HW 5

1. $g_1 = f_1 + h_2 * f_2$

$g_2 = h_1 * f_1 + f_2$

take fourier transform of these equations. By convolution property and linearity

$$G_1 = F_1 + H_2 F_2 \quad -①$$
$$G_2 = H_1 F_1 + F_2 \quad -②$$

multiply ② by H_2 and subtract:

$$G_1 - H_2 G_2 = F_1 - H_1 H_2 F_1$$

$$F_1 = \frac{G_1 - H_2 G_2}{1 - H_1 H_2}$$

back substituting, we get

$$F_2 = \frac{G_2 - H_1 G_1}{1 - H_1 H_2}$$

clearly $f_1 = \mathcal{F}^{-1}(F_1)$ (inverse fourier transform)

$$f_1 = \mathcal{F}^{-1}\left(\frac{G_1 - H_2 G_2}{1 - H_1 H_2}\right)$$

similarly

$$f_2 = \mathcal{F}^{-1}\left(\frac{G_2 - H_1 G_1}{1 - H_1 H_2}\right)$$

Problem about formula :

In expressions of both F_1 and F_2, the denominator is $1 - H_1 H_2$.
This means that the fourier transform of the images of the original scene blows up if $H_1 H_2 = 1$, which is clearly incorrect, since they are by themselves, natural images.

The condition $H_1 H_2 = 1$ means that H_2 is the inverse filter of H_1.
Intuitively, this could (approximately) be the case in a real system, since blurring the reflection in one case and blurring the scene outside in another seem to be like "inverse" operations.

If the camera hardware is such that this "intuitive inversion" assumption is true, then our formulae are rendered useless.