

Q2).b) Given:-  $f(q, \theta) = \sum_z q(z) \log \frac{P(x, z | \theta)}{q(z)}$

Explain the above approach using K-L divergence.

$$\therefore f(q, \theta) = \int q(z) \log \frac{P(x, z | \theta)}{q(z)} dz$$

~~$$= \int q(z) \log P(x, z | \theta) dz$$~~

$$= \int q(z) \log \frac{P(z | x, \theta) P(x | \theta)}{q(z)} dz$$

$$= \int q(z) \log P(x | \theta) dz + \underbrace{\int q(z) \log \frac{P(z | x, \theta)}{q(z)} dz}_{\textcircled{A}}$$

$$= L(\theta) - KL[q(z) \| P(z | x, \theta)] \quad \text{--- } \textcircled{1}$$

In equation  $\textcircled{A}$  -

The first term is the log likelihood with respect to  $\theta$ .

The second term is the KL divergence.