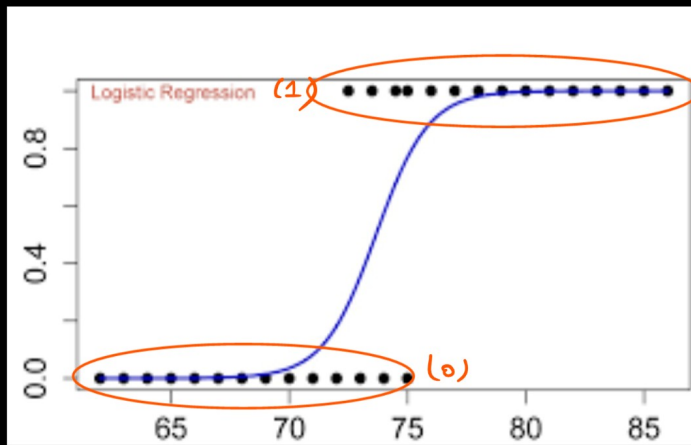
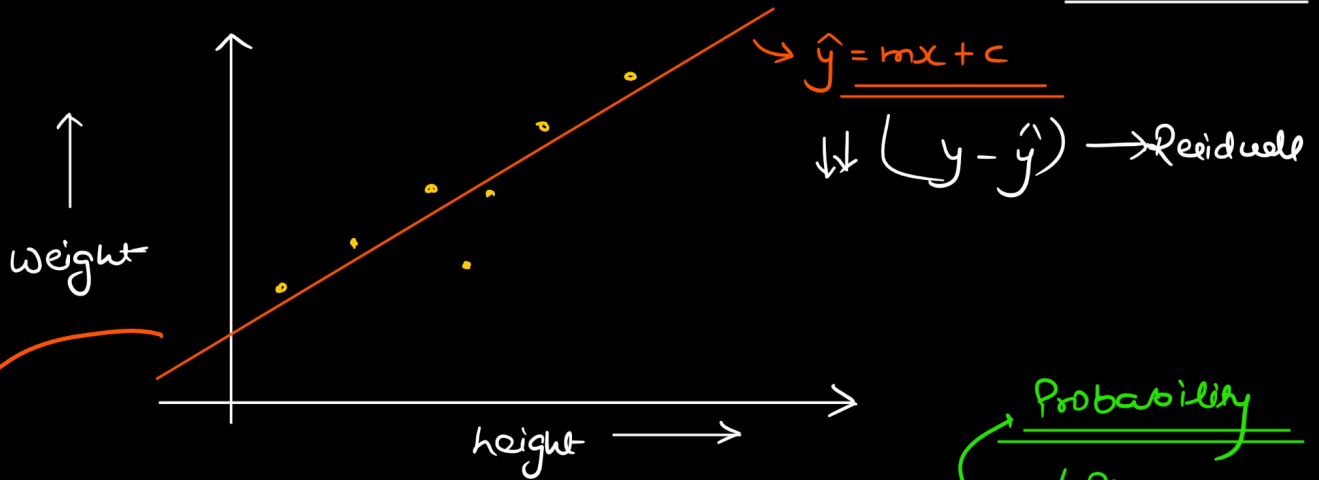
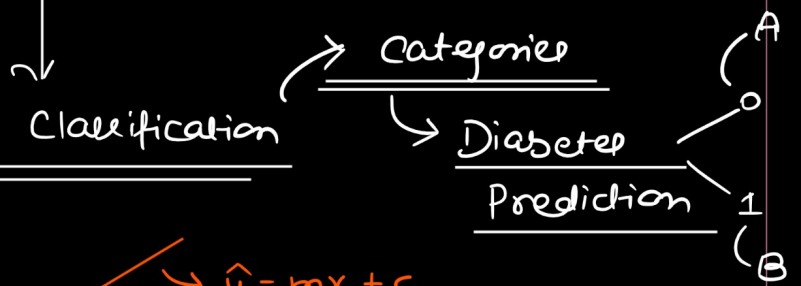


Logistic Regression → Binary Classification



Probability
(Binary Classification)
Sigmoid
activation

function ⇒

$$f(z) = \frac{1}{1 + e^{-z}}$$

$$f(\hat{y}) = \frac{1}{1 + e^{-\hat{y}}}$$

$$f(\hat{y}) = \frac{1}{1 + e^{-(mx+c)}}$$

$$f(\hat{y}) = \frac{1}{1 + \frac{1}{e^{(mx+c)}}}$$

$$f(\hat{y}) = \frac{1}{\frac{e^{mx+c} + 1}{e^{mx+c}}}$$

$$P(A) = f(\hat{y}) = \frac{e^{mx+c}}{e^{mx+c} + 1} \quad \underline{P(A) + P(B) = 1}$$

$$P(B) = 1 - P(A)$$

$$P(B) = 1 - \frac{e^{mx+c}}{e^{mx+c} + 1}$$

$$P(B) = \frac{1}{\frac{e^{mx+c}}{e^{mx+c} + 1}}$$

$$\frac{P(A)}{P(B)} = \frac{\frac{e^{mx+c}}{e^{mx+c} + 1}}{\frac{1}{\frac{e^{mx+c}}{e^{mx+c} + 1}}}$$

$$\frac{\sigma(\hat{y})}{1 - \sigma(\hat{y})} = \frac{P(A)}{P(B)} = \underline{\underline{e^{mx+c}}}$$

$$\underline{\underline{\hat{y} = mx + c}}$$

$$\log_e \left(\frac{P(A)}{P(B)} \right) = \log_e e^{mx+c}$$

$$\log_e \left(\frac{P(A)}{P(B)} \right) = \underline{\underline{mx+c}}$$

$$\boxed{\log_e \left(\frac{P(A)}{P(B)} \right) = \hat{y}} \quad \leftarrow$$

$$\begin{aligned} P(A) &= 0.8 \\ P(B) &= 0.2 \end{aligned}$$

> 0

Positive

Class

$$\underline{\underline{\hat{y} = \log_e \left(\frac{0.8}{0.2} \right) = 1.386}}$$

$$\begin{aligned} P(A) &= 0.2 \\ P(B) &= 0.8 \end{aligned}$$

< 0

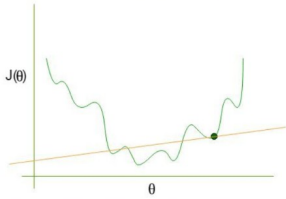
Negative

Class

$$\underline{\underline{\hat{y} = \log_e \left(\frac{0.2}{0.8} \right) = -1.386}}$$

Cost function

Non Convex Graph for Cost Function



$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

sigmoid

$$MSE = \frac{1}{n} \sum_{i=1}^n \left(y_i - \frac{1}{1 + e^{-(w x + c)}} \right)^2$$

Plot \rightarrow non-convex curve

log-likelihood function \rightarrow limited to binary-classification task

$$\text{Cost}(\hat{y}, y) = \begin{cases} -\log_e(\hat{y}) & ; y = 1 \\ -\log_e(1 - \hat{y}) & ; y = 0 \end{cases}$$

Actual Scenario $\rightarrow y = 1$

Wrong $\checkmark \hat{y} = 0 \rightarrow -\log_e(0) = \underline{\underline{-\infty}}$

Classification

Correct $\checkmark \hat{y} = 1 \rightarrow -\log_e(1) = \underline{\underline{0}}$

Classification

Actual scenario $y = 0$

$$\left\{ \begin{array}{l} \hat{y} = 0 \text{ (correct classification)} \\ \quad \hookrightarrow -\log_e(1-\hat{y}) = -\log_e(1) = 0 \\ \hat{y} = 1 \text{ (wrong classification)} = -\log_e(1-1) = \\ \quad -\log_e(0) = \infty \end{array} \right.$$

$$\text{cost}(\hat{y}, y) = \underbrace{\sum_{i=1}^n y_i \log(\hat{y}_i)}_{\text{Actual value}} + \underbrace{(1-y_i) \log(1-\hat{y}_i)}_{\text{Predicted value}}$$

$$y_i = 1$$

$$y_i = 0$$

Confusion Matrix (Binary Classification)

		<u>Actual Value</u>	
		Positive	Negative
<u>Predicted Value</u>	Positive	TP	FP
	Negative	FN	TN

Data \rightarrow Accuracy \rightarrow $\frac{TP + TN}{TP + FP + FN + TN}$
Imbalance
 Diabetic \rightarrow 0 \rightarrow 100
 1 \rightarrow 200

Healthcare

$\text{Recall} \rightarrow \frac{TP}{TP + FN}$

$\text{FN} \rightarrow$

$\text{Prediction} \rightarrow 0$

$\text{actual} \rightarrow 1$

$$\text{Specificity} \rightarrow \frac{TN}{FP + TN}$$

↑↑ Precision → $\frac{TP}{TP + FP}$ ↓↓

Spam email Classification

$$\underline{f1-Score} \Rightarrow \frac{2 * Precision * Recall}{Precision + Recall}$$