^{\frac{1.4}{0.4}}$$

$$\dfrac{p_3}{p_4} = \left(\dfrac{1000}{854.2}\right)^{3.5} = 1.736$$

Now, $\qquad\qquad \dfrac{p_3}{p_4} = \dfrac{p_3}{p_5} \times \dfrac{p_5}{p_4} = 4 \times \dfrac{p_5}{p_4}$ $\qquad\qquad\qquad \left(\because \ \dfrac{p_3}{p_5} = \dfrac{p_2}{p_1} = 4\right)$

$\therefore$ $\qquad\qquad \dfrac{p_5}{p_4} = \dfrac{1}{4}\left(\dfrac{p_3}{p_4}\right) = \dfrac{1}{4} \times 1.736 = 0.434$

Hence pressure ratio of low pressure turbine $= \dfrac{p_4}{p_5} = \dfrac{1}{0.434} = $ **2.3.** **(Ans.)**

Temperature of the exhaust from the unit, T_5 :

$$\dfrac{T_4}{T_5} = \left(\dfrac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = (2.3)^{\frac{1.4-1}{1.4}} = 1.269$$

$\therefore$ $\qquad\qquad\qquad$ $\mathbf{T_5} = \dfrac{T_4}{1.269} = \dfrac{854.2}{1.269} = $ **673 K.**

☞ **Example 4.44.** *Air is drawn in a gas turbine unit at 15°C and 1.01 bar and pressure ratio is 7 : 1. The compressor is driven by the H.P. turbine and L.P. turbine drives a separate power shaft. The isentropic efficiencies of compressor, and the H.P. and L.P. turbines are 0.82, 0.85 and 0.85 respectively. If the maximum cycle temperature is 610°C, calculate :*

$\quad$ (*i*) *The pressure and temperature of the gases entering the power turbine.*

$\quad$ (*ii*) *The net power developed by the unit per kg/s mass flow.*

$\quad$ (*iii*) *The work ratio.*

$\quad$ (*iv*) *The thermal efficiency of the unit.*

$\quad$ *Neglect the mass of fuel and assume the following :*

$\quad$ *For compression process $c_{pa} = 1.005$ kJ/kg and $\gamma = 1.4$*

$\quad$ *For combustion and expansion processes : $c_{pa} = 1.15$ kJ/kg and $\gamma = 1.333$.*

Solution. $T_1 = 15 + 273 = 288$ K, $p_1 = 1.01$ bar, pressure ratio $= \dfrac{p_2}{p_1} = 7$

$\quad \eta_{\text{compressor}} = 0.82, \ \eta_{\text{turbine (H.P.)}} = 0.85, \ \eta_{\text{turbine (L.P.)}} = 0.85,$

Maximum cycle temperature, $T_3 = 610 + 273 = 883$ K
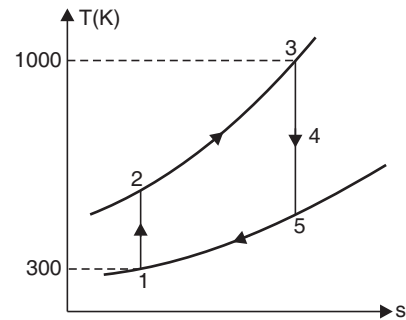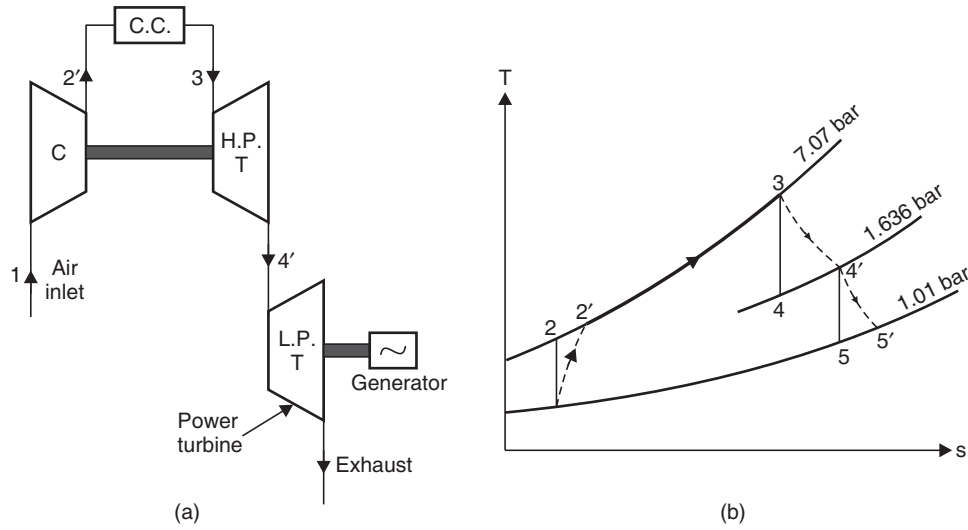
Fig. 4.60

(*i*) **Pressure and temperature of the gases entering the power turbine, $p_4{}'$ and $T_4{}'$:**

Considering isentropic compression 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (7)^{\frac{1.04-1}{1.4}} = 1.745$$

$\therefore$ $\qquad T_2 = 288 \times 1.745 = 502.5$ K

Also $\qquad \eta_{\text{compressor}} = \dfrac{T_2 - T_1}{T_2{}' - T_1}$

$\qquad 0.82 = \dfrac{502.5 - 288}{T_2{}' - 288}$

$\therefore$ $\qquad T_2{}' = \dfrac{502.5 - 288}{0.82} + 288 = 549.6$ K

$\qquad W_{\text{compressor}} = c_{pa}\,(T_2{}' - T_1) = 1.005 \times (549.6 - 288) = 262.9$ kJ/kg

Now, the *work output of H.P. turbine = work input to compressor*

$\therefore$ $\qquad c_{pg}\,(T_3 - T_4{}') = 262.9$

i.e., $\qquad 1.15(883 - T_4{}') = 262.9$

$\therefore$ $\qquad T_4{}' = 883 - \dfrac{262.9}{1.15} = 654.4$ K

i.e., *Temperature of gases entering the power turbine* = **654.4 K.** **(Ans.)**

Again, for H.P. turbine :

$$\eta_{\text{turbine}} = \frac{T_3 - T_4{}'}{T_3 - T_4}$$

i.e., $\qquad 0.85 = \dfrac{883 - 654.4}{883 - T_4}$

$\therefore$ $\qquad T_4 = 883 - \left(\dfrac{883 - 654.4}{0.85}\right) = 614$ K

Now, considering isentropic expansion process 3-4

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma}{\gamma-1}}$$

or

$$\frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{883}{614}\right)^{\frac{1.33}{0.33}} = 4.32$$

i.e.,

$$p_4 = \frac{p_3}{4.32} = \frac{7.07}{4.32} = 1.636 \text{ bar}$$

i.e.,   *Pressure of gases entering the power turbine* = **1.636 bar.**   **(Ans.)**

(*ii*) **Net power developed per kg/s mass flow, P :**

To find the power output it is now necessary to calculate T_5'.

The pressure ratio, $\dfrac{p_4}{p_5}$, is given by $\dfrac{p_4}{p_3} \times \dfrac{p_3}{p_5}$

i.e.,

$$\frac{p_4}{p_5} = \frac{p_4}{p_3} \times \frac{p_2}{p_1} \qquad\qquad (\because \quad p_2 = p_3 \text{ and } p_5 = p_1)$$

$$= \frac{7}{4.32} = 1.62$$

Then,

$$\frac{T_4'}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = (1.62)^{\frac{0.33}{1.33}} = 1.127$$

∴

$$T_5 = \frac{T_4'}{1.127} = \frac{654.4}{1.127} = 580.6 \text{ K.}$$

Again, for L.P. turbine

$$\eta_{\text{turbine}} = \frac{T_4' - T_5'}{T_4' - T_5}$$

i.e.,

$$0.85 = \frac{654.4 - T_5'}{654.4 - 580.6}$$

∴ $T_5' = 654.4 - 0.85 (654.4 - 580.6) = 591.7 \text{ K}$

$$W_{\text{L.P. turbine}} = c_{pg} (T_4' - T_5') = 1.15 (654.4 - 591.7) = 72.1 \text{ kJ/kg}$$

Hence, *net power output* (per kg/s mass flow)

$$= \textbf{72.1 kW.}   \textbf{(Ans.)}$$

(*iii*) **Work ratio :**

$$Work \ ratio = \frac{\text{net work output}}{\text{gross work output}} = \frac{72.1}{72.1 + 262.9} = \textbf{0.215. (Ans.)}$$

(*iv*) **Thermal efficiency of the unit,** η_{thermal} = ?

Heat supplied $= c_{pg} (T_3 - T_2')$

$$= 1.15(883 - 549.6) = 383.4 \text{ kJ/kg}$$

∴ $\eta_{\text{thermal}} = \dfrac{\text{net work output}}{\text{heat supplied}}$

$$= \frac{72.1}{383.4} = \textbf{0.188 or 18.8\%.   (Ans.)}$$

Example 4.45. *In a gas turbine the compressor takes in air at a temperature of 15°C and compresses it to four times the initial pressure with an isentropic efficiency of 82%. The air is then passes through a heat exchanger heated by the turbine exhaust before reaching the combustion chamber. In the heat exchanger 78% of the available heat is given to the air. The maximum temperature after constant pressure combustion is 600°C, and the efficiency of the turbine is 70%. Neglecting all losses except those mentioned, and assuming the working fluid throughout the cycle to have the characteristic of air find the efficiency of the cycle.*

Assume R = 0.287 kJ/kg K and γ = 1.4 for air and constant specific heats throughout.

Solution. T_1 = 15 + 273 = 288 K, Pressure ratio, $\dfrac{p_2}{p_1} = \dfrac{p_3}{p_4}$ = 4, $\eta_{\text{compressor}}$ = 82%.

Effectiveness of the heat exchanger, ε = 0.78,

η_{turbine} = 70%, Maximum temperature, T_3 = 600 + 273 = 873 K.

Efficiency of the cycle, η_{cycle} :

Considering the isentropic compression 1-2 ;

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{1.4-1}{1.4}} = 1.486$$

∴ T_2 = 288 × 1.486 = 428 K

Now, $\eta_{\text{compressor}} = \dfrac{T_2 - T_1}{T_2{'} - T_1}$

i.e., $0.82 = \dfrac{428 - 288}{T_2{'} - 288}$

∴ $T_2{'} = \dfrac{428 - 288}{0.82} + 288$ = 459 K

Considering the isentropic expansion process 3-4 :

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{1.4-1}{1.4}} = 1.486$$

∴ $T_4 = \dfrac{T_3}{1.486} = \dfrac{873}{1.486}$ = 587.5 K.

Fig. 4.61

Again, $\eta_{\text{turbine}} = \dfrac{T_3 - T_4{'}}{T_3 - T_4} = \dfrac{873 - T_4{'}}{873 - 587.5}$

i.e., $0.70 = \dfrac{873 - T_4{'}}{873 - 587.5}$

∴ $T_4{'}$ = 873 − 0.7(873 − 587.5) = 673 K

$W_{\text{compressor}} = c_p\,(T_2{'} - T_1)$

But $c_p = R \times \dfrac{\gamma}{\gamma - 1} = 0.287 \times \dfrac{1.4}{1.4 - 1}$ = 1.0045 kJ/kg K

∴ $W_{\text{compressor}}$ = 1.0045 (459 − 288) = 171.7 kJ/kg

$W_{\text{turbine}} = c_p\,(T_3 - T_4{'})$ = 1.0045 (873 − 673) = 200.9 kJ/kg

∴   Net work = $W_{\text{turbine}} - W_{\text{compressor}}$ = 200.9 − 171.7 = 29.2 kJ/kg.

Effectiveness of heat exchanger, $\varepsilon = \dfrac{T_5 - T_2{'}}{T_4{'} - T_2{'}}$
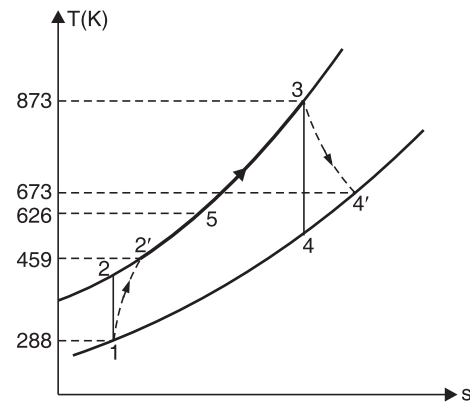
i.e., $\qquad 0.78 = \dfrac{T_5 - 459}{673 - 459}$

$\therefore \qquad T_5 = (673 - 459) \times 0.78 + 459 = 626$ K

$\therefore$ Heat supplied by fuel per kg

$\qquad = c_p(T_3 - T_5) = 1.0045\,(873 - 626) = 248.1$ kJ/kg

$\therefore \qquad \eta_{\text{cycle}} = \dfrac{\text{net work done}}{\text{heat supplied by the fuel}} = \dfrac{29.2}{248.1} = \mathbf{0.117}$ or **11.7%.   (Ans.)**

Example 4.46. *A gas turbine employs a heat exchanger with a thermal ratio of 72%. The turbine operates between the pressures of 1.01 bar and 4.04 bar and ambient temperature is 20°C. Isentropic efficiencies of compressor and turbine are 80% and 85% respectively. The pressure drop on each side of the heat exchanger is 0.05 bar and in the combustion chamber 0.14 bar. Assume combustion efficiency to be unity and calorific value of the fuel to be 41800 kJ/kg.*

Calculate the increase in efficiency due to heat exchanger over that for simple cycle.

Assume c_p is constant throughout and is equal to 1.024 kJ/kg K, and assume $\gamma = 1.4$.

For simple cycle the air fuel ratio is 90 : 1, and for the heat exchange cycle the turbine entry temperature is the same as for a simple cycle.

Solution. Simple Cycle. Refer to Fig. 4.62.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}} = 1.486$$

$\therefore \qquad T_2 = 293 \times 1.486 = 435.4$

Also, $\qquad \eta_{\text{compressor}} = \dfrac{T_2 - T_1}{T_2' - T_1}$
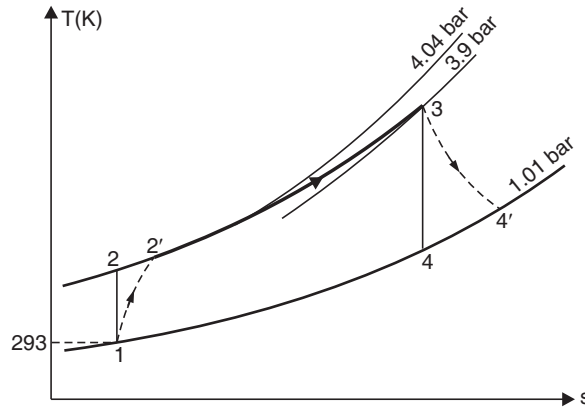
Fig. 4.62

$$0.8 = \frac{435.4 - 293}{T_2' - 293}$$

$\therefore \qquad T_2' = \dfrac{435.4 - 293}{0.8} + 293 = 471$ K

Now $\qquad m_f \times C = (m_a + m_f) \times c_p \times (T_2 - T_2')$

$\qquad\qquad$ [m_a = mass of air, m_f = mass of fuel]

$\therefore \qquad T_3 = \dfrac{m_f \times C}{c_p(m_a + m_f)} + T_2' = \dfrac{1 \times 41800}{1.024(90 + 1)} + 471 = 919.5$ K

Also,
$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_4 = T_3 \times \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = 919.5 \times \left(\frac{1.01}{3.9}\right)^{\frac{1.4-1}{1.4}} = 625 \text{ K}$$

Again,
$$\eta_{\text{turbine}} = \frac{T_3 - T_4{}'}{T_3 - T_4}$$

$\therefore$
$$0.85 = \frac{919.5 - T_4{}'}{919.5 - 625}$$

$\therefore$
$$T_4{}' = 919.5 - 0.85\,(919.5 - 625) = 669 \text{ K}$$

$$\eta_{\text{thermal}} = \frac{(T_3 - T_4{}') - (T_2{}' - T_1)}{(T_3 - T_2{}')}$$

$$= \frac{(919.5 - 669) - (471 - 293)}{(919.5 - 471)} = \frac{72.5}{448.5} = \mathbf{0.1616} \text{ or } \mathbf{16.16\%.} \quad \textbf{(Ans.)}$$

Heat Exchange Cycle. Refer Fig. 4.63 (a), (b)

$T_2{}' = 471$ K (as for simple cycle)

$T_3 = 919.5$ K (as for simple cycle)

To fine $T_4{}'$:

$p_3 = 4.04 - 0.14 - 0.05 = 3.85$ bar

$p_4 = 1.01 + 0.05 = 1.06$ bar
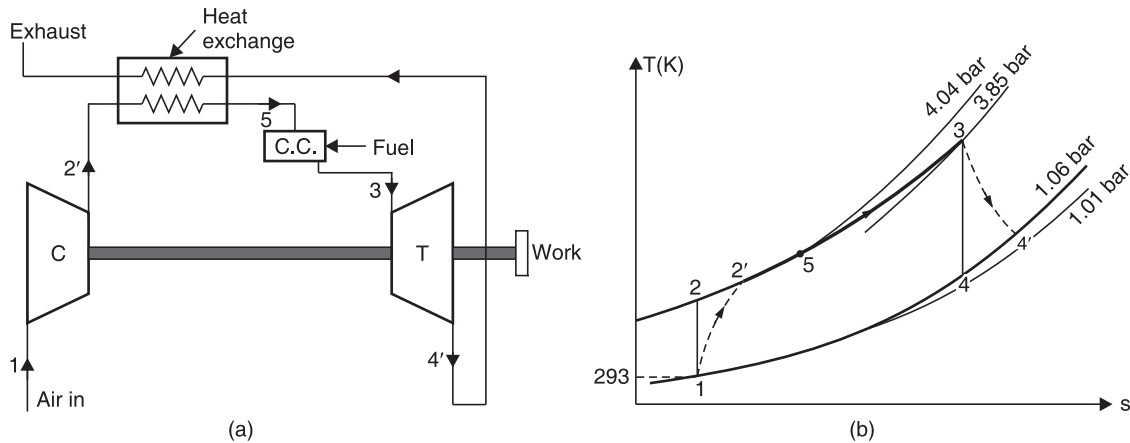
Fig. 4.63

$\therefore$
$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1.06}{3.85}\right)^{\frac{1.4-1}{1.4}} = 0.69$$

i.e.,
$$T_4 = 919.5 \times 0.69 = 634 \text{ K}$$

$$\eta_{\text{turbine}} = \frac{T_3 - T_4{}'}{T_3 - T_4}; \quad 0.85 = \frac{919.5 - T_4{}'}{919.5 - 634}$$

$\therefore$
$$T_4{}' = 919.5 - 0.85\,(919.5 - 634) = 677 \text{ K}$$

To find T_5 :

Thermal ratio (or effectiveness),

$$\varepsilon = \frac{T_5 - T_2{}'}{T_4{}' - T_2{}'} \qquad\qquad \therefore \quad 0.72 = \frac{T_5 - 471}{677 - 471}$$

$\therefore$ $\qquad\qquad\qquad\qquad T_5 = 0.72\,(677 - 471) + 471 = 619\ \text{K}$

$$\eta_{\text{thermal}} = \frac{(T_3 - T_4{}') - (T_2{}' - T_1)}{(T_3 - T_5)}$$

$$= \frac{(919.5 - 677) - (471 - 293)}{(919.5 - 619)} = \frac{64.5}{300.5} = \textbf{0.2146 or 21.46\%}$$

$\therefore$ **Increase in thermal efficiency** $= 21.46 - 16.16 = \textbf{5.3\%. (Ans.)}$

☞ **Example 4.47.** *A 4500 kW gas turbine generating set operates with two compressor stages, the overall pressure ratio is 9 : 1. A high pressure turbine is used to drive the compressors, and a low-pressure turbine drives the generator. The temperature of the gases at entry to the high pressure turbine is 625°C and the gases are reheated to 625°C after expansion in the first turbine. The exhaust gases leaving the low-pressure turbine are passed through a heat exchanger to heat the air leaving the high pressure stage compressor. The compressors have equal pressure ratios and intercooling is complete between the stages. The air inlet temperature to the unit is 20°C. The isentropic efficiency of each compressor stage is 0.8, and the isentropic efficiency of each turbine stage is 0.85, the heat exchanger thermal ratio is 0.8. A mechanical efficiency of 95% can be assumed for both the power shaft and compressor turbine shaft. Neglecting all pressure losses and changes in kinetic energy calculate :*

   (i) The thermal efficiency   *(ii) Work ratio of the plant*

   (iii) The mass flow in kg/s.

   Neglect the mass of the fuel and assume the following :

   For air : c_{pa} = 1.005 kJ/kg K and γ = 1.4

   For gases in the combustion chamber and in turbines and heat exchanger, c_{pg} = 1.15 kJ/kg K and γ = 1.333.

Solution. Refer to Fig. 4.64.

$$T_1 = 20 + 273 = 293\ \text{K},\ T_6 = T_8 = 625 + 273 = 898\ \text{K}$$

Efficiency of each compressor stage   = 0.8

Efficiency of each turbine stage   = 0.85, $\eta_{\text{mech.}}$ = 0.95, ε = 0.8

(i) Thermal efficiency, η_{thermal} = ?

Since the pressure ratio and the isentropic efficiency of each compressor is the same then the work input required for each compressor is the same since both compressors have the same air inlet temperature *i.e.,* $T_1 = T_3$ and $T_2{}' = T_4{}'$.

Also, $\qquad\qquad\qquad \dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ and $\dfrac{p_2}{p_1} = \sqrt{9} = 3$

$\therefore$ $\qquad\qquad\qquad T_2 = (20 + 273) \times (3)^{\frac{1.4-1}{1.4}} = 401\ \text{K}$

Now, $\qquad \eta_{\text{compressor}}\ (\text{L.P.}) = \dfrac{T_2 - T_1}{T_2{}' - T_1}$

$$0.8 = \frac{401 - 293}{T_2{}' - 293}$$
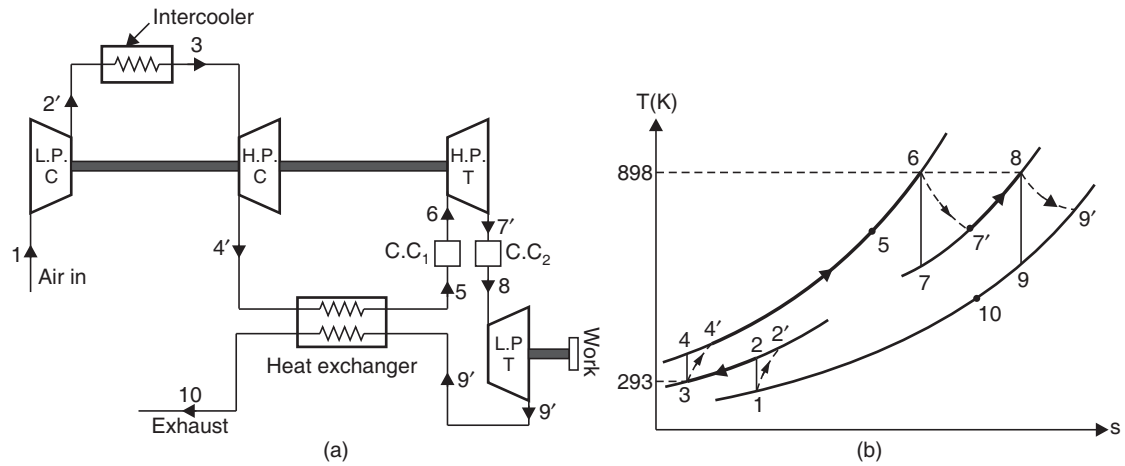
Fig. 4.64

i.e.,
$$T_2' = \frac{401 - 293}{0.8} + 293 = 428 \text{ K}$$

Work input per compressor stage
$$= c_{pa} (T_2' - T_1) = 1.005 (428 - 293) = 135.6 \text{ kJ/kg}$$

The H.P. turbine is required to drive both compressors and to overcome mechanical friction.

i.e., Work output of H.P. turbine $= \dfrac{2 \times 135.6}{0.95} = 285.5 \text{ kJ/kg}$

$\therefore$ $c_{pg} (T_6 - T_7') = 285.5$

i.e., $1.15 (898 - T_7') = 285.5$

$\therefore$ $T_7' = 898 - \dfrac{285.5}{1.15} = 650 \text{ K}$

Now, $\eta_{\text{turbine (H.P.)}} = \dfrac{T_6 - T_7'}{T_6 - T_7}; \quad 0.85 = \dfrac{898 - 650}{898 - T_7}$

$\therefore$ $T_7 = 898 - \left(\dfrac{898 - 650}{0.85} \right) = 606 \text{ K}$

Also, $\dfrac{T_6}{T_7} = \left(\dfrac{p_6}{p_7} \right)^{\frac{\gamma - 1}{\gamma}}$

or $\dfrac{p_6}{p_7} = \left(\dfrac{T_6}{T_7} \right)^{\frac{\gamma}{\gamma - 1}} = \left(\dfrac{898}{606} \right)^{\frac{1.333}{0.333}} = 4.82$

Then, $\dfrac{p_8}{p_9} = \dfrac{9}{4.82} = 1.86$

Again, $\dfrac{T_8}{T_9} = \left(\dfrac{p_8}{p_9} \right)^{\frac{\gamma - 1}{\gamma}} = (1.86)^{\frac{1.333 - 1}{1.333}} = 1.16$

$\therefore$ $T_9 = \dfrac{T_8}{1.16} = \dfrac{898}{1.16} = 774 \text{ K}$

Also, $\eta_{\text{turbine (L.P.)}} = \dfrac{T_8 - T_9'}{T_8 - T_9}$; $0.85 = \dfrac{898 - T_9'}{898 - 774}$

∴ $T_9' = 898 - 0.85 \, (898 - 774) = 792.6$ K

∴   Net work output $= c_{pg} \, (T_8 - T_9') \times 0.95$

                             $= 1.15 \, (898 - 792.6) \times 0.95 = 115.15$ kJ/kg

Thermal ratio or effectiveness of heat exchanger,

$$\varepsilon = \frac{T_5 - T_4'}{T_9' - T_4'} = \frac{T_5 - 428}{792.6 - 428}$$

i.e., $0.8 = \dfrac{T_5 - 428}{792.6 - 428}$

∴ $T_5 = 0.8 \, (792.6 - 428) + 428 = 719.7$ K

Now, Heat supplied $= c_{pg} \, (T_6 - T_5) + c_{pg} \, (T_8 - T_7')$

                          $= 1.15 \, (898 - 719.7) + 1.15 \, (898 - 650) = 490.2$ kJ/kg

∴ $\eta_{\text{thermal}} = \dfrac{\text{net work output}}{\text{heat supplied}} = \dfrac{115.15}{490.2}$

                          $= \mathbf{0.235}$ or **23.5%.**   **(Ans.)**

(ii) **Work ratio** = ?

Gross work of the plant $= W_{\text{turbine (H.P.)}} + W_{\text{turbine (L.P.)}}$

                          $= 285.5 + \dfrac{115.15}{0.95} = 406.7$ kJ/kg

∴   **Work ratio** $= \dfrac{\text{net work output}}{\text{gross work output}} = \dfrac{115.15}{406.7} = \mathbf{0.283.}$ **(Ans.)**

(iii) **Mass flow rate, m = ?**

Let the mass flow rate be m, then $m \times 115.15 = 4500$

∴ $m = \dfrac{4500}{115.15} = 39.08$ kg/s

i.e.,   **Mass flow = 39.08 kg/s.   (Ans.)**

Example 4.48. *In a closed cycle gas turbine there is a two stage compressor and a two stage turbine. All the components are mounted on the same shaft. The pressure and temperature at the inlet of the first stage compressor are 1.5 bar and 20°C. The maximum cycle temperature and pressure are limited to 750°C and 6 bar. A perfect intercooler is used between the two stage compressors and a reheater is used between the two turbines. Gases are heated in the reheater to 750°C before entering into the L.P. turbine. Assuming the compressor and turbine efficiencies as 0.82, calculate :*

   (i) The efficiency of the cycle without regenerator.

   (ii) The efficiency of the cycle with a regenerator whose effectiveness is 0.70.

   (iii) The mass of the fluid circulated if the power developed by the plant is 350 kW.

   The working fluid used in the cycle is air. For air :

               $\gamma = 1.4$ and $c_p = 1.005$ kJ / kg K.

Solution. $T_1 = 20 + 273 = 293$ K, $T_5 = T_7 = 750 + 273 = 1023$ K, $p_1 = 1.5$ bar,

               $p_2 = 6$ bar, $\eta_{\text{compressor}} = \eta_{\text{turbine}} = 0.82$,

Effectiveness of regenerator, $\varepsilon = 0.70$, power developed, $P = 350$ kW.

For air : $c_p = 1.005$ kJ/kg K, $\gamma = 1.4$
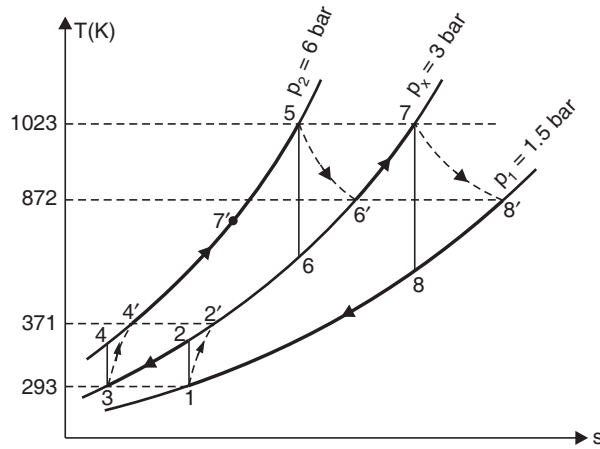
Fig. 4.65

As per given conditions : $T_1 = T_3$, $T_2' = T_4'$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{and} \quad p_x = \sqrt{p_1 p_2} = \sqrt{1.5 \times 6} = 3 \text{ bar}$$

Now
$$T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 293 \times \left(\frac{3}{1.5}\right)^{\frac{1.4-1}{1.4}} = 357 \text{ K}$$

$$\eta_{\text{compressor (L.P.)}} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.82 = \frac{357 - 293}{T_2' - 293}$$

$\therefore$
$$T_2' = \frac{357 - 293}{0.82} + 293 = 371 \text{ K}$$

i.e.,
$$T_2' = T_4' = 371 \text{ K}$$

Now
$$\frac{T_5}{T_6} = \left(\frac{p_5}{p_6}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_x}\right)^{\frac{1.4-1}{1.4}} \qquad \left[\begin{matrix} \because \ p_5 = p_2 \\ p_6 = p_x \end{matrix}\right]$$

$$\frac{1023}{T_6} = \left(\frac{6}{3}\right)^{0.286} = 1.219$$

$\therefore$
$$T_6 = \frac{1023}{1.219} = 839 \text{ K}$$

$$\eta_{\text{turbine (H.P.)}} = \frac{T_5 - T_6'}{T_5 - T_6}$$

$$0.82 = \frac{1023 - T_6'}{1023 - 839}$$

$\therefore$
$$T_6' = 1023 - 0.82 (1023 - 839) = 872 \text{ K}$$
$$T_8' = T_6' = 872 \text{ K as } \eta_{\text{turbine (H.P.)}} = \eta_{\text{turbine (L.P.)}}$$
and
$$T_7 = T_5 = 1023 \text{ K}$$

Effectiveness of regenerator, $\varepsilon = \dfrac{T' - T_4'}{T_8' - T_4'}$

where T' is the temperature of air coming out of regenerator

$\therefore$ $\qquad 0.70 = \dfrac{T' - 371}{872 - 371}$

i.e., $\qquad T' = 0.70\,(872 - 371) + 371 = 722$ K

Net work available,

$$W_{net} = [W_{T(H.P.)} + W_{T(L.P.)}] - [W_{C(H.P.)} + W_{C(L.P.)}]$$

$= 2[W_{T(L.P.)} - W_{C(L.P.)}]$ as the *work developed by each turbine is same and work absorbed by each compressor is same.*

$$W_{net} = 2c_p\,[(T_5 - T_6') - (T_2' - T_1)]$$
$$= 2 \times 1.005\,[(1023 - 872) - (371 - 293)] = 146.73 \text{ kJ/kg of air}$$

Heat supplied per kg of air *without regenerator*

$$= c_p\,(T_5 - T_4') + c_p\,(T_7 - T_6')$$
$$= 1.005\,[(1023 - 371) + (1023 - 872)] = 807 \text{ kJ/kg of air}$$

Heat supplied per kg of *air with regenerator*

$$= c_p\,(T_5 - T') + c_p\,(T_7 - T_6')$$
$$= 1.005\,[(1023 - 722) + (1023 - 872)] = 454.3 \text{ kJ/kg}$$

(*i*) $\eta_{\text{thermal (without regenerator)}} = \dfrac{146.73}{807} = 0.182$ or **18.2%.   (Ans.)**

(*ii*) $\eta_{\text{thermal (with regenerator)}} = \dfrac{146.73}{454.3} = 0.323$ or **32.3%.   (Ans.)**

(*iii*) **Mass of fluid circulated, m = ?**

Power developed, $\qquad P = 146.73 \times \dot{m}$ kW

$\therefore$ $\qquad\qquad 350 = 146.73 \times \dot{m}$

i.e., $\qquad\qquad \dot{m} = \dfrac{350}{146.73} = 2.38$ kg/s

i.e., **Mass of fluid circulated = 2.38 kg/s.   (Ans.)**

$\boxed{\textbf{HIGHLIGHTS}}$

1. A cycle is defined as a repeated series of operations occurring in a certain order.
2. The efficiency of an engine using air as the working medium is known as an '**Air standard efficiency**'.
3. Relative efficiency, $\quad \eta_{\text{relative}} = \dfrac{\text{actual thermal efficiency}}{\text{air standard efficiency}}$.
4. **Carnot cycle** efficiency,

$$\eta_{\text{Carnot}} = \frac{T_1 - T_2}{T_1}$$

5. **Otto cycle** efficiency, $\quad \eta_{\text{Otto}} = 1 - \dfrac{1}{(r)^{\gamma - 1}}$.

Mean effective pressure,

$$p_{m(\text{Otto})} = \frac{p_1 r[(r^{\gamma-1} - 1)(r_p - 1)]}{(\gamma - 1)(r - 1)}$$

6. **Diesel cycle** efficiency,

$$\eta_{\text{Diesel}} = 1 - \frac{1}{\gamma(r)^{\gamma-1}}\left[\frac{\rho^\gamma - 1}{\rho - 1}\right]$$

Mean effective pressure,

$$p_{m(\text{Diesel})} = \frac{p_1 r^\gamma[\gamma(\rho - 1) - r^{1-\gamma}(\rho^\gamma - 1)]}{(\gamma - 1)(r - 1)}$$

7. **Dual cycle** efficiency,

$$\eta_{\text{dual}} = 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{(\beta.\rho^\gamma - 1)}{(\beta - 1) + \beta\gamma(\rho - 1)}\right]$$

Mean effective pressure,

$$p_{m(\text{dual})} = \frac{p_1 r^\gamma[\beta(\rho - 1) + (\beta - 1) - r^{1-\gamma}(\beta\rho^\gamma - 1)]}{(\gamma - 1)(r - 1)} \, .$$

8. Atkinson cycle efficiency, $\eta_{\text{Atkinson}} = 1 - \gamma . \dfrac{(r - \alpha)}{r^\gamma - \alpha^\gamma}$

where α = compression ratio
         r = expansion ratio.

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. The air standard Otto cycle comprises
   (*a*) two constant pressure processes and two constant volume processes
   (*b*) two constant pressure and two constant entropy processes
   (*c*) two constant volume processes and two constant entropy processes
   (*d*) none of the above.

2. The air standard efficiency of Otto cycle is given by
   (*a*) $\eta = 1 + \dfrac{1}{(r)^{\gamma+1}}$ (*b*) $\eta = 1 - \dfrac{1}{(r)^{\gamma-1}}$
   (*c*) $\eta = 1 - \dfrac{1}{(r)^{\gamma+1}}$ (*d*) $\eta = 2 - \dfrac{1}{(r)^{\gamma-1}}$.

3. The thermal efficiency of theoretical Otto cycle
   (*a*) increases with increase in compression ratio (*b*) increases with increase in isentropic index γ
   (*c*) does not depend upon the pressure ratio   (*d*) follows all the above.

4. The work output of theoretical Otto cycle
   (*a*) increases with increase in compression ratio (*b*) increases with increase in pressure ratio
   (*c*) increases with increase in adiabatic index γ
   (*d*) follows all the above.

5. For same compression ratio
   (*a*) thermal efficiency of Otto cycle is greater than that of Diesel cycle
   (*b*) thermal efficiency of Otto cycle is less than that of Diesel cycle
   (*c*) thermal efficiency of Otto cycle is same as that for Diesel cycle
   (*d*) thermal efficiency of Otto cycle cannot be predicted.

6. In air standard Diesel cycle, at fixed compression ratio and fixed value of adiabatic index (γ)
   (a) thermal efficiency increases with increase in heat addition cut off ratio
   (b) thermal efficiency decreases with increase in heat addition cut off ratio
   (c) thermal efficiency remains same with increase in heat addition cut off ratio
   (d) none of the above.

7. Thermal efficiency of a gas turbine plant as compared to Diesel engine plant is
   (a) higher   (b) lower
   (c) same   (d) may be higher or lower.

8. Mechanical efficiency of a gas turbine as compared to internal combustion reciprocating engine is
   (a) higher   (b) lower
   (c) same   (d) un-predictable.

9. For a gas turbine the pressure ratio may be in the range
   (a) 2 to 3   (b) 3 to 5
   (c) 16 to 18   (d) 18 to 22.

10. The air standard efficiency of closed gas turbine cycle is given by (r_p = pressure ratio for the compressor and turbine)

    (a) $\eta = 1 - \dfrac{1}{(r_p)^{\gamma-1}}$ (b) $\eta = 1 - (r_p)^{\gamma-1}$

    (c) $\eta = 1 - \left(\dfrac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}$ (d) $\eta = (r_p)^{\frac{\gamma-1}{\gamma}} - 1$.

11. The work ratio of closed cycle gas turbine plant depends upon
    (a) pressure ratio of the cycle and specific heat ratio
    (b) temperature ratio of the cycle and specific heat ratio
    (c) pressure ratio, temperature ratio and specific heat ratio
    (d) only on pressure ratio.

12. Thermal efficiency of closed cycle gas turbine plant increases by
    (a) reheating   (b) intercooling
    (c) regenerator   (d) all of the above.

13. With the increase in pressure ratio thermal efficiency of a simple gas turbine plant with fixed turbine inlet temperature
    (a) decreases   (b) increases
    (c) first increases and then decreases   (d) first decreases and then increases.

14. The thermal efficiency of a gas turbine cycle with ideal regenerative heat exchanger is
    (a) equal to work ratio   (b) is less than work ratio
    (c) is more than work ratio   (d) unpredictable.

15. In a two stage gas turbine plant reheating after first stage
    (a) decreases thermal efficiency   (b) increases thermal efficiency
    (c) does not affect thermal efficiency   (d) none of the above.

16. In a two stage gas turbine plant, reheating after first stage
    (a) increases work ratio   (b) decreases work ratio
    (c) does not affect work ratio   (d) none of the above.

17. In a two stage gas turbine plant, with intercooling and reheating
    (a) both work ratio and thermal efficiency improve
    (b) work ratio improves but thermal efficiency decreases
    (c) thermal efficiency improves but work ratio decreases
    (d) both work ratio and thermal efficiency decrease.

18. For a jet propulsion unit, ideally the compressor work and turbine work are
    (a) equal   (b) unequal
    (c) not related to each other   (d) unpredictable.

19. Greater the difference between jet velocity and aeroplane velocity
 (*a*) greater the propulsive efficiency (*b*) less the propulsive efficiency
 (*c*) unaffected is the propulsive efficiency (*d*) none of the above.

Aanswers

1. (*b*)	**2.** (*b*)	**3.** (*d*)	**4.** (*d*)	**5.** (*a*)	**6.** (*b*)
7. (*b*)	**8.** (*b*)	**9.** (*d*)	**10.** (*d*)	**11.** (*a*)	**12.** (*b*)
13. (*b*)	**14.** (*b*)	**15.** (*d*)	**16.** (*d*)	**17.** (*a*)	**18.** (*a*)
19. (*c*).					

THEORETICAL QUESTIONS

1. What is a cycle ? What is the difference between an ideal and actual cycle ?
2. What is an air-standard efficiency ?
3. What is relative efficiency ?
4. Derive expressions of efficiency in the following cases :
 (*i*) Carnot cycle (*ii*) Diesel cycle (*iii*) Dual combustion cycle.
5. Explain "Air standard analysis" which has been adopted for I.C. engine cycles. State the assumptions made for air standard cycles.
6. Derive an expression for 'Atkinson cycle'.
7. Explain briefly Brayton cycle. Derive expression for optimum pressure ratio.
8. Describe with neat sketches the working of a simple constant pressure open cycle gas turbine.
9. Discuss briefly the methods employed for improvement of thermal efficiency of open cycle gas turbine plant.
10. Describe with neat diagram a closed cycle gas turbine. State also its merits and demerits.

UNSOLVED EXAMPLES

1. A Carnot engine working between 377°C and 37°C produces 120 kJ of work. Determine :
 (*i*) The heat added in kJ.
 (*ii*) The entropy change during heat rejection process.
 (*iii*) The engine thermal efficiency. [**Ans.** (*i*) 229.5 kJ ; (*ii*) 0.353 kJ/K; (*iii*) 52.3%]
2. Find the thermal efficiency of a Carnot engine whose hot and cold bodies have temperatures of 154°C and 15°C respectively. [**Ans.** 32.55%]
3. Derive an expression for change in efficiency for a change in compression ratio. If the compression ratio is increased from 6 to 8, what will be the percentage increase in efficiency ? [**Ans.** 8%]
4. The efficiency of an Otto cycle is 50% and γ is 1.5. What is the compression ratio ? [**Ans.** 4]
5. An engine working on Otto cycle has a volume of 0.5 m³, pressure 1 bar and temperature 27°C at the commencement of compression stroke. At the end of compression stroke, the pressure is 10 bar. Heat added during the constant volume process is 200 kJ. Determine :
 (*i*) Percentage clearance (*ii*) Air standard efficiency
 (*iii*) Mean effective pressure
 (*iv*) Ideal power developed by the engine if the engine runs at 400 r.p.m. so that there are 200 complete cycles per minutes. [**Ans.** (*i*) 23.76% ; (*ii*) 47.2% ; (*iii*) 2.37 bar; (*iv*) 321 kW]
6. The compression ratio in an air-standard Otto cycle is 8. At the beginning of compression process, the pressure is 1 bar and the temperature is 300 K. The heat transfer to the air per cycle is 1900 kJ/kg of air. Calculate :
 (*i*) Thermal efficiency (*ii*) The mean effective pressure.
  [**Ans.** (*i*) 56.47% ; (*ii*) 14.24 bar]

7. An engine 200 mm bore and 300 mm stroke works on Otto cycle. The clearance volume is 0.0016 m^3. The initial pressure and temperature are 1 bar and 60°C. If the maximum pressure is limited to 24 bar, find :

    (*i*) The air-standard efficiency of the cycle   (*ii*) The mean effective pressure for the cycle.

    Assume ideal conditions.   [**Ans.** (*i*) 54.08% ; (*ii*) 1.972 bar]

8. Calculate the air standard efficiency of a four stroke Otto cycle engine with the following data :

    Piston diameter (bore) = 137 mm ; Length of stroke = 130 mm ;

    Clearance volume 0.00028 m^3.

    Express clearance as a percentage of swept volume.   [**Ans.** 56.1% ; 14.6%]

9. In an ideal Diesel cycle, the temperatures at the beginning of compression, and the end of compression and at the end of the heat addition are 97°C, 789°C and 1839°C. Find the efficiency of the cycle.

                                                                       [**Ans.** 59.6%]

10. An air-standard Diesel cycle has a compression ratio of 18, and the heat transferred to the working fluid per cycle is 1800 kJ/kg. At the beginning of the compression stroke, the pressure is 1 bar and the temperature is 300 K. Calculate : (*i*) Thermal efficiency, (*ii*) The mean effective pressure.

                                                                       [**Ans.** (*i*) 61% ; (*ii*) 13.58 bar]

11. 1 kg of air is taken through a Diesel cycle. Initially the air is at 15°C and 1 ata. The compression ratio is 15 and the heat added is 1850 kJ. Calculate : (*i*) The ideal cycle efficiency, (*ii*) The mean effective pressure.   [**Ans.** (*i*) 55.1% ; (*ii*) 13.4 bar]

12. What will be loss in the ideal efficiency of a Diesel engine with compression ratio 14 if the fuel cut off is delayed from 6% to 9% ?   [**Ans.** 2.1%]

13. The pressures on the compression curve of a diesel engine are at $\dfrac{1}{8}$ th stroke 1.4 bar and at $\dfrac{7}{8}$ th stroke 14 bar. Estimate the compression ratio. Calculate the air standard efficiency of the engine if the cut off occurs at $\dfrac{1}{15}$ of the stroke.   [**Ans.** 18.54 ; 63.7%]

14. A compression ignition engine has a stroke 270 mm, and a cylinder diameter of 165 mm. The clearance volume is 0.000434 m^3 and the fuel ignition takes place at constant pressure for 4.5 per cent of the stroke. Find the efficiency of the engine assuming it works on the Diesel cycle.   [**Ans.** 61.7%]

15. The following data belong to a Diesel cycle :

    Compression ratio = 16 : 1 ; Heat added = 2500 kJ/kg ; Lowest pressure in the cycle = 1 bar ; Lowest temperature in the cycle = 27°C.

    Determine :

    (*i*) Thermal efficiency of the cycle.

    (*ii*) Mean effective pressure.   [**Ans.** (*i*) 45% ; (*ii*) 16.8 bar]

16. The compression ratio of an air-standard Dual cycle is 12 and the maximum pressure in the cycle is limited to 70 bar. The pressure and temperature of cycle at the beginning of compression process are 1 bar and 300 K. Calculate : (*i*) Thermal efficiency, (*ii*) Mean effective pressure.

    Assume : cylinder bore = 250 mm, stroke length = 300 mm, c_p = 1.005, c_v = 0.718 and γ = 1.4.

                                                                       [**Ans.** (*i*) 61.92% ; (*ii*) 9.847 bar]

17. The compression ratio of a Dual cycle is 10. The temperature and pressure at the beginning of the cycle are 1 bar and 27°C. The maximum pressure of the cycle is limited to 70 bar and heat supplied is limited to 675 kJ/kg of air. Find the thermal efficiency of the cycle.   [**Ans.** 59.5%]

18. An air standard dual cycle has a compression ratio of 16, and compression begins at 1 bar, 50°C. The maximum pressure is 70 bar. The heat transferred to air at constant pressure is equal to that at constant volume. Determine :

    (*i*) The cycle efficiency.

    (*ii*) The mean effective pressure of the cycle.

    Take : c_p = 1.005 kJ/kg K, c_v = 0.718 kJ/kg K   [**Ans.** (*i*) 66.5% ; (*ii*) 4.76 bar]

19. In an air standard gas turbine engine, air at a temperature of 15°C and a pressure of 1.01 bar enters the compressor, where it is compressed through a pressure ratio of 5. Air enters the turbine at a temperature of 815°C and expands to original pressure of 1.01 bar. Determine the ratio of turbine

work to compressor work and the thermal efficiency when the engine operates on ideal Brayton cycle. Take : $\gamma = 1.4$, $c_p = 1.005$ kJ/kg K. **[Ans. 2.393 ; 37.03%]**

20. In an open cycle constant pressure gas turbine air enters the compressor at 1 bar and 300 K. The pressure of air after the compression is 4 bar. The isentropic efficiencies of compressor and turbine are 78% and 85% respectively. The air-fuel ratio is 80 : 1. Calculate the power developed and thermal efficiency of the cycle if the flow rate of air is 2.5 kg/s.

    Take $c_p = 1.005$ kJ/kg K and $\gamma = 1.4$ for air and $c_{pg} = 1.147$ kJ/kg K and $\gamma = 1.33$ for gases. $R = 0.287$ kJ/kg K. Calorific value of fuel = 42000 kJ/kg.   **[Ans. 204.03 kW/kg of air ; 15.54%]**

21. A gas turbine has a pressure ratio of 6/1 and a maximum cycle temperature of 600°C. The isentropic efficiencies of the compressor and turbine are 0.82 and 0.85 respectively. Calculate the power output in kilowatts of an electric generator geared to the turbine when the air enters the compressor at 15°C at the rate of 15 kg/s.

    Take : $c_p = 1.005$ kJ/kg K and $\gamma = 1.4$ for the compression process, and take $c_p = 1.11$ kJ/kg K and $\gamma = 1.333$ for the expansion process.   **[Ans. 920 kW]**

22. Calculate the thermal efficiency and the work ratio of the plant in example 21 (above), assuming that c_p for the combustion process is 1.11 kJ/kg K.   **[Ans. 15.8% ; 0.206]**

23. The gas turbine has an overall pressure ratio of 5 : 1 and a maximum cycle temperature of 550°C. The turbine drives the compressor and an electric generator, the mechanical efficiency of the drive being 97%. The ambient temperature is 20°C and the isentropic efficiencies of the compressor and turbine are 0.8 and 0.83 respectively. Calculate the power output in kilowatts for an air flow of 15 kg/s. Calculate also the thermal efficiency and the work ratio.

    Neglect changes is kinetic energy, and the loss of pressure in combustion chamber.

    [Ans. 655 kW ; 12% ; 0.168]

24. Air is drawn in a gas turbine unit at 17°C and 1.01 bar and the pressure ratio is 8 : 1. The compressor is driven by the H.P. turbine and the L.P. turbine drives a separate power shaft. The isentropic efficiencies of the compressor, and the H.P. and L.P. turbines are 0.8, 0.85, and 0.83, respectively. Calculate the pressure and temperature of the gases entering the power turbine, the net power developed by the unit per kg/s of mass flow, the work ratio and the thermal efficiency of the unit. The maximum cycle temperature is 650°C.

    For the compression process take $c_p = 1.005$ kJ/kg K and $\gamma = 1.4$.

    For the combustion process and expansion process, take

    $c_p = 1.15$ kJ/kg K and $\gamma = 1.333$

    Neglect the mass of fuel.   **[Ans. 1.65 bar, 393°C ; 74.5 kW ; 0.201 ; 19.1%]**

25. In a gas turbine plant, air is compressed through a pressure ratio of 6 : 1 from 15°C. It is then heated to the maximum permissible temperature of 750°C and expanded in two stages each of expansion ratio $\sqrt{6}$, the air being reheated between the stages to 750°C. A heat exchanger allows the heating of the compressed gases through 75 per cent of the maximum range possible. Calculate : (*i*) The cycle efficiency (*ii*) The work ratio (*iii*) The work per kg of air.

    The isentropic efficiencies of the compressor and turbine are 0.8 and 0.85 respectively.

    [Ans. (*i*) 32.75%; (*ii*) 0.3852; (*iii*) 152 kJ/kg]

26. At the design speed the following data apply to a gas turbine set employing the heat exchanger : Isentropic efficiency of compressor = 75%, isentropic efficiency of the turbine = 85%, mechanical transmission efficiency = 99%, combustion efficiency = 98%, mass flow = 22.7 kg/s, pressure ratio = 6 : 1, heat exchanger effectiveness = 75%, maximum cycle temperature = 1000 K.

    The ambient air temperature and pressure are 15°C and 1.013 bar respectively. Calculate :

     (*i*) The net power output   (*ii*) Specific fuel consumption

    (*iii*) Thermal efficiency of the cycle.

    Take the lower calorific value of fuel as 43125 kJ/kg and assume no pressure-loss in heat exchanger and combustion chamber.   **[Ans. (*i*) 2019 kW; (*ii*) 0.4799 kg/kWh; (*iii*) 16.7%]**

27. In a gas turbine plant air at 10°C and 1.01 bar is compressed through a pressure ratio of 4 : 1. In a heat exchanger and combustion chamber the air is heated to 700°C while its pressure drops 0.14 bar. After expansion through the turbine the air passes through a heat exchanger which cool the air through 75% of maximum range possible, while the pressure drops 0.14 bar, and the air is finally exhausted to atmosphere. The isentropic efficiency of the compressor is 0.80 and that of turbine 0.85.

    Calculate the efficiency of the plant.   **[Ans. 22.76%]**

28. In a marine gas turbine unit a high-pressure stage turbine drives the compressor, and a low-pressure stage turbine drives the propeller through suitable gearing. The overall pressure ratio is 4 : 1, and the maximum temperature is 650°C. The isentropic efficiencies of the compressor, H.P. turbine, and L.P. turbine are 0.8, 0.83 and 0.85 respectively, and the mechanical efficiency of both shafts is 98%. Calculate the pressure between turbine stages when the air intake conditions are 1.01 bar and 25°C. Calculate also the thermal efficiency and the shaft power when the mass flow is 60 kg/s. Neglect kinetic energy changes, and pressure loss in combustion.   [**Ans.** 1.57 bar ; 14.9% ; 4560 kW]

29. In a gas turbine unit comprising L.P. and H.P. compressors, air is taken at 1.01 bar 27°C. Compressor in L.P. stage is upto 3.03 bar followed by intercooling to 30°C. The pressure of air after H.P. compressor is 58.7 bar. Loss in pressure during intercooling is 0.13 bar. Air from H.P. compressor is transferred to heat exchanger of effectiveness 0.60 where it is heated by gases from L.P. turbine. The temperature of gases supplied to H.P. turbine is 750°C. The gases expand in H.P. turbine to 3.25 bar and are then reheated to 700°C before expanding in L.P. turbine. The loss of pressure in reheater is 0.1 bar. If isentropic efficiency of compression in both stages is 0.80 and isentropic efficiency of expansion in turbine is 0.85 calculate : (*i*) Overall efficiency (*ii*) Work ratio (*iii*) Mass flow rate when the gas power generated is 6500 kW. Neglect the mass of fuel.

Take, for air : c_p = 1.005 kJ/kg K, γ = 1.4

for gases : c_{pg} = 1.15 kJ/h K, γ = 1.3.   [**Ans.** (*i*) 16.17%; (*ii*) 0.2215; (*iii*) 69.33 kg of air/sec.]

30. In a gas turbine installation, air is taken in L.P. compressor at 15°C, 1.1 bar and after compression it is passed through intercooler where its temperature is reduced to 22°C. The cooled air is further compressed in H.P. unit and then passed in the combustion chamber where its temperature is increased to 677°C by burning the fuel. The combustion products expand in H.P. turbine which runs the compressor and further expansion is continued in the L.P. turbine which runs the alternator. The gases coming out from L.P. turbine are used for heating the incoming air from H.P. compressor and then exhausted to atmosphere.

Taking the following data determine : (*i*) Power output (*ii*) Specific fuel consumption (*iii*) Thermal efficiency :

Pressure ratio of each compressor = 2, isentropic efficiency of each compressor stage = 85%, isentropic efficiency of each turbine stage = 85%, effectiveness of heat exchanger = 0.75, air flow = 13 kg/sec., calorific value of fuel = 45000 kJ/kg, c_p (for gas) = 1 kJ/kg K, c_p (for gas) = 1.15 kJ/kg K, γ (for air) = 1.4, γ (for gas) = 1.33.

Neglect the mechanical, pressure and heat losses of the system and fuel mass also.

[**Ans.** (*i*) 1849.2 kW; (*ii*) 0.241 kg/kWh; (*iii*) 33.17%]

5

Internal Combustion Engines

5.1. Heat engines. 5.2. Development of I.C. engines. 5.3. Classification of I.C. engines. 5.4. Applications of I.C. engines. 5.5. Basic idea of I.C. engine. 5.6. Different parts of I.C. engines. 5.7. Terms connected with I.C. engines. 5.8. Working cycles. 5.9. Indicator diagram. 5.10. Four stroke cycle engines. 5.11. Two stroke cycle engines. 5.12. Comparison of four stroke and two stroke cycle engines. 5.13. Comparison of spark ignition and compression ignition engines. 5.14. Comparison between a petrol engine and a diesel engine. 5.15. How to tell a two stroke cycle engine from a four stroke cycle engine ? —*Theoretical Questions.*

5.1. HEAT ENGINES

Any type of engine or machine which derives heat energy from the combustion of fuel or any other source and converts this energy into mechanical work is termed as a heat engine.

Heat engines may be classified into two main classes as follows :

1. External Combustion Engines.

2. Internal Combustion Engines.

1. External Combustion Engines (E.C. Engines)

In this case, *combustion of fuel takes place outside the cylinder* as in case of *steam engines* where the heat of combustion is employed to generate steam which is used to move a piston in a cylinder. Other examples of external combustion engines are *hot air engines, steam turbine* and *closed cycle gas turbine.* These engines are generally used for driving locomotives, ships, generation of electric power etc.

2. Internal Combustion Engines (I.C. Engines)

In this case, *combustion of the fuel with oxygen of the air occurs within the cylinder* of the engine. The internal combustion engines group includes engines employing mixtures of combustible gases and air, known as *gas engines*, those *using lighter liquid fuel* or spirit known as *petrol engines* and those using heavier liquid fuels, known as *oil compression ignition* or *diesel engines.*

The 'external combustion engines' *claim the following advantages over internal combustion engines :*

1. Starting torque is generally high.

2. Because of external combustion of fuel, cheaper fuels can be used. Even solid fuels can be used advantageously.

3. Due to external combustion of fuel it is possible to have flexibility in arrangement.

4. These units are self-starting with the working fluid whereas in case of internal combustion engines, some additional equipment or device is used for starting the engines.

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'Reciprocating internal combustion engines' offer the following *advantages over external combustion engines :*

1. Overall efficiency is high.
2. Greater mechanical simplicity.
3. Weight to power ratio is generally low.
4. Generally lower initial cost.
5. Easy starting from cold conditions.
6. These units are compact and thus require less space.

5.2.   DEVELOPMENT OF I.C. ENGINES

Many experimental engines were constructed around 1878. The first really successful engine did not appear, however until 1879, when a German engineer Dr. Otto built his famous Otto gas engine. The operating cycle of this engines was based upon principles first laid down in 1860 by a French engineer named Bea de Rochas. The majority of modern I.C. engines operate according to these principles.

The development of the well known Diesel engine began about 1883 by Rudoff Diesel. Although this differs in many important respects from the Otto engine, the operating cycle of modern high speed Diesel engines is thermodynamically very similar to the Otto cycle.

5.3.   CLASSIFICATION OF I.C. ENGINES

Internal combustion engines may be classified as given below :

1. **According to cycle of operation :**
   (*i*) Two stroke cycle engines
   (*ii*) Four stroke cycle engines.

2. **According to cycle of combustion :**
   (*i*) Otto cycle engine (combustion at constant volume)
   (*ii*) Diesel cycle engine (combustion at constant pressure)
   (*iii*) Dual-combustion or Semi-diesel cycle engine (combustion partly at constant volume and partly at constant pressure).

3. **According to arrangement of cylinder :**
   (*i*) Horizontal engine   (*ii*) Vertical engine
   (*iii*) *V*-type engine   (*iv*) Radial engine etc.

4. **According to their uses :**
   (*i*) Stationary engine   (*ii*) Portable engine
   (*iii*) Marine engine   (*iv*) Automobile engine
   (*v*) Aero-engine etc.

5. **According to the fuel employed and the method of fuel supply to the engine cylinder :**
   (*i*) Oil engine   (*ii*) Petrol engine
   (*iii*) Gas engine   (*iv*) Kerosene engine
   (*v*) Carburettor, hot bulb, solid injection and air injection engine.

6. **According to the speed of the engine :**

 (*i*) Low speed engine   (*ii*) Medium speed engine

(*iii*) High speed engine.

7. **According to method of ignition :**

 (*i*) Spark ignition engine   (*ii*) Compression ignition engine.

8. **According to method of cooling the cylinder :**

 (*i*) Air-cooled engine   (*ii*) Water-cooled engine.

9. **According to method of Governing :**

 (*i*) Hit and miss governed engine   (*ii*) Quality governed engine.

(*iii*) Quantity governed engine.

10. **According to valve arrangement :**

 (*i*) Over head valve engine   (*ii*) *L*-head type engine

(*iii*) *T*-head type engine   (*iv*) *F*-head type engine.

11. **According to number of cylinders :**

 (*i*) Single cylinder engine   (*ii*) Multi-cylinder engine.

5.4. APPLICATIONS OF I.C. ENGINES

The I.C. engines are generally used for :

 (*i*) Road vehicles (*e.g.*, scooter, motorcycle, buses etc.)

 (*ii*) Air craft

(*iii*) Locomotives

 (*iv*) Construction in civil engineering equipment such as bull-dozer, scraper, power shovels etc.

 (*v*) Pumping sets

 (*vi*) Cinemas

(*vii*) Hospital

(*viii*) Several industrial applications.

Note. Prime movers in all *construction equipment* are invariably I.C. engines, unless of course, when drive is electric. Use of steam source for this equipment is almost absolete.

5.5. BASIC IDEA OF I.C. ENGINE

The basic idea of internal combustion engine is shown in Fig. 5.1. The cylinder which is closed at one end is filled with a mixture of fuel and air. As the crankshaft turns it pushes cylinder. The piston is forced up and compresses the mixture in the top of the cylinder. The mixture is set alight and, as it burns, it creates a gas pressure on the piston, forcing it down the cylinder. This motion is shown by arrow '1'. The piston pushes on the rod which pushes on the crank. The crank is given rotary (turning) motion as shown by the arrow '2'. The fly wheel fitted on the end of the crankshaft stores energy and keeps the crank turning steadily.
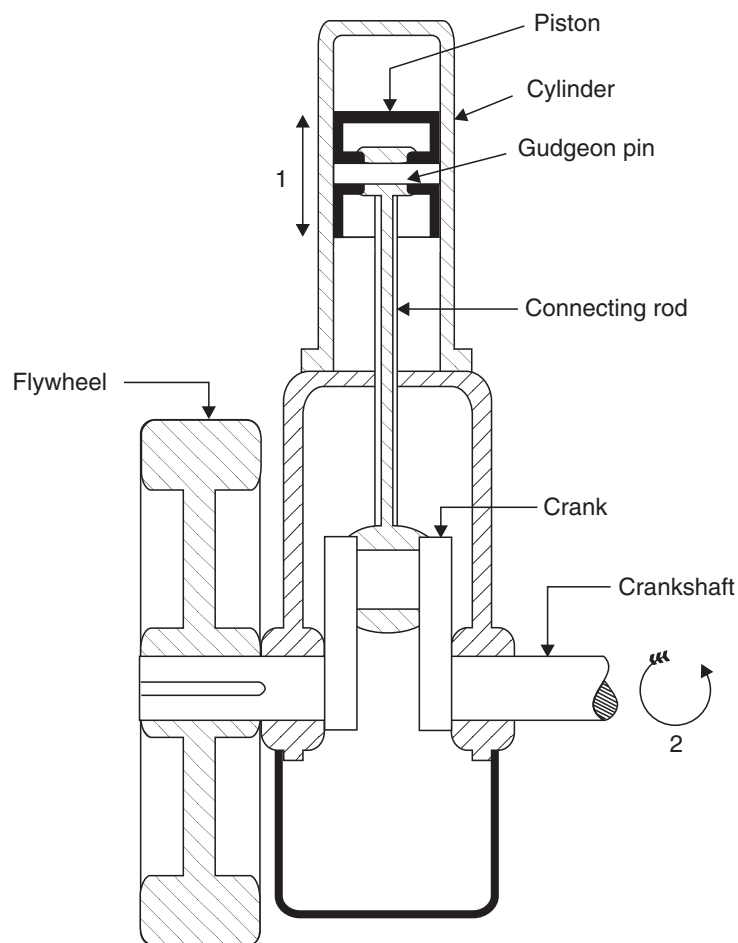
Fig. 5.1. Basic idea of I.C. engine.

5.6. DIFFERENT PARTS OF I.C. ENGINES

Here follows the detail of the various parts of an internal combustion engine.

A cross-section of an air cooled I.C. engine with principal parts is shown in Fig. 5.2.

A. Parts common to both petrol and diesel engine :

1. Cylinder	2. Cylinder head
3. Piston	4. Piston rings
5. Gudgeon pin	6. Connecting rod
7. Crank	8. Crankshaft
9. Engine bearing	10. Crankcase
11. Flywheel	12. Governor
13. Valves and valve gears.	

B. Parts for petrol engines only :

1. Spark plugs	2. Carburettor
3. Fuel pump.	

C. Parts for Diesel engine only :

1. Fuel pump.   2. Injector.
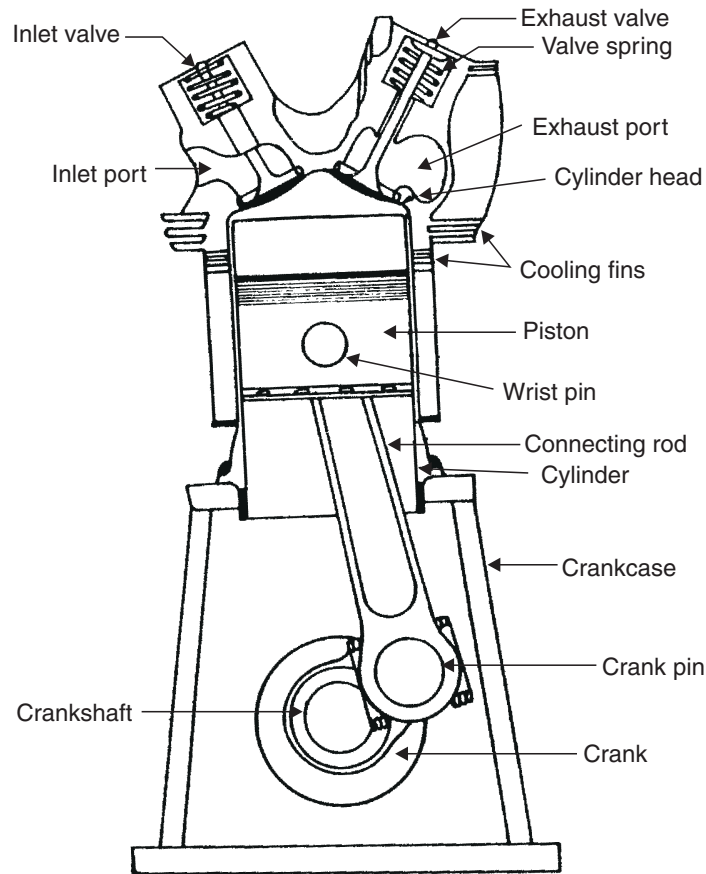
Fig. 5.2. Air-cooled I.C. engine.

A. Parts common to both petrol and diesel engines :

1. **Cylinder**

The cylinder contains gas under pressure and guides the piston. It is in direct contact with the products of combustion and it must be cooled. The ideal form consists of a plain cylindrical barrel in which the piston slides. The movement of the piston or stroke being in most cases, longer than the bore. This is known as the "*stroke bore ratio*". The upper end consists of a combustion or clearance space in which the ignition and combustion of the charge takes place. In practice, it is necessary to depart from the ideal hemispherical shape in order to accommodate the valves, sparking plugs etc. and to control the combustion. Sections of an air cooled cylinder and a water cooled cylinder are shown in Fig. 5.3 and 5.4 respectively. *The cylinder is made of hard grade cast-iron and is usually, cast in one piece.*

2. **Cylinder Head**

One end of the cylinder is closed by means of a *removable cylinder head* (Fig. 5.3) which usually contains the inlet or admission valve [Fig. 5.5 (*a*)] for admitting the moisture of air and fuel and exhaust valve [Fig. 5.5 (*b*)] for discharging the product of combustion. Two valves are kept
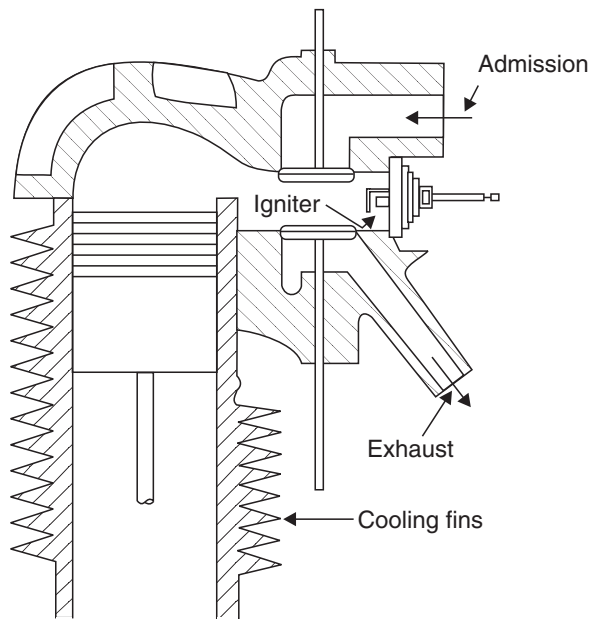
Fig. 5.3. Air-cooled cylinder.
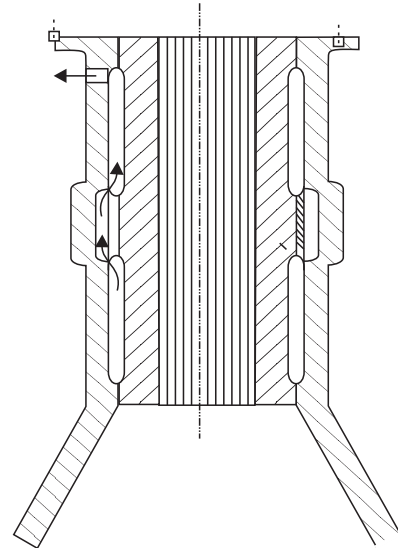
Fig. 5.4. Water-cooled cylinder.

closed, by means of cams (Fig. 5.6) geared to the engine shaft. The passage in the cylinder head leading to and from the valves are called *ports*. The pipes which connect the inlet ports of the

(a) Inlet valve   (b) Exhaust valve
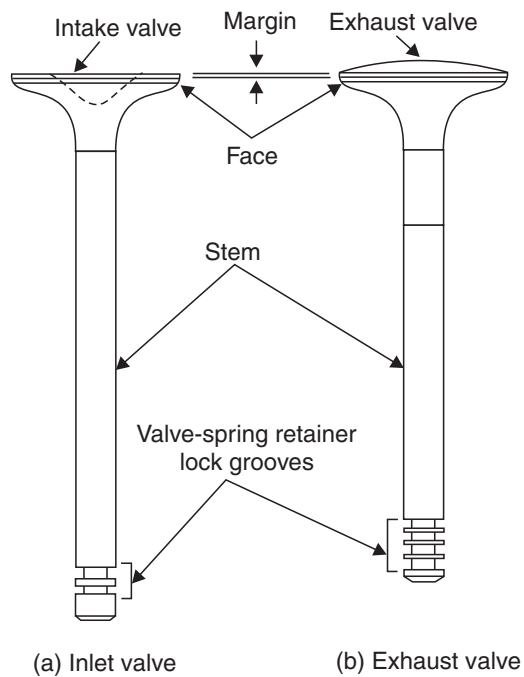
Fig. 5.5

various cylinders to a common intake pipe for the engine is called the inlet *manifold*. If the exhaust ports are similarly connected to a common exhaust system, this system of piping is called *exhaust manifold*.
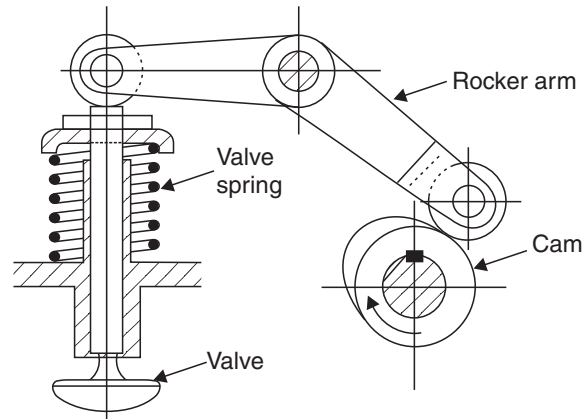
Fig. 5.6. Cam and rocker arm.

The main purpose of the cylinder head is to seal the working ends of the cylinders and not to permit entry and exit of gases on cover head valve engines. The inside cavity of head is called the *combustion chamber*, into which the mixture is compressed for firing. Its *shape controls the direction and rate of combustion*. Heads are drilled and tapped with correct thread to take the ignition spark plug. All the combustion chambers in an engine must be of same shape and size. The shape may be in part controlled by the piston shape.

The cylinder head is usually made of cast-iron or aluminium.

3. **Piston**

A piston is fitted to each cylinder *as a face to receive gas pressure and transmit the thrust to the connecting rod.*

The piston must (*i*) give gas tight seal to the cylinder through bore, (*ii*) slide freely, (*iii*) be light and (*iv*) be strong. The thrust on the piston on the power stroke tries to tilt the piston as the connecting rod swings, sideways. The piston wall, called the skirt must be strong enough to stand upto this side thrust. *Pistons are made of cast-iron or aluminium alloy for lightness*. Light alloy pistons expand more than cast iron one therefore they need large clearances to the bore, when cold, or special provision for expansion. Pistons may be solid skirt or split skirt. A section through a split skirt piston is shown in Fig. 5.7.

4. **Piston Rings**

The piston must be a fairly loose fit in the cylinder. If it were a tight fit, it would expand as it got hot and might stick tight in the cylinder. If a piston sticks it could ruin the engine. On the other hand, if there is too much clearance between the piston and cylinder walls, much of the pressure from the burning gasoline vapour will leak past the piston. This means, that the push on the piston will be much less effective. It is the push on the piston that delivers the power from the engines.

To provide a good sealing fit between the piston and cylinder, pistons are equipped with piston rings, as shown in Fig. 5.7. The rings are usually made of cast iron of fine grain and high elasticity which is not affected by the working heat. Some rings are of alloy spring steel. They are split at one point so that they can be expanded and slipped over the end of the piston and into ring
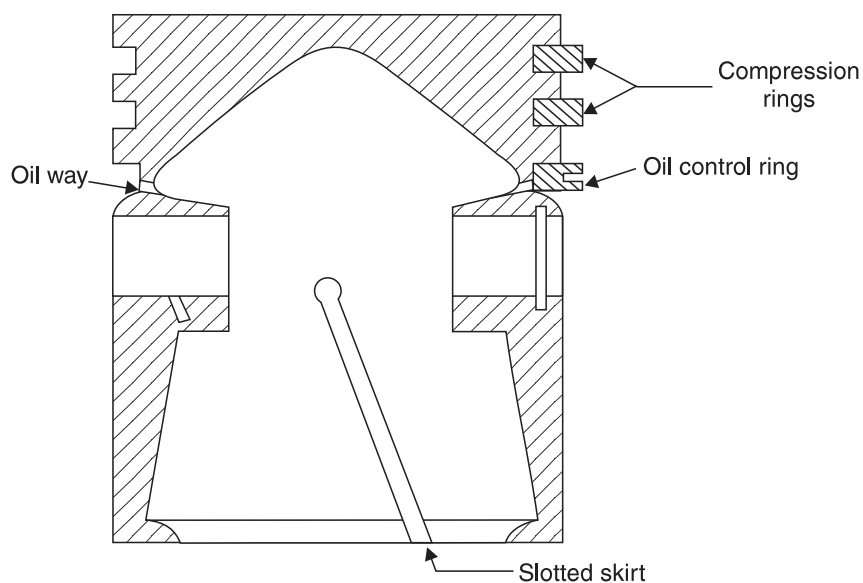
Fig. 5.7. Section through a split skirt piston.

grooves which have been cut in the piston. When the piston is installed in the cylinder the rings are compressed into ring grooves which have been cut in the piston. When the piston is installed in the cylinder, the rings are compressed into the ring grooves so that the split ends come almost together. The rings fit tightly against the cylinder wall and against the sides of the ring grooves in the piston. Thus, they form a good seal between the piston and the cylinder wall. The rings can expand or contact as they heat and cool and still make a good seal. Thus they are free to slide up and down the cylinder wall.
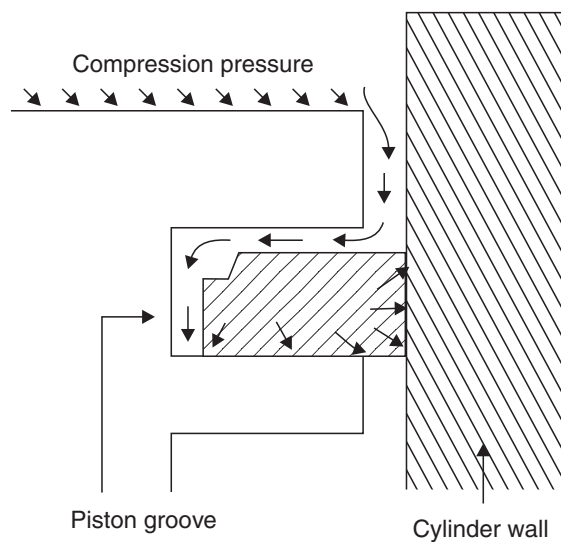
Fig. 5.8. Working of a piston ring.

Fig. 5.8 shows how the piston ring works to hold in the compression and combustion pressure. The arrows show the pressure above the piston passing through clearance between the piston and the cylinder wall. It presses down against the top and against the back of the piston rings as shown by the arrows. This pushes the piston ring firmly against the bottom of the piston ring groove. As a result there are good seals at both of these points. The higher the pressure in the combustion chamber, the better the seal.

Small two stroke cycle engines have two rings on the piston. Both are compression rings (Fig. 5.9). Two rings are used to divide up the job of holding the compression and combustion pressure. This produces better sealing with less ring pressure against the cylinder wall.
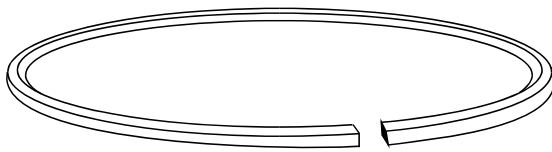
Fig. 5.9. Compression ring.   **Fig. 5.10.** Oil ring.
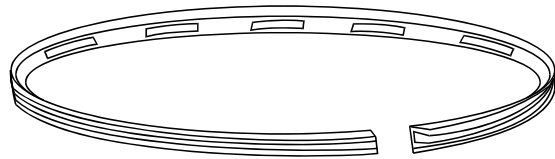
Four stroke cycle engines have an extra ring, called the oil control ring (Fig. 5.10). Four stroke cycle engines are so constructed that they get much more oil in the cylinder wall than do two stroke cycle engines. This additional oil must be scraped off to prevent it from getting up into the combustion chamber, where it would burn and cause trouble.

Refer Figs. 5.9 and 5.10, the compression rings have a rectilinear cross-section and oil rings are provided with a groove in the middle and with through holes spaced at certain interval from each other. The oil collected from the cylinder walls flows through these holes into the piston groove whence through the holes in the body of the piston and down its inner walls into the engine crank case.

5. **Gudgeon Pin** (or wrist pin or piston pin)

These are *hardened steel parallel spindles* fitted through the piston bosses and the small end bushes or eyes to allow the connecting rods to swivel. Gudgeon pins are a press fit in the piston bosses of light alloy pistons when cold. For removal or fitting, the piston should be dipped in hot water or hot oil, this expands the bosses and the pins can be removed or fitted freely without damage.

It is made hollow for lightness since it is a reciprocating part.

6. **Connecting Rod**

Refer Fig. 5.11. The connecting rod transmits the piston load to the crank, causing the latter to turn, thus converting the reciprocating motion of the piston into a rotary motion of the crankshaft. The lower or "big end" of the connecting rod turns on "crank pins".

The connecting rods are made of *nickel, chrome and chrome vanadium steels*. For small engines the material may be *aluminium*.

7. **Crank**

The piston moves up and down in the cylinder. This up and down motion is called reciprocating motion. The piston moves in a straight line. The straight line motion must be changed to rotary, or turning motion, in most machines, before it can do any good. That is rotary motion is required to make wheels turn, a cutting blade spin or a pulley rotate. To change the reciprocating motion to rotary motion a crank and connecting rod are used (Figs. 5.12 and 5.13). The connecting rod connects the piston to the crank.
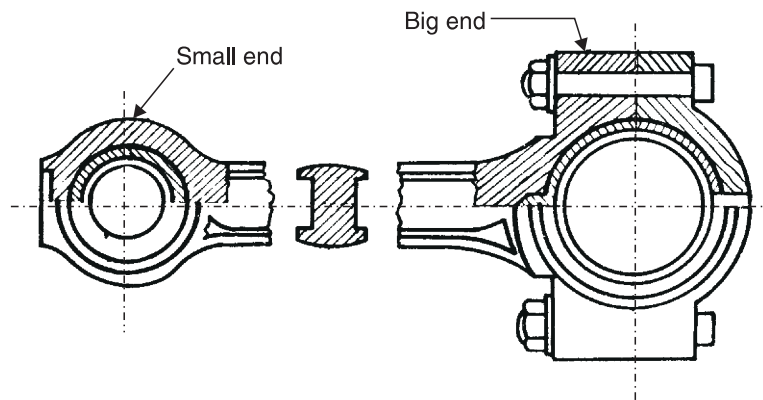
Fig. 5.11. Connecting rod.
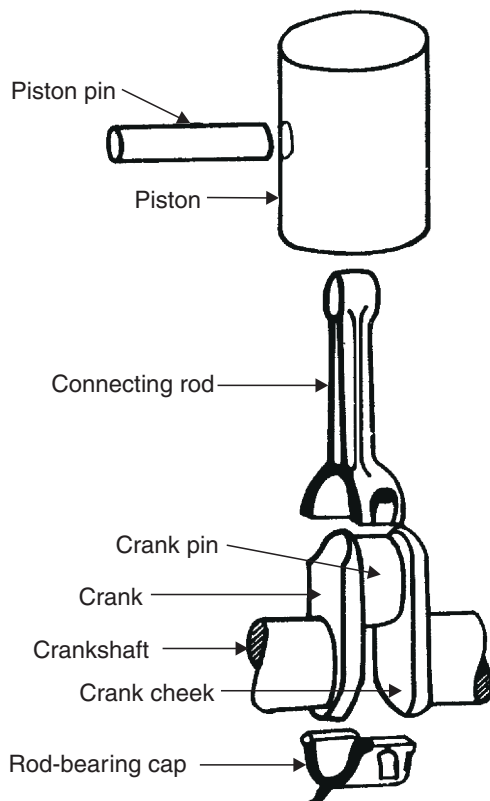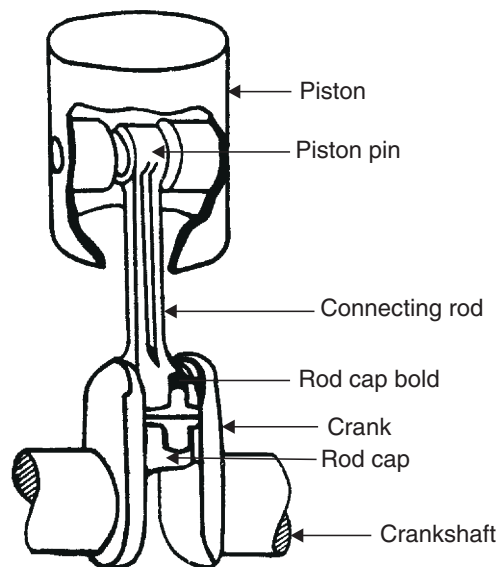
Fig. 5.12   **Fig. 5.13**

Note. The crank end of the connecting rod is called rod "*big end*". The piston end of the connecting rod is called the rod "*small end*".

8. Crankshaft

The crank is part of the crankshaft. The crankshaft of an internal combustion engine receives via its cranks the efforts supplied by the pistons to the connecting rods. All the engines auxiliary mechanisms with mechanical transmission are geared in one way or the another to the crankshaft. *It is usually a steel forging, but some makers use special types of cast-iron such as spheroidal graphitic or nickel alloy castings* which are cheaper to produce and have good service life. Refer Fig. 5.14. The crankshaft *converts the reciprocating motion to rotary motion*. The crankshaft mounts in bearings which, encircle the journals so it can rotate freely.
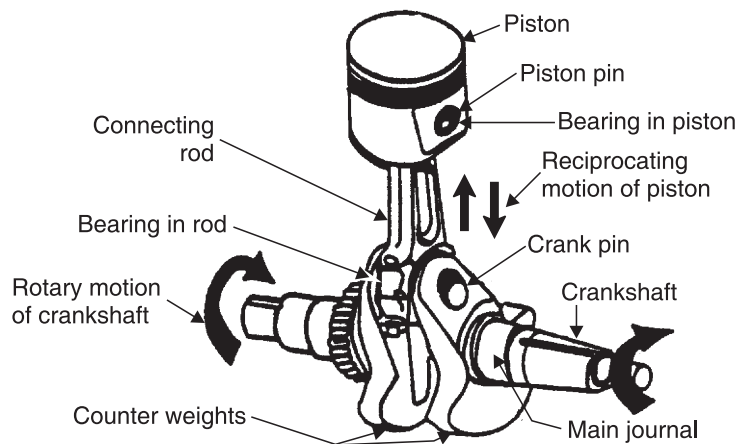
Fig. 5.14. Crankshaft and other parts.

The shape of the crankshaft *i.e.,* the mutual arrangement of the cranks depend on the number and arrangement of cylinders and the turning order of the engine. Fig. 5.15 shows a typical crankshaft layout for a four cylinder engine.
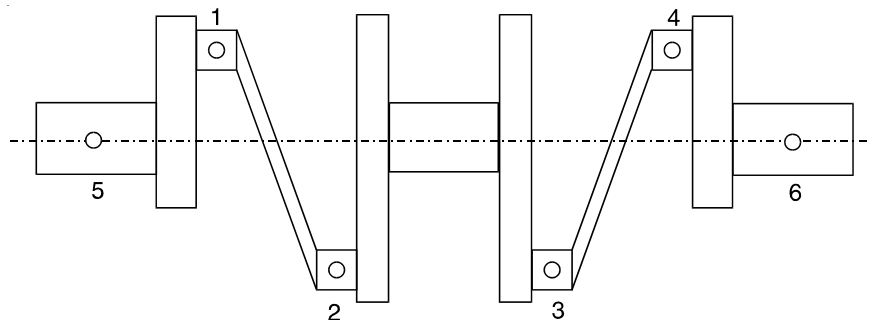
Fig. 5.15. Typical crankshaft layout.

9. Engine Bearing

The crankshaft is supported by bearing. The connecting rod big end is attached to the crank pin on the crank of the crankshaft by a bearing. A piston pin at the rod small end is used to attach the rod to the piston. The piston pin rides in bearings. Every where there is rotary action in the engine, bearings are used to support the moving parts. The purpose of bearing is to reduce the friction and allow the parts to move easily. Bearings are lubricated with oil to make the relative motion easier.

Bearings used in engines are of two types : *sliding* or *rolling* (Fig. 5.16).

The sliding type of bearings are sometimes called bushings or sleeve bearings because they are in the shape of a sleeve that fits around the rotating journal or shaft. The sleeve-type connecting rod big end bearings usually called simply rod bearings and the crankshaft supporting bearings called the *main bearings* are of the split sleeve type. They must be split in order to permit
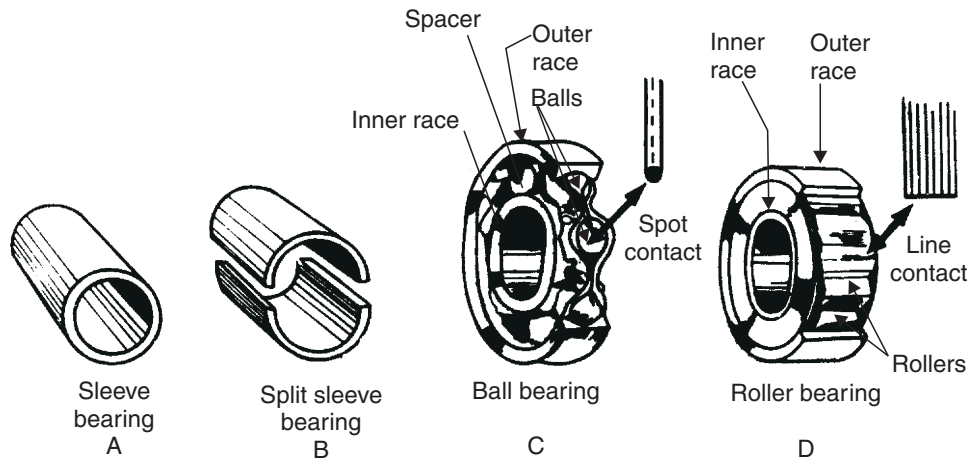
Fig. 5.16. Bearings.

their assembly into the engine. In the rod bearing, the upper half of the bearing is installed in the rod, the lower half is installed in the rod bearing cap. When the rod cap is fastened to the rod shown in Fig. 5.13 a complete sleeve bearing is formed. Likewise, the upper halves of the main bearings are assembled in the engine and then the main bearing caps, with the lower bearing halves are attached to the engine to complete the sleeve bearings supporting the crankshaft.

The typical bearing half is made of steel or bronze back to which a lining of relatively soft bearing material is applied. Refer Fig. 5.17. This relatively soft bearing material, which is made of
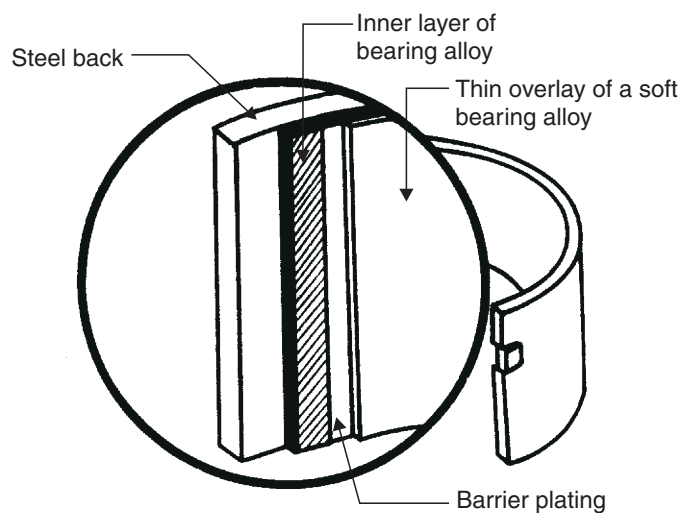
Fig. 5.17. Bearing half (details).

several materials such as copper, lead, tin and other metals, has the ability to conform to slight irregularities of the shaft rotating against it. If wear does take place, it is the bearing that wears and the bearing can be replaced instead of much more expansive crankshaft or other engine part.

The rolling-type bearing uses balls or rollers between the stationary support and the rotating shaft. Refer Fig. 5.16. Since the balls or rollers provide rolling contact, the frictional resistance to movement is much less. In some roller bearing, the rollers are so small that they are hardly bigger than needles. These bearings are called *needle bearings*. Also some rollers bearings have the rollers set at an angle to the races, the rollers roll in are tapered. These bearings are called *tapered roller bearings*. Some ball and roller bearings are sealed with their lubricant already in place. Such bearings require no other lubrication. Other do require lubrication from the oil in the gasoline (two stroke cycle engines) or from the engine lubrication system (four stroke cycle engines).

The type of bearing selected by the designers of the engine depends on the design of the engine and the use to which the engine will be put. *Generally, sleeve bearings, being less expensive and satisfactory for most engine applications, are used. In fact sleeve bearings are used almost universally in automobile engines. But you will find some engines with ball and roller bearings to support the crankshaft and for the connecting rod and piston-pin bearings.*

10. Crankcase

The main body of the engine to which the cylinders are attached and which contains the crankshaft and crankshaft bearing is called *crankcase*. This member also holds other parts in alignment and resists the explosion and inertia forces. It also protects the parts from dirt etc. and serves as a part of lubricating system.

11. Flywheel

Refer Figs. 5.1 and 5.18. A flywheel (steel or cast iron disc) secured on the crankshaft performs the following functions :

(*a*) Brings the mechanism out of dead centres.

(*b*) Stores energy required to rotate the shaft during preparatory strokes.

(*c*) Makes crankshaft rotation more uniform.

(*d*) Facilitates the starting of the engine and overcoming of short time over loads as, for example, when the machine is started from rest.
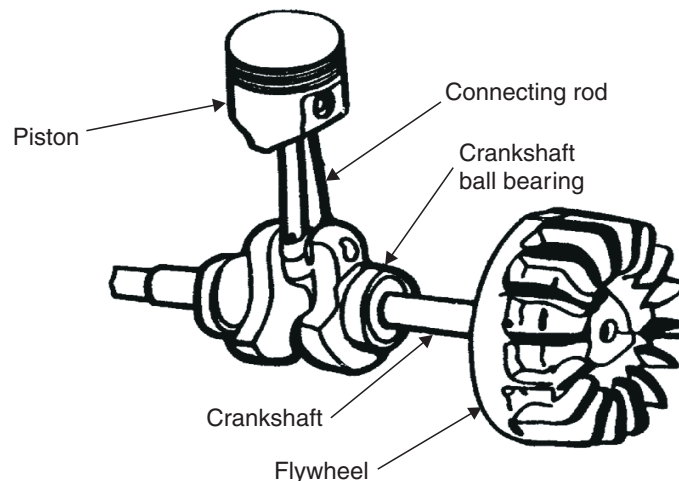
Fig. 5.18. Flywheel secured on crankshaft.

The weight of the flywheel depends upon the nature of variation of the pressure. The flywheel for a double acting steam engine is lighter than that of a single acting one. Similarly, the flywheel for a two-stroke cycle engine is lighter than a flywheel used for a four-stroke cycle engine. *Lighter flywheels are used for multi-cylinder engines.*

12. **Governor**

A governor may be defined *as a device for regulating automatically output of a machine by regulating the supply of working fluid*. When the speed decreases due to increase in load the supply valve is opened by mechanism operated by the governor and the engine therefore speeds up again to its original speed. If the speed increases due to a decrease of load the governor mechanism closes the supply valve sufficiently to slow the engine to its original speed. *Thus the function of a governor is to control the fluctuations of engine speed due to changes of load*.

Comparison between a Flywheel and a Governor

	Flywheel	*Governor*
1.	It is provided on engines and fabricating machines *viz.*, rolling mills, punching machines ; shear machines, presses etc.	It is provided on prime movers such as engines and turbines.
2.	Its function is to store the available mechanical energy when it is in excess of the load requirement and to part with the same when the available energy is less than that required by the load.	Its function is to regulate the supply of driving fluid producing energy, according to the load requirement so that at different loads almost a constant speed is maintained.
3.	It works continuously from cycle to cycle.	It works intermittently *i.e.*, only when there is change in load.
4.	In engines it takes care of fluctuations of speed during thermodynamic cycle.	It takes care of fluctuations of speed due to variation of load over long range of working engines and turbines.
5.	In fabrication machines it is very economical to use it in that it reduces capital investment on prime movers and their running expenses.	But for governor, there would have been unnecessarily more consumption of driving fluid. Thus it economies its consumption.

Types of governor :

Governors are classified as follows :

1. **Centrifugal governor**

(*a*) *Gravity controlled*, in which the centrifugal force due to the revolving masses is largely balanced by gravity.

(*b*) *Spring controlled*, in which the centrifugal force is largely balanced by springs.

2. **Inertia and flywheel governors**

(*a*) *Centrifugal type*, in which centrifugal forces play the major part in the regulating action.

(*b*) *Inertia governor*, in which the inertia effect predominates.

The *inertia type* governors are fitted to the crankshaft or flywheel of an engine and so differ radically in appearance from the centrifugal governors. The balls are so arranged that the inertia force caused by an angular acceleration or retardation of the shaft tends to alter their positions. The amount of displacement of governor balls is controlled by suitable springs and through the

governor mechanism, alters the fuel supply to the engine. The inertia governor is more sensitive than centrifugal but it becomes very difficult to balance the revolving parts. For this reason *centrifugal governors are more frequently used*. We shall discuss centrifugal governors only.

Important centrifugal governors are :

1. Watt governor   2. Porter governor
3. Proell governor   4. Hartnell governor.

1. **Watt governor**

It is the primitive governor as used by Watt on some of his early steam engines. It is used for a very slow speed engine and this is why it has now become obsolete.

Refer Fig. 5.19. Two arms are hinged at the top of the spindle and two revolving balls are fitted on the other ends of the arms. One end of each of the links are hinged with the arms, while the other ends are hinged with the sleeve, which may slide over the spindle. The speed of the crankshaft is transmitted to the spindle through a pair of bevel gears by means of a suitable arrangement. So the rotation of the spindle of the governor causes the weights to move away from the centre due to the centrifugal force. This makes the sleeve to move in the upward direction. This movement of the sleeve is transmitted by the lever to the throttle valve which partially closes or opens the steam pipe and reduces or increases the supply of steam to the engine. So the engine speed may be adjusted to a normal limit.
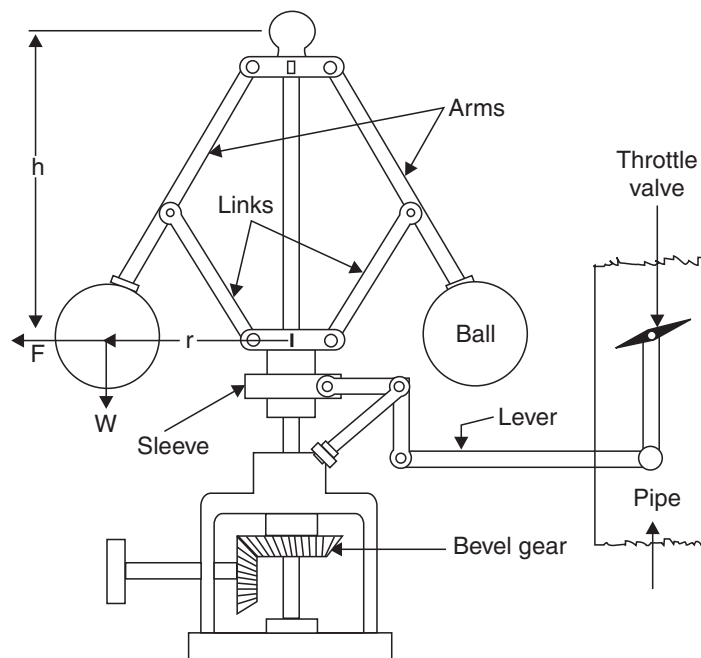
Fig. 5.19. Watt governor.

2. **Porter governor**

Fig. 5.20 shows diagrammatically a Porter governor where two or more masses called the governor balls rotate about the axis of the governor shaft which is driven through suitable gearing from the engine crankshaft. The governor balls are attached to the arms. The lower arms are attached to the *sleeve which acts as a central weight*. If the speed of the rotation of the balls increases

owing to a decrease of load on the engine, the governor balls fly outwards and the sleeve moves upwards thus closing the fuel passage till the engine speed comes back to its designed speed. If the engine speed decreases owing to an increase of load, the governor balls fly inwards and the sleeve moves downwards thus opening the fuel passage more for oil till the engine speed comes back to its designed speed. The engine is said to be running at its designed speed when the outward inertia or centrifugal force is just balanced by the inward controlling force.
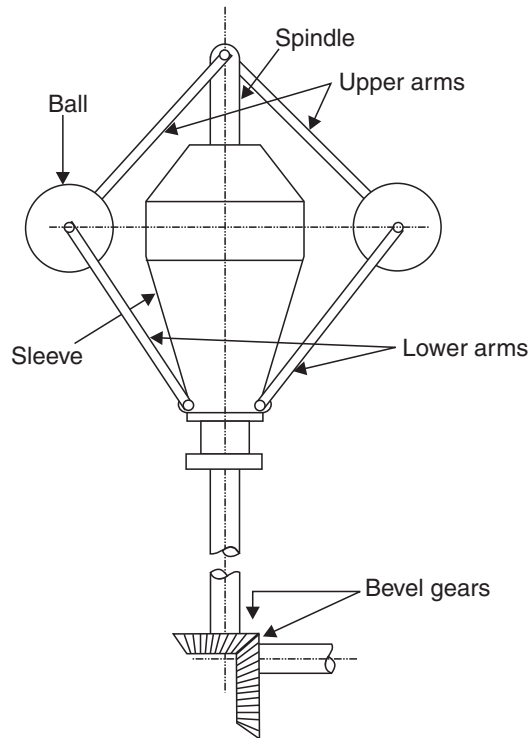
Fig. 5.20. Porter governor.

3. **Proell governor**

Refer Fig. 5.21. It is a modification of porter governor. The governor balls are carried on an *extension of the lower arms*. For given values of weight of the ball, weight of the sleeve and height of the governor, a Proell governor runs at a *lower speed* than a Porter governor. *In order to give the same equilibrium speed a ball of smaller mass may be used in Proell governor*.

4. **Hartnell governor**

The Hartnell governor is a spring loaded governor in which the controlling force, to a great extent, is provided by the spring thrust.

Fig. 5.22 shows one of the types of Hartnell governors. It consists of casing fixed to the spindle. A compressed spring is placed inside the casing which presses against the top of the casing and on adjustable collars. The sleeve can move up and down on the vertical spindle depending upon the speed of the governor. Governor balls are carried on bell crank lever which are pivoted on the lower end of the casing. The balls will fly outwards or inwards as the speed of the governor shaft increases or decreases respectively.
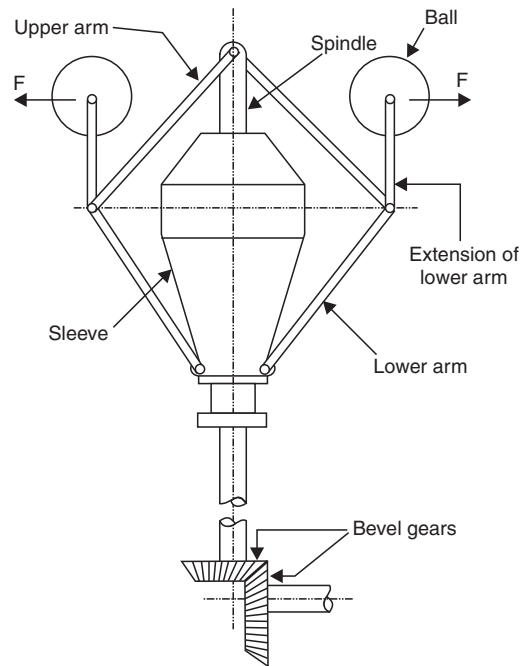
Fig. 5.21. Proell governor.

13. **Valves and Valve Gears**

With few exceptions the inlet and exhaust of internal combustion engines are controlled by poppet valves. These valves are held to their seating by strong springs, and as the valves usually open inwards, the pressure in the cylinder helps to keep them closed. The valves are lifted from their seats and the ports opened either by cams having projecting portion designed to give the period of opening required or by eccentrics operating through link-work. Of these two methods the *cam gear is more commonly used*, but in either case it is necessary that the valve gear shaft of an engine should rotate but once from beginning to end of a complete cycle, however many strokes may be involved in the completion of that cycle. This is necessary to secure a continuous regulation of the valve gear as required. For this purpose the cams or eccentrics of four-stroke engines are mounted on shafts driven by gearing at half the speed of the crankshaft. The curves used for the acting faces of the cams depend on the speed of the engine and rapidity of valve opening desired.
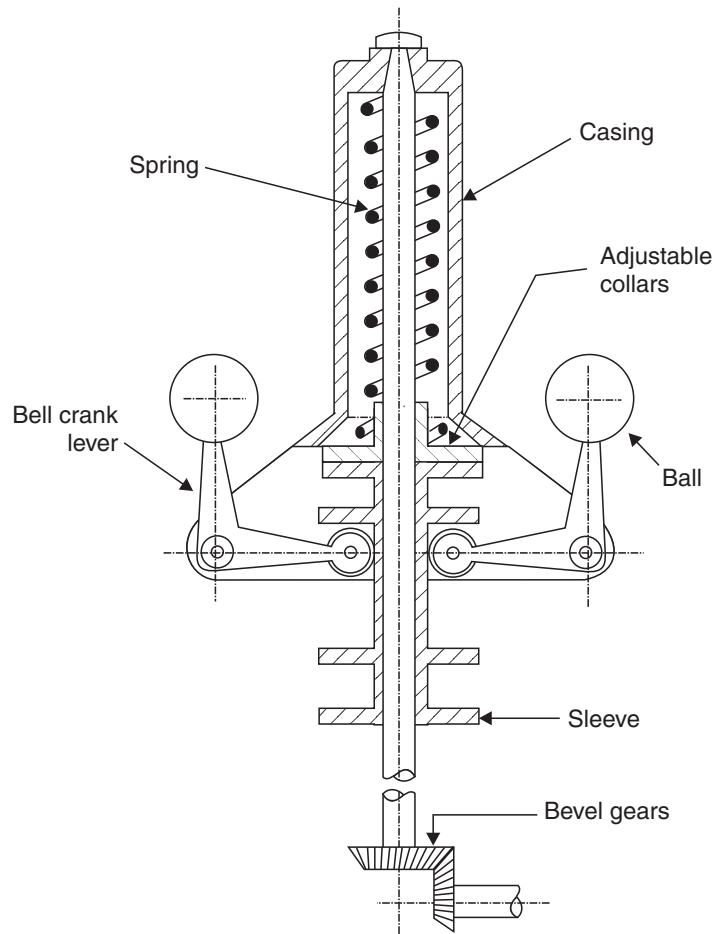
Fig. 5.22. Hartnell governor.

Fig. 5.23 shows a valve gear for I.C. engine. It consists of poppet valve, the steam bushing or guide, valve spring, spring retainer, lifter or push rod, camshaft and half speed gear for a four-stroke engine. The poppet valve, in spite of its shortcomings of noise and difficulties of cooling is commonly used due to its simplicity and capacity for effective sealing under all operating conditions. The valve is subjected to very heavy duty. It holds in combustion chamber and is exposed to high temperatures of burning gases. Exhaust valve itself may attain a high temperature while external cooling is not available. Special heat resisting alloys are therefore used in the construction of the exhaust valve and it may sometimes have a hollow construction filled with mineral salts to provide for heat dissipation. The salts become liquid when valve is working and transfer heat from the head to the stem from which it is carried through the stem guide to the cylinder block.

The timing of the valves *i.e.*, their opening and closing with respect to the travel of the piston is very important thing for efficient working of the engine. The drive of the camshaft is arranged through gears or chain and sprocket (called timing gear or timing chain). Any wearing of the gears or chain and sprocket would result in disturbing the precise timing of the valves. It is desirable, therefore, to avoid use of multiple gears of long chains in the camshaft drive.
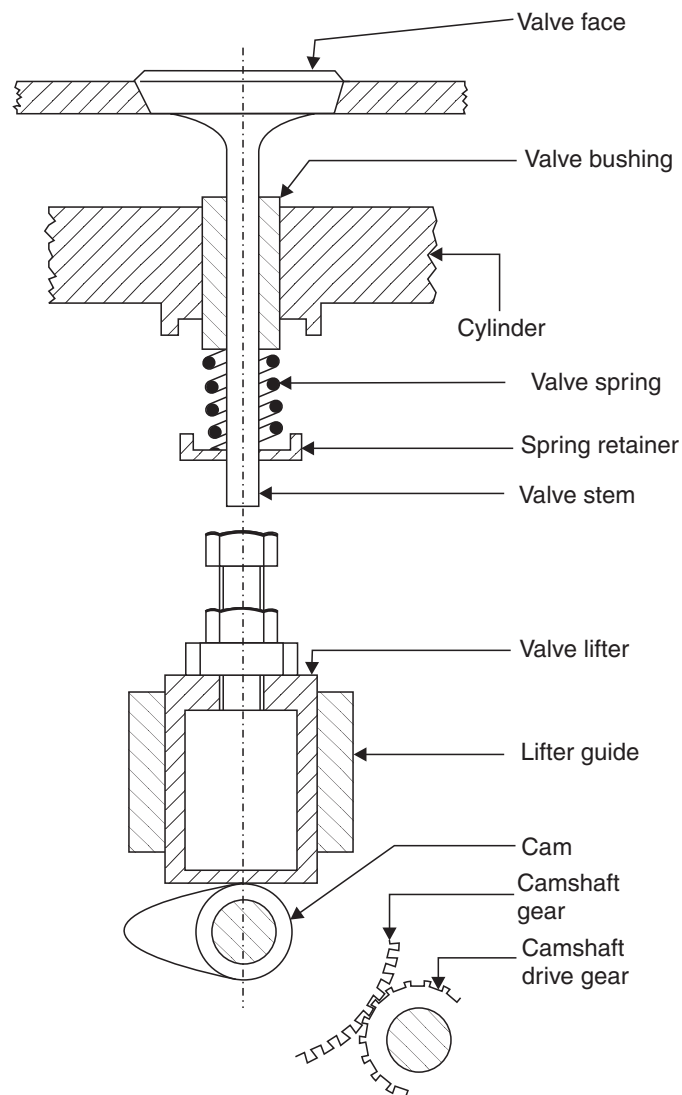
Fig. 5.23. Valve gear for I.C. engine.

Valve Timing

Theoretically the valves open and close at top dead centre (T.D.C.) or at bottom dead centre (B.D.C.) but practically they do so some time before or after the piston reaches the upper or lower limit of travel. There is a reason for this. Look at the inlet valve, for example. It normally opens several degrees of crankshaft-rotation before T.D.C. on the exhaust stroke. That is the intake valve begins to open before the exhaust stroke is finished. This gives the valve enough time to reach the fully open position before the intake stroke begins. Then, when the intake stroke starts, the intake valve is already wide open and air fuel mixture can start to enter the cylinder, immediately. Likewise the intake valve remains open for quite a few degrees of crankshaft rotation after the piston has passed B.D.C. at the end of the intake stroke. This allows additional time for air-fuel mixture to continue to flow into the cylinder. The fact that the piston has already passed B.D.C. and is moving up or the compression stroke while the intake valve is still open does not effect the movement of air

fuel mixture into the cylinder. Actually air fuel mixture is still flowing in as the intake valve starts to close.

This is due to the fact that air fuel mixture has inertia. That is, it attempts to keep on flowing after it once starts through the carburettor and into the engine cylinder. The momentum of the mixture then keeps it flowing into the cylinder even though the piston has started up on the compression stroke. This packs more air-fuel mixture into the cylinder and results in a stronger power stroke. In other words, this improves *volumetric efficiency*.

For a some what similar reason, the exhaust valve opens well before the piston reaches B.D.C. on the power stroke. As the piston nears B.D.C., most of the push on the piston has ended and nothing is lost by opening the exhaust valve towards the end of the power stroke. This gives the exhaust gases additional time to start leaving the cylinder so that exhaust is well started by the time the piston passes B.D.C. and starts up on the exhaust stroke. The exhaust valve then starts opening for some degrees of crankshaft rotation after the piston has passed T.D.C. and intake stroke has started. This makes good use of momentum of exhaust gases. They are moving rapidly towards the exhaust port, and leaving the exhaust valve open for a few degrees after the intake stroke starts giving the exhaust gases some additional time to leave the cylinder. This allows more air-fuel mixture to enter on the intake stroke so that the stronger power stroke results. That is, it improves volumetric efficiency.

The actual timing of the valves varies with different four stroke cycle engines, but the typical example for an engine is shown in Fig. 5.24. Note that the inlet valve opens 15° of crankshaft
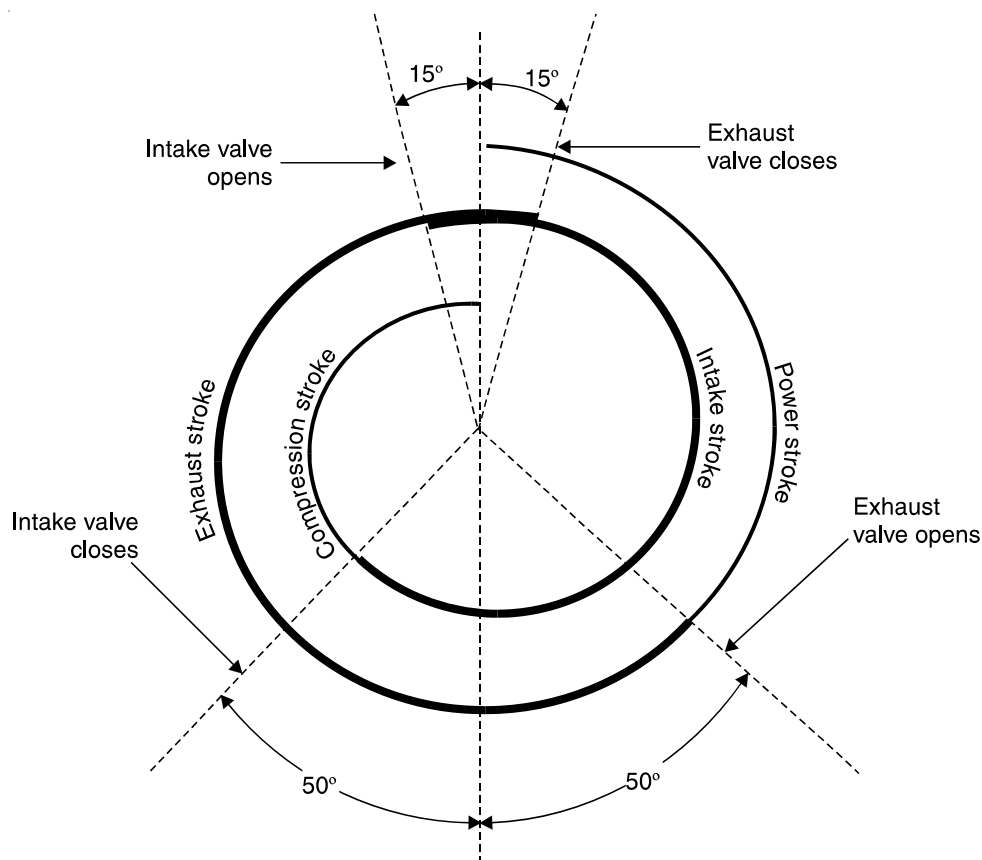
Fig. 5.24. Typical valve timing diagram.

rotation before T.D.C. on the exhaust stroke and stays open until 50° of crankshaft rotation after B.D.C. on the compression stroke. The exhaust valve opens 50° before B.D.C. on the power stroke and stays open 15° after T.D.C. on the inlet stroke. This gives the two valves an overlap of 30° at the end of exhaust stroke and beginning of the *compression stroke*.

B. Parts common to petrol engine only :

Spark Plug

The main function of a spark plug is to conduct the high potential from the ignition system into the combustion chamber. It provides the proper gap across which spark is produced by applying high voltage, to ignite the combustion chamber.

A spark plug entails the following *requirements* :

 (*i*) It must withstand peak pressures up to atleast 55 bar.

 (*ii*) It must provide suitable insulation between two electrodes to prevent short circuiting.

 (*iii*) It must be capable of withstanding high temperatures to the tune of 2000°C to 2500°C over long periods of operation.

 (*iv*) It must offer maximum resistance to erosion burning away of the spark points irrespective of the nature of fuel used.

 (*v*) It must possess a high heat resistance so that the electrodes do not become sufficiently hot to cause the preignition of the charge within the engine cylinder.

 (*vi*) The insulating material must withstand satisfactorily the chemical reaction effects of the fuel and hot products of combustion.

 (*vii*) Gas tight joints between the insulator and metal parts are essential under all operating conditions.

Refer Fig. 5.25. The spark plug consists of a metal shell having two electrodes which are insulated from each other with an air gap. High tension current jumping from the supply electrode produces the necessary spark. Plugs are sometimes identified by the heat range or the relative temperature obtained during operation. The correct type of plug with correct width of gap between the electrodes are important factors. The spark plug gap can be easily checked by means of a feeler gauge and set as per manufacturer's specifications. It is most important that while adjusting the spark plug it is the outer earthed electrode *i.e.*, tip which is moved in or out gradually for proper setting of the gap. No bending force should be applied on the centre-electrode for adjusting the gap as this can cause crack and fracture of insulation and the plug absolutely useless.

Porcelain is commonly used as insulating material in spark plugs, as it is cheap and easy to manufacture. Mica can also be used as insulating material for spark plugs. Mica, however, cannot withstand high temperatures successfully.

Simple Carburettor

The function of a carburettor is to atomise and metre the liquid fuel and mix it with the air at it enters the induction system of the engine, maintaining under all conditions of operation fuel-air proportions appropriate to those conditions.

All modern carburettors are based upon Bernoulli's theorem,

$$C^2 = 2gh$$

where C is the velocity in metres/sec, g is the acceleration due to gravity in metre/sec^2 and h is the head causing the flow expressed in metres of height of a column of the fluid.

The equation of mass rate of flow is given by,

$$m = \rho A \sqrt{2gh}$$

where ρ is the density of the fluid and A is the cross-sectional area of fluid stream.
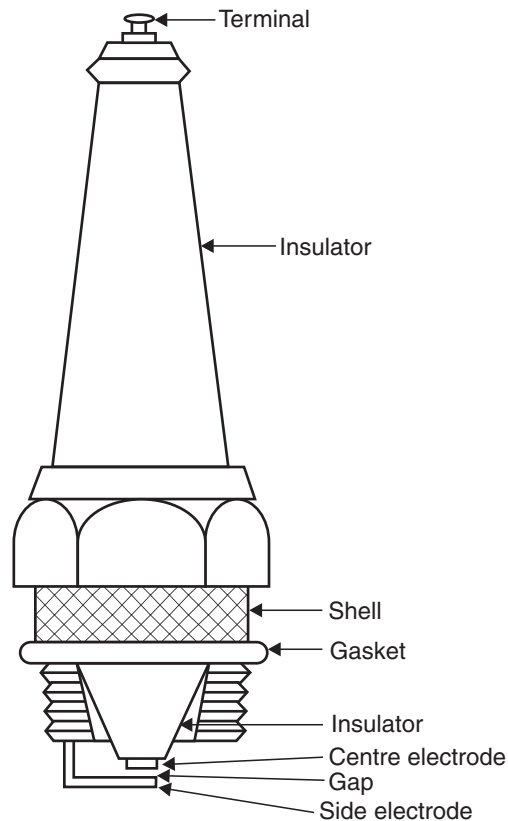
Fig. 5.25. Spark plug.

In Fig. 5.26 is shown simple carburettor. *L* is the float chamber for the storage of fuel. The fuel supplied under gravity action or by fuel pump enters the float chamber through the filter *F*. The arrangement is such that when the oil reaches a particular level the float valve *M* blocks the inlet passage and thus cuts off the fuel oil supply. On the fall of oil level, the float descends down, consequently intake passage opens and again the chamber is filled with oil. Then the float and the float valve maintains a constant fuel oil level in the float chamber. *N* is the jet from which the fuel is sprayed into the air stream as it enters the carburettor at the inlet *S* and passes through the throat or venturi *R*. The fuel level is slightly below the outlet of the jet when the carburettor is inoperative.

As the piston moves down in the engine cylinder, suction is produced in the cylinder as well as in the induction manifold *Q* as a result of which air flows through the Carburettor. The velocity of air increases as it passes through the construction at the venturi *R* and pressure decreases due to conversion of a portion of pressure head into kinetic energy. Due to decreased pressure at the venturi and hence by virtue of difference in pressure (between the float chamber and the venturi) the jet issues fuel oil into air stream. Since the jet has a very fine bore, the oil issuing from the jet is in the form of fine spray ; it vapourises quickly and mixes with the air. This air fuel mixture enters the engine cylinder ; its quantity being controlled by varying the position of the throttle valve *T*.
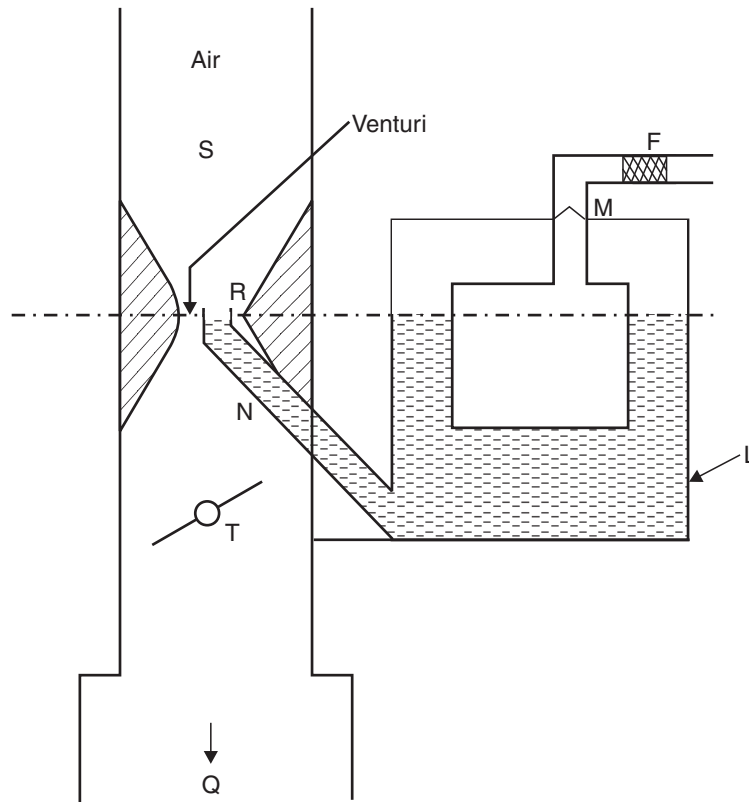
Fig. 5.26. Simple carburettor.

Limitations :

 (*i*) Although theoretically the air-fuel ratio supplied by a simple (single jet) carburettor should remain constant as the throttle goes on opening, actually it provides increasingly richer mixture as the throttle is opened. This is because of the reason that the density of air tends to decrease as the rate of flow increases.

 (*ii*) During idling, however, the nearly closed throttle causes a reduction in the mass of air flowing through the venturi. At such low rates of air flow, the pressure difference between the float chamber and the fuel discharge nozzle becomes very small. It is not sufficient to cause fuel to flow through the jet.

 (*iii*) Carburettor does not have arrangement for providing rich mixture during starting and warm up.

   In order to *correct for faults* :

   (*i*) number of *compensating devices* are used for (*ii*) an *idling jet* is used which helps in running the engine during idling. For (*iii*) *choke arrangement* is used.

   Fuel pump (for carburettor-petrol engine).

   Refer Fig. 5.27. This type of pump is used in petrol engine for supply of fuel to the carburettor. Due to rotation of the crankshaft the cam pushes the lever in the upward direction. One end of the lever is hinged while the other end pulls the diaphragm rod with the *diaphragm*. So the diaphragm comes in the downward direction against the compression of the spring and thus a

vacuum is produced in the pump chamber. This causes the fuel to enter into the pump chamber from the *glass bowl* through the *strainer* and the inlet valve, the impurities of the fuel if there is any, deposit at the bottom of the glass bowl. On the return stroke the spring pushes the diaphragm in the upward direction forcing the fuel from the pump chamber into the carburettor through the *outlet valve*.
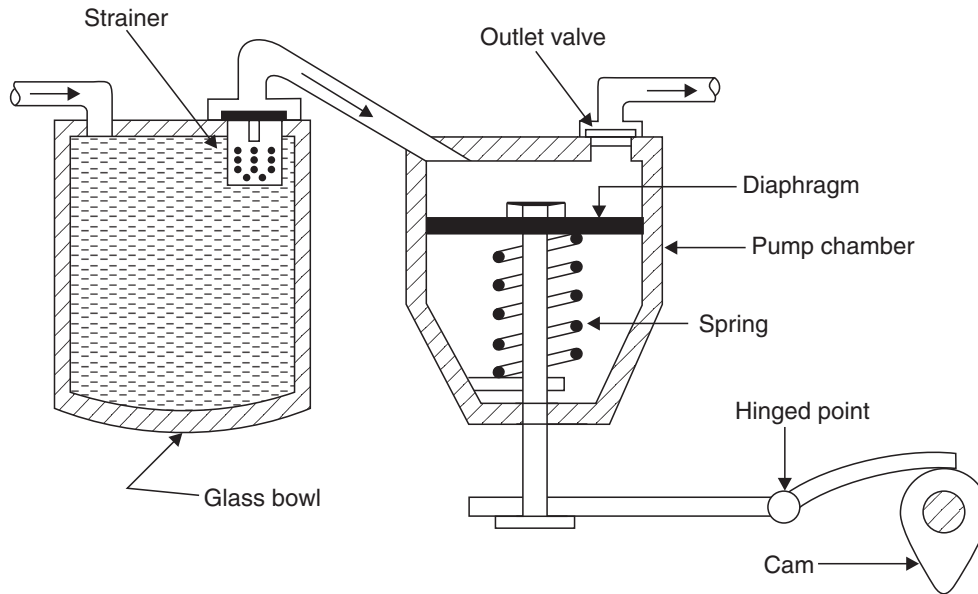
Fig. 5.27. Fuel pump for carburettor.

Parts for Diesel engine only :

Fuel Pump

Refer Fig. 5.28. *L* is the plunger which is driven by a cam and tappet mechanism at the bottom (not shown in the figure) *B* is the barrel in which the plunger reciprocates. There is the rectangular vertical groove in the plunger which extends from top to another helical groove. *V* is the delivery valve which lifts off its seat under the liquid fuel pressure and against the spring force (*S*). The fuel pump is connected to fuel atomiser through the passage *P*, *SP* and *Y* are the spill and supply ports respectively. When the plunger is at its bottom stroke the ports *SP* and *Y* are uncovered (as shown in the Fig. 5.28) and oil from low pressure pump (not shown) after being filtered is forced into the barrel. When the plunger moves up due to cam and tappet mechanism, a stage reaches when both the ports *SP* and *Y* are closed and with the further upward movement of the plunger the fuel gets compressed. The high pressure thus developed lifts the delivery valve off its seat and fuel flows to atomiser through the passage *P*. With further rise of the plunger, at a certain moment, the port *SP* is connected to the fuel in the upper part of the plunger through the rectangular vertical groove by the helical groove ; as a result of which a sudden drop in pressure occurs and the delivery valve falls back and occupies its seat against the spring force. The plunger is rotated by the rack *R* which is moved in or out by the governor. *By changing the angular position of the helical groove (by rotating the plunger) of the plunger relative to the supply port, the length of stroke during which the oil is delivered can be varied and thereby quantity of fuel delivered to the engine is also varied accordingly.*
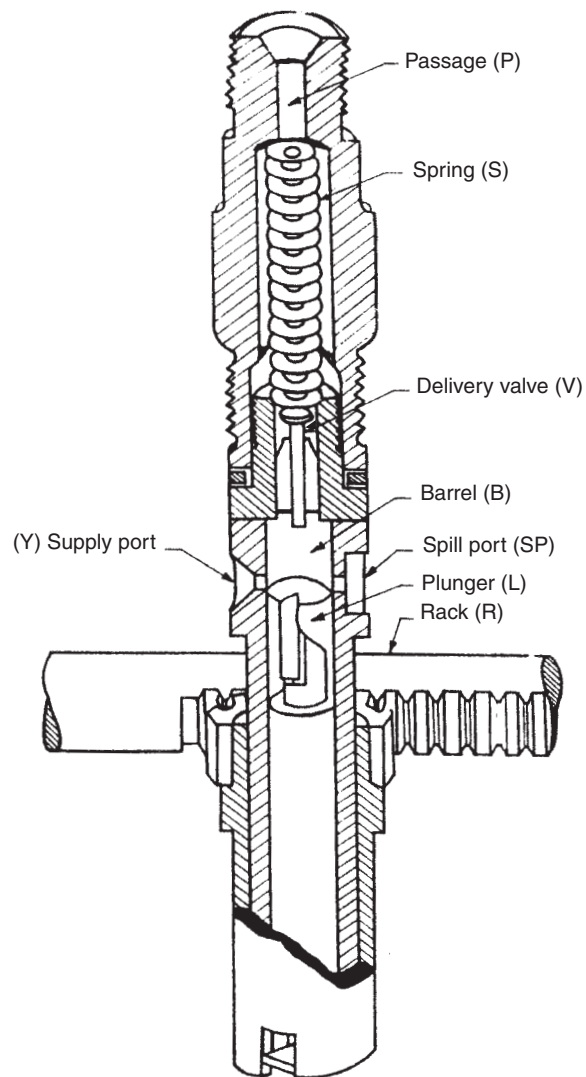
Fig. 5.28. Fuel pump.

Fuel Atomiser or Injector

Refer Fig. 5.29. It consists of a nozzle valve (*NV*) fitted in the nozzle body (*NB*). The nozzle valve is held on its seat by a spring 'S' which exerts pressure through the spindle *E*. '*AS*' is the adjusting screw by which the nozzle valve lift can be adjusted. Usually the nozzle valve is set to lift at 135 to 170 bar pressure. *FP* is the feeling pin which indicates whether valve is working properly or not. The oil under pressure from the fuel pump enters the injector through the passages *B* and *C* and lifts the nozzle valve. The fuel travels down the nozzle *N* and injected into the engine cylinder in the form of fine sprays. When the pressure of the oil falls, the nozzle valve occupies its seat under the spring force and fuel supply is cut off. Any leakage of fuel accumulated above the valve is led to the fuel tank through the passage *A*. The leakage occurs when the nozzle valve is *worn out*.
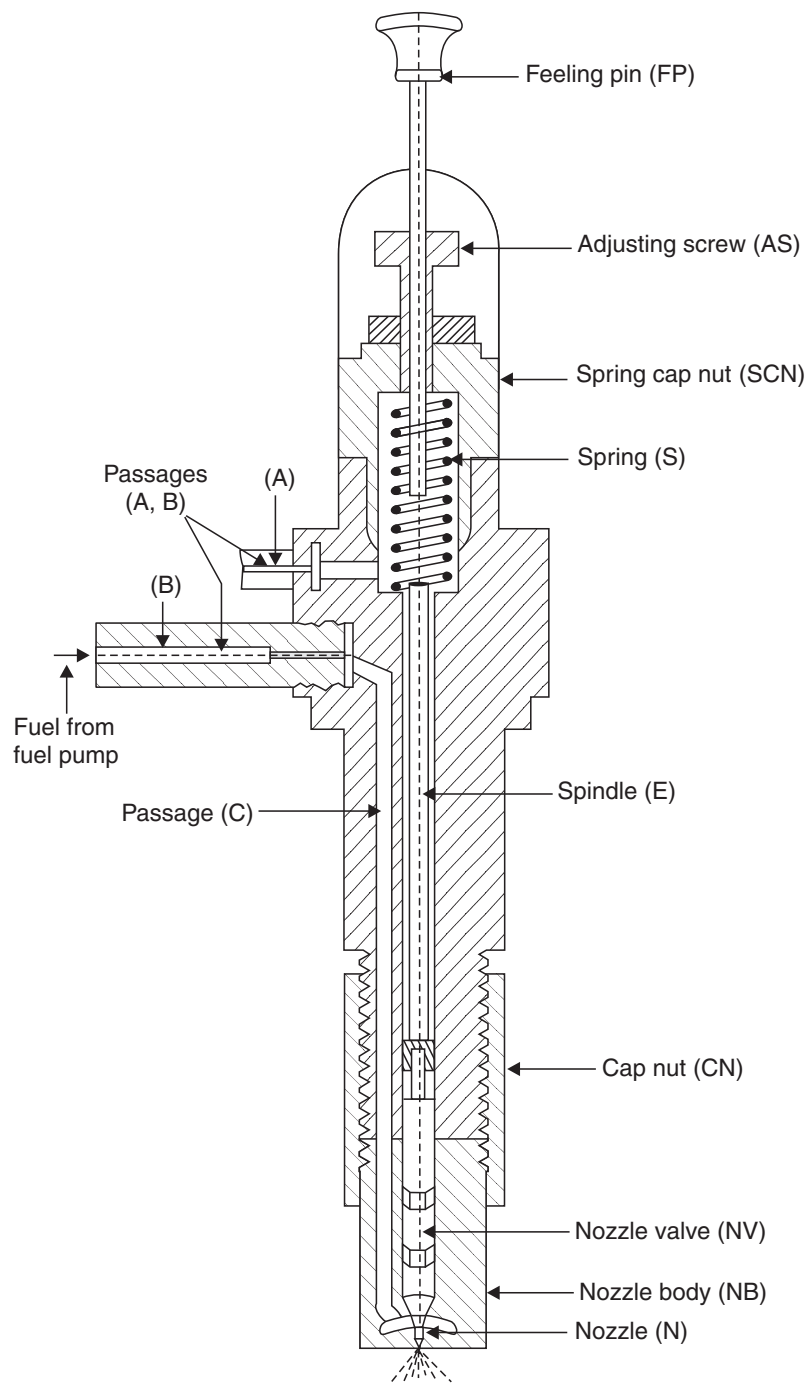
Fig. 5.29. Fuel atomiser or injector.

List of engine parts, materials, method of manufacture and functions :

	Name of the part	Material	Function	Method of manufacture
1.	*Cylinder*	Hard grade cast-iron	Contains gas under pressure and guides the piston.	Casting.
2.	*Cylinder head*	Cast-iron or aluminium	Main function is to seal the working end of the cylinder and not to permit entry and exit of gases on overhead valve engines.	Casting, forging.
3.	*Piston*	Cast-iron or aluminium alloy	It acts as a face to receive gas pressure and transmits the thrust to the connecting rod.	Casting, forging.
4.	*Piston rings*	Cast-iron	Their main function is to provide a good sealing fit between the piston and cylinder.	Casting.
5.	*Gudgeon pin*	Hardened steel	It supports and allows the connecting rod to swivel.	Forging.
6.	*Connecting rod*	Alloy steel ; for small engines the material may be aluminium	It transmits the piston load to the crank, causing the latter to turn, thus converting the reciprocating motion of the piston into rotary motion of the crankshaft.	Forging.
7.	*Crankshaft*	In general the crankshaft is made from a high tensile forging, but special cast-irons are sometimes used to produce a light weight crankshaft that does not require a lot of machining.	It converts the reciprocating motion of the piston into the rotary motion.	Forging.
8.	*Main bearings*	The typical bearing half is made of steel or bronze back to which a lining of relatively soft bearing material is applied.	The function of bearing is to reduce the friction and allow the parts to move easily.	Casting.
9.	*Flywheel*	Steel or cast-iron.	In engines it takes care of fluctuations of speed during thermodynamic cycle.	Casting.
10.	*Inlet valve*	Silicon chrome steel with about 3% carbon.	Admits the air or mixture of air and fuel into engine cylinder.	Forging.
11.	*Exhaust valve*	Austenitic steel	Discharges the product of combustion.	Forging.

5.7.   TERMS CONNECTED WITH I.C. ENGINES

Refer to Fig. 5.30.
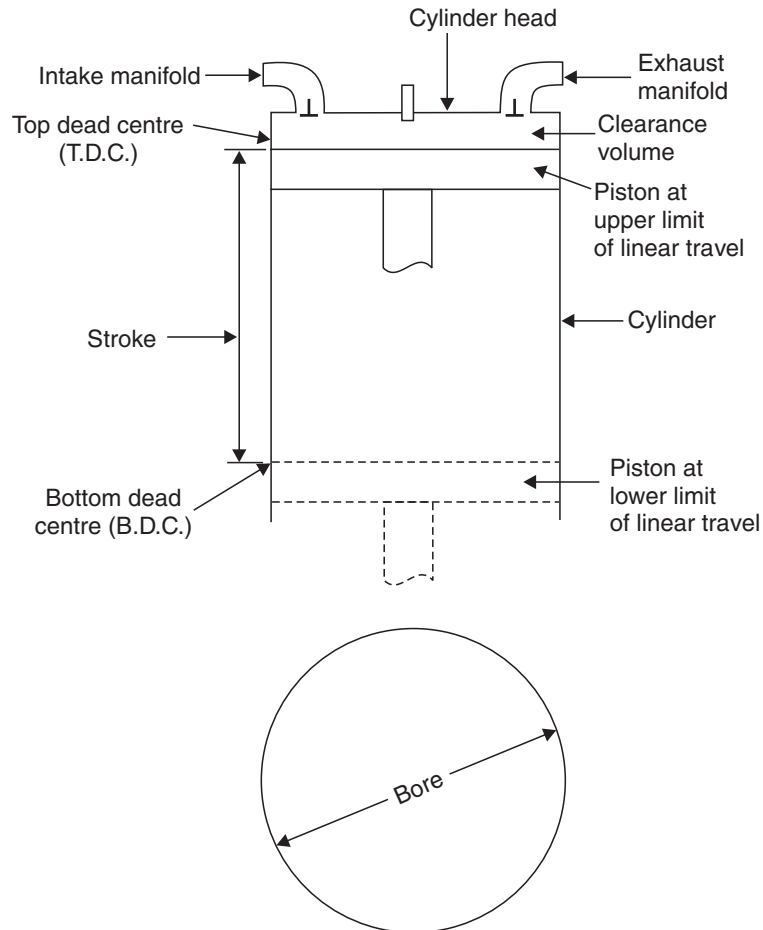
Fig. 5.30. Terms relating I.C. engines.

Bore. *The inside diameter of the cylinder is called **bore.***

Stroke. As the piston reciprocates inside the engine cylinder, it has got limiting upper and lower positions beyond which it cannot move and reversal of motion takes place at these limiting positions.

The linear distance along the cylinder axis between two limiting positions, is called "stroke".

Top Dead Centre (T.D.C.). *The top most position of the piston towards cover end side of the cylinder is called **top dead centre***. In case of horizontal engines, this is known as *inner dead centre*.

Bottom Dead Centre (B.D.C.). *The lowest position of the piston towards the crank end side of the cylinder is called **bottom dead centre***. In case of horizontal engines it is called outer *dead centre*.

Clearance volume. *The volume contained in the cylinder above the top of the piston, when the piston is at top dead centre, is called the **clearance volume***.

Swept volume. *The volume swept through by the piston in moving between top dead centre and bottom dead centre, is, called* ***swept volume*** *or* ***piston displacement***. Thus, when piston is at bottom dead centre, total volume = swept volume + clearance volume.

Compression ratio. *It is ratio of total cylinder volume to clearance volume.*

Refer Fig. 5.30. Compression ratio (r) is given by

$$r = \frac{V_s + V_c}{V_c}$$

where V_s = Swept volume

V_c = Clearance volume.

The compression ratio varies from 5 : 1 to 11 : 1 (average value 7 : 1 to 9 : 1) in *S.I. engines* and from 12 : 1 to 24 : 1 (average value 15 : 1 to 18 : 1) in *C.I. engines*.

Piston speed. *The average speed of the piston is called piston speed.*

Piston speed = 2 *LN*

where L = Length of the stroke.

N = Speed of the engine in r.p.m.

5.8. WORKING CYCLES

An internal combustion engine can work on any one of the following cycles :

(*a*) Constant volume or Otto cycle

(*b*) Constant pressure or Diesel cycle

(*c*) Dual combustion cycle.

These may be either *four stroke cycle or two stroke cycle engines*.

(*a*) **Constant volume or Otto cycle.** The cycle is so called because heat is supplied at constant volume. Petrol, gas and light oil engines work on this cycle. In the case of a petrol engine the proper mixing of petrol and air takes place in the carburettor which is situated outside the engine cylinder. The proportionate mixture is drawn into the cylinder during the suction stroke. In a gas engine also, air and gas is mixed outside the engine cylinder and this mixture enters the cylinder during the suction stroke. In light oil engines the fuel is converted to vapours by a vapouriser which receives heat from the exhaust gases of the engine and their mixture flows towards engine cylinder during suction stroke.

(*b*) **Constant pressure or Diesel cycle.** In this cycle only air is drawn in the engine cylinder during the suction stroke, this air gets compressed during the compression stroke and its pressure and temperature increase by a considerable amount. Just before the end of the stroke a metered quantity of fuel under pressure adequately more than that developed in the engine cylinder is injected in the form of fine sprays by means of a fuel injector. Due to very high pressure and temperature of the air the fuel ignites and hot gases thus produced throw the piston downwards and work is obtained. *Heavy oil engines* make use of this cycle.

(*c*) **Dual combustion cycle.** This cycle is also called semi-diesel cycle. It is so named because heat is added *partly at constant volume and partly at constant pressure*. In this cycle only air is drawn in the engine cylinder during suction stroke. The air is then compressed in hot combustion chamber at the end of the cylinder during the compression stroke to a pressure of about 26 bar. The heat of compressed air together with heat of combustion chamber ignites the fuel. The fuel is injected into the cylinder just before the end of compression stroke where it ignites immediately. The fuel injection is continued until the point of cut off is reached. The burning of fuel at first takes place at constant volume and continues to burn at constant pressure during the first part of expansion or working stroke. The field of application of this cycle is *heavy oil engines*.

5.9. INDICATOR DIAGRAM

An indicator diagram is a graph between pressure and volume ; the former being taken on vertical axis and the latter on the horizontal axis. This is obtained by an instrument known as *indicator*. The indicator diagrams are of two types : (*a*) Theoretical or hypothetical, (*b*) Actual. The theoretical or hypothetical indicator diagram is always longer in size as compared to the actual one, since in the former losses are neglected. The ratio of the area of the actual indicator diagram to the theoretical one is called *diagram factor*.

5.10. FOUR STROKE CYCLE ENGINES

Here follows the description of the four stroke otto and diesel-cycle engines.

Otto engines. The Otto four stroke-cycle refers to its use in petrol engines, gas engines and light oil engines in which the mixture of air and fuel are drawn in the engine cylinder. Since ignition in these engines is due to a spark, therefore they are also called *spark ignition engines*.

The various strokes of a four stroke (Otto) cycle engine are detailed below.

Refer Fig. 5.31.

I.V = Intel valve, E.V. = Exhaust valve, E.C. = Engine cylinder, C.R. = Connecting rod
C = Crank, S.P. = Spark plug.
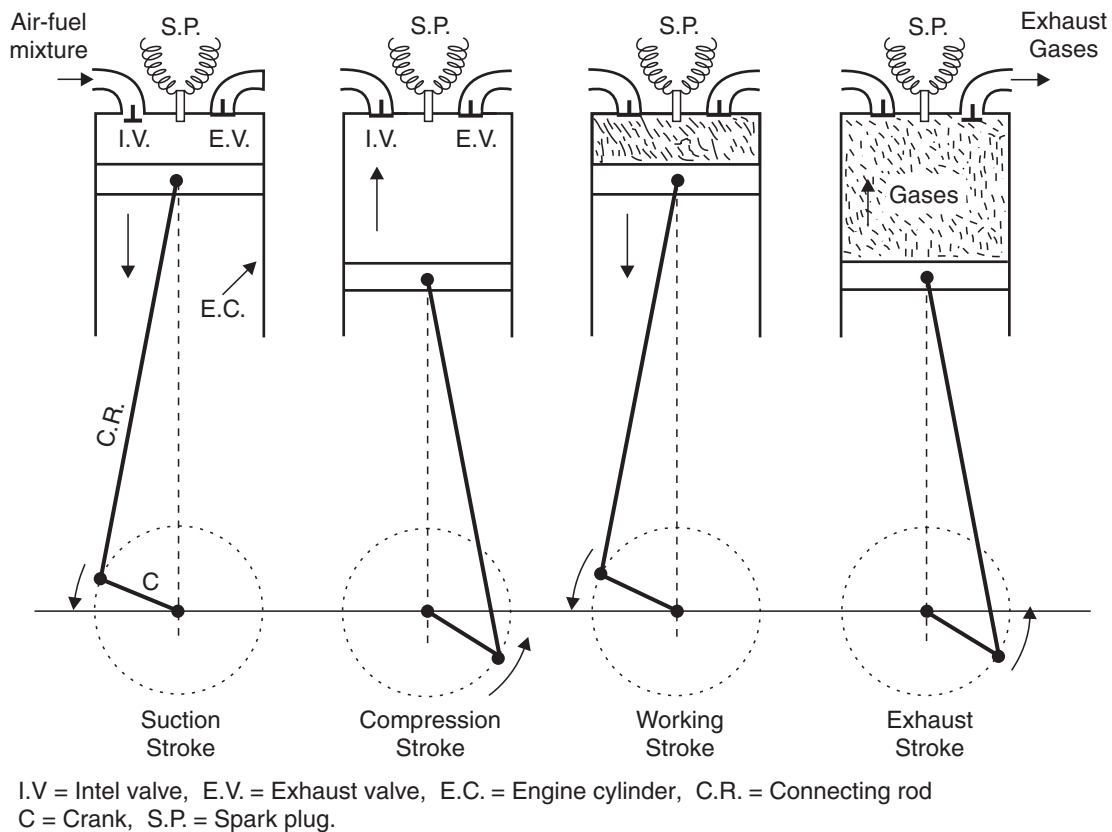
Fig. 5.31. Four stroke Otto cycle engine.

1. Suction stroke. During this stroke (also known as induction stroke) the piston moves from top dead centre (T.D.C.) to bottom dead centre (B.D.C.) ; the inlet valve opens and proportionate fuel-air mixture is sucked in the engine cylinder. This operation is represented by the line 5-1 (Fig. 5.32). The exhaust valve remains closed throughout the stroke.

2. Compression stroke. In this stroke, the piston moves (1-2) towards (T.D.C.) and compresses the enclosed fuel-air mixture drawn in the engine cylinder during suction. The pressure of the mixture rises in the cylinder to a value of about 8 bar. Just before the end of this stroke the operating-plug initiates a spark which ignites the mixture and combustion takes place at constant volume (line 2-3) (Fig. 5.32). Both the inlet and exhaust valves remain closed during the stroke.
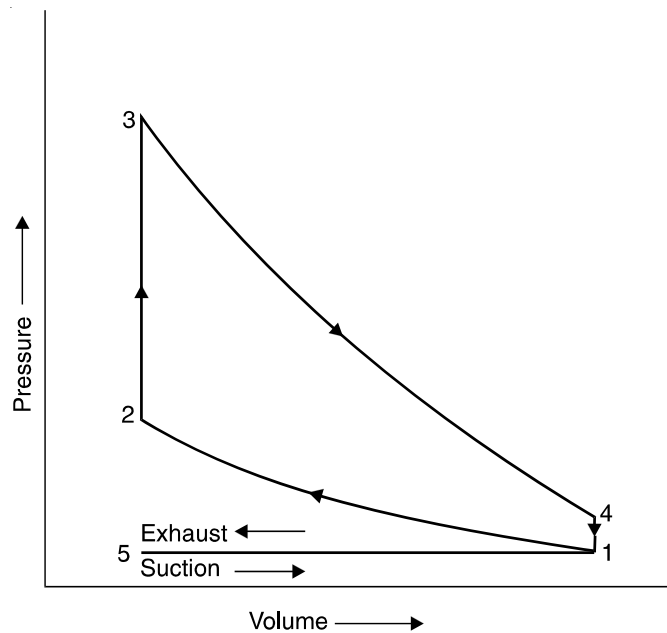
Fig. 5.32. Theoretical *p-V* diagram of a four stroke Otto cycle engine.

3. Expansion or Working stroke. When the mixture is ignited by the spark plug the hot gases are produced which drive or throw the piston from T.D.C. to B.D.C. and thus the work is obtained in this stroke. It is during this stroke when we get work from the engine ; the other three strokes namely suction, compression and exhaust being idle. *The flywheel mounted on the engine shaft stores energy during this stroke and supplies it during the idle strokes*. The expansion of the gases is shown by 3-4. (Fig. 5.32). Both the valves remain closed during the start of this stroke but when the piston just reaches the B.D.C. the exhaust valve opens.

4. Exhaust stroke. This is the last stroke of the cycle. Here the gases from which the work has been collected become useless after the completion of the expansion stroke and are made to escape through exhaust valve to the atmosphere. This removal of gas is accomplished during this stroke. The piston moves from B.D.C. to T.D.C. and the exhaust gases are driven out of the engine cylinder ; this is also called *scavenging*. This operation is represented by the line (1-5) (Fig. 5.32).

Fig. 5.33 shows the actual indicator diagram of four stroke Otto cycle engine. It may be noted that line 5-1 is *below the atmospheric pressure line. This is due to the fact that owing to restricted area of the inlet passages the entering fuel air mixture cannot cope with the speed of the piston*. The exhaust line 4-5 is slightly *above the atmospheric pressure line. This is due to restricted exhaust passages which do not allow the exhaust gases to leave the engine-cylinder quickly.*

The loop which has area 4-5-1 is called *negative loop* ; it gives the pumping loss due to admission of fuel-air mixture and removal of exhaust gases. The area 1-2-3-4 is the total or gross work obtained from the piston and net work can be obtained by subtracting area 4-5-1 from area 1-2-3-4.
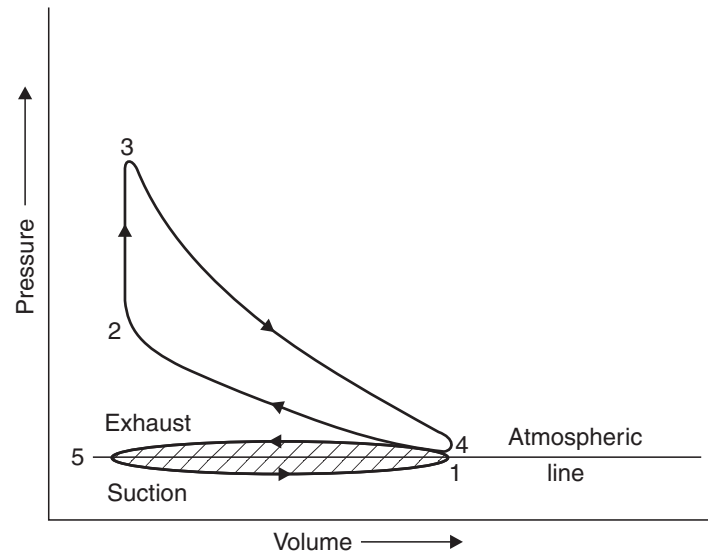
Fig. 5.33. Actual *p-V* diagram of a four stroke Otto cycle engine.

Diesel engines (four stroke cycle). As is the case of Otto four stroke ; this cycle too is completed in four strokes as follows. (Refer Fig. 5.34).

| *F.I.* = Fuel injector | *I.V.* = Inlet valve | *E.V.* = Exhaust valve |
| *E.C.* = Engine cylinder | *C* = Crank | *C.R.* = Connecting rod |
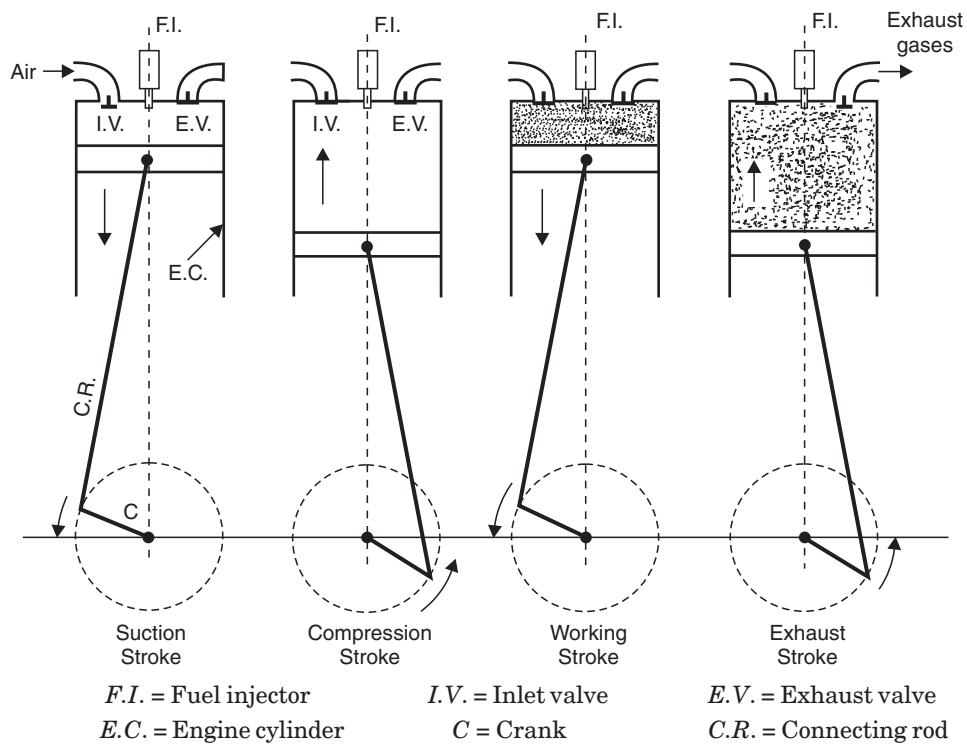
Fig. 5.34. Four stroke Diesel cycle engine.

1. Suction stroke. With the movement of the piston from T.D.C. to B.D.C. during this stroke, the inlet valve opens and the air at atmospheric pressure is drawn inside the engine cylinder ; the exhaust valve however remains closed. This operation is represented by the line 5-1 (Fig. 5.35).

2. Compression stroke. The air drawn at atmospheric pressure during the suction stroke is compressed to high pressure and temperature (to the value of 35 bar and 600°C respectively) as the piston moves from B.D.C. to T.D.C. This operation is represented by 1-2 (Fig. 5.35). Both the inlet and exhaust valves do not open during any part of this stroke.

3. Expansion or Working stroke. As the piston starts moving from T.D.C. a metered quantity of fuel is injected into the hot compressed air in fine sprays by the fuel injector and it (fuel) starts burning at constant pressure shown by the line 2-3. At the point 3 fuel supply is cut off. The fuel is injected at the end of compression stroke but in actual practice the ignition of the fuel starts before the end of the compression stroke. The hot gases of the cylinder expand adiabatically to point 4, thus doing work on the piston. The expansion is shown by 3-4 (Fig. 5.35).
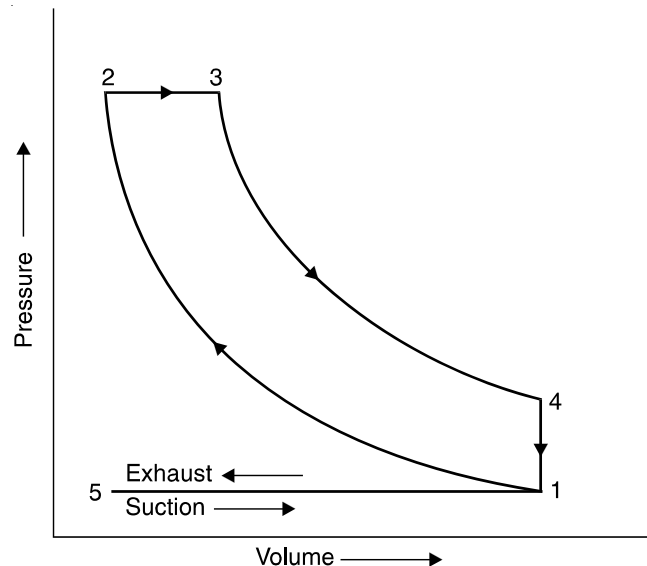
Fig. 5.35. Theoretical *p-V* diagram of a four stroke Diesel cycle.

4. Exhaust stroke. The piston moves from the B.D.C. to T.D.C. and the exhaust gases escape to the atmosphere through the exhaust valve. When the piston reaches the T.D.C. the exhaust valve closes and the cycle is completed. This stroke is represented by the line 1-5 (Fig. 5.35).

Fig. 5.36 shows the actual indicator diagram for a four-stroke Diesel cycle engine. It may be noted that line 5-1 is below the atmospheric pressure line. This is due to the fact that owing to the restricted area of the inlet passages the entering air can't cope with the speed of the piston. The exhaust line 4-5 is slightly above the atmospheric line. This is because of the restricted exhaust passages which do not allow the exhaust gases to leave the engine cylinder quickly.

The loop of area 4-5-1 is called negative loop ; it gives the pumping loss due to admission of air and removal of exhaust gases. The area 1-2-3-4 is the total or gross work obtained from the piston and net work can be obtained by subtracting area 4-5-1 from area 1-2-3-4.
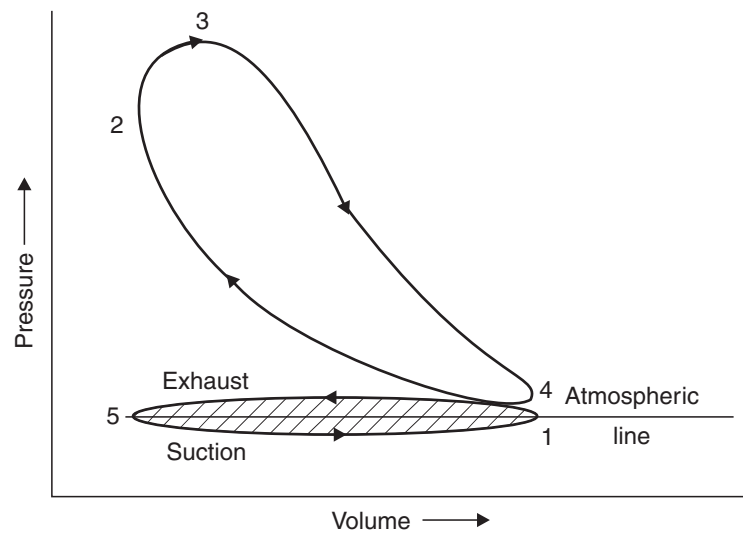
Fig. 5.36. Actual *p*-*V* diagram of four stroke Diesel cycle.

Valve Timing Diagrams (Otto and Diesel engines)

1. Otto engine. Fig. 5.37 shows a theoretical valve timing diagram for *four stroke "Otto*
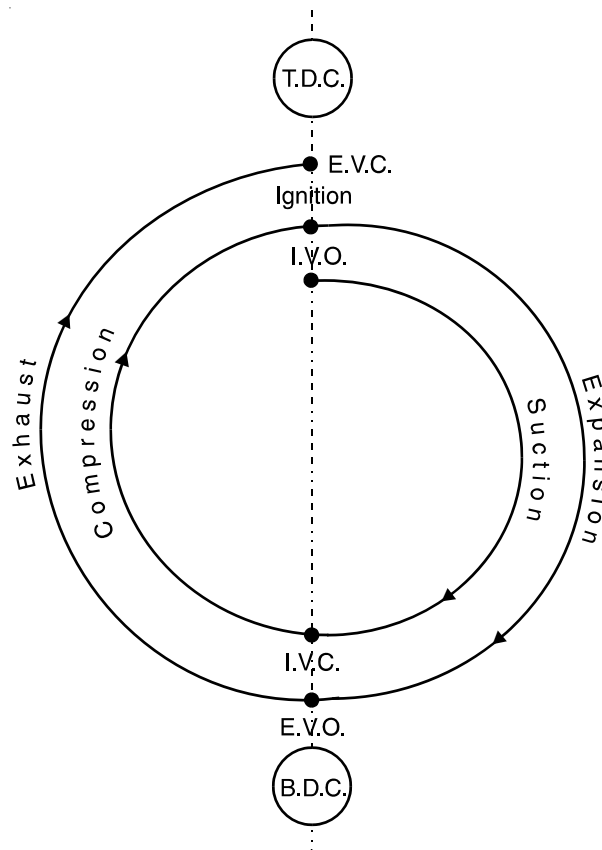
Fig. 5.37. Theoretical valve timing diagram (four stroke Otto cycle engine).

cycle" engines which is self-explanatory. In actual practice, it is difficult to open and close the valve instantaneously ; so as to get better performance of the engine the valve timings are modified. In Fig. 5.38 is shown an actual valve timing diagram. The inlet valve is opened 10° to 30° in advance of the T.D.C. position to enable the fresh charge to enter the cylinder and to help the burnt gases at the same time, to escape to the atmosphere. The suction of the mixture continues up to 30°-40° or even 60° after B.D.C. position. The inlet valve closes and the compression of the entrapped mixture starts.
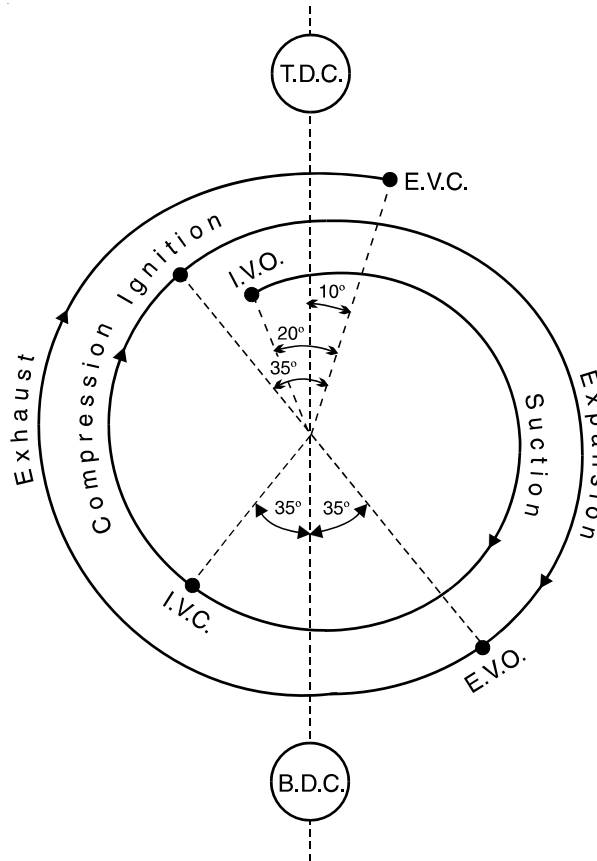
Fig. 5.38. Actual valve timing diagram (four stroke Otto cycle engines).

The sparking plug produces a spark 30° to 40° before the T.D.C. position ; thus fuel gets more time to burn. The pressure becomes maximum nearly 10° past the T.D.C. position. The exhaust valve opens 30° to 60° before the B.D.C. position and the gases are driven out of the cylinder by piston during its upward movement. The exhaust valve closes when piston is nearly 10° past T.D.C. position.

2. Diesel engines. Fig. 5.39 shows the valve timing diagram of a *four stroke "Diesel cycle" engines* (theoretical valve timing diagram, is however the same as Fig. 5.37). Inlet valve opens 10° to 25° in advance of T.D.C. position and closes 25° to 50° after the B.D.C. position. Exhaust valve opens 30° to 50° in advance of B.D.C. position and closes 10° to 15° after the T.D.C. position. The fuel injection takes place 5° to 10° before T.D.C. position and continues up to 15° to 25° near T.D.C. position.
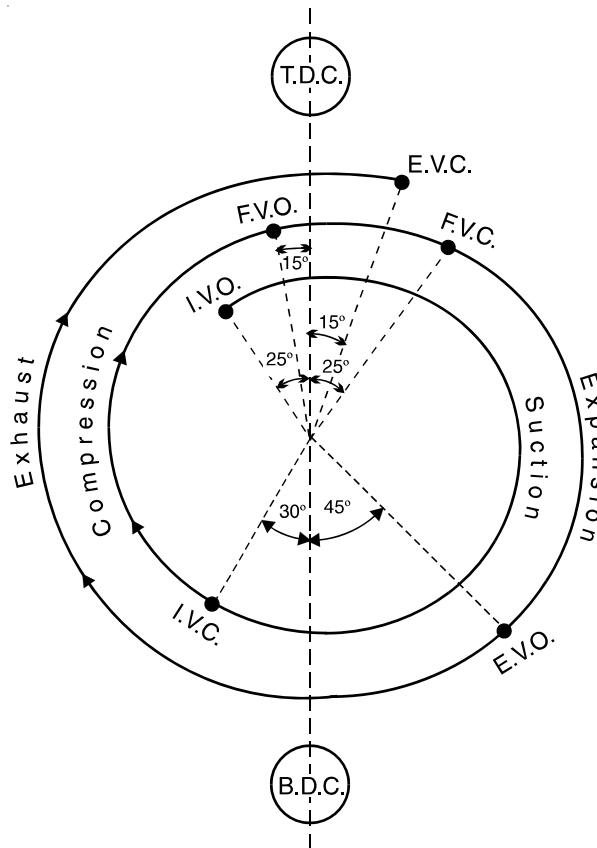
Fig. 5.39. Actual valve timing diagram (four stroke Diesel cycle engines).

5.11. TWO STROKE CYCLE ENGINES

In 1878, Dugald-clerk, a British engineer introduced a cycle which could be completed in two *strokes of piston rather than four strokes* as is the case with the four stroke cycle engines. The engines using this cycle were called two stroke cycle engines. In this engine suction and exhaust strokes are eliminated. Here *instead of valves, ports are used. The exhaust gases are driven out from engine cylinder by the fresh charge of fuel entering the cylinder nearly at the end of the working stroke.*

Fig. 5.40 shows a two stroke petrol engine (used in scooters, motor cycles etc.). The cylinder L is connected to a closed crank chamber C.C. During the upward stroke of the piston M, the gases in L are compressed and at the same time fresh air and fuel (petrol) mixture enters the crank chamber through the valve V. When the piston moves downwards, V closes and the mixture in the crank chamber is compressed. Refer Fig. 5.40 (*i*), the piston is moving upwards and is compressing an explosive change which has previously been supplied to L. Ignition takes place at the end of the stroke. The piston then travels downwards due to expansion of the gases [Fig. 5.40 (*ii*)] and near the end of this stroke the piston uncovers the exhaust port (E.P.) and the burnt exhaust gases escape through this port [Fig. 5.40 (*iii*)]. The transfer port (T.P.) then is uncovered immediately, and the compressed charge from the crank chamber flows into the cylinder and is deflected upwards by the hump provided on the head of the piston. It may be noted that the incoming air petrol mixture helps the removal of gases from the
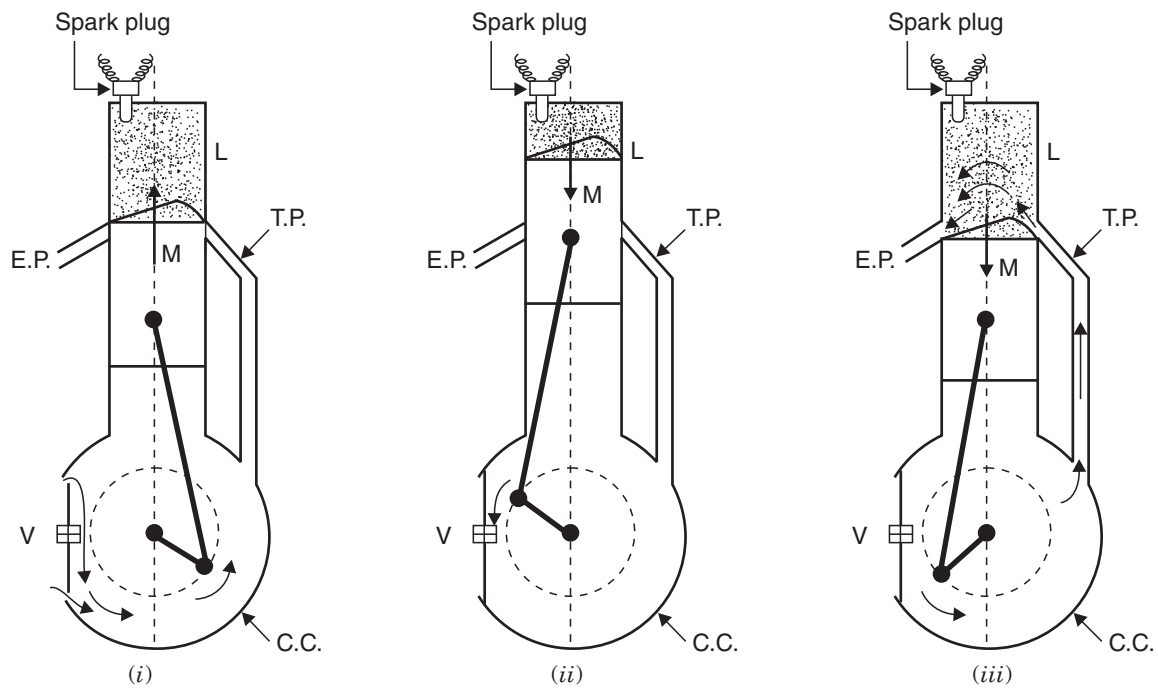
Fig. 5.40. Two stroke cycle engine.

engine-cylinder ; if, in case these exhaust gases do not leave the cylinder, the fresh charge gets diluted and efficiency of the engine will decrease. The piston then again starts moving from B.D.C. to T.D.C. and the charge gets compressed when E.P. (exhaust port) and T.P. are covered by the piston ; thus the cycle is repeated.
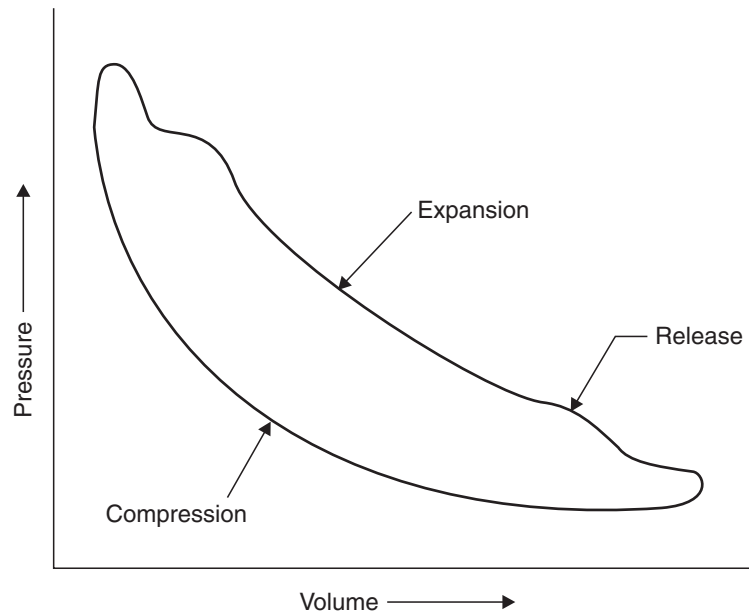
Fig. 5.41. *p-V* diagram for a two stroke cycle engine.

Fig. 5.41 show the *p-V* diagram for a two stroke cycle engine. It is only for the main cylinder or the top side of the piston. Fig. 5.42 shows self-explanatory port timing diagram for a two stroke cycle engine.

I.V.U. = Inlet valve uncovers   *I.V.C.* = Inlet valve closes   *E.P.U.* = Exhaust port uncovers
T.P.U. = Transfer port uncovers *T.P.C.* = Transfer port closes   *E.P.C.* = Exhaust port closes
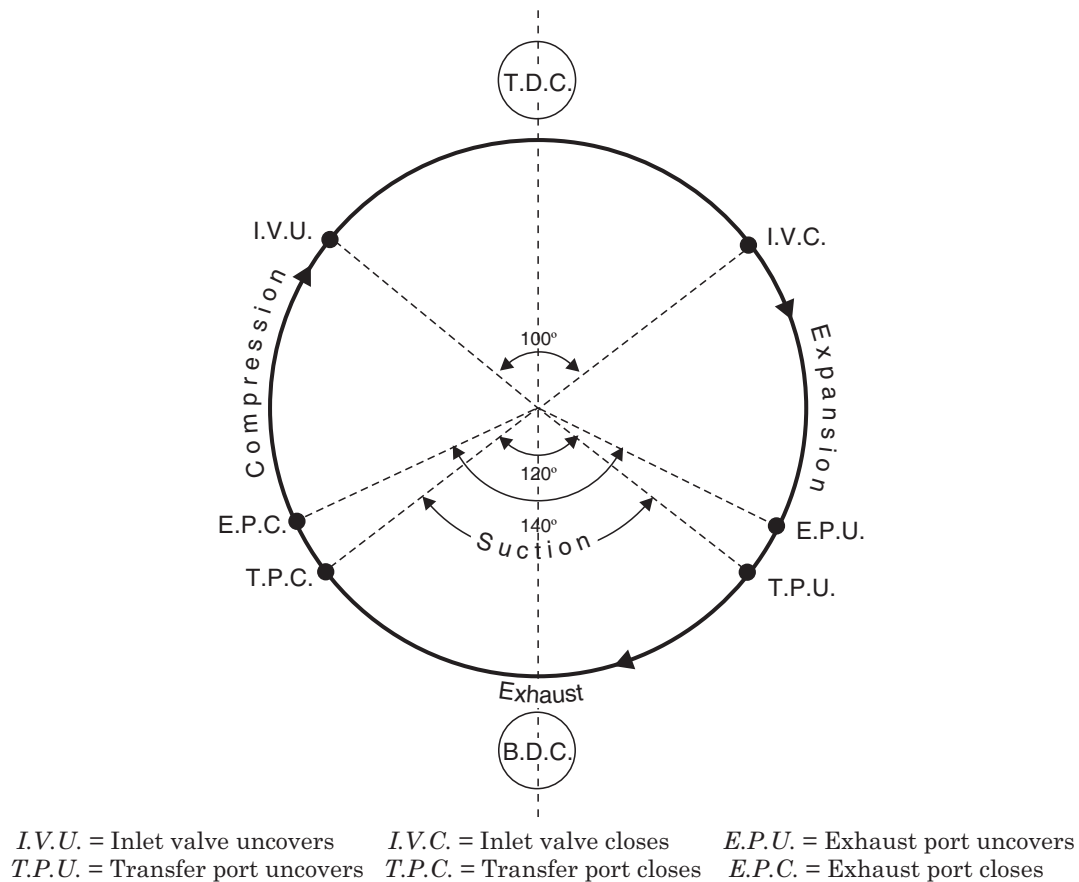
Fig. 5.42. Port timing diagram.

In a two stroke Diesel cycle engine all the operations are the same as in the spark ignition (Otto cycle) engine with the differences ; firstly in this case, only air is admitted into cylinder instead of air-fuel mixture and secondly fuel injector is fitted to supply the fuel instead of a sparking plug.

5.12. COMPARISON OF FOUR STROKE AND TWO STROKE CYCLE ENGINES

S.No.	Aspects	Four Stroke Cycle Engines	Two Stroke Cycle Engines
1.	*Completion of cycle*	The cycle is completed in *four strokes of the piston* or in *two revolutions of the crankshaft*. Thus one power stroke is obtained in every two revolutions of the crankshaft.	The cycle is completed in *two strokes of the piston* or in *one revolution of the crankshaft*. Thus one power stroke is obtained in each revolution of the crankshaft.

S.No.	Aspects	Four Stroke Cycle Engines	Two Stroke Cycle Engines
2.	*Flywheel required -heavier or lighter*	Because the turning-movement is not so uniform therefore, *heavier* flywheel is needed.	More uniform turning movement and hence *lighter* flywheel is needed.
3.	*Power produced for same size of engine*	Again because of one power stroke for two revolutions, power produced for same size of engine is *small* or for the same power the engine is heavy and bulky.	Because of one power stroke for one revolution, power produced for same size of engine in *more* (theoretically twice, actually about 1.3 times) or for the same power the engine is light and compact.
4.	*Cooling and lubrication requirements*	Because of one power stroke in two revolutions *lesser* cooling and lubrication requirements. Lesser rate of wear and tear.	Because of one power stroke in one revolution *greater* cooling and lubrication requirement. Great rate of wear and tear.
5.	*Valve and valve mechanism*	The four stroke engine *contains* valve and valve mechanism.	Two stroke engines have *no* valves but only ports (some two stroke engines are fitted with conventional exhaust valves).
6.	*Initial cost*	Because of the heavy weight and complication of valve mechanism, *higher* is the initial cost.	Because of light weight and simplicity due to absence of valve mechanism, *cheaper* in initial cost.
7.	*Volumetric efficiency*	Volumetric efficiency *more* due to more time of induction.	Volumetric efficiency *less* due to lesser time for induction.
8.	*Thermal and part-load efficiencies*	Thermal efficiency higher, part load efficiency better than two stroke cycle engine.	Thermal efficiency lower, part load efficiency lesser than four stroke cycle engine.
9.	*Applications*	Used where efficiency is important ; in *cars, buses, trucks tractors, industrial engines, aeroplane, power generators etc*.	In two stroke petrol engine some fuel is exhausted during scavenging. Used where (*a*) *low cost*, and (*b*) *compactness and light weight important*. Two stroke (air cooled) petrol engines used in very small sizes only, *lawn movers, scooters motor cycles* (lubricating oil mixed with petrol). Two stroke diesel engines used in *very large sizes* more than 60 cm bore, for *ship propulsion* because of low weight and compactness.

5.13. COMPARISON OF SPARK IGNITION (S.I.) AND COMPRESSION IGNITION (C.I.) ENGINES

S.No.	Aspects	S.I. engines	C.I. engines
1.	*Thermodynamic cycle*	Otto cycle	Diesel cycle ...... For slow speed engines Dual cycle ...... For high speed engines
2.	*Fuel used*	Petrol	Diesel.

S.No.	Aspects	S.I. engines	C.I. engines
3.	*Air-fuel ratio*	10 : 1 to 20 : 1	18 : 1 to 100 : 1.
4.	*Compression ratio*	upto 11 ; Average value 7 to 9 ; Upper limit of compression ratio fixed by *anti-knock quality of fuel.*	12 to 24 ; Average value 15 to 18 ; Upper limit of compression ratio is limited by *thermal and mechanical stresses*.
5.	*Combustion*	Spark ignition	Compression ignition.
6.	*Fuel supply*	By carburettor ...... cheap method	By injection ...... explosive method.
7.	*Operating pressure* (*i*) Compression pressure (*ii*) Maximum pressure	7 bar to 15 bar 45 bar to 60 bar	30 bar to 50 bar 60 bar to 120 bar.
8.	*Operating speed*	High speed : 2000 to 6000 r.p.m.	Low speed : 400 r.p.m. Medium speed : 400 to 1200 r.p.m. High speed : 1200 to 3500 r.p.m.
9.	*Control of power*	Quantity governing ...... by throttle	Quality governing ...... by rack.
10.	*Calorific value*	44 MJ/kg	42 MJ/kg.
11.	*Cost of running*	high	low.
12.	*Maintenance cost*	Minor maintenance required	Major overall required but less frequently.
13.	*Supercharging*	Limited by *detonation*. Used only in *aircraft engines*.	Limited by *blower power and mechanical and thermal stresses. Widely used.*
14.	*Two stroke operation*	*Less suitable*, fuel loss in scavenging. But small two stroke engines are used in mopeds, scooters and motorcycles due to their *simplicity and low cost.*	No fuel loss in scavenging. *More suitable*.
15.	*High powers*	No	Yes.
16.	*Uses*	Mopeds, scooters, motorcycles, simple engine passenger cars, aircrafts etc.	Buses, trucks locomotives, tractors, earth moving machinery and stationary generating plants.

5.14. COMPARISON BETWEEN A PETROL ENGINE AND A DIESEL ENGINE

S.No.	Petrol engine	Diesel engine
1.	Air petrol mixture is sucked in the engine cylinder during suction stroke.	Only air is sucked during suction stroke.
2.	Spark plug is used.	Employs an injector.
3.	Power is produced by spark ignition.	Power is produced by compression ignition.
4.	Thermal efficiency up to 25%.	Thermal efficiency up to 40%.

S.No.	Petrol engine	Diesel engine
5.	Occupies less space.	Occupies more space.
6.	More running cost.	Less running cost.
7.	Light in weight.	Heavy in weight.
8.	Fuel (Petrol) costlier.	Fuel (Diesel) cheaper.
9.	Petrol being volatile is dangerous.	Diesel is non-dangerous as it is non-volatile.
10.	Pre-ignition possible.	Pre-ignition not possible.
11.	Works on Otto cycle.	Works on Diesel cycle.
12.	Less dependable.	More dependable.
13.	Used in *cars* and *motor cycles*.	Used in heavy duty vehicles like *trucks, buses* and *heavy machinery*.

5.15. HOW TO TELL A TWO STROKE CYCLE ENGINE FROM A FOUR STROKE CYCLE ENGINE ?

S.No.	Distinguishing features	Four stroke cycle engine	Two stroke cycle engine
1.	*Oil sump and oil-filter plug*	It has an oil sump and oil-filter plug.	It does not have oil sump and oil-filter plug.
2.	*Oil drains etc.*	It requires oil drains and refills periodically, just an automobile do.	In this type of engine, the oil is added to the gasoline so that a mixture of gasoline and oil passes through the carburettor and enters the crankcase with the air.
3.	*Location of muffler (exhaust silencer)*	It is installed at the head end of the cylinder at the exhaust valve location.	It is installed towards the middle of the cylinder, at the exhaust port location.
4.	Name plate	If the name plate mentions the type of oil and the crankcase capacity, or similar data, it is a four stroke cycle engine.	If the name plate tells to mix oil with the gasoline, it is a two stroke cycle engine.

THEORETICAL QUESTIONS

1. Name the two general classes of combustion engines and state how do they basically differ in principle ?
2. Discuss the relative advantages and disadvantages of internal combustion and external combustion engines.
3. What are the two basic types of internal combustion engines ? What are the fundamental differences between the two ?
4. What is the function of a governor ? Enumerate the types of governors and discuss with a neat sketch the Porter governor.
5. Differentiate between a flywheel and a governor.
6. (*a*) State the function of a carburettor in a petrol engine.
   (*b*) Describe a simple carburettor with a neat sketch and also state its limitations.

7. Explain with neat sketches the construction and working of the following :
    (*i*) Fuel pump   (*ii*) Injector.
8. Explain the following terms as applied to I.C. engines :
    Bore, stroke, T.D.C., B.D.C., clearance volume, swept volume, compression ratio and piston speed.
9. Explain with suitable sketches the working of a four stroke otto engine.
10. Discuss the difference between ideal and actual valve timing diagrams of a petrol engine.
11. In what respects four stroke diesel cycle (compression ignition) engine differs from four stroke cycle spark ignition engine ?
12. Discuss the difference between theoretical and actual valve timing diagrams of a diesel engine.
13. What promotes the development of two stroke engines ? What are the two main types of two stroke engines.
14. Describe with a suitable sketch the two stroke cycle spark ignition (SI) engine. How its indicator diagram differs from that of four stroke cycle engine ?
15. Compare the relative advantages and disadvantages of four stroke and two stroke cycle engines.

6

Engineering Materials

6.1. Classification of materials—Classification of electrical engineering materials—Biomaterials—Advanced materials—Smart materials—Nanotechnology and nanomaterials. **6.2. Mechanical properties of metals. 6.3. Ferrous metals and alloys**—Introduction—Pig iron—Cast iron—Wrought iron—Composition, properties and uses of carbon steels—Comparison of cast iron, wrought iron, mild steel and hard steel—Alloy steels. **6.4. Non-ferrous metals and alloys**—Aluminium—Copper—Copper alloys—Aluminium alloys. **6.5. Polymers/plastics**—Introduction—Classification of plastics—Thermoplastic materials—thermosetting materials—Trade names and typical applications of some important plastics—Laminated plastics—Fiber glass reinforced plastics. **6.6. Ceramic materials**—Introduction—Classification of ceramics—Advantages of ceramic materials—Applications of ceramics—Properties of ceramic materials—Glass—Cements—Advanced ceramics. **6.7. Composite materials/composites**—General aspects—Classification—Particle-reinforced composites—Fibre-reinforced composites—Structural composites. **6.8. Conductors, semiconductors and insulators**—Conductors—Semiconductors—Insulators (or dielectrics). **6.9. Selection of materials**—*Theoretical Questions.*

6.1. CLASSIFICATION OF MATERIALS

The engineering materials may be *classified* as follows :

1. **Metals** (*e.g.,* iron, aluminium, copper, zinc, lead etc.).

2. **Non-metals** (*e.g.,* leather, rubber, plastics, asbestos, carbon etc.).

Metals may be further subdivided as :

(*i*) *Ferrous metals* (*e.g.,* cast iron, wrought iron and steel) and *alloys* (*e.g.,* silicon steel, high speed steel, spring steel etc.)

(*ii*) *Non-ferrous metals* (*e.g.,* copper, aluminium, zinc, lead etc.) and *alloys* (brass, bronze, duralumin etc.).

Metals

The iron containing metals which include all types of iron and steel are called *ferrous metals,* whilst others are specified as *non-ferrous*.

Non-metals

The commonly adopted *non-metallic materials* are *leather, rubber, asbestos* and *plastics.*

Leather is used for belt drives and as packing or as washers. It is very flexible and will stand considerable wear under suitable conditions. The modulus of elasticity varies according to load.

Rubber is used as packing, belt drive and as an *electric insulator*. It has a high bulk modulus and must have lateral freedom if used as a packing ring.

Asbestos is used for lagging around steam pipes and steam boilers.

Plastics is a term applied to a large class of mouldable organic compounds which are sold under different trade names and are being discovered constantly. They are used for bushing, steering

wheels, tubes for oil and water, automobile tyres etc. Plastics are divided roughly in two classes, called *thermoplastic* and *thermosetting plastics*. Materials in the former group becomes soft and pliable when heated to moderate temperatures and then hardened when cooled. They will *soften every time when heat is applied and reworked as often as desired. Thermosetting plastics soften the first time they are heated, hardened when cooled and cannot be softened by reheating. Plastics can be moulded, cast, folded into sheets and extended.*

Table 6.l shows the *difference* between metals and non-metals.

Table 6.1. Difference between Metals and Non-metals

S.No.	Property	Metals	Non-metals
1.	Structure	All solid metals have crystalline structure	They exist in amorphic or mesomorphic forms
2.	Excitation of valence electron by e.m.f. (electromotive force)	Easy	Difficult
3.	State	Generally solids at room temperature	Gases and solids at ordinary temperature
4.	Lustre	Possess metallic lustre.	Do not possess metallic lustre (except iodine and graphite)
5.	Conductivity	Good conductors of heat and electricity	Bad conductors of heat and electricity.
6.	Malleability	Malleable	Not malleable
7.	Ductility	Ductile	Not ductile
8.	Hardness	Generally hard	Hardness varies
9.	Electrolysis	Form anions	Form anions
10.	Density	High density	Low density

Engineering materials may also be **classified** as follows :

1. Metals and alloys
2. Ceramic materials
3. Organic polymers.

1. Metals and alloys :

Metals are polycrystalline bodies consisting of a great number of fine crystals (10^{-1} to 10^{-4} cm size) differently oriented with respect to one another. Depending upon the mode of crystallization, these crystals may be of various irregular shapes, and, in contrast to crystals of regular shape, are called *crystallites or grains* of the *metal*. Metals in the solid state and, to some extent, in the liquid state possess *high thermal and electrical conductivity, and a positive temperature coefficient of electrical resistivity*. The general resistance of pure metals *increases with the temperature*. Many metals display superconductivity; at temperatures near absolute zero, their electrical resistance drops abruptly to extremely low values. Besides, all metals are capable of *thermionic emission, i.e.,* the emission of electrons upon being heated; they are good reflectors of light and lend themselves well to plastic deformation.

Pure metals *are of low strength and in many cases, do not possess the required physiochemical and technological properties for some definite purpose*. Consequently, they are *soldom* used in engineering. The overwhelming majority of metals used are *alloys*.

Alloys are produced by *melting or sintering two or more metals,* or metals and a non-metal, together. Alloys possess typical properties inherent in the metallic state, the substances that make up the alloy are called its components. An alloy can consist of two or more components.

Examples of metals and alloys : Steels, copper, aluminium, brasses, bronze, invar, superalloys etc.

2. Ceramic materials :

These materials are non-metallic solids made of inorganic compounds such as oxides, nitrides, borides, silicides and carbides. They are fabricated by first shaping the powder with or without the application of pressure into a compact which is subsequently subjected to a high temperature treatment, called *sintering. Traditional ceramics* were made from crude naturally occurring mixtures of materials having inconsistent purity. These have been used essentially in the manufacture of pottery, porcelain, cement and silicate glasses. *New ceramics* possess exceptional electrical, magnetic, chemical, structural and thermal properties. Such ceramics are now extensively used in the *electronic control devices, computers, nuclear engineering and aerospace fields.*

Examples of ceramics : MgO, CdS, ZnO, SiC, $B_aT_iQ_2$, silica, sodalime, glass, concrete, cement, ferrites, garnets, etc.

3. Organic materials :

These materials are derived *directly from carbon*. They usually consist of carbon chemically combined with hydrogen, oxygen or other non-metallic substances. In many instances their structures are fairly complex.

Common organic materials are : *Plastics and Synthetic rubbers*. These are termed *"polymers"* because they are formed by polymerization reaction in which relatively simple molecules are chemically combined into massive long-chain molecules or "three dimensional" structures.

Examples of organic materials : Plastics : PVC, PTFE, polythene; Fibers : terylene, nylon, cotton; Natural and synthetic rubbers, leather, etc.

Examples of composites :

1. *Metals and alloys and ceramics :*
   (*i*) Steel reinforced concrete
   (*ii*) Dispersion hardened alloys.
2. *Metals and alloys and organic polymers :*
   (*i*) Vinyl-coated steel
   (*ii*) Whisker-reinforced plastics.
3. *Ceramics and organic polymers :*
   (*i*) Fibre-reinforced plastics
   (*ii*) Carbon-reinforced rubber.

6.1.1. Classification of Electrical Engineering Materials

The electrical engineering materials may be *classified* into the following *four* types :

1. Conductors
2. Semiconductors
3. Insulators (or dielectrics)
4. Magnetic materials.

1. Conductors :

- **Conductors** may be defined, as the *materials which have free valence electrons in plenty for electric conduction*. The commonly used conductors are *copper, aluminium, tungsten, iron and steel, lead, nickel, tin etc*. In this case the valence and conduction

bands *overlap*. Since there is no physical distinction between the two bands, therefore, a large number of free electrons (conduction) are available.

- The conductors are used in *electric devices, instruments and all kinds of electrical machine windings*. They are also employed in manufacturing of *cables and wires,* for the *distribution of electrical energy* over long distances and *telephone and telegraph circuits*.

2. Semiconductors :

Semiconductors are *solid materials, either non-metallic elements or compounds which allow electrons to pass through them so that they conduct electricity in much the same way as the metals*. They occupy an intermediate position between conductors and insulators. In this case, the *valence band is almost filled but conduction band is almost empty;* they are *separated by a small energy gap*. The valence band is completely filled at 0°K and no electron is available for conduction. But as the temperature is increased the width of energy gap decreases and some of the electrons are liberated into the conduction band. In other words the conductivity of semiconductors increases with temperature. Semiconductors usually have *high resistivity, negative temperature coefficient of resistance* and are *generally hard and brittle*.

The main difference between a conductor and semiconductor relates to the dependence of their conductivity on the degree of purity of metals. The conductivity of a good conductor *increases* with purification whereas that of semiconductor generally *decreases* with purification.

Examples of elements which are semiconductors are : Boron (B), Carbon (C), Silicon (Si), Germanium (Ge), Phosphorus (P), Arsenic (As), Antinomy (Sb), Sulphur (S), Selenium (Se), Iodine (I). A number of semiconducting compounds in the form of oxides, alloys, sulphides, halides and solenoids are also available.

Semiconductors are ***used*** in different fields of electrical engineering, *e.g., telecommunication and radio communication, electronics and power engineering. They also render their services as amplifiers, rectifiers, photocells, special sources of electric current etc.*

3. Insulators :

Insulators are *those materials in which valence electrons are very tightly bound to their parent atoms thus requiring very large electric field to remove them from attraction of nuclei*. They are not governed by electrodynamic phenomena involving the direction flow of number of electric charges by the electrostatic phenomena associated with the presence of an electric field. They have (*i*) a full valence band, (*ii*) an empty conduction band, and (*iii*) a large energy gap between them; for conduction to take place, electrons must be given sufficient energy to jump from valence band to conduction band. At ordinary temperature the probability of electrons from full valence band gaining sufficient energy so as to surmount energy gap and becoming available for conduction in conduction band is *slight. But increase in temperature enables electrons to go to conduction band.*

In *electric circuits and devices the insulators insulate one current-carrying part from another.*

The insulating materials may be of *three* types :

1. *Solid :* Mica, micanite, porcelain, asbestos, slate, marble, bakelite, rubber, PVC, polythene, paper, glass, cotton, silk, wood, valcanised fibre, ceramic, aluminium oxide.
2. *Liquid :* Natural resin varnishes, bituminous varnishes, phenolic varnishes, shellac varnishes, etc.
3. *Gaseous :* Air, nitrogen freon.

4. Magnetic materials :

- ***Magnetic materials*** are *those materials in which a state of magnetisation can be induced*. In accordance with the value of relative permeability the materials may be classified in the following three ways :

(*i*) **Ferromagnetic materials.** The relative permeability of these materials is *much greater than unity* and is dependent on the field strength. The principal ferromagnetic elements are : *Iron, cobalt and nickel*. Gadolinium however, also comes under this classification. They have *high susceptibility*.

(*ii*) **Paramagnetic materials.** They have relative permeability *slightly greater than unity* and are magnetised slightly. *Aluminium, platinum and oxygen* belong to this category.

(*iii*) **Diamagnetic materials.** The relative permeability of these materials is *slightly less than unity*. The examples are *bismuth, silver, copper and hydrogen*.

- *The magnetic properties of materials arise from the spin of electrons and orbital motion of electrons around the atomic nuclei*. In several atoms the opposite spin neutralises one another, but when there is an excess of electrons spinning in one direction, a magnetic field is produced. All substances, except ferromagnetic materials which can form permanent magnets, exhibit magnetic effects only when subjected to an external electromagnetic field.

- Since magnetic materials strengthen the magnetic field in which they are placed and possess high magnetic permeability, they claim wide field of applications in *the form of magnetic waves, magnetic screens and permanent magnets*.

6.1.2. Biomaterials

- **Biomaterials** are *employed in components implanted into the human body for replacement of diseased or damaged body parts*.

- These materials must not produce toxic substances and must be compatible with body tissues (*i.e.,* must not cause adverse biological reactions).

- All of the above materials—metals, ceramics, polymers, composites and semiconductors may be used as biomaterials.

6.1.3. Advanced Materials

- *Materials that are utilised in high-technology (or high-tech) applications are sometimes "Advanced materials"*. By high technology we mean a device or product that operates or functions using relatively intricate and sophisticated principles : *Examples* include *electronic equipment* (VCRs, CD players etc.) *computers, fiberoptic systems, spacecraft, aircraft, and military rocketry*.

- These advanced materials are typically *either traditional materials whose properties have been enhanced or newly developed, high-performance materials*. Furthermore, they may be of all materials types (*e.g.,* metals, ceramics, polymers) and are normally *relatively expensive*.

6.1.4. Materials of Future—"Smart Materials"

- *Smart (or intelligent) materials* are a group of new and state-of-the-art materials now being developed that will have a significant influence on many of our technologies. The adjective **"smart"** *implies that these materials are able to sense changes in their environments and then respond to these changes in predetermined manners-traits that are also found in living organisms*. In addition, this *"smart"* concept is *being extended to rather sophisticated systems that consist of both smart and traditional materials*.

- Components of a smart material (or system) include some types of *sensor* (that detects an input signal), and an *actuator* (that performs a responsive and adaptive function). Actuators may be called upon to change shape, position, frequency, or mechanical characteristics in response to changes in temperature, electric fields, and/or magnetic fields.

Following *four types of materials* are commonly used for **actuators :**

(*i*) *Shape memory alloys*. These are metals that, after having been deformed, revert back to their original shapes when temperature is changed.

(*ii*) *Piezoelectric ceramics*. These expand and contract in response to an applied electric field (or voltage); conversely they also generate an electric field when their dimensions are altered.

(*iii*) *Magnetostrictive materials*. The behaviour of these materials is analogous to that of piezoelectrics, except that they are responsive to magnetic fields.

(*iv*) *Electrorheological / magnetorheological fluids*. These are liquids that experience dramatic changes in viscosity upon the application of electric and magnetic fields, respectively.

Materials / devices employed as **sensors** include the following :

(*i*) Optical fibers.

(*ii*) Piezoelectric materials (including some polymers).

(*iii*) Microelectromechanical devices.

Example. One type of smart system is used in helicopters to reduce aerodynamic cockpit noise that is created by the rotating rotor blades. *Piezoelectric sensors* inserted into the blades, monitor blade stresses and deformations; feedback signals from these sensors are fed into a computer-controlled adaptive device, which generates noise-cancelling antinoise.

6.1.5. Nanotechnology and Nanomaterials

Nanotechnology

- The general procedure utilised by scientists to understand the chemistry and physics of materials, until recent times, has been to begin by studying large and complex structures, and then to investigate the fundamental building blocks of these structures that are smaller and simpler. This approach is sometimes termed **"top-down"** *science*.

- However, with the advent of scanning probe microscopes, which permit observation of individual atoms and molecules, it has become possible *to manipulate and move atoms and molecules to form new structures, and, thus design new materials that are built from atomic level constituents, (i.e., materials by design)*. This ability to carefully arrange atoms provides opportunities to develop mechanical, electrical, magnetic and other properties that are not otherwise possible. This is termed as **"bottom-up"** *approach and the study of the properties of these materials is termed* **"nanotechnology";** the "nano" prefix denotes that the dimensions of these structural entities are on the order of a nanometer (10^{-9} m) as a rule, less than 100 nanometers (equivalent to approximately 500 atom diameters).

— One *example* of a material of this type is the ***carbon nanotube.***

Nanomaterials :

Introduction :

- *"Nanomaterials" (nanocrystalline materials) are the materials which have grain sizes of the order of a billionth of metre.*

- They entail extremely fascinding and useful properties, which can be used for a variety of structural and non-structural applications.

- These materials consist of grains, which in turn comprise many atoms. Normally these grains are visible to the naked eye, depending upon their size.

- The grains of conventional materials vary in size from 100's of microns (μm) to millimetres (mm) whereas a *nanocrystalline materials has grains on the order of 1–100 nm* (1 nm = 10^{-9} m). A nanometer comprises 10 Å, and hence in one *nm,* there may be 3 to 5 atoms, depending on the atomic radii.

Characteristics of nanomaterials :

Following are the characteristics/properties of nanomaterials :

1. Chemically very active.
2. Exceptionally strong, hard and ductile at high temperatures.
3. Wear-resistant.
4. Erosion-resistant.
5. Corrosion-resistant.
6. Much more formable than their conventional, commercially available counterparts.

Methods of producing nanomaterials :

Following are the *widely known methods of producing nanomaterials :*

1. Inert gas condensation.
2. Sol-gel synthesis.
3. Plasma synthesis.
4. Electrodeposition.
5. Mechanical alloying or high-energy ball milling.

Nanocomposites :

A ***nanocomposite*** *is formed when phase mixing occurs on a nanometer length scale.*

- One successful method to achieve such nanocomposite is the *in-situ polymerisation of metal oxides in organic matrices via the sol-gel process.* Inorganic components, specially silica have been formed by the *hydrolysis and condensation of nanonuclear precursor* such as tetraethoxysilane (TEDS) in many polymer systems. The drawback of this method is that due to the loss of volatile byproducts formed in the hydrolysis/condensation reaction, it is difficult to control sample shrinkage after moulding.

  Another method of synthesis of nanocomposites is *direct dispersion of nanoparticles in polymer matrix.*

- Now-a-days technologies are available for synthesis of a wide variety of nanomaterials *like silicon whiskers, carbon nanotubes etc.*

However, the following factors *limit* the application of such nanocomposites :

 (*i*) High cost involved;

(*ii*) The difficulty associated with dispersion of such materials, in a polymer matrix;

(*iii*) Health hazards.

- A *new class of nanocomposite* include *"Polymer/clay nanocomposites".* These materials are *cheap* and well known *filler* for polymer materials. Another nanocomposite included in this category is a nyon-6/clay nanocomposite, which results in dramatic improvement of properties compared to pristine polymer.

Advantages of nanocomposites :

The following are the *advantages* of nanocomposites :

1. Lower residual stress.
2. Flame retardancy.
3. Decreased thermal expansion coefficients.
4. Increased solvent resistance.
5. Improved mechanical properties.
6. Reduced gas permeability.

Applications of nanomaterials :

Nanomaterials can be used for a wide variety of applications; these applications include :

1. Low-cost float-panel displays.
2. High power magnets.
3. Tougher and harder cutting tools.
4. *Next-generation computer chips.*
5. Longer-lasting satellites.
6. Aerospace components with enhanced performance characteristics.
7. Better and future weapon platforms.
8. Longer-lasting medical implants.
9. Kinetric energy penetrators with enhanced lethality.
10. Phosphors for high-definition TV.
11. Automobiles with great fuel efficiency.
12. Ductile, machinable ceramics.
13. Large electrochromic display devices.
14. Better insulation materials.

Applications of nanocomposite :

Nanocomposites find *applications* in the following industries :

1. Electronics
2. Packaging
3. Automotive
4. Aerospace.

6.2. MECHANICAL PROPERTIES OF METALS

1. **Strength :** *The strength of metal is its ability to withstand various forces to which it is subjected during a test or in service.* It is usually defined as tensile strength, compressive strength, proof stress, shear strength, etc. *Strength of materials is a general expression for the measure of capacity of resistance* possessed by solid masses or pieces of various kinds to any cause tending to produce in them a permanent and disabling change of form or positive fracture. Materials of all kinds owe their strength to the action of the forces residing in and about the molecules of the bodies (the molecular forces) but mainly to that ones of these known as *cohesion;* certain modified results of cohesion as toughness or tenacity, hardness, stiffness and elasticity are also important elements, and strength is in relation of the toughness and stiffness combined.

2. **Elasticity :** A material is said to be *perfectly elastic* if the whole of the stress produced by a load *disappears completely* on the removal of the load, the modulus of elasticity of Young's modulus (E) is the proportionality constant between stress and strain for elastic materials. *Young's modulus is the indicative of the property called* **stiffness;** *small values of E indicate flexible materials and large value of E reflect stiffness and rigidity.* The property of spring back is a function of modulus of elasticity and refers to the extent to which metal springs back when an elastic deforming load is removed. In metal cutting, modulus of elasticity of the cutting tools and toolholder affects their rigidity. Values of modulus of elasticity for some important metals are given below in Table 6.2.

Table 6.2. Modulus of Elasticity of some Important Metals

S.No.	Metals	Young's modulus of elasticity, E (GN/m²)
1.	Cast iron	98
2.	Wrought iron	197
3.	Mild steel	210
4.	Aluminium	72
5.	Copper	120
6.	Zinc	100
7.	Tungsten	430
8.	Molybdenum	350
9.	Tin	42
10.	Lead	18

3. **Plasticity :**

- *Plasticity is the property that enables the formation of permanent deformation in a material.* It is reverse of elasticity; a plastic material will retain exactly the shape it takes under load, even after the load is removed. Gold and lead are the highly plastic materials. *Plasticity is used in stumping images on coins and ornamental work.*

- During plastic deformation there is the displacement of atoms within metallic grains and consequently the shapes of the metallic components change. It is because of this property that certain synthetic materials are given the name "plastics". These materials can be changed into required shape easily.

4. **Ductility :** *It is the ability of a metal to withstand elongation or bending.* Due to this property, wires are made by drawing out though a hole. The material shows a considerable amount of plasticity during the ductile extension. This is a valuable property in chains, ropes etc., because they do not snap off, while in service, without giving sufficient warning by elongation.

5. **Malleability :** *This is the property by virtue of which a material may be hammered or rolled into thin sheets without rupture.* This property generally *increases with the increase of temperature.* The metals in order of their ductility and malleability (at room temperature) are given below in Table 6.3.

Table 6.3. Common Metals in Order of their Ductility and Malleability

Ductility	Malleability
Gold	Gold
Silver	Silver
Platinum	Copper
Iron	Aluminium
Nickel	Tin
Copper	Platinum
Aluminium	Lead
Zinc	Zinc
Lead	Nickel

6. **Toughness (or Tenacity) :** *Toughness (or tenacity) is the strength with which the material opposes rupture.* It is due to the attraction which the molecules have for each other; giving them power to resist tearing apart.

   The area under the stress-strain curve indicates the toughness (i.e., energy which can be absorbed by the material upto the point of rupture). Although the engineering stress-strain

curve is often used for this computation, a more realistic result is obtained from a *true-stress curve*. Toughness is expressed as energy absorbed (Nm) per unit volume of material participating in absorption (m^3) or Nm/m^3. This result is obtained by multiplying the ordinate by the abscissa (in appropriate units) of stress-strain plot.

7. **Brittleness :** *Lack of ductility is brittleness*. When a body breaks easily when subjected to shocks it is said to be *brittle*.

8. **Hardness :**
   - *Hardness is usually defined as resistance of material to penetration*. Hard materials resist scratches or being worn out by friction with another body.
   - Hardness is primarily a function of the elastic limit (*i.e.,* yield strength) of the material and to a lesser extent a function of the work hardening co-efficient. The modulus of elasticity also exerts a slight effect on hardness.
   - In the most generally accepted test, an indentor is pressed into the surface of the material by slowly applied known load, and the extent of the resulting impression is measured mechanically or optically. A large impression for a given load and indentor indicates soft material, and the opposite is true for small impression.
   - The converse of hardness is known as *softness*.

9. **Fatigue :**
   - When subjected to fluctuating or repeating loads (or stresses), materials tend to develop a characteristic behaviour which is different from that (or materials) under *steady loads. Fatigue is the phenomenon that leads to fracture under such conditions*. Fracture takes place under repeated or fluctuating *stresses whose maximum value is less than the tensile strength of the material* (under steady load). Fatigue fracture is progressive, beginning as minute cracks that grow under the action of the fluctuating stress.
   - *Fatigue fracture starts at the point of highest stress*. This point may be determined by the shape of the part; for instant, by stress concentration in a groove. It can also be caused by surface finish, such as tool marks or scratches, and *by internal voids such as shrinking cracks and cooling in castings and weldments and defects introduced during mechanical working and by defects, stresses introduced by electroplating*. It must be remembered that surface and internal defects are stress raisers, and the point of highest actual stress may occur at these rather than at the minimum cross-section of highest normal stress. Thus, processing methods are extremely important as they affect fatigue behaviour.

10. **Creep :**
    - *"Creep" is the slow plastic deformation of metals under constant stress or under prolonged loading usually at high temperature. It can take place and lead to fracture at static stresses much smaller than those which will break the specimen by loading it quickly*. Creep is specially taken care of while designing I.C. engines, boilers and turbines.
    - The creep at a room temperature is known as *low temperature creep* and occurs in load pipes, roofings, glass as well as in white metal bearings. The creep at high temperature is known as *high temperature creep*. It mainly depends upon metal, service temperature to be encountered and the *stress involved*. For studying its effects, the specimens are put under a constant load; the creep is measured during various time intervals and results then plotted to get a *creep curve*.

11. **Machinability :**
    Machinability *may be defined as the "ease with which a material can be machined"*.
    The more important *criteria* for *measuring* both tool performance and machinability are :
    (*i*) The rate at which the material can be removed.
    (*ii*) The smoothness and accuracy of the machined surface obtained.

(*iii*) The life of cutting tool, or how long the cutting edge cuts satisfactorily before resharpening is required.

(*iv*) The power required for making the cut.

The *relative machinability* of a metal is expressed in *percentage*. All machinable metals are compared to a *basic standard steel* which is given an *arbitrary machinability rating of 100%*. The standard steel is a free cutting steel containing 0.13% carbon, 0.06 to 1.1% manganese and 0.08% to 0.03% sulphur. The machinability index of a metal is given by the following relationship (this is a way to compare machinability of metals).

$$\text{Machinability index} = \frac{\text{Cutting speed of metal for 20 min. tool life}}{\text{Cutting speed of standard steel for 20 min. tool life}}.$$

12. Weldability :

Weldability is *defined as the capacity of a metal to be welded under the fabrication conditions imposed in a specific suitably designed structure and to perform satisfactorily in the intended service.*

The real *criteria* in deciding on the weldability of a metal is the *weld quality* and the *ease with which it can be machined.*

Weldability is of significant importance for fabrication of metals into various structures.

- The following materials have good weldability in the *ascending order* : (*i*) Stainless steel; (*ii*) Low alloy steel ; (*iii*) Cast iron; (*iv*) Carbon steel ; and (*v*) Iron.

6.3. FERROUS METALS AND ALLOYS

6.3.1. Introduction

The **ferrous metals** are *iron base metals which include all varieties of irons and steels.* These metals go a long way in bringing prosperity to a country.

Ordinarily the terms, *iron, cast* iron and steel in reference to a metal in which the element iron (Fe) is the major element, do not refer to a specific metal or alloy, but are loosely used to indicate as general type of iron alloy. The term *iron* should be used only when reference is made to the element iron (Fe). In speaking of the commercial forms of iron such terms as *Pig iron, grey cast iron, wrought iron* etc. may be used. Each of these terms represents some commercial form of element iron, and each form may occur in many variations of chemical composition which influence the functions within each class. Due to tremendous production tonnage of these metals, and to their many forms and varied uses, a detailed study is not a simple and easy task.

Fig. 6.1 shows *flow sheet for production of iron and steel.*

Steel (master metal) is obtainable in great quantities, both in wrought and cast form. Its plasticity, whether at room temperature or at elevated temperatures, allows it to be worked either hot or cold. Its combination of strength with plasticity makes it the most important metal for use in large structures. By varying the carbon content and by suitable heat treatments, we can alter the properties from a very soft workable steel of the type used, in pressed metal parts, wire and similar materials to a hard, strong steel suitable for use in tools and machinery where great strength and hardness are required.

Wrought iron is the oldest form of iron made by man. It was originally produced by slow reduction of the metal from the ore in the forge fire. This reduction process resulted in a very impure iron which required further refining by mechanical working, that is by hammering or shaping to the form in which it is used. Wrought iron is a metal containing high purity iron and iron silicate in physical association. *It is very low in carbon and the iron silicate or slag is distributed throughout the base metal in fibers which gives it a woody or fibrous appearance when fractured.*
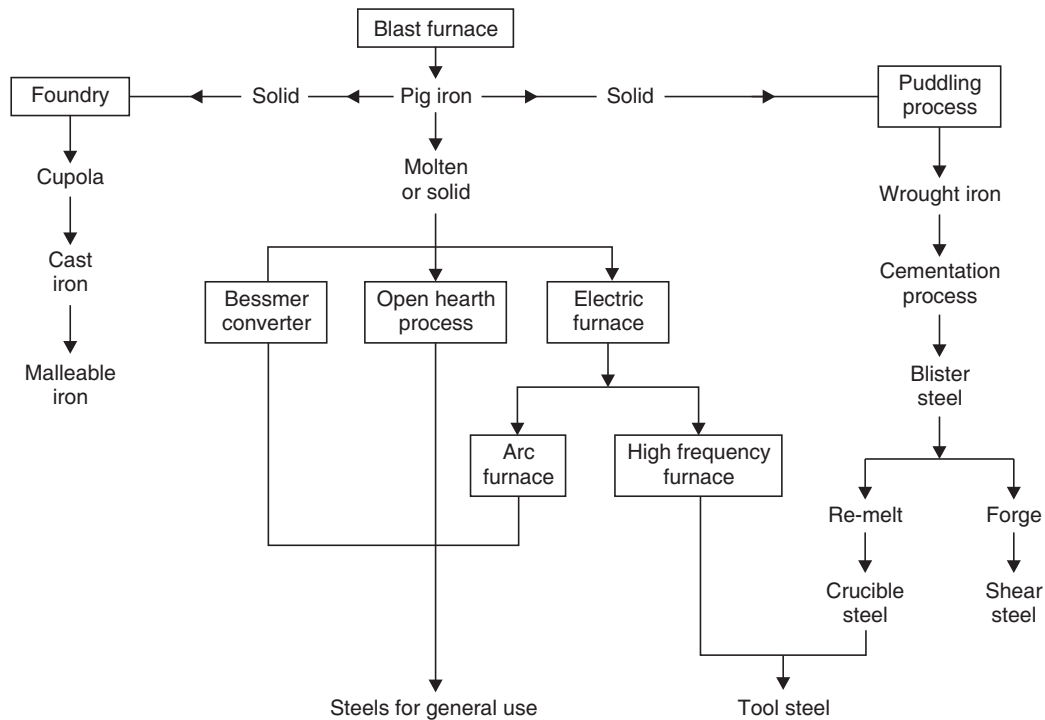
Fig. 6.1. Flow sheet for production of iron and steel.

Cast iron is fundamentally an alloy whose chief elements are iron, silicon and carbon. Cast irons are available with a wide range of properties. Pig iron, grey cast iron, white cast iron, chilled cast iron and malleable iron are all referred to as cast iron, chiefly because these forms of irons are not plastic enough, even when hot to be forgeable; therefore they are always produced commercially by a process of melting and casting into shape, the commercial form of each of these metals is in castings.

Iron ores :

The principal ores which yield ferrous metals, the percentage of iron content and the countries where available are given in Table 6.4.

Table 6.4. Iron Ores

S.No.	Name of the ore	Iron content	Chemical formula	Countries where available
1.	*Magnetite*	72.5%	Fe_3O_4	India—Salem district (Chennai); Sweden; U.S.A., U.S.S.R.
2.	*Haematite*	65 to 70%	Fe_2O_3	India (Bihar, Odisha, Andhra Pradesh, Madhya Pradesh, Mysore), U.S.A.
3.	*Limonite*	60%	$2Fe_2O, 3H_2O$	France, U.K., Spain, India.
4.	*Siderite*	40 to 44%	$FeCO_3$	U.K., Russia, India—Raniganj (West Bengal).

6.3.2. Pig Iron

Pig iron is the *basic material from which, wrought iron and steel are manufactured.* It is extracted from the abovementioned ores in a *tall, continuous working furnace called **'Blast Furnace'.*** The product obtained from the blast furnace is *crude and impure iron.*

Composition of pig iron :

In addition to iron, pig iron contains varying quantities of other elements amongst which carbon, silicon, manganese, sulphur and phosphorus are the most important. These may amount to as much as 10% of the weight and 25% of the volume of pig iron.

Effects of impurities on iron :

The impurities (such as silicon, phosphorus, sulphur and manganese) affect the iron in the following ways :

1. Silicon :
- Increases the fluidity of metal.
- Induces softness in the iron.
- Reduces the melting point but enhances the percentage content of uncombined carbon.
- Produces castings which are free from blow holes.

2. Phosphorus :
- Increases fluidity of metal.
- Gives rise to cold shortness (brittleness at ordinary temperature).

3. Sulphur :
- Encourages the formation of blow holes and makes the casting unsound.
- Exercises an injurious effect on the metal, therefore its percentage should be kept below 0.1%.
- Causes red shortness (brittleness at high temperature).

4. Manganese :
- Increases hardness and brittleness.
- Checks the bad effect of sulphur by forming MnS which is not injurious in small quantity

Classification of pig iron :

Pig iron is classified by chemical composition into the following *three grades :*

1. Basic pig iron :
- Basic pig iron must be low in sulphur (0.04%) since sulphur is an active impurity in steel and is not eliminated in the refining furnaces.
- Carbon content varies from 3.5 to 4.4%. Phosphorus is normally held to less than 1% and manganese to a range of 1 to 2%.
- It is *used for steel making* and is *low in silicon* (1.5% max.) to *prevent attack of the refractory linings of refining furnaces and to control slag formation.*

2. Foundry pig iron :
- It includes all the types that are used for the production of iron castings.
- Foundry iron contains : C = 3 to 4.5%, Si = 0.5 to 3.5%, Mn = 0.4 to 1.25%, S = upto 0.05%, P = 0.035 to 0.9% and Fe = remainder.

3. Ferro-alloys :
- These are alloys of pig iron, each rich in one specific element.
- These are used as *additives,* in iron and steel industries, *to control or alter the properties of iron and steel.*

Examples:

(*i*) *Ferromanganese*—pig iron that contains from 74 to 82% manganese.

(*ii*) *Ferrosilicon*—pig iron with 5 to 17% of silicon content.

6.3.3. Cast Iron

The product of the blast furnace *i.e.,* pig iron is unsuitable for castings as it contains impurities in high percentage. To render it suitable for desired purpose it is refined in the furnace known as *cupola.* The refined product is termed as *cast iron*.

Classification, composition, properties and uses of cast iron :

Cast iron may be *classified* as follows :

1. Grey cast iron   2. White cast iron

3. Mottled cast iron   4. Nodular cast iron

5. Malleable cast iron   6. Alloy cast iron.

1. Grey cast iron :

Composition : It is basically an *alloy of carbon and silicon with iron.* It contains C = 2.5–3.8% ; Si = 1.1–2.8% ; Mn = 0.4–1%; P = 0.15% and S = 0.10%.

Properties :

(*i*) It is characterised by presence of a large portion of its carbon in the *form of graphite flakes.* Graphite flakes occupy about 10% of the metal volume.

(*ii*) When fractured, a bar of grey cast iron gives grey appearance.

(*iii*) It is readily cast into a desired shape in a sand mould.

(*iv*) It possesses *lowest* melting point of the ferrous alloys.

(*v*) It possesses machinability better than steel.

(*vi*) It possesses high fluidity and hence it can be cast into complex shapes and thin sections.

(*vii*) It has high *resistance to wear.*

(*viii*) It possesses *high vibration damping capacity.*

(*ix*) It has *low* ductility and low impact strength as compared with steel.

(*x*) It associates *low cost combined with hardness and rigidity*.

(*xi*) It possesses *high compressive strength.*

(*xii*) It possesses excellent casting qualities for producing simple and complex shapes.

Uses :

(*i*) Manhole covers.

(*ii*) Machine tool structures, *e.g.,* bed, frame and details.

(*iii*) Frames for electric motors.

(*iv*) Rolling mill and general machinery parts.

(*v*) Cylinder blocks and heads for I.C. engines.

(*vi*) Gas or water pipes for underground purposes.

(*vii*) Ingot moulds.

(*viii*) Sanitary works.

(*ix*) Household appliances.

(*x*) Tunnel segment etc.

2. White cast iron :

Composition : Iron = 94% ; Graphite carbon = 0.5% ; Combined carbon = 3.5% and the remainder other impurities.

Properties :

(*i*) Hard, brittle and cannot be machined.

(*ii*) So called because the *carbon is mostly in combined iron.*

(*iii*) Highly resistant to wear.

(*iv*) Tensile strength is good.

(*v*) Obtained by rapid cooling of metal.

(*vi*) Due to its *poor fluidity* it does not fill the mould freely.

(*vii*) White iron castings can be made in sand moulds.

(*viii*) White iron can also be made on the surface of a grey iron casting provided the material is of special composition.

(*ix*) White iron of a particular composition is the first step in the production of malleable iron castings.

Uses :

(*i*) Used for parts subjected to excessive wear, *e.g.,* in rim of freight car wheel or railway brake blocks.

(*ii*) Used for inferior castings and does not rust so much as grey variety.

(*iii*) Used for making malleable castings also.

3. Mottled cast iron :

Composition : Iron = 93.5%, Graphite = 1.75%, Combined carbon = 1.75%; the remainder impurities.

Properties :

(*i*) Mixture of the two states, grey cast iron and white cast iron.

(*ii*) Strength and hardness varies according to the ratio of free carbon to combined carbon.

(*iii*) Less tendency to rust than grey variety.

(*iv*) Fluidity is good.

(*v*) Hard and brittle.

Uses :

(*i*) Used for manhole covers and pipes.

(*ii*) Also employed for making fire plugs and lamp posts.

4. Nodular cast iron :

Composition : Nodular or ductile cast iron contains : C = 3.2 – 4.2% ; Si = 1.1 – 3.5% ; Mn = 0.3 – 0.8% ; P = 0.08% and S = 0.2%. It is produced by ladle addition of nickel magnesium alloy. It is cheaper to make and behaves like steel.

Properties :

(*i*) In nodular cast iron graphite appears as rounded particles, or nodules or spheroids (unlike long flakes as in grey cast iron).

(*ii*) It possesses very good machinability.

(*iii*) Soft annealed grades of nodular cast iron can be turned at very high feeds and speeds.

(*iv*) It possesses damping capacity intermediate between cast iron and steel.

(*v*) It possesses excellent castability and wear resistance.

(*vi*) The properties of nodular cast iron depend upon metal composition and cooling rate.

Uses :

(*i*) Steel mill rolls and mill equipment.

(*ii*) Power transmission equipment.

  (*iii*) Valves and fittings.

  (*iv*) Internal combustion engines.

  (*v*) Pumps and compressors.

  (*vi*) Paper industries and machinery.

 (*vii*) Construction machinery.

(*viii*) Farm implements and tractors.

5. Malleable cast iron :

It is obtained by giving a long time annealing to cast iron. To get high duty cast irons, steel scrap 10 to 40% may be added to lower the total carbon contents.

Composition : Malleable cast iron contains : C = 2 – 3%, Si = 0.6 – 1.3%, Mn = 0.2 –0.6%, P = 0.15% and S = 0.10%.

The procedure of obtaining malleable cast iron is as follows :

The white cast iron casting along with silica is packed in a steel pot in heated ruffle oven or continuous type thermal kiln. The temperature is maintained at 870°C for 60 hours and casting is cooled slowly in the furnace, iron carbide dissociates as $Fe_3C \rightarrow Fe + C$ and thus reduces to malleable cast iron.

Malleable cast iron is of two types : (*i*) *Black heart,* (*ii*) *White heart.* White heart malleable cast iron can be obtained by keeping temperature and time comparatively of high values.

Properties :

  (*i*) It possesses high yield strength.

  (*ii*) It can be hammered and rolled to different shapes.

 (*iii*) It has high Young's modulus and low coefficient of thermal expansion.

 (*iv*) It possesses good wear resistance and vibration damping capacity.

  (*v*) It has shrinkage of 1.5 mm/100 mm.

 (*vi*) It has low to moderate cost.

(*vii*) It is soft, tough and easily machined.

Uses :

  (*i*) Differential and steering gear housing.

 (*ii*) Brake pedals.

(*iii*) Tractor springs.

 (*iv*) Hangers.

  (*v*) Washing machine parts.

 (*vi*) Agriculture implements.

 (*vii*) Universal joint yoke.

(*viii*) Automotive crankshafts.

 (*ix*) Rail road, etc.

6. Alloy cast iron :

Because the cast iron is supposed to be very hard, brittle, lacking in tensile and transverse strength and weak to withstand shocks it is alloyed with other metals to improve its properties. Amongst the alloying metals, *nickel is predominating alloying constituent whose addition to the extent of 0.5 to 1.5% avoids the tendency of chilling or hard spots.*

Two typical examples of alloy cast iron are : (*i*) **Acicular,** (*ii*) **Spheroidal.**

- Acicular cast iron has nickel and molybdenum as the leading constituents and is employed for crankshafts.

- *Spheroidal cast iron* has graphite content in spheroidal form, which is converted from flaky form by alloying with a small amount of magnesium and cesium. *This change in graphite increases the tensile strength and produces a tough metal which can undergo bending and twisting.*

6.3.4. Wrought Iron

- Wrought iron is virtually *pure iron, containing a large number of minute threads of slag lying parallel to each other, thereby giving the metal a fibrous appearance when broken.*
- *It contains practically no carbon and therefore, does not harden when quenched in water* (this property distinguishes it from steels).
- *Chemical composition :* A representative analysis range of wrought iron would be : C = 0.02–0.03%, Si = 0.02–0.10%, S = 0.008–0.02%, Mn = Nil–0.02%, P = 0.05–0.25%, slag = 0.15 – 1.50%, Iron = balance.

6.3.5. Composition, Properties and Uses of Carbon Steels

Carbon steels are classified as :

(*i*) Low carbon steels or mild steels.

(*ii*) Medium carbon steels.

(*iii*) High carbon steels.

Table 6.5. Composition, Properties and uses of Carbon Steels

Types of steel	Carbon (%age)	Tensile strength (MN/m^2)	Uses
Mild Steels (good for welding)	Below 0.1	320 to 400	Galvanised plates, tin plates and wires.
	0.1 to 0.18	400 to 450	Boiler plates, ships plates, casehardening for gear wheels, cams.
	0.15 to 0.25	450 to 500	General engineering purposes, crank axles, shafting.
	0.25 to 0.35	500 to 600	Hydraulic cylinders, rams, turbine motor shafts, spindles.
Medium carbon steels	0.35 to 0.45	550 to 700	Rails, turbine discs, connecting rods, railway and tram axles.
	0.45 to 0.55	700 to 850	Rifle barrels, gun parts, shells, gear wheels.
	0.55 to 0.65	800 to 950	Die blocks, gears, wheel tyres and mandrels.
High carbon steels	0.65 to 0.75	950 to 1100	Hammers, crusher rolls, general tools.
	0.75 to 0.85	950 to 1100	Hand chisels, scissors and ball mill parts.
	0.85 to 1.5	950 to 1100	Drills, taps, ball rates, dies and wood tools.
	1.0 to 1.3	850 to 1050	Razors, drills, tools and wire dies.

6.3.6.   Comparison of Cast Iron, Wrought Iron, Mild Steel and Hard Steel

Table 6.6. Comparison of Cast Iron, Wrought Iron, Mild Steel and Hard Steel

Cast iron	Wrought iron	Mild steel (Low carbon steel)	Hard steel (High carbon steel)
1. It contains 2 to 5% carbon	Purest form of iron containing 0 to 0.25% carbon	Contains 0.08 to 0.35% carbon.	Contains 0.55 to 1.3% carbon.
2. It has a cryst-alline coarse granular structure.	Fibrous structure of bluish colour.	Bright fibrous structure.	Fine granular structure.
3. Hard and brittle.	Tough and more elastic than cast iron.	Tough and more elastic than wrought iron.	Tough and more elastic than mild steel.
4. Can be hardened by heating and sudden cooling but cannot be tempered.	Can neither be hardened nor tempered.	Can be hardened and tempered but not easily.	Can be hardened and tempered but readily.
5. Can neither be forged nor welded.	Can be easily forged and welded.	Can be readily forged and welded.	Can be easily forged and welded.
6. Cannot be magnetised.	Can be temporarily magnetised.	Can be permanently magnetised.	Can be permanently magnetised.
7. Melting point = 1200°C.	Melting point = 1530°C.	Melting point = 1400°C.	Melting point = 1300°C.
8. Neither malleable nor ductile	Very malleable and ductile.	Malleable and ductile.	Brittle and less ductile
9. Does not rust easily.	Rusts more rapidly than cast iron.	Rusts readily.	Rusts rapidly.
10. Cannot absorb shocks.	Can stand sudden and excessive shocks.	Absorbs shocks.	Absorbs shocks.
11. Tensile strength fair and compressive strength good.	Tensile strength better and compressive strength less than cast iron.	Tensile strength is better than cast iron and wrought iron while compressive strength better than wrought iron but less than cast iron.	Both tensile and compressive strength better than cast iron, wrought iron and mild steel.
12. Becomes soft in salty water. Uses: (i) It is used for making bed plates, columns, rail chairs, brackets	Stands salty water better than cast iron. (i) It is used for making rolled iron joints, angle irons etc.	Not much affected by salty water. (i) Used for all kinds of structural work in bridges and buildings for making channels,	Not much affected by salty water. (i) Used for dies, cutlery and edge tools.

		griders, rails, angle iron etc.	
and machine parts not subjected to heavy shocks or tension. (*ii*) As it does not rust easily it is used for making sewers, drain pipes, water pipes etc. (*iii*) As it is poor in tension therefore, least suitable for structural purposes.	(*ii*) Since it can with-stand shocks it may be used for crane hooks chains, railway couplings etc. (*iii*) Also used for small size water pipes fitting, corrugated sheets, core of electromagnets etc.	(*ii*) Also used for rivets, bolts, wire tapes and for making sheets.	(*ii*) Also used in prestress concrete.

6.3.7. Alloy Steels

When certain special properties are desired some elements such as nickel, chromium, manganese, vanadium, tungsten etc. are added to the carbon steels. The steels thus obtained are called *alloy steels*.

The first investigation on the effect of alloying elements in steel were made from 1875 to 1890. But the use of alloyed steel found little application until 1901, when reduced cost of alloys made their use practicable.

Purposes of alloying :

The alloying elements are added to *accomplish one or more of the following :*

1. *To impart a fine grain size to steel.*
2. *To improve casehardening properties.*
3. *To improve elasticity.*
4. *To improve corrosion and fatigue resistance.*
5. *To improve hardness, toughness and tensile strength.*
6. *To improve machinability.*
7. *To strengthen the ferrite.*
8. *To improve high or low temperature stability.*
9. *To improve cutting ability.*
10. *To improve wear resistance.*
11. *To improve ductility.*

The effects of alloying elements :

Metal	Remarks
Nickel	(*i*) Increases toughness.
	(*ii*) Improves response to heat treatment especially in large sections.
	(*iii*) In large amounts provides special electrical and magnetic properties.
	(*iv*) Improves forming properties of stainless steel.
Chromium	(*i*) Provides stainless property in steel.
	(*ii*) Used widely in tool steels and in electric plates.

Vanadium	(*i*) Improves response to heat treatment.
	(*ii*) Provides control of structure.
	(*iii*) Used in high speed tool steels.
Tungsten	(*i*) Retention of hardness and toughness at high temperatures.
	(*ii*) Used in tools, dies, valves, magnets etc.
Silicon	(*i*) High electrical resistance and magnetic permeability.
	(*ii*) Used in electrical machinery.
Copper	(*i*) In small amounts improves atmospheric corrosion resistance.
	(*ii*) Acts as a strengthening agent.
Carbon	(*i*) Affects melting point.
	(*ii*) Affects tensile strength, hardness and machinability.
Silicon	(*i*) Improves oxidation resistance.
	(*ii*) Strengthens low alloy steels.
	(*iii*) Acts as a deoxidiser.
Titanium	(*i*) Prevents formation of austenite in high chromium steels.
	(*ii*) Reduces martensitic hardness and hardenability in medium chromium steels.
	(*iii*) Prevents localized depletion of chromium in stainless steel during long heating.
Molybdenum	(*i*) Enhances corrosion resistance in stainless steels.
	(*ii*) Makes steel usually tough at various hardness levels.
	(*iii*) Promotes hardenability of steel.
	(*iv*) Forms abrasion resisting particles.
	(*v*) Raises tensile and creep strength at high temperatures.
	(*vi*) Makes steel fine grained.
	(*vii*) Counteracts tendency towards temper brittleness.
Manganese	(*i*) Counteracts brittleness from sulphur.
	(*ii*) Increases strength and hardness markedly.
	(*iii*) Lowers both ductility and malleability if it is present in high percentage with high carbon content in steel.
Boron	(*i*) Increases hardenability or depth to which steel will harden when quenched.
Aluminium	(*i*) Acts as a deoxidiser.
	(*ii*) If present in an amount of about 1%, it helps promoting nitriding.
Cobalt	(*i*) Refines the graphite and pearlite.
	(*ii*) Improves heat resistance.
	(*iii*) Contributes to red-hardness by hardening ferrite.
	(*iv*) It is a mild stabilizer of carbides.
	(*v*) Improves mechanical properties such as tensile strength, fatigue strength and hardness.

Some of the important alloys are discussed below :

1. Silicon steel :

- Silicon improves considerably the electrical properties of steel.
- Silicon imparts fatigue strength and resistance to steel.
- Steel containing silicon is more ductile than a plain carbon steel.
- Steel containing 3–5 per cent silicon has very low magnetic hysteresis.

Uses :

1. Steel with Mn = 1%, Si = 2%, C = 0.4 to 0.6% has very high elastic limit and is used for springs.
2. With Cr = 5 to 7%, Si = 2 to 4%, C = 0.4 to 0.5%, steel retains its hardness and resistance to oxidation even at red heat. Such steel is used for internal combustion engines.
3. Silicon steel containing 2.5 to 4% silicon and low percentage of carbon and manganese is employed for making *laminations of electrical machines.*
4. 13% silicon content steel has a *very high corrosion resistance* and so is used in *chemical industries.*
5. Steel containing 0.6 to 1% silicon and 0.75 to 0.95% manganese is used for structural purposes.

2. High speed tool steels :

- High speed tool steels are widely used for cutting of metals where hardness *must be retained at elevated temperatures.*
- These steels are obtained by alloying tungsten, chromium, vanadium, cobalt and molybdenum with steel. This alloying produces metals which remain hard at temperatures at which normal steels become quite soft. *A common analysis is 18% tungsten, 4% chromium and 1% vanadium with a carbon content of 0.6 to 0.7%.* This alloy is termed as *18 : 4 : 1* while an increase of vanadium to 2% produces 18 : 4 : 2 steels. *Cobalt* is often added to 18 : 4 : 1 and *18 : 4 : 2* steels as it improves the *red hardness* and *cutting ability;* from 5 to 10% is used.
- Another class of high speed steel contains a lower percentage of tungsten, this being compensated by the addition of molybdenum; steel of this type is being increasingly used today.
- In addition to heat resistance, high speed steels have the desirable properties of high hardness, high compressive strength and outstanding wear resistance. They are close competitors to carbides for metal cutting-tool material such as drills, reamers, milling cutters etc.

3. Heat resisting steels :

- Steels which must be *resistant to creep at high temperatures must contain molybdenum. Silicon and chromium impart resistance to oxidation and scaling.*
- Steels which are satisfactory upto about 700°C operating temperature are : C = 0.15% max., Si = 0.5 to 20%, Mn = 0.5% max., Ni = none, Cr = 1.0 to 6%, Mo = 0.5%.

Uses : These are used in the as-rolled or as-forged condition, particularly *for the valves of internal combustion engines. For higher temperatures upto 1000°C, steels containing upto 22% nickel and 26% chromium are used.*

4. Spring steels :

(*i*) *Carbon-manganese spring steels*

- C = 0.45 to 0.65%, Si = 0.1 to 0.35%, Mn = 0.5 to 1.0%
- These steels are *quenched and tempered* to give a Brinell hardness of about 350.

Uses : They are widely used for *laminated springs for railway and general purposes.*

(*ii*) **Hyper-eutectoid spring steels**
- C = 0.9 to 1.2%, Si = 0.30% max., Mn = 0.45 to 0.70%
- These steels are oil quenched and tempered at a low temperature.

    Uses : They are *used for volute and helical springs.*

(*iii*) **Silicon-manganese spring steels**
- C = 0.33 to 0.6%, Si = 1.5 to 2%, Mn = 0.6 to 1.0%
- These steels are hardened and tempered to give a Brinell hardness of about 450.

    Uses : Employed for the manufacture of *railway and road springs generally.*

5. **Stainless steel :**

    The stainless steels are classified in two groups :
    1. Plain chromium and high chromium low nickel steel.
    2. Chromium nickel steel.

    (*i*) **Plain chromium and high chromium low nickel steel**
    - Out of this group the former has C = 0.8% and Cr = 12 to 20%, whilst the latter has C = 0.1 to 0.2%; Cr = 12 to 20% and Ni = 2%.
    - These steels can be *heat-treated.*
    - Used for dies, valves and cutlery.

    (*ii*) **Chromium nickel steel**
    - These steels are non-magnetic and cannot be hardened.
    - They have varieties due to the varying contents of chromium and nickel respectively, *e.g.,* 18 : 8, 12 : 2 and 18 : 9 (called *stay bride).* Small quantity of copper, tungsten and molybdenum is also added to these steels.
    - They have *high resistance to corrosion and may be cold or hot worked, pressed welded, brazed or soldered.*
    - The percentage of carbon in these steels is kept upto 0.5.
    - These steels are *poor conductors of heat and electricity.*

6. **Chrome vanadium steels :**
   - These steels usually contain 0.8 to 1.1% Cr, 0.25 to 0.33 C and less than 0.25% V. Even such a small amount of V has a marked effect on the properties of steel. It helps in producing cleaner steel because of the deoxidizing action. Vanadium is distributed both between the ferrite and the carbide involving both Cr intensifies the effect of V more than that of Ni.
   - Chrome-vanadium steels are used where *great strength, toughness and resistance to fatigue are required such as axles and shafts of aeroplanes, automobiles and locomotives.*

7. **Nickel steel :**
   - The percentage of nickel varies from 2 to 40.
   - Nickel from 3 to 5% *raises elastic limit and improves toughness.*
   - Steel containing 20% nickel has *very high tensile strength.*
   - If nickel is present upto 27% it makes the steel *non-magnetic and noncorrodible.*
   - Nickel containing 3.5% nickel and 0.15% to 0.45% carbon has good ductility, high elastic ratio and resistance to fatigue. It is used for *long span bridges.*
   - *Invar* (Ni = 36%) and *super-invar* (Ni = 31%) are the popular materials for least co-efficient of expansion and are used *for measuring instruments, surveyor tapes and clock pendulums.*

- Nickel *tends to retard the grain growth* in steel, resulting in a wide range of heating, without damage to the steel, or it may be maintained above the critical range for long periods of time without great damage.
- The addition of nickel to annealed carbon steel apparently strengthens the ferrite, with the result that the *endurance ratio is raised.* The full effect of nickel on the endurance ratio, however, is quenching and tempering.

8. Vanadium steel :

- Addition of vanadium even in small proportion to an ordinary low carbon steel considerably *raises its elastic limit and improves the fatigue resistance property.*
- When vanadium is added upto 0.25%; the elastic limit of the steel is raised by 50%, becomes exceedingly tough and can *resist high alternating stresses and severe shocks.*
- Vanadium makes the steel *strong and more ductile.*
- Steel with C = 0.15% to 1.1%, Cr = 0.5 to 1.5% and V = 0.15 to 0.3% has *high tensile strength, elastic limit, endurance limit and ductility.*
- Improves response to heat treatment.
- Provides control of structure.

Uses :

 1. It is widely used for making tools.
 2. It may be used for shafts, springs, gears and drop forged parts.

9. Manganese steel :

- Manganese from 1 to 1.5% makes the steel strong and tough to withstand severe duty but manganese content from 1.5 to 5% renders the steel hard and brittle.
- When manganese is from 11 to 14%, carbon 0.8 to 1.5%, steel becomes very hard, tough, non-magnetic and possesses considerably high tensile strength.
- More manganese *reduces strength and ductility.*
- Presence of manganese even upto 2% reduces the formation of sulphide and deoxidation of molten metal.
- Manganese steels show *high percentage of elongation.*
- It may be forged easily but difficult to machine and thus it is usually ground.
- The austenitic manganese steel is practically non-magnetic having a permeability of less than 2.0, so that it has special applications for parts that have to combine non-magnetism with high strength and resistance to wear.
- Heat treated cast manganese steel in bar form is *so ductile that it can be bent double when cold, without fracture* ; similarly castings of this metal will deform when fracturing. This deformation, however, is not continuous owing to the fact that cold-working raises the yield point in the stressed areas.
- Specific gravity = 7.9; melting point = 1.343°C.
- Manganese steel *can be forged* but special care is necessary to avoid degrading of the steel. After forging the steel should be heat-treated by raising it to temperature of 1010°C and quenching in water.
- Manganese steel is *weldable,* a nickel manganese welding rod being employed for repairing fractures, an alternative is an 18/8 stainless steel rod. Care is necessary in welding to avoid overheating, as this tends to cause degrading of the steel.

Uses :

 1. The properties of *work-hardening* renders this steel eminently suitable for such parts as the *jaws of stone and ore crushers, tramway and railway points and crossings, etc.*

2. Manganese steel has been much used for *helmets and shields;* it is also a good material for *"burglar proof" safes and vaults* as it is too hard to be cut with ordinary explosives.

3. Manganese steel in the hot worked condition is much used for *rails.*

4. Manganese steel in the form of bars is now widely use for screening coke. Ordinary steel wears away rapidly under the impact of the coke, but with manganese steel it has a life about a *hundred times as long.*

5. Owing to its *non-magnetic property* it has an important used in the *cover plates of lifting magnets* for handling heavy iron steel articles, and also for ship structure situated near the magnetic compass, since it has no effect on the latter.

6. Other applications include *agricultural implements* such as shovels; these are made from rolled manganese steel plates.

10. Tungsten steel :

- Tungsten when added to steel improves its *magnetic properties and hardenability.*

- Addition of tungsten to an extent of 6% to high carbon steel enables it to retain the magnetic properties to high degree and produce field more intense than an ordinary steel.

- 8% tungsten attributes so much hardness in steel that it can *scratch glass.*

- Steel containing 1.5% is too hard to be machined by ordinary methods.

- High percentages of tungsten makes the steel to *retain its hardness even at high temperatures.*

- Tungsten used in certain known percentages, *imparts air hardening properties to the steel.*

    Uses : It is used for *making permanent magnets and high speed cutting tools.*

11. Magnet steels :

Carbon, chromium, tungsten and cobalt steels are used for permanent magnets. The superiority, as well as price, increases in the order given above. These steels are used in the hardened condition. The carbon steels contain from 0.80 to 1.20% carbon and are, in fact, carbon tool steels. The chromium magnet steels contain from 0.70% to 1.0% carbon and 2% to 3% chromium. The tungsten steels contain about 0.70% to 1.0% carbon and 2% to 3% chromium. The tungsten steel contains about 0.7% carbon and 5% tungsten. The best and most expensive cobalt steels contain 35% cobalt together with several-percent of both chromium and tungsten. An important permanent magnet alloy (Alnico) contains approximately 60% iron, 20% nickel, 8% cobalt and 12% aluminium. This alloy cannot be forged and is used as a casting hardened by precipitation heat treatments.

6.4. NON-FERROUS METALS AND ALLOYS

6.4.1. Aluminium

Although aluminium is very abundant in the earth it is never found free, nor can it, by present methods be obtained commercially from clay in which it exists in such great quantities.

There is but one commercial ore called *'bauxite'. Bauxite is hydrated aluminium oxide* generally believed to consist of mixture of monohydrate $Al_2O_3 . H_2O$ and trihydrate $Al_2O_3 . 3H_2O$. The chief impurities are oxide, silica, clay and titanium oxide. It is found in districts of Bihar, Chennai and Madhya Pradesh.

Manufacture : Bauxite is first purified and then dissolved in fused cryolite (double fluoride of aluminium and sodium). The aluminium is then separated from this solution by electrolysis at about 910°C.

Physical and mechanical properties :

(*i*) Pure aluminium has silvery colour and lustre, while the commercial grades show a characteristic bluish tinge.

(*ii*) The high purity aluminium has a much greater resistance to corrosion than the ordinary steel.

(*iii*) It is ductile and malleable.

(*iv*) Its specific gravity is 2.7.

(*v*) In proportion to its weight it is quite strong.

(*vi*) Melting point = 658°C, boiling point = 2057°C.

(*vii*) Its electrical resistivity is 2.669 micro ohms/cm^3 at 20°C.

(*viii*) Its tensile strength varies from 95 to 157 MN/m^2.

(*ix*) It is a good conductor of heat and electricity.

(*x*) It forms useful alloys with iron, copper, zinc and other metals.

(*xi*) It is unaffected by ordinary atmospheric influences but is corroded in sea water. It is soluble in solutions of caustic alkalies and in hydrochloric acid. *When there is an excess of silicon present in the metal it does not withstand atmospheric actions.*

(*xii*) Aluminium is electron positive to most other metals *e.g.,* iron, chromium, zinc, copper, nickel, tin, lead etc. Care is necessary therefore, to prevent it from coming into metallic contact with other metals under conditions where moisture is present in order to avoid electrolytic action *i.e.,* corrosion.

Uses :

1. Because of its softness and difficulty of making sound castings, *little pure aluminium is used in the cast form. The largest quantity is employed after it has been mechanically worked in some manner,* as by rolling, wire drawing, drop forging or extruding. After being rolled into sheets, it may be stamped into a variety of shapes.

2. It is employed, often alloyed with small amounts of other metals, in the manufacture of *furniture, rail-road and trolley cars, automobile bodies and pistons, electric cables and bus bars, rivets, kitchen utensils and collapsible tubes for pastes.*

3. In a finely divided flake form, aluminium is employed as a *pigment in paint.* Aluminium paint is used as a priming coat for wood, as a protective coat for metals.

4. Aluminium is used in *deoxidizing molten iron and steel,* especially in the top of the ingot when steel is poured in the ingot mould. In a similar manner it is used to prepare the metals from their oxides by heating a mixture of powdered aluminium and the oxide of the metal to be reduced. The mixture is known as *Thermit.*

5. *Aluminium foil is used as silver paper for packing chocolates etc.*

Annealing :

Aluminium sheet which has been hardened by cold working, such as hammering or rolling can be annealed by heating it to about 350°C and afterwards cooling in air or water. The period of heating need only be for a few minutes. As a rough guide to correct temperature of heating for annealing, the surface may be rubbed, during the heating process, with a dry matchstick from time to time, the heating being stopped when wood begins to char.

The annealing of aluminium wire for electrical purposes require more care, since the aim is to obtain a high electrical conductivity by obtaining the correct structure. For high conductivity a long exposure at a lower temperature, namely 250°C to 300°C appears to give the best results. In passing, it may be remarked that the *aluminium alloys require higher annealing temperature, from about 350°C to 400°C.*

6.4.2. Copper

Copper is one of the comparatively few metals that are found in the metallic state. It occurs in some minerals such as copper glance (Cu_2S), copper pyrites ($CuFeS_2$), malachite ($CuCO_3.CuO_2H_2$) and azurite ($2CuCO_3.CuO_2H_2$). Copper ores are found in Burma, Sikkim and, Singhbhoom (Bihar).

Manufacture :

- Copper is extracted from its ores by several different methods that chosen depending upon the character of the ore and local conditions.

- Refining of the metal is usually considered to begin when the copper is in the *blister* stage, the surfaces of the cast material being irregular and blistered due to the generation of gases during cooling. This copper is 99% pure and is *further refined in the furnace by oxidation process which removes sulphur and other impurities.* The excess of oxygen is removed from the metal by operation known as **poling.** Green wooden poles or tree trunks are thrust under the surface of the molten metal, which is covered with charcoal, coke or similar material rich in carbon. Although this process may seem crude it is still almost invariably used, owing to its cheapness and efficiency. Poling is discontinued when sample casting indicates that the oxygen content has been reduced to 0.08–0.025 per cent. The copper is then known as *tough pitch copper.*

- Deoxidized copper is needed for intricate castings, welding and certain other processes. Special reducing agents such as phosphorus, silicon, lithium, magnesium, beryllium or calcium are added to the molten metal to eliminate the oxygen just sufficient of the deoxidizer being used to prevent any residue in the metal.

- *Arsenic,* in amounts upto approximately *5 percent* is added to improve the strength and toughness of the metal, and most copper products, other than electrical gear, are manufactured from arsenical copper.

Physical and mechanical properties :

(*i*) Copper is a *reddish-brown* metal.

(*ii*) Although pure copper is *one of the best conductors of heat and electricity,* its electrical conductivity is highly sensitive to the presence of impurities.

(*iii*) If copper is heated to red heat and cooled slowly it becomes brittle; but if cooled readily it is soft, malleable and ductile. The brittleness is due to the coarsely crystalline structure that develops during slow cooling.

(*iv*) Copper *can be welded at red heat.*

(*v*) Like aluminium, *pure copper does not cast well.* When molten it absorbs gases, such as carbon monoxide, hydrogen and sulphur dioxide which separate out on cooling and cause *blow holes.*

(*vi*) Melting point = 1084°C, boiling point = 2595°C.

(*vii*) Specific gravity = 8.9, Electrical resistivity = 1.682 microhms per cm.

(*viii*) It is *highly resistant to corrosion by liquids.*

(*ix*) Its tensile strength varies from 300 to 470 MN/m^2.

(*x*) It forms important alloys like *bronze and gun metal.*

(*xi*) It is strongly attacked by nitric acid but only very slowly by dilute hydrochloric and sulphuric acids in the absence of air; ammonical solutions also attack copper.

For copper that is to be worked, lead is sometimes added in order that metal may be worked more easily, but if more than about 0.5 percent is employed, it causes the copper to be brittle. In copper alloys, for casting, as much as 10 to 20 per cent may be added to cheapen the product but this lead largely separates out in globular masses on cooling. At the melting point of lead, copper is

practically insoluble in liquid lead and less than 0.05 per cent of the lead enters into the solid solution in the copper.

Uses :

1. It is largely used in *making electric cables and wires and electric machinery and appliances.*
2. Used in *electroplating, electrotyping and/or soldering iron bits.*
3. Used as a *damp proof material and for making alloys.*
4. It is used for *sheeting, roofing, spouts, boilers, condensers and other purposes where corrosion resistance with fair strength and flexibility is essential.*

Annealing temperature :

The annealing temperature of copper varies between 200°C and 600°C, according to the impurities present and condition of the metal, *i.e.,* upon the amount of cold work it has been subjected to. Work-hardened copper of high purity may be partly softened at temperature as low as 120°C, but the *minimum softening temperature is usually at least 200°C.* For most commercial copper a temperature of 500°C is employed but for heavy sections the furnace temperature may be raised to 600°C. After raising to the annealing temperature it may be cooled in any convenient manner, namely, in air or by water quenching; the latter method facilitates the removal of dirt and scale.

6.4.3. Copper Alloys

The copper alloys can be divided into two principal classes : *(i) Brasses* and *(ii) Bronzes.* The *'brasses' are primarily alloys of copper and zinc and 'bronzes' of copper and tin,* but there are numerous modifications of them produced by the addition of smaller amounts of other elements.

In addition there are cupro-nickels, containing principally copper and nickel, and nickel silvers (German silvers) in which zinc of brass is partially replaced by nickel.

I. Brasses : With range of composition from 5 to 45 per cent of zinc, the brasses are among the most useful alloys. *They possess excellent mechanical properties, and are corrosion resistant and are readily machinable.*

The composition of these alloys is given in Table 6.7.

Table 6.7. Classification of Brasses

Name	Composition		Colour	Typical uses
	Cu	Zn		
Muntze metal	9	41	Reddish	Architectural work, welding rod, condenser tubes, valve stems.
Cartridge or spinning brass	70	30	Typical brass colour	Cartridges, tubes, spinning, drawing.
Brazing brass	75	25	Typical brass colour	Drawing, spinning, springs, particularly suitable for brazing.
Red brass	85	15	Red	Hardware, radiator cores, plumbing pipe, condenser tubes, flexible hose.
High brass	66	34	Typical brass colour	Stamping, blanking, drawing, spinning, springs, rivets, chains.
Low brass	80	20	Red gold	Drawing, forming, flexible hose.

Modified brasses. The mechanical properties or corrosion resistance of the brasses is in some cases greatly improved by the introduction of relatively small quantities of one or more other metals. Some of the more common of these are given below :

1. *Manganese brass :* The alloy referred to under this head is generally spoken of as manganese bronze, but it is in reality a brass since the main constituents are copper and zinc. The composition varies considerably, but the following percentages may be regarded as typical ; copper = 60% ; zinc = 38 to 42% ; tin = 0.5 to 1.5% ; iron = 0.5 to 1.5%; manganese 0 to 0.5% and sometimes a little aluminium. The manganese is generally added in the form of ferromanganese; this accounts for the presence of iron, which in itself has important effects.

Properties :

  (*i*) It is exceedingly tough.

 (*ii*) It has high resistance even in sea water.

(*iii*) It is very active in reducing the oxides of other metals, an action that is highly useful.

 (*iv*) In addition to its action as deoxidiser, manganese hardens and strengthens the alloy.

  (*v*) In cast form, the tensile strength of the manganese brass lies in the neighbourhood of 500 MN/m^2 and this is somewhat improved by working, as by rolling and forging.

Uses : It is used *for pump rods, hydraulic rams, valves and cylinders, tubes, propellers, nuts, bolts etc.*

2. *Iron brass :* One of the most widely used alloys of this type is known as **delta metal,** its composition is : Cu = 60%, Zn = 37%, Fe = 3% (also some quantity of Ni or Mn).

Properties :

  (*i*) It is hard, strong and tough.

 (*ii*) It is easily cast.

(*iii*) Its tensile strength is about two-fifths greater than brass of similar composition with the iron omitted.

 (*iv*) It resists corrosion.

Uses : It is used for mild steel if corrosion is to be resisted.

3. *Tin brass* : Tin is one of the most useful metals added to brass. It increases the hardness and tensile strength but amount employed should not exceed about 2%, because with higher amounts the ductility begins to decrease. Its composition is Cu = 60%, Zn = 39%, Sn = 1%. It is also some time called *naval brass* since it is used in naval construction.

Properties :

  (*i*) It has *high strength and hardness.*

 (*ii*) It has *excellent corrosion resistance property.*

(*iii*) *Its tenacity is high and has good working qualities.*

4. *Lead brass :* In brass that is intended for filing or turning, 1 to 2% of lead is employed to prevent fouling of tools and to cause the turnings to break more readily; this prevents logging of automatic machines. *Lead increases the softness of the brass.* It is sometimes added to the brass that is to be worked, but care should be taken to add but a small amount, because it reduces the ductility and strength. Only about 3% will alloy with the brass; if more than this is *added* it has a tendency to liquate. On this account *'leaded'* brasses should be chilled quickly when cast.

5. *Aluminium brass :* The amount of aluminium added to brass does not usually exceed 3%. It raises the tensile strength but decreases the ductility. The product has a deep golden colour and resists corrosion better than ordinary brass. It casts well and may be forged or rolled.

II. Bronzes : These are essentially alloys of *copper and tin.* The effect of composition on physical properties is given below :

- The tensile strength of bronze increases gradually with the amount of tin, reaching a maximum with about 20% of tin, but as the tin increases beyond this amount the tensile strength very rapidly decreases.

- Bronze is *most ductile when it contains about 5% of tin,* with this amount it may be rolled satisfactorily at red heat. Bronze is used chiefly for casting.

- As the amount of tin increases above 5%, the ductility gradually lessens and *practically disappears with about 20% of tin;* since ductility is co-ordinate with toughness, these alloys are very brittle. They are also *very hard.*

- The most useful of bronzes are those that contain from 8 to 11% of tin, since the maximum combined strength and toughness are secured with about these amounts. Bronze containing tin within these limits *was formerly,* known as gun metal, since because of this strength it was used for making guns, but now *steel has entirely replaced it for this purpose.* At the present time, the term *gun metal is very loosely used and cannot be said to have definite significance.*

- As cast the alloy containing 9% of the tin has a tensile strength of about 215 MN/m^2.

- The bronze containing 4 to 8% tin is called *'coinage bronze'* and used for making coins and metals.

- The copper-tin series of alloys containing 15 to 25% of tin is known as *'bell metal'.* Such *alloys are very hard and brittle, but are sonorous and are employed, therefore, in making bells.*

- By alloying 2 parts copper with 1 of tin, a very hard, brittle white alloy is produced called **'spectrum metal'.** When highly polished it serves excellently for *mirrors and reflectors.*

1. *Modified bronzes* : The zinc bronzes usually contain : Cu = 88%, Sn = 8 to 10% and Zn = 4 to 2%.

   The presence of the zinc in the bronzes has the following effects :

   - It increases the fluidity of the melted metal and in this way tends to prevent gas flaws in the casting.

   - It also increases the strength and ductility.

   - The specific gravity when cast in sand is about 8.58; the melting point is about 980°C.

2. *Phosphor bronze :*

   - It is certain, however, that whatever good qualities the bronzes might have, they are *greatly increased by the addition of phosphorus.* The tensile strength, the elasticity and the resistance to fatigue are enormously increased; so much so that it may also be considered as an entirely new alloy.

   - Phosphorus is best added to bronze in the form of phosphorus copper, a hard brittle substance of white fracture, containing about 8% phosphorus. Upon being added to molten bronze nearly all of the phosphorus oxidizes and escapes, so that most phosphorus bronzes contain no more than a few tenths of 1% . The crystalline structure of two bronzes, one with and the other without phosphorus, as shown by microscope, seems to be the same in both cases. Phosphor bronze is then essentially a deoxidized bronze.

   - Although phosphorus produces very beneficial results through its activity as deoxidising agent, if *there is much of it left in the alloy, it may be decided determent.*

   - Phosphor bronze of *proper composition can be forged, drawn, cold rolled and cast.* It seems to resist corrosion better than ordinary bronze, especially by sea water, so that it is much used for *propeller blades.*

- On account of its toughness, elasticity and strength it may serve as a substitute for steel as in the manufacture of *corrosion-resistant mine cables, ship sheathing, valve parts, springs etc.*

3. **Silicon bronzes :**
   - These are copper alloys having 4% of silicon and upto 1% of manganese with small amounts of tin, zinc, iron and aluminium.
   - These are obviously not bronzes since tin is very small.
   - They are available in the market under various trade names.
   - These always *have strength of soft steel and at the same time are corrosion resistant.*
   - They are used in *screws, belts, tubings, pumps etc.*

4. **Aluminium bronze :**
   Its composition is : Cu = 88% ; Al = 8% ; Fe = 3% ; Sn = 0.5%. Thus it is primarily a copper-aluminium alloy.

   Properties :
   (*i*) This alloy has a *high strength.*
   (*ii*) It has *marked resistance to corrosion.*
   (*iii*) Its melting point is about 1040°C.
   (*iv*) Somewhat difficult to cast due to oxidation.
   (*v*) Castings of metal are sound and not liable to segregation, but the aluminium used must be free from impurity, a small quantity of which is found to produce a very marked deterioration.
   (*vi*) The commercial aluminium bronzes are practically non-magnetic, but certain complex bronzes containing appreciable proportions of iron are magnetic.

   Uses:
   1. Aluminium bronze is at present most satisfactory copper alloy for die casting, since it causes less corrosion of dies than brass.
   2. It is used in making hot stampings although it is not easy to work in this respect as brass or manganese.
   3. It is also used in marine work, bearing metal in locomotive.

6.4.4. Aluminium Alloys

- The principal elements which are alloyed with pure aluminium to improve its tensile strength and hardness are copper, silicon, manganese, zinc, magnesium and nickel. One element may be used alone but often two, three or four additions are made to the base metal to produce a metal having specific physical properties.
- *Copper* (for instance) is the *main hardening element* added while the addition of a small percentage of magnesium to an aluminium copper alloy still further improves the hardness and strength after heat treatment. Strength may also be improved by addition of small quantities of manganese and nickel to a copper-aluminium alloy.
- *Silicon,* is next in importance to copper as a main alloying element, since in combination with magnesium it forms a hard compound known as *magnesium silicide* which is largely responsible for the hardness obtained on the treatment.
- *Manganese* acts as a strengthening agent and prevents the formation of a course crystalline structure during heat treatment. Some of the important aluminium alloys are :

1. **Duralumin :**
   Composition : Al = 94%, Cu = 4%, Mg, Mn, Si, Fe = 0.5% each.

Properties :

(*i*) It can be *cast, forged and stamped.*

(*ii*) It has *high tensile strength.*

(*iii*) It possesses *high electrical conductance.*

(*iv*) It hardens spontaneously when exposed to room temperature.

(*v*) The alloy is soft enough for a workable period after it has been quenched.

(*vi*) The temperature employed for the solution heat treatment of the alloy is the lowest that is applicable to any commercial light alloy.

(*vii*) Specific gravity = 2.8, specific heat = 0.214.

(*viii*) Melting point = 650°C.

(*ix*) Brinell hardness : Annealed = 60, age hardened = 100.

Uses :

1. It is widely used for *sheets, tubes, forgings, rivets, nuts, bolts and similar parts.*

2. Used in making *cables.*

3. It is also extensively used for *air planes and other machines where weight is a deciding factor.*

4. It is also employed in *surgical and orthopaedic* work and for non-magnetic and other instrument parts.

Heat treatment :

- Annealing is carried out at 360-400°C and the metal is cooled in air. It is then ductile and can be cold worked. Normalising is carried out at a temperature of 490°C plus or minus 10 degrees. After normalising the metal is quenched in clean cold water.

- A salt bath is used for heat treatment generally consisting of 50% silver nitrate, with 50% potassium nitrate. For built up fittings a muffle furnace is sometimes used.

2. Y-alloy :

Composition : Al = 92.5% ; Cu = 4% ; Ni = 2% ; Mg = 1.5%.

Properties :

(*i*) Its strength at 200°C is better than aluminium.

(*ii*) It retains its high strength and hardness at high temperature.

(*iii*) It can be easily cast and hot worked.

Uses :

1. It is extensively used for such components as *piston cylinder heads and crankcases of internal combustion engines.*

2. It is also used for *die casting, pump rods and in sparking chisel in place of steel.*

Heat treatment and age hardening :

- The heat treatment range of Y-alloy is 5000°C – 5200°C; this is higher than for aluminium.

- It has been found that *muffle furnace* is best medium for heat treating Y-alloy. The period of treatment at 500°C – 520°C required to bring about solution of age-hardening constituents depends on the fineness of grain and distribution of constituents. For chilled cast bars, 2.5 cm diameter, about 6 hours are required. For coarser grain material a longer period becomes necessary, whilst for large castings 24 hours or more may be required. In all cases the period of treatment at 500°C – 520°C (solution treatment) must be regulated in relation to the micro-structure. Treatment at the temperature 500°C to 520°C is followed by *quenching in boiling water.* After quenching, *age hardening* at room temperature, substantially completes in 5 days.

- Age hardening can be accelerated by retaining the alloy immersed in the water at boiling temperature after quenching. Under these conditions age hardening is substantially complete in from 1/2 hour for the wrought to 2 hours for the cast alloy.
- By subjecting normally heat-treated material, cast or wrought, to temperatures between 150°C to 250°C the tensile strength and hardness are substantially increased, but the ductility is diminished. Above 250°C permanent softening results.

Note. Y-alloy is annealed in the wrought state by heating it to 350°C to 400°C and allowing it to cool in the air.

3. Hindalium :

- Hindalium is an alloy of *aluminium, magnesium, manganese, chromium and silicon etc.,* and is the trade name of the aluminium alloy produced by Hindustan Aluminium Corporation Ltd., Renukoot Distt. Mirzapur (U.P.).
- It is manufactured as a rolled product (16 gauge) mainly for *anodized utensile manufacture.* During processing special care is taken to maintain the necessary mechanical and surface characteristics.

Hindalium utensils possess the following advantages :

  1. Strong and hard.

  2. Cannot be easily scratched.

  3. Can take fine finish.

  4. Do not absorb much heat and thus save fuel while cooking.

  5. Can be easily cleaned.

  6. Do not react with the food acids.

  7. Low cost (about one-third of stainless steel).

4. Magnelium :

It is an alloy of aluminium, magnesium, copper, nickel and tin etc.

Typical composition of magnelium is :

      Al = 85 to 95%

      Cu = 0 to 25%

      Mg = 1 to 5.5%

      Ni = 0 to 1.2%

      Sn = 0 to 3%

      Fe = 0 to 0.9%

      Mn = 0 to 0.03%

      Si = 0.2 to 0.6%.

Properties :

(i) Light weight.

(ii) Tensile strength—annealed state : 200 N/mm^2 ; cold worked state : 280 N/mm^2.

(iii) Elongation—annealed state : 30% ; cold worked state : 7%.

(iv) Alloy is brittle.

(v) Castability poor.

(vi) Machinability good.

(vii) Can be welded.

Uses : Mostly used in the aircraft and automobile industries. Some of *parts* made from magnelium are :

Gearbox housings, vehicle door handles, luggage racks, coffee-grinder parts and ornamental fixtures.

6.5.   POLYMERS/PLASTICS

6.5.1.   Introduction

Polymers *are organic materials having carbon as the common element in their make-up.*

"Organic materials" are those materials which are derived directly from carbon and consist of carbon chemically combined with hydrogen, oxygen or other non-metallic substances, invariably having a complex structure. *Examples :* Wood, animal fibers, natural rubber, coal etc., having biological origin and synthetic fibers, *plastics*, soaps, cutting oil etc.

6.5.2.   Classification of Plastics

A **plastic** in *broadest sense is defined as any non-metallic material that can be moulded to shape.* The most common definition for plastics is that they are *natural or synthetic resins, or their compounds, which can be moulded, extrudend, cast or used as films or coatings.* Most of the plastics are of organic nature composed of hydrogen, oxygen, carbon and nitrogen. The synthetic plastic development dates from 1900 when Dr. Beekland announced the production of phenol-formaldehyde. Since then several new plastics have been developed.

Plastics, most commonly, are classified as :

  1. Thermoplastic, and

  2. Thermosetting.

"Thermoplastic materials" *are those which soften on the application of heat with or without pressure, but they require cooling to set them to shape.*

They can be *heated and cooled any number of times, only they should not be heated above their decomposition temperatures.*

- They are main long chain straight or slightly branched molecules and the chains are held close to each other by secondary weak forces of type van der Waal's forces. During heating, as the temperature increases the secondary forces are reduced and the sliding of these long chain molecules can easily occur one over the other at a reduced stress level.
- They are highly plastic and are easy for moulding or shaping.
- They have low melting temperatures and are not so strong as the thermosetting plastics.
- Since they can be repeatedly used, they have a resale value.
- Some commercial thermoplastics are : *Polythene, Polyvinyl chloride (PVC), Polystre Polytetrafluroethylene (PTFE) etc.*

"Thermosetting materials" *are those plastics which require heat and pressure to mould them tin shape.*

- They cannot be resoftened once they have set and hardened.
- They are ideal for moulding into components which require rigidity, strength and some resistance to heat.
- In general, resins formed by condensation are thermosetting.
- Thermosetting resins have *three-dimensional molecular structure and have very high molecular weights.*
- Due to *cross-linking* thermosetting resins are hard, tough, non-swelling and brittle. Hence they cannot be softened or remoulded as in the case of thermoplastic resins. Moulding and casting are the processes often used with such materials.
- Some important commercial examples of this type are : *Phenolics, Polyesters, Expoxies, Silicones etc.*

The *difference between thermoplastic and thermosetting materials* may be explained *in terms of molecular structure.* The thermoplastics are essentially long chain macromolecules with a limited

number of cross links. When heated and compressed, the chains glide over each other and fluid materials take the shape of any mould in which they are placed. The thermosetting plastics are characterised by strong cross links between the chains; once these are formed by heat and pressure, the plastics set to a rigid infusible solid. Figure below represents the two structures where R stands for monomer unit.

(a) Thermoplastic   (b) Thermosetting
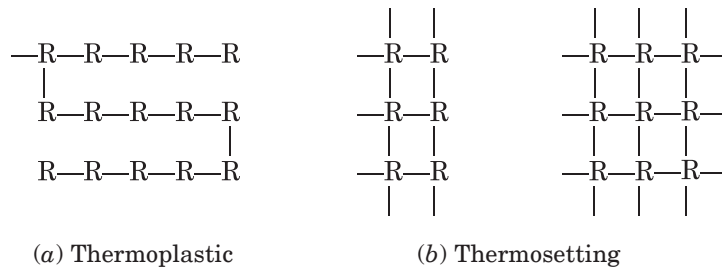
Structure of plastics.

Table 6.8. Differences between Thermosetting and Thermoplastic Materials

Thermosetting materials	*Thermoplastic materials*
1. They have three dimensional network of primary covalent bonds with cross-linking between chains.	They are linear polymers without cross-linking and branching.
2. They are more stronger and harder than thermoplastic resins.	They are comparatively softer and less strong.
3. Once hardened and set they do not soften with the application of heat.	They can be repeatedly softened by heat and hardened by cooling.
4. Objects made by thermosetting resins can be used at comparatively higher temperature without damage.	Objects made by thermoplastic resins cannot be used at comparatively higher temperatures as they will tend to soften under heat.
5. They are usually supplied in a monomeric or partially polymerized form in which they are either liquid or partially thermoplastic solids.	They are usually supplied as granular materials.
6. It is difficult to fill an intricate mould with such plastics.	They can fill the complicated mould quite easily.
7. They cannot be recycled.	The scrap of these plastics can be recycled again and thus, economical.
Uses : *Telephone receivers, electric plugs, radio and TV cabinets, camera bodies, automobile parts, circuit breaker switch panels etc.*	**Uses :** *Toys, combs, toilet goods, photographic films, insulating tapes, hoses, electric insulation, etc.*

6.5.3.   Thermoplastic Materials

Important thermoplastic materials are :

1. Polyethylene or polythene $(C_2H_4)_n$

   (a) High density polyethylene (HDPE)

   (b) Low density polyethylene (LDPE)

2. Polyvinylchloride (PVC)
3. Polypropylene (PP)
4. Teflon or polytetrafluroethylene (PTFE)
5. Polystyrene
6. Acrylics (Polymethyle methacrylate PMMA)
7. ABS (Acrylonitric butadine styrene)
8. Silicons
9. Polyvinylidene chloride
10. Polyamides
11. Bitumen.

6.5.4. Thermosetting Materials

1. Phenol formaldehyde (PF)—Bakelite
2. Amine formaldehyde (Urea and melamine formaldehyde)
3. Polysters (unsaturated)
4. Epoxy resins (epoxies)

6.5.5. Trade Names and Typical Applications of Some Important Plastics

S. No.	Material type	Trade name	Typical Applications
	A. Thermoplastics		
1.	Acrylics	Lucite, Plexiglas	—Outdoor signs
			—Lenses
			—Transparent aircraft enclosures
2.	Nylons	Zytel, Plaskon	—Handles
			—Bearings
			—Cams
			—Gears
3.	Polyethylene	Alathon, Petrothene, Hi-fax	—Tumblers
			—Ice trays
			—Toys
			—Flexible bottles
4.	Polystrene	Styron, Lustrex, Rexolite	—Appliance housings
			—Battery cases
			—Wall tiles
5.	Polystere (PET)	Mylar, Colanar, Dacron	—Automatic tyre cords
			—Magnetic recording tapers
			—Clothing
6.	Vinyls	PVC, Tygon, Saran	—Phonograph records
			—Floor coverings
			—Garden hose
	B. Thermosetting		
7.	Phenolics	Bakelite, Resinox, Durez	—Auto distributors
			—Telephones
			—Motor housings

8.	*Epoxies*	Araldite, Epon	—Adhesives
			—Electrical mouldings
			—Sinks
9.	*Polysters*	Laminac, Selectron, Paraplex	—Fans
			—Chairs
			—Helmets.

6.5.6. Laminated Plastics

These are also called *plastic laminates* and are formed by impregnating sheets of fibrous materials such as paper, linen, canvas or silk with a synthetic resin and then compressing the sheets together with application of heat. The synthetic resins may be phenolic resin, urea formaldehyde or a vinyl resin. The resin is usually, dissolved in alcohol.

The material in roll form is immersed in the resin solution at atmospheric pressure at room temperature and then run through a drier at 150°C. The rolls are next cut into sheets of given size, which are arranged into stacks. These stacks finally are compressed in a hydraulic press at about 170°C under a pressure of 200 bar. The sheets are thus bonded to one another.

Properties : The laminated plastics have the following *properties :*

 (*i*) Light and strong.

 (*ii*) Machinable.

 (*iii*) Resistance to wear, acids and alkalis.

 (*iv*) Impervious to water and oil.

 (*v*) Have a high dielectric constant.

Uses : The laminated plastics are used for :

 (*i*) Electric insulation.   (*ii*) Making *silent gears.*

 (*iii*) Water lubricated bearings.   (*iv*) Pulley wheels.

 (*v*) Pump parts   (*vi*) Press tools.

 (*vii*) Decorative purposes in wall panelling, transluscent panelling and table and counterparts.

6.5.7. Fiber Glass Reinforced Plastics

- The Fiber glass reinforced plastic (or FRP) is formed by using two materials in conjunction with each other to form a composite material of altogether different properties.
- In FRP, the glass *fibers provide stiffness and strength while resin provides a matrix to transfer load to the fibers.*

Properties of FRP : Following are the properties which have made the FRP the most commercially successful composite material of construction :

 (*i*) Aesthetic appeal.

 (*ii*) Dimensional stability.

 (*iii*) Light weight.

 (*iv*) Easy to repair.

 (*v*) Durable.

 (*vi*) Corrosion resistant.

 (*vii*) Requires less energy for production.

 (*viii*) Least maintenance required.

(*ix*) The tooling is inexpensive and fast.

(*x*) FRP products transmit a great deal of light.

Applications : Following are the *applications of FRP :*

(*i*) Water storage tanks.

(*ii*) Roof sheets.

(*iii*) Domes.

(*iv*) Structural sections.

(*v*) Doors and window frames.

(*vi*) Concrete shuttering.

(*vii*) Internal partitions and wall panelling.

(*viii*) Temporary shutters.

6.6. CERAMIC MATERIALS

6.6.1. Introduction

Ceramic materials *are defined as those containing phases that are compounds of metallic and non-metallic elements.*

The *science of ceramics,* nearly as old as mankind, is the processing of earthly materials by heat. The crude cooking utensils of early man were the first application of the materials now used in jet engines and atomic reactors. All the early ceramic products were made from clay because the ware could be easily formed. It was then dried and fired to develop the permanent structure. Because the other ceramic materials lacking plasticity also have desirable properties, other methods of forming and processing have been developed. Other forming methods used for ceramic materials are *injection moulding, sintering* and *hot pressing.* In other cases the formed materials are allowed to harden on the job by the addition of water, as in case of cements.

6.6.2. Classification of Ceramics

A. Classification of ceramic materials

Ceramic materials are *classified* as follows :

1. Functional classification :

(*i*) Abrasives	:	Alumina, carborundum
(*ii*) Pure oxide ceramics	:	MgO, Al_2O_3, SiO_2
(*iii*) Fire-clay products	:	Bricks, tiles, procelain etc.
(*iv*) Inorganic glasses	:	Window glass, lead glass etc.
(*v*) Cementing materials	:	Portland cement, lime etc.
(*vi*) Rocks	:	Granites, sandstone etc.
(*vii*) Minerals	:	Quartz, calcite, etc.
(*viii*) Refractories	:	Silica bricks, magnesite, etc.

2. Structural classification :

(*i*) *Crystalline ceramics :* Single-phase like MgO or multi-phase from the MgO to Al_2O_3 binary system.

(*ii*) *Non-crystalline ceramics :* Natural and synthetic inorganic glasses.

(*iii*) *"Glass-bonded" ceramics :* Fire clay products-crystalline phases are held in glassy matrix.

(*iv*) *Cements :* Crystalline and non-crystalline phases.

B. Classification of ceramic products

A general classification of 'ceramic products' is difficult to make because of the great versatility of these materials, but the following list includes the major groups :

1. Whitewares.
2. Bricks and tiles.
3. Chemical stonewares.
4. Cements and concretes.
5. Abrasives.
6. Glass.
7. Insulators.
8. Porcelain enamel.
9. Refractories.
10. Electrical porcelain.
11. Mineral ores.
12. Slags and fluxes.

6.6.3. Advantages of Ceramic Materials

The ceramic materials entail the following *advantages :*

1. The ceramics are hard, strong and dense.
2. They have high resistance to the action of chemicals and to the weathering.
3. Possess a high compression strength compared with tension.
4. They have high fusion points.
5. They offer excellent dielectric properties.
6. They are good thermal insulators.
7. They are resistant to high temperature creep.
8. Availability is good.
9. Good sanitation.
10. Better economy.

6.6.4. Applications of Ceramics

The applications of ceramics are listed below :

1. The ***whitewares (older ceramics)*** are largely used as/in :
   - Tiles ;
   - Sanitary wares ;
   - Low and high voltage insulators ;
   - High frequency applications ;
   - *Chemical industry—as* crucibles, jars and components of chemical reactors ;
   - *Heat resistant applications*—pyrometers, burners, burner tips, and radiant heater supports.
2. **Newer ceramics** *(e.g.,* borides, carbides, nitrides, single oxides, mixed oxides, silicates metalloid and intermetallic compounds) which have the *high hardness values* and *heat and oxidation values* are largely used as/in :
   - *Refractories* for industrial furnaces.
   - *Electrical and electronic industries*—as insulators, semiconductors, dielectrics, ferroelectric crystals, piezoelectric cystals, glass, porcelain, alumina, quartz and mica etc.

- *Nuclear applications*—as fuel elements, fuel containers, moderators, control rods and structural parts. Ceramics such as UO_2, UC, UC_2 are employed for all these purposes
- *Ceramic metal cutting tools*—made from glass free Al_2O_3.
- *Optical applications*—*Ytralox,* a comparative newcomer in the ceramic material field, is useful since it is as transparent as window glass and can resist very high temperature.

3. **Advanced ceramics** *(e.g.,* SiC, Si_3N_4, ZrO_2, B_4C, SiC , TiB_2 etc.)

   The advanced ceramics are utilized as/in :
   - Internal combustion engines and turbines, as armor plate ;
   - Electronic packaging ;
   - Cutting tools ;
   - Energy conversion, storage and generation.

6.6.5. Properties of Ceramic Materials

1. Mechanical properties :

The ceramic materials possess the following *mechanical properties :*

(*i*) The compressive strength is several times more than the tensile strength.

(*ii*) *Non-ductile/brittle.* Stress concentration has little or no effect on compressive strength.

(*iii*) The ceramic materials possess ionic and covalent bonds which impart high modulus of elasticity. The modulus decreases with increase in temperature (due to increase in interatomic distance at elevated temperature).

(*iv*) As compared to pure metals, more force is required to cause slip in diatomic ceramic materials, because diatomic material consists of a mixture of positively and negatively charged ions which have strong forces of attraction between them.

(*v*) Below recrystallisation temperature, non-crystalline ceramics are fully brittle. The cleavage failure occurs along crystallographic planes and propagation of the crack takes place at high speed.

(*vi*) At high temperature rigidity is high.

(*vii*) In case of alloy consisting of two or more metals, each phase may have appreciable difference of coefficient of thermal expansion which generate stress. This stress may then cause the metal to fail.

2. Electrical properties :

The electrical properties of ceramic products vary from the low loss, high frequency dielectrics to semiconductors. Electrical insulators fall into two general classifications, the classical electrical porcelain for both high and low tension service and the special bodies such as steatite, rutile, cordierite, high alumina, and clinoestatite for high frequency insulation.

Dielectric constant :

- *Dielectric constant is the ratio of the capacitance of a dielectric compared to the capacitance of air under the same conditions.*
- A low dielectric constant contributes to low power loss and low loss factor; a high dielectric constant permits small physical size.
- The dielectric constant for electrical porcelain varies between 4.1 and 11.0. Some special bodies have reported values of several thousands.
- Porcelain has large positive temperature coefficient.
- Rutile bodies have large negative coefficients.
- By combining capacitor dielectrics having different temperature coefficients it is possible to reduce effect of the temperature change.

Dielectric strength :

- The *dielectric strength* of a material is defined as the *ability of a material to withstand electrical breakdown.*
- The specific values of dielectric strength vary from 100 V per mil for low-tension electrical porcelain to 500 V per mil for some special bodies.
- Rutile bodies show higher breakdown strength at higher frequencies.

Volume and surface resistivity :

- A *volume resistivity* of 10^6 ohms/cm^3 is considered the lower limit for an insulating material. At room temperature practically all ceramic materials exceed this lower limit. As the temperature of ceramic materials is raised, the volume resistivity decreases; the volume resistivity of soda-lime glasses decreases rapidly with temperature, whereas some special bodies are good insulators (above 10^6 ohm/cm^3) at 700°C. Crystallised alumina has a volume resistivity of 500 ohms/cm^3 at 1600°C.
- *Surface resistivity* for dry, clean surface is 10^{12} ohms/cm^2. At 98% humidity, the surface resistivity may be 10^{11} ohms/cm^2 for a glazed piece or 10^9 ohms/cm^2 for an unglazed piece. The presence of dissolved gases and other deposits also tends to decrease the surface resistivity of ceramic materials.

3. Thermal properties :

Since the ceramic materials contain relatively few electrons, and ceramic phases are transparent to radiant type energy, their thermal properties *differ amply from that of metals.* The following are the most important thermal properties of ceramic materials (which vary from material to material and from condition to condition) :

(*i*) Thermal capacity.

(*ii*) Thermal conductivity.

(*iii*) Thermal shock resistance.

(*i*) Thermal capacity :

- The specific heats of fine clay bricks are 0.25 and 0.297 at 1000°C and 1400°C respectively
- Carbon bricks possess specific heats of about 0.812 at 200°C and 0.412 at 1000°C.

(*ii*) Thermal conductivity :

- The ceramic materials possess a vary low thermal conductivity since they do not have enough electrons (for bringing about thermal conductivity). The conduction of heat takes place by phonon conductivity and the interaction of lattice vibration, while at elevated temperatures conduction takes place by the transfer of radiant energy.
- The impurity content, porosity and temperature decrease the thermal conductivity.
- In order to have maximum thermal conductivity, it is imperative to have maximum density which most of the ceramic materials do not possess.

(*iii*) Thermal shock resistance :

"Thermal shock resistance" is the ability of a material to resist cracking or disintegration of the material under abrupt or sudden changes in temperature.

Thermal shock is developed primarily because of thermal expansion or contraction, which is largely a function of internal structure particularly the inter-atomic bonding. Loosely packed structures can provide *internal expansion.* Thus the coefficient of expansion is low.

- *Lithium compounds* are used in many ceramic compounds to *reduce thermal expansion* and to *provide excellent thermal shock resistance.*
- Common ceramic materials graded in order of decreasing thermal shock resistance are given below :

(*i*) Silicon nitride   (*ii*) Fused silica

(*iii*) Cordierite   (*iv*) Zircon

(*v*) Silicon carbide   (*vi*) Beryllia

(*vii*) Alumina   (*viii*) Porcelain

(*ix*) Steatite.

4. Chemical, optical and nuclear properties :

Chemical properties :

- Several ceramic products are highly resistant to all chemicals except hydrofluoric acid and to some extent, hot caustic solutions. They are not affected by the organic solvent.
- Oxidic ceramics are completely resistant to oxidation, even at very high temperature.
- Ziconia, magnesia, alumina, graphite etc., are resistant to certain molten metals and acids thus employed for making crucibles and furnace linings.
- Where resistance to attack from acids, bases and salt solutions is required, ceramics lime glass are employed.

Optical properties :

- Several types of glasses have been employed for the production of windows, subject to high temperatures and optical lenses.
- Special glasses, in large number, have also been used for selective transmission absorption of particular wavelengths such as infrared and ultraviolet.

Nuclear properties :

As ceramics are refractory, chemically resistant and because different compositions offer a wide range of neutron capture and scatter characteristics, they are finding nuclear applications and are being used as : *Fuel elements, moderators, controls and shielding.*

6.6.6. Glass

Introduction

- **Glass** is *any substance or mixture of substances that has solidified from the liquid state without crystallization.* Elements, compounds and mixture of wide varying composition can exist in the glass state, but the term *"glass"* as ordinarily used *refers to material which is made by the fusion of mixture of silica, basic oxides and a few other compounds that react either with silica or with the basic oxides.* No definite chemical compounds can be identified in glass. Many of its properties correspond to those of a supercooled liquid whose ingredients cannot be identified because they have not separated from the solution in crystalline form.
- *Glass* may also be defined as a *hard, brittle, transparent or transluscent material chiefly compound of silica, combined with varying proportions of oxides of sodium, potassium, calcium, magnesia, iron and other minerals.*
- Glass is an amorphous substance having a homogeneous texture.

Structure of glass :

The *glass is a random arrangement of molecules, the great majority of which are oxygen ions bounded together with the network forming ions of silicon, boron or phosphorus.* A glass made of silica alone has many desirable characteristics but unfortunately the high temperatures involved make it expensive, and difficult to prepare. In order to reduce the temperature, required network-modifying ions are added. *Sodium, potassium, and calcium* are the most common. The network-modifying ions increase the competition for the oxygen ions, thus loosening the Si-O bonds. Certain other ions may substitute for either the network-forming ions or network-modifying ions; *aluminium, zinc, beryllium, lead and ion* are a few of these intermediate ions.

Constituents of glass and their functions :

The various *constituents of glass and their functions* are described below :

1. **Silica :**
   - It is the principal constituent of glass.
   - If silica alone is used in the manufacture of glass, it could be fused only at a very high temperature but it would give a good glass on cooling. However, it is imperative to add some alkaline materials (sodium or potassium carbonate) and lime in suitable proportions to make the molten silica glass sufficiently viscous to make it amply workable and resistant against weathering agencies.

2. **Sodium or potassium carbonate :**
   - It is an alkaline material and forms an essential component of glass.
   - It is added in suitable proportion to *reduce the melting point of silica and to impart viscosity to the molten glass.*

3. **Lime :**
   - It is added in the form of chalk.
   - It imparts durability to the glass.

   In place of lime, sometimes, *lead oxide* is also added; it makes the glass *bright and shining.*

4. **Manganese dioxide :**
   - It is added in suitable proportion to *correct the colour of glass due to the presence of iron in raw materials of glass.*
   - It is also called 'Glass maker' soap.

5. **Cullet :**
   - It is the old broken glass of the same type as that which is intended to be prepared.
   - It is added in small quantity to provide body to the glass.

6. **Colouring substance :**

   While manufacturing a coloured glass, a suitable colouring substance is added at fusion stage to provide the desired colour to the glass.

   The various colouring substances for manufacturing glass of different colours are given below :

Colour	*Colouring substance*
1. *Black*	Cobalt, nickel and manganese oxide
2. *Green*	Chromic oxide
3. *Red*	Cuprous oxide, selenium
4. *Violet*	Manganese dioxide
5. *White*	Cryolite, tin oxide
6. *Yellow*	Cadmium sulphate.

Properties of glass :

Following are the *properties* of glass :

 1. No definite crystalline structure.
 2. No sharp melting point.
 3. Absorbs, refracts or transmits light.
 4. Affected by alkalies.
 5. An excellent electrical insulator at elevated temperatures.
 6. Extremely brittle.
 7. Available in beautiful colours.

8. Not affected by air or water.

9. Not easily attacked by ordinary chemical reagents.

10. Capable of being worked in several ways.

11. Can take up a high polish (and may be used as substitute for very costly gems).

12. Possible to weld pieces of glass by fusion.

13. As a result of advancement made in the science of glass production, it is possible to make glass lighter than cork or softer than cotton or stronger than steel.

14. Glass can be cleaned easily by any of the following methods :

   (*i*) Applying methylated spirit.

   (*ii*) Rubbing finely powdered chalk.

  (*iii*) Rubbing damp salt for cleaning paint spots.

  (*iv*) Painting the glass panes with lime-wash and leaving it to dry and then washing with clean water.

Commercial glass must meet the following requirements :

  (*i*) The material must melt at commercially obtainable temperature. Fused silica cools to a glass that is superior to ordinary glass, but the temperature required to melt it is so high that its production is expensive and its use restricted.

  (*ii*) The molten mixture must remain in the amorphous or non-crystalline condition after cooling.

 (*iii*) The fluidity of molten glass must presist to a sufficient extent to permit the formation of *desired shapes while the glass is cooling.*

 (*iv*) The glass must be reasonably permanent in the use for which it is intended. Glass with a high proportion to sodium oxide is more readily attacked by water and acids than glass that contains less sodium oxide and more lime and magnesia. Glasses low in basic oxides are less readily attacked.

Fabrication of glass :

The various processes involved in the fabrication of glass are enumerated and described below :

1. Blowing	2. Flat drawing
3. Rolling	4. Pressing into moulds
5. Casting	6. Spinning.

Classification of glass :

As per composition and properties glass may be *classified* as :

1. Soda-lime or crown glass

2. Flint glass

3. Pyrex or heat-resistant glass.

Glass-fibre or glass-wool :

- The usual *composition* of glass-fibres is that of a *soda-lime glass* but it may be varied for different purposes.

- The *glass-fibres are made by letting the molten glass drop through tiny orifices and blowing with air or steam to attenuate the fibres.*

- They have *very high tensile strengths*, upto about 2750 N/mm^2.

- Glass-fibre or glass-wool differs from mineral wool in that it is a glass made to a definite formulation with a uniformity not found in mineral wool.

Uses of glass :

Besides other uses, some of the important uses of glass, based on the recent development in the glass industry, are as follows :

1. The fibre glass reinforced with plastics can be used in the construction of furniture, cars, trucks, lampshades, bath room fittings etc.
2. Glass is used to form a rifle barrel which is lighter and stronger than conventional type.
3. Thousands of items in the body of a guided missile are made of glass.
4. Glass is used in the construction of noses of deep-diving vehicles.
5. Optical glass is finding wide application for the development and advancement of sciences of astronomy and bacteriology.
6. The glass linings are applied on equipments likely to be affected by the chemical corrosion such as valves, pipes, pumps etc.
7. Hollow glass blocks can be used for the construction of the walls and ceilings of the modern homes.
8. These days, it is possible to prepare the colour-changing glass; a window with such a glass will be transparent during the day and it will be a source of light at night.

6.6.7. Cements

Introduction :

- Cement may be prescribed as a *material with adhesive and cohesive properties which make it capable of bonding mineral fragments into a compact whole.* The definition embraces a large variety of cementing materials.

- For constructional purposes the meaning of the term cement is restricted to the *bonding materials used with stones, sand, bricks, building blocks etc.* The principal constituents of this type of cement are *compounds of lime,* so that in building and civil engineering we are concerned with calcareous *cement.* The cements of interest in the making of *concrete* have *property* of *setting* and of interest in *under water by virtue of a chemical reaction with it* and are therefore called *hydraulic cement.*

Classification :

Cements can be broadly classified as :

1. Natural cement
2. Artificial cement.

1. Natural cement :

- It is manufactured from stones containing 20 to 40 per cent of clay, the remainder being carbonate of lime mixed with carbonate of magnesia. The stones are first burnt and then crushed.
- It possesses a brown colour and sets rapidly when mixed with water.
- It does not find much use in India.

2. Artificial cement :

- It may be *Portland cement* or *special cement.*
- *Portland cement is so named because a paste of cement with water, after it sets hard, resembles in colour and hardness a Portland stone, a limestone quarried in Dorset.*
- It is prepared in different varieties.

Properties of cement :

A good cement possesses me following properties (which depend upon its chemical composition, thoroughness of burning and fineness of grinding).

1. Provides strength to masonry.
2. Stiffens or hardens early.
3. Possesses good plasticity.
4. An excellent building material.
5. Easily workable.
6. Good moisture-resistant.

Uses of cement :

Following are the various *uses* of cement :

1. It is used in cement mortar for *masonry work, plastering, pointing,* etc.
2. It is used for making *joints for pipes, drains* etc.
3. It is used in concrete for laying floors, roofs and constructing lintels, beams, stairs, pillars etc.
4. It is employed for manufacturing *precast pipes, piles, fencing posts* etc.
5. It is used in the construction of important engineering structures such as *bridges, culverts, dams, tunnels, light houses* etc.
6. It is used in the preparation of *foundations, water tight floors, footpaths* etc.
7. It is employed for the construction of wells, water tanks, tennis courts, lamp posts, telephone cabins, roads etc.

Chemical constituents of cement

- Percentage of various ingredients for the manufacture of Portland cement should be as follows :

Ingredient		Proportion
Lime (CaO)	....	63.0%
Silica (SiO_2)	....	22.0%
Alumina (Al_2O_3)	....	6.0%
Iron oxide (Fe_2O_3)	....	3.0%
Magnesium oxide (MgO)	....	2.50%
Sulphur trioxide (SO_3)	....	1.75%
Loss on ignition	....	1.50%
Insoluble residue	....	0.25%

- Besides the above ingredients, calcium sulphate ($CaSO_4$) commonly known as *gypsum* is also added by 3 to 4% during the grinding process for *controlling the initial setting time of cement* (larger the proportion of gypsum, more will be the initial setting time and vice versa).

Manufacture of Portland cement :

The process of manufacture of cement consists essentially of *grinding the raw materials* (calcareous and argillaceous stones containing silica, alumina and iron oxide), *mixing them* intimately in a certain proportion and *burning in a large rotary kiln* at a temperature of approximately 1300°C to 1900°C, when the material *sinters* and partially fuses into balls known as **clinker**. The clinker is cooled and ground to fine powder, with *some gypsum added,* and the resulting product is the commercial Portland cement so widely used throughout the world.

There are *two processes* employed for the manufacture of the cement :

1. Wet process
2. Dry process

Both the processes involve the following steps in the manufacture of Portland cement :

(*i*) Collection of raw materials.

(*ii*) Crushing, grinding and mixing of raw materials.

(*iii*) Burning.

(*iv*) Grinding of clinker.

Testing of Portland cement :

For normal setting or ordinary Portland cement the following tests (laboratory) are carried out :

1. Chemical composition.

2. Fineness test.

3. Consistency test.

4. Setting times test.

5. Soundness test.

6. Tensile strength test.

7. Compressive strength test.

Types of cements :

Following are the various types of cements available in the market.

1. **Portland cements :**

(*i*) Ordinary Portland cement

(*ii*) Modified Portland cement

(*iii*) Rapid hardening Portland cement

(*iv*) Low heat Portland cement

(*v*) Sulphate resisting Portland cement

(*vi*) Water-repellent Portland cement

(*vii*) Water-proof Portland cement.

2. **Other varieties of cement :**

(*i*) High alumina cement

(*ii*) Quick setting cement

(*iii*) Blast furnace slag cement

(*iv*) White cement

(*v*) Coloured cement

(*vi*) Acid resistant cement

(*vii*) Expanding cement

(*viii*) Hydrophobic cement

(*ix*) Portland Pozalana cement

(*x*) Supersulphated cement

(*xi*) Masonry cement.

6.6.8. Advanced Ceramics

1. **Glass ceramics :**

- These are special glass compositions that are thermally treated prior to forming operations to *divertify or precipitate a crystalline phase from the material ;* this phase gives that material special properties such as *zero thermal expansion for applications involving high thermal-shock application.*

- The compositions (typical of glasses) in which nucleation and crystallization have been commerically produced are : MgO-Al_2O_3-SiO_2; LiO_2-Al_2O_3-SiO_2; LiO-MgO-SiO_2.

Characteristics :

(*i*) Very low coefficient of thermal expansion.

(*ii*) Relatively high mechanical strengths.

(*iii*) High thermal conductivities.

(*iv*) Can be easily fabricated (conventional glass-forming techniques may be employed conveniently in the mass production of nearly pore-free ware).

Uses :

 (*i*) Owing to their excellent resistance to thermal shock and their high conductivity, glass ceramics are used as *ovenware* and *tableware.*

(*ii*) As insulators.

(*iii*) As substrates for printed circuit boards.

2. **Dielectric ceramics :**

- The use of ceramic materials is made both as *electrical insulators* and as *functional parts* of an electrical circuit. Since the electrical insulators can breakdown under high electrical voltages, the insulators are *designed with lengthened surface paths to decrease the possibility of surface shorting.* Since internal pores and cracks provide opportunity for additional *surface* failure, the insulators are *glazed* to make them non-absorbent.

- *Non-linear dielectric ceramics* are suitable in the miniaturization of electronic parts which have led to the development of increasingly sophisticated electrical circuitry.

- These ceramics are also used in capacitors.

- Some typical non-linear dielectric ceramics are : Lead zirconate-titanate, lead niobates, barium titanate, etc.

3. **Electronic ceramics :**

Ferrites ferroelectric ceramics etc. are the ceramic materials with unusual properties that are of specific use in electronic circuits.

- *Ferrites* are mixed-metal-oxide ceramics (almost completely crystalline). They assimilate high electric resistivity and strong magnetic properties. *Soft ferrites* can be used for specific uses such as *memory cores for computers* and cores for radio and television loop antennas. *Barium and lead ferrites* are widely used in permanent-magnet motors in automobiles, portable electrical tools and small appliances.

- *Ferroelectric ceramics* can convert electrical signal into mechanical energy (such as sound); and can also change sound, pressure or motion into electrical signals. Thus, they function as *transducers.*

  Examples : *Barium titanate* (most common), *tantalates, zirconates, niobates,* etc.

4. **Cermets :**

- Cermets are ceramic-metal composites.

- Cermets contain alumina (Al_2O_3) and chromium in varying proportions.

- These are used in *brake shoe linings, oxidation-resistant parts and inject engines.*

- The most common cermet is *cemented carbide* and such like composites are extensively used as cutting tools for hardened steels.

6.7. COMPOSITE MATERIALS/COMPOSITES

6.7.1. General Aspects

- **A composite material** is *a combination of two or more materials having compositional variations and depicting properties distinctively different from those of the individual materials of the composite.* The composite material is generally better than any of the individual components as regards their strength, heat resistance or stiffness.

- Composites include the following :
    - (*i*) Multiphase metal alloys ;
    - (*ii*) Ceramics ;
    - (*iii*) Polymers.

Examples of composites are :

- (*i*) **Pearlitic steels :** The *pearlitic steels* have a microstructure consisting of alternating layers of a ferrite and cementite ; *the ferrite phase is soft and ductile,* whereas *cementite is hard and very brittle.* The combined mechanical characteristics of the pearlite, reasonably high ductility and strength, are superior to those of either of the constituent phases.
- (*ii*) **Wood :** It is one of the examples of the composites that occur in nature. It consists of strong and flexible cellulose fibers surrounded and held together by a stiffer material called *lignin.*

    Plywood is the composite of thin sheets of wood with grains of alternate sheets perpendicular to each other and bonded together by a polymer in between them.
- (*iii*) **R.C.C. :** It has steel rods embedded in the concrete mix, which, itself, is a composite of cement, sand aggregate and water. The resulting R.C.C. structure can take loads which, otherwise, cannot be carried by the concrete alone. Steel rods can be of different shape, size and provided in various directions.
- (*iv*) **Vehicle tyres :** The vehicle tyres are rubber reinforced with woven cords.
    - Each of the materials in composites serves one or more specific functions. The properties of composites are affected by the following :
        - (*i*) The size and distribution of the constituents in relation to each other ;
        - (*ii*) The bond strength between them ;
        - (*iii*) The shape, size, amount and properties of each material.
    - *The base material surrounding other materials is normally present in higher percentage and is called* **matrix.** *Other materials which reinforce the properties of base material are called* **reinforcements.** *Cohesion between the matrix and reinforcement is essential and may take place in any or combination of the following ways :*
        - (*i*) Chemical reaction at the interfaces of the constituents.
        - (*ii*) Mechanical keying between the matrix and the reinforcement.
        - (*iii*) Mechanical bonding between the matrix and reinforcement by van der Waals' forces acting between the surface molecules of the various constituents.
    - The selection of matrix and reinforcement is made in such a way that their *mechanical properties complement each other, while deficiencies are neutralised.* In particular cases there may be more than one type of reinforcement present at the same time. The *effect of reinforcement is to increase both the tensile strength and tensile modulus of the composites.*

6.7.2. Classification

The composite materials may be classified as follows : Refer to Fig. 6.2.

1. **Particle reinforced :**
    - (*a*) Large particle
    - (*b*) Dispersion strengthened.
2. **Fiber-reinforced :**
    - (*a*) Continuous (aligned)
    - (*b*) Discontinuous (short)
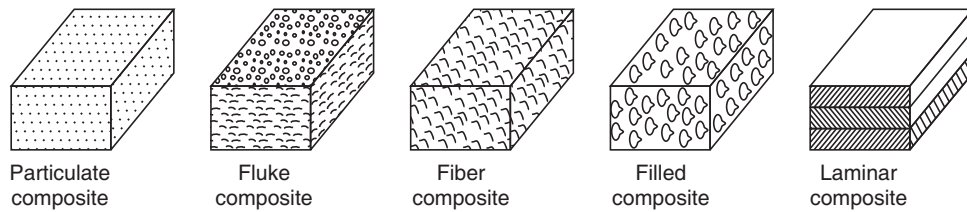   - (*i*) Aligned
        - (*ii*) Randomly oriented.

Particulate composite · Fluke composite · Fiber composite · Filled composite · Laminar composite

Fig. 6.2

3. **Structural :**

(*a*) Laminates

(*b*) Sandwich panels.

6.7.3. Particle-reinforced Composites

(*a*) Large-particle composites :

- These composites are utilized with all three types of materials, *viz., metals, polymers and ceramics.*

Examples :

(*i*) *Automobile tyres*—containing 15 to 30% volume of spherical particles of carbon with 20–50 μm diameter in vulcanized rubber which improves the tensile strength, toughness and resistance against corrosion.

(*ii*) *Concrete*—consisting of cement (the matrix), and sand and gravel (the particulates).

(*iii*) *Cements, e.g., cemented carbide*—consisting of extremely hard particles of a refractory carbide ceramic (such as tungsten carbide or titanium carbide) embedded in a matrix of a metal such as cobalt or nickel (load is shared between ceramics and metals). These composites are widely used for *cutting tools* for hardened steels, *drills, electrical contacts, magnets* and *rocket nozzles* etc. Other applications of cemented carbides are *burner nozzles, gauges and plugs used for inspection of materials, grinding balls and liners in grinding mills.* The *cermets are prepared by powder metallurgy techniques.*

- In the case of *two-phase composites,* the following two mathematical expressions have been formulated for the dependence of the elastic modulus on the volume fraction of the constituent phases :

$$E_c \text{ (upper bound)} = E_m V_m + E_p V_p \qquad \qquad ...(i)$$

$$E_c \text{ (lower bound)} = \frac{E_m E_p}{V_m E_p + V_p E_m} \qquad \qquad ...(ii)$$

where, E and V denote elastic modulus and volume fractions respectively ; and c, m, p represent composite, matrix and particulate phases respectively.

Dispersion-strengthened composites :

- The metals and metal alloys may be strengthened by the *uniform dispersion of several volume percent of fine particles of a very hard and inert material.* The dispersed phase may be metallic or non-metallic; oxide materials are often used.

- The strength of nickel alloys at elevated temperature may be enhanced considerably by the addition of about 3 volume percent of thoria (ThO_2) as finely dispersed particles, this material is known as *thoria dispersed (or TD) nickel.*

6.7.4.   Fiber-reinforced Composites

The most important composites, technologically, are those in which the disperse phase is in the form of the fiber. Fiber-reinforced composites with high specific strengths and moduli have been produced that utilize low-density fiber and matrix materials.

The strength and other properties of these composites are influenced by the following :

 (*i*) The arrangement or orientation of the fibers relative to one another;

 (*ii*) The fiber concentration;

 (*iii*) The fiber distribution.

When the *fiber distribution is uniform, belter overall composite properties are realized.*

- **The fiber phase** : The fibers, on the basis of diameter and character, are grouped into three different classifications : *whiskers, fibers and wires.*

Whiskers :

— These are very thin crystals having extremely large length-to-diameter ratios (crystal size ranges from 0.5 to 2.0 microns in diameter to 20 mm length). As a consequence of their small size, they have a high degree of crystalline perfection and are virtually free from flaws, which accounts for their exceptionally high strengths (tensile strength of whisker is approx. 21 GN/m^2 compared to 3 GN/m^2 for carbon fiber); they are strongest known materials.

— They are *difficult and costly to manufacture* (as such whiskers are not employed extensively as a reinforcement medium).

— Whisker materials include graphite, silicon carbide, silicon nitride and aluminium oxide.

— Boron and carbon whiskers are used in polymeric matrix. Alumina whiskers are used to reinforce the metal nickel.

Fibers :

— Fibers are either *polycrystalline or amorphous and have small diameters.*

— The fibrous materials are generally either polymers or ceramics.

Wires :

— Wires are used as a radial steel reinforcement in automobile tyres, in filament-wound rocket castings, and in wire-wound high-pressure hoses.

— Fine wires have relatively large diameter; typical materials include steel, tungsten and molybdenum.

- **The matrix phase :** In fibrous composites, the matrix phase may be a metal, polymer, or ceramic. The metals and polymers, in general, are employed as matrix materials because some *ductility is desirable.* In case of ceramic-matrix composites, the reinforcing component is added to *improve fracture toughness.*

The matrix phase, in case of fiber-reinforced composites, serves the following purposes/ functions :

 (*i*) *It binds the fibers together* and acts as a medium by which an externally applied stress is transmitted and distributed to the fibers; only a very small proportion of an applied load is sustained by the matrix phase.

 (*ii*) It *protects* the individual fibers *from surface damage* as a result of mechanical abrasion or chemical reactions with the environment.

 (*iii*) The matrix separates the fibers and, by virtue of its relative softness and plasticity *prevents the propagation of brittle cracks from fiber to fiber,* which could result in catastrophic failure.

In order to minimize fiber pull-out it is essential that *adhesive forces between fiber and matrix be high.*

Some *important fiber-reinforced composites are :*
- Polymer-matrix composites
    — Glass fiber-reinforced polymer (GFRP) composites
    — Carbon fiber-reinforced polymer (CFRP) composites
    — Aramid fiber-reinforced polymer composites.
- Metal-matrix composites
- Ceramic-matrix composites
- Carbon-carbon composites
- Hybrid composites.

6.7.5. Structural Composites

(*a*) Laminar composites :
- These composites are made from the *two dimensional* sheets or panels that have a *preferred high strength direction* (such as found in wood and continuous and aligned fiber reinforced plastics).
- The layers are sacked and subsequently cemented together such that the orientation of the high strength direction varies with each successive layer *(e.g.,* in *plywood,* adjacent wood sheets are aligned with the *grain direction at right angles to each other)*.

(*b*) Sandwich panels :
- These composites are composed of *two strong outer sheets or faces* (typical face materials include aluminium alloys, fiber-reinforced plastics, titanium steel and plywood), separated by a *layer of less-dense material* or *"core"* (typical core materials include foamed polymers, synthetic rubbers, inorganic cements and wood) which has lower stiffness and lower strength. The core, structurally perform the following *two functions :*
    - (*i*) It separates the face and resists deformation perpendicular to the face plane.
    - (*ii*) It provides a certain degree of shear rigidity along planes which are perpendicular to the faces.
- Sandwich panels find wide applications in the following :
    - (*i*) Roofs, floors and walls of building;
    - (*ii*) In aircraft for wings, fuselage and tailplane skins.

Tensile modulus of composite :

Consider a composite cylinder made up of continuous fibers, all parallel to the axis of the cylinder in a matrix. Each component in the composite shares the applied force. Thus,

Total force = Force on matrix + force on fiber

= Stress on matrix × area of matrix + stress on fiber × area of fiber

[∵ Force = stress × area]

$$\therefore \quad \frac{\text{Total force}}{\text{Total area}} = \left[\text{Stress on matrix} \times \frac{\text{area of matrix}}{\text{total area}}\right] + \left[\text{stress on fiber} \times \frac{\text{area of fiber}}{\text{total area}}\right]$$

∴   Stress on composite = (Stress on matrix × % area of matrix) + (stress on fiber × % area of fiber)   ...(1)

Strain on composite = Strain on matrix = strain on fiber   ...(2)

From eqns. (1) and (2), we get

Tensile modulus of composite = Modulus of matrix × % area of matrix
+ modulus of fiber × % area of fiber   ...(3)

6.8.   CONDUCTORS, SEMICONDUCTORS AND INSULATORS

6.8.1.   Conductors

Conductors *may be defined as the materials which have free valence electrons in plenty for electric conduction.* **Examples :** *Silver, copper, aluminium etc.*

Resistance :

It is defined as the property of a substance due to which it opposes the flow of electricity through it. The practical unit of resistance is ohm and a conductor is said to have a resistance of one ohm if it allows one ampere current to flow through it when one volt is applied across its terminals.

Specific resistance :

Specific resistance or resistivity of the material may be defined as *"the resistance between the opposite faces of a cm. cube of that material".* It is usually represented by ρ.

Superconductivity :

At very low temperature some metals acquire *zero electrical resistance and zero magnetic induction ;* the property known as *superconductivity.* Some of the important superconducting elements are :

  (*i*) Aluminium,   (*ii*) Zinc,

(*iii*) Cadmium,   (*iv*) Mercury, and

  (*v*) Lead.

The characteristic temperature at which a metal becomes superconducting depends on (*i*) the strength of magnetic field, and (*ii*) upon whether the field is applied externally or is the result of current used to measure the resistance.

An interesting feature to note is that metals such as copper, silver, and gold which are very good conductors do not show superconducting properties whilst metals and compounds which are superconducting are rather bad conductors at room temperature.

The *superconductivity can be destroyed by the application of a strong magnetic field.* When the magnetic field exceeds a certain critical value the superconducting state disappears, the magnetic field penetrates the material and electrical resistance is restored. *The disappearance of superconductivity by means of a strong magnetic field is the principle on which switching elements like the "Cryotrons" operate.*

Superconductivity is known to be sensitive to structure but its mechanism is still under research.

Applications of superconductors :

Some of the *important applications* of superconductors are as follows :

1. The *Cryotron.*
2. *Electrical machines :* It is possible to manufacture electrical generators and transformers in exceptionally small sizes having efficiency of 99.99%.
3. *Electromagnet ;* It is possible to produce superconducting solenoids which do not produce heat during operation. By the use of superconductivity, it is possible to design electromagnets for use in laboratories for lower temperature devices (like *maser*).
4. *Power cables.*

Classification of electrical conductors :

Materials serving as electrical conductors can be mainly divided into two groups :

1. Materials of *low resistivity* (and of high conductivity)
2. Materials of *high resistivity.*

The *former are employed for making conductors for all kinds of windings needed in electrical machines, apparatus and devices as well as for transmission and distribution of electric energy.* Important *examples* being *copper, aluminium and steel.*

The latter find their use in the making of *thermocouples, resistance heating devices etc.* Materials (alloys) like manganin, constantan, nichrome, carbon etc. belong to this category.

6.8.2. Semiconductors

Semiconductors *are solid materials, either non-metallic elements or compounds, which allow electrons to pass through them so that they conduct electricity in much the same way as metal.*

Characteristics :

Semiconductors possess the following *characteristics :*

 (*i*) The resistivity is usually high.

(*ii*) The temperature co-efficient of resistance is always *negative.*

(*iii*) The contact between semiconductor and a metal forms a layer which has a *higher resistance in one direction than the other.*

(*iv*) When some suitable *metallic impurity* (*e.g.* Arsenic, Gallium etc.) is added to a semiconductor, its *conducting properties change appreciably.*

 (*v*) They exhibit a rise in conductivity in the increasing temperature, with the decreasing temperatures their conductivity falls off, and at low temperatures semiconductors become dielectrics.

(*vi*) They are usually metallic in appearance but (unlike metals) are *generally hard and brittle.*

Both the resistivity and the contact effect are as a rule very sensitive to small changes in physical conditions, and the great importance of semiconductors for a wide range of uses apart from rectification depends on the *sensitiveness.*

Of all the elements in the periodic table, *eleven* are semiconductors which are listed below.

S.No.	Element	Symbol	Group in the periodic table	Atomic no.
1.	Boron	B	III	15
2.	Carbon	C	IV	6
3.	Silicon	Si	IV	14
4.	Germanium	Ge	IV	32
5.	Phosphorus	P	V	15
6.	Arsenic	As	V	33
7.	Antimony	Sb	V	51
8.	Sulphur	S	VI	
9.	Selenium	Se	VI	
10.	Tellurium	Te	VI	
11.	Iodine	L	VIII	

Examples of semi-conducting compounds are given below :

 (*i*) Alloys : $Mg_3 Sb_2$, ZnSb, Mg_2Sn, CdSb, AlSb, InSb, GeSb.

(*ii*) Oxide : ZnO, Fe_3O_4, Fe_2O_3, Cu_2O, CuO, BaO, CoO, NiO, Al_2O_3, TiO_2, UO_2, Cr_2O_3, WO_2, MoO_3.

(*iii*) Sulphides : Cu_2S, Ag_2S, PbS, ZnS, CdS, HgS, MoS_2.

(*iv*) Halides: AgI, CuI.

 (*v*) Selenides and Tellurides.

PbS is used in photo conductive devices, BaO in oxide coated cathodes, caesium antimonide in photomultipliers etc.

Intrinsic semiconductor :

A pure semiconductor is called **intrinsic semiconductor.** Here no free electrons are available since all the covalent bonds are complete. *A pure semiconductor, therefore behaves as an insulator.* It exhibits a peculiar behaviour even at room temperature or with rise in temperature. *The resistance of a semiconductor decreases with increase in temperature.*

When an electric field is applied to an intrinsic semiconductor at a temperature greater than 0° K, conduction electrons move to the anode and, the holes (when an electron is liberated into the conduction band, a positively charged hole is created in valence band) move to cathode. Hence semiconductor current consists of movement of electrons in opposite direction.

Extrinsic semiconductor :

In a pure semiconductor, which behaves like an insulator under ordinary conditions, if small amount of certain *metallic impurity is added it attains current conducting properties.* The impure semiconductor is then called *impurity semiconductor* or *extrinsic semiconductor.* The process of adding *impurity (extremely in small amounts, about 1 part in 10^8) to a semiconductor to make it extrinsic (impurity) semiconductor is called* **Doping.**

Generally following *doping agents* are used :

 (*i*) *Pentavalent atom* having five valence electrons (arsenic, antimony, phosphorus) ... called **donor atoms.**

(*ii*) *Trivalent atoms* having three valence electrons (gallium, aluminium, boron) ... called **acceptor atoms.**

With the addition of suitable impurities to semiconductor, two types of semiconductors obtained are :

 (*i*) N-type semiconductor, and

(*ii*) P-type semiconductor.

Applications of semiconductors :

1. Copper oxide and selenium, were the first materials to be used to serve as rectifiers.

2. Germanium and silicon came into commercial use somewhat later after copper-oxide and selenium. Germanium rectifiers found application earlier than silicon ones. One of the reasons for this is that it is easier and simpler to produce germanium monocrystals, although the process involves considerable technological difficulties. The Germanium and silicon semiconductors find wide use in both *high-frequency and commercial-frequency circuits* particularly as non-controlled rectifiers (diodes) and controlled rectifiers (for example, silicon controlled rectifiers).

3. *Non-linear resistors.* These are also called *varistors.* These are the semiconductors whose resistance is marked by dependence on the applied voltage, due to which the current rises non-linearity with rise in voltage.

   These are made mainly from *silicon carbide* obtained by electrically heating a mixture of quartz sand with carbon to temperature of about 2000°C. This is commonly known as *synthetic (electrical) carborundum.*

4. *Temperature-sensitive resistors.* These are also called *Thermistors.* They possess a *negative temperature resistivity of high absolute value.*

   They are made from oxides of certain metals such as copper, manganese, cobalt, iron and zinc. Thermistors are produced in the form of discs, short rods, beads etc. by ceramic techniques.

5. *Photo-conductive and photo-voltaic cells.* These are prepared from the materials which possess high sensitivity to light. The materials are sulphides, selenides and tellurides. Sometimes germanium and silicon are also used for the purpose.

These days considerable use is made of silicon photo-voltaic cells, more commonly called *solar cells* which serve to convert solar radiant energy into electric energy for spacecraft power supply, etc.

Photo-voltaic cells find wide *applications* in the following :

(*i*) Automatic control systems.

(*ii*) Television circuits.

(*iii*) Sound motion picture recording and reproducing equipment.

6.8.3. Insulators (or Dielectrics)

Introduction :

- **'Electrical insulating materials'** are defined as *materials which offer a very large resistance to flow of current, and for that reason they are used to keep the current in its proper path along the conductor.*

- A large number of substances and materials may be classified as insulators, many of which have to be employed in practice, as no single substance or material can satisfy all the requirements involved in the numerous and varied applications of insulators in electrical engineering. Such requirements involve consideration of physical properties, reliability, cost, availability, adaptability to machining operations etc. Thus in applications, the insulating material in addition to its function as an insulator may have to act as a rigid mechanical support to the conductor and may be installed out of doors, in which case the insulating qualities must be retained under all atmospheric conditions. In other cases extreme flexibility is required. Again, in electric heaters the insulating materials must maintain their insulating qualities over a wide range of temperatures extending in some cases to 1100°C, and for radio purposes the insulating qualities must be maintained upto very high frequencies. In electrical machines and transformers the insulating materials applied to the conductors are required to be flexible, to have high specific electric strength (to reduce thickness to minimum) and ability to withstand unlimited cycles of heating and cooling.

Characteristics of good electrical insulating material :

A good insulating material should possess the following *characteristics :*

1. Large insulation resistance.

2. High dielectric strength.

3. Uniform viscosity—it gives uniform electrical and thermal properties.

4. Should be uniform throughout—it keeps the electric losses as low as possible and electric stress uniform under high voltage differences.

5. Least thermal expansion.

6. When exposed to arcing should be non-ignitable.

7. Should be resistant to oils or liquids, gas fumes, acids and alkalies.

8. Should have no deteriorating effect on the material, in contact with it.

9. *Low dissipation factor (loss tangent).*

10. High mechanical strength.

11. High thermal conductivity.

12. Low permittivity.

13. High thermal strength.

14. Free from gaseous insulation to avoid discharges (for solids and gases).

15. Should be homogeneous to avoid local stress concentration.

16. Should be resistant to thermal and chemical deterioration.

Classification of electrical insulating materials :

The insulating materials can be *classified* in the following *two ways :*

1. Classification according to substances and materials.

2. Classification according to temperature.

    1. Classification according to substances and materials :

      (*i*) **Solids (Inorganic and organic) :**

        Mica, wood, slate, glass, porcelain, rubber, cotton, silk, rayon, terelyne, paper and cellulose materials.

      (*ii*) **Liquids (Oils and Varnishes) :**

        Linseed oil, refined hydrocarbon mineral oils, spirit, synthetic varnishes etc.

      (*iii*) **Gases:**

        Dry air, carbon dioxide, argon, nitrogen etc.

    2. Classification according to temperature :

Class	Insulating materials included	Assigned limiting insulating temperature
Y (Formerly O)	Cotton, silk, paper, cellulose, wood, etc. neither impregnated nor immersed in oil Materials of Y class are *unsuitable for electrical machines and apparatus as they deteriorate rapidly and are extremely hygroscopic.*	90°C
A	Materials of class Y impregenated with natural resins, cellulose esters, insulating oils etc. Also included in this list are laminated wool, varnished paper.	105°C
E	Synthetic resin enamels, cotton and paper laminates with formaldehyde bonding etc.	120°C
B	Mica, glass fibres, asbestos with suitable bonding substances, built up mica, glass fibre, and asbestos laminates.	130°C
F	Materials of class B with bonding materials of higher thermal stability.	155°C
H	Glass fibre and asbestos materials, and built up mica, with silicon resins.	180°C
C	Mica, ceramics, glass, quartz without binders or with silicon resins of higher thermal stability.	above 180°C

6.9.   SELECTION OF MATERIALS

General considerations for selection of materials are enumerated and described as follows :

1. Mechanical strength

2. Ductility

3. Design

4. Stability

5. Availability

6. Fabricability

7. Corrosion resistance

8. Cost.

1. **Mechanical strength :** While the primary selection criterion is often strength it may also be toughness, corrosion resistance, electrical conductivity, magnetic characteristics, thermal conductivity, specific gravity, strength-weight ratio or other properties.

   Examples. In household usage with relativity low water pressure, weaker and more expensive copper tubing may actually be a better choice than stronger steel pipe. One major difference lies in installation, since steel pipe comes in sections and is joined by threaded connections with elbows at corner, whereas soft copper can be obtained in coils and can be threaded around corners. *The lower installation cost of copper could overcome its higher material cost.* Also since copper has adequate strength, the greater strength of steel is not necessary. Furthermore, in *the event of freezing, copper tends to yield instead of burst.*

2. **Ductility :** Ductility is related to strength. Considerable ductility is generally obtained at a sacrifice of strength. For example, during *cold working there is gain in strength and loss in ductility.* It may be seen that some ductility is always required, and the more ductility obtainable without great loss in strength, the better. This is often true, but, at the same time, many metals and alloys have ductility and may not need much.

   In some cases, *brittleness* may be an asset, for example, the use of readily replaceable fragile members that are intended to fail first and protect the rest of the system. At the same time *appreciable ductility or plasticity is required for fabrication* by rolling, drawing, extrusion, and other mechanical working processes.

3. **Design :** In selection of materials design is closely related to strength and ductility. It is also quite widely recognised that a large portion or service failures are due *to fatigue.* Study of fatigue of materials is the joint duty of metallurgical engineering and production departments. There is no definite line between mechanical and metallurgical factors that contribute to fatigue.

   There are several cases in which the search for a substitute material led to feasible design modifications which were much more advantageous than a change in alloy composition.

4. **Stability :** Stability of material in service is related to :

   (*i*) temperature,

   (*ii*) fluctuations in temperature, and

   (*iii*) length of time at temperature.

   In some application exposure to radiation may also be important condition.

   Temperature not only directly affects strength and creep, but it can also produce changes in the microstructure of the material.

   - Obviously time is important in determining the extent to which these phenomena occur and, consequently, in the stringency of stability requirements. For example, a rocket motor may be required to operate only briefly, whereas a steam turbine is expected to operate for many years.

   - In several components it is desirable to have characteristics which produce shutting down for repairs. In other components, especially those subject to mechanical wear,

replacement at regular intervals is anticipated, and the part is made to be readily detachable. For example, in a nuclear reactor the problem of stability is far less drastic in a reactor that is to be operated for some months to test design feasibility than it is in a central power station reactor that is operated for a long period (several years).

- Other aspect of stability is the question of *seriousness of failure.* For example, a leak in tea kettle may have, only nuisance value but a leak in a vessel containing an inflammable or radioactive fluid is entirely different matter.

It may be noted that any design for long time operation may be an extrapolated or educated guess, since the best available data are often for times much shorter than anticipated in long term operation.

5. **Availability :** If a material is not available, irrespective of the merits of a material, it is not reasonable to base a design on it. This question involves availability of material at an appropriate cost and availability in the desired form. Obviously a material obtainable only in castings cannot be used in applications requiring tubing wirecloth etc.

6. **Fabricability :** There is a closer relation between fabricability and availability. A material may not be commercially available in the desired state of fabrication, but it may be possible, with relatively small-scale development type operations, to produce in it the desired from. This, of course, entails considerable expense, but circumstances may justify the cost. The development of production and fabrication procedures of beryllium and zirconium for nuclear reactor use provides *two specific examples.*

7. **Corrosion resistance :** A material may or may not be regarded as corrosion resistant depending on the particular service requirement. The criteria for corrosion resistance can be considered in three degrees :

   (*i*) Avoiding contamination (*e.g.,* food products).

   (*ii*) Preventing leaks of closed containers or conduits.

   (*iii*) Maintaining strength and other properties during corrosion attack.

   The possibility of corrosion should always be considered in any design. We may obtain acceptable resistance to corrosive attack under a particular set of conditions only to discover that some change in condition gives a new or modified problem.

8. **Cost :** The initial cost of a price of equipment involves *raw material, fabrication and installation costs,* in the form of replacements due to failure, shut down expenses while undergoing repair or replacement, and the economic damage of production losses.

   No industry is immune to savings through more effective application of materials. At least *three major approaches* may be taken to *reduce cost* through better use :

   (*i*) Reconsider the material selected.

   (*ii*) Reconsider the form of material.

   (*iii*) Redesign to take full advantage of properties.

   In several situations *definite savings can be realized by the simple expedient of changing from one material to another without substantial change in form or processing procedure.*

   - Ample savings can often be realized by changing fabrication procedure or the form in which material is used.

THEORETICAL QUESTIONS

1. How are materials classified ?
2. What are the differences between 'metals' and 'non-metals' ?
3. What are organic materials ? Give examples.

4. Give the classification of electrical engineering materials.
5. Discuss briefly the following materials :
   (i) Biomaterials ;   (ii) Advance materials.
6. What are 'Smart materials' ? Explain briefly.
7. What is 'Nanotechnology' ?
8. Discuss briefly 'Nanomaterials'.
9. State the applications of 'Nanomaterials'.
10. Explain briefly any three of the following mechanical properties of metals :
    (i) Elasticity ;   (ii) Toughness ;
    (iii) Creep ;   (iv) Machinability ;
    (v) Weldability.
11. Give the classification of cast iron.
12. Give the comparison between cast iron, wrought iron, wild steel and hard steel.
13. What are alloy steels ?
14. What are the purposes of alloying ?
15. What are the effects of the following alloying elements on steel ?
    (i) Chromium ;   (ii) Tungsten ;
    (iii) Silicon ;   (iv) Nickel.
16. Explain briefly the following alloy steels :
    (i) High speed steels ;   (ii) Stainless steel ;
    (iii) Nickel steel.
17. State the difference between 'brass' and 'bronze'.
18. Discuss briefly the following aluminium alloys :
    (i) Duralumin ;   (ii) Y-alloy.
19. What is a polymer ?
20. What is a plastic ?
21. How are plastics classified ?
22. Explain briefly 'Thermoplastic materials' and 'Thermosetting materials'.
23. Give examples of thermoplastic and thermosetting materials.
24. Give the comparison between 'Thermoplastic and thermosetting materials'.
25. What are 'ceramic materials' ?
26. Give the classification of ceramics.
27. State the advantages and applications of ceramic materials.
28. What is a glass ? State its properties and uses.
29. What is cement ? Give its properties and uses.
30. What are the ingradients of cement ?
31. Write short note on 'Advanced ceramics'.
32. What is a composite materials ?
33. Give the classification of composite materials.
34. State differences among conductors, semiconductors and insulators.
35. What are the general considerations for selection of materials ? Explain briefly.

7

Centre of Gravity and Centroid

7.1. Centre of gravity of a body. 7.2. Determination of centre of gravity. 7.3. Centroid. 7.4. Positions of centroids of plane geometrical figures. 7.5. Positions of centre of gravity of regular solids. 7.6 (*a*). Centroids of composite areas. 7.6 (*b*). Centre of gravity of simple solids. 7.7. Areas and volumes—Centroid method. 7.8. Centre of gravity in a few simple cases—*Highlights—Objective Type Questions—Exercises—Theoretical Questions—Unsolved Examples.*

7.1. CENTRE OF GRAVITY OF A BODY

A body comprises of several parts and its every part possesses weight. *Weight is the force of attraction between a body and the earth and is proportional to mass of the body*. The weights of all parts of a body can be considered as parallel forces directed towards the centre of the earth. Therefore, they may be combined into a resultant force whose magnitude is equal to their algebraic sum. If a supporting force, equal and opposite to the resultant, is applied to the body along the line of action of the resultant, the body will be in equilibrium. This line of action will pass through the centre of gravity of the body. Thus, centre of gravity of the body may be defined as the *point through which the whole weight of a body may be assumed to act*. The centre of gravity of a body or an object is usually denoted by c.g. or simply by *G*. The position of c.g. depends upon shape of the body and this may or may not necessarily be within the boundary of the body.

Fig. 7.1

420
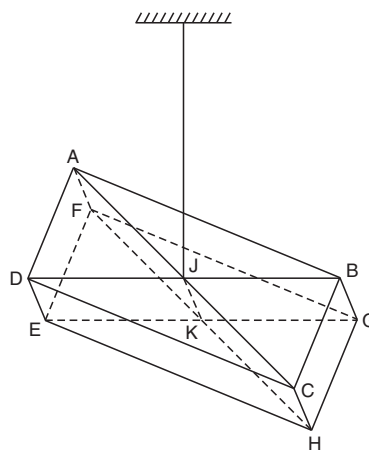
7.2. DETERMINATION OF CENTRE OF GRAVITY

The centre of gravity of some objects may be found by *balancing the object on a point*. Take a thin plate of thickness t, shown in Fig. 7.1. Draw the diagonals of the upper and lower forces to intersect at J, and K respectively. It the plate is placed on point at K, the plate will not fall. That is, it is balanced. If suspended from J, the plate will hang horizontally. The centre of gravity of the plate is at the centre of the line JK.
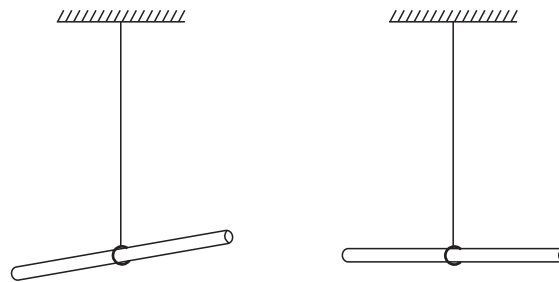
Fig. 7.2

Again, if we suspend a uniform rod by a string (Fig. 7.2) and move the position of the string until the rod hangs vertically, we can determine that the centre of gravity of the rod lies at its centre. Through the use of similar procedures it can be established that a body which has an axis, or line, symmetry has its centre of gravity located on that line, or axis. Of course, if a body has more than one axis of symmetry, the centre of gravity must lie at the intersection of the axes.

Another method for determining the centre of gravity is by *suspension*. Take an object, the section of which is shown in Fig. 7.3. Suspend it from point L. The body will not come to rest until its resultant weight is vertically downward from A. Through L, draw a vertical line LN. Then suspend the body from a point M, and let come to rest. Through M, draw a vertical line MT. The point of LN and MT is the position of the centre of intersection of gravity.
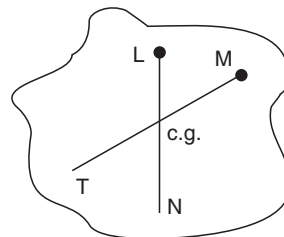
Fig. 7.3
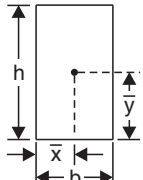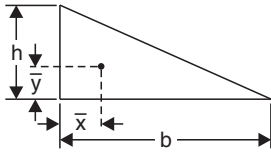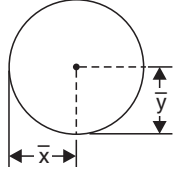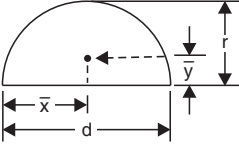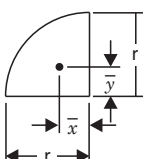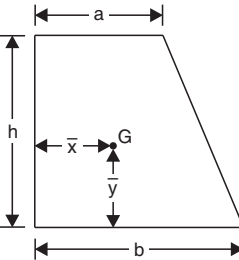
7.3. CENTROID

The centroid or centre of area is defined as the point where the whole area of the figure is assumed to be concentrated. Thus, centroid can be taken as quite analogous to centre of gravity when bodies have area only and not weight.

7.4. POSITIONS OF CENTROIDS OF PLANE GEOMETRICAL FIGURES

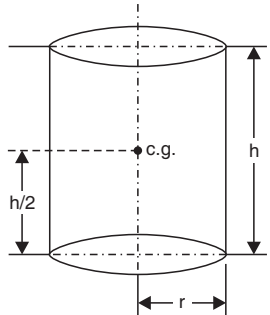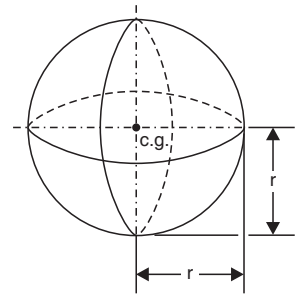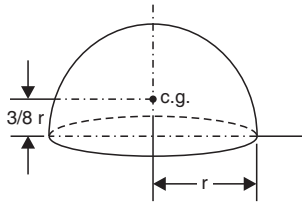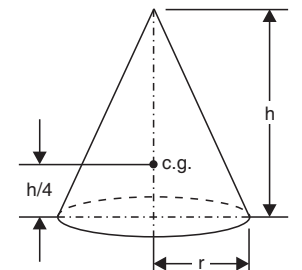Table 7.1 gives the positions of centroids of some plane geometrical figures.

Table 7.1 Centroids of Plane Geometrical Figures

Shape	Area	$\bar{x}$	$\bar{y}$	Figures
Rectangle	bh	$\dfrac{b}{2}$	$\dfrac{h}{2}$	 Fig. 7.4
Triangle	$\dfrac{bh}{2}$	$\dfrac{b}{3}$	$\dfrac{h}{3}$	 Fig. 7.5
Circle	$\dfrac{\pi}{4}d^2$	$\dfrac{d}{2}$	$\dfrac{d}{2}$	 Fig. 7.6
Semicircle	$\dfrac{\pi}{8}d^2$	$\dfrac{d}{2}$	$\dfrac{4r}{3\pi}(=0.424\,r)$	 Fig. 7.7
Quadrant	$\dfrac{\pi}{16}d^2$	$0.424\,r$	$0.424\,r$	 Fig. 7.8
Trapezium	$(a+b)\dfrac{h}{2}$	$\dfrac{a^2+b^2+ab}{3(a+b)}$	$\dfrac{(2a+b)}{(a+b)}\times\dfrac{h}{3}$	 Fig. 7.9

7.5. POSITIONS OF CENTRE OF GRAVITY OF REGULAR SOLIDS

Table 7.2 gives the positions of centre of gravity of regular solids.

Table 7.2. C.G. of Regular Solids

Shape	*Volume*	*Regular solids*
Cylinder	$\pi r^2 h$	 **Fig. 7.10**
Sphere	$\dfrac{4}{3}\pi r^3$	 **Fig. 7.11**
Hemisphere	$\dfrac{2}{3}\pi r^3$	 **Fig. 7.12**
Right circular cone	$\dfrac{1}{3}\pi r^2 h$	 **Fig. 7.13**

7.6. (*a*) CENTROIDS OF COMPOSITE AREAS

The location of the centroid of a plane figure can be thought of as the average distance of the area to an axis. Usually the axes involved will be the X and Y-axes. In determining the location of the centroid it is found advantageous to place the X-axis through the lowest point and the Y-axis through the left edge of the figure. This places the plane area entirely within the first quadrant where x and y distances are positive (Fig. 7.14). Then divide the area into simple areas such as rectangles,
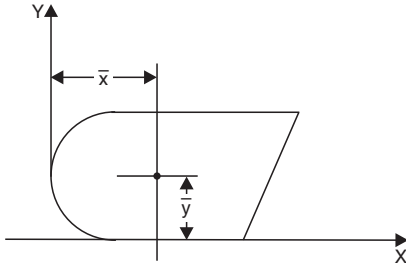
Fig. 7.14. Centroid of a composite area.
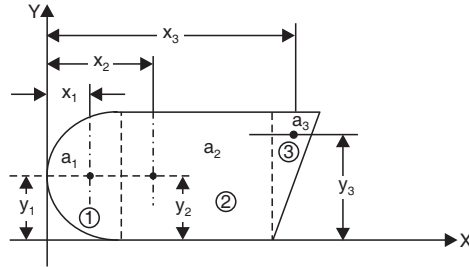
Fig. 7.15. Composite area divided into simple areas.

triangles, etc. (Fig. 7.15). Take the moment of each single area about the Y-axis. Sum up the moments about the Y-axis. Since the centroid of the composite figure is the point at which the entire area about the Y-axis must be equal to the moments of its component parts about the Y-axis, therefore

$$(a_1 + a_2 + ... + a_n)\, \overline{x} = a_1 x_1 + a_2 x_2 + ... + a_n x_n$$

or

$$\overline{x} = \frac{a_1 x_1 + a_2 x_2 + ... + a_n x_n}{a_1 + a_2 + ... + a_n}$$

or

$$\overline{x} = \frac{\Sigma ax}{\Sigma a} \qquad\qquad ...(7.1)$$

Following the same procedure for moments about the X-axis

$$\overline{y} = \frac{a_1 y_1 + a_2 y_2 + ... + a_n y_n}{a_1 + a_2 + ... + a_n}$$

or

$$\overline{y} = \frac{\Sigma ay}{\Sigma a} \qquad\qquad ...(7.2)$$

Note. *If a hole exists in the plane figure, treat it as a negative area. The moment of a negative area will be negative provided that the entire figure lies in the first quadrant.*

7.6. (*b*) CENTRE OF GRAVITY OF SIMPLE SOLIDS

The weight of the body is a force acting at its own centre of gravity and directed towards the centre of the earth. The position of the centres of bodies weighing W_1, W_2, W_3 etc. is found in the same manner as the resultant of parallel forces.

$$\left.\begin{array}{l} \overline{x} = \dfrac{\Sigma Wx}{\Sigma W} \\[2mm] \overline{y} = \dfrac{\Sigma Wy}{\Sigma W} \\[2mm] \overline{z} = \dfrac{\Sigma Wz}{\Sigma W} \end{array}\right\} \qquad\qquad ...(7.3)$$

If all the bodies are of the same material and have the same density (ρ), then
$$W = \rho V_1, \; W_2 = \rho V_2, \; W_3 = \rho V_3 \text{ etc.}$$
Substituting in eqn. (7.3) above, we have

$$\left.\begin{array}{l} \bar{x} = \dfrac{\Sigma \rho V x}{\Sigma \rho V} = \dfrac{\Sigma V x}{\Sigma V} \\[3mm] \bar{y} = \dfrac{\Sigma \rho V y}{\Sigma \rho V} = \dfrac{\Sigma V y}{\Sigma V} \\[3mm] \bar{z} = \dfrac{\Sigma \rho V z}{\Sigma \rho V} = \dfrac{\Sigma V z}{\Sigma V} \end{array}\right\} \qquad ...(7.4)$$

That is, if the bodies are made of the same material and are of the same density throughout, the centre of gravity of the bodies is their centre of volume. If the bodies are of the same cross-section but perhaps of different lengths,
$$V = al_1, \; V_2 = al_2, \; V_3 = al_3 \text{ etc.}$$
Substituting in eqn. (7.4) above, we get

$$\left.\begin{array}{l} \bar{x} = \dfrac{\Sigma al x}{\Sigma al} = \dfrac{\Sigma l x}{\Sigma l} \\[3mm] \bar{y} = \dfrac{\Sigma al y}{\Sigma al} = \dfrac{\Sigma l y}{\Sigma l} \\[3mm] \bar{z} = \dfrac{\Sigma al z}{\Sigma al} = \dfrac{\Sigma l z}{\Sigma l} \end{array}\right\} \qquad ...(7.5)$$

If the bodies are parts of a wire, pipe or rod of constant cross-section, then their centre of gravity may be found from the centre of their lengths.

Positions of centre of gravity of some regular solids are given in Table 7.2.

7.7. AREAS AND VOLUMES—CENTROID METHOD

Since the centre of gravity of an area or a body is the point at which the area or mass of the body may be assumed to be concentrated, it can be said that the distance through which an area or a body moves is the same as the distance described by its centre of gravity. This relation is used in finding areas and volumes. Thus, a line moving parallel to its original position is said to generate an area that is equal to the *length of the line multiplied by the distance through which its centroid moves*. That is, area is equal to length times width. Also, an area moving parallel to its original position is said to develop the same volume of prism that is equal to the *area multiplied by the distance through which the centroid moves*. That is, a *volume is equal to the area of the base times the altitude*.

Similarly, a line rotating about one end will develop the area of a circle. A right triangle rotating about either leg will develop the volume of a cone. *In each case a line or area moves through a distance equal to length of a path described by the centroid of either the line or the area.* Many determinations of areas or volumes are simplified by the use of this method.

7.8. CENTRE OF GRAVITY IN A FEW SIMPLE CASES

1. C.G. of a solid right circular cone

Refer to Fig. 7.16. Let *ABC* be the cone and *AD* its axis. Consider an elementary circular plate *PQ* cut off by two planes parallel to the base *BC* at distance y and $y + dy$ from *A*, and having its centre at *M*.

Let $AD = h, BD = r, PM = r'$

Triangles *APM, ABD* are similar.

$\therefore$ $\dfrac{AM}{MP} = \dfrac{AD}{BD}$

$\therefore$ $\dfrac{y}{r'} = \dfrac{h}{r}$ *i.e.,* $r' = \dfrac{yr}{h}$

Fig. 7.16

If w be the density of the material, mass of *PQ*

$$= \pi r'^2 \, dy \cdot w$$

$$= \frac{\pi r^2 y^2}{h^2} \cdot dy \cdot w$$

The c.g. of *PQ* is at *M*. Hence, the distance of c.g. of the cone from *A*

$$= \frac{\displaystyle\sum_{y=0}^{y=h} \frac{\pi^2 y^2}{h^2}\, dy \cdot w \cdot y}{\displaystyle\sum_{y=0}^{y=h} \frac{\pi r^2 y^2}{h^2}\, dy \cdot w} = \frac{\displaystyle\int_0^h y^3 dy}{\displaystyle\int_0^h y^2 dy} \qquad \begin{bmatrix} w = \text{weight density)} \\ w = \rho g, \text{where } \rho \text{ is} \\ \quad \text{mass density} \end{bmatrix}$$

$$= \frac{\dfrac{h^4}{4}}{\dfrac{h^3}{3}} = \frac{3h}{4}$$

$$= \left(h - \frac{3h}{4} \right) = \frac{h}{4} \text{ from the base.} \qquad\qquad ...(7.6)$$

Hence, the c.g. of a solid cone lies on the axis at a height one-fourth of the total height from the base.

2. C.G. of a thin hollow right circular cone

Refer to Fig. 7.17. Let *ABC* be the cone and *AD* its axis. Consider a circular ring cut off by planes *PQ* and *P'Q'* to parallel to the base *BC* at distances y and $y + dy$ from *A*.

Let the radius of *PQ* = r'

$$BD = r, AD = h$$

Semi-vertical angle of the cone
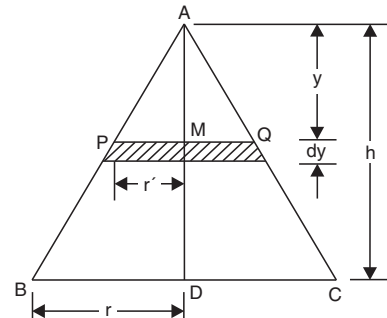
$$= \angle BAD = \alpha$$

Clearly, $\qquad PP' = dy \sec \alpha$

Also, $\qquad r' = \dfrac{ry}{h} \qquad$ ...(Refer previous case)

Area of elementary ring

$$= 2\pi r' PP' = 2\pi \cdot \dfrac{ry}{h}\, dy \sec \alpha$$

If w be the weight per unit area of the material, weight of the ring

$$= 2\pi \dfrac{ry}{h} \cdot dy \sec \alpha \cdot w$$

The c.g. of the ring lies on AD at distance y from A.

Hence, the distance of the c.g. of the cone from A

$$= \dfrac{\displaystyle\sum_{y=0}^{y=h} 2\pi \dfrac{ry}{h}\, dy \sec \alpha\, w \cdot y}{\displaystyle\sum_{y=0}^{y=h} 2\pi \dfrac{ry}{h}\, dy \sec \alpha \cdot w} = \dfrac{\displaystyle\int_0^h y^2 dy}{\displaystyle\int_0^h y\, dy} = \dfrac{\dfrac{h^3}{3}}{\dfrac{h^2}{2}}$$

$$= \dfrac{2}{3}h$$

$$= \left(h - \dfrac{2}{3}y\right) = \dfrac{h}{3} \text{ from the base} \qquad\qquad ...(7.7)$$

Hence the c.g. of a thin hollow cone lies on the axis at a height one-third of the total height above the base.

3. C.G. of a solid hemisphere

Refer to Fig. 7.18. Let ACB be the hemisphere of radius r, and OC its central radius. Consider an elementary circular plate PQ cut off by planes parallel to AB at distances y and $y + dy$ from AB.

$$PM^2 = OP^2 - OM^2$$
$$= r^2 - y^2$$

Weight of $PQ \qquad = \pi\, (r^2 - y^2) \times dy \cdot w$

where w is the weight of unit volume of the material.

The c.g. of PQ lies on OC at distance y from O.

Therefore, the distance of the c.g. of the hemisphere from O

$$= \dfrac{\displaystyle\sum_{y=0}^{y=r} \pi\, (r^2 - y^2)\, dy \cdot w \cdot y}{\displaystyle\sum_{y=0}^{y=r} \pi\, (r^2 - y^2)\, dy \cdot w}$$
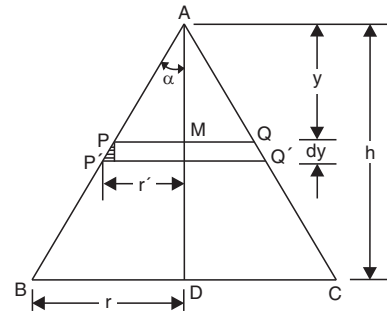
Fig. 7.17
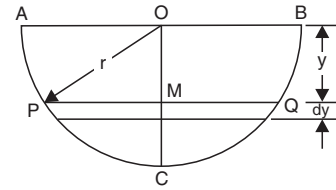
Fig. 7.18

$$
= \frac{\displaystyle\int_0^r (r^2 - y^2)\, y\, dy}{\displaystyle\int_0^r (r^2 - y^2)\, dy} = \frac{\left| \dfrac{r^2 y^2}{2} - \dfrac{y^4}{4} \right|_0^r}{\left| \left(r^2 y - \dfrac{y^3}{3} \right) \right|_0^r}
$$

$$
= \frac{\dfrac{r^4}{4}}{\dfrac{2r^3}{3}} = \frac{3r}{8} \ . \qquad\qquad\qquad ...(7.8)
$$

Hence c.g. of a solid hemisphere lies on the central radius at distance $\dfrac{3r}{8}$ *from the plane base, where r is the radius of the hemisphere.*

4. C.G. of a thin hollow hemisphere

In Fig. 7.18, if the hemisphere is hollow of negligible thickness, then PQ is a ring whose area $= 2\pi r\, dy$, by mensuration.

$$\text{Weight of } PQ = 2\pi r\, dy \cdot w$$

where w is the weight per unit area of the material.

∴   Distance of the c.g. of the hemisphere from $O = \bar{y}$ (say)

$$
= \frac{\displaystyle\int_0^r 2\pi r\, dy\, w \cdot y}{\displaystyle\int_0^r 2\pi r\, dy \cdot w} = \frac{\displaystyle\int_0^r y\, dy}{\displaystyle\int_0^r dy} = \frac{\dfrac{r^2}{2}}{r}
$$

$$
= \frac{r}{2} \ . \qquad\qquad\qquad ...(7.9)
$$

Hence the c.g. of a hollow hemisphere bisects the central radius.

5. C.G. of a semi-circular lamina

Let Fig. 7.18 represents a semi-circular plate of radius r.

The length of the elementary strip $PQ = 2 \times PM = 2\sqrt{(r^2 - y^2)}$

$$\text{Area of } PQ = 2\sqrt{r^2 - y^2}\ dy$$

If w be the weight per unit area of the material, weight of

$$PQ = 2\sqrt{r^2 - y^2}\ dy \cdot w$$

The c.g. of the PQ lies on OC at distance y from O.

$$
\frac{\displaystyle\int_0^r 2\sqrt{r^2 - y^2}\ dy \cdot wy}{\displaystyle\int_0^r 2\sqrt{r^2 - y^2}\ dy \cdot w} = \frac{\displaystyle\int_0^r y\sqrt{r^2 - y^2}\ dy}{\displaystyle\int_0^r \sqrt{r^2 - y^2}\ dy}
$$

Now, $\displaystyle\int_0^r y\sqrt{r^2 - y^2}\ dy = -\frac{1}{2} \int_0^r (r^2 - y^2)\,(-2y)\, dy$

$$= -\frac{1}{2} \left| \frac{2}{3} (r^2 - y^2)^{\frac{3}{2}} \right|_0^r = \frac{r^3}{3}$$

and

$$\int_0^r \sqrt{r^2 - y^2} \, dy = \left| y \frac{(r^2 - x^2)}{2} + \frac{r^2}{2} \sin^{-1} \frac{y}{r} \right|_0^r$$

$$= \frac{r^2}{2} \times \frac{\pi}{2} = \frac{\pi r^2}{4}$$

$$\therefore \qquad \bar{y} = \frac{\dfrac{r^3}{3}}{\dfrac{\pi r^2}{4}} = \frac{4r}{3\pi} \ . \qquad\qquad\qquad ...(7.10)$$

Hence the c.g. of a semi-circular lamina lies on the central radius at distance $\dfrac{4r}{3\pi}$ *from the bounding diameter, where r is the radius of the plate.*

6. C.G. (or centroid) of semi-circular arc

Refer to Fig. 7.19. Let OC be the central radius and P an element of arc subtending angle $d\theta$ at O.

By symmetry, the c.g. of the arc lies on OC.

Length of elemental arc $P = rd\theta$, where r is the radius of the arc.

Distance of P from $AB = r \cos \theta$

$\therefore$ The distance of c.g. of the whole arc from AB
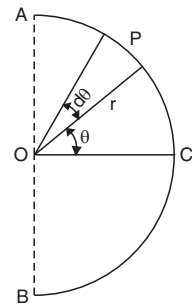
$$= \frac{\displaystyle\int_{-\pi/2}^{\pi/2} rd\theta \cdot r \cos \theta}{\displaystyle\int_{-\pi/2}^{\pi/2} r \, d\theta}$$

$$= \frac{r \left| \sin \theta \right|_{-\pi/2}^{\pi/2}}{\left| \theta \right|_{-\pi/2}^{\pi/2}}$$

$$= \frac{2r}{\pi} \ . \qquad\qquad\qquad ...(7.11)$$

Fig. 7.19

WORKED EXAMPLES

Example 7.1. *Find out the position of the centroid of L section as shown in Fig. 7.20.*

Sol. Refer to Fig. 7.20.

Divide the composite figure into two simple areas :

 (*i*) Rectangle (16 cm × 4 cm)
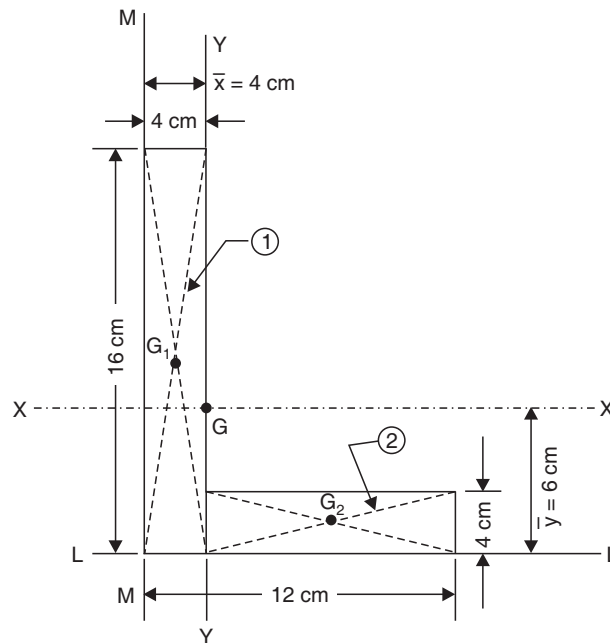 (*ii*) Rectangle (8 cm × 4 cm)

...(1)

...(2)

Fig. 7.20

To determine the location of the centroid of the plane figure we have the following table :

Components	Area a (cm²)	Centroidal distance 'x' from MM (cm)	Centroidal distance 'y' from LL (cm)	ax (cm³)	ay (cm³)
Rectangle (1)	16 × 4 = 64	2	8	128	512
Rectangle (2)	8 × 4 = 32	8	2	256	64
	96 (Σa)	—	—	384 (Σ ax)	576 (Σ ay)

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{384}{96} = \textbf{4 cm. (Ans.)}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{576}{96} = \textbf{6 cm. (Ans.)}$$

Example 7.2. *Determine the position of the centroid of I-section as shown in Fig. 7.21.*

Sol. Refer to Fig. 7.21.

Divide the composite figure into there simple areas :

 (*i*) Rectangle (10 cm × 2 cm) – top flange   ...(1)

(*ii*) Rectangle (25 cm × 2 cm) – web   ...(2)

(*iii*) Rectangle (15 cm × 3 cm) – bottom flange   ...(3)
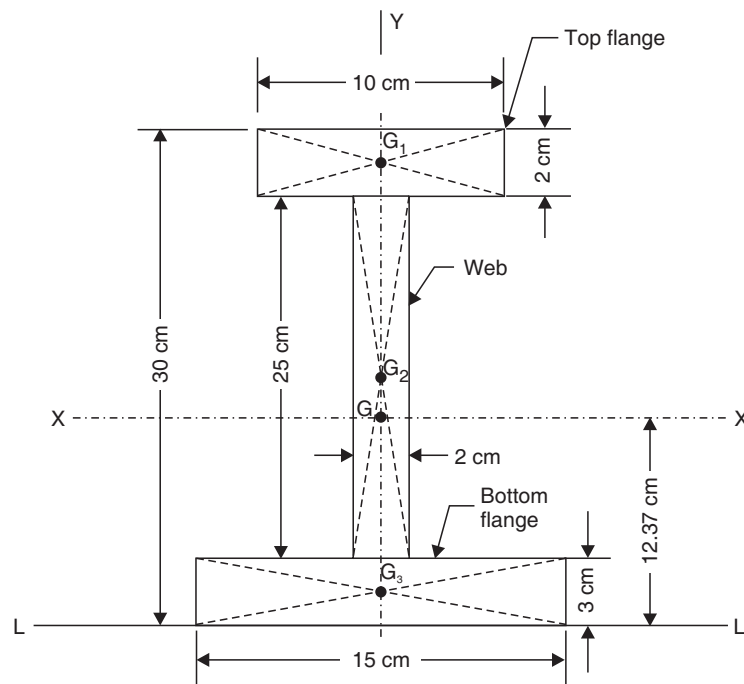
Fig. 7.21

To determine the location of the centroid of the plane figure we have the following table:

Components	Area 'a' (cm²)	Centroidal distance 'y' from LL (cm)	ay (cm³)
Rectangle (1)	10 × 2 = 20	29	580
Rectangle (2)	25 × 2 = 50	15.5	775
Rectangle (3)	15 × 3 = 45	1.5	67.5
	115 (Σa)	—	1422.5 (Σay)

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{1422.5}{115} = \textbf{12.37 cm. (Ans.)}$$

Example 7.3. *Using the analytical method, determine the centre of gravity of the plane uniform lamina shown in Fig. 7.22.*

Sol. Refer to Fig. 7.22.

The lamina may be divided into three parts :

 (*i*) A triangle marked (1)

(*ii*) A semi-circle marked (2)

(*iii*) A rectangle marked (3)
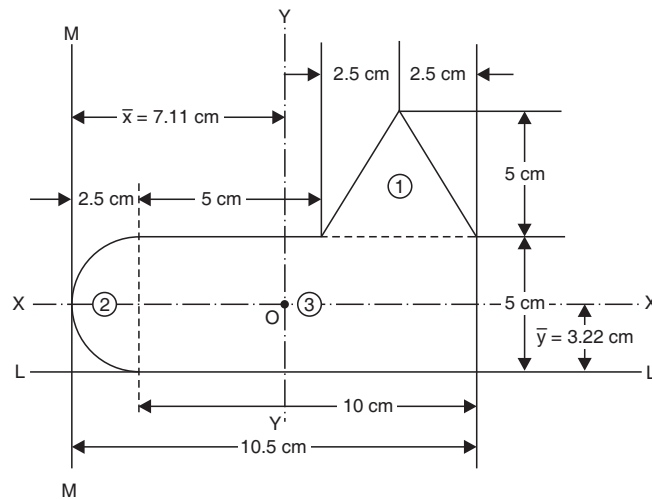
Fig. 7.22

The area of these components, their centroidal distances from the *LL*-axis and *MM*-axis and the moments of the areas of individual components about *LL*-axis and *MM*-axis are tabulated below :

Components	Area (a) (cm²)	Centroidal distance 'x' from MM (cm)	Centroidal distance 'y' from LL (cm)	ax (cm³)	ay (cm³)
Triangle (1)	$\frac{5 \times 5}{2} = 12.50$	2.5 + 5 + 2.5 = 10	5 + 5/3 = 6.67	125	83.4
Semicircle (2)	$\frac{\pi \times 2.5^2}{2} = 9.82$	$2.5 - \frac{4 \times 2.5}{3\pi}$ = 1.44	2.5	14.14	24.55
Rectangle (3)	10 × 5 = 50.00	2.5 + 5 = 7.5	2.5	375	125
Total	72.32 (Σ a)	—	—	514.14 (Σ ax)	232.95 (Σ ay)

Distance of the centroid from *MM*-axis,

$$\overline{x} = \frac{\Sigma ax}{\Sigma a} = \frac{514.13}{72.32} = \textbf{7.11 cm. (Ans.)}$$

Distance of the centroid from *LL*-axis

$$\overline{y} = \frac{\Sigma ay}{\Sigma a} = \frac{232.95}{72.32} = \textbf{3.22 cm. (Ans.)}$$

Example 7.4. *Determine the location of the centroid of the plane figure shown in Fig. 7.23.*

Sol. Refer to Fig. 7.23.

Divide the composite figure into three simple areas :

(*i*) a rectangle (7 cm × 5 cm)   ...(1)

(*ii*) a quadrant (2 cm radius)   ...(2)

(*iii*) a circle (3 cm dia.)   ...(3)
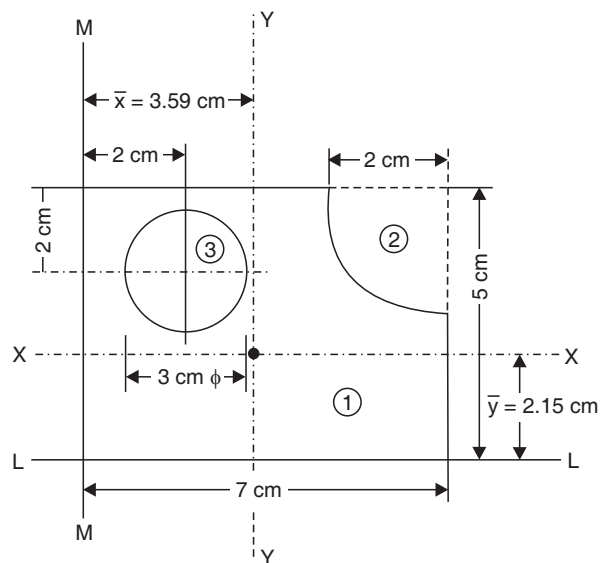
Fig. 7.23

The rectangle is a positive area. The quadrant and hole are treated as negative areas.

To determine the location of the centroid of the plane figure, we have the following table :

Components	Area 'a' (cm²)	Centroidal distance 'x' from M-M (cm)	Centroidal distance 'y' from L-L (cm)	ax (cm³)	ay (cm³)
Rectangle (1)	$7 \times 5 = 35$ (+)	3.5	2.5	122.5(+)	87.5(+)
Quadrant (2)	$\frac{\pi}{16}d^2 = \frac{\pi}{16} \times 4^2$ $= 3.14$ (–)	6.15	4.15	19.31(–)	13.03(–)
Circle (3)	$\frac{\pi}{4}d^2 = \frac{\pi}{4} \times 3^2$ $= 7.07$ (–)	2	3	14.14(–)	21.21(–)
Total	24.79 (Σa)	—	—	89.05 ($\Sigma\, ax$)	53.26 ($\Sigma\, ay$)

Distance of centroid from *MM*-axis

$$\bar{x} = \frac{\Sigma ay}{\Sigma a} = \frac{89.05}{24.79} = \textbf{3.59 cm. (Ans.)}$$

Distance of centroid from *LL*-axis

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{53.26}{24.79} = \textbf{2.15 cm. (Ans.)}$$

Example 7.5. *Determine the centroid of the lamina shown in Fig. 7.24.*
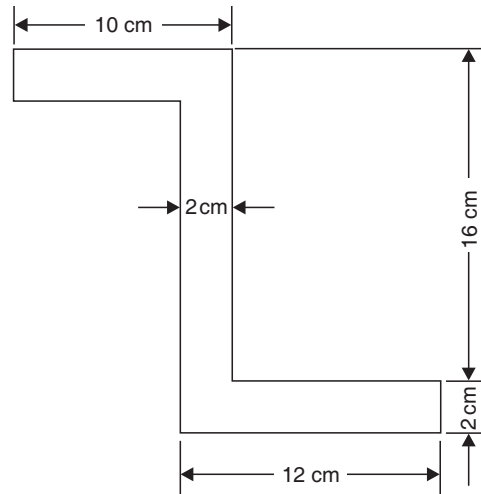
Fig. 7.24

Sol. (*a*) **Analytical method :**

Refer to Fig. 7.25.
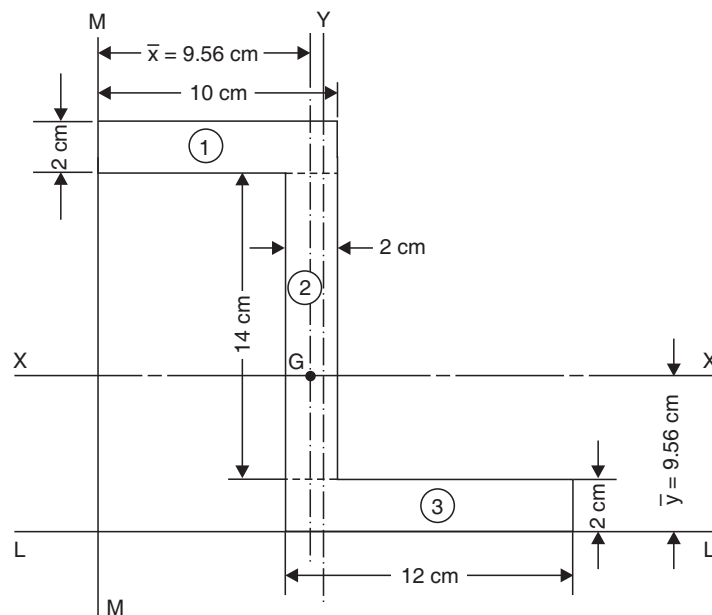
Fig. 7.25

Divide the figure into three areas :

(*i*) Rectangle (10 × 2)   ...(1)

(*ii*) Rectangle (14 × 2)   ...(2)

(*iii*) Rectangle (12 × 2)   ...(3)

To determine the location of the centroid of the plane figure we have the following table :

Components	Area 'a' (cm²)	Centroidal distance 'x' from MM (cm)	Centroidal distance 'y' from LL (cm)	ax (cm³)	ay (cm³)
Rectangle (1)	10 × 2 = 20	5	17	100	340
Rectangle (2)	14 × 2 = 28	9	9	252	252
Rectangle (3)	12 × 2 = 24	14	1	336	24
Total	72 (Σ a)	—	—	688 (Σ ax)	616 (Σ ay)

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{688}{72} = \textbf{9.56 cm. (Ans.)}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{616}{72} = \textbf{8.56 cm. (Ans.)}$$

(b) **Graphical method.** Refer to Fig. 7.26.
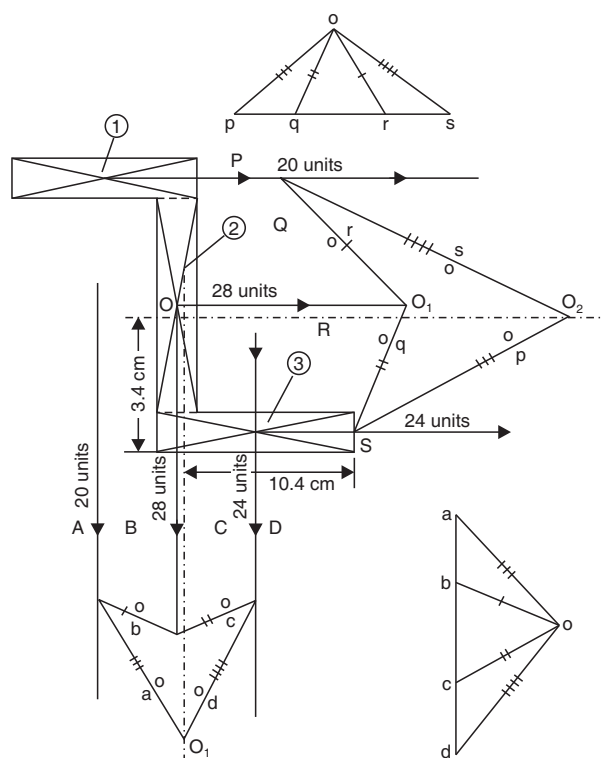
Fig. 7.26

To find $\overline{x}$. The given section is split upto into three rectangles as shown. The areas of the three rectangles are respectively 20 cm^2, 28 cm^2, 24 cm^2.

Now forces of magnitudes 20 units, 28 units and 24 units are assumed acting vertically through the centroids of the respective rectangles. Let the forces be represented by *AB, BC* and *CD*. Adopting a convenient scale mark off *ab* = 20 units. *bc* = 28 units and *cd* = 24 units. A pole *O* is chosen and *ao, bo, co* and *do* are joined. Starting from any point on the load line *AB* the funicular polygon is drawn and the point O_1, is obtained. Through O_1, a vertical line is drawn. This line represents the vertical centroidal axis. This axis is at a distance of 9.56 cm from left end.

To find $\overline{y}$. Similarly, taking three horizontal forces of 20 units, 28 units and 24 units through the centroids of the respective rectangles, a polar diagram and the corresponding furnicular polygon are drawn and the point O_2 is obtained. Now a horizontal line is drawn through O_2. This represents the horizontal centroids axis. This axis is found to be 8.56 cm above the bottom edge.

Example 7.6. *A square hole is punched out of a circular lamina of diameter 'a' with one diagonal of the square coinciding with a radius of the circle. Show that the centre of remainder is at*

a distance of $\dfrac{a}{8\pi - 4}$ *from the centre of the circle.*

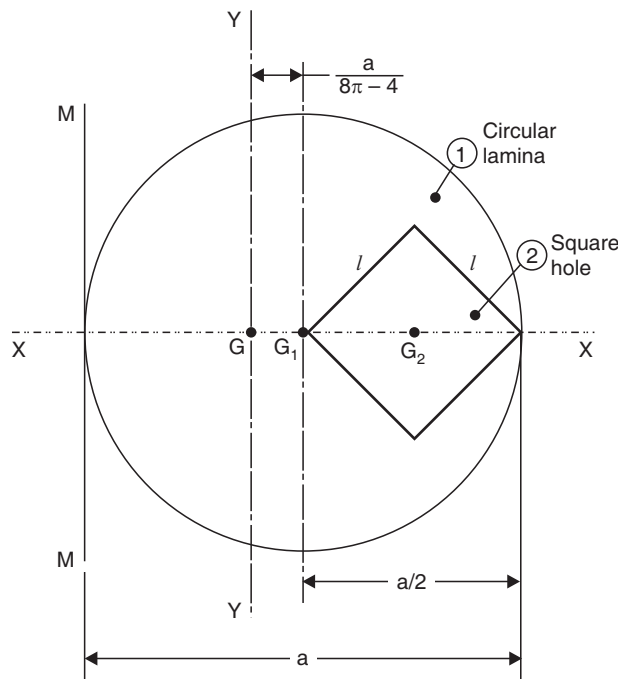**Sol.** Refer to Fig. 7.27. Since, the remaining body is symmetrical about *XX*-axis.

$$\overline{y} = 0$$

Fig. 7.27

For finding out location of centroid, we have the following table :

Components	Area 'A'	Centroidal distance 'x' from MM	Ax
Circular lamina (1)	$\dfrac{\pi a^2}{4}$	$\dfrac{a}{2}$	$\dfrac{\pi a^3}{8}$
Square hole (2)	$-\dfrac{a^2}{8}\,(=l^2)$ $\left[\begin{array}{l}\because\ l^2+l^2=\dfrac{a^2}{4}\\ \text{or}\qquad l^2=\dfrac{a^2}{8}\end{array}\right]$	$\dfrac{3a}{4}$	$\dfrac{-3a^3}{32}$
	$\Sigma a=\dfrac{\pi a^2}{4}-\dfrac{a^2}{8}=a^2\left(\dfrac{\pi}{4}-\dfrac{1}{8}\right)$	—	$\Sigma ax=\dfrac{\pi a^3}{8}-\dfrac{3a^3}{32}$

$$\bar{x}=\frac{\Sigma Ax}{\Sigma A}=\frac{\dfrac{\pi a^3}{8}-\dfrac{3a^3}{32}}{a^2\left(\dfrac{\pi}{4}-\dfrac{1}{8}\right)}\qquad \text{from } MM$$

$$=\frac{\dfrac{a^3}{8}\left(\pi-\dfrac{3}{4}\right)}{a^2\left(\dfrac{\pi}{4}-\dfrac{1}{8}\right)}\qquad \text{from } MM$$

$$=\frac{\dfrac{a^3}{8}\left(\dfrac{4\pi-3}{4}\right)}{a^2\left(\dfrac{2\pi-1}{8}\right)}\qquad \text{from } MM$$

$$=\frac{a}{4}\left(\frac{4\pi-3}{2\pi-1}\right)\qquad \text{from } MM$$

Centre of the remainder from the centre of the circle

$$=\frac{a}{2}-\frac{a}{4}\left(\frac{4\pi-3}{2\pi-1}\right)=\frac{a}{4}\left(2-\frac{4\pi-3}{2\pi-1}\right)$$

$$=\frac{a}{4}\left(\frac{4\pi-2-4\pi+3}{2\pi-1}\right)=\frac{\mathbf{a}}{\mathbf{8\pi-4}}.\ \textbf{(Ans.)}$$

Example 7.7. *From a uniform circular disc of radius 'a' is cut off a circular portion of a radius a/2. Find the distance between the centres of the disc and hole if the centre of the mass of the remainder is on the circumference of the hole.*

Sol. Refer to Fig. 7.28. Since, the remaining body is symmetrical about X-axis, therefore, $\bar{y}=0$.
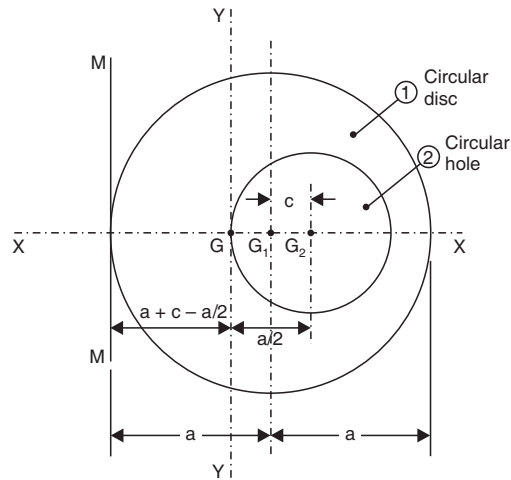
Fig. 7.28

Let 'c' be the distance between the centre of the disc and hole. For finding the value of 'c' we have the following table :

Components	Area 'A'	Centroidal distance 'x' from MM	Ax
Circular disc (1)	πa^2	a	πa^3
Circular hole (2)	$-\dfrac{\pi a^2}{4}$	$(a + c)$	$-\dfrac{\pi a^2}{4}(a + c)$
	$\Sigma A = \dfrac{3\pi a^2}{4}$	—	$Ax = \pi a^3 - \dfrac{\pi a^2}{4} \times (a + c)$

$$\overline{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{\pi a^3 - \dfrac{\pi}{4} a^2 (a + c)}{\dfrac{3\pi a^2}{4}}$$

But

$$\overline{x} = \left(a + c - \frac{a}{2}\right)$$

∴

$$\left(a + c - \frac{a}{2}\right) = \frac{\pi a^2 \left(a - \dfrac{a + c}{4}\right)}{\dfrac{3\pi a^2}{4}}$$

or

$$\left(\frac{a}{2} + c\right) = \frac{\left(\dfrac{3a - c}{4}\right)}{\dfrac{3}{4}} = \frac{3a - c}{3}$$

$$\frac{3a}{2} + 3c = 3a - c$$

$$4c = \frac{3a}{2}$$

or

$$c = \mathbf{\frac{3a}{8}} . \textbf{(Ans.)}$$

Example 7.8. *Where must a circular hole of 1 metre radius be punched out of a circular disc of 3 metres radius so that the centre of gravity of the remainder be 2 cm from the centre of the disc ?*

Sol. Refer to Fig. 7.29. Since, the remaining body is symmetrical about *XX*-axis, therefore $\overline{y} = 0$.
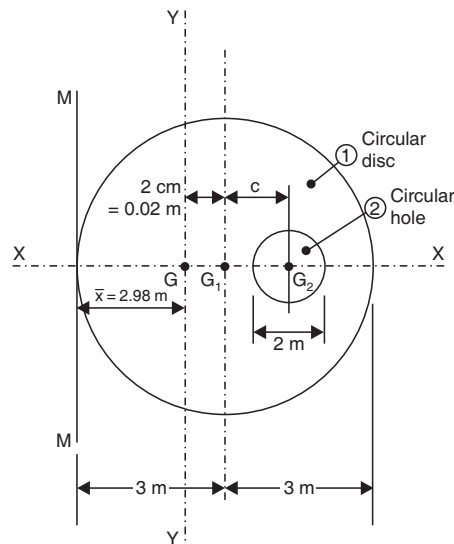
Fig. 7.29

Let '*c*' be the distance of the centre of the circular hole from the centre of the circular disc. For finding out the value of '*c*' we have the following table :

Components	Area '*a*' (m^2)	Centroidal distance '*x*' from *MM* (m)	*ax* (m^3)
Circular disc (1)	$\pi \times 3^2 = 9\pi$	3	27π
Circular hole (2)	$- \pi \times 1^2 = - \pi$	$(3 + c)$	$- \pi (3 + c)$
	$\Sigma a = 8\pi$	—	$\Sigma ax = 27\pi - \pi (3 + c)$

$$\overline{x} = \frac{\Sigma ax}{\Sigma a} = \frac{27\pi - \pi (3 + c)}{8\pi} = \frac{24 - c}{8}$$

But $\quad\quad \overline{x} = 2.98$ m = (given) from *MM*

$\therefore \quad\quad\quad 2.98 = \dfrac{24 - c}{8}$

or $\quad\quad 24 - c = 23.84$

or $\quad\quad c = \textbf{0.16 m. (Ans.)}$

Example 7.9. *A square hole is punched out of a circular lamina, a diagonal of such a square being along any radius of the circle with one vertex at the centre of the circular lamina. It is said that the length of the said diagonal is equal to the radius of circular lamina. Find the centre of gravity of the remainder, if r be the radius of the circle.*

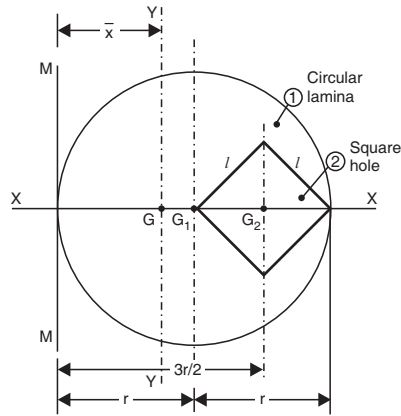**Sol.** Refer to Fig. 7.30. Since, the remaining body is symmetrical about X-axis, therefore, $\bar{y} = 0$.

Fig. 7.30

To find out the centre of gravity of the remainder, we have the following table :

Components	Area 'a'	Centroidal distance 'x' from MM	ax
Circular lamina (1)	πr^2	r	πr^3
Square hole (2)	$-\dfrac{r^2}{2}$ $\begin{bmatrix} l^2 + l^2 = r^2 \\ \therefore \quad l^2 = \dfrac{r^2}{2} \end{bmatrix}$	$\dfrac{3r}{2}$	$-\dfrac{3r^3}{4}$
	$\Sigma a = \pi r^2 - \dfrac{r^2}{2}$	—	$\Sigma ax = \pi r^3 - \dfrac{3r^3}{4}$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{\pi r^3 - \dfrac{3r^3}{4}}{\pi r^2 - \dfrac{r^2}{2}} = \frac{r^2\left(\pi - \dfrac{3}{4}\right)}{r^2\left(\pi - \dfrac{1}{2}\right)}$$

$$= \frac{r\,(4\pi - 3)}{2\,(2\pi - 1)} \text{ from } MM$$

$$= \frac{\mathbf{r\,(4\pi - 3)}}{\mathbf{4\pi - 2}} \textbf{ from MM. (Ans.)}$$

Example 7.10. *From the circular lamina of radius 'a' a smaller circular hole, one quarter the size of the given one is punched out so that its centre bisects a radius of the larger circle. Find the centre of gravity of the remainder.*

Sol. Refer to Fig. 7.31.
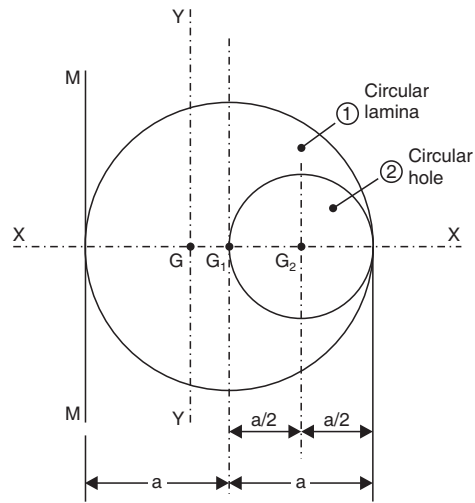
Fig. 7.31

Since, the remaining body is symmetrical about x-axis therefore, we shall find out $\bar{x}$ only $(\bar{y} = 0)$.

For finding out $\bar{x}$, we have the following table :

Components	Area 'a'	Centroidal distance 'x' from MM	ax
Circular lamina (1)	πa^2	a	πa^3
Circular hole (2)	$-\pi \times \left(\dfrac{a}{2}\right)^2 = -\dfrac{\pi a^2}{4}$	$a + \dfrac{a}{2} = \dfrac{3a}{2}$	$-\dfrac{3\pi a^3}{8}$
	$\Sigma a = \pi a^2 - \dfrac{\pi a^2}{4}$	—	$\Sigma ax = \pi a^3 - \dfrac{3\pi a^3}{8}$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{\pi a^3 - \dfrac{3\pi a^3}{8}}{\pi a^2 - \dfrac{\pi a^2}{4}} = \frac{\pi a^2 \left(a - \dfrac{3a}{8}\right)}{\pi a^2 \left(1 - \dfrac{1}{4}\right)} = \frac{\dfrac{5a}{8}}{\dfrac{3}{4}}$$

$$= \frac{5a}{6} \text{ from MM. (Ans.)} \qquad \left[\text{or } \left(a - \frac{5a}{6}\right) = \frac{a}{6} \text{ from the centre.}\right]$$

Example 7.11. *Determine the position of the c.g. of the shaded area of spandrel OLM, the curve OM being a parabola with axis vertical.*

Sol. Refer to Fig. 7.32.

Let the equation of *OM* be $x^2 = ky$

The point *M*, whose co-ordinates are (a, b) lies on it.

$\therefore \qquad\qquad a^2 = kb$

or $\qquad\qquad k = a^2/b$
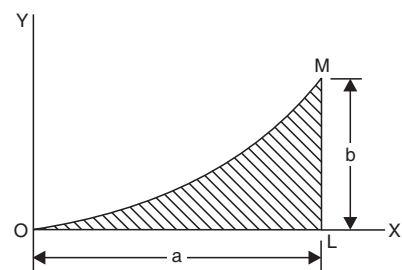
Fig. 7.32

$$x^2 = \frac{a^2}{b} y$$

$$\bar{x} = \frac{\int_0^a y dx \cdot x}{\int_0^a y dx} = \frac{\dfrac{b}{a^2} \int_0^a x^3 \, dx}{\dfrac{b}{a^2} \int_0^a x^2 dx}$$

$$\frac{a^4/4}{a^3/3} = \frac{\textbf{3a}}{\textbf{4}} \text{ . (Ans.)}$$

$$\bar{y} = \frac{\int_0^a y dx \cdot \dfrac{y}{2}}{\int_0^a y dx} = \frac{\dfrac{1}{2} \int_0^a \dfrac{b^2}{a^4} x^4 \, dx}{\int_0^a \dfrac{b}{a^2} x^2 \, dx}$$

$$\frac{b}{2a^2} \cdot \frac{\dfrac{a^5}{5}}{\dfrac{a^3}{3}} = \frac{\textbf{3b}}{\textbf{10}} \text{ . (Ans.)}$$

Example 7.12. *Determine the c.g. of the area of a sector of angle* α *of a circle of radius r as shown in Fig. 7.33.*

Sol. Refer to Fig. 7.33.

Area $OPQ = \dfrac{1}{2} \, r^2 \, d\theta$. Its c.g. will be on OP at a distance $\dfrac{2r}{3}$ from O, so that its distance from OY will be

$\dfrac{2r}{3} \cos \theta$
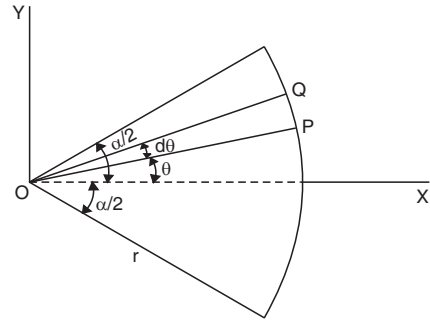
Fig. 7.33

$$\bar{x} = \frac{\int_0^{\alpha/2} \dfrac{2r}{3} \cos \theta \, \dfrac{r^2}{2} \, d\theta}{\int_0^{\alpha/2} \dfrac{r^2}{2} \, d\theta} = \frac{\dfrac{2r}{3} \int_0^{\alpha/2} \cos \theta \, d\theta}{\int_0^{\alpha/2} d\theta}$$

$$= \frac{2r}{3} \cdot \frac{\sin \alpha/2}{\alpha/2} = \frac{\textbf{4r}}{\textbf{3}\alpha} \cdot \textbf{sin} \, \frac{\alpha}{\textbf{2}} \text{. (Ans.)}$$

$\bar{\textbf{y}} = \textbf{0.}$ **(Ans.)**

Example 7.13. *Find the position of centre of gravity of the plane lamina in the form of a quarter of an ellipse, shown in Fig. 7.34.*

Sol. Refer to Fig. 7.34.
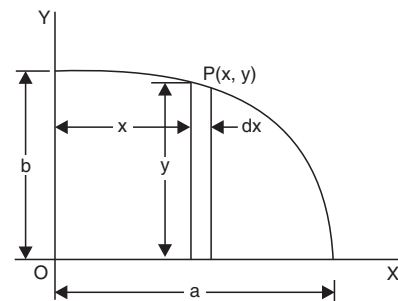
$$\bar{x} = \frac{\int_0^a xy \, dx}{\int_0^a y \, dx}$$

Fig. 7.34

$$\bar{y} = \frac{\int_0^a \frac{y}{2} \, y \, dx}{\int_0^a y \, dx}.$$

The equation of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

From which
$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} = \frac{a^2 - x^2}{a^2}$$

$$y = b/a \, \sqrt{a^2 - x^2}$$

$\therefore$
$$\int_0^a xy \, dx = \int_0^a x \cdot \frac{b}{a} \, \sqrt{a^2 - x^2} \, dx$$

$$= -\frac{b}{2a} \int_0^a (a^2 - x^2)^{1/2} \, (-2x) \, dx$$

$$= -\frac{b}{2a} \cdot \frac{2}{3} \left| (a^2 - x^2)^{3/2} \right|_0^a = \frac{b}{3a} \cdot a^3 = \frac{a^2 b}{3}$$

$$\int_0^a y \, dx = \int_0^a \frac{b}{a} \, \sqrt{a^2 - x^2} \, dx$$

$$= \frac{b}{a} \left| \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right|_0^a = \frac{b}{a} \cdot \frac{a^2}{2} \quad \frac{\pi}{2} = ab \, \frac{\pi}{4}$$

$\therefore$
$$\bar{x} = \frac{a^2 b}{3} \times \frac{4}{ab\,\pi}$$

or
$$\bar{x} = \frac{\mathbf{4a}}{\mathbf{3\pi}}. \quad \textbf{(Ans.)}$$

Similarly,
$$\bar{y} = \frac{\mathbf{4b}}{\mathbf{3\pi}}. \quad \textbf{(Ans.)}$$

Example 7.14. *A hemisphere of diameter 60 mm is placed on the top of a cylinder, whose diameter is also 60 mm. The height of the cylinder is 75 mm. Find the common C.G. of the composite body.*

Sol. Refer to Fig. 7.35.

Since, the composite solid is symmetrical about Y-axis, so the c.g. will lie on Y-axis and we shall, therefore, find out $\bar{y}$ only.

Volume of hemisphere,

$$V_1 = \frac{2}{3} \, \pi r^3 = \frac{2}{3} \times \pi \times 30^3 = 18000 \, \pi \text{ mm}^3$$
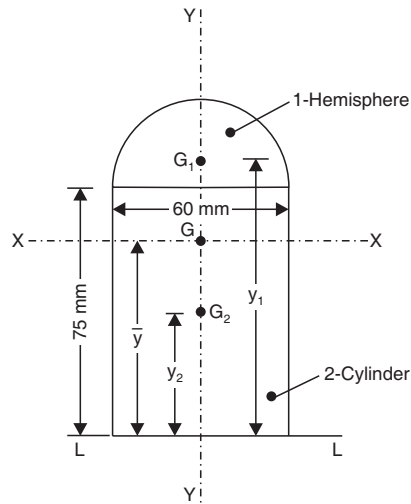
Fig. 7.35

Volume of cylinder,

$$V_2 = \pi r^2 h = \pi \times 30^2 \times 75 = 67500 \ \pi \ mm^3.$$

To determine $\bar{y}$ of composite solid, we have the following table :

Components	Volume (mm^3) V	Centroidal distance 'y' from LL (mm)	$V . y$ (mm^4)
Hemisphere (1)	18000π	$y_1 = 75 + \dfrac{3r}{8} = 75 + \dfrac{3 \times 30}{8} = 86.25$	1552500π
Cylinder (2)	67500π	$y_2 = \dfrac{75}{2} = 37.5$	2531250π
Total	85500π (ΣV)	—	4083750π ΣVy

$$\bar{y} = \frac{\Sigma V . y}{\Sigma V} = \frac{4083750\pi}{85500\pi}$$

= 47.76 mm. (Ans.)

Hence the c.g. of the composite body lies at a distance of 47.76 mm from *LL*, the axis of reference.

Note. In this case since the individual components (hemisphere and cylinder) are made of same material so we have considered volumes only (instead of weights) but in case the components are made of different materials then we should take weights instead of volumes (because densities are different).

Example 7.15. *A body consists of a right circular solid cone of height 18 cm and radius 15 cm placed on a solid hemisphere of radius 15 cm of the same material. Find the position of c.g.*

Sol. Refer to Fig. 7.36.

Since, the composite body is symmetrical about *Y*-axis, so the c.g. will lie on *Y*-axis and we shall, therefore, find out $\bar{y}$ only.

Volume of right circular cone,

$$V_1 = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 15^2 \times 18 = 1350\ \pi\ cm^3$$

Volume of hemisphere,

$$V_2 = \frac{2}{3}\pi r^3 = \frac{2}{3}\ \pi \times 15^3 = 2250\ \pi\ cm^3$$

To determine $\bar{y}$ of composite body, we have the following table :

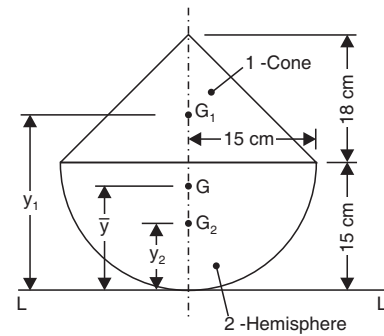**Fig. 7.36**

Components	Volume (cm³) V	Centroidal distance 'y' from LL	V.y (cm⁴)
Cone (1)	1350π	$y_1 = 15 + \dfrac{h}{3} = 15 + \dfrac{18}{4} = 19.5$ cm	32265π
Hemisphere (2)	2250π	$y_2 = 15 - \dfrac{3r}{8}$ $= 15 - \dfrac{3 \times 15}{8} = 9.375$ cm	21093.75π
Total	3600π (ΣV)	—	47418.75π (ΣV.y)

$$\bar{y} = \frac{\Sigma V.y}{\Sigma V} = \frac{47418.75\pi}{3600\pi} = 13.17\ cm.$$

Hence, the c.g. of the composite body lies at a distance of **13.17 cm from LL**, the axis of reference.   **(Ans.)**

Example 7.16. *The frustrum of a solid cone has base diameter 10 cm, the top diameter 5 cm and height 7.5 cm. Determine the c.g. of the solid.*

Sol. Refer to Fig. 7.37 (*a* and *b*)
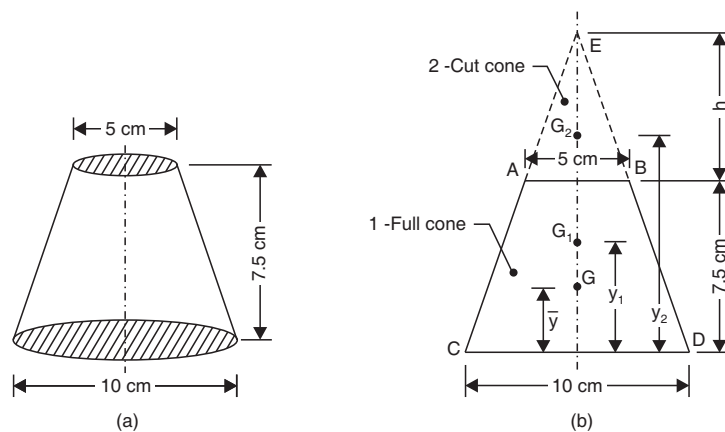
Fig. 7.37

To find h, from similar Δs *EFB* and *EGD*, we have

$$\frac{EF}{EH} = \frac{FB}{HD}$$

$$\frac{h}{h + 7.5} = \frac{2.5}{5}$$

or $\qquad 5h = 2.5\,(h + 7.5) = 2.5h + 18.75$

or $\qquad 2.5\,h = 18.75$

$$h = 7.5 \text{ cm}$$

Volume of full cone *CDE*,

$$V_1 = \frac{1}{3} \times \pi \times 5^2 \times (7.5 + 7.5) = 125\pi$$

Volume of cut cone *ABE*,

$$V_2 = \frac{1}{3} \times \pi \times 2.5^2 \times 7.5 = 15.625\pi.$$

Since the solid is symmetrical about *Y*-axis, we shall find out $\overline{y}$ only as given below :

Components	Volume (cm³) V	Centroidal distance 'y' from CD (cm)	Vy
Full cone *CDE* (1)	125π	$y_1 = \dfrac{15}{4} = 3.75$	468.74π
Cut cone *ABE* (2)	− 15.625π	$y_2 = 7.5 + \dfrac{7.5}{4} = 9.375$	− 146.48π
Total	109.375π (ΣV)	—	322.26π (ΣVy)

$$\overline{y} = \frac{\Sigma Vy}{\Sigma V} = \frac{322.26\pi}{109.375\pi} = \textbf{2.946 cm.} \quad \textbf{(Ans.)}$$

Example 7.17. *A hemisphere and a cone have their bases jointed together, the two bases being of the same size. Find the ratio of the height of the cone to the radius of the base, so that their common c.g. may be at the centre of the common base.*

Sol. Refer to Fig. 7.38.

Let *O* be the centre of the common base, *r* its radius, *h* the height of the cone, G_1 the c.g. of the cone and G_2 the c.g. of the hemisphere.

Then $\qquad OG_1 = 1/4\,h$

and $\qquad OG_2 = 3/8\,r$

Now, masses of the cone and hemisphere are as

$$\frac{1}{3}\,\pi r^2\,h : 2/3\,\pi r^3$$

or $\qquad h : 2r$

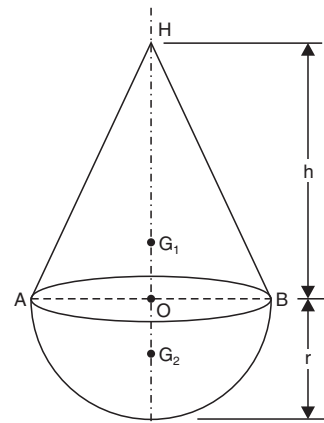**Fig. 7.38**

Let the mass of the cone = kh and the mass of the hemisphere,

$$k \times 2r = 2rk$$

∴ Distance of the c.g. of the whole from O, taking OH as positive

$$= \frac{kh \times h/4 - 2rk \times 3/8\,r}{kh + 2rk} = 0 \qquad\qquad (\because \quad O \text{ is the c.g.})$$

∴

$$\frac{kh^2}{4} - \frac{3r^2 k}{4} = 0$$

$$h^2 - 3r^2 = 0$$

$$h^2 = 3r^2 \quad \text{or} \quad \frac{h^2}{r^2} = 3$$

$$\frac{h}{r} = \sqrt{3}, \text{ which is the required ratio.} \quad \textbf{(Ans.)}$$

Example 7.18. *A solid right circular cylinder has its base scooped out so that the hollow is a right circular cone on the same base and having the same height as the cylinder. Find the c.g. of the remainder.*

Sol. Refer to Fig. 7.39.

The portion scooped out is shown shaded in the figure. O is the centre of the base, G is the c.g. of the cylinder, G_1 is the c.g. of the portion removed, and G_2 is the c.g. of the remainder. Let r be the radius of the base and h the height of the cylinder.

Weight of the cylinder = $\pi r^2\, hw$

Here w is the weight of unit volume of the material.

Weight of the portion scooped out

$$= \frac{1}{3}\, \pi r^2\, hw \text{ acting at } G_1$$

Weight of the remainder

Fig. 7.39

$$= \pi r^2\, h.w - \frac{1}{3}\, \pi r^2\, h.w$$

$$= \frac{2}{3}\, \pi r^2\, h.w \text{ acting at } G_2$$

$$OG_1 = \frac{1}{4}\, h,\ OG = \frac{1}{2}\, h$$

$$GG_1 = \frac{1}{2}\, h - \frac{1}{4}\, h = \frac{1}{4}\, h$$

G is the c.g. of a weight $\frac{1}{3}\, \pi r^2\, hw$ acting at G_1 and a weight $\frac{2}{3}\, \pi r^2\, hw$ acting at G_2.

∴

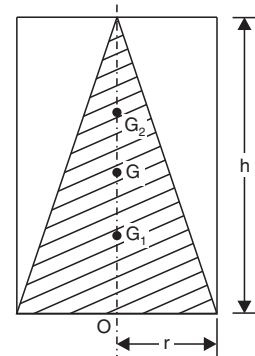$$\frac{1}{3}\, \pi r^2\, hw \times GG_1 = \frac{2}{3}\, \pi r^2\, hw \times GG_2$$

$$GG_2 = \frac{1}{2}\ GG_1 = \frac{1}{8}\ h$$

$$OG_2 = OG + GG_2$$

$$= \frac{1}{2}\ h + \frac{1}{8}\ h = \frac{\mathbf{5}}{\mathbf{8}}\ \mathbf{h.}\quad \textbf{(Ans.)}$$

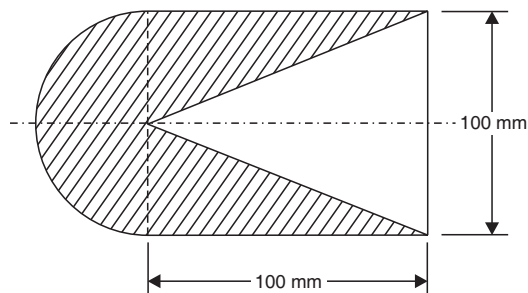☞ **Example 7.19.** *Determine the centre of gravity of a homogeneous solid body of revolution as shown in Fig. 7.40.*
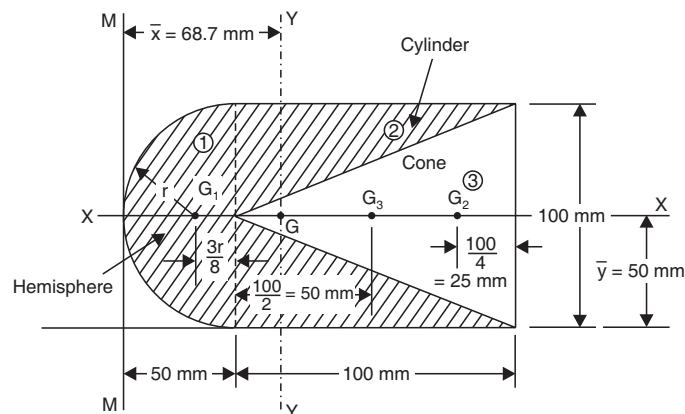

Fig. 7.40

Sol. Refer to Fig. 7.41.

Fig. 7.41

The solid body consists of the following three parts :

  (*i*) Hemisphere   ...(1)

  (*ii*) Cylinder   ...(2)

(*iii*) Cone   ...(3)

The solid body is obtained by *removing* the mass corresponding to the cone (3) from the composite body consisting of hemisphere and the cylinder.

Since, the body is symmetrical about X-axis, therefore we shall find out $\bar{y}$ only with the following table.

Components	Volume (mm³) V	Centroidal distance 'x' from MM (mm)	Vx (mm⁴)
Hemispherical (1)	$\frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times 50^3 = 8.3333\pi(+)$	$50 - \frac{3r}{8} = 50 - \frac{3\times 50}{8} = 31.25$	$2604156\pi(+)$
Cylinder (2)	$\pi r^2 h = \pi \times 50^2 \times 100 = 250000\pi(+)$	$50 + \frac{100}{2} = 100$	$25000000\pi(+)$
Cone (3)	$\frac{1}{3}\pi r^2 h = \frac{1}{3}\times \pi \times 50^2 \times 100 = 83333\pi(-)$	$50 + 100 - \frac{100}{4} = 125$	$10416625\pi(-)$
Total	$\Sigma V = 250000\pi$	—	$\Sigma Vx = 17187531\pi$

$$\bar{x} = \frac{\Sigma Vx}{\Sigma V} = \frac{17187531\pi}{250000\pi} = \textbf{68.7 mm.} \quad \textbf{(Ans.)}$$

and

$$\bar{y} = \frac{100}{2} = \textbf{50 mm. (Ans.)}$$

Example 7.20. *A frustrum of a solid right circular cone having base circle diameter 100 mm, height 100 mm and top diameter 50 mm has an axial hole of diameter 25 mm. Find the c.g. of the solid.*

Sol. Refer to Fig. 7.42. The frustrum of cone can be obtained by removing cone (2) of base diameter 50 mm and height 100 mm from another cone (1) of base diameter 100 mm and height 200 mm. The given solid will be obtained by removing a cylinder of (3) of diameter 25 mm and height 100 mm from this frustrum.
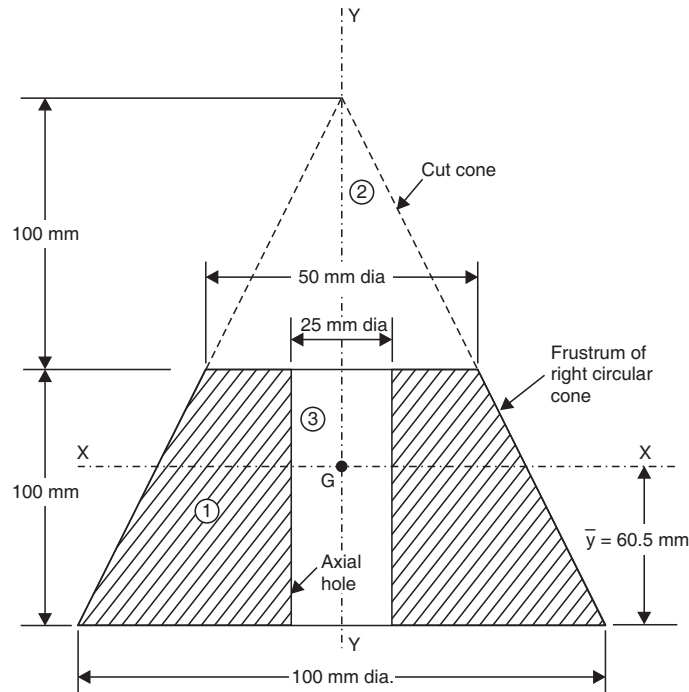

Fig. 7.42

Since, the solid is symmetrical about y-axis, we shall out $\bar{y}$ only with the following table :

Components	Volume (mm³) V	Centroidal distance 'y' from base (mm)	Vy (mm⁴)
Cone (1)	$\frac{1}{3} \times \pi \times 50^2 \times 200 = 166666\ \pi(+)$	$\frac{200}{4} = 50$	$8333300\pi\ (+)$
Cone (2)	$\frac{1}{3} \times \pi \times 25^2 \times 100 = 20833\ \pi(-)$	$\left(\frac{100}{4} + 100\right) = 125$	$2604125\pi\ (-)$
Cylinder (3)	$\pi \times (12.5)^2 \times 100 = 15625\pi\ (-)$	$\frac{100}{2} = 50$	$781250\pi\ (-)$
Total	$\Sigma V = 130208\pi$	—	$\Sigma Vy = 4947925\pi$

$\therefore$ $$\bar{y} = \frac{\Sigma Vy}{\Sigma V} = \frac{4947925\,\pi}{130208\,\pi} = \textbf{38 mm. (Ans.)}$$

HIGHLIGHTS

1. Weight is force of attraction between a body and the earth and is proportional to mass of the body.
2. Centre of gravity of the body is the point through which the whole weight of the body may be assumed to act.
3. The centre of gravity of some objects may be found by balancing the object on a point.
4. The *centroid* or *centre of area* is defined as the point, where the whole area of the figure is assumed to be concentrated.
5. Centroid of a triangle lies at intersection of radius ($h/3$ from base) of the triangle.
6. Centroid of semi-circle lies at $\frac{4r}{3\pi} = 0.424r$ from base, where r is radius of the semicircle.
7. C.G. of a cylinder lies on the axis at a height one-half of the total height from the base.
8. C.G. of a solid cone lies on the axis at a height one-fourth of the total height from the base.
9. C.G. of thin hollow cone lies on the axis at a height one-third of the total height above the base.
10. C.G. of a solid hemisphere lies on the central radius at distance $3r/8$ from the plane base, where r is radius of hemisphere.
11. C.G. of a hollow hemisphere bisects the central radius.

OBJECTIVE TYPE QUESTIONS

Fill in the blanks :

(i) ...... is the force of attraction between a body and the earth and is proportional to mass of the body.

(ii) ...... is the point through which the whole weight of a body may be assumed to act.

(iii) ...... is the point where the whole area of the figure is assumed to be concentrated.

(iv) Centroid of a triangle lies at .... from the base, where h is the height of triangle.

(v) Centroid of a semicircle lies at a distance of ...... from the base, where r is the radius of semicircle.

(vi) Area of a quadrant is given by ......, where r is the radius.

(vii) C.G. of a cylinder lies on the axis at a height ...... of the total height from the base.

(viii) C.G. of a solid cone lies on the axis at a height ...... of the total height from the base.

(ix) C.G. of a hollow cone lies on the axis at a height ...... of the total height above the base.

(x) C.G. of a solid hemisphere lies on the central radius at a distance ...... from the plane base, where r is the radius of hemisphere.

Answers

(*i*) Weight   (*ii*) Centre of gravity   (*iii*) Centroid   (*iv*) $h/3$

(*v*) $0.424r$ (*vi*) $\pi r^2/4$ (*vii*) $h/2$ (*viii*) one-fourth

(*ix*) $h/3$ (*x*) $\dfrac{3r}{8}$.

$$\boxed{\textbf{EXERCISES}}$$

Theoretical Questions

1. Define the terms 'Centre of gravity' and 'Centroid'.
2. Explain briefly how centre of gravity of a body can be determined.
3. Write down the positions of centroids of the following plane geometrical figures :
    Rectangle, circle, semicircle, quadrant and trapezium.
4. Where do the C.Gs of the following regular solids lie ?
    Cylinder, sphere, hemisphere, right circular cone.
5. Derive expressions for centre of gravity in the following cases :
    (*i*) Solid right circular cone
    (*ii*) Thin hollow right circular cone
    (*iii*) Semicircular lamina.

Unsolved Examples

1. Locate the centroid of the piece of sheet metal shown in Fig. 7.43.   [**Ans.** $\bar{x} = 4.59$ cm, $\bar{y} = 3.19$ cm]

3 cm   4 cm   3 cm   6 cm
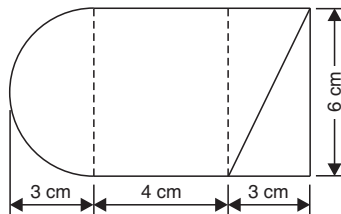
Fig. 7.43

2. Find the centroid of the lamina shown in Fig. 7.44.   [**Ans.** $\bar{x} = 2.50$ cm, $\bar{y} = 3.50$ cm]

2 cm   10 cm   8 cm   2 cm
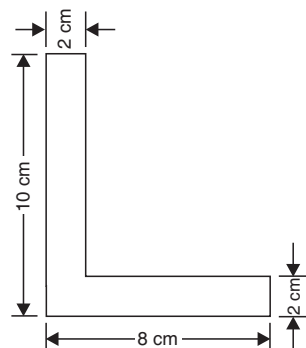
Fig. 7.44

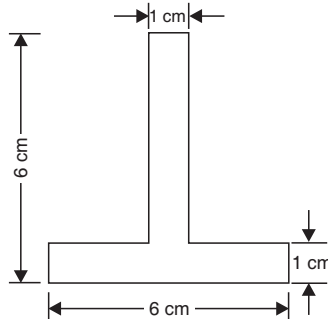3. Find the centroid of the lamina shown in Fig. 7.45.   [**Ans.** $\overline{y}$ = 1.864 cm from the bottom]

Fig. 7.45

4. Fig. 7.46 shows an unsymmetrical I section. The upper flange is 6 cm × 3/4 cm, the lower flange is 12 cm × 1 cm, the overall depth is 16 cm and the thickness of the metal at the web is $\frac{1}{2}$ cm. Find the distance of c.g. from the bottom of the lower flange.   [**Ans.** 5.68 cm]
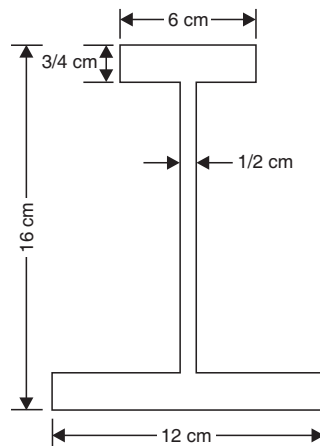

Fig. 7.46

5. A square sheet of metal has a square of one quarter of the original area cut from one corner as shown in Fig. 7.47. Calculate the position of the c.g. of the remaining portion of the sheet.

[**Ans.** Distance of c.g. from *AB* or *AD* = 5/12 *AB*]
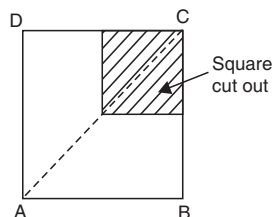
Fig. 7.47

6. From a circular plate of diameter 6 cm is cut out a circle whose diameter is a radius of the plate. Find the c.g. of the remainder.   [**Ans.** 0.5 cm from the centre]

7. A hollow cost iron column has 30 cm external diameter and 22.5 cm inside diameter. In casting, the bore got eccentric such that the thickness varies from 2.5 cm on one side and 5 cm on the other side. Find the position of c.g. of the section.   [**Ans.** 16.67 cm]

8. From a rectangular sheet of metal *ABCD,* in which *AB* = 40 cm and *BC* = 60 cm, a triangular piece *ABX* is removed, such that *AX* = *BX* = 25 cm. Calculate the distance of centre of gravity of the remainder.

[Ans. 33.57 cm from the base]

9. Find the c.g. of a semicircular section having the outer and inner radii 20 cm and 16 cm respectively.

[Ans. $\bar{y}$ = 11.2 cm]

10. Where must a hole of radius 0.25 cm be punched out of a circular disc of radius 1 m, if c.g. of the remainder is 5 cm from centre of disc. **[Ans.** 0.625 m]

11. *ABCD* is a square piece of paper of side 10 cm and *E* and *F* are mid-points of *AB* and *AD.* Find the c.g. of the portion left when Δ*AEF* is cut off as shown in Fig. 7.48. **[Ans.** $\bar{x}$ = 5.476 cm ; $\bar{y}$ = 5.476 cm]
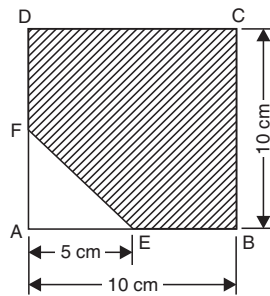

Fig. 7.48

12. Determine the position of centroid of an area enclosed by two semicircles and their common diameter as shown in Fig. 7.49. Take r_1 = 180 mm and r_2 = 135 mm. **[Ans.** $\bar{x}$ = 0, $\bar{y}$ = 103 mm]
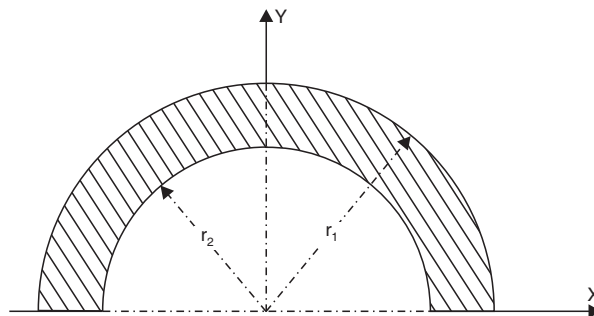

Fig. 7.49

13. A semicircle of diameter 100 mm is drawn on a paper sheet. A triangle with diameter of the semicircle as base is cut out of the semicircle as shown in Fig. 7.50. **[Ans.** $\bar{x}$ = 0, $\bar{y}$ = 29.2 mm]
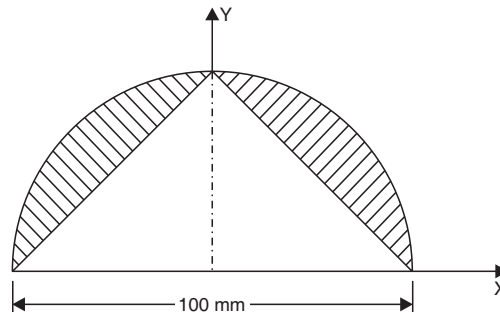

Fig. 7.50

14. Find the centroid of the shaded area shown in Fig. 7.51.   [**Ans.** $\bar{x}$ = 47 mm, $\bar{y}$ = 64.44 m]
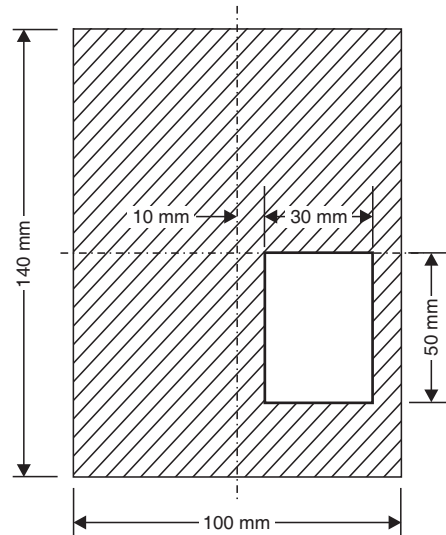

Fig. 7.51

15. Determine by calculation the position of the c.g. of the section shown in Fig. 7.52.
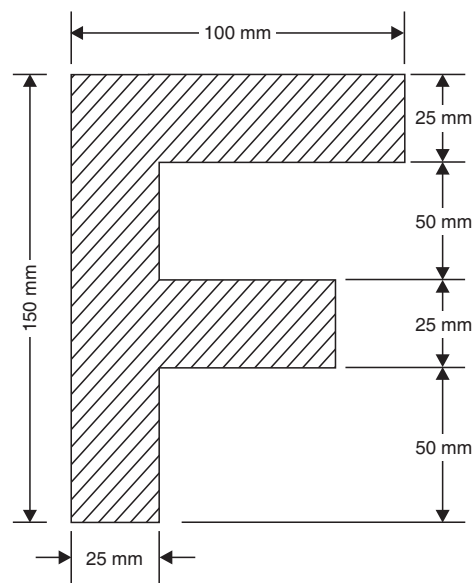

Fig. 7.52

16. Find the co-ordinates of the centroid of the area obtained after removing a semicircular of radius 100 mm from a quadrant of a circle of radius 200 mm. (See Fig. 7.53). [**Ans.** 69.8 mm, 127.2 mm]
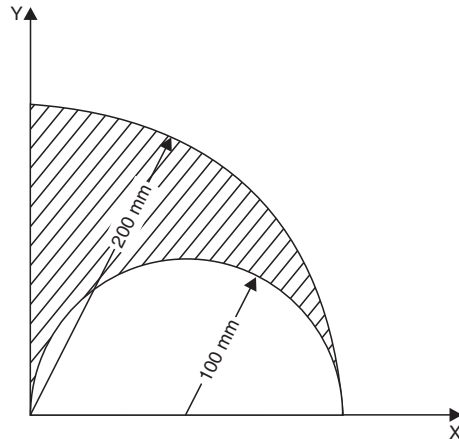

Fig. 7.53

17. A solid right circular cylinder has its base scooped out (Fig. 7.54) so that the hollow is a right circular cone on the same base and having the same height as the cylinder. Find the c.g. of the remainder.
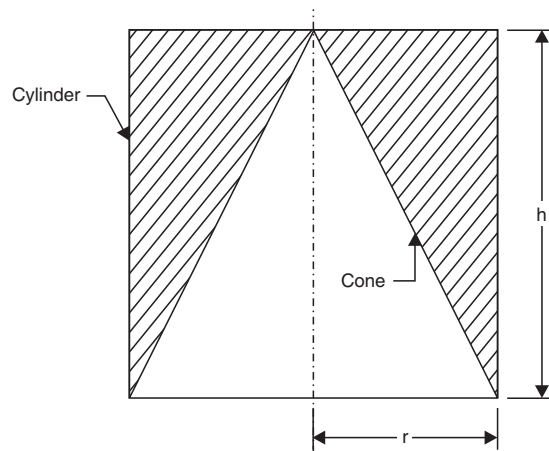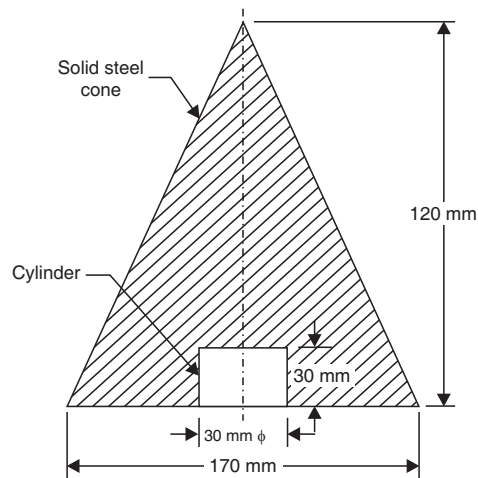
[**Ans.** 5/8*h*]

Fig. 7.54

18. A solid steel cone is made lighter by removing a part of the material from it in the form of a cylinder as shown in Fig. 7.55. Determine the position of the c.g. of the remaining metal. [**Ans.** 37.6 mm]

Fig. 7.55

19. A plate of uniform thickness is in the form of an isosceles triangle with base 24 cm and height 30 cm. A hole of 25 cm^2 is cut from it, with its centre 8 cm above the base and 2 cm to the left of vertex. Determine the centroid of the remaining plate. [**Ans.** $\bar{x}$ = 12.15 cm, $\bar{y}$ = 10.15 cm]

20. A body consists of a solid hemisphere of radius 10 cm and a right circular solid cone of height 12 cm. The hemisphere and cone have a common base and are made of the same material. Find the position of the c.g. of the compound body. [**Ans.** 13.22 cm]

21. A solid right circular cone has its base scooped out, so that the hollow is right cone on the same base. Find the height upto which the cone should be scooped out so that the c.g. of the remainder coincide with the vertex of the hollow. [**Ans.** $h/3$]

22. A right circular cylinder and a hemisphere, having the same radius of 6 cm are joined together face to face. The density of the material of the hemisphere is twice that of the material of the cylinder. Find the greatest height of the cylinder so that the compound body may rest with any point of the hemisphere in contact with a horizontal plane. [**Ans.** 6 cm]

8

Moment of Inertia

8.1. MOMENT OF INERTIA

The cumulative product of area and square of its distance from an axis is called the moment of inertia of a section about that axis. It can be expressed as

$$I_x = \int^a y^2 \, da \qquad \qquad ...(8.1)$$

where I_x = moment of inertia (M.O.I.) of the section about the *x*-axis, and

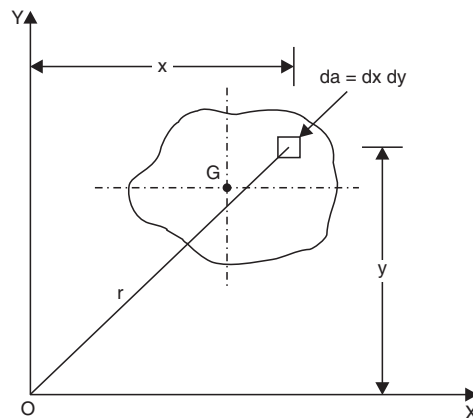y = the distance of infinitesimal area *da* from the *x*-axis as shown in Fig. 8.1.

Fig. 8.1

Similarly, the moment of inertia of a section about the *y*-axis is given by
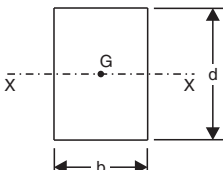
$$I_y = \int^a x^2 \, da \qquad \qquad ...(8.2)$$
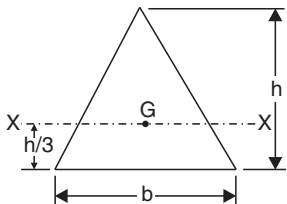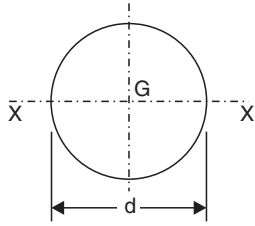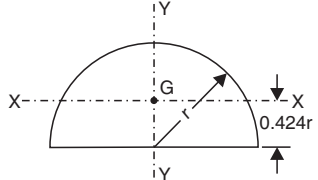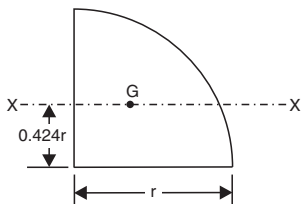
The cumulative product of area and square of its distance from a point is known as the polar moment of inertia. It is given by

$$I_p = \int^a r^2 \, da \qquad \qquad ...(8.3)$$

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$$= \int (x^2 + y^2) \, da$$

$$= I_x + I_y \qquad \qquad \qquad \qquad ...(8.4)$$

Table 8.1 gives the centroidal moments of inertia for simple areas.

Table 8.1. Moments of Inertia for Simple Areas

Shape	Moment of inertia	Simple areas
Rectangle	$I_{xx} = \dfrac{bd^3}{12}$	 **Fig. 8.2**
Triangle	$I_{xx} = \dfrac{bh^3}{36}$	 **Fig. 8.3**
Circle	$I_{xx} = \dfrac{\pi d^4}{64}$	 **Fig. 8.4**
Semicircle	$I_{xx} = 0.11 r^4$ $I_{yy} = \dfrac{\pi d^4}{128}$	 **Fig. 8.5**
Quadrant	$I_{xx} = 0.055 r^4$	 **Fig. 8.6**

8.2. THEOREM OF PARALLEL AXES

The theorem of parallel axes states as follows :

"*The moment of inertia of a lamina about any axis in the plane of the lamina equals the sum of moment of inertia about a parallel centroidal axis in the plane of lamina and the product of the area of the lamina and square of the distance between the two axes.*"

In Fig. 8.7 is shown a lamina of area A. Let LM be the axis in the plane of lamina about which the moment of inertia of the lamina is required to be found out. Let XX be the centroidal axis in the plane of the lamina and parallel to the axis LM. Let 'h' be the distance between the two XX and LM.
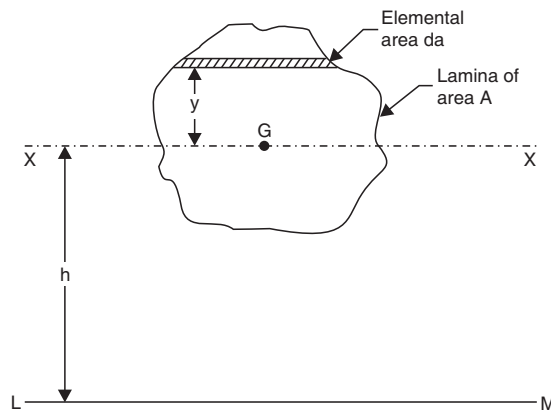

Fig. 8.7

It may be assumed that the lamina consists of an infinite number of small elemental components parallel to the axis XX. Consider one such elemental component at a distance y from the axis XX. The distance of the elemental component from the axis LM will be $(h + y)$ according as the elemental component and the axis LM are on opposite sides of XX or on the same side of XX.

Moment of inertia of the elemental component about the axis

$$LM = da \, (h \pm y)^2$$

∴   Moment of inertia of the whole lamina about the axis LM

$$= I_{lm} = \Sigma da \, (h \pm y)^2$$
$$= \Sigma dah^2 + \Sigma day^2 \pm 2\Sigma dahy$$
$$= h^2\Sigma da + \Sigma da \, y^2 \pm 2h\Sigma da \, y$$

But $\Sigma da = A, \; h^2 \, \Sigma da = Ah^2$

Σday^2 = moment of inertia of the lamina about the axis XX

$\Sigma day = 0$ since XX is a centroidal axis

∴ $I_{lm} = I_{xx} + Ah^2$ ...(8.5)

8.3. THEOREM OF PERPENDICULAR AXES

The theorem of perpendicular axes states as follows :

"*If I_{ox} and I_{oy} be the moments of inertia of a lamina about mutually perpendicular axes OX and OY in the plane of the lamina and I_{oz} be the moment of inertia of the lamina about an axis (oz) normal to the lamina and passing through the point of intersection of the axes OX and OY,*

then $I_{oz} = I_{ox} + I_{oy}$

Refer to Fig. 8.8. Let OX and OY be the two mutually perpendicular axes lying is the plane of the lamina. Let OZ be axis normal to the plane of the lamina and passing through O.
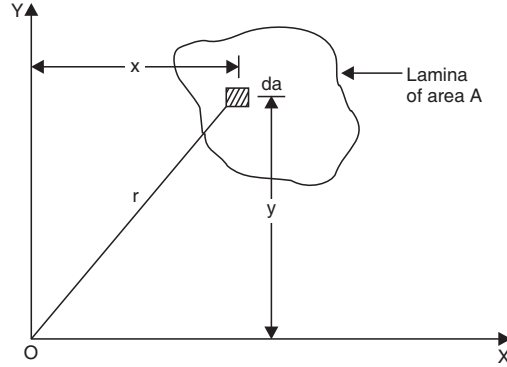

Fig. 8.8

Consider an elemental component of area da of the lamina. Let the distance of this elemental component from the axis OZ, *i.e.* from O be r.

∴   Moment of inertia of the elemental component OZ,

$$= da \times r^2$$

If the co-ordinates of the elemental component be (x, y) referred to the axes OX and OY, we have

$$r^2 = x^2 + y^2$$

Moment of inertia of the elemental component about the axis OZ

$$= da(x^2 + y^2)$$
$$= da\, x^2 + da\, y^2$$

∴   Total moment of inertia of the lamina about the axis OZ

$$= I_{oz} = \Sigma(da\, x^2 + da\, y^2)$$
$$= \Sigma da\, x^2 + \Sigma da\, y^2$$

But Σdax^2 = moment of inertia of the lamina about the axis $OY = I_{oy}$ ...(8.6)

and Σday^2 = moment of inertia of the lamina about the axis $OX = I_{ox}$

Hence, $I_{oz} = I_{ox} + I_{oy}$.

8.4. RADIUS OF GYRATION OF THE SECTION

One of the properties of cross-section which influence the structural behaviour of the members is *radius of gyration*.

$$k_i = \sqrt{\frac{I_i}{A}}$$ ...(8.7) $(\because \quad I_i = Aki^2)$

where I_i = moment of inertia about ith axis ; and

k_i = radius of gyration of area about ith axis.

Members, when subjected to axial forces tend to buckle. The load at which members will buckle is proportional to the square of the radius of the gyration. The radius of gyration is usually referred to *with respect to centroidal axes system of the reaction.*

8.5. MOMENT OF INERTIA OF LAMINAE OF DIFFERENT SHAPES

1. Rectangular lamina. Refer to Fig. 8.9. Let a rectangular lamina be b units wide and d units deep.
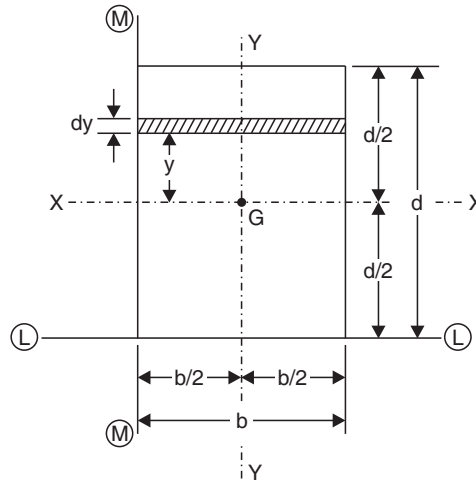
Fig. 8.9

(*a*) **Moment of inertia about centroidal axis XX parallel to the width.** Consider an elemental component of lamina at a distance y from the axis XX and having a depth dy.

Area of the elemental component

$$= da = b \times dy.$$

∴   Moment of inertia of the elemental component about the axis

$$XX = da \times y^2 = b \times dy \times y^2 = by^2 \ dy$$

∴   Total moment of inertia of the lamina about the axis XX,

$$I_{xx} = 2 \int_0^{d/2} by^2 \, dy = 2b \times \frac{1}{3} \times \frac{d^3}{8}$$

$$= \frac{bd^3}{12} \qquad\qquad\qquad\qquad\qquad\qquad ...(8.8)$$

Similarly, moment of inertia about the centroidal axis YY parallel to the depth,

$$I_{yy} = \frac{db^3}{12} \ . \qquad\qquad\qquad\qquad\qquad ...(8.9)$$

(*b*) **Moment of inertia about an axis LL passing through the bottom edge or top edge.** By the theorem of parallel axis, the moment of inertia about the axis LL is given by

$$I_{ll} = L_{xx} + Ah^2$$

In this case $A = bd$

$$h = \frac{d}{2}$$

$$I_{xx} = \frac{bd^3}{12}$$

$\therefore$
$$I_{ll} = \frac{bd^3}{12} + bd \times \left(\frac{d}{2}\right)^2$$

$$= \frac{bd^3}{12} + \frac{bd^3}{4} = \frac{bd^3}{3} \qquad \qquad ...(8.10)$$

Similarly, the moment of inertia about the axis MM is given by,

$$I_{mm} = \frac{db^3}{3} \qquad \qquad ...(8.11)$$

If G be the centroid of the lamina, the axis through the centroid and normal to the plane of the lamina is called the *polar axis*. Let I_p be the moment of inertia about the polar axis. I_p is called the polar moment of inertia.

By the theorem of perpendicular axes,

$$I_p = I_{xx} + I_{yy} = \frac{bd^3}{12} + \frac{db^3}{12} \qquad \qquad ...(8.12)$$

2. Rectangular lamina with a centrally situated rectangular hole. Refer to Fig. 8.10. Let in a rectangular lamina $B \times D$, a rectangular hole $b \times d$ be made centrally.

Moment of inertia of the lamina about any axis = moment of inertia of bigger rectangle – moment of inertia of the smaller rectangle.

For example,

$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12} . \qquad \qquad ...(8.13)$$

Fig. 8.10

3. Moment of inertia of a triangular lamina. Refer to Fig. 8.11. Let ABC be a triangular lamina of base b and altitude h.
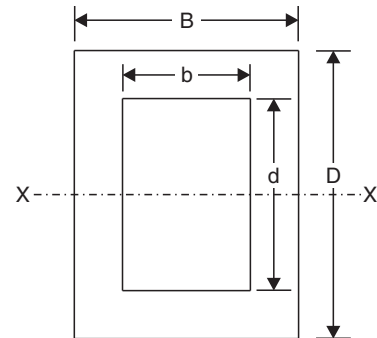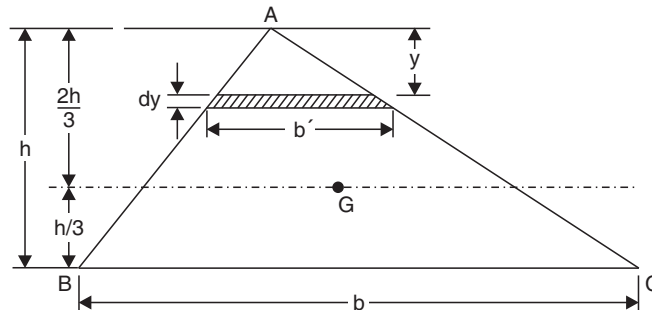

Fig. 8.11

(*a*) **Moment of inertia of a triangle about an axis LL through the vertex and parallel to the axis.** The triangle may be considered to consist of a number of infinitely small elemental components parallel to the base. Consider one such elemental component at a distance y from the vertex and of thickness dy. Width of elemental component

$$= b' = \frac{b}{h}\, y$$

∴ Area of elemental component

$$= b'\, dy = \frac{b}{h}\, y\, dy$$

∴ Moment of inertia of the elemental component about the axis LL

$$= \frac{b}{h}\, y\, dy\, y^2 = \frac{b}{h}\, y^3\, dy$$

∴ Moment of inertia of the lamina about the axis LL,

$$I_{ll} = \frac{b}{h} \int_0^h y^3 dy = \frac{bh^3}{4} \qquad \qquad \dots(8.14)$$

(*b*) **Moment of inertia of a triangle about the centroidal axis parallel to the base.** Let

XX be the centroidal axis. This axis is at a distance of $\frac{2}{3}h$ from the vertex.

Applying the theorem of parallel axes, we have

$$I_{ll} = I_{xx} + A\left(\frac{2}{3}h\right)^2$$

∴ $$\frac{bh^3}{4} = I_{xx} + \frac{bh}{2} \times \frac{4}{9}h^2$$

∴ $$I_{xx} = \frac{bh^3}{4} - \frac{2}{9}bh^3$$

or $$I_{xx} = \frac{bh^3}{36} \qquad \qquad \dots(8.15)$$

(*c*) **Moment of inertia of a triangle about the base.** Applying the theorem of parallel axis again, we have

$$I_{bc} = I_{xx} + A\left(\frac{h}{3}\right)^2$$

$$= \frac{bh^3}{36} + \frac{bh}{2} \times \frac{h^2}{9} = \frac{bh^3}{36} + \frac{bh^3}{18}$$

$$= \frac{bh^3}{12} \qquad \qquad \dots(8.16)$$

4. Moment of inertia of a circular lamina. Fig. 8.12 shows a circular lamina of radius R. The lamina may be considered as consisting of infinite number of elemental concentric rings. Consider one such elemental ring at a radius r and having a thickness dr.
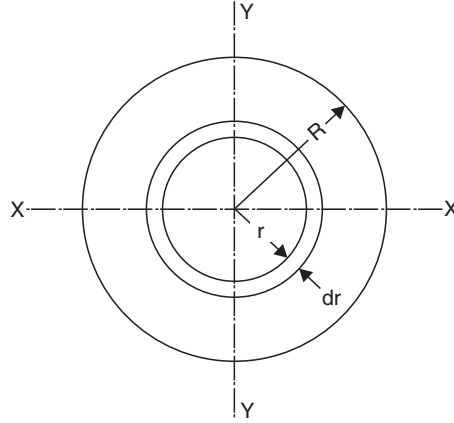
Fig. 8.12

The moment of inertia of elemental ring about the polar axis

$$= \text{area of ring} \times (\text{radius})^2$$

$$= 2\pi r \, dr \times r^2 = 2\pi r^3 \times dr$$

∴ Polar moment of inertia of the whole lamina,

$$I_p = \int_0^R 2\pi r^3 \, dr$$

∴ $$I_p = \frac{2\pi R^4}{4} = \frac{\pi R^4}{2}$$

If D be the diameter of the lamina

$$D = 2R$$

∴ $$I_p = \frac{\pi}{2}\left(\frac{D}{2}\right)^4 = \frac{\pi}{32} D^4 \qquad\qquad\qquad ...(8.17)$$

But $$I_{xx} = I_{yy}$$

and $$I_{xx} + I_{yy} = I_p = \frac{\pi D^4}{32}$$

∴ $$I_{xx} = I_{yy} = \frac{\pi D^4}{64} \qquad\qquad\qquad ...(8.18)$$

5. Moment of inertia of a circular lamina with a centrally situated circular hole. Refer to Fig. 8.13. Let D and d be the external and internal diameters of the lamina respectively.
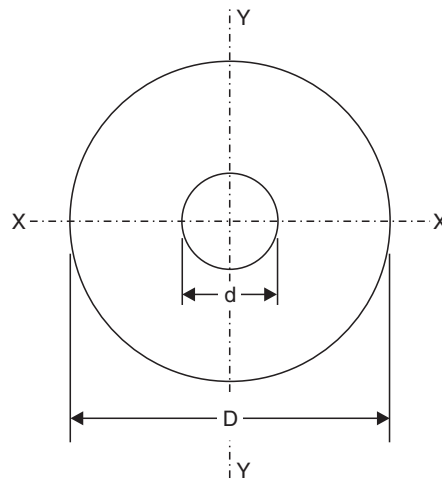
Fig. 8.13

Polar moment of inertia,

I_p = polar moment of inertia of the bigger circle-polar moment of inertia of the smaller circle

$$\therefore \qquad I_p = \frac{\pi D^4}{32} - \frac{\pi d^4}{32}$$

$$I_{xx} = I_{yy} = \frac{I_p}{2} = \frac{\pi D^4}{64} - \frac{\pi d^4}{64}.$$

6. Moment of inertia of a semi-circular lamina. In Fig. 8.14 is shown a semicircle of radius R. Let LM be the base of the semicircle.
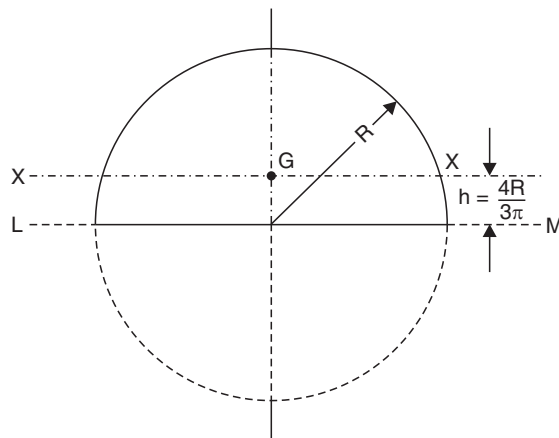
Fig. 8.14

The moment of inertia of a circular lamina about a diameter

$$LM = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$$

∴   Moment of inertia of the semicircle about LM

$$I_{lm} = \frac{\pi R^4}{8} = \frac{\pi D^4}{128} \qquad \qquad ...(8.19)$$

Let XX be centroidal axis parallel to the base LM. Let h be the distance between the axis XX and LM. We have

$$h = \frac{4R}{3\pi} = \frac{4}{3\pi} \times \frac{D}{2} = \frac{2D}{3\pi}$$

By theorem of parallel axis, we have

$$I_{lm} = I_{xx} + Ah^2$$

∴ $$\frac{\pi R^4}{8} = I_{xx} + \frac{\pi R^2}{2} \times \left(\frac{4R}{3\pi}\right)^2$$

From which, $$I_{xx} = \frac{\pi R^4}{8} - \frac{8\pi R^4}{9\pi^2}$$

or, $$I_{xx} = \frac{\pi R^4}{8} - \frac{8R^4}{9\pi}$$

or, $$I_{xx} = 0.11R^4 \qquad \qquad ...(8.20)$$

$$I_{yy} = \frac{\pi R^4}{8} = \frac{\pi}{128} D^4 \qquad \qquad ...(8.21)$$

7. Moment of inertia of a quadrant. Refer to Fig. 8.15. Let LON be a quadrant of a circular plate of radius R.
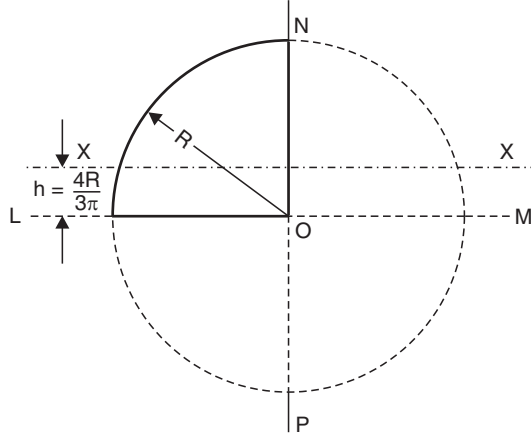
Fig. 8.15

Moment of inertia of the area LON = one-fourth of the moment of inertia of the a circular area about axis LM.

∴ $$I_{lm} = \frac{1}{4} \times \frac{\pi R^4}{4} = \frac{\pi R^4}{16}$$

Consider the semi-circle LNM. Distance of its centroid from LM

$$= \frac{4R}{3\pi}$$

The distance of the centroid of the quadrant *LON* from *LM* is also equal to $\dfrac{4R}{3\pi}$, *i.e.*, the axis *XX* is centroidal axis of the quadrant *LON* as well as the semi-circle *LNB*.

∴   Moment of inertia of the quadrant about axis *XX*,

I_{xx} = one-half of the moment of inertia of the semi-circle about *XX*.

∴ $I_{xx} = \dfrac{1}{2}(0.11R^4)$

$\qquad = 0.055R^4$ ...(8.22)

8. Moment of inertia of a thin ring. Fig. 8.16 shows a ring of mean radius *r* and of thickness *t*. Consider an elemental component *da* of the lamina. The moment of inertia of this elemental component about the polar axis of the lamina = dar^2. Since all the elemental components are at the same distance *r* from the polar axis, moment of inertia of the ring about the polar axis,

$I_p = \Sigma da\ r^2 = r^2\ \Sigma da$

$\qquad = r^2 \times$ area of the whole ring

$\qquad = r^2 \times 2\pi\ rt$ ...(8.23)
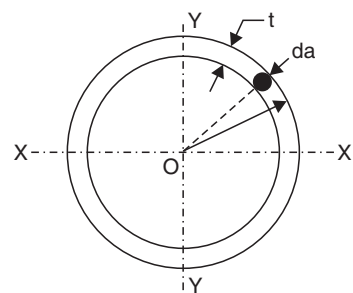
Moment of inertia about the axis *XX*

$\qquad$ = moment of inertia about the *YY*

$\qquad = \dfrac{I_p}{2}.$

Fig. 8.16

WORKED EXAMPLES

Example 8.1. *Find the moment of inertia about the centroidal axes XX and YY of the section shown in Fig. 8.17.*
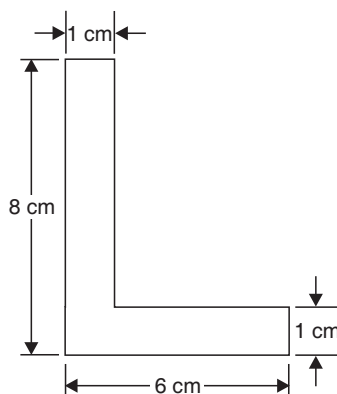
Fig. 8.17

Sol. Refer to Fig. 8.18.
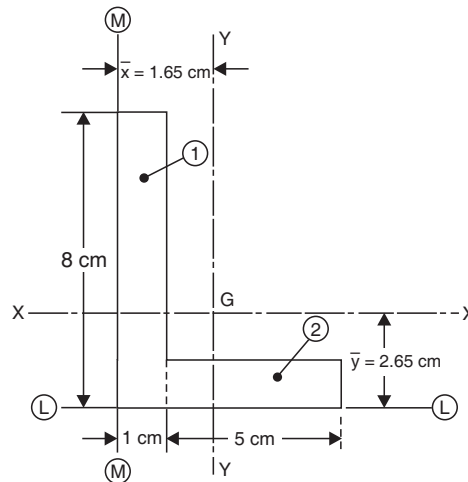
Fig. 8.18

To determine the location of centroid of the section we have the following table :

Components	Area 'a' cm²	Centroidal distance 'x' from MM (cm)	Centroidal distance 'y' from LL (cm)	ax (cm³)	ay (cm³)
Rectangle (1)	8 × 1 = 8	0.5	4	4	32
Rectangle (2)	5 × 1 = 5	3.5	0.5	17.5	2.5
Total	Σa = 13	—	—	Σax = 21.5	Σay = 34.5

$\therefore$ $\qquad\qquad\qquad \bar{x} = \dfrac{\Sigma ax}{\Sigma a} = \dfrac{21.5}{13} = 1.65$ cm

and $\qquad\qquad\qquad \bar{y} = \dfrac{\Sigma ay}{\Sigma a} = \dfrac{34.5}{13} = 2.65$ cm.

To find I_{xx} and I_{yy} using theorem of parallel axes, we have

$$I_{xx} = \left[\frac{1 \times 8^3}{12} + 8 \times 1 \times (4 - 2.65)^2\right] + \left[\frac{5 \times 1^3}{12} + 5 \times 1 \times (2.65 - 0.5)^2\right]$$

$$= (42.66 + 14.58) + (0.416 + 23.112)$$

$$= 57.24 + 23.53 = \textbf{80.77 cm}^4\textbf{. (Ans.)}$$

$$I_{yy} = \left[\frac{8 \times 1^3}{12} + 8 \times 1 \times (1.65 - 0.5)^2\right] + \left[\frac{1 \times 5^3}{12} + 5 \times 1 \times (3.5 - 1.65)^2\right]$$

$$= (0.66 + 10.58) + (10.42 + 17.11)$$

$$= 11.24 + 27.53 = \textbf{38.77 cm}^4\textbf{.   (Ans.)}$$

Example 8.2. *Calculate the moment of inertia about horizontal and vertical gravity axes* (I_{xx} *and* I_{yy}) *of the section shown in Fig. 8.19.*
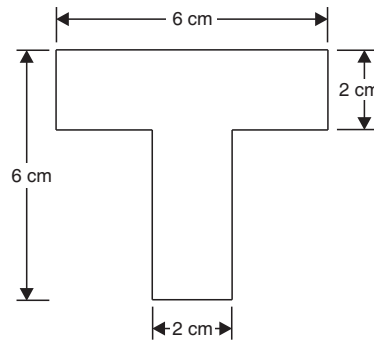
Fig. 8.19

Sol. Refer to Fig. 8.20.
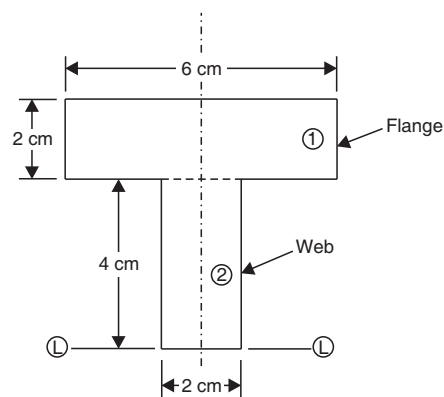
Fig. 8.20

Since, the section is symmetrical about *Y*-axis, therefore we shall find and $\bar{y}$ only, for which we have the following table :

Components	Area 'a' (cm²)	Centroidal distance 'y' from LL (cm)	ay (cm³)
Rectangle (1)	6 × 2 = 12	$4 + \dfrac{2}{2} = 5$	60
Rectangle (2)	4 × 2 = 8	$\dfrac{4}{2} = 2$	16
Total	$\Sigma a = 20$	—	$\Sigma ay = 76$

$$\therefore \qquad \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{76}{20} = 3.8 \text{ cm from } LL.$$

To find I_{xx} and I_{yy} using the theorem of parallel axes, we get

$$I_{xx} = \left[\frac{6 \times 2^3}{12} + 6 \times 2(5 - 3.8)^2 \right] + \left[\frac{2 \times 4^3}{12} + 4 \times 2 \times (3.8 - 2)^2 \right]$$

$$= (4 + 17.28) + (10.66 + 25.92)$$

$$= 21.28 + 36.58 = \textbf{57.86 cm}^4. \quad \textbf{(Ans.)}$$

$$I_{yy} = \left[\frac{2 \times 6^3}{12} + \frac{4 \times 2^3}{12} = 36 + 2.67 \right]$$

$$= \textbf{38.67 cm}^4. \quad \textbf{(Ans.)}$$

Example 8.3. *A T-beam is made up of two plates and two angles as shown in Fig. 8.21. Determine the moment of inertia of T-section about an axis passing through the centroid of the section and parallel to the top plate.*
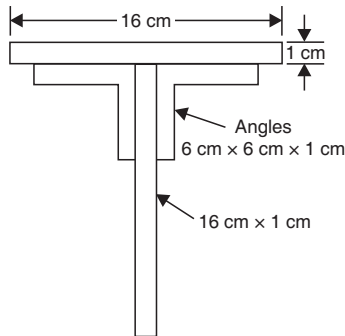
Fig. 8.21
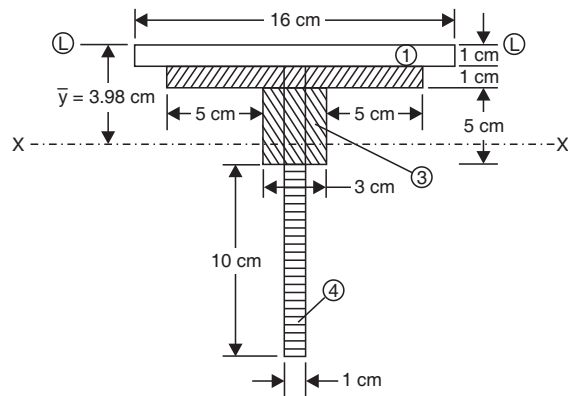
Fig. 8.22

Sol. Refer to Fig. 8.22.

The given section is divided into four rectangles as shown. Since the section is symmetrical about y-axis we shall find out $\bar{y}$ for which we have the follows table :

Components	Area 'a' (cm²)	Centroidal distance 'y' from LL (cm)	ay (cm³)
Rectangle (1)	$16 \times 1 = 16$	$\frac{1}{2} = 0.5$	8
Rectangle (2)	$13 \times 1 = 13$	$1 + \frac{1}{2} = 1.5$	19.5
Rectangle (3)	$3 \times 5 = 15$	$\frac{5}{2} + 1 + 1 = 4.5$	67.5
Rectangle (4)	$1 \times 10 = 10$	$\frac{10}{2} + 5 + 1 + 1 = 12$	120
Total	$\Sigma a = 54$	—	$\Sigma ay = 215$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{215}{54} = 3.98 \text{ cm from } LL$$

$$I_{xx} = ?$$

$$I_{xx} = \left[\frac{16 \times 1^3}{12} + 16 \times 1 \times (3.98 - 0.5)^2\right] + \left[\frac{13 \times 1^3}{12} + 13 \times 1 \times (3.98 - 1.5)^2\right]$$

$$+ \left[\frac{3 \times 5^3}{12} + 3 \times 5 \times (4.5 - 3.98)^2\right] + \left[\frac{1 \times 10^3}{12} + 10 \times 1 \times (12 - 3.98)^2\right]$$

$$= (1.33 + 193.76) + (1.08 + 79.95) + (31.25 + 4.056) + (83.33 + 643.2)$$

$$= 195.09 + 81.03 + 35.31 + 726.53$$

$$= \textbf{1037.96 cm}^4. \quad \textbf{(Ans.)}$$

Example 8.4. *A compound section is formed by riveting 200 mm × 10 mm flat plates, one on each flange of (ISL 300) 300 mm × 150 mm I-section girder. Find for the compound section :*

(*a*) I_{xx},   (*b*) I_{yy} *and*

(*c*) *The least radius of gyration.*

The properties of I-section are as follows :

    $I_{xx} = 7332.9\ cm^4,\ I_{yy} = 376.2\ cm^4,\ area = 48.08\ cm^2.$

Sol. Refer to Fig. 8.23.
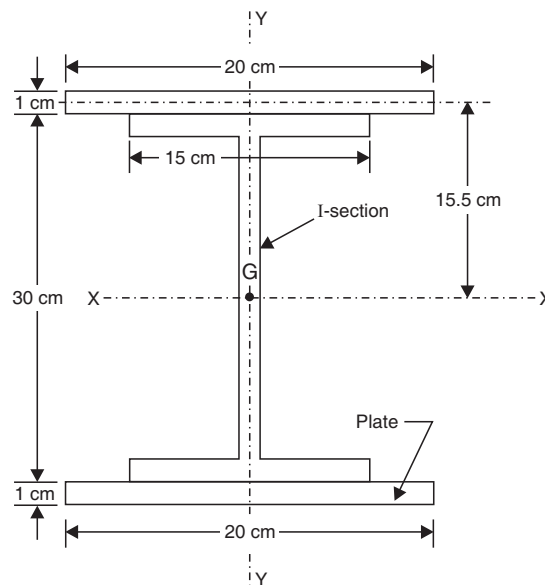
Fig. 8.23

Since, the section is symmetrical about both *xx* and *yy*-axes, therefore the c.g. of the whole section will lie at *G* as shown in Fig. 8.23.

(*a*) **Moment of inertia about *xx*-axis,**

$$I_{xx} = I_{xx} \text{ for } I\text{-section} + I_{xx} \text{ for two plates}$$

$$= 7332.9 + 2\left[\frac{20 \times 1^3}{12} + 20 \times 1 \times \left(15 + \frac{1}{2}\right)^2\right]$$

$$= 7332.9 + 2(1.67 + 4805)$$

$$= \textbf{16946.24 cm}^4. \quad \textbf{(Ans.)}$$

(*b*) **Moment of inertia about *yy*-axis,**

$$I_{yy} = I_{yy} \text{ for } I\text{-section} + I_{yy} \text{ for two plates}$$

$$= 376.2 + 2 \times \left(\frac{1 \times 20^3}{12} \right)$$

$$= \textbf{1709.53 cm}^4. \quad \textbf{(Ans.)}$$

(*c*) **Least value of radius of gyration,**

$$k_{least} = \sqrt{\frac{\text{Least value of moment of inertia}}{\text{Total area of the section}}}$$

$$= \sqrt{\frac{1709.53}{48.08 + 2 \times 20 \times 1}} = \sqrt{\frac{1709.53}{88.08}}$$

$$= \textbf{4.4 cm.} \quad \textbf{(Ans.)}$$

Example 8.5. *A fabricated girder system consists of two plates 30 cm × 1.5 cm and two ISLC 300 channels 30 cm × 10 cm set 10 cm apart as shown in Fig. 8.24. The properties of a 30 cm × 10 cm channel section are as follows :*

$I_{xx} = 6047.9$ cm^4, $I_{yy} = 346.0$ cm^4, area = 42.11 cm^2, axis yy from back of channel = 2.55 cm.

Calculate the value of moment of inertia and radii of gyration about the horizontal and vertical axes through the centroid of the combined section. Neglect the effect of rivets etc.

Sol. Refer to Fig. 8.24.
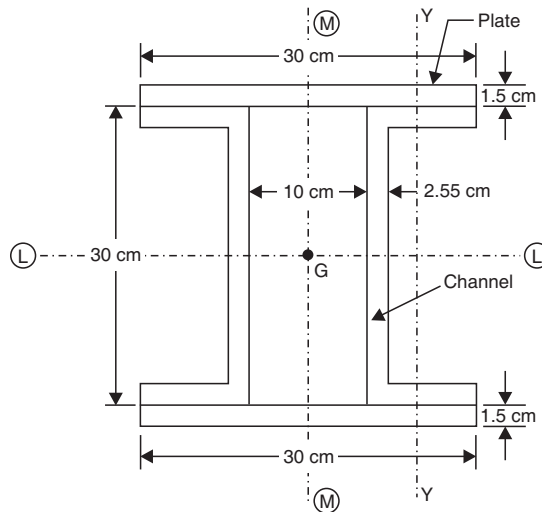
Fig. 8.24

Since, the section is symmetrical the centroid will lie on the intersection of two axes of symmetry, *LL* and *MM* as shown.

$$I_{ll} = I_{ll} \text{ or } I_{xx} \text{ for channels} + I_{ll} \text{ for plates}$$

$$= 2 \times 6047.9 + 2 \left[\frac{30 \times (1.5)^3}{2} + 30 \times 1.5 \times \left(15 + \frac{1.5}{2} \right)^2 \right]$$

$$= 12095.8 + 22342.5$$

$$= \textbf{34438.3 cm}^4. \quad \textbf{(Ans.)}$$

$$I_{mm} = I_{mm} \text{ for channels} + I_{mm} \text{ for plates}$$
$$= 2(I_{yy} \text{ for channels} + Ah^2) + I_{mm} \text{ for plates}$$

$$= 2\left[346 + 42.11\left(\frac{10}{2} + 2.55\right)^2\right] + 2 \times \frac{1.5 \times 30^3}{12}$$

$$= 5492.75 + 6750$$
$$= \mathbf{12242.75 \ cm^4.} \quad \textbf{(Ans.)}$$

Radius of gyration about *LL* axis,

$$k_{ll} = \sqrt{\frac{I_{ll}}{\text{area}}} = \sqrt{\frac{34438.3}{2 \times 42.11 + 2 \times 30 \times 1.5}}$$

$$= \mathbf{14.06 \ cm.} \quad \textbf{(Ans.)}$$

Radius of gyration about *MM* axis,

$$k_{mm} = \sqrt{\frac{I_{mm}}{\text{area}}} = \sqrt{\frac{12242.75}{2 \times 42.11 + 2 \times 30 \times 1.5}}$$

$$= \mathbf{8.38 \ cm.} \quad \textbf{(Ans.)}$$

Example 8.6. *A stanchion is built up of three ISMB 225 as shown in Fig. 8.25. Calculate second moment of area of the built-up-section about the principal axes LL and MM, and the radius of gyration.*

Sol. Refer to Fig. 8.25.
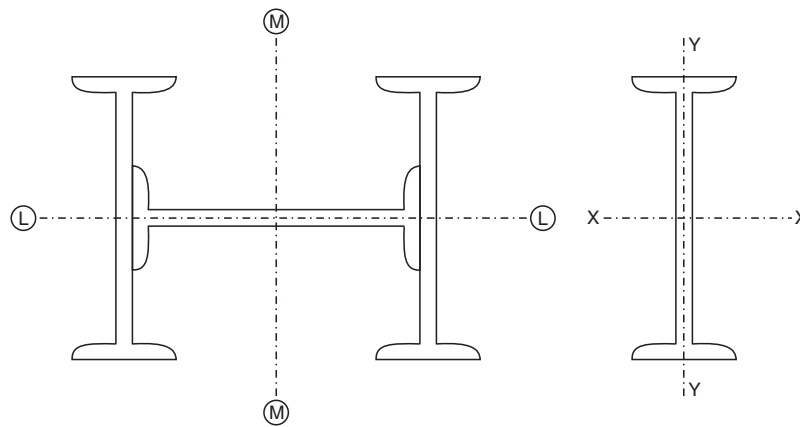
Fig. 8.25

From steel tables for one *ISMB 225*

Area, $a = 39.72 \ cm^2$
Web thickness $= 6.5 \ mm$
$I_{xx} = 3441.8 \ cm^4$
$I_{yy} = 218.3 \ cm^4$

Since, axis *LL* passes through the centroids of all the three beams,

$$I_{ll} = I_{xx} \text{ for two extreme beams} + I_{yy} \text{ for central beam}$$
$$= 2 \times 3441.8 + 218.3$$
$$= \mathbf{7101.9 \ cm^4.} \quad \textbf{(Ans.)}$$

Axis *MM* passes through the centroids of central beam but is parallel to axis *YY* of two extreme beams.

$\therefore$ $I_{mm} = I_{xx}$ for central beam + 2[I_{yy} + area × (distance between

axis *MM* and *YY* of extreme beam)^2]

$$= 3441.8 + 2 \left[218.3 + 39.72 \left(\frac{22.5}{2} + \frac{0.65}{2} \right)^2 \right]$$

$$= \textbf{14521.82 cm}^4. \quad \textbf{(Ans.)}$$

Least radius of gyration

$$= \sqrt{\frac{\text{least of } l_{ll} \text{ or } I_{mm}}{\text{area of compound beam}}} = \sqrt{\frac{7101.9}{3 \times 39.72}}$$

$$= \textbf{7.72 cm.} \quad \textbf{(Ans.)}$$

SELECTED QUESTIONS FROM EXAMINATION PAPERS

Example 8.7. *Find the moment of inertia and radius of gyration of the area shown in Fig. 8.26 about the axis AB.*

Sol. $a_1 = \frac{1}{2} \times 50 \times 15 = 375$ mm^2

$a_2 = 50 \times 20 = 1000$ mm^2

$a^3 = \frac{\pi \times 15^2}{2} = 353.43$ mm^2
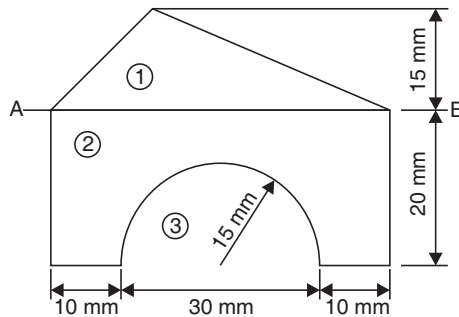
Fig. 8.26

M.O.I. about the axis AB :

$$I_{AB} = \left[\frac{50 \times 15^3}{36} + 375 \times \left(\frac{15}{3} \right)^2 \right] + \left[\frac{50 \times 20^3}{12} + 1000 \times 10^2 \right]$$

$$- \left[0.11 \times 15^4 \times 353.43 \times \left(20 - \frac{4 \times 15}{3\pi} \right)^2 \right]$$

$$= 10^4 [(0.469 + 0.937) + (3.333 + 10) - (0.557 + 6.569)]$$

$$= 7.613 \times 10^4 \text{ mm}^4$$

Hence, $I_{AB} = \textbf{7.613} \times \textbf{10}^4 \textbf{ mm}^4. \quad \textbf{(Ans.)}$

Example 8.8. *Find the centroidal moment of inertia of the shaded area shown in Fig. 8.27.*
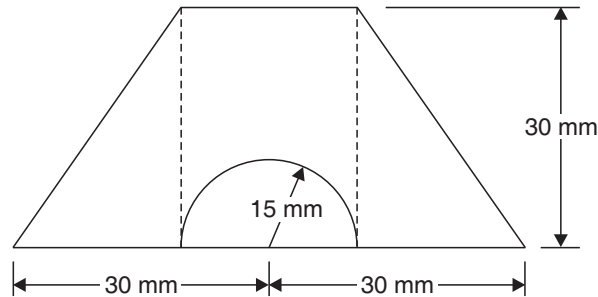
Fig. 8.27

Sol. Refer to Fig. 8.28.
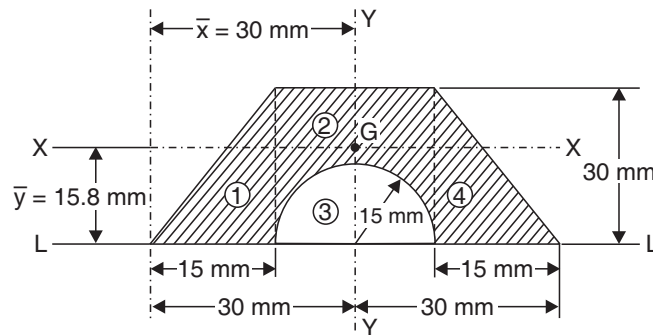
Fig. 8.28

Since the figure is symmetrical about YY-axis, therefore we shall find $\bar{y}$ for which we have the table given on the previous page.

I_{xx} :

$$I_{xx} = I_{xx_1} + I_{xx_2} - I_{xx_3} + I_{xx_4}$$

$$I_{xx} = \left[\frac{15 \times (30)^3}{36} + \frac{1}{2} \times 15 \times 30 \left(15.8 - \frac{30}{3} \right)^2 \right] + \left[\frac{30 \times 30^3}{12} + 30 \times 30 \times (15.8 - 15)^2 \right]$$

$$- \left[0.11 \times 15^4 + \frac{\pi \times 15^2}{2} \times \left(15.8 - \frac{4 \times 15}{3\pi} \right)^2 \right]$$

$$+ \left[\frac{15 \times 30^3}{36} + \frac{1}{2} \times 15 \times 30 \left(15.8 - \frac{30}{3} \right)^2 \right]$$

$$= 18819 + 68076 - 37023 + 18819$$

$$= 68691 \text{ mm}^4$$

i.e., $\mathbf{I_{xx} = 68691 \text{ mm}^4.}$ **(Ans.)**

$\mathbf{I_{yy} = ?}$

$$I_{yy} = I_{yy_1} + I_{yy_2} - I_{yy_3} + I_{yy_4}$$

$$I_{yy} = \left[\frac{30 \times 15^3}{36} + \frac{1}{2} \times 30 \times 15 \times \left(30 - \frac{2}{3} \times 15 \right)^2 \right]$$

$$+ \left[\frac{30 \times 30^3}{12} \right] - \left[\frac{\pi \times 30^4}{64 \times 2} \right]$$

$$+ \left[\frac{30 \times 15^3}{36} + \frac{1}{2} \times 30 \times 15 \times \left(30 - \frac{2}{3} \times 15 \right)^2 \right]$$

$$= 92812.5 + 67500 - 19880.4 + 92812.5$$

$$= 233244.6 \text{ mm}^4$$

i.e., $I_{yy} = 233244.6 \text{ mm}^4.$ **(Ans.)**

Example 8.9. *For the shaded area shown in Fig. 8.29, find the following :*

(*i*) *The position of the centroid*

(*ii*) *The second moment of area about the base*

(*iii*) *The radius of gyration about the base.*
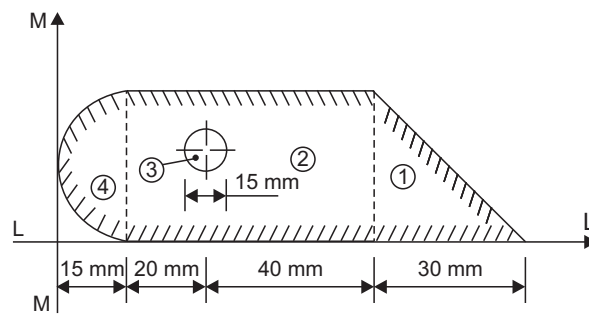
Fig. 8.29

Sol.

(*i*) **The position of the centroid :**

To determine the location of the centroid of the shaded area we have the table given below :

(*ii*) **The second moment of area (M.O.I.) about the base, I_{LL} :**

Component	Area 'a' (mm²)	Centroidal distance 'x' from MM (mm)	Centroidal distance 'y' from MM (mm)	ax (mm²)	ay (mm²)
Triangle (1)	$\frac{1}{2} \times 30 \times 30 = 450$	$15 + 20 + 40 + \frac{30}{3} = 85$	$\frac{30}{3} = 10$	38250	4500
Rectangle (2)	$60 \times 30 = 1800$	$15 + \frac{60}{2} = 45$	$\frac{30}{2} = 15$	81000	27000
Circle (3)	$\pi \times 7.5^2 = 176.71 \ (-)$	$15 + 20 = 35$	$\frac{30}{2} = 15$	6184.8 (–)	2650.6 (–)
Semicircle (4)	$\frac{\pi \times 15^2}{2} = 353.43$	$15 - \frac{4 \times 15}{3\pi} = 8.63$	$\frac{30}{2} = 15$	3050	5301.4
	$\Sigma a = 2426.72$			$\Sigma ax = 116115.2$	$\Sigma ay = 34150.8$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{116115.2}{2426.72} = \textbf{47.84 mm.} \quad \textbf{(Ans.)}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{34150.8}{2426.72} = \textbf{14.07 mm.} \quad \textbf{(Ans.)}$$

Applying parallel axes theorem :

$$I_{LL} = I_G + Ah^2$$

where I_G = M.O.I. about C.G.,

A = area of the lamina, and

h = distance between the C.G. and the reference line (*i.e., LL* in this case)

∴ M.O.I. about base *LL*,

$\textbf{I}_{LL}$ = M.O.I. of triangle (1) + M.O.I. of

Rectangle (2) – M.O.I. of *Circle* (3) + M.O.I. of *Semicircle* (4)

$$= \left(\frac{30 \times 30^3}{2}\right) + \left[\frac{60 \times 30^3}{12} + 60 \times 30 \times 15^2\right] - \left[\frac{\pi \times 15^4}{64} + \frac{\pi \times 15^2}{4} \times 15^2\right]$$

$$+ \left[\frac{\pi \times 30^4}{64 \times 2} + \frac{\pi \times 30^2}{8} \times 15^2\right]$$

$$= 67500 \times 540000 - 42245.8 + 99401.9$$

$$= 669656.1 \text{ mm}^4$$

$$= \textbf{664656.1 mm}^4. \quad \textbf{(Ans.)}$$

(*iii*) **The radius of gyration about the base, k$_{LL}$:**

$$I_{LL} = A\, k_{LL}{}^2$$

∴ $$k_{LL} = \sqrt{\frac{I_{LL}}{A}}$$

$$= \sqrt{\frac{664656.1}{2426.72}} = \textbf{16.55 mm.} \quad \textbf{(Ans.)}$$

Example 8.10. *Determine the moment of inertia of the shaded area about the edge LM as shown in Fig. 8.30.*
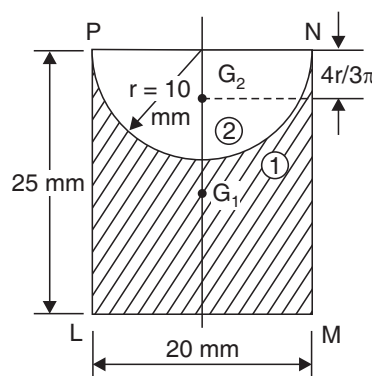
Fig. 8.30

Sol. I_{LM} :

I_{LM} = M.O.I. of *Rectangle* (1) − M.O.I. of *Semicircle* (2)

$$= \left[\frac{20 \times 25^3}{12} + 20 \times 25 \times \left(\frac{25}{2} \right)^2 \right] - \left[0.11 \times 10^4 + \frac{\pi \times 10^2}{2} \times \left(25 - \frac{4 \times 10}{3\pi} \right)^2 \right]$$

= 104166 − 68771

= **35395 mm⁴.   (Ans.)**

Example 8.11. *(i) With reference to the shaded area of Fig. 8.31 locate the centroid w.r.t. OL and OM.*

(ii) Determine the moment of inertia of the area about the horizontal axis passing through the centroid.
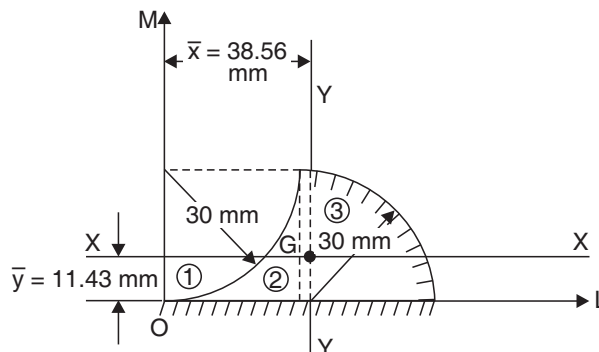
Fig. 8.31

Sol. (*i*) **Centroid of the area :**

For location of the centroid of the shaded area we have the following table.

Component	Area 'a' (m^2)	Centroidal distance 'x' from OM (mm)	Centroidal distance 'y' from OL (mm)	ax (mm^2)	ay (mm^2)
Quarter Circle (1)	$\frac{\pi \times 30^2}{4} = 706.85(-)$	$\frac{4 \times 30}{3\pi} = 12.73$	$30 - \frac{4 \times 30}{3\pi} = 17.27$	8998.2 (−)	12207.3 (−)
Square (2)	$30 \times 30 = 900$ (+)	$\frac{30}{2} = 15$	$\frac{30}{2} = 15$	13500	13500 (+)
Quarter Circle (3)	$\frac{\pi \times 30^2}{4} = 706.85$ (+)	$30 + \frac{4 \times 30}{3\pi} = 42.73$	$\frac{4 \times 30}{3\pi} = 12.73$	30203.7	8998.2 (+)
	$\Sigma a = 900$	—	—	$\Sigma ax = 34705.5$	$\Sigma ay = 10290.9$

∴ $\bar{x} = \dfrac{\Sigma ax}{\Sigma a} = \dfrac{34705.5}{900} = \textbf{38.56 mm.   (Ans.)}$

$\bar{y} = \dfrac{\Sigma ay}{\Sigma a} = \dfrac{10290.9}{900} = \textbf{11.43 mm.   (Ans.)}$

(*ii*) **M.O.I. about XX axis :**

$$I_{xx} = -I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$= -\left[0.055 \times 30^4 + \frac{\pi \times 30^2}{4} \times \left(18.57 - \frac{4 \times 30}{3\pi}\right)^2\right]$$

$$+ \left[\frac{30 \times (30)^3}{12} + 30 \times 30 \times (15 - 11.43)^2\right]$$

$$+ \left[0.055 \times 30^4 + \frac{\pi \times 30^2}{4} \times \left(\frac{4 \times 30}{3\pi} - 11.43\right)^2\right]$$

$$- [44550 + 24091] + [67500 + 11470] + [44550 + 1198]$$

$$= -68641 + 78970 + 45748 = 56077 \text{ mm}^4$$

Hence, $\qquad$ **I_{xx} = 56077 mm^4. (Ans.)**

<div align="center">

HIGHLIGHTS

</div>

1. The cumulative product of area and square of its distance from an axis is called the *moment of inertia* of a section about that axis.

2. *Moment of inertia* of simple areas :

   Rectangle : $\qquad\qquad\qquad\qquad I_{xx} = \dfrac{bd^3}{12}$

   $\qquad\qquad\qquad\qquad\qquad\qquad I_{yy} = \dfrac{db^3}{12}$

   Triangle : $\qquad\qquad\qquad\qquad I_{xx} = \dfrac{bh^3}{36}$

   About the base : $\qquad\qquad\qquad = \dfrac{bh^3}{12}$

   Circle : $\qquad\qquad\qquad\qquad\quad I_{yy} = \dfrac{\pi d^4}{64}$

   Semicircle : $\qquad\qquad\qquad\quad I_{xx} = 0.11r^4$

   $\qquad\qquad\qquad\qquad\qquad\qquad I_{yy} = \dfrac{\pi d^4}{128}$

   Quadrant : $\qquad\qquad\qquad\qquad I_{xx} = 0.055r^4.$

3. *Theorem of parallel axis :* It states that *moment of inertia of a lamina about any axis in the plane of lamina equals the sum of moment of inertia about a parallel centroidal axis in the plane of lamina and the product of area of the lamina and square of the distance between the two axes.*

4. *Theorem of perpendicular axes :* It states that *if I_{ox} and I_{oy} be the moment of inertia of a lamina about mutually perpendicular axes OX and OY in the plane of the lamina and I_{oz} be the moment of inertia of the lamina about an axis (OZ) normal to the lamina and passing through the point of intersection of the axes OX and OX, then*

$$I_{oz} = I_{ox} + I_{oy}.$$

5. *Radius of gyration of the section* (k_i) :

$$k_i = \sqrt{\frac{I_i}{A}}$$

where I_i = moment of inertia about its axis, and

A = area of cross-section.

OBJECTIVE TYPE QUESTIONS

Fill in the blanks :

1. The cumulative product of area and square of its distance from an axis is called the ...... of a section about that axis.

2. M.O.I. of a circle = ... where d is the diameter of the circle.

3. M.O.I. of a triangle about an axis passing through its c.g. = ..., where b and h are the base and height of the triangle respectively.

4. Moment of inertia (M.O.I.) of a quadrant about axis XX (passing through c.g.) = ..., where R is radius of the quadrant.

Answers

1.   M.O.I.   **2.** $\dfrac{\pi d^4}{64}$ **3.** $\dfrac{bh^3}{36}$ **4.** $0.055R^4$.

EXERCISES

Theoretical Questions

1. Define the term "Moment of inertia".

2. State and explain "Theorem of parallel axes".

3. Enunciate "Theorem of perpendicular axes".

4. Derive expressions for M.O.I. of the following :

(*i*) Rectangular lamina   (*ii*) Circular lamina

(*iii*) Semicircular lamina   (*iv*) Thin ring.

5. What is "Radius of gyration" ?

Unsolved Examples

1. Find I_{xx} and I_{yy} for the unequal angle section 12.5 × 9.5 × 1 cm shown in Fig. 8.32.

[**Ans.** I_{xx} = 330.8 cm⁴, I_{yy} = 161.3 cm⁴]
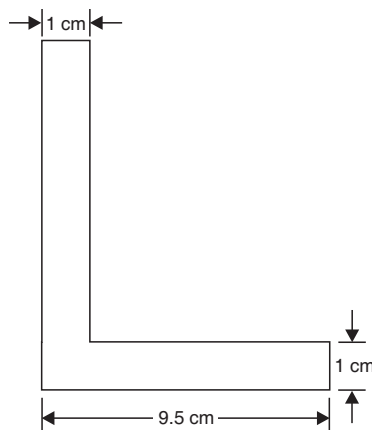
Fig. 8.32

2. Find the moment of inertia about the horizontal axis through the c.g. of the section shown in Fig. 8.33.

[**Ans.** I_{xx} = 590.4 cm⁴]
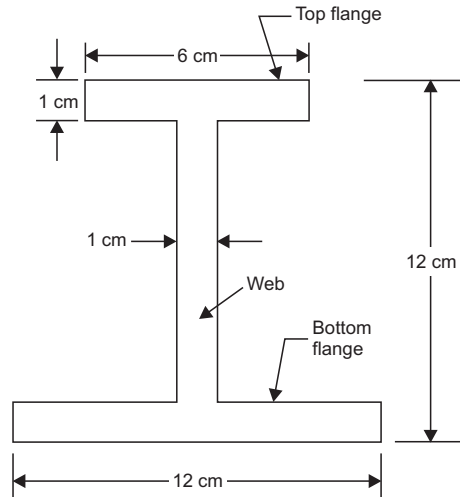
Fig. 8.33

3. Determine the moment of inertia of a *T*-section (as shown in Fig. 8.34) 10.16 × 10.16 × 1.27 cm about an axis passing through the centre of the section and perpendicular to the stem or vertical leg.

[**Ans.** I_{xx} = 231.4 cm⁴]
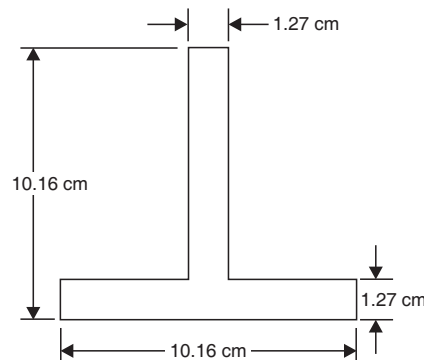
Fig. 8.34

4. Calculate the second moment of area of the built up section shown in Fig. 8.35 about *XX* and *YY*, *XX* and *YY* are the axes passing through the centroid of the section.   [**Ans.** I_{xx} = 1152.35 cm⁴, I_{yy} = 6463 cm⁴]
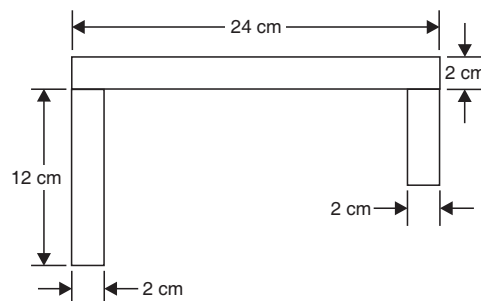
Fig. 8.35

5. Determine the moment of inertia of the Z-section in Fig. 8.36 about *XX* and *YY* axes, where *XX* and *YY* are the axes passing through the centroid of the figure.   [**Ans.** I_{xx} = 1548 cm^4, I_{yy} = 2668 cm^4]
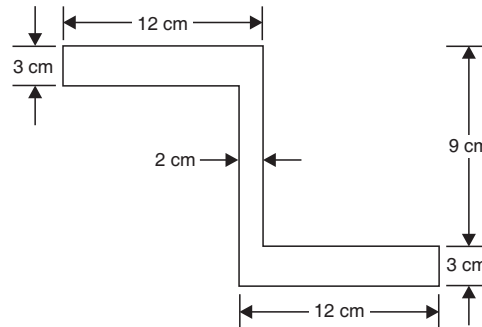
Fig. 8.36

6. Find the moment of inertia about the centroidal axis *XX* for the lamina shown in Fig. 8.37.

[**Ans.** I_{xx} = 4015.38 cm^4]
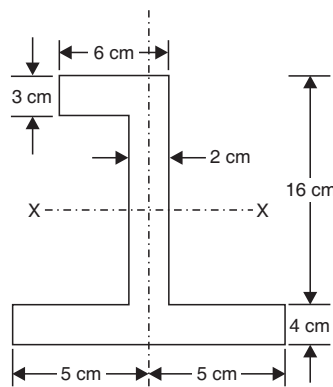
Fig. 8.37

7. Find I_{xx} of the channel section shown in Fig. 8.38.   [**Ans.** I_{xx} = 6300 cm^4]
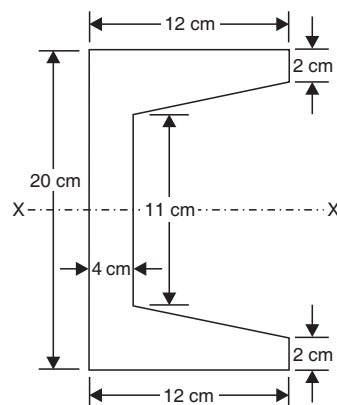
Fig. 8.38

8. Find the centroidal axis of the lamina shown in Fig. 8.39 parallel to the base. Find also the moment of inertia about this centroidal axis. [**Ans.** 4.08 cm ; 385.33 cm^4]
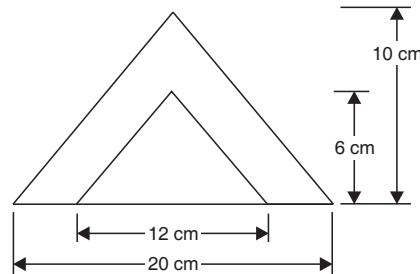
Fig. 8.39

9. A channel section $16 \times 8 \times 2$ cm stands with flanges horizontal. Determine I_{xx} and I_{yy} passing through the centroid of the section. [**Ans.** I_{xx} = 1863.7 cm^4 ; I_{yy} = 302 cm^4]

10. A short column is built by riveting flanges of T-sections of size 30 mm × 60 mm × 5 mm to form across of 60 mm × 60 mm. Determine I_{xx}, I_{yy} and the least radius of gyration square. Neglect the effect of rivets.

[**Ans.** I_{xx} = 9.458 cm^4 ; I_{yy} = 18.052 cm^4 ; k^2_{least} = 1.113 cm^2]

11. A steel stanchion is built up of two rolled steel joists *ISJB* 150 placed 10 cm apart from centre to centre of webs, united by plates 15 cm wide and 1 cm thick, one on each side of the top and bottom flanges. If for *ISJB* 150 *RSJ* I_{xx} = 322.1 cm^4, I_{yy} = 9.2 cm^4 and area A = 9.01 cm^2, find the moment of inertia about (*a*) horizontal axis *LL* passing through the centroid ; and (*b*) vertical axis *MM*, which also passes through the centroid of the section. [**Ans.** I_{LL} = 2566.7 cm^4 ; I_{mm} = 1031 cm^4]

12. Calculate the moment of inertia of the built up section shown in Fig. 8.40 about the centroidal axis *XX*.

[**Ans.** I_{xx} = 336995.12 cm^4]
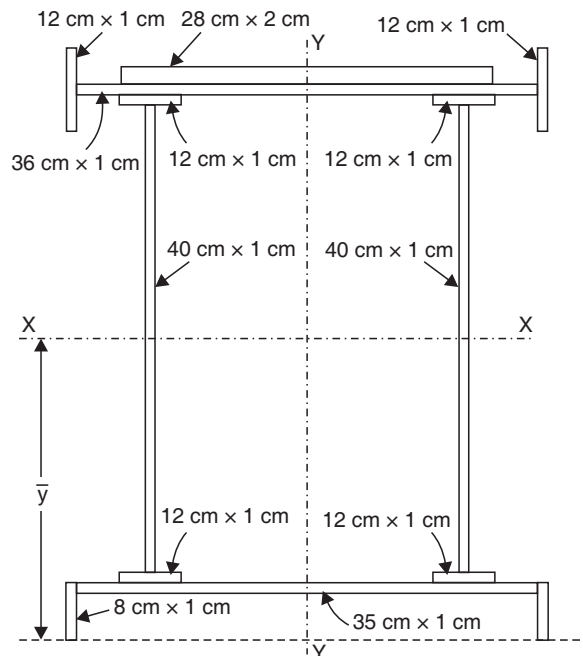
Fig. 8.40

ADDITIONAL TYPICAL WORKED EXAMPLES

ADDITIONAL TYPICAL WORKED EXAMPLES
First Law of Thermodynamics and Entropy

Example A-1. *Derive the equation of state for a perfect gas.*

Solution. To derive the equation of state for a perfect gas let us consider a *unit mass* of a perfect gas to change its state in the following two successive processes (Fig. 1) :

(*i*) Process 1–2′ at constant pressure, and

(*ii*) Process 2′–2 at constant temperature.

For the *process 1–2′*, applying Charle's law, we get

$$\frac{v_1}{T_1} = \frac{v_2{}'}{T_2{}'}$$

and, since $T_2{}' = T_2$, we may write

$$\frac{v_1}{T_1} = \frac{v_2{}'}{T_2} \qquad \qquad ...(i)$$

For the *process 2′–2,* using Boyle's law, we have

$$p_2{}'v_2{}' = p_2 v_2$$

and, since

$$p_2{}' = p_1$$

$$p_1 v_2{}' = p_2 v_2$$

i.e.,

$$v_2{}' = \frac{p_2 v_2}{p_1} \qquad \qquad ...(ii)$$

Fig. 1. Formulation of equation of state of a perfect gas.

Substituting the value of $v_2{}'$ from eqn. (*ii*) in eqn. (*i*), we get

$$\frac{v_1}{T_1} = \frac{p_2 v_2}{p_1 T_2} \quad \text{or} \quad \frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

i.e.,

$$\frac{pv}{T} = \text{constant} \qquad \qquad ...(1)$$

The magnitude of this constant depends upon the particular gas and it is denoted by R, where R is called the **specific gas constant.** Then

$$\frac{pv}{T} = R$$

The equation of the state for a perfect gas is thus given by the equation

$$pv = RT \qquad \qquad ...(2)$$

or for m kg, occupying V m³,

$$pV = mRT \qquad \qquad ...(3)$$

If the mass is chosen to be numerically equal to the molecular weight of the gas then 1 mole of the gas has been considered, *i.e.*, 1 kg mole of oxygen is 32 kg oxygen, or 1 kg mole of hydrogen is 2 kg hydrogen.

The equation may be written as

$$pV_0 = MRT \qquad \qquad ...(4)$$

where, V_0 = Molar volume, and

M = Molecular weight of the gas.

Avogadro discovered that V_0 is the same for all gases at the same pressure and temperature and therefore it may be seen that MR = a constant ; R_0 and thus

$$pV_0 = R_0 T \qquad \qquad \qquad ...(5)$$

R_0 is called the **molar or universal gas constant** and its value is **8.3143 kJ/kg mol K.**

If there are n moles present then the ideal gas equation may be written as

$$pV = nR_0 T \qquad \qquad \qquad ...(6)$$

where, V is the volume occupied by n moles at pressure p and temperature T.

Example A-2. *0.1 m^3 of an ideal gas at 300 K and 1 bar is compressed adiabatically to 8 bar. It is then cooled at constant volume and further expanded isothermally so as to reach the condition from where it started. Calculate :*

  (*i*) *Pressure at the end of constant volume cooling.*

 (*ii*) *Change in internal energy during constant volume process.*

(*iii*) *Net work done and heat transferred during the cycle. Assume*

C_p = *14.3 kJ / kg K and c_v = 10.2 kJ / kg K.*

Solution. *Given :* V_1 = 0.1 m³ ; T_1 = 300 K ; p_1 = 1 bar ; C_p = 14.3 kJ/kg K ; C_v = 10.2 kJ/kg K. Refer to Fig. 2.

(*i*) **Pressure at the end of constant volume cooling, p_3 :**

$$\gamma = \frac{c_p}{c_v} = \frac{14.3}{10.2} = 1.402$$

Characteristic gas constant,

$$R = c_p - c_v = 14.3 - 10.2 = 4.1 \text{ kJ/kg K}$$

Considering *process 1-2*, we have :

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

$$V_2 = V_1 \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = 0.1 \times \left(\frac{1}{8}\right)^{\frac{1}{1.402}} = 0.0227 \text{ m}^3$$

Also, $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\dfrac{8}{1}\right)^{\frac{1.402-1}{1.402}} = 1.815$

Fig. 2

or $T_2 = T_1 \times 1.815 = 300 \times 1.815 = 544.5$ K

Considering *process 3–1*, we have

$$p_3 V_3 = p_1 V_1$$

$\therefore$ $p_3 = \dfrac{p_1 V_1}{V_3} = \dfrac{1 \times 0.1}{0.0227} = \textbf{4.4 bar.}$ **(Ans.)** $(\because \; V_3 = V_2)$

(*ii*) **Change in internal energy during constant volume process, ($U_3 - U_2$) :**

Mass of gas, $m = \dfrac{p_1 V_1}{R T_1} = \dfrac{(1 \times 10^5) \times 0.1}{(4.1 \times 1000) \times 300} = 0.00813$ kg

$\therefore$ Change in internal energy during *constant volume process 2–3,*

$$U_3 - U_2 = m c_v (T_3 - T_2)$$
$$= 0.00813 \times 10.2 \,(300 - 544.5) \qquad \qquad (\because \; T_3 = T_1)$$
$$= \textbf{– 20.27 kJ.} \quad \textbf{(Ans.)}$$

(–ve sign means *decrease* in internal energy)
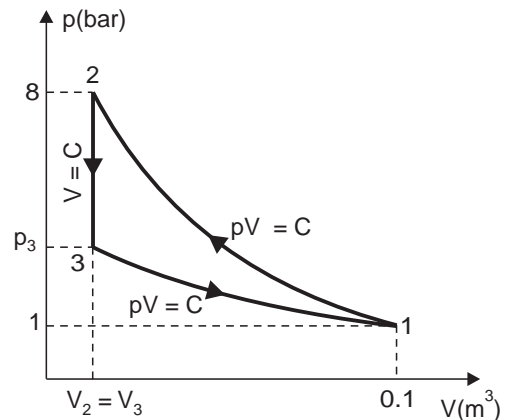
● During constant volume cooling process, temperature and hence internal energy is *reduced*. This *decrease in internal energy equals to heat flow to surroundings since work done is zero.*

(*iii*) **Net work done and heat transferred during the cycle :**

$$W_{1-2} = \frac{p_1V_1 - p_2V_2}{\gamma - 1} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{0.00813 \times 4.1(300 - 544.5)}{1.402 - 1} = -20.27 \text{ kJ}$$

$$W_{2-3} = 0 \qquad\qquad\qquad \text{... since volume remains constant}$$

$$W_{3-1} = p_3V_3 \log_e\left(\frac{V_1}{V_3}\right) = p_1V_1 \log_e\left(\frac{p_3}{p_1}\right)$$

$$= (1 \times 10^5) \times 0.1 \times \log_e\left(\frac{4.4}{1}\right) \qquad (\because \quad p_3V_3 = p_1V_1)$$

$$= 14816 \text{ Nm (or J) or } 14.82 \text{ kJ}$$

∴   **Net work done** $= W_{1-2} + W_{2-3} + W_{3-1}$

$$= (-20.27) + 0 + 14.82 = \textbf{-5.45 kJ}$$

–ve sign indicates that work has been done *on the system.*   **(Ans.)**

For a cyclic process : $\oint \delta Q = \oint \delta W$

∴   Heat transferred during the complete cycle = **– 5.45 kJ**

–ve sign means heat has been *rejected i.e., lost from the system.*   **(Ans.)**

Example A-3. *0.15 m³ of an ideal gas at a pressure of 15 bar and 550 K is expanded isothermally to 4 times the initial volume. It is then cooled to 290 K at constant volume and then compressed back polytropically to its initial state.*

Calculate the net work done and heat transferred during the cycle.

Solution. *Given :* $V_1 = 0.15 \text{ m}^3$; $p_1 = 15 \text{ bar}$; $T_1 = T_2 = 550 \text{ K}$; $\dfrac{V_2}{V_1} = 4$; $T_3 = 290 \text{ K}$

Refer to Fig. 3.

Considering the *isothermal process* 1–2, we have

$$p_1V_1 = p_2V_2 \quad \text{or} \quad p_2 = \frac{p_1V_1}{V_2}$$

or

$$p_2 = \frac{15 \times 0.15}{(4 \times 0.15)} = 3.75 \text{ bar}$$

Work done, $W_{1-2} = p_1V_1 \log_e\left(\dfrac{V_2}{V_1}\right)$

$$= (15 \times 10^5) \times 0.15 \times \log_e(4)$$

$$= 311916 \text{ J} = 311.9 \text{ kJ}$$

Considering *constant volume process 2–3*, we get

$$V_2 = V_3 = 4 \times 0.15 = 0.6 \text{ m}^3$$
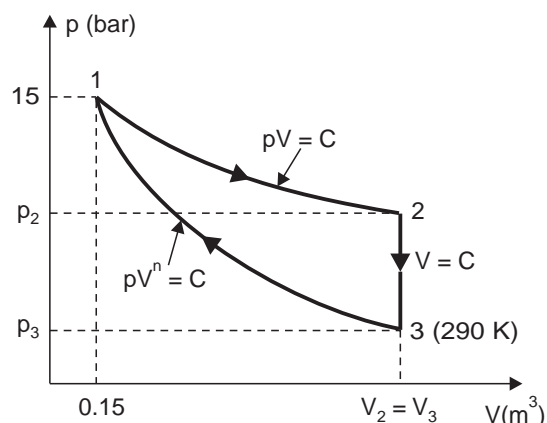
Fig. 3

$$\frac{p_2}{T_2} = \frac{p_3}{T_3} \quad \text{or} \quad p_3 = p_2 \times \frac{T_3}{T_2} = 3.75 \times \frac{290}{550} = 1.98 \text{ bar}$$

$$W_{2-3} = 0 \qquad \text{... since volume remains constant}$$

Consider *polytropic process 3–1* :

$$p_3 V_3{}^n = p_1 V_1{}^n \quad \text{or} \quad \frac{p_1}{p_3} = \left(\frac{V_3}{V_1}\right)^n$$

Taking log on both sides, we get

$$\log_e (p_1/p_3) = n \log_e (V_3/V_1)$$

or
$$n = \frac{\log_e(p_1/p_3)}{\log_e(V_3/V_1)} = \frac{\log_e (15/1.98)}{\log_e (4)} = 1.46$$

$$W_{3-1} = \frac{p_3 V_3 - p_1 V_1}{n - 1} = \frac{1.98 \times 10^5 \times 0.6 - 15 \times 10^5 \times 0.15}{(1.46 - 1)}$$

$$= -230869 \text{ J} \quad \text{or} \quad -230.87 \text{ kJ}$$

∴ *Net work done* $= W_{1-2} + W_{2-3} + W_{3-1}$

$$= 311.9 + 0 + (-230.87) = \textbf{81.03 kJ.} \quad \textbf{(Ans.)}$$

For a cyclic process, $\oint \delta Q = \oint \delta W$

∴   Heat *transferred during the cycle* = **81.03 kJ.**   **(Ans.)**

Example A-4. *A system consisting of 1 kg of an ideal gas at 5 bar pressure and 0.02 m³ volume executes a cyclic process comprising the following three distinct operations :* (*i*) *Reversible expansion to 0.08 m³ volume 1.5 bar pressure, presuming pressure to be a linear function of volume* (*p = a + bV*), (*ii*) *Reversible cooling at constant pressure and* (*iii*) *Reversible hyperbolic compression according to law pV = constant. This brings the gas back to initial conditions.*

(*i*) *Sketch the cycle on p-V diagram.*

(*ii*) *Calculate the work done in each process starting whether it is done on or by the system and evaluate the net cyclic work and heat transfer.*

Solution. *Given :* $m = 1$ kg ; $p_1 = 5$ bar ; $V_1 = 0.02$ m³ ; $V_2 = 0.08$ m³ ; $p_2 = 1.5$ bar.

(*i*) **p-V diagram :** *p-V* diagram of the cycle is shown in Fig. 4.

(*ii*) **Work done and heat transfer :**

● *Process 1-2* (*Linear law*) :

$$p = a + bV \qquad \qquad \text{...(Given)}$$

The values of constants *a* and *b* can be determined from the values of pressure and volume at the state points 1 and 2.

$$5 = a + 0.02b \qquad \qquad \text{...(i)}$$
$$1.5 = a + 0.08b \qquad \qquad \text{...(ii)}$$

From (*i*) and (*ii*), we get, $b = -58.33$ and $a = 6.167$

$$W_{1-2} = \int_1^2 p\,dV = \int_1^2 (a + bV)\,dV$$

$$= \int_1^2 (6.167 - 58.33V)\,dV$$

$$= 10^5 \left| 6.167\,V - 58.33 \times \frac{V^2}{2} \right|_{0.02}^{0.08}$$
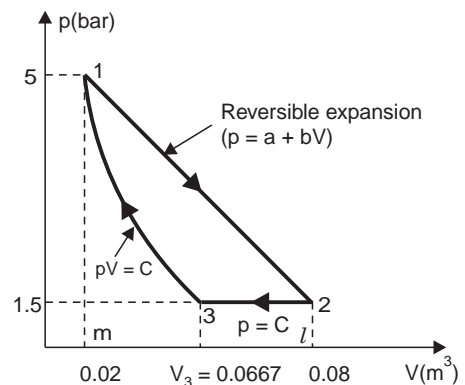
Fig. 4. *p-V* diagram.

$$= 10^5 \left| 6.167\,(0.08 - 0.02) - 58.33 \times \frac{(0.08^2 - 0.02^2)}{2} \right| \times 10^{-3}\,kJ = \textbf{19.5 kJ}$$

This is work done **by the system.   (Ans.)**

$$\left[\begin{array}{l} \textit{Alternatively :} \quad W_{1-2} = \text{area under the process line 1–2} \\ \qquad\qquad\qquad = \text{area of trapezium 1–2–}l\text{-}m \\ \qquad\qquad\qquad = \left[\dfrac{5 + 1.5}{2} \times 10^5 \right] \times (0.08 - 0.02) = 19.5\,kJ \end{array} \right]$$

● *Process 2 – 3* (*constant pressure*) :

$$p_3 = p_2 = 1.5\ \text{bar}$$

The volume V_3 can be worked out from the hyperbolic compression 3–1, as follows :

$$p_1 V_1 = p_3 V_3 \quad \text{or} \quad V_3 = \frac{p_1 V_1}{p_3} = \frac{5 \times 0.02}{1.5} = 0.0667\ m^3$$

∴ $\qquad\qquad W_{2-3} = p_2(V_3 - V_2) = 1.5 \times 10^5\,(0.0667 - 0.08) \times 10^{-3}\,kJ = \textbf{– 1.995 kJ}$

● *Process 3 – 1* (*hyperbolic process*) :

$$W_{3-1} = p_3 V_3 \log_e \left(\frac{V_1}{V_3} \right)$$

$$= (10^5 \times 1.5) \times 0.0667 \log_e \left(\frac{0.02}{0.0667} \right) \times 10^{-3}\,kJ = \textbf{– 12.05 kJ.}$$

This is the work **done on the system.   (Ans.)**

Net work done, $\qquad W_{\text{net}} = W_{1-2} + W_{2-3} + W_{3-1}$
$$\qquad\qquad\qquad = 19.5 + (-1.995) + (-12.05) = \textbf{5.445 kJ.   (Ans.)}$$

Heat transferred during the complete cycle, $\oint \delta Q = \oint \delta W = \textbf{5.455 kJ.   (Ans.)}$

Example A-5. *5 m^3 of air at 2 bar, 27°C is compressed up to 6 bar pressure following $pv^{1.3} = $ constant. It is subsequently expanded adiabatically to 2 bar. Considering the two processes to be reversible, determine the net work. Also plot the processes on T-S diagram.*

Solution. Refer to Fig. 5.

Given : $\quad V_1 = 5\ m^3$; $\quad p_1 = 2\ \text{bar}$; $\quad T_1 = 27 + 273 = 300\ K$; $\quad p_2 = 6\ \text{bar}$; $\quad p_3 = 2\ \text{bar}$

Net work :
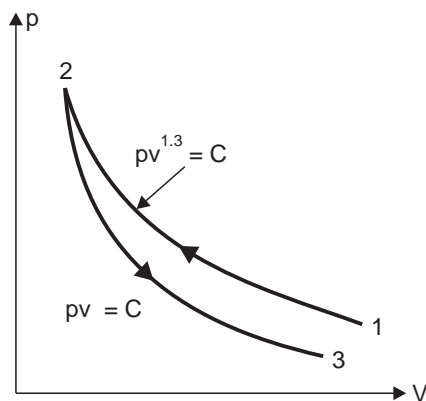
Fig. 5. *p-V* diagram.
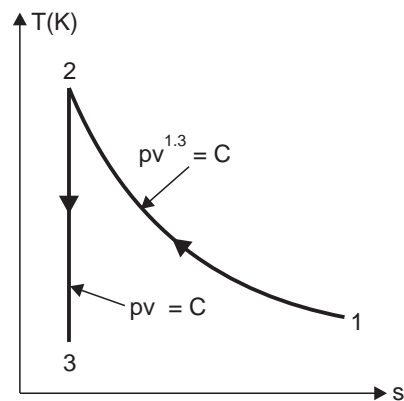
Fig. 6. *T-s* diagram.

Mass of air, $\quad m = \dfrac{p_1 V_1}{R T_1} = \dfrac{2 \times 10^5 \times 5}{287 \times 300} = 11.61$ kg.

Considering polytropic compression process 1-2, we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \quad \text{or} \quad \frac{T_2}{300} = \left(\frac{6}{2}\right)^{\frac{1.3-1}{1.3}} \quad \text{or} \quad T_2 = 386.5 \text{ K}.$$

Considering isentropic process 2-3, we get

$$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{6}{2}\right)^{\frac{1.4-1}{1.4}} = 1.369 \qquad\qquad (\because \quad p_3 = p_1)$$

$\therefore \qquad\qquad T_3 = \dfrac{T_2}{1.369} = \dfrac{386.5}{1.369} = 282.3$ K

Now, work done during polytropic *compression* 1-2,

$$W_{1-2} = \frac{mR(T_1 - T_2)}{n-1} = \frac{11.61 \times 0.287(300 - 386.5)}{1.3 - 1} = -960.7 \text{ kJ}$$

and, work done during *adiabatic expansion* 2-3

$$W_{2-3} = \frac{mR(T_2 - T_3)}{\gamma - 1} = \frac{11.61 \times 0.287(386.5 - 282.3)}{1.4 - 1} = 868 \text{ kJ}$$

$\therefore \quad$ Net work done $= W_{1-2} + W_{2-3} = -960.7 + 868 = -92.7$ kJ

Hence **net work done on the air = 92.7 kJ. (Ans.)**

The process plotted on *T-s* diagram is shown in Fig. 6.

Example A-6. *A rigid cylinder containing 0.004 m^3 of nitrogen at 1 bar and 300 K is heated reversibly until temperature becomes 400 K. Determine :*

 (*i*) *The heat supplied.*

(*ii*) *The entropy change.*

Assume nitrogen to be perfect gas (molecular mass = 28) and take γ = 1.4.

Solution. *Given :* $V_1 = 0.004$ m^3 ; $p_1 = 1$ bar ; $T_1 = 300$ K ; $T_2 = 400$ K ; *M* for $N_2 = 28$; $\gamma = 1.4$.

 (*i*) **The heat supplied :**

Gas constant $R \quad = \dfrac{R_0 \text{ (Universal gas constant)}}{M \text{ (Molecular mass)}} = \dfrac{8.314}{28} = 0.297$ kg/kg K

Mass, $m \qquad = \dfrac{p_1 V_1}{R T_1} = \dfrac{(1 \times 10^5) \times 0.004}{(0.297 \times 1000) \times 300} = 0.00449$ kg

$c_v = \dfrac{R}{\gamma - 1} = \dfrac{0.297}{1.4 - 1} = 0.742$ kJ/kg K

$\therefore \quad$ Heat supplied $\quad = m \, c_v (T_2 - T_1)$

$\qquad\qquad\qquad\quad = 0.00449 \times 0.742(400 - 300) = \mathbf{0.333}$ **kJ. (Ans.)**

(*ii*) **The entropy change :**

The entropy change, $S_2 - S_1 = m \, c_v \log_e \left(\dfrac{T_2}{T_1}\right)$

$$= 0.00449 \times 0.742 \times \log_e \left(\frac{400}{300}\right) = \mathbf{9.584 \times 10^{-4}} \text{ **kJ/kg K.** (Ans.)}$$

Example A-7. *A piston-cylinder arrangement contains 0.05 m³ of nitrogen at 1 bar and 280 K. The piston moves inwards and the gas is compressed isothermally and reversibly until the pressure becomes 5 bar. Determine :*

(*i*) *Change in entropy.*   (*ii*) *Work done.*

Assume nitrogen to be a perfect gas.

Solution. *Given :* $V_1 = 0.05$ m³ ; $p_1 = 1$ bar ; $T_1 = 280$ K ; $p_2 = 5$ bar.

(*i*) **Change in entropy, $(S_2 - S_1)$:**

Gas constant, $R = \dfrac{R_0}{M} = \dfrac{8.314}{28} = 0.297$ kJ/kg K

Mass of the gas, $m = \dfrac{p_1 V_1}{R T_1} = \dfrac{(1 \times 10^5) \times 0.05}{(0.297 \times 1000) \times 280} = 0.06$ kg

∴   Change in entropy $S_2 - S_1 = mR \log_e \left(\dfrac{p_1}{p_2} \right)$

$$= 0.06 \times 0.297 \log_e \left(\dfrac{1}{5} \right) = \mathbf{-\,0.0287\ kJ/K.}\quad \textbf{(Ans.)}$$

(*ii*) **Work done :**

Heat interaction, $Q = T(S_2 - S_1)$
$$= 280 \times (-\,0.0287) = -\,8.036\ \text{kJ}$$

∴   Work done, $W = Q = \mathbf{-\,8.036\ kJ.}\quad \textbf{(Ans.)}$ (∵   In its other process, $W = Q$)

$$\left[\begin{array}{l} Alternatively : W = p_1 V_1 \log_e \left(\dfrac{V_2}{V_1} \right) = p_1 V_1 \log_e \left(\dfrac{p_1}{p_2} \right) \\[2mm] \qquad\qquad = 1 \times 10^5 \times 0.05 \times \log_e \left(\dfrac{1}{5} \right) \times 10^{-3}\ \text{kJ} = 8.04\ \text{kJ} \end{array} \right]$$

Example A-8. *1 kg of air initially at 8 bar pressure and 380 K expands polytropically* $(pv^{1.2} = constant)$ *until the pressure is reduced to one-fifth value. Calculate :*

(*i*) *Final specific volume and temperature.*

(*ii*) *Change of internal energy, work done and heat interaction.*

(*iii*) *Change in entropy.*

Take :   R = 0.287 kJ / kg K and γ = 1.4.

Solution. *Given :* $m = 1$ kg ; $p_1 = 8$ bar ; $T_1 = 380$ K ; Law of expansion : $pv^{1.2} = $ constant ;

$$p_2 = \dfrac{p_1}{5} = \dfrac{8}{5} = 1.6\ \text{bar} ; R = 0.287\ \text{kJ/kg K} ; \gamma = 0.4.$$

(*i*) **Final specific volume and temperature, v_2, T_2 :**

$$p_1 v_1 = R T_1$$

or $v_1 = \dfrac{R T_1}{p_1} = \dfrac{(0.287 \times 10^3) \times 380}{8 \times 10^5} = \mathbf{0.1363\ m^3/kg.}\quad \textbf{(Ans.)}$

Also, $p_1 v_1{}^n = p_2 v_2{}^n$ or $v_2 = v_1 \times \left(\dfrac{p_1}{p_2} \right)^{\frac{1}{n}}$

∴ $v_2 = 0.1363 \times \left(\dfrac{8}{1.6} \right)^{\frac{1}{1.2}} = \mathbf{0.5211\ m^3/kg.}\quad \textbf{(Ans.)}$

Again, $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1} \right)^{\frac{n-1}{n}}$ or $\dfrac{T_2}{380} = \left(\dfrac{1}{5} \right)^{\frac{1.2-1}{1.2}}$

$$\therefore \qquad\qquad T_2 = 290.6 \text{ K}$$

$$\left[\textbf{Alternatively :} \ T_2 = \frac{p_2 v_2}{R} = \frac{1.6 \times 10^5 \times 0.5211}{287} = 290.6 \text{ K} \right]$$

(*ii*) **Change of internal energy, work done and heat interaction :**

Change of internal energy

$$\textbf{u}_2 - \textbf{u}_1 = c_v (\text{T}_2 - T_1) = \frac{R}{\gamma - 1} (T_2 - T_1)$$

$$= \frac{0.287}{1.4 - 1} (290.6 - 380) = \textbf{-- 64.14 kJ/kg} \ (\textit{Decrease}). \quad \textbf{(Ans.)}$$

Work done, $\quad \textbf{W}_{1\text{-}2} = \dfrac{p_1 v_1 - p_2 v_2}{n - 1} = \dfrac{\text{R}(T_1 - T_2)}{n - 1}$

$$= \frac{0.287 (380 - 290.6)}{1.2 - 1} = \textbf{128.29 kJ/kg} \ (\textit{Work done by air}). \quad \textbf{(Ans.)}$$

Heat interaction, $Q_{1-2} = (u_2 - u_1) + W = -64.14 + 128.29 = \textbf{64.15 kJ/kg} \ (\textit{Heat received}). \quad \textbf{(Ans.)}$

$$\left[\textit{Alternatively} : Q = \frac{\gamma - n}{\gamma - 1} \times W = \frac{1.4 - 1.2}{1.4 - 1} \times 128.29 = 64.15 \text{ kJ/kg} \right]$$

(*iii*) **Change in entropy, $(\textbf{s}_2 - \textbf{s}_1)$:**

$$s_2 - s_1 = c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1}$$

$$= \frac{0.287}{1.4 - 1} \times \log_e \left(\frac{290.6}{380} \right) + 0.287 \log_e \left(\frac{0.5211}{0.1363} \right)$$

$$= -0.192 + 0.385 = \textbf{0.193 kJ/kg K} \ (\textit{increase}). \quad \textbf{(Ans.)}$$

Example A-9. (*a*) *Show that approximate change of entropy during a polytropic process equals the quantity of heat transferred divided by the mean absolute temperature.*

(*b*) *One kg of air at 290 K is compressed in a cylinder according to the polytropic law $pv^{1.3}$ = constant. If the compression ratio is 16, calculate the entropy change of air during the compression process stating whether it is an increase or decrease.*

What would be the percentage error if the entropy change is calculated by dividing the quantity of heat exchanged by the mean absolute temperature during the process ?

Take $\gamma = 1.4$ and $c_v = 0.718 \ kJ/kg \ K$.

Solution. (*a*) In Fig. 7 curve 1-2 represents the polytropic process ($pv^n = c$) from state point 1 to state point 2. The area under the process curve 1-2 on *T-S* diagram represents the heat transferred during the process. The slope of the curve 1-2 is usually *small* and can be considered to be a straight line (shown dotted).

Heat transferred = Area of trapezium 1-2-3-4

$$= \text{Base} \times \text{mean ordinate}$$

$$= dS \times \left(\frac{T_1 + T_2}{2} \right)$$

= Entropy change × mean absolute temperature during the process
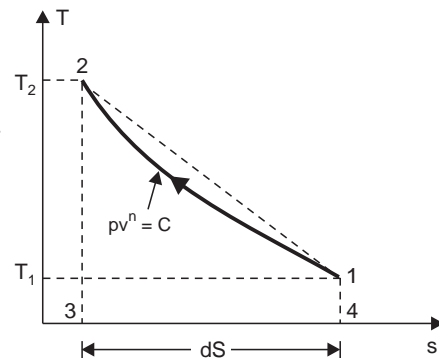
Fig. 7

or Entropy change $= \dfrac{\text{Heat transferred}}{\text{Mean absolute temperature}}$

(b) Given : $m = 1$ kg ; $T_1 = 290$ K ; $pv^{1.3} = $ constant ; $r = 16$; $\gamma = 1.4$; $c_v = 0.718$ kJ/kg K

For a polytropic process : $\dfrac{T_2}{T_1} = \left(\dfrac{v_1}{v_2}\right)^{n-1}$

or $T_2 = 290 \times (16)^{1.3-1} = 666.2$ K

Now, $s_2 - s_1 = c_v \left(\dfrac{n-\gamma}{n-1}\right) \log_e \left(\dfrac{T_2}{T_1}\right)$ ... per kg

$= 0.718 \left(\dfrac{1.3 - 1.4}{1.3 - 1}\right) \log_e \left(\dfrac{666.2}{290}\right) = $ **– 0.199 kJ/kg K. (Ans.)**

The –ve sign indicates *decrease* in entropy.

Heat transferred during the process is given by,

$Q = \dfrac{\gamma - n}{\gamma - 1} \times W = \dfrac{\gamma - n}{\gamma - 1} \times \dfrac{R(T_1 - T_2)}{n - 1} = c_v \left(\dfrac{\gamma - n}{\gamma - 1}\right)(T_1 - T_2)$ ...per kg

$= 0.718 \left(\dfrac{1.4 - 1.3}{1.3 - 1}\right)(290 - 666.2) = -90.04$ kJ/kg $\left(\because c_v = \dfrac{R}{\gamma - 1}\right)$

Mean absolute temperature, $T_{\text{mean}} = \dfrac{T_1 + T_2}{2} = \dfrac{290 + 666.2}{2} = 478.1$ K

Approximate change of entropy $= \dfrac{Q}{T_{\text{mean}}} = \dfrac{-90.04}{478.1} = -0.188$ kJ/kg K

$\therefore$ % age error $= \dfrac{0.199 - 0.188}{0.199} \times 100 = $ **5.53%. (Ans.)**

● The approximate value of entropy change is lower, because in the relation $Q = T_{av} \times dS$ actual value of heat transferred is substituted instead of approximate value (*i.e.*, Area under the straight line) which is higher.

Example A-10. *1.2 m³ of air is heated reversibly at constant pressure from 300 K to 600 K, and is then cooled reversibly at constant volume back to initial temperature. If the initial pressure is 1 bar, calculate :*

(i) The net heat flow.

(ii) The overall change in entropy.

Represent the processes on T-S plot.

Take $c_p = 1.005$ kJ / kg K and R = 0.287 kJ / kg K

Solution. Given : $V_1 = 1.2$ m³ ; $p_1 = p_2$; $T_1 = 300$ K ; $T_2 = 600$ K ; $p_1 = 1$ bar ; $c_p = 1.005$ kJ/kg K ; $R = 0.287$ kJ/kg K

Fig. 8 shows the *T-S* plot of the processes.

(i) **The net heat flow, Q :**

Mass of air, $m = \dfrac{p_1 V_1}{R T_1} = \dfrac{1 \times 10^5 \times 1.2}{(0.287 \times 1000) \times 300} = 1.394$ kg

$$Q = Q_{1\text{-}2} + Q_{2\text{-}3}$$

$$= mc_p(T_2 - T_1) + mc_v(T_3 - T_2)$$

$$= mc_p(T_2 - T_1) + mc_v(T_1 - T_2) \qquad ...(\because \quad T_1 = T_3)$$

$$= m(T_2 - T_1)(c_p - c_v) = m(T_2 - T_1) \times R$$

$$= 1.394(600 - 300) \times 0.287 = \textbf{120 kJ.} \quad \textbf{(Ans.)}$$

Fig. 8

(*ii*) **The overall change in entropy :**

Entropy change during *constant pressure process 1-2*,

$$S_2 - S_1 = mc_p \, \log_e\left(\frac{T_2}{T_1}\right)$$

$$= 1.394 \times 1.005 \, \log_e\left(\frac{600}{300}\right) = 0.9711 \text{ kJ/K}$$

Entropy change during *constant volume process 2-3 ;*

$$S_3 - S_2 = mc_v \, \log_e\left(\frac{T_3}{T_2}\right) = m\,(c_p - R) \, \log_e\left(\frac{T_1}{T_2}\right)$$

$$= 1.394 \times (1.005 - 0.287) \, \log_e\left(\frac{300}{600}\right) = -0.6938 \text{ kJ/K}$$

∴   Overall change of entropy

$$= (S_2 - S_1) + (S_3 - S_2)$$

$$= 0.9771 + (-0.6938) = \textbf{0.2833 kJ/K.} \quad \textbf{(Ans.)}$$

Example A-11. *A closed system contains air at a pressure 1 bar, temperature 300 K and volume 0.018 m³. This system undergoes a thermodynamic cycle consisting of the following three processes in series : (i) Constant volume heat addition till pressure becomes 5 bar, (ii) Constant pressure cooling, and (iii) Isothermal heating to initial state.*

Represent the cycle on T-S and p-V plots and evaluate the change in entropy for each process.

Take c_p = 0.718 kJ / kg K and R = 0.287 kJ / kg K.

Solution. *Given :* p_1 = 1 bar ; T_1 = 300 K ; V_1 = 0.018 m³ ; p_2 = 5 bar ;

c_v = 0.718 kJ/kg K ; R = 0.287 kJ/kg K.
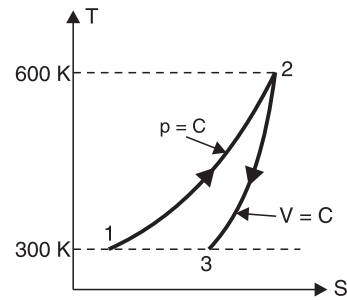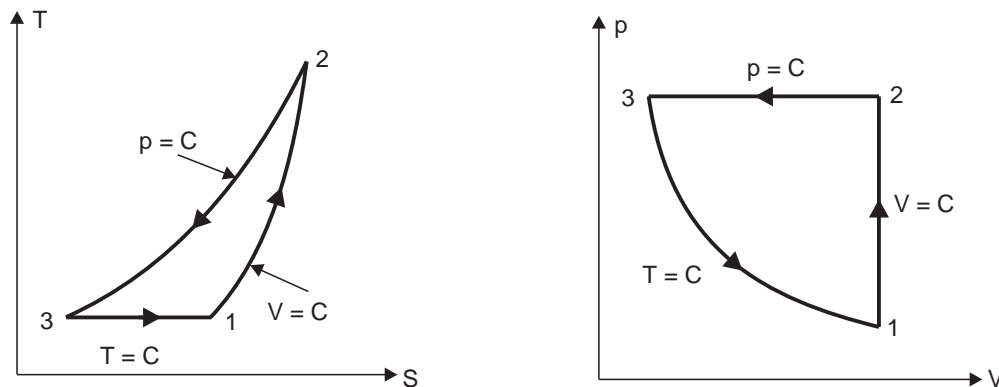
Fig. 9. *T-S* and *p-V* diagrams.

Mass of air, $m = \dfrac{p_1 V_1}{R T_1} = \dfrac{(1 \times 10^5) \times 0.018}{(0.287 \times 1000) \times 300} = 0.0209$ kg

Refer to Fig. 9.

● *Constant volume process 1-2 :*

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad \text{or} \quad T_2 = T_1 \times \frac{p_2}{p_1} = 300 \times \frac{5}{1} = 1500 \text{ K}$$

∴   Change in entropy, $\mathbf{S_2 - S_1} = m c_v \log_e \left(\dfrac{T_2}{T_1} \right)$

$$= 0.0209 \times 0.718 \times \log_e \left(\frac{1500}{300} \right) = \mathbf{0.0241 \text{ kJ/K.}} \quad \textbf{(Ans.)}$$

● *Constant pressure process 2-3 :*

$$T_3 = T_1 = 300 \text{ K}$$

Now, change in entropy,

$$\mathbf{S_3 - S_2} = m c_p \log_e \left(\frac{T_3}{T_2} \right) = m(c_v + R) \log_e \left(\frac{T_3}{T_2} \right)$$

$$= 0.0209 \times (0.718 + 0.287) \times \log_e \left(\frac{300}{1500} \right)$$

$$= \mathbf{-0.0338 \text{ kJ/K.}} \quad \textbf{(Ans.)}$$

● *Constant temperature (isothermal) process 3-1 :*

$$p_3 = p_2 = 5 \text{ bar}$$

Change in entropy,

$$\mathbf{S_1 - S_3} = mR \log_e \left(\frac{p_3}{p_1} \right)$$

$$= 0.0209 \times 0.287 \times \log_e \left(\frac{5}{1} \right) = \mathbf{0.00965 \text{ kJ/K.}} \quad \textbf{(Ans.)}$$

Example A-12. *Derive expressions for entropy change with variable specific heat.*

Solution. Let us assume that the specific heats of a gas vary with temperature according to the linear relations :

$$c_p = a + kT, \quad \text{and} \quad c_v = b + kT$$

where, a, b and k = Constants, and T = Temperature, K.

For *unit mass* of gas, $Q = T\,ds = du + p\,dv$

$$= c_v dT + p\,dv$$

or $$ds = c_v \frac{dT}{T} + \frac{p\,dv}{T} = c_v \frac{dT}{T} + R \frac{dv}{v} \qquad\qquad (\because \quad pv = RT)$$

Now, $$R = c_p - c_v = (a + kT) - (b + kT) = a - b$$

∴ $$ds = (b + kT) \frac{dT}{T} + (a - b) \frac{dv}{v}$$

This is the differential form of entropy change.

Integrating both sides between limits 1 and 2, we get

$$s_2 - s_1 = b \log_e \left(\frac{T_2}{T_1} \right) + k(T_2 - T_1) + (a - b) \log_e \left(\frac{v_2}{v_1} \right) \qquad\qquad …(1)$$

For the entropy change the following expressions can be obtained by suitable manipulations to *eqn.* (*1*) :

1. *Expression for entropy change in terms of* **temperature** *only* :

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1}$$

or

$$\log_e \frac{T_2}{T_1} = (n-1) \log_e \left(\frac{v_1}{v_2}\right) = -(n-1) \log_e \left(\frac{v_2}{v_1}\right)$$

∴

$$s_2 - s_1 = b \log_e \left(\frac{T_2}{T_1}\right) + k(T_2 - T_1) - \left(\frac{a-b}{n-1}\right) \log_e \left(\frac{T_2}{T_1}\right) \qquad \text{...[From eqn. (1)]}$$

or

$$\mathbf{s_2 - s_1} = \left(b - \frac{a-b}{n-1}\right) \log_e \left(\frac{T_2}{T_1}\right) + k(T_2 - T_1) \qquad \text{...(i)}$$

2. *Expression for entropy change in terms of* **pressure, volume and temperature.**
From eqn. (1), we have

$$s_2 - s_1 = b \log_e \left(\frac{T_2}{T_1}\right) + k(T_2 - T_1) + a \log_e \left(\frac{v_2}{v_1}\right) - b \log_e \left(\frac{v_2}{v_1}\right)$$

$$= a \log_e \left(\frac{v_2}{v_1}\right) + b \log_e \left(\frac{T_2}{T_1} \times \frac{v_1}{v_2}\right) + k(T_2 - T_1)$$

or

$$\mathbf{s_2 - s_1} = a \log_e \left(\frac{v_2}{v_1}\right) + b \log_e \left(\frac{p_2}{p_1}\right) + k(T_2 - T_1) \qquad \text{...(ii)}$$

3. *Expression for entropy change in terms of* **pressure and temperature only.**
Again, from eqn. (1), we have

$$s_2 - s_1 = b \log_e \left(\frac{T_2}{T_1}\right) + k(T_2 - T_1) + a \log_e \left(\frac{v_2}{v_1}\right) - b \log_e \left(\frac{v_2}{v_1}\right)$$

$$= a \log_e \left(\frac{T_2}{T_1} \times \frac{p_1}{p_2}\right) + b \log_e \left(\frac{T_2}{T_1} \times \frac{v_1}{v_2}\right) + k(T_2 - T_1)$$

$$= a \log_e \left(\frac{T_2}{T_1}\right) - a \log_e \left(\frac{p_2}{p_1}\right) + b \log_e \left(\frac{p_2}{p_1}\right) + k(T_2 - T_1)$$

or

$$\mathbf{s_2 - s_1} = a \log_e \left(\frac{T_2}{T_1}\right) + (b-a) \log_e \left(\frac{p_2}{p_1}\right) + k(T_2 - T_1) \qquad \text{...(iii)}$$

● Derivation of the formula $T^b v^{a-b} e^{kT}$ = constant for the *adiabatic expansion of gas* :

We know that, $ds = (a-b) \dfrac{dv}{v} + (b + kT) \dfrac{dT}{T}$

$s_2 - s_1 = a \log_e v - b \log_e v + b \log_e T + kT$

$\qquad = a \log_e v + b \log_e \left(\dfrac{T}{v}\right) + kT$

$\qquad = a \log_e v + b \log_e \left(\dfrac{p}{R}\right) + kT$ = 0 for adiabatic expansion

$\qquad = a \log_e v + \log_e \left(\dfrac{p}{a-b}\right)^b + kT$

This gives : $v^a \, p^b \, e^{kT}$ = constant

$pv^{a-b} \, e^{kT}$ = constant

$T^b \, v^{a-b} e^{kT}$ = constant

The above expressions can be obtained by taking kT on right-side and taking the antilog of the resulting expressions.

Example A-13. *Determine the entropy change of 4 kg of a perfect gas whose temperature varies from 127°C to 227°C during a constant volume process. The specific heat varies linearly with absolute temperature and is represented by the relation :*

c_v = *(0.48 + 0.0096 T) kJ / kg K.*

Solution. *Given :* m = 4 kg ; T_1 = 127 + 273 = 400 K ; T_2 = 227 + 273 = 500 K ;

c_v = (0.48 + 0.0096 T) kJ/kg K.

Entropy variation for a constant volume process is given by :

$$dS = mc_v \, \frac{dT}{T} \quad \text{or} \quad dS = 4 \times (0.48 + 0.0096T) \, \frac{dT}{T}$$

Integrating both sides, we get

$$S_2 - S_1 = 4 \times 0.48 \int_{T_1}^{T_2} dT + 4 \times 0.0096 \int_{T_1}^{T_2} dT$$

$$= 1.92 \log_e \left(\frac{T_2}{T_1} \right) + 0.0384 \, (T_2 - T_1)$$

$$= 1.92 \log_e \left(\frac{500}{400} \right) + 0.0384(500 - 400) = 4.268 \text{ kJ/K}$$

i.e., $S_2 - S_1$ = **4.268 kJ.**   **(Ans.)**

Example A-14. *The specific heats of a gas vary linearly with absolute temperature according to the following relations :*

c_p = *(0.85 + 0.00025 T) kg / kg K, and*

c_v = *(0.56 + 0.00025 T) kJ/kg K*

If the entropy of the gas at 1 bar pressure and 273 K is zero, find the entropy of the gas at 25 bar and 750 K temperature.

Solution. *Given :* c_p = (0.85 + 0.00025 T) kJ/kg K ; c_v = (0.56 + 0.00025 T) kJ/kg K ;

p_1 = 1 bar ; T_1 = 273 K ; p_2 = 25 bar ; T_2 = 750 K.

We know that, $ds = c_v \, \dfrac{dT}{T} + \dfrac{p}{T} \, dv = c_v \, \dfrac{dT}{T} + R \, \dfrac{dv}{v}$

Integrating both sides, we get

$$s_2 - s_1 = \int c_v \, \frac{dT}{T} + R \log_e \left(\frac{v_2}{v_1} \right)$$

$$= \int c_v \, \frac{dT}{T} + (c_p - c_v) \log_e \left(\frac{p_1}{p_2} \times \frac{T_2}{T_1} \right)$$

$$= \int \left(\frac{0.56}{T} + 0.00025 \right) dT + 0.29 \log_e \left(\frac{1}{25} \times \frac{750}{273} \right)$$

$$= \left[0.56 \log_e \left(\frac{T_2}{T_1} \right) + 0.00025 \, (T_2 - T_1) \right] - 0.6404$$

$$= 0.56 \log_e \left(\frac{750}{273} \right) + 0.00025(750 - 273) - 0.6404 = 0.0448 \text{ kJ/kg K}$$

i.e., $\textbf{s}_2 - \textbf{s}_1 = \textbf{0.0448 kJ/kg K.}$ **(Ans.)**

Example A-15. *An insulated vessel of 0.5 m^3 capacity is divided by a rigid conducting diaphragm into two chambers A and B, each having a capacity of 0.25 m^3. Chamber A contains air at 1.4 bar pressure and 290 K temperature and the corresponding parameters for air in chamber B are 4.2 bar and 440 K. Calculate :*

   (*i*) *Final equilibrium temperature,*

  (*ii*) *Final pressure on each side of the diaphragm, and*

 (*iii*) *Entropy change of system.*

 For air take c_v = 0.715 kJ / kg K and R = 0.287 kJ / kg K.

 Solution. *Given :* $V_A = 0.25 \text{ m}^3$; $p_{Ai} = 1.4 \text{ bar}$; $T_{Ai} = 290 \text{ K}$; $V_B = 0.25 \text{ m}^3$; $p_{Bi} = 4.2 \text{ bar}$;
 $T_{Bi} = 440 \text{ K}$; $c_v = 0.715 \text{ kJ/kg K}$.

   (*i*) **Final equilibrium temperature, T_f :**

Mass of air, $m_A = \dfrac{p_{Ai} V_A E}{R T_{Ai}} = \dfrac{(1.4 \times 10^5) \times 0.25}{(0.287 \times 1000) \times 290} = 0.4205 \text{ kg}$

$$m_B = \frac{p_{Bi} V_B}{R T_{Bi}} = \frac{(4.2 \times 10^5) \times 0.25}{(0.287 \times 1000) \times 440} = 0.8315 \text{ kg}$$

Let T_f be the final equilibrium temperature (K). Since the diaphragm is conducting,

Heat gained by air in chamber *A* = Heat lost by air in chamber *B*

$$m_A c_v (T_f - 290) = m_B c_v (440 - T_f)$$

or $0.4205 \times (T_f - 290) = 0.8315(440 - T_f)$

or $0.4205 \, T_f - 121.94 = 365.86 - 0.8315 \, T_f$

   $\therefore$ $T_f = \textbf{389.6 K.}$ **(Ans.)**

  (*ii*) **Final pressure on each side of the diaphragm : p_{Af} ; p_{Bf}**

$$p_{Af} = \frac{1.4 \times 389.6}{290} = \textbf{1.88 bar.}   \textbf{(Ans.)}$$

$$p_{Bf} = \frac{4.2 \times 389.6}{440} = \textbf{3.72 bar.}   \textbf{(Ans.)}$$

 (*iii*) **Entropy change of the system :**

Entropy change for chamber $A = m_A c_v \log_e \left(\dfrac{T_f}{T_{Ai}} \right)$

$$= 0.4205 \times 0.715 \times \log_e \left(\frac{389.6}{290} \right) = 0.0888 \text{ kJ/K}$$

Entropy change for chamber $B = m_B c_v \log_e \left(\dfrac{T_f}{T_{Bi}} \right)$

$$= 0.8315 \times 0.715 \times \log_e \left(\frac{389.6}{440} \right) = -0.0723 \text{ kJ/K}$$

$\therefore$ Net change of entropy = 0.0888 + (− 0.0723) = **0.0165 kJ/K.**   **(Ans.)**

EXAMINATION PAPERS

P.T.U., JALANDHAR

THEORY EXAMINATION, MAY 2009

B. Tech. 1 and 2 Semesters

SUBJECT CODE : ME - 101 (2K4 and Onwards)

Paper ID : (A0114)

Time: 3 Hours   *Maximum Marks: 60*

Instructions to Candidates:

(1) Section - A is Compulsory.
(2) Attempt any *Five* questions from Section - B & C.
(3) Select at least *Two* questions from Section - B & C.

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Section A   **[Marks : 2 Each]**

</div>

1. (*a*) What do you understand by Quasi-static process? How it is achieved?

    (*b*) Discuss concept of thermal equilibrium.

    (*c*) Write and explain analytical expression applicable for a process and cycle.

    (*d*) What are non-flow processes? Give suitable examples.

    (*e*) Discuss equivalence of various statements of second law of thermodynamics.

    (*f*) What is air standard efficiency?

    (*g*) Differentiate between mechanism and machine.

    (*h*) What is Poisson's ratio?

    (*i*) Describe creep and fatigue.

    (*j*) What do you understand by throttling process?

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Section B   **[Marks : 8 Each]**

</div>

2. (*a*) Define work. Show that work done $dW = pdV$.

    (*b*) The pressure volume correlation for a non-flow reversible process is given by $p = (8 - 4V)$ bar, where V is in m^3. If 150 kJ of work is supplied to the system, determine the final pressure and volume. Take initial volume = 0.6 m^3.

3. Air initially at 60 kPa pressure, 800 K temperature and occupying a volume of 0.1 m^3 is compressed isothermally till the volume is halved and further it goes compression at constant pressure till the volume is halved again. Sketch the process on *p-V* diagram and make calculations for total work done and total heat interaction for the two processes. Assume ideal gas behaviour for air and take c_p = 1.005 kJ/kg.

4. A centrifugal pump delivers 2750 kg of water per minute from initial pressure of 0.8 bar absolute to a final pressure of 3.8 bar absolute. The suction is 2 m below and delivery is 5 m above the centre of pump. If the suction and delivery pipes are 15 cm and 10 cm diameter respectively, make calculations for power required to the pump. Density of water = 1000 kg/m^3.

5. Two reversible heat engines E_1 and E_2 are arranged in series between a hot reservoir at temperature T_1 of 600 K and a cold reservoir at temperature T_2 of 300 K. Engine E_1 receives 500 kJ of heat from reservoir at T_1. Presuming that both engines have equal thermal efficiency determine:

(a) The temperature at which heat is rejected by E_1 and is received by engine E_2.

(b) Thermal efficiency of each engine.

(c) Work done by engine E_1 and E_2.

(d) The heat rejected by engine E_2 to cold reservoir.

<p align="center">**Section C**   **[Marks : 8 Each]**</p>

6. A reversible heat engine operates between 875 K and 310 K and drives a reversible refrigerator operating between 310 K and 255 K. The engine receives 2000 kJ of heat and the network output from the arrangement equals 350 kJ. Make calculations for cooling effect *i.e.*, heat.

7. (a) What is cut-off ratio? How it affects the air standard efficiency of diesel cycle?

(b) With the help of p-V and T-S diagram show that for same maximum pressure and heat input:

$$\eta_{diesel} > \eta_{dual} > \eta_{otto}$$

8. (a) Define velocity ratio, mechanical advantage and efficiency of a machine.

(b) The upper block of the differential pulley block has two pulleys of diameter 250 mm and 200 mm. What load will be lifted by this machine by the application of an effort equal to 20 N? Take efficiency of the system as 60 per cent.

9. Two vertical rods of steel and copper are firmly secured at their upper ends and lie at a distance of 100 cm apart. Each rod is of 3 m length and 30 mm in diameter. A horizontal rigid cross bar connects the lower ends of the rod and on it is placed a load of 40 kN. Determine:

(a) Location of the load on the cross bar if it remains horizontal even after being loaded.

(b) What will be the inclination of the cross bar if the load is hung at its mid span? Take E for steel = 2×10^5 N/mm^2, E for copper = 1×10^5 N/mm^2.

P.T.U., JALANDHAR

THEORY EXAMINATION, DEC. 2009

B. Tech. 1 and 2 Semesters

SUBJECT CODE : ME - 101 (2K4 and Onwards)

Paper ID : (A0114)

Time: 3 Hours   *Maximum Marks: 60*

Instructions to Candidates :

(1) Section - A is Compulsory.

(2) Attempt any *Five* questions from Section - B & C.

(3) Select at least *Two* questions from Section - B & C.

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Section A   **[Marks : 2 Each]**

</div>

1. (*a*) What are similarities and dissimilarities between heat and work?

 (*b*) What is reversible process? Also write conditions for reversibility.

 (*c*) Why gases have two specific heats?

 (*d*) What is PMM1 (perpetual motion machine of first kind)?

 (*e*) How COP of heat pump and refrigerator are related?

 (*f*) What is mean effective pressure and what is its role in gas power cycles?

 (*g*) List different systems of pulleys.

 (*h*) What are temperature stresses?

 (*i*) What is significance of elastic constant?

 (*j*) What are the limitations of first law of thermodynamics?

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Section B   **[Marks : 8 Each]**

</div>

2. (*a*) Define and explain Zeroth law of thermodynamics. Why it is so called?

 (*b*) A new temperature scale in degrees N is desired with freezing point at l00°N and the boiling point at 400°N. Establish a correlation between degrees Celsius and degrees N. What would be the absolute temperature at 0°N?

3. Air initially at 90 kPa pressure, 700 K temperature and occupying a volume of a 0.5 m^3 is compressed isothermally till the volume is halved and further it goes compression at constant pressure till the volume is halved again. Sketch the process on *p-V* diagram and make calculations for total work done and total heat interaction for the two processes. Assume ideal gas behaviour for air and take c_p = 1.004 kJ/kg K.

4. (*a*) Explain and derive steady flow energy equation.

 (*b*) A perfect gas flows through a nozzle where it expands in a reversible adiabatic manner. The inlet conditions are – 22 bar, 500°C, 38 m/s. At the exit the pressure is 2 bar. Determine the velocity and exit area if the flow rate is 4 kg/s. Take *R* = 190 J/kg K and *Y* = 1.35.

5. (*a*) State and explain Carnot theorem.

 (*b*) Write various statements of second law of thermodynamics and also *Q* show their equivalence.

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Section C   **[Marks : 8 Each]**

6. A reversible heat engine operates within the higher and lower temperature limits of 1400 K and 400 K respectively. The entire output from this engine is utilized to operate a heat pump. The pump works on reversed Carnot cycle, extracts heat from a reservoir at 300 K and delivers it to the reservoir at 400 K. If 100 kJ/s of net heat is supplied to the reservoir at 400 K, calculate the heat supplied to the engine by the reservoir at 1400 K.

7. In an air standard diesel cycle, the compression ratio is 15 and the pressure and temperature of the air at the beginning of the compression are 1 bar and 288 K. The peak temperature in the cycle is 2700 K. Calculate:

(*a*) Heat supplied;

(*b*) Work done;

(*c*) Cycle efficiency;

(*d*) Peak pressure of the cycle;

(*e*) Cut-off ratio; and

(*f*) Mean effective pressure.

8. (*a*) Draw stress-strain diagram and explain various salient features.

(*b*) A conical bar tapers uniformly from a diameter of 4 cm to 15 cm in a length of 40 cm. If an axial force of 80 kN is applied at each end, determine the elongation of the bar. Take E = 200 GPa.

9. (*a*) Define kinematic link, kinematic pair and kinematic chain.

(*b*) In a differential wheel and axle, the diameter of the larger axle is 250 mm and that of the smaller is 225 mm. The diameter of the effort wheel is 500 mm. Find the velocity ratio. If an effort of 150 N lifts a load of 3.5 kN, what is the efficiency and effort lost in friction?

P.T.U., JALANDHAR

THEORY EXAMINATION, MAY 2010

B. Tech. 1 and 2 Semesters

SUBJECT CODE : ME - 101 (2K4 and Onwards)

Paper ID : (A0114)

Time: 3 Hours	*Maximum Marks: 60*

Instructions to Candidates :
  (1) Section - A is Compulsory.
  (2) Attempt any *Five* questions from Section - B & C.
  (3) Select at least *Two* questions from Section - B & C.

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Section A   **[Marks : 2 Each]**

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1. (*a*) Define enthalpy of the system. How it is related to internal energy?

    (*b*) Define compression ratio.

    (*c*) Is it possible that $W \neq 0$ even if $dV = 0$? If so give an example.

    (*d*) What is entropy principle?

    (*e*) Draw *P-V* and *T-S* diagrams for Brayton cycle.

    (*f*) What is PMM1?

    (*g*) Give the relation between COP of heat pump and refrigerator.

    (*h*) Differentiate between machine and mechanism.

    (*i*) State the uses of Oldham coupling.

    (*j*) Define temperature stresses and strains.

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Section B   **[Marks : 8 Each]**

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2. (*a*) Differentiate between temperature, heat and internal energy.

    (*b*) A non-flow reversible process occurs for which pressure and volume are correlated by the expression $P = (V^2 + 5/V)$, where P is in bar and V is in m³. What amount of work will be done when volume changes from 3 to 6 m³?

3. (*a*) What is throttling process? Point out its salient aspects.

    (*b*) The centrifugal pump delivers 50 kg of water per second. The inlet and outlet pressures are 1 bar and 4.2 bar respectively. The suction is 2.2 m below the center of the pump and delivery is 8.5 m above the center of the pump. The suction and delivery pipe diameters are 20 cm and 10 cm respectively. Determine the capacity of the electric motor to run the pump.

4. (*a*) Draw *P-V* diagram for isobaric, isochoric and isothermal processes.

    (*b*) 0.2 m³ of an ideal gas at a pressure of 2 MPa and 600 K is expanded isothermally to 5 times the initial volume. It is then cooled to 300 K at constant volume and then compressed back polytropically to its initial state. Determine the network done and heat transfer during the cycle.

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5. State Kelvin-Planck and Clausius statements of second law of thermodynamics. Explain the equivalence of Kelvin-Planck and Clausius statements.

Section C   **[Marks : 8 Each]**

6. Explain the working principle of dual cycle with the help of *P-V* and *T-S* diagrams. Derive an expression for the air standard efficiency of the dual cycle in terms of the compression ratio, pressure ratio, cut-off ratio and the adiabatic index.

7. (*a*) Explain the working of elliptical trammel.

(*b*) A machine raised a load of 500 N through a distance of 300 mm. The effort of 500 N moved 2.7 m during the process. Determine:

  (*i*) Mechanical advantage;

 (*ii*) Velocity Ratio;

(*iii*) Efficiency;

(*iv*) Effect of friction.

8. (*a*) Derive the relation between Bulk modulus (*K*), Young's modulus (*E*) and Poisson's ratio (1/*m*).

(*b*) In order to evaluate various mechanical properties, a steel specimen of 20 mm diameter and 20 cm length was tested in a standard tension test and data collected are: yield load = 150 kN; maximum load = 225 kN; fracture load = 175 kN; gauge length at fracture = 25 cm; extension at load of 30 kN = 0.08 N/m. Determine:

  (*i*) Yield point stress;

 (*ii*) Ultimate tensile stress;

(*iii*) Percentage elongation;

(*iv*) Modulus of elasticity;

 (*v*) Breaking strength.

9. Write short notes on the following:

(*a*) Reversible processes;

(*b*) Carnot cycle;

(*c*) Creep;

(*d*) Working of four stroke diesel engine.

P.T.U., JALANDHAR

THEORY EXAMINATION, DEC. 2010

B. Tech. 1 and 2 Semesters

SUBJECT CODE : ME - 101 (2K4 and Onwards)

Paper ID : (A0114)

Time: 3 Hours   *Maximum Marks: 60*

Instructions to Candidates :

(1) Section - A is Compulsory.

(2) Attempt any *Five* questions from Section - B & C.

(3) Select at least *Two* questions from Section - B & C.

Section A	[Marks : 2 Each]

1. (*a*) How is the property of system classified?

    (*b*) What do we understand by energy in Transit?

    (*c*) Differentiate between the adiabatic and polytropic process.

    (*d*) Why c_p is greater than c_v?

    (*e*) What are the assumptions for steady flow process?

    (*f*) Sketch the Carnot cycle on *P-V* and *T-S* diagram.

    (*g*) Define the mean effective pressure for Reciprocating engine.

    (*h*) State the law of a lifting machine.

    (*i*) What is kinematic link?

    (*j*) Sketch a Stress-strain for ductile and brittle materials and show the salient points on it.

Section B	[Marks : 8 Each]

2. (*a*) Define and explain the Zeroth law of thermodynamics. Why is it so a called?

    (*b*) A non-flow reversible process occurs for which pressure and volume are correlated by the expression $PV = 150$, where *V* is the volume in m^3 and *p* is the pressure in bar. Make calculations for the work done on or by the system as pressure increases from 10 bar to 100 bar. Indicate the nature of process, whether expansion or compression.

3. (*a*) How are the state parameters *P*, *V* and *T* related in a polytropic process? What are the values of polytropic exponent for isochoric, isothermal and adiabatic processes?

    (*b*) Show that the internal energy is a property of the system.

4. The gas leaving the turbine jet engine flows steadily into the Jet pipe with an enthalpy 960 kJ/kg and velocity 250 m/s relative to the pipe. The exit from the pipe is at enthalpy 860 kJ/kg, and the exhaust is in line with intake. Neglecting heat loss from the system, determine the relative velocity of gas leaving the pipe.

5. Explain the establishment of thermodynamic temperature scale. Why is thermodynamic temperature scale called the absolute temperature scale?

Section C   **[Marks : 8 Each]**

6. Derive an expression for the air standard efficiency and mean effective pressure of an Otto cycle. State the assumptions made.

7. (*a*) Discuss briefly the working of a 4 stroke diesel engine.

   (*b*) Make a labelled sketch of Oldham coupling.

8. Discuss the various inversions of double slider crank chain mechanism.

9. An unknown weight falls 4 cm on to a collar rigidly attached to the lower end of a vertical bar 4 m long and 8 cm^2 in section. If the maximum instantaneous extension is found to be 0.42 cm, find the corresponding stress and the value of unknown weight, $E = 200$ kN/mm^2.

P.T.U., JALANDHAR

THEORY EXAMINATION, MAY 2011

B. Tech. 1st Semester

SUBJECT CODE : ME - 101

Paper ID : (A0123)

Time: 3 Hours   *Maximum Marks: 60*

Instructions to Candidates :

(1) Section - A is Compulsory.

(2) Attempt any *Five* questions from Section - B & C.

(3) Select at least *Two* questions from Section - B & C.

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Section A   **[Marks : 2 Each]**

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1. (*a*) With same compression ratio, an engine working on Dual! Diesel! Otto Cycle is most efficient .... (Select the correct Cycle).

(*b*) Starting from the same initial state 1, plot constant volume and constant pressure heat addition processes I_2 and I_3 on the T-S—Co-ordinates I along with arrow-heads indicating the direction of these processes and identity these two processes on the T-S Chart.

(*c*) What is the fundamental and basic difference between the terms enthalpy and total energy associated with the mass of a thermodynamic medium?

(*d*) Justify by assigning some numerical values by way of a practical example, to justify that $d\phi > \%$ for an irreversible heat exchange process, say, by taking the case of irreversible heat addition Q to a Heat Engine.

(*e*) Starting from the same initial state 1 plot reversible and irreversible adiabatic expansion processes on T-S Co-ordinates along with arrow-A heads indicating the direction of the two processes. Show the area which is representative of the fraction of energy which became unavailable during the irreversible process.

(*f*) Can the COP of a Refrigerator be greater than/less than/both, more than as well as less than unity? Select the Correct Answer. Justify your reply by assigning some numerical values to the cooling effect and the compressor work by way of presenting a simple live practical example.

(*g*) Can throttling process be plotted on P-V co-ordinates? Give answer in YES or NO and give reasons for the same.

(*h*) Superimpose a Carnot Cycle on a Diesel Cycle plotted on the T-S co-ordinates, such that the end-states representing the lowest and highest temperatures for the two Cycles are coinciding with each other. Justify with the aid of T-S co-ordinates that the Diesel Cycle operating between the same lowest and highest temperature limits will be less efficient than the corresponding Carnot Cycle.

(*i*) Temperature decreases/increases/remains constant during free expansion process. .... (Select correct Answer)

(*j*) Define temperature stresses and strains.

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Section B   **[Marks : 8 Each]**

2. (*a*) How enthalpy differs from the term heat transfer? Clearly explain the difference between the two in detail.

   (*b*) "One of the essential requirements for accomplishing any reversible expansion process is that there should be complete thermal equilibrium between the system and the thermal reservoir exchanging heat during the process. Justify the above statement by thermodynamic or any other logics.

3. (*a*) $dQ = dH - Vdp$ ...Is this equation valid for a process occurring:

   (*i*) In an Open System,

   or

   (*ii*) In a Closed System,

   or

   (*iii*) In both Systems? .... (Select the correct Answer).

   (*b*) The properties of a certain gas are related as follows:

   $$E = 196 + 0.718\ t$$
   $$pv = 0.287\ (t + 273)$$

   where E is the specific internal energy (kJ/kg), 't' is in –°C, p is pressure in kN/m^2 and v is specific volume (m^3/kg). Determine the two specific heats c_v, c_p for this gas.

4. (*a*) Select suitable co-ordinates for plotting free expansion process and plot it on these co-ordinates along with an arrow-head indicating the direction of the process.

   (*b*) Air at 1 bar and 7°C is heated at constant volume in a cylinder till its temperature has risen to 827°C. It is then expanded isentropically to 1 bar and subsequently heat is rejected at constant pressure until the temperature is again equal to 7°C. Determine the following per kg of air:

   (*i*) Pressure, volume and temperature at the end of each process;

   (*ii*) Heat supplied to the cycle;

   (*iii*) Work delivered by the cycle;

   (*iv*) Efficiency of the cycle.

5. A reversible engine takes 1200 kW from a reservoir at 700 K and develops 200 kW of work when executing complete cycle. The engine rejects heat to two reservoirs at 600 K and 500 K. Determine heat rejected to both the reservoirs. Draw a schematic line sketch for the Engine and plot the Cycle for this Engine on T-S co-ordinates.

Section C   **[Marks : 8 Each]**

6. (*a*) Explain and justify how isothermal heat addition process contributes in optimizing the output and hence efficiency of the Carnot Cycle.

   (*b*) Which is more effective (*i.e.*, better) method of increasing the efficiency of a Carnot Cycle out of the following two possibilities:

   (*i*) By increasing T_1 while maintaining T_2 constant.

   (*ii*) By decreasing T_2 while maintaining T_1 as constant ($T_2 > T_1$).

   Arrive at your decision with the help of T-S Chart.

7. In an air standard Brayton Cycle, the air enters the compressor at 1 bar and 25°C. The pressure after compression is 3 bar. The temperature at turbine inlet is 650°C. Calculate per kg of air:

   (*a*) Heat supplied;

   (*b*) Heat rejected;
   (*c*) Net work done;
   (*d*) Temperature of air leaving the turbine;
   (*e*) Air-standard efficiency of the cycle;
   (*f*) Work Ratio of the Unit.
8. Write brief notes on:
   (*a*) Differential wheel and axle;
   (*b*) Lifting Machines;
   (*c*) Worm and Worm wheel.
9. (*a*) Make a labelled sketch of the Oldham coupling and list its use(s).
   (*b*) Define kinematic link, kinematic pair and kinematic chain.

P.T.U., JALANDHAR

THEORY EXAMINATION, DEC. 2011

B. Tech. 1st Semester

SUBJECT CODE : BTME - 101 (2011 Batch)

Paper ID : (1107)

Time: 3 Hours   *Maximum Marks: 60*

Instructions to Candidates:

 (1) Section - A is Compulsory.

 (2) Attempt any *Five* questions from Section - B & C.

 (3) Select at least *Two* questions from Section - B & C.

Section A   **[Marks : 2 Each]**

1. (*a*) An ideal gas goes through an expansion process where the volume doubles. Which process will lead to the larger work output: an isothermal process or a polytrophic process with $n = 1.25$?

 (*b*) In an effort to conserve energy in a heat engine cycle, somebody suggests incorporating a refrigerator that will absorb some of the waste energy qL and transfer it to the energy source of the heat engine. Is this a smart idea? Explain.

 (*c*) Air at 500 K, 500 kPa is expanded to 100 kPa in two steady flow cases. Case one is a throttle and case two is a turbine. Which has the highest exit T? Why?

 (*d*) Assume a heat engine with a given QH. Can you say anything about QL if the engine is reversible? If it is irreversible?

 (*e*) Why is the carnot cycle not suitable as an ideal cycle for all power producing cycle devices?

 (*f*) A substance is compressed adiabatically so P and T go up. Does that change S?

 (*g*) For the same maximum pressure and heat input, compare Otto cycle and Diesel cycle on P-V and T-S diagrams.

 (*h*) What are the technological properties of metals and alloys? Name any four.

 (*i*) What are the smart materials? Name any two along with their applications.

 (*j*) State the purpose of camshaft and flywheel in an I.C. engine.

Section B   **[Marks : 8 Each]**

2. Fluid is confined in a cylinder by a spring loaded, frictionless piston so that the pressure in the fluid is linear function of volume ($p = a + bV$). The internal energy of the fluid is given by the following equation $U = 34 + 3.15\ PV$, where U is in kJ, P is in kPa, and V is in m³. If the fluid changes from an initial state of 170 kPa , 0.03 m³ to a final state of 400 kPa, 0.06 m³, with no work other than that done on the piston, find the direction and magnitude of the work and heat transfer.

3. Air at 9 bar and 200°C is throttled to 5 bar before being expanded through a nozzle to a pressure of 1.1 bar. Assuming that the flow through the nozzle is reversible steady flow

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process, and that no heat is rejected. Calculate the velocity of air at nozzle outlet when the inlet velocity is 80 m/s.

4. (a) A reversible engine operates between temperatures T_1 and T $(T_1 > T)$. The energy rejected from the engine is received by a second reversible engine at the same temperature T. The second engine rejects energy at temperature T_2 $(T_2 < T)$. Show that the temperature T is the geometric mean of temperatures T_1 and T_2 if the engines have the same mechanical efficiencies.

(b) A source X can supply energy at the rate of 11000 kJ/min at 320°C. Another source Y can supply at the rate of 11000 kJ/min at 60°C. Which source X or Y would you choose, to supply energy to an ideal reversible engine, that is to produce a large amount of power, if the temperature of the surroundings is 4°C.

5. (a) State and prove clausius inequality.

(b) 1 kg of water at 0°C is brought into contact with a heat reservoir at 100°C. When the water has reached at 100°C, find:

  (i) Entropy change of water;

  (ii) Entropy change of heat reservoir;

  (iii) Entropy change of universe.

Section C   [Marks : 8 Each]

6. (a) Derive an expression for mean effective pressure of diesel cycle.

(b) A diesel engine has a compression ratio of 14. The fuel is cut-off at 0.08 of stroke. The relative efficiency is 52%. Find the mass of fuel of calorific value 41800 kJ/kg which would be required per kWh.

7. (a) Prove the parallel axis theorem in the determination of moment of inertia of areas with the help of a neat sketch.

(b) Find the centroid of a semicircular lamina of radius r.

8. (a) What are ER and MR fluids? Discuss their properties and areas of applications.

(b) Give the composition, properties and uses of following alloy steels:

  (i) Heat resisting steels;   (ii) Spring steels;

  (iii) Stainless steels;   (iv) High speed steels.

9. (a) Discuss the difference between theoretical and actual P-V diagrams for two stroke S.I. and C.I. engines.

(b) An ideal air standard Brayton cycle operates with air. At the compressor inlet, the air is at 305 K and 1 bar. It is further compressed to 5.5 bar. The maximum cycle temperature is limited to 1050 K. The heat supplied is 10.5 MW. Find:

  (i) Thermal efficiency of the cycle;

  (ii) Work ratio;

  (iii) Power output.

ABOUT THE BOOK

This book has been written specially to meet the exhaustive requirements of the subject **"Elements of Mechanical Engineering"** of B.E. 1st year Examination of Punjab Technical University, Jalandhar and other Courses of B.E., B.Tech., B.Sc. (Engg.), U.P.S.C., A.M.I.E.

Salient Features:

- The presentation of the subject matter is very systematic and the language of text is in a lucid, direct and easy to understand manner.

- The book provides a comprehensive treatment of the subject matter under wide range of topics mentioned in the syllabus common to the above mentioned universities, including a large number of solved examples to support the text wherever required.

- A large number of solved examples, properly graded, have been added in various chapters to enable the students to attempt different types of questions in the examination without any difficulty.

- At the end of each chapter Highlights, Objective Type Questions, Theoretical Questions and Unsolved Examples have been added to make the book a complete unit in all respects.

ABOUT THE AUTHOR

Er. R.K. Rajput, born on 15th September, 1944 (coincident with Engineer's Day) is a multi-disciplinary engineer. He obtained his *Master's degree* in **Mechanical Engineering** (with Hons.–Gold Medal) from Thapar Institute of Engineering and Technology, Patiala. He is also a *Graduate Engineer* in **Electrical Engineering.** Apart from this he holds memberships of various professional bodies like Member Institution of Engineers (MIE), Member Indian Society of Technical Education (MISTE) and Member Solar Energy Society of India (MSESI). He is also a Chartered Engineer (India). He has served for several years as Principal of "Punjab College of Information Technology, Patiala" and "Thapar Polytechnic, Patiala".

He has more than 40 years of experience in *teaching* different subjects of Mechanical and Electrical Engineering disciplines. He has published/presented a large number of technical papers. He is the author of several books on the important subjects of Mechanical as well as Electrical Engineering disciplines.

He has earned, by dint of hard work and devotion to duty, the following **awards/honours:**

*Best Teacher (Academic) Award * Jawahar Lal Nehru Memorial Gold Medal for an outstanding research paper (Institution of Engineers) * Distinguished Author Award *Man of Achievement Award.*

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