$$T(n) = \begin{cases} 1 & n = 1 \\ T(n-1) + \frac{1}{n} & n > 1 \end{cases}$$

$T(n-1)$

$$= T(n-2) + \frac{1}{n-1}$$

$$T(n-2) = T(n-3) + \frac{1}{n-3}$$

$$n - k = 1$$

$$T(n-k) + \frac{1}{n-(k+1)} + T(n-(k+1)) + \frac{1}{n-(k+2)} + T(n-(k+2)) + \frac{1}{n-(k+3)}$$

$$T(n-(n-1)) + \frac{1}{n-(n-1+1)} + T(n-(n)) + \frac{1}{n-(n+1)}$$

$$T(1) + T(2) + \frac{1}{0} +$$

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \dots + \frac{1}{n}$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \dots + \frac{1}{n}$$

$$= \frac{1}{d} \ln \left(\frac{2a + (2n-1)d}{2a - d} \right)$$

$$= \ln \left(\frac{2 + (2n-1)}{1} \right)$$

$$= \ln(2n+1) \Rightarrow \boxed{O(\log n)}$$

$$\begin{cases} 1 & n=0 \\ T(n-2)+n^2 & n>0 \end{cases}$$

$T(n) = T(n-2) + n^2$

$T(n-1) = T(\cancel{n}(n-3) + (n-1)^2$

$T(n-2) = T(n-4) + (n-2)^2$

$n-k=0$

$k=n$

$T(n) = T(n-2) + n^2$

$= T(n-4) + (n-2)^2 + n^2$

$= T(n-6) + (n-4)^2 + (n-2)^2 + n^2$

$= T(n-k) + (n-(k-2))^2 + (n-(k-4))^2 + (n-(k-6))^2 + \cdots$

$T = 1 + (n-(n-2))^2$

$= 1 + 2^2 + 4^2 + 6^2 + 8^2 + 10^2 + \cdots$

$= 1 + 2^2(1 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + \cdots)$

$= 1 + 2^2 \dfrac{(n)(n+1)(2n+1)}{6}$

$\boxed{O(n^3)}$

$n-k=0$

$$T(n) = T(n-2) + \log n$$

$$T(n-2) = T(n-4) + \log(n-2)$$

$$T(n-4) = T(n-6) + \log(n-4)$$

$$T(n) = T(n-2k) + \log(n-2) + \log(n-4) \dots + \log(n-2k)$$

$n - 2k = 0$ $\quad \log(n) + \log(n-2) + \log(n-4) + \log(n-6) \dots + \log(0)$

$2k = n$

$k = n/2$ $\quad \log(0) + \log(1) + \log(2) \dots \log(n)$

$\log(n-k)$ $\quad \log(2 \times 4 \times 6 \times 8 \times 10 \times 12 \dots)$

$$= \log\left[\frac{n}{2}(2a + (n-1)d)\right]$$

$$= \log\left[\frac{n}{2}(2 + (n-1)2\right]$$

$$= \log(n/\cancel{1} + n - \cancel{1})$$

$$= \log_2(2n)$$

$T(n) = 1.2 + 2.2^2 + 3.2^3 + \ldots + (n-1)2^{n-1} + n.2^n + 0$

$= \cancel{e^2 + e^2}$ Log$_2$ both sides

$\log_2 (T(n)) = 1 + 4.\log_2 2 + 3^2 + \ldots (n-1)^2 \log_2 n + n^2 \log_2$

$\log_2 T(n) = 1 + 2^2 + 3^2 + 4^2 + \ldots n^2$

$= \cancel{1+} \dfrac{(n)(n+1)(2n+1)}{6}$

$T(n) = 2^{\frac{n(n+1)(2n+1)}{6}}$

$$\boxed{T(n) = 2^{\frac{n(n+1)(2n+1)}{6}}}$$

$= \cancel{O(2^{n^3})}$ $T(n) = \boxed{2^{n^3}}$

$f(n) = \Theta\, g(n)$

$f(n) = n \qquad g(n) = n^{(1+\sin n)}$

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$f(n) \geq c_1 g(n)$

$n \geq c_1\, n^{(1+\sin n)}$

$1 \geq c_1 (1 + \sin(n))\ 1$

$\frac{1}{1+\sin n} \geq c_1$

$\frac{1-\sin n}{\cos^2 n}$

$\boxed{\sec^2 n - \sec n \tan n \geq c_1}$

$\boxed{\sec^2(n) - \sec(n)\tan(n) \geq c_2}$

$f_1(n) = 2^n$ — 4

$f_2(n) = n^{3/2}$ — 2

$f_3(n) = n \log_2 n$ — 1

$f_4(n) = n^{\log_2 n}$ — 3