DIP Assignment 5

Q2.

$$g = h * f$$

Where h is the gradient kernel (1D kernel) and f is the original image (1D image). If we look at it the the frequency domain, the convolution will turn into simple product of spectrograms. This can be written as

$$G = H \times F$$

$$\Rightarrow F = \frac{G}{H}$$

As we are taking the gradient of a 1D image, we an write the kernel h as either of these one depending on the approximation we use (forward, backward or center approximation):

$$h \in [1, -1], [-1,1], \left[\frac{1}{2}, 0, \frac{-1}{2}\right]$$

The fourier transforms of all there are given by

$$[1, -1] \rightarrow \left(1 - e^{\frac{-2\pi \mathrm{u} j}{N}}\right) = H_{\mathrm{f}}(u)$$

$$[-1,1] \rightarrow \left(e^{\frac{2\pi \mathrm{u} j}{N}} - 1\right) = H_{\mathrm{b}}(u)$$

$$\left[\frac{1}{2}, 0, \frac{-1}{2}\right] \rightarrow \left(\frac{e^{\frac{2\pi \mathrm{u} j}{N}}}{2} - \frac{e^{\frac{-2\pi \mathrm{u} j}{N}}}{2}\right) = H_{\mathrm{c}}(u)$$

If we are given g and h and we want to recover f from it, we can take the Fourier transform of the gradient image and the kernel and divide them in the frequency domain. This can be done for all values of u except $u = 0$. When $u = 0$, the above transforms will result in value 0.

Note: If we assume Dirichlet boundary condition (0 padding), then we will be able to recover the original image with the DC value, but this will result in step discontinuity at the boundary. On the contrary, if we assume the Neumann boundary condition then although we will get a smoother edge, we won't be able to get the DC component as the anti-derivative is not unique.

In case of 2D image, let $f_x(\mathrm{x}, \mathrm{y}), f_y(\mathrm{x}, \mathrm{y})$ denote the derivatives in x and y direction.

We can write

$$F_x(u, v) = F(u, v) \times H(u)$$

$$F_{\mathrm{y}}(u, v) = F(u, v) \times H(\mathrm{v})$$

Where H is one of the H_f, H_b, H_c.

To recover F from the x and y derivative, we can again use the Fourier domain approach. In this approach we will again face the same problem as that with 1D case i.e. we won't be able to calculate the DC component (0, 0) as the magnitude of the filter component is zero there.

One more point to note is that this approach will work only if the gradient image are noise free. If the noise is added to the gradients, it may result poorer reconstruction of the original image.