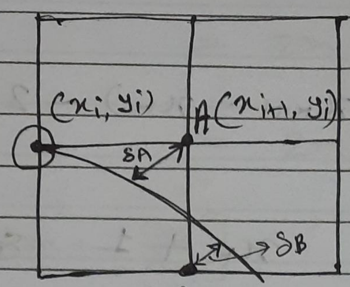
Page

## Bresenham's Circle Drawing Algorithm :-




Circle eqn $\Rightarrow$ $\boxed{x^2+y^2=r^2}$

$$r = \sqrt{x^2+y^2}$$

The Distance of pixels A & B from the origin are given as

$$D_A = \sqrt{(x_{i+1})^2 + (y_i)^2}$$

$$D_B = \sqrt{(x_{i+1})^2 + (y_{i-1})^2}$$

Now, the distance of pixels A & B from the true circle are given as

$$\delta_A = D_A - r$$

$$\delta_B = D_B - r$$

now, take

$$\delta_A = D_A^2 - r^2$$

$$\delta_B = D_B^2 - r^2$$

We can Observe that $S_A$ is always positive & $S_B$ always negative. Therefore, we can define decision variable $d_i$ as

$$d_i = S_A + S_B$$

$$d_i = (x_{i+1})^2 + (y_i)^2 - r^2 + (x_{i+1})^2 + (y_{i-1})^2 - r^2$$

$$d_i = 2(x_{i+1})^2 + y_i^2 - 2r^2 + (y_{i-1})^2$$

$$\boxed{d_i = 2(x_i+1)^2 + y_i^2 - 2r^2 + (y_i - 1)^2} \quad \text{— (1)}$$

Now,

$$\boxed{d_{i+1} = 2(x_{i+1}+1)^2 + y_{i+1}^2 - 2r^2 + (y_{i+1} - 1)^2}$$

$d < 0$ [ $S_A < S_B$, then only $x$ is incremented ]

$$y_{i+1} = y_i$$
$$x_{i+1} = x_i + 1$$

$$d_{i+1} = 2(x_i+2)^2 + y_i^2 - 2r^2 + (y_i - 1)^2$$

$$= 2(x_i+2)^2 + d_i - 2(x_i+1)^2$$

$$= 2(x_i^2 + 4x_i + 4) + d_i - 2(x_i^2 + 2x_i + 1)$$

$$= \cancel{2x_i^2} + 8x_i + 8 + d_i - \cancel{2x_i^2} - 4x_i - 2$$

Date
Page

$$\boxed{d_{i+1} = d_i + 4x_i + 6}$$

Case II :- $d \geq 0$

$$x_{i+1} = x_i + 1$$
$$y_{i+1} = y_i - 1,$$

$$d_{i+1} = 2(x_i+2)^2 + (y_i-1)^2 - 2r^2 + (y_i-2)^2$$

$$= 2(x_i^2+4x_i+4) + (y_i^2-2y_i+1) - 2r^2$$
$$+ (y_i^2 - 4y_i + 4)$$

$$= 2x_i^2 + 8x_i + 8 + y_i^2 - 2y_i + 1 - 2r^2$$
$$+ y_i^2 - 4y_i + 4$$

$$= 2x_i^2 + 8x_i + 8 + 2y_i^2 - 6y_i + 5 - 2r^2$$

$$= 2x_i^2 + 2y_i^2 + 8x_i - 6y_i + 13 - 2r^2$$

$$= 2x_i^2 + 2y_i^2 + 8x_i - 6y_i + 13 + [d_i - 2(x_{i+1})^2$$
$$- y_i^2 - (y_i-1)^2]$$

Date
Page

$= 2x_i^2 + 2y_i^2 + 8x_i - 6y_i + 13 + [d_i$
$- 2(x_i^2 + 2x_i + 1) - y_i^2 - (y_i^2 - 2y_i + 1)]$

$= \cancel{2x_i^2} + \cancel{2y_i^2} + 8x_i - 6y_i + 13 + d_i$
$- \cancel{2x_i^2} - 4x_i - 2 - \cancel{y_i^2} - \cancel{y_i^2} + 2y_i - 1$

$= 4x_i - 4y_i + d_i + 10$

$$\boxed{d_{i+1} = d_i + 4x_i - 4y_i + 10}$$

Initialize the desision variable

put $x = 0$ & $y = r$ in eq (1)

$d = 2(0+1)^2 + r^2 - 2r^2 + (r-1)^2$

$= 2 + \cancel{r^2} - \cancel{2r^2} + \cancel{r^2} - 2r + 1$

$$\boxed{d = 3 - 2r}$$