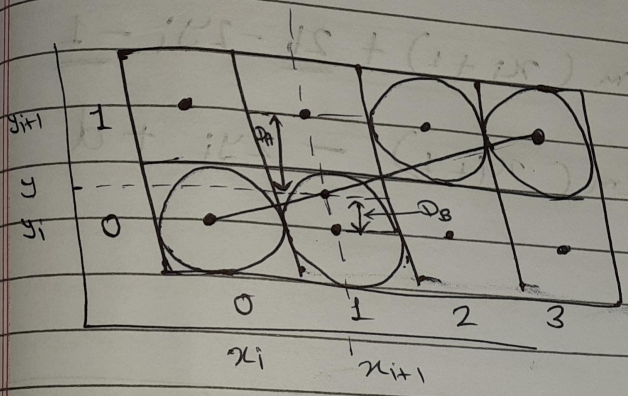
Bresenham's line Algorithm :-

$m < 1$



D_A = Distance Above
D_B = Distance Below

error or decision variable :-

$$e = D_B - D_A \quad \text{or} \quad D_A - D_B$$

now,

$$D_A = y_{i+1} - y$$
$$D_B = y - y_i$$

$$e = D_B - D_A$$
$$= (y - y_i) - (y_{i+1} - y)$$
$$= y - y_i - y_{i+1} + y$$
$$= 2y - y_i - y_{i+1}$$

as we know
$y = mx + b$

$$e = 2[m x_{i+1} + b] - y_i - y_{i+1}$$

$$= 2m x_{i+1} + 2b - y_i - y_{i+1}$$

$$e = 2m(x_i+1) + 2b - y_i - (y_i+1)$$

$$= 2m(x_i+1) + 2b - 2y_i - 1$$

$$= 2m(x_i+1) - 2y_i + d$$

$\left[\because m = \frac{\Delta y}{\Delta x}\right]$

$$(D_B - D_A) = \frac{2\Delta y}{\Delta x}(x_i+1) - 2y_i + d$$

$$D_B - D_A = 2\frac{\Delta y}{\Delta x}(x_i+1) - 2y_i + d$$

$$\Delta x(D_B - D_A) = 2\Delta y(x_i+1) - 2\Delta x y_i + d$$

$m \geq 0$	$m < 0$
$y_{i+1} = y_i + 1$	$y_{i+1} = y_i$
$x_{i+1} = x_i + 1$	$x_{i+1} = x_i + 1$

A decision parameter P_k for the k^{th} step

$$P_k = \Delta x(D_B - D_A)$$

$$= 2\Delta y(x_{k+1}) - 2\Delta x y_k + d$$

$$P_{k+1} = 2\Delta y \cdot x_{k+1} - 2\Delta x\, y_{k+1} + d$$

$$P_{k+1} - P_k = 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$$

$\because x_{k+1} = x_k + 1$ & that

$$P_{k+1} = P_k + 2\Delta y - 2\Delta x(y_{k+1} - y_k)$$

$m \geq 0$	$m < 0$
$y_{k+1} = y_k + 1$	$y_{k+1} = y_k$
then $P_{k+1} = P_k + 2\Delta y - 2\Delta x$	$P_{k+1} = P_k + 2\Delta y$